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## RADC NONELECTROME RELIABILITY MOTEBOOK

Hughes Aircraft Compeny

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Cnief, Reliability $\bar{y}$ Compatibility Division

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## PREFACE

This Notebook is the result of research conducted at Hughes Aircraft Company, Ground Systems Group, Pullerton, California, for Rome Air Development Center under contract number F30602-82-C-0127, covering the period Jul: 1982 through August 1985. The RADC project engineer for this effort was Mr. James A. Collins (RADC/RBES). This research was undertaken within the Systems Projects Section of the Systems Effectiveness Department of Hughes under the direction and supervision of Dr. Ray E. Schafer until his untimely death in September 1983. At that time, direction of the research was taken over by Dr. John E. Angus with support and assistance from Mr. Tom F. Pliska, Systems Effectiveness Department Assistant Manager, and Mr. Larry E. James, Systems Projects Section Fead.

Several individuals made significant technical contributions to this research. Dr. Mal Yerasi, working closely with Dr. Schafer, collected and compiled the entire database of nonelectronic part failure data. The statistical analyses and report generation for the Part Failure Characteristics Section of the Notebook was undertaken by Dr. Angus with extensive computer programming support from Mr. Shick P. Jue. Under a subcontract, Mr. Donald W. Fulton of RAC/IITRI (and past Rome project engineer on a previous edition of this Notebook) wrote the section on Special Application Methods for Reliability Prediction. Finally, the sections on Reliability Demonstration and Specification were written by Dr. Angus with assistance from Mr. Jack M. Finkelstein who also reviewed the entire Notebook. The fughes report number for this document is FR84-16-446 Rev B.

This document replaces RADC-TR-75-22, Nonelectronic Reliability Notebook. Although RADC's interest in nonelectronic/mechanical components is limited to those used in electronic systems, this revised rotebook contains fallure data and reliability methods pertaining to a variety of applications. $\bar{L}$


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### 1.0 INTRODUCTION

The purpose of the RADC Nonelectronic Reliability Notebook is twofold. First, it serves as a reference document for the reliability characteristics of the most comonly used, nonelectronic parts based on industry supplied failure data; and secondly, to present the most useful reliability and life data analysis methods applicable to nonelectronic parts. These analysis methods are presented without regard to rigorous mathematical derivation and with an emphasis on making them accessible to reliability practitioners possessing moderate statistical/mathematical training.

The suggested use of this Notebook is described by the table below where each reliability task is associated with a section of this Notebook. The use of section 5.0, Part Fallure Characteristics, requires aome olaboration. In the majority of cases, the nonelectronic parts covered in this Notebook are adequately described in the reliabllity sense by a constant failure rate. Thus, mainly, section 5.0 will be used to look up a failure rate for a particular device. Sometimes, however, either the part will exhibit nonconstant failure rate, or the analyst will simply wish to use a Weibull analysis of the part's reliability characteristics. For these purposes, section 5.0 also nresents Welbull analyses for selected nonelectronic parts based on the availability of actual failure times in the database. These Weibull analyses are based on data from three different projects, two in the ground mobile application environment, and one in the ground fixed environment. In many cases, the same part type occurs in more than one Weibull analysis. In these cases, if it is desired to use the Weibull analysis for modeling, the analysis in which the most failures were recorded should be used. In some instances, the estimated parameters in the Weibull a alyses of the same part type will differ greatly. These differences are explained by differences between project applications (even though the projects have the same use environment) of the parts, and differences between parts of the same name and type due to lack of data which would better characterize the parts (i,e., two parts of the same generic part name and type can be, nevertheless, different). As the results of these analyses indicate the vast majority of the time, the constant fallure rate tables will be adequate. In spite of this result, the Weibull analyses have been included for reference.

Reliability Task

Specification

Prediction

Demonstration

Nonelectronic Reliability Notebook Sections (s)

Section 3.0
Suction 5.0, if part is represented there;
Section 2.0, if failure data is available; Section 4.0, if no failure data is available.

Section 6.0

Section 2.0 of this Notebook describes the selection and application of several failure distributions which are used for describing the life characteristics of nonelectronic parts, given part failure data. The remainder of this section 18 devoted to methods of operating on failure data once a fallure distribution has been found to, or is assumed to, describe nonelectronic part failure times. The general format used includes methods of point and interval estimation for the reliability parameters of the proven or assumed fallure distribution based on empirical data.

Section 3.0 of this notebook presents guidelines and criteria for specifying reliability for nonelectronic parts and equipments. Specifications appropriate for the nonparametric reliability demonstration teat plans presented in section 6.0 are included.

The next section of the notebook, section 4.0 , addresses reliability prediction, and is intended to supplement section 5.0. It gives rules for using specific prediction models which are known to have application to certain nonelectronic parts. This section is oriented towards strength of alloys, grease and oil lubricated rolling bearings, and spur gear systems. It explains and gives examples on the use of stress-strength interference theory. A new subsection addressing reliability prediction based on minimal vendor information and no life data is also included.

Section 5.0 is Part Failure Characteristics. This section describes the results of the statistical analyses of failure data from more than 250 distinct nonelectronic parts collected from recent conmercial and military projects. This data was collested in-house (from operations and maintenance reports) and from industry wide sources, all of whom are aware of the importance of this Notebook. Tables, alphabetized by part class/part type, are presented for easy reference to part failure rates assuming that the part lives are exponentially distributed (as in previous editions of this notebook, the majority of data available included total operating time, and total number of fallures only). For parts for which the actual life times for each part under test were included in the database, further tables are presented which describe the results of testing the fit of the exponential and Weibull distributions. A quick reference index for locating the beginning page of the Tables for each part class is presented in Table 1.1 in this introduction. The results show that the exponential distribution is adequate for a large majority of the nonelectronic parts for which its fit was tested. A small number of nonelectronic parts exhibited life times which were better described by the Weibull distribution. Recommendations for approximating these cases by the exponential distribution are presented. part malfunction data which was available when the part failure data was collected is presented in Table 5.6.1. See the contract Final Technical Report describing the study and investigation for more details on data and data analysis (RADC-TR-85-66 dated April 1985, AD Al57242).

Section 6.0 presents reliabilty demonst:ation test plans applicable to nonelectronic parts. Both attributes and variables types of demonstration plans are described. Attributes plans are used to demonstate whether
or not a finite population of items possesses an acceptable fraction of items which have a particular attribute, while pariables plans are used to demonstrate whether or not a particular type of item possesses an acceptable level of some pre-specified reliability quantity. Whenever the exponential distribution is judged appropriate for the life distribution of an item of interest, test plans are documented in Mil-sid-781C and Mil-HDBR-108 and are not reproduced in this section. When the exponential distribution is not appropriate, the variables test plans of this section are nonparametric (i.e., not dependent on the form of the underlying life distribution) and based on one dimensional reliability specifications. Because of this, these test plans are easy to design and use, and operating characteristic curves can be developed and used.

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*NOTE: A listing under this column indicates that a Weibull analysis for one or more part types and environments under this part class is included in Sections 5.3-5.5.

## PART CLASS

SHAPT
SHOCK ABSORBERS
SLIP RING-BRU8A
SLIP RINGS
SOLEMOIDS
SPRING
SPROCRET
STEAMBOILER
STON PIN
SWITCR
SWITCHBOARD CONTROL SYNCRO
SYHCRO ASSEMBLY
SYNCRO/RESOLVER
TACHOMETER
TANK
TELESCOPE
TERMINAL BOARDS
THERMOCOUPLE
TRACR BALL
TRANSMISSION
TRUNNION ASSFMBLY
VALVE
WASRER
WATER DEMINERALIZER
WINDLASB

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### 2.0 APPLICABLE STATISTICAL METHODS FOR NONELECTRONIC RELIABILITY*

### 2.1 Statistical Failure Models.

2.1.1 The Hazard Rate Concept. The measure of an equipment's reliability is the infrequency with which failures uccur in time. A failure distribution represents an atterapt to describe mathematically the length of life of a material, a structure, or device. There are many physical rauses that individually or collectively may be responsible for the failure of a device at any particular instant. The present atate-of-the-art does not permit isolation of chese physical causes and mathematical account for all of chem, and, as consequence, the choice of a failure distribution is still an art. If one tries to rely on actual observations of time to failure to distinguish among the various nonsymmetricsl probability functions, one is still faced with a difficulty because nonsymmetric distributions are significantly different at the tails and actual observations are sparse, particularly at the right-hand tail, because of limited sample size.

In view of these difficulties, it is often necessary to hypothesire the type of failure distribution on the basis of knowledge of the physical failure process. For example, fatigue failure of nonelectronic parts is usually assumed to follow a Weibull probability distribution because the theoretical development of this distribution was based on fatigue type failures.

One useful characteristic of failure distributions is the hazard rate or failure rate.

$$
\text { Hazard rate, } \begin{aligned}
h(x) & =\frac{f(x)}{1-F(x)} \\
\text { where } & f(x)
\end{aligned}
$$

Hazard rate is the probability that a device already in service for time $x$, will fail in the next instant of time, given no failure up to $x$.

Each absolutely continuous probability distribution can be characterized by the hazard rate. Physical systems can also be clasified in the same manner. Thus the nature of the failure rates in a physical syatem suggests the type of probability distribution to be assumed.

[^1]To assist the choice of $h(x)$ three types of failures generally have been recognized as having a time characteristic. The first one, called the initial failure, manifests itself shortly after cime $x=0$. The frequency of failures ot tins type decreases during the initial period of operation. A good example of this is the standerd human mortality table, in which it is assumed that up to the age of 10 years a child can die of hereditary defects, but having lived past this age, it is almost free ot such defects. The second rype occurs during the "chance failure period," in which the device exhibits a constant faslure rate, genarally lower than during the iaitial period. The cause of this failure is attribured to unusually severe and unpredictable environmentai conaitions occurring during the operating tiae of the device. In the example of human mortality tables, it is assumed that deaths between the ages of 10 and 30 years are generally due to accidents. The thira type is called the wearout failure period, and is associated with the gradual depletion of a material, or an accumulation of shocks, fatigue, and so on. In the human mortality tables discussed before, after an age of 30 years an increasing proportion of deachs are attributed to "ola age." The chree types ot failures have been classically represented by the "bathtub" curve, wherein each one of the three segments of the curve represents the three time periods of initial, cnance, and wearout.

Tne discussion in che previous paragraphs applies to the theory of life testing in general and may not apply strictly to every case where tie life characteristics of nonelectronic parts are involved. For example the wearout process begins immediately in many types of nonelectronic parts.

It was stated before that given the functional form of $h(x)$, the density function $t(x)$ and the cumulative aistribution function $F(x)$ coula be easily determined. The development of the foliowing two results is straightiorward and can be found in Barlow and Proschan (1964).

$$
\begin{equation*}
1-f(x)=\exp \left(-\int_{0}^{x} h(x) d x\right) \tag{2.1.1}
\end{equation*}
$$

and

$$
\begin{equation*}
t(x)=h(x) \exp \left(-\int_{0}^{x} h(x) d x\right) \tag{2.1.2}
\end{equation*}
$$

In the sections to follow a use will be made of this technique to develop the commonly used failure distributions.
2.1.2 The Poisson Process and the Exponential Distribution. In reliability studies, the exponential distribution plays a role analogous to that of the normal astribution in other areas of statistics. An acceptable justification for the assumption ot an exponential distribution to life
studies was initially discussed by Epstein in 1953. More recently a mathematical argument has been advanced to support the plausibility of the exponential as the failure iaw or complex equipment (Barlow \& Proschan, 1964; p. 18). Although many life distributions, especially those pertaining to the nonelectronic devices, cannot be adequacely described by the exponential distribution, an understanding of the theory in che exponential case facilitates the treatment of the more general cases. The desirability of the exponental distribution is because of its simplicity and its inherent association with the well developed theory of Poisson Processes (Feller, 1968). The applicability of the exponential distribution is limited because of its lack of memory property; this property requires that the previous use does not affect its future life length, and the exponential distribution is the only continuous distribution with this property (Feller, 1968).
2.1.2.1 The Poisson Process. The exponential distribution corresponds to a purely random tailure pattern, and mathematically chis means that whatever is causing the failure occurs according to a Poisson Process with some parameter $\lambda$. Tl. z zoisson probability law can be derived from rigorous machematical considerations, and the interested reader is referred to feller (1yob). Briefly, the postulates of a Poisson Process are stated below.

Consider a system (or a unit) subjected to instantaneous changes due to the occurrence of random events (shocks). All random events are assumed to be of the same kind, and one is interested in their cotal number. Let $P_{m}(t)$ be the probadility that exactly m random events occur during a time interval of length $t$.

Tne physical process tnat induces the ofcurrence of che random events is characterized by the rollowing two postulates:
i) the process is time homogeneous and the future occurrences of the random event are independent ot its past occurrences.
ii) the simultaneous occurrences of two or more events is excluded.

The above postulates lead to a system of differential equations for $P_{m}(t)$, which lead to

$$
P_{m}(t)=\frac{e^{-\lambda t}(\lambda t)^{m}}{m!}, m=0,1,2, \ldots
$$

2.1.2.2 The txponential Distribution. The probability density function ot the exponential distribution can be obtained trom either the hazard rate concept, or by conoidering tae wating time between arrivals in a Poisson Process. Consider che latter situation tirst.

Suppose that the device under consideration is subjected to an environment in which snocks occur according to the Poisson distribution, with a poisson rate $A$. Ine device will fail only if a shock occurs and will not fall otherwise. Let $X$ be the lite of che device.

$$
\begin{aligned}
\text { Let } R(x) & =\operatorname{Pr}(x>x)=\operatorname{Pr} \text { lno shocks occur during }(0, x) \text { J } \\
& =e^{-\lambda x}, \text { by putting } m=0 . \\
\operatorname{Pr}(X \leq x) & =1-e^{-\lambda x} \text { or } \\
f(x) & =\lambda e^{-\lambda x},
\end{aligned}
$$

The same expression for the probability density function of $X$ could be obtalned from the hazard rate concept, since the assumption of random shocks with a constant poisson rate $\lambda$ implies a constant failure rate $h(x)=\lambda$, for $x \geq 0$.

Substicuting $h(x)=\lambda$ in equations $2.1 . i$ and 2.1 .2 one has

$$
F(x)=1-e^{-\lambda x}
$$

This section will be concluded by emphasizing the fact that the exponential distribution can be chosen as a failure distribution if and only if the assumption of a constant hazard rate can be justified. Shis assumption imples that the failure of a device is not because of its deterioration aue to wear, but is due to random shocks which occur according to the postulates or a foisson Process. This fact is of importance in nonelectronic parts consideration, since invariably the failure of these is due to either a pure wear or due to a combination of wear and shocks.
2.1.3 The Weibull Distribution. Recently, the Weibull distribution has emerged to be a popular parametric family ot failure aistribucions. Its applicability to a wide variety of failure situations was discussed by Weibull ( 1 ySl); it has been used to describe vacuum tube failure by kao (1958) and a ball bearing failure by Lieblein et. al. (1956). While the applicability of the exponential distribution in limited because of the assumption of a constant hazard rate, the family ot Weibull distributions can be written ro include the increasing and the decreasing hazard rates as well. Since many mechanical or electromechanical components have an increasing failure rate ( $1 . e$. , due to deterioration or wear), the Weibull distribution is more palatable in describing the failure pattern of such devices.
lf the hazara rate $h(x)$ is some monotone tunction ot $x$, say if

$$
h(x)=\frac{p}{\alpha}(x-\gamma)^{p-1}, \beta, \alpha>0, \gamma \geq 0, x \geq \gamma
$$

then equations 2.1 .1 , and 2.1 .2 give

$$
\begin{aligned}
F(x) & =1-\exp \left[-\frac{(x-\gamma)^{p}}{\alpha}\right] \text { for } x \geq \gamma \text { and } \\
f(x) & =\frac{Q}{\alpha}(x-\gamma)^{p-1} \exp \left[-\frac{(x-\gamma)^{b}}{\alpha}\right] \quad \ldots x \geq \gamma \\
& =U \text { otnerwise. }
\end{aligned}
$$

D, $u$, and $\gamma$ are the shape, the scale, and the location parameters respectively. In Section 5 of this notebook (and other sources) the scale parameter is $\hat{a}^{l / \sigma}$.

The hazard rate for the Weibull distribution is increasing in ( $x-y$ ) if $D>1$, and is independent of $x$ if $b=1$. When $b=1$, the Weibull distribution becomes the exponential distribution with location paramerer $\gamma$, and when $p<1$, the Weibull distribution reduces to nat is called the nyper exponential distribution. When $b<1$, the hazard rate decreases in ( $x-\gamma$ ), and such a hazard rate is useful in characterizing phenomenon such as work hardening or ocher pnenomenon associated with the improvement of relradzlity such as debugging, etc.

Experience in the use of the Weibull distribution in describing the life characteristics of nonelectronic parts leads to the conclusion that very often the location parameter $\gamma$ can be assumed to be zero. This leads to the fallure model referred to in many of the methods presented in this and succeeding sections of this notebook as the two parameter weibull distribution.
2.1.4 Tne Normal Distribution. A fundamencal derivation of chis aistribution is not attempted nere because of is familiarity.

It $X$ aenotes the time to failure ranaom variable of a device which tails accoralny to the normal or Gaussian law, then the probability sensity function of $X$ is given by

$$
f(x)=\frac{1}{\sqrt{2 \pi} 0} e^{-1 / 2\left(\frac{x-\mu}{\sigma}\right)^{2}} \quad-\infty<x<\infty
$$

$H$ and o are the parameters, commonly referred to as the mean and the scanard deviation respectively. The failure rate of the normal distribution is increasing in $x$, and hence, this distribution can be used co characterize wear. Since it is not possible co observe negative lifetimes, the use of this distribution $i s$ limitea to positive random variables.
2.1.5 The Log-Normal Distribution. The log-Normal Distribution can sometimes be used as failure model when failure is due co fracture. Since failures due to fracture occur quite commonly in practice, especially for nonelectronic devices, a study of the Log-Normal Distribution is warranted.

The Log-Normal Distribution implies that the logarithms of the lifetimes are normally distributed, and hence, it can be easily derived by a simple logarichmic transtormation. It can also be derived more fundamentally by consıaering a physical process wherein tailure is aue to fatigue cracks, and the incerested reader is reterred to kao (ly65). The prubability density tunction of the Log-Normal Distribution is given by

$$
\begin{aligned}
f(x) & =\frac{1}{x \cup \sqrt{2 \pi}} \exp \left\{-1 / 2\left(\frac{108 x-\mu}{\sigma}\right)^{2}\right\}, x>0,0>0,-\infty<\mu<\infty \\
& =0 \text { ocnerwise. }
\end{aligned}
$$

$\mu$ and $\sigma$ are the (usually) unknown parameters.
2.1.6 txtreme Value Distribution. The Largest Extreme Value (L.E.V.) aistribution is a two parameter right-skewed probability distribution, similar in appearance to the garma, log-normal, or Werbull distribution. The L.E.V. distribution has been successfully titted co failure data, particularly where failures are caused by fluctuation of a random load variable such as stress or voltage. For example, a component with tensile strength $Y$ is suojected to a stress during each mission. Let $X$ be the largest stress observed in $n$ missions. If $n$ is large, the random variable, $X$, will have a L.E.V. alstribution, and reliability for $n$ missions will be given by $P(X<Y)$, assuminy tnat the theory of cumulative damage does not apply.

The L.E.V. distribution reliability function is

$$
k(x)=1-\exp \left(-e^{-p(x-t)}\right) \quad b>0, \quad-\infty<x<\infty
$$

and the density function is

$$
t(x)=p e^{-p(x-m)} \exp \left(-e^{-p(x-m)}\right)
$$

The Smallest extreme Value (S.E.V.) distribution is the "mirror image" of tne L.i. V. distribution and represents the distribution of tne smallest observation in a large number or trials. It is unique amont the many lite distributions available in that the probadility distribution ls skewed to lle
left. One obvious application for the S.E.V. distribution is the "chain" model, a series system of $n$ components where $X$ is the lowest strength among the components. For large $n, X$ has a S.E.V. distribution.

The reliability function is

$$
R(x)=\exp \left(-e^{B(x-m)}\right) \quad B>0, \quad-\infty<x<\infty
$$

and the density function

$$
f(x)=B e^{B(x-m)} \exp \left(-e^{B(x-m)}\right)
$$

2.1.7 Summary. In the preceding secticns several failure models were proposed as possible candidates for describing the life characteristics of nonelectronic parts. In practice, it is very difficult to identify a particular model as the suitable one, because of the considerations given in Section 2.1.1. However, some broad guidelines for the applicability of certain models were given in the other sections and these can be presented in the table below.

| Model | Applicability Conditions | Comments |
| :---: | :---: | :---: |
| Exponential | Failure due to exactly 1 random shock | Does not characterize wear |
|  | Systems comprised of many components |  |
| Weibull | Applicable under a variety of conditions, especially mechanical parts that fall due to wear | Characterizes wear or work hardening |
| Normal | Failures occur due to wearout | Describes many life processes as well as many manufacturing processes |
| Log-Normal | Failure due to fatigue cracks | Characterizes wear |
| Extreme Value | Failure due to extreme value of some variable | Corrosion is one example |

The remainder of this section of the Notebook is devoted to methods of operating on failure data once one of the previously discussed failure distributions is found to or is assumed to describe nonelectronic part failure times. The section is divided by failure distribution and follows a similar pattern for each. Methods are described for calculating point estimates of the rellability or of other parameters of the proven or assumed failure distribution based on empirical data. Where they are available and have utility for the users of the Notebook, graphical methods for estimating these
same values are presented. The general format used includes methods of point and interval estimation for the reliability parameters ot each failure distribution. Numerical examples are presented and the user of the Notebook is furnished with references for theorecical development and adaitional examples of eacn of the methods presented.

### 2.2 Design of Statistical Experiments.

2.2.1 Introduction. Since the state of the art of failure data collection for nonelectronic parts does not give sufficient information for proper analysis, it is necessary to generace the needed information in a systematic manner.

In collecting failure data for estimating reliatility characteristics for nonelectronic parts, one is frequently faced with insutficient information. Thas problem arises from the manner in which the data is coilected. It is comon practice to include only the operating time of the equipments or systems ana the number of failures observed during the operational period covered by a failure report. Incividual part failure times are generally not recorded.

Tinis section of the Notebook, therefore, is devoted to describing the methodoloyy or the principles to be followed in the event that one has the opportunity to generate failure data for a nonelectronic part. It describes in a loyical manner the discrete guidelines for setting up an experiment which will yıeld an evaluation of the important factors or combinations of factors which affect the lite characteristics of the parts of interest.

The general steps for planning and conducting an experiment plus the procedures to be followed to most efficiently analyze the results of a test program for nonelectronic parts are outlined below. The remainder of this subsection describes these steps in detail and references sources which cari be used as patterns to be followed in generating and analyzing the types of information which are required to allow a complete and detailed reliability analysis of nonelectronic parts and of the operating and environmental stresses which atfect tneir life characteristics.

When these guidelines are utilized in the generation and analysis of rellability data for nonelectronic parts, a full and complete analysis of all che data should be possible and no assumptions or guesses should be required which might dilute the power ot the conclusions that can be reached trom a proper treatment of failure data.

The major steps discussed in the surceeding pages are:

1. Uetermination of Stresses
2. Determination of Stress Levels
3. Statistacal Teot Designs
4. Physical Test vesigns
5. Analysis of Experimental Data


#### Abstract

2.2.2 Determination of Stresses and Stress Levels. In setting up an expermment to generate reliability data on nonelectronic parts, those stresses snoula be evaluated which experience andor failure mode analysis indicates have the greatest etfect on part iffe for the applications of interest. tuen if a part is operated in a very benign environment, there are still many factors competing in combination to cause deterioration of ita life characteristics. The rate of deterioration is a function ot the level or concentration of a given stress. It is well-known that certain factora affect a product more than others. For example, a tire's life is reduced by vioration, radation, corrosive agencs, type of road surface, and temperature to name but a few factors. It is well-known, however, that the effects of temperature greatly exceed the eftects of other envirommental atresses to the point tnat scting up an experiment to evaluate the life characteristics of thas part without incluang this factor would render useless or distorted results. Some factors work in combination in such a manner that their effect together is greater than the sum of their individual effects acting separately. For example, ozone increases the tendency of rubber to crack, but ozone combined with high temperature creates an even greater amount of damage in most cases. The selection of factors for an experiment then should not only be directed toward including the most important factors affecting life but shoula seek to apply them in combination since this allows the evaluation of synergistic effects and more closely simulates actual operating conditions.


The goal of an experiment or rest program to generate reliability inrormation is therefore co determine the manner in which a part's life characteristics vary over the envelope of environmental and operating stresses to which it will be subjected during normal application. lf all the stresses to whicn it is co be subjected during normal applications are included in the experment and are evaluated, the size and expense of the experiment required wuld be prohibitive in nearly every case. The decision as to which stresses to incluce must therefore be based on experience, knowledge of failure theory, nistorical data, failure mode analysia, or predictea values in order to yield the most information for a reasonable expenditure ot monies and time.

In addition to being certain to anclude the most important operatiny or environmental stresses for inclusion in a reliability data generation program, it is important to evaluate the effects of various levels of the stresses. It is expected in most cases that very litcle degradation occurs in a so-called laboratory environment. However, the rates of deterioration for a given stress usually vary in some systematic manner over che range of atresses to be encounterea in a given application. Therefore, it is desirable co investigate what happens at several polnts in the environmental and operational envelope.

In determining the most significant stress levels to investigate there are several points to consider. The first is to relace failure mode to stress level. This can be done either by prior knowledge or by running some short cime screening tests to locate the seneral screac jevel at which failure mode changes can be expected. If these itress levels can be located, they may even be usea in influencing tne operating or environmental limits to be specified in order to effect a meaningful improvement in product life. Another point to
consider in stress level selection is the concept of endurance limit associated with many nonelectronic parts. This assumes that below certain stress levels life can be considered co be infinite or at least extremely long. Therefore, in order to observe failures in a reasonable time these stress levels must be avoided.

This presents the usual dilemma which can best be solved by step-stress testing as described by Dodson and Howard and by Prot. It consists of testing a specimen or group of specimens for a fixed time at a fixed stress level. The survivors are continued on test at the next increment of higher stress for the same fixed time. This procedure continues until it is posible to select several stress levels suitable for evaluation. The stress levels should not be spaced so closely that no differences can be detected but should be selected to sufficiently cover the spectrum of interest.

The objective of the test program is to generate a mathematical model that demonstrates how life characteristics change as operational or operating stresses or combinations of stresses vary. In the case of nonelectronic parts the dimension of time must also be included in the model gince the probability of failure increases as the part sees more service.

In summary, this copic discusses the general ground rules for selecting logical stresses and stress levels when setting up an experiment or test program for evaluating the reliability of nonelectronic paris. The objectives of the guidelines are the generation of more useful data for analysis than is now generally available.
2.2.3 Statistical and Physical Test Design. The goal of the designer of an experiment is the generation of accurate and useful conciusions based on economical test program, effirient data collection methods and the proper selection of statistical methods which will lead to the attainment of the goals.

In generating or gathering reliability data on nonelectronic parts, it is possible to simply put a part or group of parts on test at nominal operating and environmental conditions and collect information on operating times and failure times. From this, it will be possible to estimate the parameters of the assumed fallure distribution exhibited by the parts with the methods described in Sections 2.3 and 2.4 of this Notebook.

Lf a greater degree of diversity is desired because the part of interest frequently may encounter stresses other than nominal in different applications then it is probably wise to attempt to evaluate the effect of a iven stress or part life when applied af several different levels. Also, it is probably prudent to investigate the effects of several stresses that are thought to be major factors affecting part life again investigating each of these at several levels. More importantly, it is reasonable to evaluate these stresses when they are applied in combination since this comes the closest to simulating the real life situation.

When it is desired to evaluate stresses applied in combination on a
the most efficient type of statistical experiment to use is some form of factorial design. It is true that it would be possible to evaluate the effect of contact current on the life of a switch by holding all environmental and operating conditions constant while varying the stress of interest, in this case contact current over a desired range of values. This same procedure could then be followed for actuation rate, vibration, temperature and the infinite number of other operating and environmental stresses that could and do affect the life of switches. The obvious result would be a series of tests that would take a rather long time to perform. More importantly, hovever, is the fact that there would be no measure of how two or more of the stresses might act when both were at levels other than nominal. In other words, if a synergistic effect was brought about by a given set of stresses or stress levels, the aforementioned procedure would not be able to paluate it. The solution to the problem lies in the use of some form of factorial experiment. There are full factorial experiments in which every combination of stresses and stress levels is tested and perhaps replicated. This type of experimental design yields the maximum amount of information regarding main effects and the effects of interactions of stresses. The price paid for the complete information is paid in the cost of parts placed on test and in the test time required to gather the requisite amount of data needed to perform the analysis.

Fractional factorial experiments can be performed in which some of the test cells in the experiment are omitted in order to reduce test time and expenses. The disadvantages associated with the omission of some stress combinations is that the higher order interactions cannot be evaluated. Therefore, the judgment as to which treatments to omit must be based on experience or opinion. Naturally, those that are not felt to be significant will be omitted. There are several other special types of experimental designs which can be used such as central composites, latin squares and many more. For further information regarding the details of how to set up and analyze this type of experiment the reader is referred to "The Design and Analysis of Industrial Experiments," by 0. W. Davies, Hafner Publishing Company, 1954. An example of an application of a full factorial experiment is presented in RADC Technical Report 65-46 "Accelerated Reliability Test Methods for Mechanical and Electromechanical Parts," July 1965. Its Defense Documentation Center number is AD 621074. This report details all the steps necessary in selecting the stresses, stress levels, the development of the statistical test design, the physical test design and all the mathematical analyses required to evaluate the effects of environmental and operating stresses and their interactions on part life. Included also is the methodology for fitting the failure distribution and for estimating the values of the parameters of the Weibull distribution. lie details presented in the subject report will serve as a complete example sor designing and performing a statistical experiment. The methodology used can be followed in almost a step-by-step manner and can therefore be used in a similar manner as this Notebook. An example of a central composite experimental design is technical Report ECOM-01433-F "Multi-Pole Relay Evaluation Study," December 1968. The central composite analysis yields a response surface in terms of regression equations involving the main stress effects and certain interactions.

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$$

With regard to the physical test deaign there are a few guidelines that should be followed. The test equipment for generating reliability data for nonelectronic parts should simulate actual operating conditions accurately as possible. In addition to this the test equipment should be economical to builc and use and should yield accurate measuraments of the parameters which determine whether or not a part has failed. Finally, the physical test design should include a simple and accurate data collection aytem. When a large number of parts are tested the record-keepiag probleas can quickly become quite complex. Therefore, automation of deta collection is a comendable goal.

In sumary, this section has presented the ajor types of statistical designs that will yield accurate and useful data and refers the reader to typical sources that will aid in the specification of useful, efficient and economical statistical and physical test designs.

### 2.3 Fitting Failure Distributions.

2.3.1 Introduction. The two topics immediately following this one deal with specific methods of fitting failure distributions. The placement of this discussion in the overall table of contents is logical because when empirical daca are observed the first logical step in its analysis is to attempt to determine the underlying distribution of fallure cimes. While a method for small sample sizes is presented as well as one for large ample aizes it is a fact of life that must be accepted that tests based on small samples are simply not very powerful. Therefore, the methodology is presented here for compleceness but very likely a more logical approach is to first make an assumption regarding the failure distribution based on engineering judgment or on nistorical data or on knowledge of the failure characteristics of similar parts. Once the failure distribution has been assumed the test can be performed for goodness-of-fit for that particular distribution. It the hypothesized distribution is shown not to fit, it is likely that the assumed distrioution was not the one from whicn the samples were selected. If, however, the goodness-of-fit test shows that the data could have come from the hypothesized distribution, then it is likely that other tests for fit vould yield like results.

In summary then, it must be realized that the tests presented in the next two items have limitations. The only cure for these limitations is a larger number of observations. If this proves uneconomical or not feasible from the stanapoint of cest time required to generate the desired number of failures, then the only alternative is to use the results of sooll sample size analyses wath proper discretion.

> 2.3.2 Small Sample Sizes (Kolmogorov-Smirnov).

1. When to Use

When failure times from a relatively small sample have been observed and it is desired to derermine the underlying distribution ot failure times.
2. Conditions for Use
a. Uaually nistorical data or engineering juagment suggests thar part failure times ot interest are from a given statistical failure aistribution. This cest then follows the step of assuming a given tallure distribution and is useful to determine if empirical data disproves tnis hypothesis.
b. The Kolmogocov-Smirnov test for goodness of fit is distribution free and can theretore be used regardless of the tailure distribution that the data is assumed to follow.
c. Ine aiscriminating ability of the statistical test is dependent on sample size so naturally the larger the sample aize the more reliade the results. Where large sample sizes are available the $X^{2}$ test for Goodness-of-Fit shoula be used. Where sample sizes are small the kolmogorov-Smirnov test provides some assurance.

$$
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$$

d. Strictly speaking, this test method requires prior knowledge of the parameters. If the parameters are estimated from the sample the exact error risks are given in Lawless (1982). However, the values from a kolmogorov-Smirnov table will provide an adequate approximation in most circumscances.
3. Method
a. Observe and record part failure thers.
D. Assume a distribution of fallure times based on historical information or on engineering juagment.
c. Estimate the parameters of the assumed distribution from the observed data.
d. Calculate the probability of fallure for each observation from the cumulative failure function tor the assumed distribution.

## Example

a. Given the following 20 failure times in hours

| 92 | 640 |
| ---: | ---: |
| 130 | 700 |
| 233 | 710 |
| 260 | 770 |
| 320 | 830 |
| 325 | 1010 |
| 420 | 1020 |
| 430 | 1280 |
| 465 | 1330 |
| 518 | 1690 |

b. Assume failure times are distributed according to the two parameter Weibull distribution.
c. By the method of least squares (see Section 2.4.2.1.1) the weibull shape parameter (G)=1.50 and the Weibull scale parameter $(a)=28400$.
d. For the weibull distribution the cumulative failure function 18

$$
\hat{F}(x)=1-\exp \left(-\frac{x^{k}}{a}\right)
$$

where $x=$ observed fasbure came pel.5 Weibull snape parameter $\alpha \mathbf{2 8 4 0 0}$ = Weibull scale parameter $F(x)$ probability of tallure at cr before time $x$.

## 3. Method

e. Calculate the percentile for each of (i) failure times by the relationship
$F(i)=\frac{i}{n}$ and subtract tnese
respective values from tnose of step d. above. Record the absolute valua of the difference. Also, shift i to i-l and compute the difterences once again.

## Example

d. (Continued)

For the 20 observations of this example, the probability of failure at the respective $x^{\prime} s$ is:

| $\underline{x}$ | $\widehat{\boldsymbol{F}(x)}$ |
| ---: | ---: |
| 92 | 0.03 |
| 130 | 0.05 |
| 233 | 0.12 |
| 260 | 0.14 |
| 320 | 0.16 |
| 325 | 0.19 |
| 420 | 0.26 |
| 430 | 0.27 |
| 465 | 0.30 |
| 518 | 0.24 |
| 640 | 0.43 |
| 700 | 0.48 |
| 710 | 0.49 |
| 770 | 0.53 |
| 830 | 0.57 |
| 1010 | 0.66 |
| 1020 | 0.88 |
| 1280 | 0.80 |
| 1330 | 0.82 |
| 1690 | 0.91 | results:


| $\underline{F(x)}$ | F(i) | F(i-1) | $\left\|\begin{array}{r} \hat{F}(x) \\ -f(i) \end{array}\right\|$ | $\left\|\begin{array}{l}\vec{F}(x) \\ \underline{F}(i-1)\end{array}\right\|$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.03 | 0.05 | 0 | 0.02 | 0.03 |
| 0.05 | U. 10 | 0.05 | U.US | 0.00 |
| 0.12 | 0.15 | 0.10 | 0.03 | 0.02 |
| 0.14 | 0.20 | 0.15 | 0.00 | 0.01 |
| 0.18 | 0.25 | U. 20 | 0.07 | 0.02 |
| 0.19 | 0.30 | 0.25 | 0.11 | 0.06 |
| 0.26 | 0.35 | 0.30 | 0.04 | 0.04 |
| 0.27 | 0.40 | 0.35 | 0.15 | 0.08 |
| 0.30 | 0.45 | 0.40 | 0.15 | 0.10 |
| 0.34 | 0.50 | 0.45 | 0.10 | 0.11 |
| 0.43 | 0.55 | 0.50 | 0.12 | U.07 |
| 0.48 | 0.60 | U. 55 | 0.12 | U.U7 |
| 0.49 | 0.65 | 0.60 | U. 10 | U.11 |
| 0.53 | 0.70 | 0.65 | 0.17 | U. 12 |

## Example

e. (Continued)

| $\hat{F}(x)$ | $\underline{F}(1)$ | $\underline{F(1-1)}$ | $\hat{F}(x)$ <br> $-F(1)$ | $\hat{F}(x)$ <br> $-F(i-1)$$\|$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.57 | 0.75 | 0.70 | 0.18 | 0.13 |
| 0.68 | 0.80 | 0.75 | 0.12 | 0.07 |
| 0.68 | 0.85 | 0.80 | 0.17 | 0.12 |
| 0.80 | 0.90 | 0.85 | 0.10 | 0.05 |
| 0.82 | 0.95 | 0.90 | 0.13 | 0.08 |
| 0.91 | 1.00 | 0.95 | 0.09 | 0.04 |

f. Compare the largest difference from step e. with a value at the desired significance level in the Kolmogorov-Smirnov Tables to test for goodness-of-fit. If the tabled value is not exceeded then it is not possible to reject the hypothesis that the fallure times are from the assumed distribution.
f. The largest difference in step e. was .18. From the KolmogorovSmirnov Table for a significance uf .05 and for a sampie of size 20 a difference of greater than .29 must be observed before it can be said that the data could not have come from a Weibull distribution with $B=1.5$, $\alpha=28400$.
4. For Further Information

The example presented here illustrates how to test the hypothesis that the fallure data came from the Weibull distribution. The KolmogorovSolrinov Test can also be used for other fallure distributions by properly estimating the parameters in step $c$. for the appropriate distribution and by using the appropriate cumulative distribution function in step d. Kolmogorov-Smirnov Tables are available on pages 321 and 322 of the Handbook of Tables for Probability and Statistics, Edited by W.H. Beyer, published by the Chemical Rubber Company, Cleveland, Ohio, 1966, and ir. many texts on statistics.

### 2.3.3 Large Sample Sizes ( $X^{2}$ Test)

1. When to Use

When fallure times are avallable from a relatively large sample and it is desired to determine the underlying distribution of failure times.

## 2. Conditions for Use

a. In the statistical analysis of failure data it is common practice to assume that failure times follow a given failure distribution family. This assumption can be based on historical data or on engineering judgment. This test for goodness of fit is used to determine if the empirical data disproves the hypothesis of fit to the assumed distribution.
b. The $x^{2}$ test for goodness-of-fit is asymptotically distribution free and can therefore be used regardless of the failure distribution that che data is assumed to tollow when samples are large.
c. Tn2s test is not directly depencent on sample size but on the number of intervals into which the scale of failure times is divided with che restriction that no interval should be so narrow that there are not at least 5 theoretical failures within the interval. Therefore, che teat is only useful if a relatively large number of failures has been observed.
d. A table of $\chi^{2}$ percentage points is required.
3. Method
a. Ubserve and record part failure times.

## Example

a. The following is the number of cycles to failure for a group of 50 relays on a life test:

| 1283 | 6820 | 16306 |
| :--- | ---: | :--- |
| 1887 | 7733 | 17621 |
| 1888 | 8025 | 17807 |
| 2357 | 8185 | 20747 |
| 3137 | 8559 | 21990 |
| 3606 | 8843 | 23449 |
| 3752 | 9305 | 28940 |
| 3914 | 9460 | 29254 |
| 4394 | 9595 | 30824 |
| 4398 | 10247 | 36319 |
| 4865 | 11492 | 41554 |
| 5147 | 12913 | 42870 |
| 5350 | 12937 | 62690 |
| 5353 | 13210 | 63910 |
| 5410 | 14833 | 68888 |
| 5530 | 14840 | 73475 |
| 6499 | 14986 |  |

b. Assume a distribution of ratlure times based on historical information or on engineering juagment.
c. Estimate the parameters of the assumed distribution from the observed data.
b. Assume failure times are distributed according to th, two parameter Weibull distribution.
c. By the method of least aquares (see Section 2.4.2.1.1) the weibull shape parameter $B=1.21$ and che weibull scale parameter © 127y78.
3. Method
d. Divide the spectrum of failure times into intervals of such a width that the theoretical number of failures in each interval will be at least five. The width of intervals need not be equal.

Example
d. Divide the relay cycles to fallure into the following intervals:

$$
\begin{array}{r}
0-4000 \\
4001-7200 \\
7201-13000 \\
13001-18000 \\
18001-25000 \\
25001-\infty
\end{array}
$$

e. Calculate the theoretical number e. The expected number of failures of fallures for each interval.
in each interval is obtained as follows:

For the Weibull distribution the cumulative fallure funccion is
$F(x)=1-\exp \left(-\frac{x^{\beta}}{\alpha}\right)$
where $x$ = observed failure zimes
$\beta=$ Welbull shape parameter
$\alpha=$ Weibull scale parameter

Then $F\left(x_{n}\right)-F\left(x_{n-1}\right)=$ probability a failure time falls within the interval.

Then for each interval the probability of failure in that interval multiplied by the sample size the theoretical number of fallures for each interval.
3. Method

## Example

e. (Continued)

\begin{tabular}{|c|c|c|c|}
\hline \begin{tabular}{l}
(1) \\
Upper Boundary of Intervel
\end{tabular} \& (2)
\(F(x)\) \& \((3)\)

$F\left(x_{n}\right)$

$-F\left(x_{n-1}\right)$ \& | (4) |
| :--- |
| Theoretical |
| Failure Frequency (Col. $3 \times 50$ ) | <br>

\hline 4000 \& 0.10 \& 0.16 \& 8 <br>
\hline 7200 \& 0.30 \& 0.14 \& 7 <br>
\hline 13000 \& 0.52 \& 0.22 \& 11 <br>
\hline 18000 \& 0.06 \& U. 14 \& 7 <br>
\hline 25000 \& 0.80 \& 0.14 \& 7 <br>
\hline $\infty$ \& 1.00 \& 0.20 \& 10 <br>
\hline \multicolumn{4}{|l|}{NUTE: The theoretical trequency must not be less than $)$ for any interval.} <br>
\hline
\end{tabular}

f. Calculate the $x^{2}$ statistic


$$
\begin{aligned}
\text { Where } k= & \text { number of intervals } \\
f= & \text { observed frequency/ } \\
& \text { interval } \\
F= & \text { theoretical frequency/ } \\
& \text { interval }
\end{aligned}
$$

8. Determine if the $X^{2}$ statiscic indicates tnat the data could nave come from the hypothesized distribution using $X^{2}$ cables and ( $k-1$ ) - $\rho$ degrees of freedom.
where
$k=$ number of intervals
0 - number of parameters estimated from data
f.

| Upper <br> Boundary of <br> Interval | $F$ | $f$ | $\frac{\left(f_{i}^{-F}\right)^{2}}{F}$ |
| :---: | :---: | :---: | :---: |
| 4000 | 8 | 8 | 0 |
| 7200 | 7 | 10 | 1.29 |
| 13000 | 11 | 12 | 0.11 |
| 18000 | 7 | 7 | 0 |
| 25000 | 7 | 3 | 2.29 |
|  | 10 | $\frac{10}{}$ | 0 |
|  | so | 50 | $x^{2} 3.69$ |

8. The degrees of freedom for this example are calculated as:

$$
\begin{aligned}
& \text { d.f. }=(k-1)=\rho \\
& \text { a.f. }=(b-1)-2=3
\end{aligned}
$$

The value from the $X^{2}$ table for s degrees of freedom and 0.05 level of signiticance is 7.015 . Since $3.6 y$ does not exceed the tablea value, then the hypothesis that this data came from a Weibull distribution cannot be rejected at the $5 \%$ level of significance.

The example presented here illustrates how to test the hypothesis tnat the observea failure daca came from the Weibull distribution. Ine $X^{2}$ test can also be used for other distributions by properly estimating the parameters in atep $c$. for the appropriate distribution and by using the appropriate cumulative distribution function in step c. In step g. the selection of the $5 \pi$ level of significance was arbitrary and will depena on tne researchers willingness to risk a wrong decision in rejecting the hypothesized distribution. There is also a risk of accepting the distribution wrongly whach for chis cesc cannot be specified. There are several versions of $x^{2}$ cables but the one used with this example is from "New Tables of the Incomplete Gama-Function Ratio and of Percentage Points of the Chi-Square and Betz Distributions," by H. Leon Harter, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, 1964.

### 2.4 Estimation Mechods

### 2.4.1 The Exponential Distribution

2.4.1.1 Analytical Point Estimation

1. When to Use

To estimate the parameter $\theta$, mean-time-between-failures, in the exponential distilibution function
$F(x)=1-e^{-x / \theta}$
this computation may be performed. Also, the estimation of reliability is described.
2. Conditions for Use
a. This estimation method may be used when units are selected at random and placed on test, whether or not all units are allowed to fail, and whether or not failed units are replaced.
b. No burn-in or wear-out type fallures occur. Use of the exponential distribution assumes a constant failure rate.
c. Total test time and total number of failures must be collected.
3. Method
a. Sum together the test time accumulated on each unit tested to get the total test time. Whether failed units are replaced or not does not affect the calculacion, nor does it matter whether all units are allowed to fail. Only compute the total operating time of parts on test.

## Example

a. Suppose 10 units are placed on test for 80 hours and the failed units are not replaced. Failures occur at $20,30,35$, 45,70 and 75 hours. So, test tine accumulates as follows:

| Unit 1 | 20 hours |
| :--- | :--- |
| Unit 2 | 30 hours |
| Unit 3 | 35 hours |
| Unit 4 | 45 hours |
| Unit 5 | 70 hours |
| Unit 6 | 75 hours |
| Unit 7 | 80 hours |
| Unit 8 | 80 hours |
| Unit 9 | 80 hours |
| Unit 10 | 80 hours |

Total Test Time $=595$ hours
b. Divide total test time by total number of fallures to get an estimate of $\theta=$ mean-time-between-failures.
c. The reliability is given by

$$
R(x)=1-\left(1-e^{-x / \theta}\right)
$$

## Example

b. Since the cotal number of failures is 6, divide

$$
\frac{595}{6}=99.1 \text { hours. }
$$

c. The reliability for 30 hours is estimated co be:

$$
\begin{aligned}
& R(30)=e^{-30 / 99.1} \\
& R(30)=0.74
\end{aligned}
$$

## 4. For Further Intormation

Additional examples on the use of the exponential distribution are presentea in "Reliability Theory and Practice," by Igor Bazovsky, Prentice-Hall, 1461.

### 2.4.1.2 Interval Estimation

2.4.1.2.1 Two-Sided Confidence Limits

1. When to Use

To compute upper and lower confidence limits on the exponential distribution parameter $\theta$ (mean-time-between-failures), this method is useg.
2. Conaicions for Use
a. A confidence level, say $1-\alpha$, must be specified.
b. Total test time and total number of failures must be collected, whether ur not tailed units are replaced.
c. A table of $X^{2}$ percentage points is required.
3. Mernoa
a. It the cest is failure truncated, ratner than time truncated, then the lower two-sided confidence limit is

$$
\frac{2 T}{x_{\angle 5,1-a / 2}^{2}}
$$

## Example

a. Suppose $\zeta$ units are placed on Life test and fail at $20,30,35$, 45 , and 70 hours. If the $90 \%$ confidence limits are desired, then

$$
1-\alpha=0.90
$$

3. Nethod
a. (Continued)
where
$T=$ cotal test cime
$1-\alpha=$ configence level desired
$r$ - tocal number of iailures
$\begin{aligned} x_{2 r, 2-w / 2}^{2}= & 1-\alpha / 2 \text { quantile or } \\ & \text { che chi-square dis- } \\ & \text { tribution with } \\ & 2 r \text { degrees of freedom. }\end{aligned}$
b. The corresponding upper twosidea confidence limit is

$$
\frac{2 I}{x_{2 r, \alpha / 2}^{2}}
$$

## Example

a. (Continued)

$$
\begin{aligned}
& I=200 \text { hours } \\
& \hat{\theta}=T / r=40 \text { hours }
\end{aligned}
$$

So, the lower two-sided confidence limit is

$$
\frac{2 \times 200}{x_{10,0.45}^{2}}=21.85
$$

4. For Further Iniormation

For a time cruncated test, the lower two-sided confidence limit is computeo with $2 r+2$ degrees of treedom:

$$
\frac{2 I}{x_{2 r+2,1-a / 2}^{2}}
$$

Tine upper twc-sided contidence limit is the same as in a failure truncated test, with $2 r$ aegrees of freedom.

Adaitional examples demonstrating this method are presented in "Reliability Ineory and Practice" by Lgor bazovsky, Prentice-Hall, 196l. 2.4.1.2.2 One-Sided Confidence Limits

## 1. When to Use

Use chis method to compute a lower onesided confidence limit on the exponentiai digtribution parameter $\theta$ (mean-time-between-tailures).
a. A confidence level, say $1-\alpha$, must be specified.
b. Total teat $t$ ame and totel number of failures must be collected, whether or not failed units are replaced.
c. Even if no failuxes have occurred, this method may be used.
d. A table of $X^{2}$ percentage points is required.
3. Mechod
a. If the test is failure truncated, rather than time cruncated,* then the lower one-sided conficence limit is
$\frac{2 I}{x_{2 r, 1-\alpha}^{2}}$
where

$$
\begin{aligned}
T= & \text { total test time } \\
1-\alpha= & \text { confidence level } \\
& \text { desired } \\
\tau= & \text { total number of } \\
& f a l i u r e s \\
= & 1-\alpha \text { quantile of } \\
X_{L r, 1-\alpha=}^{2} & \text { che chi-square dis- } \\
& \text { tribution with } \\
& 2 r \text { degrees of freedom. }
\end{aligned}
$$

FFor a cime truncated test, the lower one-sidea confidence limit is computed with $2 x+2$ degrees of treedom.

$$
\frac{2 I}{x_{2 r+2,1-\alpha}^{2}}
$$

NOCE: It no failures have occurred, the lower one-sided contidence limit is $T /(-1 n a)$.

## Example

a. Suppose $S$ units are placed on life test and fail at 20, 30,35 , 45 , and 70 hours. If the $90 \%$ lower confidence limit is desired, then

$$
1-\alpha=0.90
$$

$r=5$
$T=200$ hours
$\hat{\theta}=T / r=4 U$ hours
So, the lower $90 \%$ one-sided confidence limic is
$\frac{2 \times 200}{x_{10,0.90}^{2}}=25.02$

Addational examples demonstrating this method are presented in "Keliability Theory and Practice" by Lgor Bazovaky, Prentice-Hall, 196l.

### 6.4.2 The Weabull Distribution

2.4.2.1 Analytical Point Estimation
2.4.2.1.1 The Mechod of Least Squares

1. When coUse

Estimating Weibull snape ana scale parameters may be accomplished by fatting a least squares line to transformed Weibull data, provided that the location parameter $\gamma$ is known or assumed. $\gamma$ is often assumed to be zero. bf not the transtormation $X^{\prime}=X-Y$ will reduce to the 2 parameter form. This section of the Notebook out the Weibull distribution contains three methods of estimating the shape ana scale parameters. The method of least squares is basically a more accurate version of the graphical method. It takes more calculations to estimate $a$ and $p$ than the graphical method and hence the added cost of these calculations must be balanced against the costs associated with using a less accurate graphical method and being subject to estimation error. While a computer helps reduce calculation time, the least squares method does not require as complex a computer program as the maximum likelihood method alchough it may not result in as accurate an estimate.
2. Conditions for Use
a. Failure times must be collected.
b. A computer if helpful.
c. A table of median ranks is required. It is provided in Appendix 11 , Table I.
3. Method
a. For $n$ ordered Weibull failure themes $t_{1}, t_{2}, \ldots t_{n}$, find wedian rank values $F\left(t_{1}\right)$ from Table $L$ in the Appendix.

## Example

a. Table 1 in the Appendix gives the median ranks for these Weibull failure tames.

| Failure Time | Median Rank |
| :---: | :---: |
| 92 Hours | 0.0341 |
| 130 Hours | 0.0831 |
| 233 Hours | 0.1326 |
| 260 Hours | 0.1814 |
| 320 Hours | 0.2302 |
| 325 Hours | 0.2793 |
| 420 Hours | 0.3283 |
| 430 Hours | 0.3774 |
| 465 Hours | 0.4264 |
| 518 Hours | 0.4755 |
| 640 Hours | 0.5245 |
| 700 Hours | 0.5730 |

3. method

Example
a. (Continuea)

Eailure Time
Median Kank

| 710 Hours | 0.6226 |
| ---: | ---: |
| 770 Hours | 0.6717 |
| 830 Hours | 0.7207 |
| 1010 Hours | 0.7698 |
| 1020 Hours | 0.8188 |
| 1280 Hours | 0.8678 |
| 1330 Hours | 0.9164 |
| 1690 Hours | 0.9659 |

b. An adapted computer prograin for fitting least squares lines gives
$b=1.52$
$\alpha=2.36 \times 10^{4}$.

d. *Compute

$$
\alpha=\exp \left[-\frac{\sum_{i=1}^{n}\left(x_{i}\right)^{2} \sum_{i=1}^{n} y_{i}-\sum_{i=1}^{n} x_{i} \sum_{i=1}^{n} x_{i} y_{i}}{n \sum_{i=1}^{n}\left(x_{i}\right)^{2}-\left(\sum_{i=1}^{n} x_{i}\right)^{2}}\right]
$$

*NUTE: Steps $c$ and $d$. would be lengthy calculations by hand, but a computer program to fit a line $Y=b X+c$ by least squares may be eadily adapted to ficting a Weibull line by substituting $\ln t$ for $X$ and $\ln \ln (1 /(1-F(t)))$ for Y. Then $b$ will be $f$ and $c$ will be - $\ln$ a.

## Example

b. (Continued)
where

> n = sample size
$x_{i}=$ ith failure time
It when the equation is solved $F=0$, then $C$ represents the maximum likelihood estimate ot the shap parameter. If $F \neq 0$ go to Step c.
c. Take the derivative (with respect $c$. Using $C=1.5, F^{\prime}=-13.99$. to C) of the equation in Step b:

$$
F^{\prime}=-\frac{n}{c^{2}}-\frac{\left\{2 x_{i}^{c} 2\left(x_{i}^{c} \ln x_{i}^{2}\right)-\left[\sum x_{i}^{c} \ln x_{i}\right]^{2}\right\} \cdot n}{\left(2 x_{i}^{c}\right)^{2}}
$$

d. Set $b=C=\left(E / F^{\prime}\right)$.
e. Set $\varepsilon$ some small number, determined by the accuracy desired in the answer. It accuracy to $K$ places is desired, then set $\varepsilon=10^{-(K+1)}$.
t. Li $\left|E / F^{\prime}\right| \geq E$, set $C=b$ and repeat Steps b. - $f$.
8. Now it is necessary to apply the unbiasing factor to the maximum likelihood estimate. Appendix Table XI gives tactors to be multipliea to the maximum likelinood estimate.
d. $b=1.5+0.115=1.615$.
e. Suppose 2 place accuracy ${ }^{i} s$
desired, then set $\varepsilon=10^{-3}$.
3. Mechod
n. To solve for the Weibull scale parameter 4

$$
\hat{a}=\sum \dot{x}_{i}^{\hat{c}} / n
$$

## Example

h. The estimate of the Weibull scale parameter is

$$
\hat{\alpha}=4.47 \times 10^{4}
$$

4. For Further Information

The statistical theory developing the use of this method is presented in "Inferences on the Parameters of the Weibull Distribution," by Thoman, Bain and Antle, lechnometrics, Vol. Il, No. 3, August 1969, pp. 445-460.
2.4.2.2 Grapnical Poinc Estimation

1. When to Use

Estimates of the Weibull shape and acale parameters may be obtained graphically by using specially prepared Weibull probability paper. The decision co use this method over those described in the two previous topics shoula be based wholly on the accuracy desired. This method is the least accurate but can be done quickly and casily.
2. Conditions for Use
a. Failure times must be collected.
b. Median rank tables are required. They are provided in the Appendix, Table I.
c. Weibull probability paper is required. See Figure 2.4.2.2.

## 3. Method

a. To plot the ith tailure tame in a set of $n$ ordered failure times, tind the median rank plotting position on the left-hand ordinate by consulting the table of median ranks at $n$, $i$. To obtasn median ranks tor $n$ greater than cwenty, the following formula may be used:

Median rank $(n, i)=\frac{i-0.3}{n+0.4}$,
where
$i=$ order number of failure
$n=$ number of tailures.

## Example

a. As an example of plotting failure times on Weibull probability paper, consider a case in which 20 items are all tested to failure; the 20 failure tames, in ascending order, are given below in the left-hand column. In the right-hand column are the median rank plotting positions for each failure time, obtained trom the table of median ranks for $n=20$ in the Appendix, Table I.

| Failure Times <br> (Hours) | Median <br> Kanks |
| :---: | :---: |
| 92 <br> 130 | 0.0341 |

a. (Ciontinued)

| Faj.lure Times <br> (Hours) | Median <br> Kanks |
| :---: | :---: |
| 233 | 0.1322 |
| 260 | 0.1812 |
| 320 | 0.2302 |
| 325 | 0.2793 |
| 420 | 0.328 s |
| 430 | 0.3774 |
| 465 | 0.4264 |
| 518 | 0.4755 |
| 640 | 0.5245 |
| 700 | 0.5736 |
| 710 | 0.6226 |
| 770 | 0.6717 |
| 830 | 0.7207 |
| 1010 | 0.7698 |
| 1020 | 0.8188 |
| 1280 | 0.8678 |
| 1330 | 0.9169 |
| 1090 | $0.965 y$ |

Before plotting the data, it is necessary to perform a transformation on the bottom scale to accommodate the large failure times. The axis must be multiplied by $10^{-2}$ in order for the failure data to fit on the paper. So, the bottom scale is properly labeled HoUks $X 10^{-2}$.
b. The Weibull line, labeled $x_{1}$ in Figure 2.4.2.2 is drawn as described.
using the last point plotted as a reference point for a atraight-edge and dividing the rest ot the points into two equal groups above and below the line.
c. To estimate $k$, parallei to the weibull line draw a line passing cnrougn the small circled point on the $f$ feper.
d. Horizontel projection of che point where chis line intersects the principal ordinate to the right-hand cale gives - $\beta$. The principal ordinate terminates in 0.0 on the upper scale.

- Sometimes in order to plot the tailure data it is necessary to convert the bottom scale to hand:e larger numbers. The scales used on this axis are selectea for the purpose of convenience in presenting the data on the graph. It the bottom scale has been multiplied by $K$, then read $-\ln a_{K}$ at the horizontal projection to the right-hand axis of the incersection of the Weibull line and the principal orainate.
$f$. Find the value of $\alpha_{k}$ by using a calculator. The computed OK is a coded value which is dependent on the time scale used.

8. To convert $a_{K}$ to an uncoded state that is independent of the time scale used on the probability paper, divade $\alpha_{K}$ by $K^{b}$.
where s is the previously obtained snape parameter.

## Example

d. The point where $l_{2}$ intersects $\ell_{3}$, the principsi ordinate, is projected horizontally to the right-hand axis and $-\beta$ read off es -1.5. So, B - 1.5.
e. To find $a$, the intersection of $\ell_{1}$ and $\ell_{3}$ is projected horizontally to the right-hand axis. The value read off the axis, -2.9 , is $-1 n a_{k}$, and wast be converted.
f. The value of $\alpha_{K}$ is found to be 16.2.
8. $a_{K}$ i: converted to an uncoded state by dividing by $\mathrm{K}^{\mathrm{B}} 0^{-2 \mathrm{~B}}$ So, divide 18.2 by
giving

$$
\alpha=\frac{18.2}{10^{-2 B}}=18.2 \times 10^{2} \times 1.5
$$

$$
=1.82 \times 10^{4}
$$



## 4. For Furtner Information

If desirea, the median ranks may be replaced by the unbiased estimate $i /(n+1)$, where

1 = oraer number of failure
$n=$ number of failures.
2.4.2.3 Interval Estimation
2.4.2.3.1 Weibull Parameters

1. When to Use

Use this method to obtain confidence intervals on the shape (c) and scale $(\alpha)$ parameters of the two parameter Weibull cumulative aistribution function given by $F(x)=1-\exp \left(-\left(x^{c}\right) / \alpha\right)$.
2. Conditions for Use
a. Polnt estimates of $c$ and $\alpha$ must be mace using the method of

Section 2.4.2.1.2.
b. Tne tables required in the calculation are provided in Appendix II, Tables $V$ and $V I$.
3. Method
a. To compute $100(1-\gamma)$ percent contidence limits on $c$, locate in Table $V$ the column labeled with the value of $\gamma / 2$. Kead crf the cable value at $N=$ sample size and call it Ll. Locste the column labeled with the value of $1-Y / 2$. Kead oft the table value at $N=$ sample size and call it $L_{2}$. Then, tne confidence interval on $c$ is of the form $\left(\widehat{c} / L_{2}, \widehat{c} / L_{1}\right)$, where $c$ was obtained by the method of Section 2.4.2.1.2.
b. To compute $l \cup U(1-\gamma)$ percent confidence limits on $\alpha$, locate in Table VI the column labeled with the value of $\gamma / 2$. Read oft the table value at $N=$ sample size and call it $t_{1}$. Locate the column labeled with the value

## Example

a. Suppose it is desired to compute $90 \%$ confidence limits on the Weibull paraweters for the example given in Section 2.4.2.1.2. Then $N=20, Y=0.10, c=1.51$ and $\bar{\alpha}=4.47 \times 10^{4}$. Erom Table $V$, at $N=20$ in the 0.05 column the value of $L_{1}$ is read as 0.791 and at $N=20$ in the 0.95 column the value at $L_{2}$ as 1.449. So the interval on $\widehat{c}$ is $(1.51 / 1.449,1.51 / 0.791)=(1.04$, 1.91).
b. The $90 \%$ confidence interval on a requires consulting Table VI:
$t_{1}=-0.428$
$t_{2}=0.421$
3. Mechod

## Example

D. (Continued)
D. (Continuea)
at $1-\gamma / 2$. Reaa off the table value at $N=$ sample size and call it $t_{2}$. Then, the confidence interval on $\alpha$ is of the form

$$
\left(\hat{\alpha} \exp \left(-t_{2}\right), \hat{\alpha} \exp \left(-t_{1}\right)\right)
$$

The intervad on $\alpha$ is given by

$$
\begin{aligned}
& {\left[4.47 \times 10^{4} \exp (-0.421)\right.} \\
& \left.4.47 \times 10^{4} \exp (0.428)\right] \\
& =\left(2.93 \times 10^{4}, 6.87 \times 10^{4}\right)
\end{aligned}
$$

where $\hat{\alpha}$ was ootained by the mechod of Section 2.4.2.1.2.
4. For further Information

This method is from "Interences on the Parameters of the Weibull Distrıbution," Thoman, Bain and Antle, Technometrics, Vol. II, No. 3, August 1964, pp. 445-460.
2.4.2.3.2 Keliability
2.4.2.3.2.1 Uncensored Samples

1. When to Use

Use this method to estimate $90 \%$ confidence limits on reliability for the Werbull distribution.
2. Conaicions for Use
a. A plot of the fallure times must be prepared on Weibull probability paper. See Section 2.4.2.2 for the methodology.
b. A table of $5 \%$ ranks and one of $95 \%$ ranks are required. These are provided in Appendix LI, Tables II and LII.
3. Mechod
a. Draw the Weibull line for the observed data, as described in section 2.4.2.2.
b. Locate the medran ranks on the left-iana axis, project tnem horizontally and mark their intersection with the Weibull line.

## Example

a. Refer to the example ot 20 failure times used in section 2.4.2.2. The Welbull line on Weibull probability paper is presented here again, Figure 2.4.2.3.2.1.
b. The following median ranks, for $n=2 U$, have been marked on the Weibull line:
\$2242.2P


Figure 2.4.2.3.2.1. Graphical Method for Interval Estimation of Reliability for the Weibull Distribution
3. Method

## Example

b. (Continued)

Median Rank Graph Notation

| 0.0341 | A |
| :--- | :--- |
| 0.0831 | B |
| 0.1322 | C |
| 0.1812 | D |
| 0.2302 | E |
| 0.2793 | F |
| 0.3283 | G |
| 0.3774 | H |
| 0.4264 | I |
| 0.4755 | J |
| 0.5245 | K |
| 0.5736 | L |
| 0.6226 | M |
| 0.6717 | O |
| 0.7207 | Q |
| 0.7698 | R |
| 0.8188 | I |

c. The tables show a $5 \%$ rank value of 0.0183 and a $95 \%$ rank value of 0.2182 for $n=20, j=2$. So, these plots have been plotted on the vertical line through point $B$.
d. Curves are drawn through the plotted points.
e. To find limits on reliatility at a particular cime $t$, locate $t$ on the lower axis and read off upper and lower limits on reliability from the two curves by referring to the left-hana axis.

## 4. For Further Information

Tables II and III for $5 \%$ and $95 \%$ ranks are from Electromechanical Component Keliability, May 1963, Chernowitz, et.al., RADC-TDR-63-295, American Power Jet, Rragefield, N.J.

If confiaence intervals other than the $90 \%$ for $n \leq 20$ given here are desirea, any percentile ranks may be obtained in the following manner for sample sizes up to 50. Tables of the Incouplete Beta Function are requirea. The notation used here is found in Tables of the Incomplete Beta Function, K. Pearson, Cambridge University Press, 1956.

## Method

a. For the $j^{\text {th }}$ failure of $n$ failuses, compute

$$
k=n-j+1
$$

For $J \geq k$ let $p=j, q=k$.
For $j<k$ let $p=k, q=j$.
b. To compute $100(1-\alpha)$ percent confidence limits, locate values of Ix $(p, q)$ in Table $I$ of the zeterenced tades, such that $\alpha / 2$ and $1-\alpha / 2$ are approximated as closely as possible. For $j \geq k$, the $\gamma$ percentile tank, where $\gamma=\alpha /<, 1-\alpha / 2$, is given by the $x$ in the lefthand column on the same row as the value of $I_{x}(p, q)=\gamma$.

## Example

a. Suppose $n=10$ and it is required to find the $80 \%$ confidence limits on reliability. Then the follow ing table may be constructed:

| $j$ | $k$ | $\mathbf{Q}$ | P |
| ---: | ---: | ---: | ---: |
| 1 | 10 | 1 | 10 |
| 2 | 9 | 2 | 9 |
| 3 | 8 | 3 | 0 |
| 4 | 7 | 4 | 7 |
| 5 | 6 | 5 | 0 |
| 0 | 5 | 5 | 0 |
| 7 | 4 | 4 | 7 |
| 8 | 3 | 3 | 6 |
| $y$ | 2 | 2 | 5 |
| 10 | 1 | 1 | 10 |

b. To compute $80 \%$ configence limits, the 0.10 and 0.90 percentile ranks are needed. So, in Table $I$ find the $\gamma=0.10$ and $\gamma=0.90$ percentile ranks for $j-1$ on PP. 22, 23, for $p=10$ ana $q=1$. The value of $x$ opposite 0.0946828 in the table, the closest value to 0.1 given, is 0.79 , and the value of $x$ opposite 0.9043821 , closest to 0.9 , is 0.99 . So, the

Mechod
b. (Continued)

For $j<k$, the $\gamma$ percentile rank is given by $l-x$, where $x$ again is in the lefthand column on the same row as the value of $I_{x}(p, q)=\gamma$.

## Example

b. (Continued)
desirea percentile ranks for $n=10, j=1$ are 9U\%: $1-0.7 y$ - 0.21; 10\%: $1-0.99=0.01$.

The complete set of percentile ranks follows, as computed from Table I.

| 2 | $90 \%$ | $10 \%$ |
| :--- | :--- | :--- |
| 1 | 0.21 | 0.01 |
| 2 | 0.40 | 0.06 |
| 3 | 0.51 | $0.0 y$ |
| 4 | 0.55 | $0.1 y$ |
| 5 | 0.65 | 0.20 |
| 0 | 0.74 | 0.35 |
| 7 | 0.81 | 0.45 |
| 8 | 0.91 | $0.4 y$ |
| $y$ | 0.94 | 0.0 |
| 10 | $0.9 y$ | $0.7 y$ |

2.4.2.3.2 Reliability
2.4.2.3.2.2 Censored Samples

1. Wren co Use

This section descrides a procedure for calculating a lower confidence limit on reliablifty for parts which are known to have Weibull fallure distributions. Ihe method is applicable to both censored and uncensored test data.
2. Londitions for Use
A. Fallure times must be collected.
b. Certaln tables are required in the calculation. They are provided in Appendix IL, Tables VIL and VIII.
3. Metnod
a. Hor a test ct $n$ items, $r$ ot whicn are allowed to fail before termination of the test, order the $r$ failure times and for $1=150 \mathrm{r}, \operatorname{set} X_{i}=1 \mathrm{ch}$ fallure time.

Example
a. Supfose $2 U$ parts are put on test and the test terminates atter the lotn fallure. Then $n=20$, $r=10$, and suppose the following fallure times are odserved:
3. Method
b. To find the lower confidence
limit at time $t_{0}$, compute
for $1=1$ to $r$
$y_{1}=\ln X_{1}-\ln t_{0}$

Example
a. (Continued)

$$
\begin{aligned}
& x_{1}=92 \text { hours } \\
& x_{2}=130 \text { hours } \\
& x_{3}=233 \text { hours } \\
& x_{4}=260 \text { hours } \\
& x_{5}=320 \text { hours } \\
& x_{6}=325 \text { hours } \\
& x_{7}=420 \text { hours } \\
& x_{8}=430 \text { hours } \\
& x_{9}=465 \text { hours } \\
& x_{10}=518 \text { hours }
\end{aligned}
$$

b. Suppose it is desired to find the $95 \%$ lower confidence limit on $R(50)$, reliability at 50 hours. Then $\ln (50)=3.91202$ and

$$
\begin{aligned}
& Y_{1}=0.60977 \\
& Y_{2}=0.95551 \\
& Y_{3}=1.53902 \\
& Y_{4}=1.64866 \\
& Y_{5}=1.85630 \\
& Y_{6}=1.87181 \\
& Y_{7}=2.12823 \\
& Y_{8}=2.15177 \\
& Y_{9}=2.23002 \\
& Y_{10}=2.33796
\end{aligned}
$$

c. Since $p=10 / 20$, Table VII gives the following values for $a_{1}$ 's and $b_{1}$ 's.

$$
\begin{array}{ll}
a_{1}=-0.04527 & b_{1}=-0.09198 \\
a_{2}=-0.04032 & b_{2}=-0.09230 \\
a_{3}=-0.03371 & b_{3}=-0.09010 \\
a_{4}=-0.02574 & b_{4}=-0.08597 \\
a_{5}=-0.01650 & b_{5}=-0.08013
\end{array}
$$

3. Method
a. Compute

$$
\begin{aligned}
& Z_{a}=\sum_{i=1}^{r} a_{i}^{Y} Y_{i} \\
& Z_{b}=\sum_{i=1}^{r} b_{i} Y_{i}
\end{aligned}
$$

e. Compute

$$
c_{a} / \angle_{b}
$$

t. To find the lower contidence limit on reliability with contidence coetficient $Y$, use Iable VIIl and find the value of $L *\left(Z_{a} / Z_{b}\right)$ in the colurn with the desired $Y$ heading. It is the exact lower confidence bound for $k\left(t_{o}\right)$, reliability at time $t_{0}$.

## Example

c. (Continued)

| $a_{6}$ | $=-0.00596$ | $b_{0}=-0.07264$ |
| :--- | :--- | ---: | :--- |
| $a_{7}$ | $=0.00595$ | $b_{7}=-0.06345$ |
| $a_{8}=0.01935$ | $b_{y}=-0.05246$ |  |
| $a_{9}=0.03444$ | $b_{9}=-0.03948$ |  |
| $a_{10}=1.10777$ | $b_{10}=0.66851$ |  |

$$
\text { d. } \begin{aligned}
2_{a}= & (-0.04527)(0.60977) \\
& +(-0.04052)(0.95551) \\
& +(-0.03371)(1.53902) \\
& +(-0.02574)(1.64866) \\
& +(-0.01650)(1.85630) \\
& +(-0.00546)(1.87181) \\
& +(0.00595)(2.12823) \\
& +(0.01935)(2.15177) \\
& +(0.03444)(2.23002) \\
& +(1.10777)(2.33796) \\
= & 2.5188
\end{aligned}
$$

$$
i_{b}=(-0.09198)(0.60977)
$$

$$
+(-0.09230)(0.95551)
$$

$$
+(-0.09010)(1.55902)
$$

$$
+(-0.08597)(1.64866)
$$

$$
+(-0.08013)(1.85030)
$$

$$
+(-0.07264)(1.87181)
$$

$$
+(-0.06345)(2.12823)
$$

$$
+(-0.05246)(2.15177)
$$

$$
+(-0.03948)(2.23002)
$$

$$
+(0.66851)(2.33796)
$$

$$
=0.5176
$$

e. $L_{a} / L_{b}=4.87$
f. Keterring co Table Vill,

$$
L *\left(Z_{a} / L_{b}\right)=l *(4.87)
$$

$$
=0.939
$$

So, the lower $95 \%$ confidence limit on reliability at $S U$ hours 18 U.939.
3. Method
8. A point estimate of $R\left(t_{0}\right)$, relability at time $t_{0}$, is given by

## Example

g. Relisbility at 50 hours is given by

$$
\exp (-\exp (-4.87))=0.991
$$

4. For Furtner Information
a. The method given in chis section is from "An Exact Asymptotically Efficient Confidence Bound for Reliability in the Case of the Weibull Distribution," Johns and Lieberman, Technometrics, Vol. 8, No. 1, February 1966, pp. 135-175. That paper also includes an estimation method for obtaining a lower confidence limit on reliability for large sample sizes.
b. A graphical technique for obtaining two-sided confidence limits on reliability is given in Section 2.4.2.3.2.1.

### 2.4.3 The Normal Distribution

2.4.3.1 Analytical Point Estimation

1. When to Use

Use this method to obtain estimates of $\mu$ and $\sigma$, the mean and standard deviation of the Normal distribution. The choice of this method over the graphical method described in the next topic is a mater of the accuracy desired, with thas one yitlding the most accurate estimate.
2. Conditions for Use

Failure times must be collected and dats must be uncensored.
3. Method
a. The sample mean, $\bar{x}$, is an estimate of $\mu$ and is given by

$$
\bar{x}=\sum_{i=1}^{n} \frac{x_{i}}{n},
$$

where

$$
\begin{aligned}
& x_{1}=\text { ith tailure time } \\
& n=\text { samplesize. }
\end{aligned}
$$

## Example

a. Suppose 20 units are tested to failure and the following failure times obecrved:

$$
\begin{aligned}
& 175 \text { hours } \\
& 695 \text { hours } \\
& 8 ? 2 \text { hours } \\
& 1250 \text { hours } \\
& 1291 \text { hours } \\
& 1402 \text { hours } \\
& 1404 \text { hours } \\
& 1713 \text { hours } \\
& 1741 \text { hours } \\
& 1893 \text { hours } \\
& 2025 \text { hours } \\
& 2115 \text { hours }
\end{aligned}
$$

a. (Continued)

2172 hours
2418 hours
2583 hours
2725 hour 8
2844 hour 8
2980 hours
3268 hours
3538 hours
Then $n=20,80$

$$
\bar{x}=\sum_{i=1}^{20} \frac{x_{1}}{20}=1955.2 \text { hours }
$$

b. The sample standard deviation $s$ is an estimate of $\sigma$ and is given by
b. The sample standard deviation calculation gives
$s=886.6$ hours

$$
s=\left(\frac{\sum_{1=1}^{n}\left(x_{1}-\bar{x}\right)^{2}}{(n-1)}\right)^{1 / 2}
$$

where

$$
\begin{aligned}
x_{1} & =\text { ith fallure time } \\
n & =\text { sample size } \\
\bar{x} & =\text { sample mean. }
\end{aligned}
$$

An alternate form, useful in computer programing of this method is

$$
s=\left(\frac{\sum_{i=1}^{n} x_{1}^{2}}{(n-1)}-\frac{\left(\sum_{i=1}^{n} x_{1}\right)^{2}}{n(n-1)}\right)^{1 / 2}
$$

## 4. For Further Information

Additional information regarding these estimation methods can be obtained from any text on elementary statistics.

$$
2-42
$$

### 2.4.3.2 Graphical Point Estimation

1. When to Use

This method estimates $\mu$ and $\sigma$, the mean and standard deviation when failure times are normally distributed. This method yields a less accurate estimate than the method of the previous topic but requires very minimal calculations.
2. Conditions for Use
a. Failure times must be collected, but may be censored.
b. Normal probability paper 18 required.
3. Method
a. On normal probabizty paper, plot the ith failure time in a sample of $n$ ordered failure times on the lower axis vs $1 /(n+1)$ on right-hand axis.

## Example

a. The sample data used on the example for Section 2.4.3.1 is repeated here, with the necessary plotting positions.

|  | Plotting <br> Position |
| :---: | :---: |
| Failure Time | 1/(n+1) |
| 175 hours | 0.05 |
| 695 hours | 0.10 |
| 872 hours | 0.14 |
| 1250 hours | 0.19 |
| 1291 hours | 0.24 |
| 1402 hours | 0.29 |
| 1404 hours | 0.33 |
| 1713 hours | 0.38 |
| 1741 hours | 0.43 |
| 1893 hours | 0.48 |
| 2025 hours | 0.52 |
| 2115 hours | 0.57 |
| 2172 hours | 0.62 |
| 2418 hours | 0.67 |
| 2583 hours | 0.71 |
| 2725 hours | 0.76 |
| 2844 hours | 0.81 |
| 2980 hours | 0.86 |
| 3268 hours | 0.90 |
| 3538 hours | 0.95 |

b. Draw the Normal line of best fit through the plotted points by using the last point plotted as a reference point for a straight-edge and dividing the rest of the points into two
b. Pigure 2.4.3.2 is the plot of this data on normal paper. The normal line has been labeled $\ell_{1}$.


Figure 2.4.3.2. Graphical Point Estimation for the Normal Distribution

$$
2-44
$$

3. Method
b. (Continued)
equal groups above and below the line.
c. The mean, $\mu$, is estimated by projecting the $50 \%$ probability of failure point on the righthand axis to the normal line and then projecting that intersection point down to the lower axis. The estimate of $\mu, \bar{x}$, is read off there.
d. The estimate of $\sigma, s, i s$ obtained by projecting the intersection of the $84 \%$ probabillty of failure point on the right-hand axis with the normal line to the lower axis. Call that point on the lower axis $U$.
c. Repeat Step d. with the $16 \%$ point. Call the noint $L$.
f. The estimate of $\sigma$ :s

$$
s=\frac{U-L}{2}
$$

Example
c. The value of $\bar{x} 18$ read of $f s$ 1950 hours.
d. $U=2900$ hours.
e. L = 1000 hours.
f. The sample standard deviation, $s$, 18

$$
\frac{U-L}{2}=\frac{2900-1000}{2}=950 \text { hour } s
$$

4. For Further Information

Additional examples of the use of this estimation method are presented in most texts on elementary statistics.
2.4.3.3 Interval Estimation
2.4.3.3.1 Small Sample Sizes (O unknown)

1. When to Use

Use this method to obtain confidence limits on the mean of the Normal distribution for sample sizes $<30$.
2. Conditions for Use
a. Estimates of the mean and standard deviation must be available. See Section 2.4.3.1 for method of computing.
b. A table of percentiles of Student's $t$ distribution is required.
a. To find two-sided* $100(\gamma)$ per cent confidence limits on $\mu$, consult a table of the percentiles of the $t$ distribution, for the value of $t(1+Y) / 2, n-l$ where $t$ is Students $t$ with n-l degrees of freedom.

## Example

a. Consider the 20 failure times used as an example in Section 2.4.3.1,
where $\bar{x}=1955.2$ hours $s=886.6$ hours

To obtain two-sided $95 \%$ confidence limits on $\mu$, the value of $t$ is needed. from a table of percentiles of the $t$ distribution it is seen to be 2.093 .
b. The confidence limits are then
$1955.2 \pm 2.093 \times \frac{886.6}{4.47}$
$=(1540.1,2370.3)$
If it were deaired to calculate a lower one-sided $80 \%$ confidence limit on 11 , it would be given by

$$
1955.2-0.861 \times \frac{886.6}{4.47}
$$

$=1784$.

$$
\begin{aligned}
& \text { upper only }+\bar{x}+t_{Y, n-1} s / \sqrt{n} \\
& \text { lower only } \rightarrow \bar{x}-t_{Y, n-1} s / r \bar{n}
\end{aligned}
$$

4. Vor Further Information

Adaitional examples demonstrating this method are presented in
"keliability Handbook" edited by W. Grana Ireson, McGraw-Hill, 1900.
2.4.3.3.2 Large Sample Sizes ( 0 unknown)

1. When to Use

Use this method to obtain confidence limits on the mean of the Normal distribution for sample sizes $\geq 30$. If the standards deviation is unknown, Student's t distribution nolds. However, for sample sizes of 30 or more the Normal distribution approximates the r distribution.
2. Gonditions for Use
a. Estimates of the mean and standard deviation must be available. See Section 2.4.3.1 for method of computing.
b. A cable of standardized normal variates is required.
3. Method
a. To find two-sided $100(\gamma)$ confidence limits on $\mu$, consult a table of standaraized normal deviates for the value of $2(1+\gamma) / 2$, where 2 is the stanaardized normal variate and $(1+r) / 2$ is the area under the curve to be found in the table.

## Example

a. Suppose, after teating 50 items to failure, the sample mean and standard deviation are found to be

$$
\begin{aligned}
& \bar{x}=3780 \text { hours } \\
& s=1440 \text { hours }
\end{aligned}
$$

by the methods of Section 3.4.3.1. Suppose, it is desired to find two-sided $90 \%$ confidence on $H$. Then, from a table of standara normal deviates,

$$
2_{(1+0.90) / 2}=1.645
$$

b. The approximate two-sided conti- b. The confidence imits are chen dence limits are given by

$$
\begin{aligned}
\bar{x} \pm & L_{(1+\gamma) / 2} s / \sqrt{n} \\
\text { wnere } \bar{x}= & \text { sample mean } \\
s= & \text { sample standard } \\
& \text { deviation } \\
n= & \text { sample size }
\end{aligned}
$$

c. One-sided $\operatorname{lUU}(\gamma)$ percent confidence limits are given by

$$
\begin{aligned}
& \text { upper only } \bar{x}+2_{\gamma} s / \sqrt{n} \\
& \text { lower only } \bar{x}-L_{\gamma} s / \sqrt{n}
\end{aligned}
$$

c. If it were desired to calculate an upper one-sided $99 \%$ confidence limit on $\mu$, it would be given by
$3780+2.33 \times \frac{1440}{7.07}=4255$.
4. For Further Intormation

Additional examples describing the use of this method are given in "Reliability Handbook" edited by W. Grant Ireson, McGraw-Hill, 1966.


#### Abstract

2.4.4 The Log-Normal Distribution. To treat Log-Normal tailure data, one proceaure is to first cake the natural logarithm of each failure time. Then use the methods of Section 2.4.3, the Normal distribution, on the logaritnms. The advantage of chis procedure is that tables of the standardized Normal deviates may be used in working with the logarithms of the failure times, since they are Normally distributed.


An alternate procedure tor treating Log-Normal data is to use Table iV in Appendix II. Its advantage is that logarithms of the failure times do not have to be computed.

```
Use of Table IV
```

Merhod
a. For $n$ Log-Normally distributed failure trmes $x_{i}$, compute

$$
\bar{x}=\sum_{i=1}^{n} \frac{x_{i}}{n}
$$

and

$$
s=\left(\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}\right)^{1 / 2}
$$

o. To find the $\gamma$ percentile value, located in Table IV the column headea "YPercentile." Kead off the value in this column opposite the value of $s / \bar{x}$ in the left-hand column. call it $p$.
c. The $r$ percentile is estimated by $p \bar{x}$.

## Example

a. Consider the tollowing set of 6 Log-Normally alstribuced failure times:

> 5.53 hours
> 5.70 hours
> 6.62 hours 7.61 hours 8.33 hours 8.76 hours

Then

$$
\begin{aligned}
& \bar{x}=7.09 \text { hours } \\
& y=1.36 \text { hours }
\end{aligned}
$$

b. To fina the 5 th percentile, that coluran in the table is consulted and the value located opposite $s / \bar{x}=1.36 i 7.09 \quad 0.19$ is read offas $p=0.72$.
c. The 5th percentile then is $0.72 \times 7.09=5.1$. This can be interpreted as an estimate of che reliability ( $R_{0.95}$ ).

## Adicional Applications

a. It it is desired to find some percentile other than chose presented in the table the following formula will be useful:

Let $P_{i}=$ the desired percentile
$\mu_{x}=$ the mean of the observations which are assumed to be LogNormally distributed

$$
\begin{aligned}
& \sigma_{x}^{2}=\text { the variance of the observations which are assumed to be } \\
& \quad \text { Log-Normally distributed } \\
& 2_{1}=\text { the standard Normal deviate associated with } P_{i}
\end{aligned}
$$

Then

$$
p_{i}=\frac{u_{x}^{2}}{\left[u_{x}^{2}+\sigma_{x}^{2}\right]^{1 / 2}} \exp \left[2_{i}\left(\log \left[\frac{\sigma_{x}^{2}+\mu_{x}^{2}}{u_{x}^{2}}\right]\right)^{1 / 2}\right]
$$

b. If the observations are in logarithmic form and a given percentile is desired, the following formula is used:

Let $P_{i}=$ desired percentile
$H_{y}=$ the mean of the $\log _{e} x_{i}{ }^{\prime}$ s
$\sigma_{y}=$ the standard deviation of the $\log _{e} x_{i}{ }^{\prime s}$
$Z_{i}=$ the standard Normal deviate associatea with $P_{i}$
Shen

$$
P_{i}=\exp \left(\mu_{y}+z_{i} \sigma_{y}\right)
$$

## For Further Intormation

Ine mathematical theory of this distriburion is presented in "Ihe Log-Normal Distribution" by Aitchison and Brown, Cambridge University Press, 1457. The analysis methods and tables accompanying this method were developed and prepared by J.G. Erost, Hughes Aircraft Company, Fullerton, California.
2.4.5 The Extreme Value Distrabution
2.4.5.1 Analytical Point Estimation

1. when to Use

When the distribution of failure times follows the extreme value distribution, this method is applicable. The parameter estimates are calculated by the method of moments. A graphical method for parameter estimation is presenced in the next topic. The decision whether to use che analytical or graphical method rests in the accuracy desired when compared to the calculations to be performed in the method deacribed here.
2. Conditions for Use

Failure times must be known or assumed to follow the extreme value distribution. Random failure times must be observed.
3. Method
a. Observe the failure times from a randomly selected sample.
b. Calculate estimates of the mean and standard deviation from the sample observations.
c. Estimate the parameters $\beta$ and mfrom the following formula:

$$
\begin{aligned}
& \hat{B}=\frac{\sigma_{N}}{S} \\
& \hat{\mathrm{~m}}=\overline{\mathrm{x}}-\frac{\bar{Y}_{N}}{\hat{B}}
\end{aligned}
$$

## Example

a. Given the following 30 failure times from a process assuming a largest extreme value (L.E.V.) distribution:

Failure Time (Hours)
$220 \quad 453$
$230 \quad 455$
262470
288476
289517
297540
312550
$315 \quad 552$
360586
$369 \quad 588$
$394 \quad 633$
$399 \quad 637$
$412 \quad 657$
$431 \quad 69 \mathrm{C}$
$438 \quad 728$
b. Using the mettods of Secifon 2.4.3.1, caiculate estimates of the mean

$$
\bar{x}=451.6
$$

and the standard deviation
$s=139.0$
c. For $n=30$, Gumbel gives the constant:
$\sigma_{N}=1.11238$
$\bar{T}_{N}=.53662$
3. Method

Example
c. (Continued)
c. (Continued)
where $\sigma_{N}$ and $Y_{N}$ are two constants which are functions of sample size only. A table of these constants is available in Gumbel, "Statistics of Extremes," Columbia University Press, 1960.
d. Estimate the reliability for
any time $t$ from the formula
d. Estimate the reliability for
any time $t$ from the formula

Then

$$
\begin{aligned}
& \hat{B}=\frac{1.11238}{139.0}=.008 \\
& \hat{m}=451.6-\frac{.53662}{.008}=384.5
\end{aligned}
$$

$$
R(t)=1-e^{-e^{-\hat{B}(t-\tilde{m})}}
$$

d. The estimated reliability for a mission of 500 hours is

$$
\begin{aligned}
& R(500)=1-\mathrm{e}^{-\mathrm{e}^{-.008(500-384.5}} \\
& R(500)=.335
\end{aligned}
$$

4. For Further Ininrmation

The example presentet in this section is for the Largest Extreme Value (L.E.Y.) Distribution. The methods for the Smallest Extreme Value (S.E.V.) Distribution ar: eesentially equivalent.

### 2.4.5.2 Graphical Point Estimation

1. When to Use

When the distribution of failure times is known or assumed to follow the extreme value distribution, this method is applicable. This method yields somewhat less accurace estimates than the anslytical method of Section 2.4.5.1 but does not require the performance of calculations.
2. Conditions for Use

Either random or ordered censored observations may be used to estimate the parameters.
3. Method
a. Collect fallure times from a randem process and couple them with median ranks $\widehat{F}(x)$ and calculate

$$
\ln \ln x \frac{1}{5}
$$

## Example

a. Given 30 random fadlure times from a process assuming an L.E.V. distribution. Obtain median ranks for $n=30$ using the method of Section 2.4.2.2 and couple these with the failure times as follows:


Example
a. (continued)

Median
Ranks $\ln \ln \frac{1}{\pi}$ Failure Times

| $F(x)$ | $F(x)$ | (Hours) |
| :---: | :---: | :---: |
| . 123 | 1.33 | 220 |
| . 056 | 1.06 | 230 |
| . 089 | . 88 | 262 |
| . 122 | . 74 | 288 |
| . 155 | . 62 | 289 |
| . 188 | . 52 | 297 |
| . 220 | . 41 | 312 |
| . 253 | . 32 | 315 |
| . 286 | . 22 | 360 |
| . 314 | . 13 | 364 |
| . 352 | . 04 | 344 |
| . 385 | -. 05 | 399 |
| .416 | -. 14 | 412 |
| .451 | -. 23 | 431 |
| . 484 | -. 32 | 438 |
| . 510 | -. 41 | 453 |
| . 549 | -. 51 | 455 |
| . 582 | -. 61 | 470 |
| . 615 | -. 72 | 476 |
| . 648 | -. 84 | 517 |
| . 681 | -. 96 | 540 |
| . 714 | -1.09 | SSU |
| . 747 | -1.23 | 552 |
| . 780 | -1.39 | 580 |
| . 813 | -1.57 | S80 |
| . 845 | -1. 78 | 633 |
| .878 | -2.04 | 637 |
| . 911 | -2.38 | 657 |
| . 944 | -2.86 | 69 u |
| . 977 | -3.76 | 720 |

o. Plot the failure times on the $x$ axis and $\ln \ln \frac{1}{\hat{F}(x)}$ on the $y$ axis and draw a line of best fit through the points. from the graph.
b. See Figure 2.4.S for the grapi of this data.


## Example

d. (Continued)

Therefore

$$
\hat{m}=\frac{2.85}{.0076}=380
$$

NOTE: These results agree closely with the results of the analytical wethod Section 2.4.5.1.

## 4. For Further Information

The example presented in this section is for the Largest Extreme Value (L.E.V.) Distribution. The method for the Smallest Extreme Value (S.E.V.) Distribu* is essentially the same. Commercial graph paper is available for ple. . the extreme value distribution.
2.4.5.2 1 nterval Estimation

1. When to Use

When the distribution of failure times follows the extreme value distribution, this method yields an approximate confidence interval or lower confidence bound for reliability.

## 2. Conditions for Use

This wethod is applicable if it is assumed that the ample size, $n$, is large and that the graphical estimates of parameters, $B$ and m (see Section 2.4.5.2) are maximum likelihood estimates. The actual maximum likelihood estimate values can be obtained only by iteration.
3. Method
a. Obtain estimates for the parameters $\beta$ and musing graphical methods of Section 2.4.5.2.
. Specify the mission time and confidence desired.

Examples
a. From the example in Section 2.4.5.2

$$
\begin{aligned}
& \hat{\beta}=.0076 \\
& \hat{\mathrm{~m}}=380
\end{aligned}
$$

b. Por a mission time of 300 hours, find a lower 95\% confidence bound on reliability.



Figure 2.4.5. Plot of Fitted Straight Line for Use With the Extreme Value Distribution
3. Method
c. Perform the following calculations:

$$
\begin{aligned}
& \hat{v}=\hat{B}(x-\hat{m}) \\
& c=\sqrt{\frac{6}{N \pi^{2}} z_{1-\alpha}} \\
& b=\frac{\pi^{2}}{6}
\end{aligned}
$$

where

## Examples

c. The calculations provide the
foliowing results:

$$
\begin{aligned}
\hat{v} & =.0076(300-380)=-.6 \\
z_{1-.05} & =1.645 \\
c & =\sqrt{\frac{6}{30 \pi^{2}}(1.645)}=.16 \\
b & =\frac{\pi^{2}}{6}=1.64
\end{aligned}
$$

d. Calculate a lower bound for $u$ d. $v^{*}=-.52$
$v^{*}=\frac{1}{1-c^{2}}\left(-.423 c^{2}+\hat{v}+c \sqrt{(\hat{v}+.423)^{2}+\left(1-c^{2}\right) / b^{2}}\right)$
e. Calculate a lower bound for rellability as follows:
$R^{\star}=1-\exp \left[-e^{-v \star}\right]$
e. The lower $95 \%$ confidence bound for reliability at 300 hours is

$$
\begin{aligned}
& R^{\star}=1-e^{-e^{-.52}} \\
& R^{\star}=.81
\end{aligned}
$$

## 4. For Further Information

This example demonstrates the Largest Extreme Value Distribution. The method for the Smallest Extreme Value Distribution is essentially the same.

### 2.4.6 Tests for Increasing Failure Rate

### 2.4.6.1 Distribution Free Test

1. When to Use

When a number of fallure times have been observed and the probability distribution is unknown this test may be used to determine if the observations are from a distribution with a decreasing, constant or increasing failure rate. The test is non-parametric and was originally proposed hy M.G. Kendall (1938). The detalled method presented here is from the work of Henry Mann (1945), and from Barlow and Proschan (1964).

## 2. Conditions for Use

The observed failure times must be arranged in ascending order. A simple computer program will facilitate the calculations. Table $A$ in Appendax Il is used in conjunction with the calculations to define the values required to hypothesize whether the iailure rate is decreasing constant or increasing.
3. Method
a. Arrange the observed failure times ( $x_{i}{ }^{\prime} s$ ) in ascending order.
b. Compute $T_{1}$ for each adjacent parr of failure times as follows:

$$
\begin{aligned}
& T_{1}=x_{1} \\
& T_{2}=x_{2}-x_{1} \\
& T_{3}=x_{3}-x_{2} \\
& T_{n}=x_{n}-x_{n-1}
\end{aligned}
$$

## Examples

a. Given the following 8 failure times arranged in ascending order:

$$
2,6,9,12,14,16,17,18
$$

b. In this example, the $\mathrm{I}_{i}$ 's are

$$
\begin{aligned}
& I_{1}=2 \\
& T_{8}=6-2=4 \\
& T_{3}=9-6=3 \\
& T_{4}=12-9=3 \\
& T_{5}=14-12=2 \\
& T_{6}=16-14=2 \\
& T_{7}=17-16=1 \\
& T_{8}=18-17=1
\end{aligned}
$$

c. The $D_{i}{ }^{\prime} s$ are

$$
\begin{aligned}
& D_{1}=8 \cdot 2=16 \\
& D_{2}=7 \cdot 4=28 \\
& D_{3}=6 \cdot 3=16 \\
& D_{4}=5 \cdot 3=15 \\
& D_{5}=14 \cdot L=8 \\
& D_{6}=3 \cdot 2=6 \\
& D_{7}=2 \cdot 1=2 \\
& D_{8}=1 \cdot 1=1
\end{aligned}
$$

3. Mechod
d. Generate a $V_{i j}$ statistic by comparing each set of two $\nu_{i}{ }^{\prime} \mathrm{s}$.

LEI $V_{i j}=$ Lif $D_{i}>D_{j}$ for all
$i, j=1,2 \ldots n$, with $i<j$
$V_{2 j}=0$ otherwise
e. Generate the test statiatic

$$
v_{n}=\sum_{i<j} v_{i j}
$$

f. Now enter Table $X$ of the Appendix with $V_{n}$ and the desired level of significance.

## Example

d. Since $n=8$, there will be $\binom{8}{2}=\frac{8!}{2!6!}=28$ comparieons ot $D_{1}$ through $D_{y}$ as follows:

$$
v_{12}=0, \text { since } D_{1} \Phi_{2}=16<28
$$

$V_{13}=0$, since $D_{1} \Phi_{3}=10<18$
$V_{14}=1$, since $D_{1} \Phi_{4}=10>15$
In a similar manner the tollowing $V_{i j}{ }^{\prime}$ s are assigned values
of $L$ since $D_{i}>D_{j}$ :
$v_{15}, v_{16}, V_{17}, V_{18}, V_{23}, v_{24}, V_{25}$,
$v_{26}, v_{27}, v_{28}, v_{34}, v_{35}, v_{36}, v_{37}$,
$v_{38}, v_{45}, v_{46}, v_{47}, v_{48}, v_{56}, v_{57}$,
$v_{58}, v_{67}, v_{68}, v_{78}$
e. From step d

$$
v_{n}=26
$$

f. Enter Table $X$ of the Appenaix with $n=8$ and an 0.10 level of significance. Following across the row $n=8$, the closest value to . 10 is .089 which corresponds to an observed $V_{n}$ of $\delta$. Theretore, it $V_{n}$ haa been from 0 to 8 it could have been concluded at a . $08 y$ level of significance that the failure rate was decreasing.

In this example it is desired to test for an increasing tailure rate since $V_{n}=20$.

Table $X$ in the Appendix is symmetrical; therefore, an .089 level of significance corresponds

## Example

f. (Continued)

$$
\begin{aligned}
& \text { to } V_{n}=20 \text {. Since } V_{n}= \\
& 26>20 \text { it can be concluded } \\
& \text { that the failure rate is } \\
& \text { increasing. }
\end{aligned}
$$

4. For Further Information

For the full derivation of this method refer to "Nonparametric Tests Against Trend," Henry Mann, Econometrica, Vol. 13, 1945, and "Marhematical Theory of Reliabilicy," by Barlow and Proschan, John Wiley of Sons, 1904, pp. 232-233.

For an example of the use of the method on empirical data, refer to RADC TR-06-425 "Accelerated Reliabilicy Testing for Nonelectronic Parts," September 1960, AD 803484.

The table from Mann's paper which is reproduced as Table $X$ in Appendix II covers sample sizes up to $n=10$. Above $n=10$, tables of the jancardized normal distribution can be used because Mann proved that $V_{n}$ is asymptotically normally distributed with mean $\frac{n(n-1)}{4}$ and a variance or
$\frac{2 n^{3}+3 n^{2}-5 n}{72}$.
2.4.6.2 Test Based on Probability Limits and Weibull assumptions.

1. When to Use

When a set of part failure times has been generated and it is aesired to test it the underlying distribution of failure times is exponential ( $s=$ Weibull shape parameter $=1$ ). In eftect, this becomes a form of goodness-of-fit test for deciding between the exponential and Weibull distributions.
2. Conditions for Use
a. Sne sample size must be greater than 5 .
b. I'he $p$ (estimate of Weibull shape parameter) must have been calculated from the empirical data by the maximum likelihood method of Section 2.4.2.1.2.
c. Table $1 X$ in Appendix Il is requirea.
3. Method
a. Ubserve and record part failure times.
b. Calculate the Weibull shape parameter by the maximum likelinood method of Section 2.4.2.1.2.
c. Deciae on the desired significance for the test for exponentiality.
d. Enter Table $1 \times$ with the sample size ( $N$ ), the upper percentage points, and the lower percentage points of the probability interval.
e. Compare the probability limits with the calcu?ated value of ( $B$ ), the Weibull shape paramerer. If $\gamma_{L} \leq B \leq \gamma_{U}$, then it can be stated that $\&=1$ and nence, the data is from an exponential distribution.

## Example

a. Given the following : 0 failures times

| 92 | 325 | 640 | 1010 |
| ---: | ---: | ---: | ---: |
| 130 | 420 | 700 | 1020 |
| 233 | 430 | 710 | 1280 |
| 260 | 465 | 770 | 1330 |
| 320 | 518 | 830 | 1690 |

b. From this salae data and from the exarople in Section 2.4.2.1.2 the Weibull shape parameter is 1.62 .
c. For this example, it is desired to have $10 \%$ signifacance level tor the hypothesis test (an arbitrary decision).
d. Enter the table at $N=20, \gamma_{L}=.95$, Yú.05. The tablea values for a 90\% probability interval are 0.690 and 1.264.
e. Since $\widehat{\beta}=1.62$ is not contained in the incerval $0.690-1.264$, then the hypothesis tinat $\overline{\hat{B}}$ is from a distribution with $b=1$ (exponential) cannot be supported. The alternate hypothesis is that the failure times were from a distribution with an increasing hazara rate, sance $\widehat{b}>1.264$.
4. For Further Intormation

The method presented here is from a paper by D.K. Thoman, L.J. Bain and C.E. Antle. The paper was titled "Inferences on the Parameters of the Weabull Distrabution" and was published in Technometrics, vol. 11, No. 3, Auguec 1909, Pp. 445-460.
2.4.7 Outlier Tests
2.4.7.1 Early Failures

1. When to Use

When an early failure is suspected not to belong to a population of fallures which fits a particular failure distribution, this method may be
employed to determine if an early failure occurrea too early to be included in the population.

## 2. Condztaons for Use

a. There must be engineering justification for suspecting the early farlure to be unrelated to the main group of failures. Such justification might be based upon a difference in failure modes for example.
b. Failure times must be collected.
c. A sample size of at least 20 in the main group is necessary.

## 3. Method

a. For an early suspect failure time $x_{0}$, compute $F\left(x_{0}\right)$, the cumulative distribution funccion, with parameters estimated from the main group of failure times only, evaluated ac $x_{0}$.

## Example

> a. Consider a Werbull sample of size 20 with parameters, estimated from the 20 failure trmes by the method of Section 2.4 .2 .1 .1 .

$$
\begin{aligned}
\hat{B}= & 1.5 \\
\widehat{\alpha}= & 2 \times 10^{10} \text { cycles. To } \\
& \text { examine an early failure } \\
& \text { at } 10^{5} \text { cycles, compute }
\end{aligned}
$$ $F\left(10^{5}\right)=1-\exp \left(\frac{-10^{5} \times 1.5}{2 \times 10^{10}}\right)=.00150$ b. $p=(1-.00158)^{20}=0.904$.

$$
p=\left(1-f\left(x_{0}\right)\right)^{n}
$$

where

```
n= sample size of main group
    of failures.
```

c. It $p>0.95$, omit the suspect c. Since $p=0.96 y>0.95$, the failure time. It probably does early failure time does not not belong to the population determining the distribution of fallure times.
4. For Further Information

If $p \leq U . y 5$, recalculate estimates for the parameters of $F(x)$ with $x_{0}$ included in the group.

### 2.4.7.2 Late Failures

1. When to Use

When a later failure is suspected not to belong to a population of failures whach fits a particular failure distribution, this method may be employed to decermine it a late failure occurred coo late co be included in the population.
2. Conditions for Use

There must be engineering justification for suspecting the late failure to be unrelatea to the main group of failures. Such justification might be based upon a difference in failure modes for example. A sample size of at least $2 U$ in the main group is needed.
3. Methoa
a. For a late suspect failure time $x_{0}$, compute $F\left(x_{0}\right)$, the cumulative dastribution function, with parameters escimated from the wain group of failure times only, evaluated at $x_{o}$.

## Example

a. Consider a Weibull sample of size 20 with parameters, estimated from the 20 failure times by the method of Section 2.4.2.1.1,

$$
\bar{B}=1.5
$$

$$
\widehat{\alpha}=2 \times 10^{10} \text { cycles }
$$

To examine a late failure at $2 \times 10^{7}$ cycles, compute

$$
E\left(2 \times 10^{7}\right)=1-\exp \left(\frac{-2 \times 10^{7 \times 1.5}}{2 \times 10^{10}}\right)=0.980
$$

b. Compute

$$
q=\left(F\left(x_{0}\right)\right)^{n}
$$

c. If $q>0 . y 5$, omit che suspect failure time. It does not belong to the population determining the distribution of fallure tames.
d. $1 \mathrm{f} q \leq 0 . y 5$, recalculate estmates for the parameters of $F(x)$ with $x_{0}$ included in the group.
b. $q=(0.958)^{20}$
$=0.794$
c. $q=0.744 \geq 0.95$
d. Since $q=0.794 \leq 0.95$, the failure time $2 \times 10^{7-}$ cycles does belong to the population. so, it ls neceseary to repeat the estimation of the Weibull distribution parameters 5 and $\alpha$ from section 2.4.2.1.1 with, $n=21$ anc the feilure time $2 \times 10^{7}$ cycles included.

### 3.0 RELIABILITY SPECIFICATIONS

3.1 Introduction. The purpose of a reliability specification is to fix the reliability component of mission effectiveness by quantifying required reliability characteristics for the part, equipaent, or system. In order to achieve this purpose the reliability specification must be stated in complete and unambiguous terms. A reliability demonatration test will verify whether or not the requirement is satisfied.

Ambiguous requirements may result in an item that passes the demonstration but is not effective with respect to the requirements, and vice versa. For example, to state that a part must heve a life of 1000 hours is ambig uous. The intention of such a specification could be: to require all such parte to survive 1000 hours; to require all such parts to have a 1000 hour MTBF; to require $90 \%$ (on the average) of all such parts to have 1000 hours; or any number of requirements. Nureover, in an unambiguous atatement, the true intention of the specification must be accurately stated. For example, if the rellability requirement is that a part must survive 100 hours with $90 \%$ probability it is often tempting to "convert" this requirement to an MTBF specification based on exponentially distributed lifetimes. For an exponential distribution an MTBF of 1000 hours is roughly equivalent to a 100 hour survival probability of .90 (since $\exp (-100 / 1000)=\exp (-.1)=.9048$ ). Specifying a 1000 hour MTBF would be approximately equivalent to the requirement in the exponential case. However, if lifetimes are actually Weibull distributed with survival function given by

$$
\exp \left(-(t / 308)^{2}\right)
$$

then the mean of this distribution is only 27 h hours, and yet the survival probability for 100 hours is

$$
\exp \left(-(100 / 308)^{2}\right)=\exp (-.105)=.90
$$

which satisfies the original requirement. Imposing the loul hour mean life would thus cause overdesign. Also, this part actually meets the rellability requirement of survival of 100 hours with .90 probability, but would probably fall the correaponding reliability demonstration test that is designed to a $10 C U$ hour MTBF requirement.

Many of these difficulties are alleviated when the underlying life distribution is exponential. In this case, it is sufficient to specify mean life, and knowing the mean determines all other quantifiable reliability measures associated with the exponential distribution. The exponential distribution has been the life distribution of choice in the electronics industry because of compelling evidence (both empirical and theoretical) which makes it suitable for describing lifetimes for complex electronic units whose fallure modes are not wear-out related. Consequently, much effort has been apent in developing and updating MLL-STD-781C, "Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution." And, in view of the success of the exponential distribution in the analyses of section 2 of this notebook concerning its applicability to nonelectronic parts, in many cases specifying mean life will be adequate for nonelectronic parts as well.

In summary, reliability can be quantified in many ways: mean life, median life, probability of mission survival, etc. A rellability specification may be any one or more such quantities dependias on the underlying reliability requirements, and/or life distribution. The reliability specification must accurately reflect the underlying reliadility requirements, and be both semantically and quantitatively unambiguous.
3.2 Reliability Specification for the Exponential Distribution. The survival function (or rellability function) for the exponential distribution is $R(t)=\exp (-t / 0), t>0$, where $t$ is the mean life. The most direct way to specify any reliability requirement in the context of the exponential distribution is to convert the requirement to a requirement on mean life. The specification of mean life or, equivalently, MTBF, is particulariy convenient in subsequent reliability demonstration test design since MIL-STD-78iC is based on MTBF specifications. This, along with the fact that the exponential aiscribution is uniquely determined by the mean life (making MTBF an unambiguous specification) should establish MTBF as the best form of reliability specification. The following table presents formulae for converting various reliability requirements to MTBF in the exponential case.

| RELIABILITY MEASURE | CORRESPONDING MTBF |
| :--- | :--- |
| Failure Rate | $\theta=1 /($ Failure Rate) |
| Probability of survival for <br> $t$ hours $=r$ | $\theta=-t / \ln (r)$ |
| $x(p)=$ the p quantile life | $\theta=-x(p) / \ln (l-p)$ |
| (i.e. $x(p)=$ the life <br> beyond which the part will live <br> with probability l-p) |  |

The following examples should clarify these concepts.
Example 1.
The rellability requirement for an equipment is expressed as a failure rate of 250 failures per million hours. The corresponding MTBF requirement is ( $1 / 250$ )(1,000,000) hours - 4000 hours.

Example 2.
The reliability requirement for an equipment is expressed as a probability of mission ( $=1000$ hours) survival of .99. The corresponding NTBF requirement $18-1000 / \ln (.99)=99,499$ hours.

Example 3.
The . 10 quantile iffe requirement for an equipment is 1000 hours. The corresponding MTBF requirement $13-1000 / \ln (1-.10)=9,491$ hours.

In order to use MTBF specifications in a reliability demonstration tast from MIL-STD-781C, it is necessary to specify the upper (acceptable) test MTBF ( 00 ), the lower (unacceptable) test MTBF ( $\forall_{1}$ ), the producer's risk $a$, and the consumer's risik p. Further details way be found in MIL-STD-781C.
3.3 Rellability Specification for the Weibull Distribution. The two parameter Weibull survival function is given by
$R(t)=\exp \left(-(t / b)^{c}\right), t>0, b>0, c>0$.
The parameter $b$ is called the scale parameter (also, characteristic life), and the paramerer $c$ is callea the shape parameter. The following table lists common reliadlity measures in terms of these parameters.

| RELIABILITY MEASURE | FORMULA |
| :---: | :---: |
| Mean Life | b [ $(1+1 / c)$, where I is the usual Gamma function. |
| $p$ quantile, $x(p)$, 1.e. the | $b(-\ln (1-p))^{1 / c}$ |
| life beyond which the equipment will survive with probability l-p. |  |
|  |  |
| Characteristic Life, i.e.$x(.632)$ |  |
| Since the Weibull distribution is a two-parameter distribution, any unambiguous reliability specification must involve two quantities. For example, specifying ( $b, c$ ) directly would be sufficient, although not much physical significance can be attached to the parameter c. Other specifications whicn would a! so be sufficient would be mean 11 fe and the .90 quantile life; the . 50 and .90 quantiles; or characteristic life (b) and .95 quantile life. These possibilities are summarized in the following table. |  |
| RELIABILITY SPECIFICATION | CORRESPONDING VALUES OF ( $b, c$ ) |
| Mean Life, 0 , and $p$ quantile $x(p)$. | Must be found by iteratively solving: |
|  | $\theta=b r(1+1 / c)$ |
|  | $x(p)=b(\ln (1 /(1-p)))^{1 / c}$ |

## RELIABILITY SPECIFICATION

Two quantilpr. íni, x(q).

Characteristic Life, $b$, and the quantile $x(p)$.

COPRESPRNDIMS VALUES OF $(b, c)$
$c=\frac{\ln [\ln (1 /\{1-q\}) / \ln (1 /\{1-p\})]}{\ln (x(q) / x(p)]}$ $b=\frac{x(p)}{[\ln (1 /\{1-p\})]^{1 / c}}$
$b$ is the Characteristic Life

$$
c=\frac{\ln [\ln (1 /\{1-p\})]}{\ln (x(p)]-\ln (b)}
$$

It is not often practical to specify reliability in terms of the Weibull or any other two parameter distribution because the corresponding reliability demonstration test procedures are cumbersome. Moveover, since there are two parameters involved, there is no "OC curve" as in the case of MIL-STD-781C for the exponential distribution. Finally, and most importantly, there is no natural ordering of the parameter pairs (b,c). That is, given that "acceptable" values have been established for the parameters (b, c), there is no obvious logical way to assign "unacceptable" values to ( $b, c$ ). For example, the pairs $(b, c)=(308.6,2)$ and $(b, c)=(212,3)$ both determine Weibull distributions having 100 hours as the . 10 quantile. The respective mean lives are 273 hours, and 189 hours, so that the pair (b,c)=(300.6,2) appears wore acceptable. However, the .05 quantiles for the pairs $(b, c)=(308.6,2)$ and $(b, c)=(212,3)$ ere 69.9 hours and 78.8 hours, respectively. Thus, from this point of view, the pair $(b, c)=(212,3)$ is more acceptable alnce 78.8 hours are survived with probability . 95 , whereas for the pair $(b, c)=(308.6,2)$ only 69.9 hours are survived with probability . 95 .

One way to alleviate this problem is to fix one parameter for all cases. This is the same as assuming that one parameter, either $b$ or $c$, is known exactly. Since neither b nor $c$ is ever known exactly in practice, this is an unacceptable solution.
3.4 Reliability Specification Without Respect to a Particular Underlying Life Distribution. In many instances, there is no evidence to suggest a feasible parametric (e.g. exponential, Weibull) iffe distribution for the equipment on which reliability is to be specified. Also, it is of ten the case that when a two-parameter life distribution is appropriate, the reliability requirement is only sufficient to determine one ot the parameters. Moreover, in a two-parameter model, it is not always clear how to specify acceptable values for the parameters versus unacceptable values even when the reliability requirements are suiticient to determine both parameters.


### 4.0 SPECIAL APPLICATION METHODS FOR RELIABILITY PREDICTION

4.1 Introduction. This section is concerned with the estimation of reliability of mechanical and electromechanical components in service. We are not concerned herein with the classical eatimation problems faced by statisticians which are described in Section 2 but with practical examples where the engineer is required to create an estimate of reliability from whatever data he can find. The methods described in this section involve the use of "eugineering experience" or judgment to extend what is usually meager sampling information to obtain meaningful comparisons of materials, designs, and environments.
It is only through the application of engineering judgment to available data that an estimate of reliability in service can be created. The consequence of such methods will of necessity be a point estimate rather chan an interval estimate because the prabability distribution of the estimator cannot be obtalned.

### 4.2 Background for Reliability Prediction Model Development.

Nonelectronic components have numerous failure modes as compared to electronic components. Some of the more basic failure modes which affect this class of components are fatigue, creep, impact, thermal shock, corrosion, oxidation, fretting-corrosion, elastic deformation, relaxation, lubrication failure, wear, spalling, erosion, leakage, delamination, buckling, and radiation damage. Detailed discussion of these fallure modes may be found in ASME (1965). In a proper rellatility assessment the dominant mechanisms must be identified and considered since each mechanism represents a competing failure risk with its own failure distribution.

Several possible approaches are available for the model development, each of which has definite merit but is alao aubject to limitigg constraints.

One approach is through the analysis of accelerated life test results. Thif approsch presupposes that a large number of devices has been tested or is currently being tested in combinations representing the various technoloyies, processes, etc. The results of such controlled tests would provide some indication of the characteristics and peculiarities of the devices as a function of the several configurations, stresses and applications included in the test design. Hovever, the extrapolation of these accelerated test results to more normal operating conditions would be open to questions of validity due to the uncertainties regarding the extrapolation algorithm. Further, while test data under controlled accelerated conditions should aid in understandirg the reliability characteristics, it is difficult to obtain data that covers the wide range of technologies and stress conditions that would be necessary in order to place aajor dependence on this approach alone.

An alternate approach involves the development of a reliability model and its parameters based on a knowledge of fabrication techniques and the anticipated fallure modes. Also required by this approach is a thorough understanding of the fundamental physical/metallurgical/chemical/alertrical degradation mechanioms involved, as well as the proportionate welghting of these mechanisms in translating to the various configurations the component may assure.

A third approach would be to rely solely on the collection and reduction of empirical operating data where the pertinent information with respect to the model parameters would be extracted using suitable statistical techniques. This approach should provide optimal applicability since the field data reflect the actual rellability experience of the devices operating in their use environment. Yowever, it requires the collection and reduction of a large database on the entire range of device configurations and application environments in order co evaluate each of the critical factors. In some cases, particularly with new devices, the amount of data needed to provide oufficiently accurate results way not be available.

The best approach endeavors to utilize the collective data and knowledge offered by all three approaches and subject it to careful, analytical scrutiny to censor out conflicting and discrepant informstion. This approach includes the following tasks: a literature review to define the component, equipment and environmental attributes which will be considered during model development, dertvation of the preliminary model form, data collection, data reduction and analysis, development of the model parameters, and model refinement and verification.

Regardless of the approach taken, the derived model should have the following attributes:

- verified accuracy over the total range of all factors considered
- an uncomplicated approach using easily accessible information on component characteristics and environmental parameters
- dynamic, flexible expression, easily modified to accommodate new techniques
- appropriate discrimination against design and usage attributes which may degrade rellability

The simultaneous attainment of all of the above objectives is difficult, if not impossible. Often these goals are contradictory or mutually exclusive. As an example, some years ago a prediction model was proposed for microcircuits which possessed comendable accuracy over the range of parameters and was based on sound theoretical considerations. Unfortunately, to use the model, the engineer was required to input such information as metallization area, total diffusion area and other such fabrication/design information. Since this information was not available on vendor specification sheets and indeed was often vendor-proprietary, the model proved to be useless. Modifying the model to use more generic bur readily available device parameters degraded the accuracy of the model. The overall utility of the model, however, was enhanced. A discussion of modeling and model limitations can be found in flint et. al. (1982).

In reviewing the literature it becomes obvious that an abundance of models nave been advared for use in the prediction of nonelectronic componer reliability. Unfortimite!y, wost have been found to be deficient in onc or
more of the areas below and as a consequence cannot be included in this section:

- Not application-oriented
- Not yet "engineer-ready"
- Not verified
- Single-vendor specific
- Requires information not easily available
4.3 Graphical Approaches to Reliability Prediction. The graphical approach discussed below briefly provides a means of translating meager information into a reliability estimate.

The information is most likely vendor supplied and thus way be optimistic, based on imited test data, or in the worst case represents a design objective. For these reasons, unless prior experience dictates otherwise, assumptions must be conservative in nature. For example, if an $\mathrm{L}_{10}$ (ilfe time beyond which $90 \%$ survive) supplied by the vendor is based on small sample tests or if the vendor is unwiling to discuss the matter, the $\mathrm{L}_{10}$ life should be reduced by one-half as a minimum.

The ambiguity regarding life frequentiy must be clarified. Life may mean average, median, some period of time/cycles by which some percent will fail on the average, or the time to firat fallure among a population.

Weibull probability paper is a most useful tool available when the uge of a graphical solution is necessary. When the shape or slope parameter, $B$, is equal to one, the distribution reduces to the exponential; an approximation of the lognormal distribution results from values of $B$ in the range of $1.5-3$, and the normal distribution approximation results from a $B$ value of 3.5. With an estimate of some percentile of life, an assumption of the value of 6 , and the use of Weibull paper, the probability of failure before time $x$, say $F(x)$, and the reliability, $1-P(x)$, for any lifetime can be determined.

In those cases when a failure rate must be used in a prediction, it must be remembered that only for the special case of the exponential distribution, 1.e., $\beta=1$, will the failure rate be constant with all other values of $\beta$ greater than one reaulting in an increasing failure rate. In the latter case an average failure rate over a stated time period may be calculated employing the average cumulative pazard function, $H(t)$. The expression is (assuming $1-F(x)=\exp \{-(x / \alpha) \quad \mathcal{B}\}):$

$$
\bar{H}(t)=\frac{t^{\beta-1}}{\alpha^{3}} \quad \text { (Wilson, et, al (1977). }
$$

Alternate notaition for $\bar{H}(:)$ is $\bar{\lambda}(t)$.

### 4.4 General Theory of Interference. When the strength of a

component is less than the stress imposed on it, a failure can be expected to occur. Strength is the ability of a component to resist failur? when subjected to stress. Stress or load may be defined as a mechanical load, dimensional variation, environment, tempersture, etc. Since boch strength and stress are variables, they may be described by probability distributions.

The strength of nominally identical components can be expected to vary due to variations in materials, dimensions, treatments, surface conditions, and so on. This variability can be described by a diatribution function. Various approaches to estimating the strength distribution function are given in Bompass-Smith (1969), Burns (1975), Konno et. al. (1975), ASME (1965), Nilsson (1975), Thomas et. al. (1975), and Welker et. al. (1975). Most typically, however, so little will be known that both the form of the distribution and the variability about the mean will have to be assumed. Unless experience dictates otherwise, the strength variable is often assumed to be normally distributed with standard deviation equal to 5 - 15 percent of the normal value (Fulton, 1983).

The distribution of operational stresses can only be known to a reasonable certainty when the response of a reasonable number of prototypes to the full spectrum of operating conditions has been closely observed. Due to such onnstraints as time, cost, etc., the distribution of stress cannot be established and assumptions must be made. Further information regarding the estimation of the stress distribution msy be found in Kececioglu et. al. (1964, 1967), and Fiderer (1976).

If the expected distributions of stress and atrength can be estimated for a component then by employing interference thecry the probability of failure of the component can be calculated. The concept is presented in detail in Kececioglu et. al. (1964), Disney et. al. (1968), Lipson et. al. (1973), Kapur et. al. (1977), Kececioglu (1972, 1974, 1977, 1968), Kececioglu et. al. (1974), and Dhillon (1980, 1981).

The mathematical foundations of Interference Theory may be outlined as follows. It is assumed that the stress is a random variable $X$ havirig probability density function $f_{x}$ and that the strength is a random variable $Y$ having probability density $f_{Y}$. It is generally assumed that $X$ and $Y$ are statistically independent (although this assumption is not strictly necesary). The probability of failure, $p(f)$, is then

$$
p(f)=F \quad \mid X>Y_{1},
$$

i.e. the probability of failure is the probability that the stress exceeds the strength. An expression may be derived for $p(f)$ as follows.

$$
p(f)=p|x>Y|=\int_{-\infty}^{\infty} \int_{y}^{\infty} f_{X}(x) f_{Y}(y) d x d y
$$

$=$

$$
\int_{-\infty}^{\infty} f_{Y}(y)\left(1-F_{X}(y)\right) d y
$$

where

$$
E_{X}(y)=\int_{-\infty}^{y} f_{X}(x) d x
$$

is tre cumulative distribution of the random variable $X$. Thus, the probability of failure is the area beneath the curve $f_{Y}(y)\left(1-F_{X}(y)\right.$ ) as $y$ varies over all real numbers. Intuitively, ( $\left.1-F_{X}(y)\right) f_{Y}(y) d y$ is the probability that the strength is in the infinitesimal interval ( $y, y+d y$ ), and that the stress exceeds $y$. Integrating (or "summing") over all y then gives the total probability that stress exceeds strength.

The area under the curve $f_{Y}(y)\left(1-F_{X}(y)\right)$ is often referred to as the "interference zone." When the two densities $f_{X}$ and $f_{Y}$ coincide (i.e. are exactly equal), the probability of failure is exactly $1 / 2$, although this is by no means the maximum value possible. In fact, when $f_{X}$ is the normal density with mean $\mu_{X}$ and variance $\sigma_{X}^{2}$, and $f_{Y}$ is the normal density with mean $\mu_{Y}$ and variance $\sigma_{Y}^{2}$, then it can be shown that
where

$$
p(f)=\varnothing\left(\frac{\mu_{X}-\mu_{Y}}{\sqrt{\sigma_{X}^{2}+\sigma_{Y}^{2}}}\right)
$$

where

$$
\phi(u)=(\sqrt{2 \pi})^{-1} \int_{-\infty}^{u} e^{-t^{2} / 2} d t
$$

$i=$ the standard normal cumulative distribution function. Hence, for example,
(1) $p(f)=1 / 2$, if $\mu_{X}=\mu_{Y}$;
(2) $p(f) \rightarrow 1$, as $\mu_{X} \rightarrow+\infty$, $\boldsymbol{u}_{Y}$ fixed;
(3) $p(f) \rightarrow 0$, as $\mu_{Y} \rightarrow+\infty, \mu_{X}$ fixed;

Thus, $p(f)$ can take any value between zero and one. The explanation of (1) above is that when the mean stress equals the mean strength, it is equally likely for stress to exceed strength and vice-versa. In (2) above, when the mean stress is very large with respect to mean strength, the probability of failure is close to one. In (3) above, when the mean stress is very small with respect to strength, the probability of failure is very small.

The concept of interference is shown in Figure 4.4-1 where the interference zone is given as the shaded area. This illustrates the simple case where the strength distribution remains unchanged across time, i.e., is not affected by exposure to the failure causal stress distribution. Figure 4.4-2(a) and (b) illustrates the case where the strength distribution degrades as a function of time exposure to stress as the result of such failure mechanisms as fatigue, corrosion, and wear. Whether this time shift must be considered or may be dismissed depends of course on the rate of change expected. For example, in a naval environment corrosion is a rapid failure causal mechanism and the effect on the strength distribution must be considered; on the other hand, in most military ground fixed applications corrosion is a weak failure causal mechanism and the effect on the strength distribution may be ignored.


Figure 4.4-1. Illustration of the Concept of Interference


Figure 4.4-2(A) Time Varying Strength Density


Figure 4.4-2(B) Time Varying Strengrh Density

### 4.5 Applications of Interference Theory to Reliability Prediction Methodology

Source: RADC-TR-66-710 (March, 196\%) by Charles Lipson et. al., entitled "Reliability Prediction-Mechanical Stress/Strength Interference Models."

RADC-TR-68-403 (December, 1968) by Charles Lipson et. al., entitled "Reliability Prediction-Mechanical Stress/Strength Interference (Nonferrous)."
4.5.1 Purpose. The purpose of this method is to obtain a point estimate of reliability in service for mechanical components subject to fatigue failure. The method is applicable to:
a) Components subjected to completely reversed cyclic bending from axial, or torsion loads.
b) Components subjected to a combination of static and cyclic loads.
4.5.2 Description of Method. Given that strength and service stress each have a probability distribution of known type, and that the parameters of the two distributions are known, the probability that a random observation from the strength dictribution exceeds a random observation from the stress distribution is equal to the reliability. The term "interference" is used to designate an occurrence where stress exceeds strength, so that reliability is the probabllity of no interference.

The source documents provide extensive tables giving the probability of fallure as a function of the parameters for the following combinations of distributions:

| Strength Distribution | Stress Distribution |
| :--- | :---: |
| Weibull | Normal |
| Weibull | Weibull |
| Normal | Normal |
| Largest Extreme Value | Normal |
| Smallesi Extreme Value | Normal |

These tables are perfectly general, and may be used for other types of failure than fatigue. However, the two reports tabulate parameters for fatigue failure only. Nonstandard symbols for the parameters are used in these tables. Table 4.5.l shows their relationship with generally accepted symbols.

Several tables have been complled in the scurce documents giving fatigue strength parameters for virtually all of the common ferrous, nonferrous, and light metal alloys, subjected to completely reversed bending or axial stresses. Included are the effect of heat treatment, surface finish, stress concentrators, temperature, and frequency.

TABLE 4.5.1. PARAMETERS OF TABULATED PROBABILITY DISTRIBUTIONS AS USED IN RADC-TR-68-403 (LIPSON, et. al. 1968)

1. Weibull Distribution:

$x$ is the variable
3 parameters $\left\{\begin{array}{l}x_{0} \text { is the lower bound of } x \\ \theta \text { is the characteristic strength } \\ b \text { is the slope parameter }\end{array}\right.$
This compares with the usual 3 -parameter Weibull distribution

$$
R(x)=\exp \left[-\frac{(x-r)^{\beta}}{a}\right] \quad r<x<0
$$

as follows:

$$
\begin{aligned}
& r=x_{0} \\
& B=b \\
& a=\left(\theta-x_{0}\right)^{t} .
\end{aligned}
$$

2. Normal Distribution:
'The uni' normal deviate is characterized as

$$
2=\frac{x-\mu}{\sigma}
$$

which agrees with standard notation.
3. Extreme Value Distribution

For the Smallest Extreme Value (S.E.V.) distribution,

$$
\begin{gathered}
R(x)=\exp \left[-e^{B(x-M)}\right], \cdots<x<\infty \\
x \text { is the variable } \\
2 \text { parameters }\left\{\begin{array}{l}
B^{-1} \text { is the scale parameter } \\
M \text { is the locstion parometer. }
\end{array}\right.
\end{gathered}
$$

TABLE 4.5.1. PARAMETERS OF TABULATED PROBABILITY DISTRIBUTIONS AS USED IN RADC-TR-68-403 (LIPSON, et. al. 1968) (Continued)

This compares with the usual 2-parameter S.E.V. distribution,

$$
R(x)=\exp \left[-e^{a(x-u)}\right]
$$

as follows:

$$
\begin{aligned}
& a=3 \\
& \mu=M .
\end{aligned}
$$

For the Largest Extreme Value distribution, the same relationship holds, where

$$
R(x)=1-\exp \left[-e^{-8(x-M)}\right]
$$

Note: in 10,2 and 3 ) above, $R(x)$ is the reliability function.

The procedure for estimating reliability using the interference method takes several forms depending upon the assumptions made regarding the strength distribution and the stress distribution. The various methods are illustrated with numerical examples in the source documents.
4.6 Application of Interference Theory for Normally Distributed Strength and Normally Distributed Stress

Source: (1) Fiderer, Leo, "Design For Reliability in Hostile Environment," Microelectronics and Reliability, Vol. 15, Supplement, Pergamon Press, 1976, pp. 75-85.
(2) Lipson, Charles, Statistical Design and Analysis of Engineering Experiments, McGraw-Hill, 1973.
4.6.1 Purpose of the Method. To obtain a point escimate of reliability in service for non-electronic components. The method may be applied to a diverse number of failure causal factors.
4.6.2 Description of the Method. The description of interference of stress and strength distributions given in 4.4 appiles equally to this method. The difference lies in the assumption that the random variables, which may take various distribution forms, may be approximated by a normal distribution so that in practical calculations normal distributions may be assumed without excessive error.

The steps involved in the procedure are as follows:
(1) Estimate the mean, $\mu_{L}$, and standard deviation, $\sigma_{L}$ of the load, $L$, and $\mu_{B}$ and $\sigma_{g}$ of the atrength, $S$.

Where there is no information available to estimate $\sigma_{L}$ or $\sigma_{s}$ a value may be assumed from the interval $.05 \mu$ . $15 \mu$. Where the part is critical or a high reliability requirements exists, a worse-case approach should be taken, 1.e., in the range of . $10 \mu$ to . $15 \mu$. For most conditions $\sigma$ may be taken as . $09 \mu$.
(2) Define the difference between strength and load as a new random variable $D:$

$$
\begin{gathered}
D=S-L \\
\text { with mean } \mu_{D}=\mu_{B}-\mu_{L},
\end{gathered}
$$

and
standard deviation $\sigma_{D}=\sqrt{\sigma_{s}^{2}+\sigma_{L}{ }^{2}}$.
(3) The probability of failure, $P(f)$, is found by computing:

$$
\begin{aligned}
P(f)=P & \{D<0\}=P\{S-L<0\} \\
& =P\left\{\frac{D-\mu_{D}}{\sigma_{D}}<\frac{-\mu_{D}}{\sigma_{D}}\right\}
\end{aligned}
$$



Figure 4.6.2-1.
Normalized Density Function of Excess Strength Over Load.

$$
=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{-\mu_{D} / \sigma_{D}} \exp \left\{-t^{2} / 2\right\} d t
$$

Thus, $P(f)$ is found by entering a table of the cumulative distribution function of a standard normal random variable with the value $-u_{D} / \sigma_{D}$. See figure 4.6.2-1.
(4) The reliability $R$ is then

$$
R=1-P(f) .
$$

### 4.6.3 Examples

4.6.3.1 Shaft and Bushing Reliability. The reliability of a shaft and a bushing after ten years exposure (in a nonoperating state) co a heavy industrial environment is to be determined, i.e., the ability of the shaft to rotate without drag.

The dominant fallure mechanism is considered to be corrosion. Information on corrosion rates and corrosion product build-up is available from the Battelle operated Metals and Ceramics Information Center.

In the analysis it will be assumed that the part dimensional variability is normally distributed and that the maximum and minimum allowable dimensions may be taken as the upper and lower three sigma points, respectively.

## Part Specifications

```
Shaft - Sta1nless Steel
    Dia. . 123 - .124
    Corrosion Rate 3.5 x 10-5 in/year loss
    Corrosion Products 7 x 10-5 In/year buildup
    Net Gain 3.5 x 10-5 in/year
Bushing - Aluminum Chromated
    I.D. . 125 - . 127 in.
    Corrosion Rate 6 < 10-5 in/yęar loss
    Corrosion Products 7.8 x 10-5 in/year buildup
    Nec Gain 1.8 < 10 0-5 in/year
```

The maximum and minimum diameter 30 points after ten years based on the buildup of corrosion products are:

```
Shaft Max 30g}=.124+3.5\times1\mp@subsup{0}{}{-5}\times10\times2=.124
    M1n 30 s}=.123+3.5\times1\mp@subsup{0}{}{-5}\times10\times2=.123
    6\sigma
    \sigmag}=\frac{.001}{6
    Hg
```

Bushing $\quad \begin{aligned} & \operatorname{Max} 3 \sigma_{B}=.128+1.8 \times 10^{-5} \times 10 \times 2=.12764 \\ & \\ & \\ & \\ & \\ & \\ & 6 \sigma_{B}=3 \sigma_{B}=.125+1.8 \times 10^{-5} \times 10 \times 2=.12464 \\ & \\ & \sigma_{B}=\frac{.003}{6} \\ & \mu_{B}=.12614\end{aligned}$
The unreliability or probability of failure, $p(f)$, is evaluated by establishing a new random variable $D$ :
$D=B-S$
with mean $\mu_{D}=\mu_{B}-\mu_{B}=.00194$
and standard deviation $\sigma_{D}=\sqrt{\sigma_{B}{ }^{2}+\sigma_{L}^{2}}=5.27 \times 10^{-4}$
Thus,

$$
\begin{array}{rl}
P(f): P & D<\left.0\right|_{i}=P\left|\left(D-\mu_{D}\right) / \sigma_{D}<-\mu_{D} / \sigma_{D}\right| \\
& \left.=\frac{1}{\sqrt{2} \pi} \int_{-\infty}^{-\mu_{D} / \sigma_{D}} \exp \right\rvert\,-t^{2} / 2: d t .
\end{array}
$$

From a table of the standard normal cumulative distribution entered at $-\mu_{D} / \sigma_{D}=-.00194 /\left(5.27 \times 10^{-4}\right)$ -3.68, it follows that

$$
P(f)=0.0001
$$

and the reliability is:

$$
R_{1}=1-P(f)=.9999 .
$$

4.6.3.2 Lifting Eye Reliability. A lifting eye intended to be used in lifting shipboard equipment while che ship is at sea has a nominal atrength of 60,000 lbs. The dead weight load is 12,000 lbs. This is all the information available to the reliability analyst.

It will be assumed that both the strength and load can be represented by normal distributions and that the nominal strength and load are che means of the distributions.

$$
\mu_{s}=60,000 \quad \mu_{L}=12,000
$$

The variability of the tensile atrength should be controllable such that the atandard deviation may be assumed to be eight percent of the mean strength. Huwever, the load due to the dynamics of wave/ship action may be quite variable. Thus, the load standard deviation will be assumed to be twenty percent of the mean load.

$$
\begin{gathered}
\sigma_{s}=.08 \mu_{s}=4,800 \quad \sigma_{\mathrm{L}}=.2 \mu_{\mathrm{L}}=2,400 \\
4-16
\end{gathered}
$$

The probability of failure is evaluated by forming the new random variable:

$$
D=S-L
$$

where

$$
\mu_{D}=\mu_{s}-\mu_{L}=48,000, \text { and } \sigma_{D}=\sqrt{\sigma_{s}^{2}+\sigma_{L}^{2}}=5366 .
$$

thus,

$$
p(f)=P\{D<0\}=P\left\{\frac{D-\mu_{D}}{\sigma_{D}}<\frac{-\mu_{D}}{\sigma_{D}}\right\}=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{-\mu_{D} / \sigma_{D}} \exp \left|-t^{2} / 2\right| \quad d t
$$

where

$$
-\mu_{D} / \sigma_{D}=-48000 / 5366=-8.9
$$

From a table of the standard normal cumulative distribution function the value of $P(f)$ is approximately 0 .

Thus the reliability is

$$
R=1-P(f)=1
$$

4.6.3.3 Diaphragm Reliability. Calculate the reliability of a diaphragm intended to be used in a one-shot device. Each diaphragm must pass 5 cycles of a worst-case pressure-time profile proof test as acceptance criteria. The purchaser has specified the lower 3 sigma point to be 5 test cycles.

A small sampie size population has been tested to failure which resulted in an estimate of a mean iifetme of 11.44 proof tests. The sample size was not large enough to prove the lifetimes to be normally distributed.

It will be assumed that the proof test lifetiaes of the production population will be normally distributed with a mean of 11.44 test times and a $\sigma$ of 2.147 .

Denoting by $L$ the lifetime in proof tests of the diaphraga, and noting that chis is a one-shot device and therefore the diaphragm needs only to survive one test, it follows that the probability of failure is
$\left.P(f)=P \quad|L<1|_{i}=P \quad \frac{\mid-\mu}{\sigma}<\frac{1-\mu \mid}{\sigma} \right\rvert\,$ where $\frac{1-\mu}{\sigma}=(1-11.44) /$
$2.147=-4.86$. Referring to a table of the standard normal cumulative distribution function, it follows that
$P(f)=.000000605$
and then

$$
R=1-P(f)=.999999395
$$

4.7 Application of the Average Failure Rate Method for Grease Lubricated Rolling Element Bearings

```
Source: Wilson, D.S., and Smith, R., "Electric Motor Reliability Model," RADC-TR-77-408, December, 1977 (AD 050179).
```

4.7.1 Purpose of Method. To obtain an estimate of the average failure rate of grease lubricated rolling element bearings over a related time period.
4.7.2 Description of Method. The method consists of an empirical mathemarical model which provides an escimate of the Weibull characteristic life, $\alpha$, and was developed through the use of regression analysis and a large data base. Essential tables are provided.

The average failure rate is obtained by averaging the Weibull cumulative hazard function over the time period of interest.

A modification has been made in the characteristic life model so that it is valid for a single bearing rather than for a population of first failures of pairs of bearings as given in the source. The method employed is given in Ang (1970) under Suspended Item Tests.

The characteristic life model is based primarily on the effect of temperature on the lubrication qualities of grease and such secondary effects as quality, bore size, speed, grease, and load. The consideration of load as a secondary factor is consistent with good design practice which ifmits the loads on grease-lubricated bearings to 15 percent or less of rated load capacity.

The models are as follows:

where

```
t = time for which fallure rate is required (in hours)
\mp@subsup{\alpha}{B}{}}=\mathrm{ bearing characteristic life (in hours)
q= quality factor
DN - bearing bore (mm) x speed (RPM)
T - bearing operating temperature (degrees Kelvin)
Kg
N=RPM
W = load in pounds
SP = Specific dynamic capacity at 33 1/3 RPM in pounds
```

TABLE 4.7.1. QUALITY FACTORS

| Military Specification | .12 |
| :--- | :---: |
| Commercial | -.27 |

TABLE 4.7.2. $\mathrm{K}_{\mathrm{g}}$ GREASE CONSTANT

| Source | 011 | Thickener | MIL-Spec. | Max. T ${ }^{\text {c }}$ C | $\mathrm{K}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Diester | Sodium and Solid Lubricant | MIL-G-3278A | 170 | 1.35 |
| 2 | Diester | Lithium | MIL-G-3278A | 120 | 1.55 |
| 3 | Silicone | Lithium | - - - | 150 | 1.74 |
| 4 | Mineral | Sodium | MIL-G-18709A | 150 | 1.41 |
| 5 | Silicon | Lithium | MIL-L-15719A | 177 | 1.81 |
| 6 | Synthetic Hydrocarbon | Non-soap | MIL-G-81322 | 170 | 1.74 |

### 4.7.3 Examples

## Example 1

Determine the average failure rate of a bearing with the following specifications for an operating period of 10,000 hours.

Military quality
Grease 5
Bore Dia. 13 mm
Bearing Operating Temp. $30^{\circ} \mathrm{C}$
Speed 3600 RPM
Load 10 lbs
Specific Dynamic Capacity 505 lb (from bearing manufacturer's catalog)

Then:

```
q(quality factor) =.12(Table 4.7.1)
Kg(grease constant) = 1.81 (Table 4.7.2)
DN=Bore D x speed = 13 人 3600=46800
T=30+273=303}\mp@subsup{}{}{\circ}\textrm{K
```

Using Eq. 2


Using Eq. 1

$$
\begin{aligned}
& \bar{\lambda}_{10,000}=\frac{(10,000)^{1.878}}{(12378)^{2} .878}-.000054 \\
& \text { or } 54 \text { failures per million hours. }
\end{aligned}
$$

NOTE: This $\bar{i}$ is valid only if the bearing is replaced at the end of the 10,000 hours of operation.

## Example 2

Determine that period of operation, $t$, for the bearing of Example 1 that will result in the average failure rate equal to 20 failures per million hours.

Solve Eq. 1 for $t$

$$
\begin{aligned}
t & =\left(\bar{\lambda} \times a^{2.878}\right) \frac{1}{1.878} \\
& =\left[20 \times 10^{-6} \times(12378)^{2.878}\right] \frac{1}{1.878} \\
& =5886 \mathrm{hrs} .
\end{aligned}
$$

4.8 Reliability Prediction Mechod - Rolling Bearings Oil Lubricated

Source:
(1) International Organization for Standardization ISO 281/1-1977(E), "Rolling Bearings - Dynamic Load Ratings and Rated Life - Part 1: Calculation Methods."
(2) Marks' Standard Handbook for Mechanical Engineers, 8th ed., McGraw-Hill, 1977, Pp. 8-136 through 8-142.
4.8.1 Purpose of the Method. To obtain a point estimate of the reliability in service of rolling bearings.
4.8.2 Description of the Mechod. Standard formulas have been developed to predict the $\mathrm{L}_{10}$ life of a bearing under any given set of conditions. These formulas are based on an exponential relationship of load to life which has been established from extensive testing.

$$
L_{10}=\left(\frac{C}{P}\right)^{K} \times 10^{6} \mathrm{cyc}
$$

where
$L_{10}=$ the number of revolutions that 90 percent (on the average)
of apolation of bearings will complete or exceed without failure,
i.e., $R=.9$.
$C=$ basic load rating, lbs.
$p=e q u i v a l e n t$ radial load, $1 b$.
$K=3$ for all bearings, $10 / 3$ for roller bearings.
To convert to hours of life ( $\mathrm{L}_{10}$ ), this formula becomes

$$
\begin{equation*}
L_{10}=\frac{16,666.67}{N} \quad \frac{C^{K}}{\mathrm{~K}} \tag{2}
\end{equation*}
$$

where $N=$ rotacional speed, rpm.
The basic load rating, $C$, value is readily obtainable from any bearing manufacturer's catalog. All bearing loads are converted to an equivalent radial load, P. Equation 3 is the general expression used for both ball and roller bearings.

$$
\begin{equation*}
P=X R+Y T \tag{3}
\end{equation*}
$$

where
$R=$ radial load, $1 b$.
$T=$ thrust (axial) load, lb.
$X=$ radial factor
$Y=$ chrust factor.
The $X-Y$ factors may be calculated using the methods described in Source 1 or, with some loss in precision, average values may be selected from Table 3, PP. $8-140$ of Source 2 .

One further formula is necessary to adjust the $\mathrm{I}_{10}$ life for other levels of reliability and less than optimum operating conditions.

$$
\begin{equation*}
L_{n}=a_{1} a_{2} a_{3} L_{10} \tag{4}
\end{equation*}
$$

where:

$$
\begin{aligned}
& a_{1}=\text { life reliability factor } \\
& a_{2}=\text { material properties factor } \\
& a_{3}=\text { operating conditions factor. }
\end{aligned}
$$

The reliability factors are given in Table 4.8-1.
TABLE 4.8-1: LIFE ADUUSTMENT FACTOR OF RELIABILIIY, $a_{1}$, (From Source 1)

| Reliability <br> $\%$ | $\mathrm{~L}_{\mathrm{n}}$ |  |
| :---: | :---: | :---: |
| 90 | $\mathrm{~L}_{10}$ | 1 |
| 95 | $\mathrm{~L}_{5}$ | 0.62 |
| 96 | $\mathrm{~L}_{4}$ | 0.53 |
| 97 | $\mathrm{~L}_{3}$ | 0.44 |
| 98 | $\mathrm{~L}_{2}$ | 0.33 |
| 99 | $\mathrm{~L}_{1}$ | 0.21 |

The iffe adjustment factor for material, $a_{2}$, has not been quantified on the basis of material characteristics but rather on test results and bearing applications. In general an al value of one applies. However, a value greater than one may apply to bearings made of steei of low impurity content or of special analysis. Values of a should be obtained from the bearing manufacturer or from Zaretsky (1.971).

Operating cunditions which remain to be zaken into account are the adequacy of the lubrication (at the operating speed and temperature) and conditions causing changes in material properties (1.e., high temperature causing reduced hardness). The influence on bearing life of such conditions may be considered by the application of a life adjustment factor a3.

The calculation of basic dynamic load rating and basic rating life assumes that bearing life is limited principally by sub-surface fatigue, i.e., that the rolling elements and the ring (washer) raceways are sufficientiy separated by a lubricant to make the probability of fallures caused by aurface distress negligible. Where this requirement if fulfilled, $a_{3}=1$; provided a lower value does not apply, for example, because of a change in material properties caused by the operating conditions.

Reduction of as values should be considered whenever the visccsity of the lybricanc is less than $13 \mathrm{~mm} \mathrm{~m}^{2 / \mathrm{s}}\left(1 \mathrm{~mm}^{2} / \mathrm{s}=1 \mathrm{cST}\right.$ ) for ball bearings or $20 \mathrm{~mm}^{2} / \mathrm{s}$ for roller bearings at the operating temperature and/or where the rotational speed is exceptinnally low (revolutions per minute times pitch dia. in mm less than 10,000 ). Values of a3 greater than 1 may be considered only where the lubrication conditions are particularly favorable.

In most cases, discussions with the bearing manufacturer regarding the specifics of the application will help in quantifying a value for a ${ }_{3}$. Carter (1972) should also be reviewed for guidance.

### 4.9 Reliability Prediction Method - Spur Gear Systems

$$
\begin{aligned}
\text { Source: } & \text { Savage, M., C.A. Paridon, and J.J. Coy, "Reliability } \\
& \text { Model for Planetary Gear Trains," U.S. Arwy Aviation } \\
& \text { Research and Development Comand, AVRADCOM TR 82-C-6. }
\end{aligned}
$$

4.9.1 Purpose of the Method. To estimate the gear system life which will result in a 90 percent probability of survival.
4.9.2 Description of the Method and Example. In the design of a tranamission to carry power, high strength alloy steels are normally used in key element to help minimize the transmission's size for a given power and speed rating. As a result, the endurance limit of soft ductile steels is replaced by a higher capacity which gradually decreases with the load cycle count. Key elements, such as bearings and gears are designed on a life basis in order to keep their size reasonable.

This finite life design for lives greater than $10^{7}$ load cycles is common practice in the design of bearings. It is the intent of this approach to extend the Weibull reliability, life and load theory to the gears as well as the bearings and to combine the component lives in a consistent fashion in order to predict the transmission rellability and life as a function of the applied load and its critical component capabilities.

Although this theory will apply just as well to a simple gear reduction or any transmission composed of bearings and gears, the presented example will be for the planetary reduction in the main rotor bor of a light helicopter. A schematic diagram of a three planet reduction with the ring gear fixed and the sun gear as input is shown in figure 4.9-1.

In this transmission the overall ratio is $5: 1$ and the output planetary spider or arm is to be rotated at 300 RPM. The power transmitied by the transmission is to be 200 horsepower ( 150 kw ). The input speed of the sun gear is 1500 RPM . The geags are all $20^{\circ}$ full depth AGMA toothed gears with a diametrical pitch of $6 \mathrm{in}^{-1}$ ( $\mathrm{Ng} / \mathrm{Dg}$ ) (a module of 4.23 mm ). They are all made of case hardened A151 9310 vacuum arc remelt steel with a material constant of $21,000 \mathrm{pBi}$ ( 144 MPa ). The sun gear has 24 teeth and a 4 inch pitch diameter ( 102 mm ). The planet gears have 36 teeth each and a 6 inch ( 153 mm ) pitch diameter. And the ring gear which 18 internal has 96 teeth and a pitch diameter of 16 inches ( 466 mm ). All the gears have a 0.725 inch face width ( 18.4 mm ).

The planet bearings are the other key elements in the transmission. These bearings are 75-02 single row cylindrical roller bearings with a 1 inch width ( 25 mm ) and an outside diameter of $51 / 8$ inches ( 130 mm ). These bearings have a nominal basic dynamic capacity of 18,200 pourds ( 81 kn ) each.

Since the transmission is isolated from external side and thrust loads by outside bearings, these three bearings and five gears comprise the critical elements in the transuission. It is assumed that their loading is sufficiently light to prevent early tooth rupture or bearing brinelifig. It is assumed that the life of each component is based on Hertzfan stress pitting fatigue and that the strength in this mode is continually reduced with load cycles.

In order to combine the reliabilities of the transmission components into a consistent system relfability, all component load cycles will be reflected

$$
4-24
$$



Figure 4.9-1. Planetary Gear Reduction
into a common counting basis of input sun rotations. This requires a litele kinematics.

For a planetary gear train, the number of relative rotations of the plant gear with respect to the planet spider or arm in terms of input sun rotations 18:

$$
\theta_{P / A}=\frac{R_{S} R_{R}}{R_{P}\left(R_{S}+R_{R}\right)} \theta_{S}
$$

where the subscripts denote the respective gears: $S$ - sun, $R$ - ring and $P$ planet. The $R$ 's represent the reapective gear radii. Thus:

$$
\theta_{P i A}=\frac{-2(8)}{3(2+8)} \theta_{S}=-0.533 \theta_{S}
$$

The negative sign indicates rotation in the opposite direction to $\theta_{S}$. Since the speeds are proportional to the number of revolutions

$$
\omega_{P / A}=-0.533(1500)=-800 R P M
$$

Since the loads on the gears and bearings are stationary with respect to the planetary spider or ari, we are also interested in the number of relative rotations of the sua gear and the ring gear with respect to the arm in terms of input sun rotations:

$$
\begin{aligned}
& \theta_{S / A}=\frac{R_{R}}{R_{S}+R_{R}} \theta_{S}=\left(\frac{8}{2+8}\right) \theta_{S} \\
& \theta_{S / A}=0.8 \theta_{S} \\
& \omega_{S / A}=0.8(1500)=1200 \mathrm{RPM}
\end{aligned}
$$

and

$$
\begin{aligned}
& \theta_{R / A}=-\frac{R_{S}}{R_{S}+R_{R}} \theta_{S}=\frac{-2}{2+8} \theta_{S} \\
& \theta_{R / A}=-0.2 \theta_{S} \\
& \omega_{R / A}=-0.2(1500)=-300 \mathrm{RPM}
\end{aligned}
$$

The forces on the components can be found from the power and input speed.

$$
\begin{align*}
& T_{i}=\frac{\text { Power }}{\omega_{S}}\left[63025 \frac{1 b-1 n \mathrm{RPM}}{H P}\right] \\
& T_{i}=\frac{200}{1500}(63025)=8403 \mathrm{lb}-\mathrm{in}
\end{align*}
$$

where $T_{1}$ is the cotel input torque on the sun gear.
As shown in Figure 4.9-2, this torque produces equal tangential tooth loads $\mathrm{F}_{\mathrm{T}}$ and a planet bearing load of twice this value,

$$
\begin{aligned}
& F_{T}=\frac{T_{1}}{n R_{S}}=\frac{8403}{3(2)} \\
& F_{T}=1400.51 \mathrm{bs} \\
& F_{B}=2 F_{2}=28011 \mathrm{bs}
\end{aligned}
$$

This assumes equal load sharing among the planets and no dynamic loading in the gear meshes.

Given these loads, one can determine the $\ell_{10}$ lines and effective dynamic capacities of the five components in terms of their own load cycle counts. The two basic relationships for each element are:

$$
\ln \frac{1}{S}=\left[\ln \frac{1}{.9}\right]\left(\frac{\ell}{l_{10}}\right) E
$$

where $S$ is the reliability of the component for $\ell$ load cycles and $\ell 10$ is the number of cycles at this load for which the somponeat has a reliability of 90 percent and $E$ is the Weibull shape parameter for this reliability distribution. Normally E is taken as 1.2 for roller bearings (it asy be as high as 1.5 for tapered roller bearings) and as 2.5 for gears based on tenting at NASA Lewis Research Center. The second relationship is that for basic dynamic capacity, or:

$$
\ell_{10}=\left(\frac{C}{F}\right)^{p}
$$

Where $C$ is the oasic dynamic capacity of the component, or the load at which 90 percent of the units will last for $10^{6}$ load cycles. Here $F$ is the applied load, $\ell_{10}$ is the corresponding $90 \%$ reliability life and $p$ is the load life factor. The exponent $p$ is normally taken as $10 / 3$ for roller bearings and the NASA Lewis Research Center tests for gears indicates that 4. 3 is an appropriate value for gears. The dynamic capacity equation is often modified for bearings as

$$
\ell_{10} \cdot a\left(\frac{C}{V P}\right)^{p}
$$

where a is a factor used to increase the life estimate for improved material properties due to a reduction in impurities of the roller and race materials. According to the Roller Elements Comittee of the iubrication comittes of the ASME, life improvements of from 3 t. 8 times are not uncomon. This factor

Figure 4.9-2. Planet Gear Forces
'a' may also be used to derate the life of the bearing if its apeed of uperation is extremely high (DN $>400,000$ RPM-min) or extremely low (DN < $10,000 \mathrm{RPM}-\mathrm{mm})$ or if the iubrication to the bearing is inadequate. For this application the roller speed is

$$
\begin{aligned}
& \mathrm{DN} \approx\left[\frac{75 \mathrm{mmID}+130 \mathrm{mmOD}}{2}\right] 800 \mathrm{RPM} \\
& \mathrm{DN} \approx 82,000 \mathrm{RPM} \cdot \mathrm{~mm}
\end{aligned}
$$

as the operating speed should not be a problew. Although helicopter manufacturers use improved steels and lubrication in their cransmisaions and often use life improvement factors in the order of 8 in their calculations, we chose, in this example, to use a factor of $1 / 1.5$ or 0.67 to indicate unsureness of the dynamic loading on the bearing (this is equivalent to a load factor of 1.13). At best, one can say that this factor is conservative, the 1972 AFBMA standard recommends a faccor of 3 for a reasonable application, but disclaimers are also present. In addition to the life adjustaent factor, there is also a load adjustment factor, $V$. The value of 1.2 is used since the counterformal contact on the inner race produces higher strases than the conformed contact on the oucer race does. The choice of not using this factor way be justified in the wide range of the life iaprovenent factor. fiowever tor two identical bearings for which one has the load cycling on the inner race and the second has the same load cycling on the outer race, the first bearing will fail first due to its higher etress state.

With all this under advisement, the component $\ell_{\text {Blo }}$ life in bearing load cycles of a single planet bearing is:

$$
\begin{aligned}
& \ell_{B 10}=a\left(\frac{C}{V F}\right)^{P} \\
& \ell_{B 10}=\frac{1}{1.5}\left[\frac{18,200}{1.2(2801)}\right]^{3.33} \\
& \ell_{B 10}=184.8 \times 10^{6} \text { cycles }
\end{aligned}
$$

In a similar fashion, the dynamic capacity of a gear tooth is related to 1ts life as:

$$
\ell_{10}=\left(\frac{C_{T}}{F}\right)^{P}
$$

where $C_{T}$ is the dynamic capacity of a tooth and $F$ is the tangential pitch point load on that tooth. Prom tests on a particular gear material (AISI 9310

Vacuum Arc Remelt Steel) at over $9,000 \mathrm{ft} / \mathrm{min}$ pitch line velocity, the dynamic capacity, $C_{T}$, is given by:

$$
C_{T}=B_{1} \frac{F \sin }{\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)}
$$

where $B_{1}$ is the material constant determined by test to be $21,000 \mathrm{psi}, F$ is the face width of the gear, is the pitch line pressure angle and $R_{1}$ and $R_{2}$ are the pitch radii of the two gears.

For the sun-planet mesh,

$$
\begin{aligned}
& C_{S}=\frac{(21,000)(.725) \sin \left(20^{\circ}\right)}{\frac{1}{2.0}+\frac{1}{3.0}} \\
& C_{S}=6,250 \mathrm{lbs}
\end{aligned}
$$

For the ring-planet mesh,

$$
\begin{aligned}
& C_{R}=\frac{(21,000)(.725) \sin \left(20^{\circ}\right)}{\frac{1}{3}-\frac{1}{8.0}} \\
& C_{R}=25,000 \mathrm{lbs} .
\end{aligned}
$$

Unfortunately, the wide range of data available for bearing lives is not matched for gears. Since it is on the basis of this data that the life adjustment factors are established, corresponding factors do not exist in gearing. More statistical gear life data is really needed for gearing. The direct application of the NASA Lewis Research Center gear test data outside the load cycle range of the cests ( $1.2-3 \times 10^{7}$ cycles) appears to be conservative.

For a sun gear rooth and one side of a planet gear tooth

$$
\ell_{S 10}=\left(\frac{6250}{1400.5}\right)^{4.3} 10^{6}=620 \times 10^{6} \text { load cyclas. }
$$

For a ring gear tooth and the other side of the planet gear tooth

$$
\ell_{R 10}=\left(\frac{25000}{1400.5}\right)^{4.3} 10^{6}=241 \times 10^{9} \text { load cycles. }
$$

The next step is to reflect these lines and capacities to a common counting basis, input sun torque and rotation.

For a single bearing

$$
\frac{L_{B 10}}{\theta_{S}}=\frac{\ell_{B 10}}{\theta_{P / A}}
$$

$$
\begin{aligned}
& L_{B 10}=\frac{\theta_{S}}{\theta_{P / A}} \ell_{B 10}=\frac{1}{.533}\left(184.8 \times 10^{6}\right) \\
& L_{B 10}=347 \times 10^{6} \text { Revolutions } \\
& L_{B 10}=347 \times 10^{6}\left[\frac{1}{1500 \times 60}\right]=3856 \text { hours }
\end{aligned}
$$

and its dynamic capacigy as an input torque is the corque for which this bearing life equals $10^{6}$ sun rotations

$$
D_{B}=\left[\frac{R_{P}\left(R_{S}+R_{R}\right)}{R_{S} R_{R}}\right]^{1 / P}\left(\frac{n R_{S} C_{B}}{2}\right)
$$

where

$$
C_{B}=\left(\frac{C}{V}\right) a^{1 / p}
$$

is the modified bearing dynamic capacity

$$
C_{B}=\left(\frac{18200}{1.2}\right)\left(\frac{1}{1.5}\right)^{1 / 3.33}-13,4301 \mathrm{bs}
$$

and

$$
\begin{aligned}
& D_{B}=\left[\frac{3(10)}{2(8)}\right]^{1 / 3.33}\left(\frac{3(2)(13430)}{2}\right) \\
& D_{B}=48,6011 b-1 n .
\end{aligned}
$$

For the sun gear we mast combine che lives of the individual teeth. This is done by the product of probabilities of independent events:

$$
S_{S}=S_{T}{ }^{N_{S}} \text {, where } S_{S} \text { and } S_{T} \text { are the reliabilitics of the sun }
$$

gear and a single sun gear tooth, respectively.
Thus, $\quad \ln \left(\frac{1}{S_{S}}\right)=N_{S} \ln \left(\frac{1}{S_{T}}\right)$
and

$$
\ln \left(\frac{1}{S_{S}}\right)=N_{S}\left[\ln \left(\frac{1}{.9}\right)\right]\left(\frac{\ell_{S}}{l_{S} 10}\right)^{e_{G}}
$$

where $\ell_{S}$ is the life of the individual tooth in terms of its own load cycles, and $e_{G}$ is the Weibull shape parameter. In terms of rotations, counting contacts with each of $n$ planets per revolutions, this becomes:

$$
\ell_{S}=\frac{n \theta_{S / A}}{\theta_{S}} L_{S}
$$

Or

$$
\ell_{S}=3(0.8) \mathrm{L}_{S}=2.4 \mathrm{~L}_{S}
$$

so each tooth receives 2.4 load cycles for each sun rotation. In terms of L 10 lives

$$
\ln \frac{1}{S_{S}}=\left[\ln \frac{1}{.9}\right]\left(\frac{L_{S}}{L_{S 10}}{ }^{e_{G}}=N_{S}\left[\ln \frac{1}{.9}\right]{ }_{\left(\frac{2.4 L_{S}}{\ell_{S 10}}\right.}{ }^{e_{G}}\right.
$$

or

$$
\begin{aligned}
& L_{S 10}=\left(\frac{1}{N_{S}}\right)^{1 / e_{G}} \frac{\ell_{S 10}}{2.4}=\left(\frac{1}{24}\right)^{1 / 2.5} \frac{620 \times 10^{6}}{2.4} \\
& L_{S 10}=72.5 \times 10^{6} \text { Sun Rotations }
\end{aligned}
$$

or

$$
L_{S 10}=72.5 \times 10^{6}\left[\frac{1}{1500 \times 60}\right] \text { hours }
$$

$L_{S 10}=806$ hours.

Its dynamic capacity can be found from the tooth capacity in a similar fashion:

$$
\begin{aligned}
& D_{S}=\left(\frac{1}{N_{S}}\right)^{1 / e_{G} P_{G}}\left[\frac{R_{R}+R_{S}}{n R_{R}}\right]^{1 / P_{G}}\left(\pi R_{S} C_{S}\right) \\
& D_{S}=\left(\frac{1}{24}\right)^{1 / 2.5(4.3)}\left[\frac{10}{3(8)}\right]^{1 / 4.3}(3 \cdot 2 \cdot 6250) \\
& D_{S}=22,7621 b-1 \mathrm{n} .
\end{aligned}
$$

For the ring gear, the calculations are similar to those for the sun gear. The reliability of the gear in terms of its teeth reliabilities are:

$$
S_{R}=S_{T}{ }^{N_{R}}
$$

so

$$
\ln \left(\frac{1}{S_{R}}\right)=N_{R} \ln \left(\frac{1}{S_{T}}\right)
$$

and

$$
\ln \left(\frac{1}{S_{R}}\right)=N_{S}\left[\ln \left(\frac{1}{.9}\right)\right]{\frac{\ell}{l_{R 10}}}_{\left(e_{G}\right.}
$$

where $\ell_{R}$ is the iffe of the individual tooth in terme of ite own load cycles. In tems of sun rotations, counting contacts with each of planets per revolution, this becomes:

$$
\ell_{R}=\square \frac{\theta_{R / A}}{\theta_{S}} L_{R}
$$

or

$$
\ell_{R}=3(0.2) L_{R}=0.6 L_{R}
$$

so each tooth receives 0.6 load cycles for each sun rotation. In terme of $\mathrm{L}_{10}$ lives:

$$
\operatorname{In}\left(\frac{1}{S_{R}}\right)=\left[\ln \frac{1}{.9}\right]\left(\frac{L_{R}}{L_{R 10}}\right)^{e_{G}}-N_{R} \ln \left(\frac{1}{.9}\right)\left(\frac{.6 L_{R}}{L_{R 10}}\right)^{C_{G}}
$$

or

$$
\begin{aligned}
& L_{R 10}=\left(\frac{1}{N_{R}}\right)^{1 / e_{G}} \frac{\ell_{R 10}}{0.6}=\left(\frac{1}{96}\right)^{1 / 2.5}\left(\frac{241 \times 10^{9}}{.6}\right) \\
& \mathrm{L}_{R 10}=64,710 \times 10^{6} \text { Sun Rotations }
\end{aligned}
$$

or

$$
L_{R 10}=64,710 \times 10^{6}\left[\frac{1}{1500 \times 60}\right] \text { hours }=719,000 \text { hours. }
$$

Its dynamic capacity can be found from the tooth capacity in a aimilar fashion.

$$
\begin{aligned}
& D_{R}=\left(\frac{1}{N_{R}}\right)^{1 / e_{G} P_{G}\left[\frac{R_{R}+R_{S}}{n R_{S}}\right]^{1 / P_{G}}\left(n R_{S} C_{R}\right)} \\
& D_{R}=\left(\frac{1}{96}\right)^{1 / 2.5(4.3)}\left[\frac{10}{3(2)}\right]^{1 / 4.3}(3(2)(25,000)) \\
& D_{R}=110,5001 b-1 n .
\end{aligned}
$$

For the planet gears, the fact that each tooth is loaded on one side by the sun gear and on the other by the ring gear changes the calculation slightly.

The numbers of load cycles that each tooth sees from either sun gear or planet gear is the number of relative rotations of the planet with respect to the arm. In terms of sun gear rotations, this is

$$
\begin{aligned}
& \frac{\ell_{P}}{\theta_{P / A}}=\frac{L_{P}}{\theta_{S}} \\
& \ell_{P}=\frac{\theta_{P / A}}{\theta_{S}} L_{P}=0.533 L_{P}
\end{aligned}
$$

The reliability of a planet gear is the product of the reliabilities of its individusl cooth faces:

$$
S_{P}=S_{P S}{ }^{N p} \cdot S_{P R}{ }^{N p}
$$

where $S_{P S}$ is the reliability of a planet tooth face meshing with the sun and $S_{P R}$ is the rellability of a planet tooth face meshing with the ring. Thus:

$$
\ln \left(\frac{1}{S_{P}}\right)=N_{P} \ln \left(\frac{1}{S_{P S}}\right)+N_{P} \ln \left(\frac{1}{S_{P R}}\right)
$$

05

$$
\left(\frac{L_{P}}{L_{P 10}}\right)^{e_{G}}-N_{p}\left(\frac{.533 L_{P}}{L_{S 10}}\right)^{e_{G}}+N_{p}\left(\frac{.533 L_{P}}{Q_{R 10}}\right)^{G}
$$

The single tooth face lives $\ell_{S 10}$ and $\ell_{R 10}$ are the same as those of the mating teeth on the sun and ring gears. So:

$$
\begin{aligned}
& L_{P 10}=\left(\frac{1}{N_{p}}\right)^{1 / e_{G}}\left(\frac{\ell_{S 10^{\ell} R 10}}{.533}\right)\left[\frac{1}{\ell_{R 10} e_{G}+\ell_{S 10} e_{G}}\right]^{1 / e_{G}} \\
& L_{P 10}=\left(\frac{1}{36}\right)^{1 / 2.5}\left[\frac{\left(620 \times 10^{6}\right)\left(241 \times 10^{9}\right)}{.533}\right] \times \\
& {\left[\frac{1}{\left(620 \times 10^{6}\right)^{2.5}+\left(241 \times 10^{9}\right)^{2.5}}\right]^{1 / 2.5}} \\
& L_{P 10}=277 \times 10^{6} \text { Sun Revolutions } \\
& L_{P 10}=277 \times 10^{6}\left[\frac{1}{1500(60)}\right]=3078 \text { hours }
\end{aligned}
$$

The basic dynamic capacity for a single planet gear is the input torque for which its life equals $10^{\circ}$ sun rotations:

$$
\begin{aligned}
D_{P} & \left.=\left(\frac{1}{N_{P}}\right)^{1 / e_{G} P_{G}}\left[\frac{R_{P}\left(R_{S}+R_{R}\right)}{R_{S} R_{R}}\right]^{1 / P_{G}} \frac{n R_{S} C_{S} C_{R}}{\left(C_{S}{ }_{G} P_{G}+C_{R}{ }^{e}{ }_{G} P_{G}{ }^{1 / e_{G} P_{G}}\right.}\right) \\
D_{P} & =\left[\frac{1}{36}\right]^{\frac{1}{2.5(4.3)}}\left[\frac{3(10)^{\frac{1}{2}}}{2(8)}\right] \\
X & \frac{3(2)}{\left.(6250)^{2.5(4.3)}+(25,000)^{2.5(4.3)}\right)^{1 /(2.5) 4.3}} \\
D_{P} & =31,1001 \text { b-in. }
\end{aligned}
$$

At this point all the components are rated for $90 \%$ reliability life and basic dynamic capacity in terms of un rotations.

| Component | Life |  | Dynamic Capacity 1b-in |
| :---: | :---: | :---: | :---: |
|  | $10^{6}$ Sun Rotations | hrs |  |
| Plant Bearing | 347 | 3,856 | 48,661 |
| Sun | 72.5 | 806 | 22,762 |
| Ring | 64,710 | 719,000 | 110,500 |
| Planet Gear | 277 | 3,078 | 31,100 |

The combination of these lives and capacities involves the product of the probabilities of survival of all the components

$$
S_{T}=S_{B}{ }^{n} S_{S} S_{P}{ }^{n_{1}} S_{R}
$$

or

$$
\begin{aligned}
\ln \left(\frac{1}{S_{T}}\right) & =\ln \left(\frac{1}{9}\right)\left\{_{n\left(\frac{L_{T}}{L_{B 10}}\right)^{e_{B}}+\left(\frac{L_{T}}{L_{S 10}}\right)^{e_{G}}}\right. \\
& \left.+n\left(\frac{L_{T}}{L_{P 10}}\right)^{e_{G}}+\left(\frac{L_{T}}{L_{R 10}}\right)^{e_{G}}\right\}
\end{aligned}
$$

Since $e_{G}$ does not equal $e_{B}$, this relation cannot be directly set to:

$$
\ln \left(\frac{1}{S_{T}}\right)=\ln \left(\frac{1}{9}\right)\left(\frac{L_{T}}{L_{T 10}}\right)^{e_{T}}
$$

However, for values of $S_{T} f$ rom 0.5 to 0.95 , a least squares fit can be made on Weibull paper to find the values of $\theta_{T}$ and $L_{T 10}$ which best characterize the system.

For the data of this example:
$\mathrm{L}_{\mathrm{T} 10}=58.1 \times 10^{6}$ Sun Revolutions
$\mathrm{L}_{\mathrm{T} 10^{0}} 58.1 \times 10^{6}\left[\frac{1}{1500(60)}\right]=646$ hour s
and the Weibull slope for the system is

$$
e_{T}=2.12
$$

At similar situation exists for the system's dynamic capacity. To find:

$$
C_{T 10}=\left(\frac{U_{T}}{T_{1}}{ }^{P_{T}}\right.
$$

one can take the equation for $L_{T} 10$ and vary the input torque over a range of $0.1 D_{T}$ to $D_{T}$ and find the corresponding $L_{T i O}$ lives. A least square fit of this $\mathrm{L}_{\mathrm{T} 10} \mathrm{vs}$. $\mathrm{T}_{1}$ data on $\log -10 \mathrm{~g}$ paper will produce a linear curve for which

$$
\mathrm{D}_{\mathrm{T}}=22,6051 \mathrm{~b}-1 \mathrm{n}
$$

and

$$
\mathbf{e}_{T}=4.03
$$

So system Weibull and load life model for this example is

$$
\ln \left(\frac{1}{S_{T}}\right)=\ln \left(\frac{1}{.9}\right)\left[\frac{L_{T}}{58.1}\right]^{2.12}
$$

and

$$
L_{T 10}=\left(\frac{22,605}{T_{i}}\right)^{4.03}
$$

## 4. 10 Reliability Prediction Mechod - Minimum Information

The methods of this section are to be employed when there is not sufficient statistical data nor sufficient structural/analytical information concerning the nonelectronic part to allow the use of the other methods of this notebook to obtain reliability predictions.

The circumstances which lead to the necessity of using this section are partly due to rapid advancement of rechnology. New parts are constantly being introduced with hardly enough lifetime history to allow the vendor to set a warranty or service life (also called useful life). This service life is usually available from the vendor and can be used in conjunction with data from similar devices to provide reliability predictions.

The following examples illustrate the use of this type of information in determining Weibull parameters and failure rates. In the Weibull case, two quantities (usually the shape parameter or "slope" as it is often called, and the vendor supplied service life) are used. In the constant failure rate case (exponential distribution), the service life is sufficient.

### 4.10.1 Examples

Example 1. The expected service life of a hydraulic motor has been calculated by the vendor to be 12,413 hours where the service life is defined as the minimum life expected without failure of the motor section exclusive of the bearing section. The bearing section has been calculated to have an $\mathrm{L}_{10}$ life of 50,000 hours in this application, i.e., $90 \%$ probability of surviving 50,000 hours. The preceding constitutes the only information available to be reliability analyst.

In the following calculation, the motor section service life estimate will be conservatively assumed to be the tenth pecentile of failure, $L_{10}$, of a Weibull distribution. The slope, $B_{M}$, is assumed to be 2 , which is consistent with the scatter in lives to be expected when fatigue is the dominant failure mechanism. The calculated bearing section $\mathrm{L}_{10}$ is reduced to 25,000 hours to account for less chan ideal lubrication. The distribution is assumed to be Weibull with a slope, ${ }^{\beta_{B}}$, of 1.5 which is consistent with practice.

The characteristic lives of the motor section, $a_{M}$ can now be computed. Since the Lio life of the motor is 12,413 , it must be that

$$
.90=\exp \left\{-\left(L_{10} \alpha_{M}\right)^{B_{M}}\right\}
$$

or

$$
.90=\exp \left\{-\left(12413 / \alpha_{M}\right)^{2}\right\}
$$

so that

$$
\alpha_{M}=(12,413) /(-\ln (.90))^{1 / 2}=38,241 \text { hours }
$$

Similarly, $\alpha_{B}=(25,000) /\left(-\ln (.90)^{1 / 1.5}\right)=112,070$ hours
Finally, an average failure rate for a use life, $t$, is calculated using an average Weibull competing risk cumulative hazard model:

$$
\bar{\lambda}_{t}=\left(\frac{t^{G_{m}^{-1}}}{\alpha_{m}^{B_{m}}}+\frac{t_{B}^{B_{B}^{-1}}}{\alpha_{B} B_{B}}\right)
$$

where:

$$
\begin{array}{ll}
t=10,000 \text { hours } & B_{m}=2 \\
\alpha_{M}=38,241 & B_{B}=1.5 . \\
\alpha_{B}=112,070 & =\left[\frac{10,000}{(38241)^{2}}+\frac{(10,000)^{.5}}{(112070)^{1} .5}\right]=9.5 \times 10^{-6} \\
\lambda_{10,000}=
\end{array}
$$

or 9.5 fallures per million hours.
Should this failure rate be unacceptably high, a lower fallure rate can be obtained by reducing the in-use life or by redesign of the motor section to obtain a greater $\mathrm{L}_{10}$ life rating.

Example 2. Ball Screw Reliability Estimation
A ball screw is to be applied in an environment which includes vibration and salt-laden air. Vendor catalog information provides an $L_{10} 11 f e$ of 20 x $10^{6}$ inches under the conditions of loading and lubrifation of this application. Discussions with the vendor's engineering staff suggest that the given $L_{10}$ is realistic provided a life correction factor of 0.5 is used to account for the special environmental conaitions. The vendor also states that test data indicates that the lives will follow the Weibull distribution with a $\beta$ of 3 when grease lubricated as in this application. In operation there will be 60 inches of travel per cycle and 13 cycles per hour.

Estimate the reliability for a service life of 10,000 hours.

$$
\text { Life } L_{10}=20 \times 10^{6} \times 0.5 \text { inches }=10^{7} \text { inches. }
$$

cyc/hr $=13$
in/cyc $=60$
Weibull Distribution $=1-e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \quad B=3$
Service Life ( $t$ ) $=10,000$ hours.
Convert $L_{10}$ inches to $L_{10}$ hours.

$$
L_{10} \mathrm{hrs}=\frac{10^{7}}{(13)(60)}=12,821 \text { hours. }
$$

Since $B=3$, and since the $L_{10}$ iffe satisfies. $90=$ exp
$\left\{-\left(L_{10} / \alpha\right)^{\beta}\right\}$, it follows that $\alpha=(12821) /(-\ln (.9))^{1 / 3}=27,145$
hours. Thus, the reliability for $t=10,000$ hours is $R(10,000)=$ $\exp \left\{-(10000 / 27145)^{3}\right\}=0.951$.

If an average fallure rate over the 10,000 hours service life is required, it can be calculated as follows:

$$
\begin{aligned}
& \bar{\lambda}(t)=\frac{t^{\beta-1}}{\alpha^{\beta}} \\
& \bar{\lambda}(10,000)=\frac{(10,000)^{2}}{(27145)^{3}}=5.0 \times 10^{-6} \\
& \quad \text { or } S \text { failures per million hours. }
\end{aligned}
$$

Example 3. Estimation of a Constant Failure Rate Based on Service Life,
As demonstrated in this Notebook, most of the time it can be assumed that the nonelectronic parts represented in this Notebook have constant fallure rates. In this case, if the only information available is a vendor supplied "warranty" or "service" life, then a failure rate is easily estimated.

Occasionally, the vendor will indicate at which percentile the varranty is developed. That is, if a warranty or service life of 1000 hours is specified, the vendor may have done life testing which indicates that 1000 hours 1 s the life beyond which $90 \%$ of the devices will survive on the
average. This would make 1000 hours the loth percentile of the life jistribution. To compute the constant failure rate, simply set $0.90=\exp (-\lambda$ 1000) and solve for $\lambda$. In this case, $\lambda=-\ln (0.90) / 1000=0.000105$ or 105 failures per million hours. In general, use the following formula:
failure rate $=-\ln (1-p) /($ warranty time $)$
where $p$ is the quantile associated with the warranty time. Usually, $p=0.10$ but occasionally, if the vendor information is suspect, or the vendor will not say what value $p$ should be, use $p=0.05$.

### 5.0 PART FAILURE CHARACTERISTICS

5.1 Introduction. The following sections of the notebook describe the analyses of the nonelectronic part fallure data collected for this study. Section 5.2 presents the results of fitting the exponential distribation to the failure data for each part class, part type, and enviromment. Sections 5.3-5.5 present the results of testing the ift of the exponential distribution against Weibull alternatives for the data corresponding to some items. section 5.6 gives part malfunction data and frequency of occurrence for some parts. It should be pointed out that the data used for preparation of this notebook was sereened to exclude secondary failures and failures caused by maintenance personnel.

The environment abbreviations in the tables of this section follow the conventions of MIL-RDBR-217D. FOr convenience, these abbreviations are defined in table 5.1.1. More detailed descriptions may be found in MIL-EDBR-217D.

Finally, an explanation of the confidence intervals presented in sections 5.2 through 5.5 is necessary. These confidence intervals (for failure rate in the exponential cases, and for the shape and scale parameters in the Weibull analyses) have been called " 60 confjdence intervals." In the tables, however, the lower and upper bounds are labeled "BOs lower" and " 80 upper" bounds. This has been done so that either one-sided bound can be used by itaelf to form a one-sided 80 s confidence interval if desired. When the two 808 bounds are combined to form a tw-osided interval, the resultant confidence $1860 \%$.

TABLE 5.1.1 DEFINITIONS OF ENVIRONMENT ABBREVIATIONS

part class, part type, and environment of interest in the tables listed in section 5.2. The corresponding table will give a point estimate (also referred as a "prediction" in many reliability circles) of failure rate per million hours. A two-sided $60 \%$ confidence bound on the failure rate is also given in order to give the user a feel for the precision of the estimate. Also included is the number of independent sources (usually projects; which contributed to the estimates, along with total number of failures, total part operating hours and an estimate of mean life (i.e., mean operating time to failure). In a large number of cases, less than 50,000 hours of operating time were available so that care should be taken to examine the width of these confidence intervals. In cases where the total part operating hours shown are less than 1,000 hours, failure rate information is not tabulated. For these cases, the user should be very cautious in using the information presented for reliability purposes. Wherever there is more than one source contributing to an estimate, the observed significance level of a statistical test of homogeneity (also called a p-value, see Cox \& Hinkley (1974), p. 66, for further discussion) is given. This test of homogeneity was performed in order to determine if the sources reporting failures were statistically different. In general, the observed significance level is between 0 and 1 with values close to 0 indicating evidence to reject homogeneity. We recommend a threshold of 0.05 for the homogeneity test, i.e., homogeneity is rejected if the observed significance level is below 0.05 .
5.1.2 Use of Weibull Analyses. Most data collected was restricted to total operating time and total number of failures. While this approach is adequate (sufficient, in fact, in the statistical sense) when dealing with the exponential distribution, it does not provide a means of evaluating the "fit" or validity of the exponential model. The validity of the exponential distribution for describing the life distribution for nonelectronic parts was one of the central issues addressed by this study.

In order to address this important issue, data sets that contained actual part lifetimes for each failed part and total operating time were collected. In most cases, part lifetime data simply did not exist. However, for a significant number of part classes, part types, and environments, these data were available and were used to test the fit of the exponential distribution against Weibull alternatives. The Weibull family of distributions is rich enough to approximate virtually any unimodal life distribution and is therefore applicable for most nonelectronic parts. Moreover, the Weibull distribution is the resultant extreme-type distribution for describing lifetimes of nonelectronic parts which fail in accordance with a "weakest link" scenario. This technique for testing the fit of a distribution by embedding it in a parametric family of distributions is called a "smooth goodness-of-fit test," and is described in further detail in Lawless (1982), p. 438. Since the lifetime data was not collected under any of the commonly treated sampling plans, i.e., nonreplacement type I or II censoring, or complete samples, the goodness-of-fit procedure had to be developed ad hoc, and the smooth goodness-of-fit approach allowed the use of standard
likelihood ratio procedures in performing the test. The resulte of these analyses are presented in sections 5.3-5.5.

If the same part class, part type, and environment is reported in sections 5.3-5.5 a Weibull analysis was performed (this is indicated in the index to Section 5.2). Use the information in the Weibull table to decide whether to adopt the Weibull distribution, or retain the exponential distribution. For each of the part classes, part types and environments analyzed in sections 5.3-5.5, a table summarizing the results is presented. Each table contains 4 point estimate of mean life (i.e. mean time to failure in hours), and point and $60 \%$ confidence interval estimates for the Weibull scale parameter (in hours) and shape parameter (unitless). Also included are total part operating hours, and total failures. In cases where there was a predominant failure mode, this failure mode is given in the comments field, along with the observed significance level for the test of exponentiality. The observed significance level for testing exponentiality is used to decide whether to adopt the Weibull model, or to retain the exponential model. As before, we recommend that the threshold value be 0.05 , i.e. reject exponentiality if the observed significance level is less than 0.05 , and retain exponentiality otherwise. However, depending on the particular circumstances, the analyst using the observed significance level may wish to base the decision on a different threshold value, e.g. 0.10 , $0.005,0.001$, etc. In the majority of cases, the exponential model is shown to be the best fit based on the 0.05 level of significance.

If the Weibull model is shown to be the better fitting model, it may still be desirable to approximate the distribution by the exponential distribution. This approximation is useful if the nonelectronic part(s) under consideration are part of a large system in which the other elements exhibit exponentially distributed lifetimes, and it is necessary to analyze the system as a whole. For exponential approximations it is recommended that the point estimate of the mean for the Weibull be used in the exponential model whenever the Weibull shape parameter is greater than one. When the Weibull shape parameter is less than one, use the mean life estimate from the appropriate table in section 5.2. These guidelines will yield conservative results (i.e. lower bounds) when computing system reliability for the series string in the case where the Weibull shape parameter is greater than one.

A total of 145 part classes/ types were analyzed in sections 5.35.5. In 6 of those cases exponentiality would be rejected at the 0.05 level of significance. This is not statistically significant. These results suggest that the exponential distribution is an adequate life model for most nonelectronic parts that are operated for time periods smaller than those analyzed in sections 5.3-5.5, i.e., there is a time period for most nonelectronic parts during which a constant failure rate model is appropriate. This phenomenon, conjectured in previous editions of this notebook, is supported by the results of sections 5.3-5.5.

### 5.2 Constant Failure Rate Analysis <br> 5.2.1 Index to Section 5.2

| Part Class | Part Type Id | dentification Number* |
| :---: | :---: | :---: |
| ACCELEROMETER | forced balanced | 1* |
| ACCELEROMETER | PENDULUM, LINEAR | 2 |
| ACCELEROMETER | PENDULUM, SINGLE AXIS | 3 |
| ACCIMULATOR | HYDRAULIC | 4 |
| ACCUMULATOR | hYdraulic-PNEUMATIC | 5 |
| ACTUATOR | ELECTRICAL | 6 |
| ACTUATOR | ELECTROMECHANICAL (LINEAR) | 7* |
| actuator | ELECTROMECHANICAL (ROTARY) | 8 |
| ACTUATOR | electromechanical (linear) | 9* |
| ACTUATOR | HYDRAULIC-PNEUMATIC | 10 |
| ACTUATOR | mechanical | 11 |
| actuator | ROTARY | 12 |
| AIR CONDITIONER | COMPORT | 13 |
| AIR CONDITIONER | GENERAL | 14 |
| AIR CONDITIONER | PROCESS | 15 |
| antenna | communication | 16* |
| antenna | MICROWAVE (COMMUNICATION) | 17 |
| ANTE NNA | RADAR | 18 |
| AXLE | general | 19* |
| AZIMUTH ENCODER | OPTICAL | 20* |
| battery | RECHARGEABLE | 21* |
| BEAR ING | BALL | 22* |
| BEARING | ROLLER | 23* |
| BEARING | SLEEVE | 24* |
| bearing nut | general | 25 |
| BELLOWS | general | 26* |
| BELT | geared | 27 |
| BELT | TIMING | 28* |
| BELT | v-belt | 29* |
| binocular | NITROGEN PRESSURIZED | 30 |
| blade assembly | general | 31 |
| BLOWERS \& PANS | AXIAL | 32* |
| BLOWERS \& FANS | CENTRIFUGAL | 33* |
| BOOT (DUST \& MOISTURE) | general | 34 |
| brake | ELECTRUMECHANICAL | 35* |
| BRUSHES | ELECTRIC MOTOR | 36* |
| BURNER | Catalytic | 37 |
| BIJSHINGS | GENERAL | 38* |
| CAM | GENERAL | 39 |
| camera | MOTION (TV) | 40* |
| CeSilm beam tube | GENERAL | 41 |
| CIRCUIT PROTECTION |  |  |
| device | SPARK GAP | 42* |
| circuit protection |  |  |

*Note: An asterisk "*" indicates that a Weibull gnalysis is included in one or more environments in Sections 5.3-5.5.


| Patt Class | Part Type I | Identification Number* |
| :---: | :---: | :---: |
| FILTER | OPT ICAL | 84 |
| FITTINGS | GENERAL | 85 |
| FITTINGS | PERMANENT | 86* |
| FITTINGS | QUICK DISCONNECT | 87* |
| FITTINGS | THREADED | 88* |
| FLASH LAMP | GENERAL | 89 |
| FUSE HOLDER | BLOCR | 90* |
| FUSE HOLDER | EXTRACTOR POST | 91 |
| FUSE HOLDER | PLIIG | 92* |
| GAS DRYER DESICATOR | MOLECULAR SIEVE | 93 |
| GASKETS \& SEALS | DYNAMIC | 94* |
| GASKETS \& SEALS | STATIC | 95* |
| GEAR | ANTIROTATION | 96 |
| GEAR | BEVEL | 97* |
| GEAR | HELICAI, | 98* |
| GEAR | HYPOID | 99 |
| GEAR | SPUR | 100 |
| GEAR | WORM | 101 |
| GEAR BOX | MULTIPLIER | 102 |
| GEAR BOX | REDUCTION | 103 |
| GEAR TRAIN | BEVEL | 104 |
| GENERATOR | AC | 105 |
| GENERATOR | GENERAL (OXYGEN GENERATOR) | 106 |
| GLASS (SIGHT GAUGE) | general | 107 |
| GROMMET | GENERAI, | 108 |
| GIMBALS | general | 109 |
| GIMBALS | TORQUE | 110 |
| GYROSCOPE | SINGLE AXIS | 111 |
| GYROSCOPE | TWO AXIS ROTOR | 112 |
| HEAT EXCHANGERS | COPLATES | 113* |
| HEAT EXCHANGERS | GENERAL | 114 |
| HEAT EXCHANGERS | RADIA TOR | 115* |
| HEATER | WATER | 116 |
| HEATER BLANKETS | GENERAL | 117 |
| HEATER, FLEX ELEMENT | HEATER TAPE | 118 |
| HIGH SPEED PRINTER | ELECTROSTATIC | 119* |
| HIGH SPEED PRINTER | IMPACT | 120* |
| HIGH SPEED PRINTER | THERMAL | 121 |
| HOSE | FLEXIBLE | 122 |
| HOSE | FLEXIBLE, PROPELLANT | 123 |
| HOSE | GENERAL | 124* |
| HOUS ING | GENERAL | 125 |
| INCINERAIOR | PROM SEWAGE TREATMENT | 126 |
| INSTRUMENTS | AMMETER | 127* |
| INS TRUMENTS | FLOW METER | 128 |
| INSTRUMENTS | HUMIDITY INDICATOR | 129 |
| *Note: An asterisk " ${ }^{\prime \prime}$ " indicates that a Weibull analysis is included in one or more environments in Sections 5.3-5.5. |  |  |











NOTE: Low total part operating hours, develop failure data with caution




























| ! ENV | MUMBER OF SOURCES | NUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| ! 6 | 4 | 4 | 752208 | SIGNIFICANCE LEVEL FOR COMBINING SOURCES= 0.24 |
| \% GM | 2 | 8 | 209859 | SIGNIFICANCE LEVEL FOR COMBINING SOURCES: 0.07 |


























| ENV | NUMBER OF SOURCES | PARTS ${ }_{\text {NUMER }}$ Of | TOTAL PART HOURS OPERAT HOUPS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| GF | 2 | 7 | 978897 | sIonifican |
| GM | 2 |  | 150188 | SIC |
















[^2]










| ENV | NUMBER OF SOURCES | nUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS ! |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | ------------------ ! |
| I AUT | $!$ | 17 | 1878881 |  |
| ! GF | $!$ | 11 | 3337948 | SIGNIFICANCE LEVEL FOR COMBINING SOURCES= 0.16 I |
| $!\mathrm{Gm}$ | $!3$ | 7 | 1507580 | SIGNIFICANCE LEVEL FOR CONBINING SOURCES= 0.06 |
| ! NS | $!4$ | $!11$ | 5722160 | ! SIGNIFICANCE LEVEL FOR COMBININO SOURCES=0.29 |
| ! NSB | 1 | ! 1 | 46000 | ! |
| $!\times N u$ | 2 | $!$ | 630240 | ! SIGNIFICANCE LEVEL FOR COMBINING SOURCESE 0.86 |

















| ENV | NUMBER OF SOURCES | MUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS | $!$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $!$ GM | 1 | 2 | 356886 |  | I |
| ! NS | 1 | 5 | 20000 |  | 1 |









[^3]






| ENV | NUMBER OF SOURCES | $\begin{aligned} & \text { HUPBER OF } \\ & \text { PARTS FAILED } \end{aligned}$ | total part OPERATING HOURS | COMAENTS | i |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AUT | 1 | 1 | 48924 |  | $!$ |







| env |  | PARTMER |  | coments |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 532916 | sionifical |
| om |  | 3 | 313376 | SICNIFICAI |
|  |  |  | 2938632 | siontrical |
| nse |  |  | 53400 | significal |



| EnV | NUMBER Of SOURCES | NUMBER OF PARTS FAILED | total part OPERATINO HOURS | COMMENTS |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AIT | 1 | 1 | 75000 |  | - |
| ! GB | 1 | 1 | 112828 |  | - |
| ! NS | 3 | 1 | 47810 | sianifican | SOURCES 0.28 |
| NSB | 1 | 0 | 2000 |  | .-- |




| ENV | NUMBER OF SOURCES | MUMBER OF | TOTAL PART OPERATING HOURS | COMAENTS |
| :---: | :---: | :---: | :---: | :---: |
| Aut | 1 | 18 | 293544 |  |
| : 6 | 2 | 2 | 961243 | SIGNIF ICANCE LEVEL FOR COMBINING SOURCES= 0.14 ! |
| ! NS | 2 | 2 | 2769659 | SIGNIF ICANCE LEVEL FOR COMBINING SOURCES: 0.54 I |
| nSB | 2 | 2 | 20400 | SIGNIF ICANCE LEVEL FOR COMBINING SOURCES: 0.35 ! |
| ! N N | 1 | 0 | 73530 |  |















| ENV | NJMBER OF SOURCES | NUMBER OF PARTS FAILED | total part operating hours | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| aut | 1 | 6 | 97848 |  |
| Gf | 1 | 1 | 90000 |  |







| ENV | NUMBER OF SOURCES | Numbes of PARTS FAILED | tOTAL PART OPERATING MOURS | comments |
| :---: | :---: | :---: | :---: | :---: |
| NS | 1 | 8 | 1157842 |  |







| ENV | NUMEER OF SOURCES | NUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| GF | $!1$ | 1 1 | 250000 | $!$ |
| GM | ! 3 | 1 | 304158 | SIGNIFICANCE LEVEL FOR COMBININO SOURCES= 0.60 |
| NU | 1 1 | 0 | 24510 | ( |




| ENV | NUMBER OF SOURCES | NUMBER OF PARTS FAILED | total part OPERATIMG MOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| AUT | 1 | 0 | 146772 |  |















| ENV | NUMBER OF SOURCES | NUMEER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| OF | 1 | 10 | 90000 |  |



NOTE: Low total part operating hours, develop failure data with caution




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| ENV | NUMBER OF SOURCES | NUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| GF | 1 | 7 | 90000 |  |

























NOTE: Low total part operating hours, develop failure data with caution




| ENV | $\begin{aligned} & \text { NUPBER OF } \\ & \text { SOURCES } \end{aligned}$ | $\begin{aligned} & \text { NUMBER OF } \\ & \text { PARTS FAILED } \end{aligned}$ | TOTAL PAR, OPERATIMG HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| GF | 3 | 8 | 1996924 | SIGNIFICANCE LEVEL FOR COMBINIMG SOURCES= 0.32 |
| CM | 1 | 1 | 3780 |  |









| ( ENV | NUMBER OF SOURCES | NUMBER OF PARTS FAILED | TOTAL PART OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| \% 6 | 2 | 32 | 780257 | SIGNIfICANCE LEVEL FOR COMBINING SOURCES= 0.70 |
| GM | 3 | 5 | 153988 | SIGMIfICANCE LEVEL FOR COMBINING SOURCES 0 0.88! |
| $!$ NS | 2 | 21 | 2690101 | SIGNIFICANCE LEVEL FOR COMBINING SOURCES= 0.17 |


| $/$ General |  | ToEntification mumer 238 |
| :---: | :---: | :---: |
|  |  |  |
| arn : Exponential sat | 1044302 | 1101.965 ( 1163.125 |
| (env |  |  |
|  |  |  |
| Wion assemelr 1 oeneral |  |  |
|  |  |  |
|  |  |  |
|  | coit |  |
|  | -2749 |  |







| env | MUMBER OF SOURCES |  | total part HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
| ML | 1 | 0 | 42 |  |


NOTE: Low total part cperating hours, develop failure data with caution

NOTE: Low total part operating hours, develop failure dats with caution






| WATER | DEMINERALIZER | / MIX-RESIN |  |  | IDENTIFICATION |  |  | NUMBER | 255 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ! |  | $!$ |  | RATE |  |  |  |  | ! |
|  | ! ! | MEAN | ! | $80 \times$ | n! | failure |  |  |  | $!$ |
| ENV | DIST. TYPE ! | ESTIMATE | 1 | LOWER | ! | RATE | , |  |  | , |
|  | OIST. Trat | (HOURS) | ! | BOUND | $!$ | ESTIMATE | ! |  |  | ! |
|  | 1 | (Hours) | 1 |  | 1 |  | 1 |  |  | 1 |
|  | ! EXPONEMTIAL! |  |  |  | ! |  | $!$ |  |  | ! |
| NS | EXPONENTIAL! | 13100 | $!$ | 17.035 | ! | 76.338 | 1 | 228 |  | 1 |


| ENV | NUMBER OF SOURCES | $\begin{aligned} & \text { NUMBER OF } \\ & \text { PARTS FAILED } \end{aligned}$ | TOTAL PART OPERATING HOURS | COMMENTS | $!$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| NS | 1 | 1 | 13100 |  | $!$ |


5.3 Weibull Analyses -- Project 1. This section contains the results of fitting the Weibull distribution to nonelectronic part lifetime data collected on a ground mobile mortar locating radar. For each part type and slass represented, a table is presented which gives the part identification number, a point estimate of mean life in hours, a point and $60 \%$ confidence interval estimate of the Weibull scale parameter in hours, and a point and $60 \%$ confidence interval estimate of the Weibull ahape parameter. The form of the Weibull survival function considered here is given by:

$$
\operatorname{Pr}(\text { survive } t)=\exp \left[-(t / b)^{c}\right],
$$

where $b>0$ is the scale parameter, $c>0$ is the shape parameter, and $t>0$ is measured in hours. This convention is followed throughout sections 5.3-5.5.

In addition to these quantities, the total part operating hours are given, along with the total number of failures. The "comments" field contains the predominant failure mode observed (i.e. the failure mode which occurred most often in the sample) and the observed significance level for testing exponentiality. We recomend that exponentiality be rejected if the observed significance level is below 0.05 , although other thresholds may be used according to the particular application.

In some cases, less than two failures are reported. However, because there are multiple systems reporting ( 53 in this case) and one or more parts per system, the systems which have no failures were also used as "data" so that the two parameters of interest could actually be estimated. This applies to section 5.4 also.

In only one instance in project 1 is exponentiality rejected at the 0.05 significance level, namely for electrostatic high speed printers (identification number 1-2-36). The Weibull shape parameter estimate in this case was 0.748 . The corresponding Weibull distribution would, in this case, have a decreasing failure rate and its probabilicy density function would be shaped somewhat like that of the exponential distribution. One interpretation for the shape parameter being less than one is that infant failures were still taking place in the printers after they were installed. If this is true, it would be indicative of poor vendor quality control.
S.3.1 Waibull Analyoes Summaries. Following is an index of the nouelectronic parts analyzed in this section. Each part is identified by a number of the form "x-y-z". The prefix " $x$ " identifies the project from wich the data was collected (" $x$ " is 1 in this section). The number " $y$ ", being either 1,2 , or 3 indicates the sampling and censoring scheme (in the statistical sense) used in collection the data for that part. These schemes are described in the Final Technical Report of this study. The number " $z$ " is the sequence number as insted in the index that follows.

The column under "Best Fit" indicates which distribution is the better fitting distribution (assuming 0.05 significance level) with E-exponential, and $W=W e 1 b u l l$. The format for this index is used in sections 5.4 and 5.5 . also.



Indez to Protect 1 Neibull Analyees

| Part Name | Part Type | Environment | Sequence Number | $\begin{aligned} & \text { Best } \\ & \text { P1t } \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| RETAINING RING | GENERAL | GM | 46 | E |
| SHAPT | Gereral | GM | 47 | E |
| SOLENTOLDS | LINTAR | GM | 48 | B |
| SWITCH | PRESSURE (AIR FLON) | GH | 49 | E |
| SWITCH | THEPY08TATIC | GM | 50 | E |
| VALVES | Pwomatic | GM | 51 | E |








































| instrlments | / VOLTMETER |  |  |  | doentification number 1-1-39 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ENV DIST. TYPE | - ESTIMATE | 80X LOWER BOUND | $\begin{aligned} & \text { SCALE } \\ & \text { POINT } \\ & \text { ESTIMATE } \end{aligned}$ | $80 x$ BOUND BOUND | $\begin{aligned} & \text { BOX } \\ & \text { COOER } \\ & \text { BOUNE } \end{aligned}$ | $\begin{gathered} \text { SHAPE } \\ \text { ESOPNMATE } \\ \text { ESTIMAT } \end{gathered}$ | $80 x$ BOUND UPPER |
| GM Weibull | $!49694.65$ | 0.0 | 52728.06 | 112377 | 0.749 | 1.191 | 1.634 |
| ENVNUMMER OF <br> SOURCES | NUMBER OF PARTS FAILED | $\begin{aligned} & \text { TOTAL PA } \\ & \text { OPERAT } \\ & \text { OOURS } \end{aligned}$ | COMMENT |  |  |  |  |
| i GM ! $\quad 1$ | 2 | 181414 | UnKNON | ce level | testi | EXPOMENTIA | Y 00.71 |


| Joint microwave rotary $/$ general |  |  |  | Identification mumber 1-1-40 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 80X LOUER BOUND a | $\begin{gathered} \text { SCALE } \\ \text { POINT } \\ \text { ESTIMATE } \end{gathered}$ | 80x GOUED BOUNO | $\begin{aligned} & \text { 80X } \\ & \text { COER } \\ & \text { BOUNO } \end{aligned}$ | $\left\{\begin{array}{c} \text { SHAPE } \\ \text { PSOINT. } \\ \text { ESTMATE } \end{array}\right.$ | 50x UPPER BOUN |
| GM WEIBULL 53027.44 | 2131.451 | 55938.90 | 109746 | 0.808 | 1 1.168 | 1.524 |
| ENV $\underset{\substack{\text { NUMBER } \\ \text { SOURCES }}}{\text { PMUMBER }}$ OF | TOTAL PART OPERAT HOURS | COMMENT |  |  |  |  |
|  | 272121 |  | CE LEVE | test | EXPONEMTIA | -0.09 |









5.4 Weibull. Ansiyses -- Project 2. This section contains the results of fitting the Weibull distribution to nonelectronic part lifetime data collected from a ground mobile artillery locating radar. For a description of the tables in this section, refer to section 5.3 .

In only three cases is exponentiality rejected at the 0.05 level of significance. These cases are listed below.

Part Class
Type
Shape Parameter Estimate

| Accelerometer | Forced Balanced | 0.297 |
| :--- | :--- | :--- |
| Azimuth Encoder | Optical | 0.641 |
| Crank Shaft | General | 0.413 |

Note that in each of these cases, the corresponding Weibull distribution has a decreasing failure rate. This is perhaps indicative of the occurrence of infant failures which were not ploperly screened by the vendor, or, as with the Accelerometer nd Crank Shaft, a very smali sample of failures.
5.4.1 Weibull Analyses Sumaries. Following is an index of the nonelectronic parts analyzed in this section. See section 5.3 .1 for a description of the entries of this index.

Index to Project 2 Weibull Analyaen

| Part Name | Part Type Env | ronsent | Sequance Nurber | $\begin{aligned} & \text { Beet } \\ & \text { Pit } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| ACCELEROMETER | FORCED BAIANCED | GM | 1 | * |
| ACTUATOR | MigCHANICAL | GM | 2 | E |
| ANTENNA | COMPUNICATION | GM | 3 | E |
| AXIE | GETERAL | GM | 4 | $\underline{E}$ |
| AZIMUTH ENCOUER | OPTICAL | GI | 5 | W |
| BATTERY | RBCHARGEABLB | GM | 6 | 8 |
| Braring | BALI | GM | 7 | E |
| BEARING | ROLLER | GM | 8 | E |
| BEARING | SLEEVE | GM | 9 | E |
| BELLOWS | GENERAL | GM | 10 | E |
| BELT | TIMLING | GM | 11. | E |
| BELT | V-BELT | GM | 12 | E |
| BLOWERS \& PANS | AXIAL | GM | 13 | E |
| BLOWERS \& FANS | CENTRIFUGAL | GM | 14 | E |
| brares | ETLECTROMECHANICAS | GM | 15 | E |
| BRUSHES | ELECIRIC MOTOR | GM | 16 | E |
| BUSYTNGS | GENERAL | GM | 17 | E |
| CIRCUIT PROTECTION DEVICE | SPARR GAP | GM | 18 | E |
| CIRCUIT PROTECTION DEVICE | SURGE ARRESTER | GM | 19 | E |
| CLUTCH | FRICTION | GM | 20 | $\mathbf{E}$ |
| COMPUTER MASS MEYORY | MAGNETIC TAPE | GM | 21 | E |
| CRANR SHAPT | GENERAL | GM | 22 | H |
| DLAFTRAGMS BURST | generral | GM | 23 | E |
| DRIVE ETR COMPUTRR TAPES/DISCS | MAGNETIC TAPE TRANS. | GM | 24 | E |
| DRIVES | GEAR | GM | 25 | E |
| DRIVES | VARIABLE PITCH | GM | 26 | E |
| DRUM | WEAPON LOCATING UNIT | GM | 27 | E |
| DUCT | GENERAL | GM | 28 | E |
| ELECTRIC HEATEAS | RESISTANCE | GM | 29 | E |
| ELECTROMECHANIC4L TIMERS | GENERAL | GM | 30 | E |
| PILTERS | AIR | GH | 31 | E |
| PILTERS | LIQUID | GM | 32 | E |
| FITIINGS | PRRMANENT | GM | 33 | E |
| PITTINGS | QUICR DISCONNECT | GM | 34 | E |
| PITTINSS | THREADED | GM | 35 | E |
| PUSE HOLDER | BLOCR | CM | 36 | E |
| FUSE HOLDER | PLUG | GM | 37 | E |
| GASKETS \& SEALS | DINAMIC | GM | 38 | E |
| GASKETS \& SEALS | STATIC | GM | 39 | E |
| GEAR | BEVEL | GM | 40 | E |
| gear | HELPCAL | GM | 41 | E |
| GEAR | SPUR | GM | 42 | E |
| GEAR BOX | REDUCTION | GM | 43 | E |
| HPAT EXCHANGERS | RADLATOR | GM | 44 | E |
| HIGH SPEED PRINTERS | ELbCTROSTATIC | GM | 45 | E |
| HOSES | GENERAL | GM | 46 | E |

Index to Project 2 Heibull Analyees

| Part Name | Part Type E | Envirommat | Sequence Number | $\begin{aligned} & \text { Besi } \\ & \text { PIt } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Instruminnts | AnPIETER | GM | 47 | E |
| INSTRUMENTS | voltmeter | GM | 48 | E |
| JOINT, MICROWAVE ROTARY | genteral | GM | 49 | E |
| KEYBOARD | blectromichanical | GM | 50 | E |
| METAL TUBING | general | GM | 51 | E |
| MOİR, ELECTRIC | > 1 HORSE POWz, AC | C GM | 52 | E |
| MOTOR, ELECTRIC | SERVO, DC | GM | 53 | E |
| MOTOR, ELECTRIC | STEPPER | GM | 54 | E |
| POWIER CIRCUIT BREAKER | CURRENT TRIP | GM | 55 | E |
| PULLEY | GROOVRD | GM | 56 | E |
| PULLEY | V -PULLEY | GM | 57 | E |
| PUMP | HYDRAULIC | GM | 58 | E |
| PUMP | PNEUMATIC | GM | 59 | E |
| RESILIENT MOUNT | genteral | GM | 60 | E |
| RETAINING RING | genteral | GM | 61 | E |
| SHOCR ABSORBERS | COMBINATION | GM | 62 | E |
| SHOCK ABSORBERS | RESILIENT | GM | 63 | E |
| SWITCH | LIQUID PLOW | GM | 64 | E |
| SWITCH | PRESSURE (AIR PLOW) | ) GM | 65 | E |
| SWITCH | THERMOSTATIC | GM | 66 | E |
| SWITCH | WAVE GUIDE | GM | 67 | E |
| telescope | BORE SIGHT | GM | 68 | E |
| TRACK BALI | ELECTROMECHANICAL | GM | 69 | E |
| VALVES | HYDRAULIC | GM | 70 | E |

































| ENV | NUMBER OF SOURCES | NUMBER OF PARTS FAILED | total par: OPERATING HOURS | COMMENTS |
| :---: | :---: | :---: | :---: | :---: |
|  | ! | ! |  | UNKNOWN |
| GM | 1 | 5 | 1368063 | SIGNIFICANCE LEVEL FOR TESTIMG EXPONEMTIALITY $=0.773$ |













Brer




## $\stackrel{\pi}{4}$



PMMP









5.5 Weibull Analyses -a Profect 3. Thls section presents the reaults of firting the Weibull distribution to nonelectronic part lifetimes collected from an alr defense, ground fixed ayatem. Por a desciption of the tables presented in this section, refer ro section 5.3.

The part clasaes/types for which the observed aignificance level was below 0.05 (supporting the Weibull over the exponential at the 0.05 level of significance) are listed below.

| Part Clasa | Typa | Significance <br> Level | Shape Paraneter <br> Eatinate |
| :--- | :--- | :--- | :--- |
| Drive for <br> Computer Tapes | Magnecic Tape <br> Drive | 0.028 | 4.649 |
| Track Ball | Electromechanical | $<0.0005$ | 2.319 |

5.5.1 Weibull Acalyses Summariea. Pollowing is an inder of the nonelectronic parts analyzed in this section. See Section 5.3 .1 for a description of the entries of this indez.


## Index to Project 3 Woibull Analyses

| Part Nane | Part Type Eavironaent |  | Sequence Number | $\begin{aligned} & \text { Beat } \\ & \text { Fit } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| BELT | TIMING | GF | 1 | E |
| BLOTERS \& PANS | AXIAL | GF | 2 | E |
| BLOWERS 8 PANS | CENIRIPUGAI | GF | 3 | E |
| CAMERA | TV | GF | 4 | E |
| COMPUTER MASS MRMORY | PIXED GRAD DISK MEMORY | GF | 5 | E |
| COMPUTER MASS MEMORY | magnetic Tape | GF | 6 | F |
| CUMPUTER MASS MEYORY | MOVABLR HEAD DISR | GF | 7 | E |
| DRIVE FOR COMPUTER TAPES/DISCS | DISCS | GF | 8 | E |
| DRIVE FOR COMPUTER TAPES/DISCS | MAGNETIC TAPE DRIVE | GF | 9 | W |
| PILTERS | AIR | GF | 10 | E |
| HIGH SPEED PRINTERS | IMPACT | GF | 11 | E |
| KEYBOARD | BLECIROMECHANICAL | GF | 12 | E |
| LOW SPEED PRINTERS | DOT MATRIX | GF | 13 | E |
| MOTOR, ELECTRIC | SERVO, DC | GF | 14 | E |
| PULLEY | GRAR, BELT | GF | 15 | E |
| PUNP | PNEUMATIC | GF | 16 | E |
| PUMP | Vacuun | GF | 17 | E |
| EENSOR/TRANSDUCER/TRANSMITTER | Presure | GF | 18 | E |
| SENSOR/TRANSDUCER/TRANSKITTER | TEMPERATURE | GF | 19 | E |
| SWITCH | PRESSURE | GP | 20 | E |
| SWITCH | ROCRER | GF | 21 | E |
| SWITCH | THERMOSTATIC | GF | 22 | E |
| SWITCH | TKUMBWHEEL | GF | 23 | E |
| TRACK BALL | BLECTROMECHANICAL | GF | 24 | W |








| DRIVE FOR COMPUTER | Pes/oiscs |  |  |  | LoEntification number 3-1-8 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ENV DISt. TYPE | ESTIMATE | $\begin{aligned} & \text { POX } \\ & \text { LOMER } \\ & \text { BOUNO } \end{aligned}$ | SCALE POINT ESTMATE | $80 x$ UPPER BOUND | $80 x$ LOWER BOWE soun | $\begin{gathered} \text { SHAPE } \\ \text { PSTINT } \\ \text { ESTIMTE } \end{gathered}$ | 80x UPPER BOUNO |
| gr weibull | 20502.5 | 14032 | 2258 | 31135 | 1.915 | 4.111 | 6.308 |
















| SWITCH | / thumbwheel |  |  |  | identification mumber | 3-1-23 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nv |  | 80x COWER BOUND | $\begin{gathered} \text { ScALE } \\ \text { SCOLIMATE } \end{gathered}$ |  |  |  |
|  |  | 1660.29 | 46198.29 |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| track ball $/$ electromechanical |  |  |  |  | identification mumber 3-3-24 |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  | 805371 | OEFECTIV SIGNJFIC |  | STIme Exponentia | - |

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5.6 Part Malfunction Data. Table 5.6 .1 gives the malfunction data and frequency of occurrence for each part name based only on the information which was available when the part failure data was collected. The malfunction data for each part name are accumulated over all use environments and part types for the particular part name.

Not all malfunctions reported are mutually exclusive. For example, "improper adjustment" and "improperly installed" may overlap. We leave it to the reader to combine the malfunction data into categories, as needed, using the information presented.

Teble 5.6.1. PART MALFUNCTION DATA

| PART MAMR | MALFUNCTION | $\frac{\text { FREQUENC }}{\frac{O F D C C L R}{2}}$ |
| :---: | :---: | :---: |
| ACCELEROMETER |  |  |
|  | DEPRCTIVE PARTS INSIDE | 100.0 |
| ACTUATOR |  |  |
|  | Braring \& brarr rusted | 6.7 |
|  | CABLE INSULATION FRAYED | 6.7 |
|  | CABLE SIERVE NREDS PIXING | 6.7 |
|  | IMPROPER CONFIGURATION SHOULD BE -2 | 6.7 |
|  | IMPROPER CONNECTOR INSTALLED | 6.7 |
|  | REQUIRES ADJUSTMENT OF TM | 6.7 |
|  | REQUIRES OVERHAUL | 6.7 |
|  | SAPETY WIRE BRACKRT BROKEN | 6.7 |
|  | THERMAL SWITCH POUND TO BE DEPECTIVE INONOHN | $\begin{array}{r} 6.7 \\ 40.0 \end{array}$ |
| ANTEANA |  |  |
|  | UNKNOWN | 100.0 |
| AXIE |  |  |
|  | DAMAGED | 50.0 |
|  | UNKONOWN | 50.0 |
| AZIMUTH ENCODER |  |  |
|  | ANTENNA WON'I MOVE | 32.3 |
|  | CASING ROTATES | 3.2 |
|  | CRACKRED GLASS DISC | 3.2 |
|  | ENCODER MARKING SHOULD BE REMYOED | 3.2 |
|  | INCORRECT ANTENNA ROTATION | 3.2 |
|  | LAMP DESIGN DEPECTIVE | 3.2 |
|  | NO MOVEMENT BETWEEN DWELLS | 3.2 |
|  | OPTICAL ASSEMBLY DEFECTIVE | 3.2 |
|  | RESISTER IS DEPECTIVE | 3.2 |
|  | UNKNOWN | 41.9 |
| BATTERY |  |  |
|  | CONNECTOR PANEL DEPECTIVE | 20.0 |
|  | CONNECTOR PINS SHORTED | 20.0 |
|  | CONNECTOR SHORTED | 20.0 |
|  | K-I MISWIRED | 20.0 |
|  | SHORTED VR1-3 TO CHASSIS | 20.0 |

Table 5.6.1, continued. SAR: :ALFUNCTION DATA
PART NAMRMALFUNCTION
BEARING
AZIMUTH DRIVB INORERATIVE
BLOVRR BEARINGS WORN OUT ..... 4.2
BLOHER INOPRRATIVE ..... 4.2
DEPBCTIVE ..... 4.2
EXGESSIVE PLAY ..... 12.5
LUBRICATION DRIED OUT ..... 4.2
PRINTER INOPERATIVE ..... 4.2
requires overhaul ..... 12.5
UNROOWN. ..... 45.8
WLW has excessive weight ..... 4.2
BELLOWS
CRACKRD ..... 50.0
UNKNOWN ..... 50.0
BROKEN ..... 10.0
dUE TO EXCESSIVE USE ..... 60.0
HORN OUT ..... 30.0
BLOWERS \& FANS
bearings worn out ..... 15.8
DEPECTIVE SENSOR ..... 18.4
DEFECTIVE SWITCH ..... 2.6
excessive current has shorted ..... 2.6
EXCESSIVE VIBRATIONS \& BEARINGS LOOSE ..... 5.3
has excessive vibrations \& MOUNT IS LOOSE ..... 5.3
improper installation ..... 2.6
MOTOR DAMAGED ..... 2.6
NOISY DUE TO DEPECTIVE BEARINGS ..... 5.3
REVERSE WIRING ..... 2.6
SHORTED ..... 5.3
SWITCH IS LOOSE ..... 2.6
SWITCH NOT PROPERLY INSTALIED ..... 2.6
SWITCH NOT WORKING ..... 2.6
UNKNOWN ..... 23.7EREOGENCY OFQCCUBRENCE:

Table 5.6.1, continued. YART MALFUNCTION DATA

| BRAKES | MALFUNCTICN! | FPEQUENCY OF OCCURRENCE |
| :---: | :---: | :---: |
|  | ASSY SCREN TOO TIGHT | 2.3 |
|  | BRARE CABLE TOO SHORT | 2.3 |
|  | BRAKE DISCS HORN OUT | 2.3 |
|  | BRAKE PAD BUSHINGS SCREN LOOSE | 2.3 |
|  | BRAKES CORRODED | 4.7 |
|  | EXCESSIVE GAP | 18.6 |
|  | IMPROPER ADJUSTMENT | 7.0 |
|  | IMPROPER POSITIONING | 4.7 |
|  | IMPPOPERLY INSTALLED | 2.3 |
|  | LOCR SCREW NEEDS REPLACEIRENT | 2.3 |
|  | NEEDS ADJUSTMENT | 18.6 |
|  | PARTS ARE WORN OUT | 2.3 |
|  | PARTS BROKTEN | 2.3 |
|  | RECRIMP TERMINAL DKC REQUIRED | 2.3 |
|  | SCREN LOOSE | 2.3 |
|  | UNENOWN | 20.9 |
|  | WORN OUT | 2.3 |
| BRUSHES |  |  |
|  | NEEDS ADJUSTMENT | 66.7 |
|  | SHORTED | 33.3 |
| BUSHINGS |  |  |
|  | UNTONOWN | 16.7 |
|  | WORN OUT | 83.3 |
| GAMERA |  |  |
|  | DEPECTIVE POWER SUPPLY | 40.0 |
|  | PICTURE BECOMES WRAR \& BRRARS UP | 20.0 |
|  | UNKNOWN | 40.0 |
| CIRCUIT PROTECTION DEVICE |  |  |
|  | PAINT STENCILED ON ARRESTER SHORTED | 14.3 |
|  | UNKNOWN | 85.7 |
| CLUTCH |  |  |
|  | ANTENNA PAILS TO ROTATE | 62.5 |
|  | ANTENNA MOVES SLOWLY | 12.5 |
|  | NEEDS ADJUSIMENT | 25.0 |

Table 5.6.1, continued. PART MALFUNCTION DATA

| PART NAME | MALFUNCTION. | $\begin{aligned} & \text { EREUUENCY OF } \\ & \text { OCCURRENCE E } \end{aligned}$ |
| :---: | :---: | :---: |
|  | bad rerl hub assembly | 1.0 |
|  | bad Solder joints on connector pins | 1.0 |
|  | CABLE WIRES SWITCHED | 1.0 |
|  | CAPSTAN MOTOR DEPBCTIVE | 1.0 |
|  | CAPSTAN MOTOR JAMPRD | 1.0 |
|  | CONNECTOR PLNS HAVE BROKEN | 1.0 |
|  | DEFECTIVE CAPACITOR | 2.0 |
|  | depective cca | 1.0 |
|  | depective chick valve | 1.0 |
|  | defective control syster | 1.0 |
|  | DEPRCTIVE DISC | 1.0 |
|  | DEPECTIVE IC | 6.9 |
|  | DEPECTIVE MOTOR ASSEMBLY | 1.0 |
|  | depective powra supply | 1.0 |
|  | DEPECTIVE SENSOR | 14.9 |
|  | DEPECTIVE SOLDERING ON IC. | 2.0 |
|  | DEPECTIVE TRANSISTOR | 3.0 |
|  | depective transistor in servo assembly | 1.0 |
|  | defective vacuum chamber sensor | 1.0 |
|  | PLOPPY DRIVE ONE IS DEPECTIVE | 1.0 |
|  | pUSE BLOWN | 1.0 |
|  | has Low plessure | 2.0 |
|  | MTC DEFPCTIVE | 1.0 |
|  | PINS $1 \& 3$ ARE DAMAGED | 1.0 |
|  | POWER SUPPL DEPECTIVE | 1.0 |
|  | Q1 \& Q2 IMPROPERLY ORIENTED | 1.0 |
|  | READ/WRITE HRAD IS DEPECTIVE | 1.0 |
|  | RIBBON CABLE BROREN | 1.0 |
|  | SERVO AMP. DEPECTIVE | 1.0 |
|  | TWO PINS SHORTED ON CONNECiOR | 1.0 |
|  | UNTONOWN | 8.9 |
|  | WON'T ACCEPT CERTAIN PROGRAMS | 1.0 |
|  | WON'T ACCEPT VRS-131 | 1.0 |
|  | WON'T LOAD | 19.8 |
|  | WON'T RECORD | 4.0 |
|  | WON'T REWIND | 9.9 |
|  | WRONG PUSE WAS INSTALIED | 1.0 |
| COUPLING |  |  |
|  | UNTONOW | 100.0 |
| CRANK SHAFT |  |  |
|  | BRACKET BROKEN | 40.0 |
|  | 5-288 |  |

Table 5.6.1, contiaued. PART MALFUNCTION DATA

bisctric heaters

Table 5.6.1, continued. PART MALFUNCTION DATA

| PART NANE | MALFUNCTION | FREQUENCY OF |
| :--- | :--- | :--- |
|  | SHORTED | 40.0 |
|  | UNKNOWN | 60.0 |

blectromrchanical timers

| NEEDS ADTUSTMRNT | 33.3 |
| :--- | :--- |
| UNONOWN |  |

PILTER
DAMAGED 16.7
LEAKING 38.9
LINE FILTER PIN-D OPEN 5.6
NEEDS REPLACEMENT 11.1
UNIONOWN 27.8
PITTINGS
IMPROPER ADJUSTMENT 7.1
LEAKING 28.6
NREDS CLPANING 7.1
UNKNOWN 57.1
PUSE HOLDER
DAMAGED 100.0
GASKETS \& SBALS

| BAD INTERNAL SEAL IN GEAR DRIVE | 6.7 |
| :--- | ---: |
| IMPRORER INSTALLATION | 6.7 |
| LEARING | 6.7 |
| NEEDS REPLACEMENT | 46.7 |
| POPS UP DURING ANTENNA MOVEMRNT | 6.7 |
| UNCNOWN | 26.7 |

gear
UNONOWN 60.0
gear box

| NEEDS OVERHAUL | 33.3 |
| :--- | :--- |
| NO OIL IN GEAR BOX | 66.7 |

Table 5.6.1, continued. PART MALFUNCTION DATA

| PART NAME | MALFUNCTION | FRECUENCY OF |
| :---: | :---: | :---: |
| hrat exchangers |  |  |
|  | NERDS ADUSTMENT | 20.0 |
|  | NREDS RRPIACEMENT | 20.0 |
|  | NOT PROPERLI PABRICATED BY VENDOR | 20.0 |
|  | UNTSNOWA | 40.0 |


| AC INPUT ShORTED | 0.9 |
| :---: | :---: |
| AC INPUT SHORTED UNDER LOAD | 0.9 |
| Braring horn out. requires replacemient | 0.9 |
| BELT IS SLIPPING | 0.9 |
| DEPECTIVE IC | 10.5 |
| DEFECTIVE POTENTIOMETER | 0.9 |
| DEPECTIVE SENSOR | 1.8 |
| DRPECTIVE TRANSISTOR | 3.5 |
| PERDING MECHANISM NREDS REPAIR | 0.9 |
| HANPIRR BLOWER NOISY | 0.9 |
| HAMGIER DRIVER DAMAGED | 0.9 |
| HAMOIRR HRAD BROKBN | 0.9 |
| has ribbon stewing problea | 0.9 |
| LOOSE PULLEY | 2.6 |
| MOTOR \& ROLLER ARE DRPBCTIVE | 0.9 |
| MOTOR DPPECTIVE | 1.8 |
| MOTOR JAMPED | 0.9 |
| MOTOR NOT WORRING PROPERLI | 1.8 |
| MOTOR SHORTED | 0.9 |
| NEEDS OVERHAUL | 0.9 |
| NEEDS WIRE REPLACEMIRNT | 0.9 |
| NO POURR AT - 32C | 0.9 |
| OVER CURRENT, SHORTED | 1.8 |
| PAPER PEED NOT WORKING | 1.8 |
| PAPER PEEDING MrCHANISM JAMDRD | 0.9 |
| PAPER SPINDLE ROTATION SLOW | 0.9 |
| PAPER SPINDLE TENSION LOW | 1.8 |
| PARTS MISSING | 3.5 |
| PRINT PINGERS BENT | 1.8 |
| PROBIEM WITH TARE UP | 0.9 |
| PULLEY BROREN | 0.9 |
| RESISTOR R-35, IS OPEN | 0.9 |
| RIBBON MOTOR INOPRRATIVE | 0.9 |
| RIBBON HORN OUT | 1.8 |
| RIBBON WORN OUT \& BAD SWITCH | 0.9 |
| ROLLER PRESSURE IS LOW-NEEDS ADSUSTMENT | 0.9 |
| ROLLER PRESSURE NEEDS ADJUSTMENT | 0.9 |

Table 5.6.1, continued. PART MALFUNCTION DA'TA

| PART NAME | MALFUNCTION | Fr.EQUEN |
| :---: | :---: | :---: |
|  | STEPPRR MOTOR DEPECTIVE | OCCURP. |
|  | STEPPER MOTOR IS INOPERATIVE | 0.9 |
|  | tare up brel latch is inopgrative | 1.8 |
|  | TIMING BELT \& PULLEY WORN OUT | 0.9 |
|  | TIMING BELT BROKBN | 1.8 |
|  | TIMING BELT DAMAGED \& WORN OUT | 0.9 |
|  | TIMING BELT WORN OUT | 1.8 |
|  | TOP ROLLER PRESSURE NEEDS ADJUSTMENTT | 0.9 |
|  | TRANSPORMER HAS OPEN LEAD | 0.9 |
|  | UNTONOWN | 32.5 |
| HOSES |  |  |
|  | depective | 28.6 |
|  | hose has cracks due to antenna movement | 42.9 |
|  | UNKNOWN | 14.3 |
|  | WORN OUT | 14.3 |
| INSTRUMENT |  |  |
|  | SEALING DAmAGED | 20.0 |
|  | UNONOW | 80.0 |
| JOINT, MIC |  |  |
|  | derective | 40.0 |
|  | UNKNOWN | 60.0 |
| KEYBOAKD |  |  |
|  | CABLE(GP554) DEFECTIVE | 4.3 |
|  | IMPROPER CONNECTIONS | 4.3 |
|  | LED DISPLAY DEFECTIVE | 4.3 |
|  | LOCKED UP | 13.0 |
|  | U-32 DEPECTIVE | 4.3 |
|  | UNONOWN | 69.6 |
| LOW SPEED |  |  |
|  | DEFECTIVE CAPACITOR | 8.3 |
|  | DEPECTIVE IC | 16.7 |
|  | DEFECTIVE RELAY | 8.3 |
|  | DEPECTIVE SWITCH | 8.3 |
|  | PUSE BLOWN OUT | 33.3 |
|  | tear bar broken | 8.3 |

Table 5.6.1, continued. PART MALFUNCTION DATA


Table 5.6.1, continued. PART i.SLIFUNCTION DATA

| PART NAME | MALFUNCTION | FREQUENCY OF OCCURRENCE \% |
| :---: | :---: | :---: |
|  | SCREWS MISSING | 20.0 |
|  | UNRNOWN | 40.0 |

RETAINING RING

$$
\text { NEEDS ADJUSTMENT } 33.3
$$

NEEDS REPLACEMENT 33.3
UNKNOWN 33.3
SENSOR/TRANSDUCER/TRANSMITTER
NO TAPE LOADING 30.0
PRINTER INOPERATIVE 30.0
RELAY DEFECTIVE 20.0
VACUUM COLUMN NOISY 20.0
SHAPT

$$
\begin{array}{ll}
\text { HINGE REDESIGN REQUIRED } & 11.1 \\
\text { RUST UNDER GUSSET } & 11.1 \\
\text { UNIT HAD GREASE WHICH HAS FROZEN } & 11.1 \\
\text { UNKNOWN } & 66.7
\end{array}
$$

SHOCK ABSORBERS

| IMPROPER MOUNT | 50.0 |
| :--- | :--- |
| REQUIRES REFURBISHING | 25.0 |
| UNKNOWN | 25.0 |

SOLENOIDS
UNK NOWN 100.0
SWITCH
BRAKR INTERLOCK SYSTEM STUCK 1.6
CONNECTION LOOSE 1.6
CONNECTOR BASE PULLED OUT 1.6
DAMAGED 4.8
DEPECTIVE WIRING 4.8
IMPROPER CONNECTION 3.2
IMPROPER INSTALLATION 6.5
IMPROPER WIRING 9.7
IMPROPERLY BONDED 1.6
LOOSE DUE TO EXCESSIVE VIBRATIONS 1.6

Table s.6.1, continued. PART MALFUNCTION DATA
PART NANELOOSE INSTALLATIONLOW PRESSURE
FREQLENCY OF OCCURKENCE \% 1.6 1.6
NRERE REPOSITIONING NREDS REHIRING OPEN
Q-3 Lrads shorted
REVERSED Lends
SHORTED
SWITCH IS BENT UNTONOHN
4.8
1.6
1.6
1.6
1.6
3.2
SWIICH IS BENT 1.6
43.5
TELESCOPE
DAMAGED 50.0
UNXNOWN 50.0
TRACR BALL
CONSOLE CANOT ENTER MODE BLTE 3.2
DEPECTIVE COMPONENT 67.7
DEPECTIVE DIODE 3.2
DEPRCTIVE IC 3.2
DEFECTIVE LAKP 9.7
IMPROPRR SOLDERING INSIDE 3.2
LOOSE CONNECTOR 3.2
UNONOWN 6.5

## valves

CRACXS ON BODY ..... 12.5
dUE TO LOW PRESSURE ..... 12.5
IMPROPER SEALING ..... 12.5
seal worn out ..... 37.5
UNTONOWN ..... 25.0

### 6.0 RELIABILITY DEMONSTRATION TESTS

6.1 Introduction. The purpose of a rellability demonstration test is to decide if additional reliabsility design effort is necessary to achieve the specified reliability for the nonelectronic item when it is operated in the field environment. The reliability specification (see section 3.0) identifies the parameter (s) and the values to be nominally and minimally acceptable. Reliability demonatrations are etatistical hypothesis tests which lead to one of two mutually exclualve decisions:
(a) The reliability parameter(s) of the component is (are) acceptable and no additional design effort is required under the contract;
(b) The realiability parameter(s) of the component is (are) unacceptable and additional design effort is required.

The demonstration is designed to have a high probability that the decision reached is correct. When the decision is (a), the consumer runs the risk that the decision is incorrect, i.e. that (b) is true but there were an unusually low number of fallures during the test. The probability of this type of incorrect decision is called the consumer's risk (B).

Similarly, the probability that the decision is (b) when (a) is true (1.e. the component has an unusually large number of failures during the test) is called the producer's risk ( $\alpha$ ).

It is important that the demonstration simulate the field environment or that there be a known relationship between the field environment and the test environment. For example, if the component actuation rate in the field is low and the effect of actuation rate is know, it would save test time to raise the actuation rate and lover the acceptable reliability values accordingly.
6.1.1 Statistical Characteristics of a Reliability Demonstration Test. There are aly essential characteristics of reliabiliny demonstration test.
(1) The reliability parameter(s) in the apecification. If the distribution of the number of fallures in the period of time $[0, T]$ is available in a mathematical expression then the rellability parameters will be related to the distribution parameters.
(2) The acceptable values for the reliability parameter(s). For example, the upper test MTBF ( $\theta_{0}$ ) in MIL-STD-781C is the smallest desired value of MTBF.
(3) The unacceptable values for the rellability parameter(s). For example, the lower test MTBP $\left(\theta_{1}\right)$ in MIL-STD-781C is the largest unacceptable value for MTBF.
(4) The producer's risk, a.
(5) The consumer's risk, $\beta$.

### 0.0 RELIABILITY dEmonstkation tests

6. 1 Introduction. The purpose of a reliability demonstration test is to decide if additional rellability design effort is necessary to achieve the specified reliability for the nonelectronic item when it is operated in the field environment. The reliability specification (see section 3.0) fdentifies the parameter(s) and the values to be nominally and minimally acceptable. Rellability demonstrations are atatistical hypothesis tests which lead to one of two mutually exclusive decisions:
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Similarly, the probability that the decision is (b) when (a) is true (i.e. the component has an unusually large number of failures during the test) is called the producer's risk ( $\alpha$ ).

It is important that the demonstration simulate the field environment or that there be a known relationship between the field environment and the test environment. For example, if the component actuation rate in the field is 10 w and the effect of actuation rate is known, it would save test time to raise the actuation rate and lower the acceptable reliability values accordingly.
6.1.1 Statistical Characteristics of a Reliability Demonstration Test. There are six essential characteristics of a reliability demonstration test.
(1) The reliability parameter(s) in the specification. If the distribution of the number of failures in the period of time [ $0, T$ ] is available in a mathematical expression then the rellability parameters will be related to the distribution parameters.
(2) The acceptable values for the reliability parameter(s). For example, the upper test MTBF $\left(\theta_{0}\right)$ in MIL-STD-781C is the smallest desired value of MTBF.
(3) The unacceptable values for the reliability parameter(s). For example, the lower test $\operatorname{MTBF}\left(\theta_{1}\right)$ in MIL-STD-781C is the largest macceptatie value for MTBF.

$$
\begin{aligned}
& \text { (4) The producer's risk, } \alpha \text {. } \\
& \text { (;) The consumer's risk, } \beta \text {. }
\end{aligned}
$$

(0) The samplias plan which detines how and what parametere will be obacrved, and the crlterion for ending the demonstration test and reaching a decision.

In what follows, R will denote a generic rellabllity parameter (e.g. MTBF, probability of survival of a prespecified time period, etc.) larger values of which are preferred. The smallest acceptable value of $R$ in (2) above is denoted by $R U$, and the largest unacceptable value of $R$ is denoted by $R_{1}$. $R_{0}$ must be strictly greater than $R_{1}$. The values of $R$ which lie between $R_{1}$ and $R_{0}$ are called indifference values. The ratio $R_{0} / R_{1}$ is called the discrimination ratio.

The statistical characteristics of a demonstration are summarized in the operating characteristic ( $O C$ ) relationship between $R$ and the probability of passing the test when $R$ is the "true" rellability, $P(R)$. Characteristics (4) and (S) give two points in the $O C$ relationship, namely $P\left(R_{0}\right)=(1-u)$, and $P\left(R_{1}\right)=$.
6.1.2 Cost of Demonstration. The cost of demonstration is determined by the number of samples collected and the calendar time the test facility is occupied. Cost efficient demonstrations require the smallest number of samples and the least calendar time to meet the risk objectives of the demonstration.

If the reliability parameters are related to the parameters of a life distribution, then the most cost efficient demonstration is to measure the time of failure (a variable) for each sample. The demonstration is then called a variables test and the sample of failure times provides the maximum information on the reliability of the design of the component. If the reliability parameter is the fraction of components in a lot which will live beyond some time $T$, the demonstration will attribute success or failure to each sample according to whether or not the component is operational at time $T$ (attributes testing).

The rule for terminating the demonstration also affects the cost. Given the values for the characteristics in 6.1 .1 (1)-(5), it is possible to design a fixed sample size (or fixed time) or sequential demonstration test. In a fixed sample dimonstration test termination occurs when (i) all the sample values are observed or (ii) when enough failures have been observed to decide the reliability is unacceptable. Similarly, a fixed time demonstration terminates when (i) the fixed time limit is reached or (ii) wher enough fallures are observed to decide the reliability is unacceptable. During a sequential demonstration, a sequence of decision points for both acceptable and unacceptable reliability are formulated and a decision is reached the first time one of these decision points is reached. On the ave:age a sequential demonstration will require a smaller sample than a fixed sample test for the same $R_{0}, R_{1}$, $\alpha$, 1 . Sequential demonstrations are prosibibe for both variables and attributes testing.

[^4]a relisbility demonstration (i.e. the demonstration characturistics in 6.1 .1 ( 1 )-(S) are allowed to determine $N$ ). N increases as either the discrimination ratio, $\alpha$, or $B$ decrease. There is a maximum value for $N$ ( $N M A X$ ) in all practical situations. If the $N$ required by the characteristics of 6.1 .1 (1)-(5) exceeds NMAX, one of the parametera (usually $\alpha$ ) must be changed ( $R_{0}, \alpha$. or $B$ can be increased, or $R_{1}$ can bal decreased).
6.1.4 Surmary. The remainder of this section presents step-by-step instructions on the use of various types of rellability demonstration test plans. The section is asranged to present first test plans based on attributes, followed by variables test plans.

### 6.2 Attributes Demonstration Tests

6.2.1 Attributes Plans for Small Lots

1. When to use

When testing parts from a small lot where the accept/reject decision for the lot is based on attributes, the hypergeometric distribution is applicable. Attributes tests should be used when the parameter of interest is the fraction of components in a lot which possess a certain reliability attribute.

The example demonstrating the method is based on a small lot and small sample size. This situation frequently characterizes the demonstration test problem associated with nonelectronic parts. The sample size limits the discriminatory power of the demonstration test plan but frequently cost and time constraints force us into larger than desired risks.
2. Conditions for Use

The attribute of interest may be that a part survives at least $t$ hours. A "success" for a component tested would be that it survives $t$ hours. The parameter to be evaluated then is the fraction of the parts in the lot whose lives would exceed $t$ hours. The estimation of the parameter would be based on a fixed sample size and testing without replacen ent. The selection of the criteria for success ( $t$ hours) can be derived from a requirement (such as mission length, for example). If the lot size is 30 or more, then the Poisson approximation may be used to make the calculation simpler. (See Section 6.2.3).
3. Method
a. Define criterion for success/fallure, l.e. define the attribute.

## Example

a. A part that lasts 100 or more hours on a given life test is considered a success. Parts falling before 100 hours are considered failures.
b. Define acceptable lot quality level (l-po).
c. Specify producer's risk ( $\alpha$ ) (i.e, the probability that acceptable lots be rejected).
d. Define unacceptable quality level ( $1-\mathrm{P}_{1}$ ).
e. Specify the consumer's risk ( $B$ ) (i.e., the probability that unacceptable quality lots will pass the demonstration test).
f. Now that $\alpha, \beta$, $1-p_{0}$, and $1-p_{1}$ have been specified the following steps describe the calculations required to determine the sample sfze and accept/ reject criteria which will satisfy the stated risks.
g. The process consists of a trial and error solution of the hypergeometric equation using $N$, l-por $1-p_{1}$ and various sample sizes until the conditlons of $\alpha$ and $\beta$ are

## Example

b. Lots in which ( $1-\mathrm{P}_{0}$ ) $=90 \%$ of of the partis will survive 100 hours are to be accepted by this demonstration test plan with high probability.
c. Let $\alpha=$. 2 . This decision $1 s$ an engineering one based on the consequences of allowing defective lots to be accepted and based on the time and dollar constraints associated with inspecting the lot.
d. Lots in which only 1-p1 $=20 \%$ of the parts will survive 100 hours will be accepted by the demonstrations test plan with low probability.
e. Let $\beta=.022$ (Taken for convenience in calculations).
f. Given: lat size $N=10$
$1-p_{0}=.9$
$\begin{aligned} 1-p_{1} & =.2 \\ \alpha & =.2\end{aligned}$ $\beta=.022$

## Example

8. met. The equation used is
$\operatorname{Pr}(x)=\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$
first step in the trial and error procedure atsume a sample size of 2 . The possible outcomes are either 0,1 or 2 good parts. The probability of each outcome using the hypergeometric formula is
$x=\max (0, n-N+r), 1,2$ ... min( $n, r)$
where $x$ n number of successes in sample
$r=$ number of successes in 10t
N - lot size n = sample size

$$
\binom{r}{x}=\frac{r!}{x!(r-x)!}
$$

## h. Find the number of

 successes which satisfies $\alpha$ and $B$ in the calculations involving 1-Po and 1-P1.$\operatorname{Pr}(2)=\frac{\binom{9}{2}\binom{1}{0}}{\binom{10}{2}}=.8$
$\operatorname{Pr}(1)=.2$
$\operatorname{Pr}(0)=0$

The same calculations for $1-P I=.2$ results in
$\operatorname{Pr}(2)=.022$
$\operatorname{Pr}(1)=.356$
$\operatorname{Pr}(0)=.622$
h. From these 2 sets of
results it can be seen that if a sample size of 2 is apecified, then $\alpha$ and $B$ will be satisfied if the decision rule ia made that $1 f 2$ successes are observed in the sample the lot is accepted and for all other outcomes the lot 18 rejected.

If $1-P_{0}=.9$, then $\operatorname{Pr}(2)=.8$, therefore $1-.8=, 2=\alpha$ If $1-P_{1}$ - 2 , then $\operatorname{Pr}(2)=.022=B$.

NOTE: A different sample size can be traded off against different $a, B, 1-p_{0}, 1-p_{1}$.
3. Method

1. The demonstration test is then specified.

Example

1. The test procedure is as follows:
2. Test a random sample of 2 parts from a lot of 10 parts for 100 hours.
3. If both parts survive 100 hours, accept the lot.
4. If only 0 or 1 parts survive 100 hours reject the lot.
5. For Further Information.

There are "Tables of the Hypergeometric Distribution" by G. J. Lieberman and D. B. Owen, Stanford University Press, Stanford, California, 1961 to perform the mathematical calculations of Step $g$. Also if $N$ becomes large (say 30 or more) then the binomial or the Poisson distribution can be used as an approximation for the hypergeometric distribution.

### 6.2.2 Attributes Plans for Large Lots (Binomial)

## 1. When to Use

When testing parts from a large lot where the accept/reject decision for the lot is based on attributes, the binomial distribution is applicable. Strictly speaking, all reliability attributes testing should follow the hypergeometric distribution as long as individual parts are placed on test and tested to fallure without replacement. However, when the lot size is large, the binomial distribution is a good approximation for the hypergeometric and therefore the example presented in this section covers the use of the binomial. Attributes test should be used when the parameter of interest is the fraction of components in a lot whici possess a certain reliability attribute.

## 2. Conditions for Use

The attribute of interest may be that a part survives for at least $t$ hours. A "success" for a component tested would be that it survives $t$ hours. The parameter to be evaluated then is the fraction of the parts in the lot that would survive $t$ hours. The estimation of the parameter would be based on a fixed sample size and testing without replacement. The selection of the criteria for success ( $t$ hours) can be derived from a requirement (such as a mission length, for example).

## 3. Method

a. Define criterion for success/failure, i.e. define the attribute.
b. Define acceptable lot quality level ( $1-\mathrm{p}_{\mathrm{o}}$ ).
c. Specify producer's risk (a) (i.e., the probability that acceptable lots will be rejected).
d. Define unacceptable lot quality level ( $1-p_{1}$ ).
e. Specify consumer's risk (B). (i.e., the probability that lots of unacceptable quality level will be accepted).
f. Now that $\alpha, \beta$, $1-p_{0}$, and $1-p_{1}$ have been specified, the following steps describe the calculations required to determine the sample size and accept/reject criteria which will satisfy the stated risks.
g. The process now consists of a trial and error solution of the binomial equation using $1-p_{0}$, $1-p_{1}$ and various sample sites until at a given decision point, the condi$t$ ions of $a$ and $b$ are

Example
a. A part that lasts 100 or more hours on a given life test is considered a success. Parts failing before 100 hours are considered failures.
b. Lots in which $1-\mathrm{p}_{0}=.9$ (1.e., the life of $90 \%$ of the parts will exceed 100 hours) are to be accepted by this demonstration test plan with high probability.
c. let $\alpha=.01$.
d. Lots with only a true fraction of acceptable parts $1-p_{1}=.5$ are to be accepted by this demonstration test plan with low probability.
e. let $\beta=.17$ (selected for ease of calculation).
f. Given: lot size $N=$ large say > 30
$1-p_{0}=.9$
$1-\mathrm{p}_{1}=.5$
$\alpha=.01$
$\beta=.17$
g. Assume a random sample of size $n=10$ is taken from a lot whose true fraction of good parts is .9. Solve the binoalal equation for the toral number of consecutive outcomes whose summed probabilities equal
3. Method
8. satiafied. The binomial equation 1a:
$\operatorname{Pr}(x)=\binom{n}{x}(1-p)^{x}(p)^{n-x}$
where n - ample ise $x=$ observed successes in sample $p=$ lot fraction defective

Example
a starting at 0
successes. The calculations for this decieion point are:
$\operatorname{Pr}(10)=\binom{10}{10}(.9)^{10}(.1)^{0}=.3486$
$\operatorname{Pr}(9)=\quad .387$
$\operatorname{Pr}(8)$ is . 1935
$\operatorname{Pr}(7)=\quad .0574$
$\operatorname{Pr}(7$ or more $)=\quad .9865$

Then
$\operatorname{Pr}(6$ or less $)=1-\mathrm{Pr}$ (7 or more)
$=1.0-.9865$
$\approx .01$ (which
satisfles the $\alpha$ risk).
Perform the same type of calculations assuming the true fraction defective is .5. In this instance sum the probubilities starting at 10 successes until succeeding consecutive probabilities sum to the value of 6. This yielde the following results:
$\operatorname{Pr}(10)=\binom{10}{10}(.5)^{10}(.5)^{0}=.001$
$\operatorname{Pr}(9)=.010$
$\operatorname{Pr}(8)=.044$
$\operatorname{Pr}(7)=\quad .117$
$\operatorname{Pr}(7$ or more) $\approx .17$ (which satisfles the $\beta$ risk)
h. The demonstration test is then specified.
h. The test procedure is as follows:

1. Test a random sample of 10 parts for 100 hours.
2. Method
h.

Example

> 2. If 7 or more parts survive 100 hours, accept the lot.
> 3. If 6 or less auccesses are observed, reject the lor.
4. For Further Information

There are several published tables for use in determiniag binowial probabilities in the event that the sample size makes calculations too lengthy. One of these is Tables of the Binomial Probability Distribution, National Bureau of Standards, Applled Mathematics Series 6, Washington, D.C., 1950. It gives individual terms and the distribution function for $p=.01$ to $p=.501 n g r a d u a t i o n s$ of .01 and $n=2$ to $n$ $=49$ in graduations of 1 .
6.2.3 Attributes Demonstration Test Plans for Large Lota (The Poisson Approximation Method)

1. When to Use

In attributes demonstration teat plans if the lot size gets much above 100 the calculations required to generate a demonstration test plan become very time consuming. The Poisson distribution can be used as an approximation of both the hypergeometric and the binomial distributions if the lot size is large and if the fraction defective in the lot is small. This method can therefore be used in lieu of the previous two methods in many cases.
2. Conditions for Use

If the lot size is large and the fraction defective is small, this method is applicable. Its use is initiated by specifying a desired producer's risk, consumer's risk, acceptabie lot fraction defective and unacceptable lot fraction defective. As before, it is also necessary to specify the characteristics that constitute a defective part since this is an attributes type test.
3. Method
a. Define criterion for success/failure.

## Example

a. A part that lasts 100 or more hours on a given life test is considered a success. Parts failing before 100 hours are considered failures.
b. Define acceptable lot quality level (1-Po).
b. Lots in which 1- $P_{0}=.9$
(the life of $90 \%$ of the parts in the lot will exceed 100 hours) are to be accepted by this demonstration test plan with high probability.
c. Specify the producer's

```
c. Select a=.05.
```

risk ( $\alpha$ ) (i.e., the probability that acceptable lots will be rejected).
d. Define unacceptable lot quality level ( $1-p_{1}$ ).
d. Lots with only a true fracticn of acceptable parts l-p .75 are to be accepted by this demonstration test plan with low probability.
e. Specify the Consumer's e. Select $B=.02$.
risk B (i.e. the probability that lots of unacceptable quality level will be accepted by this plan).
f. Now that $\alpha, \beta$,
f. Given: lot size $N=1000$
$1-P_{0}$, and 1-P1 have been specified, the accept/reject criteria
$1-P_{0}=.90$ are determined by the
$1-P_{1}=.75$ following formulas:

$$
\begin{aligned}
1-\alpha & =\sum_{x=0}^{c} \frac{\left(n p_{0}\right)^{x} \exp \left(-n p_{0}\right)}{x!} \\
\beta & =\sum_{x=0}^{c} \frac{\left(n p_{1}\right)^{x} \exp \left(-n p_{1}\right)}{x!}
\end{aligned}
$$

8. The solution now consists g. of trying various values of $n$ in the above formulas until they are approximately satisfied.

Assume $n=$ (sample size)
= 100 .
Then,
$n p 0=100(.10)=10$
$n p_{1}=100(.25)=25$.
Using a digital computer to
3. Method
h. The demonstration is then fully specified.

## Example

compute the formulas in (f) above leade to cm 15 , and
$\alpha=.049$
$B=.022$.
The decision criterion is now specified as cols or less fallures.

The demonstration test procedure is as follows:

1) Take a random sample of 100 parts from the lot of size 1000 and test each part for 100 hours.
2) If 15 or less fail to survive 100 hours, accept the lot. If more than 15 parta fall to survive 100 hours, reject the lot.
4. For additional examples using this method, refer to E.B. Grant, Statistical Quality Control, McGraw Rill, 1964.

### 6.2.4 Attributes Sampling Using MIL-STD-10SD

1. When to Use

When the accept/reject criteria for a part is based on attributes decisions MIL-STD-105D is a useful tool. These sampling plans are keyed to fixed AQL's (Acceptable Quality Level) and are expressed in lot size, sample size, $A Q L$ and acceptance number. Plans are avallable for single sampling, double sampling and multiple sampling. The decision as to which type to use 18 based on a trade-off between the average amount of inspection, the administrative cost and the information yielded regarding lot quality. For example, single sampling usually results in the grestest amount of inspection, but this can be offset by the fact that it requires less training of personnel, and record keeping is simpler, and it gives a greater amount of information regarding the lot being sampled. The main difference between MIL-STD-105D plans and the previous plans is that the unacceptable quality level need not be specified.

## 2. Conditions for Use:

The user of a MLL-STD-105D sampling plan must have items a and $b$ below. MIL-STD-105D will determine items $c, d$, and e below, for a given type of sampling type (i.e. single, double, multiple, etc.):
a. Lot size
b. Acceptable Qcality Level
c. Sample Size
d. Acceptance Number
e. Criterie for Acceptance or Rejection.

The apecification of the AQL is an engineering decision based on the fraction defective that a user of parts considers acceptable. Lote with this percent defective will be accepted a high fraction of the time. Operating characteristic curves are supplied with aach ampling plan and these can be used to evaluate the protection afforded by the plan fiot various quality levele.

MIL-STD-105D also contains plans for normal, tightened and reduced inspection plans which can be invoked if the fraction defactive of lots seems to be varying or trending.

## 3. Method

a. Determine lot size and specify AQL and type of sampling.
b. Enter the table with lot size and select the sample size code letter.
c. Enter the single sampling plan table for normal inspection with the code number from Step b.
d. Enter the same table in the proper column for the specified AQL.

Exampla
a. Given a lot containing 100 parts and an AQL is specifiad at $6.5 \%$ with ingle ampling specified.
b. From Table I (Sample Size Code Letters) on page 9 , MIL-STD-10SD, find the sample size code letter for a lot of a1ze 100. For this example and for normal sampling, the epecified code number is $F$ (General inspection level II is the default).
c. Enter Table II-A (Single Sampling Plans for Normal Inepection) page 10 with code letter F. Under the column titled sample size, find the number 20 in the same row as the letter $F$. This is the number of parts to be randomly selected and inspectad.
d. Find the column in

Table II-A page 10
corresponding to an AQL of 6.5\%.
3. Method
e. Proceed horizontally along the Sample Size Code Number row until it intersects with the AQL column to obtain the acceptance number.
f. The Single Sampling Plan from MIL-STD-105D is to select a random sample of size $n$ from a lot of size $N$, inspect it and accept the lot if the number of defectives in the lot is equal to or less than the Acceptance Number. If the observed number of defects is equal to or greater than the rejection number, the lot is rejected.

## Example

e. At the intersection of row $F$ and column $6.5 \%$, the acceptance number is 3 and the rejection number is 4 .
f. For the single sampling plan $N=100, A Q L=6.5 \%$, select $a$ random sample of size $n=20$ and inspect it for attributes criteria. If 3 or less
f. defectives are found in the sample accept the lot. If 4 or more defectives are found in the sample reject the lot.

## 4. For Further Information

In addition to the example discussed above, MIL-STD-105D contains other plans for any lot size and for selected AQL's from. 01 to $1000 \%$ (AQL's over $10 \%$ are defects per hundred units, rather than percent of defective units). MIL-STD-105D also presents operating characteristic curves for each sampling plan.

### 6.2.5 Sequential Binomial Test Plans

1. When to Use

When the accept/reject criterion for the parts on test is based on attributes, and when the exact test time available and sample size to be used are not known or specified then this type of test plan is useful. The test procedure consists of testing parts one at a time and classifying the tested parts as good or defective. After each part is tested, calculations are made based on the test dala generated to that point and the decision is made either that the test has been passed, failed, or that another observation should be made. A sequential test will result in a shorter average number of parts tested than efther failure truncated or time truncated tests when the lot tested has a fraction defective at or close to $p_{0}$ or $p_{1}$.
2. Conditions for Use
a. The parts subjected to test will be classified as either good or defective. In other words, testing will be by attributes.
b. The acceptable fraction defective in the lot $P_{0}$, the unacceptable fraction defective $p_{1}$, the producer's risk $\alpha$, and consumer's risk $B$ aust be specified.
c. The teat procedure will be to test one part at a rime. After the part falls or its test time is sufficient to classify it as a success, the decision to accept, reject or continue testing the lot will be made.
d. The part lot alze must be large (greater than 100).
3. Method
a. Specify PO, $P_{1}, \alpha$, 8

## Example

a. Given lot of parts to be tested by attributes. Lots having only $\mathrm{PO}=.04$ fraction defective parts are to be accepted by the demonetration tent plan 95\% of the time (1.e., $\alpha$. .05). Lots having $P_{1}=$ . 10 iraction defective are to be accepted $10 \%$ of the time (1.e., $B=.10$ ).
b. Calculate decieion points with the following formula $\frac{1-\beta}{\alpha}$ and $\frac{\beta}{1-\alpha}$
b. The deciaion points are:

$$
\begin{aligned}
& \frac{1-\beta}{\alpha}=\frac{1-.10}{.05}=18 \\
& \frac{\beta}{1-\alpha}=\frac{.10}{1-.05}=.105
\end{aligned}
$$

c. As each part is tested, clasalfy it as a fallure or a success and evaluate the expression:
$\left(p_{1} / p_{0}\right)^{t}\left(\left(1-p_{1}\right) /\left(1-p_{0}\right)\right)^{t}$
where $f$ - total number of fallures - - total number of successen.
If at some point, this expreseion exceedn (1- - )/ a reject the lot. If at some point, this expression 1s less than $\beta /(1-\alpha)$ accept the lot. Continue
3. Method

$$
\begin{aligned}
& \text { c. sampling as long as neither } \\
& \text { of these conditions arises. } \\
& \text { d. The operating character- } \\
& \text { istic curve (1.e. the pro } \\
& \text { bability of acceptance as } \\
& \text { a function true fraction } \\
& \text { defective) can be roughly } \\
& \text { sketched from the follow } \\
& \text { ing points: }
\end{aligned}
$$

Probability of

| $P$ | Acceptance <br> Acobabilat |
| :--- | :---: |
| 0 | 1 |
| $P_{0}$ | $1-\alpha$ |
| $P_{1}$ | $B$ |
| 1 | 0 |
| $P^{\prime}$ | $P a$ |

Example
d. The five points on the $O C$ curve are as follows:

| P | Prob. of Accept |
| :---: | :---: |
| 0.00 | 1.00 |
| .04 | .95 |
| .10 | .10 |
| 1.00 | 0.00 |
| .063 | .56 |

The last point above is calculated as follows:
$\ln (.94) /(\ln (.94)-\ln (2.5))$
-.063;
$\ln (18) /(\ln (18)-\ln (.105))$ - . 56
where:

$$
\begin{aligned}
& p^{\prime}=\frac{\ln \left(\left(1-p_{1}\right) /\left(1-p_{0}\right)\right)}{\ln \left(\left(1-p_{1}\right) /\left(1-p_{0}\right)\right)-\ln \left(p_{1} / p_{0}\right)} \\
& P a=\frac{\ln ((1-\beta) / \alpha}{\ln ((1-\beta) / \alpha)-\ln (\beta /(1-\alpha))}
\end{aligned}
$$

### 6.3 Variables Demonstration Tests

6.3.1 Introduction. Reliability demonstration tests conducted in industrial applications are virtually always constrained by time. It is almost never the case that a demonstration test is carried out by placing n items on test, and waiting until all (in the complete sample case) or $r$ $<n$ (in the failure ceasored case) items have failed and recording their respective lifetimes. In practice, such sampling schemes are not used because the time necessary to complete the test is random, making it impossible for management to allocate the correct amount of time and resources to conduct the test. Instead, a time truncated test is appropriate (and often easier to administer) because an upper bound on the time to complete the test is known in advance of testing. Such tests were developed in MIL-STD-781C for the exponential distribution, and have been used almost exclusively in industry for electronic equipaent.

Another aspect of sampling for rellability demonstration tests is replacement versus nonreplacement tests. That is, if $n$ items are placed on life test initially, should failed items be replaced (or repaired to new working order) or not. Just as in the failure truncated case discussed above, the replacement life test presents a problem with respect to pianaing, since the ultimate number of items needed to complete the test 18 random and thus impossible to plan exactly in advance. Moreover, except in the exponential case, replacement tests are mathematically extremely difficult to levelop in the time truncated case. Replacement tests are appropriate, however, when the item under test is a complete system, and "replacement" signifies "repair/restore co new working condition." Indeed, the MIL-STD-781C time truncated tests are replacement tests. Whenever the item to be tested is a complete, complex system in which the predominant failure modes are due to electronic (or other constant failure rate) equipments, then the MIL-STD-781C the truncated tests can be used. However, if the system is primarily composed of nonelectronic parts having increasing failure races and the predominant failure modes are associated with these parts, then a replacement (by repair to new working order) test is out of the question, since in order to restore the system to new working order at each fallure, each wear-out related part would have to be replaced with a new part whether failed or not.

In summary, when the exponential distribution is assumed, the time truncated tests presented in MIL-STD-781C are recommended in the replacement case. In the nonreplacement case, ML-HDBK-108 (H 108) contains time truncated test plans for the exponential case. In view of the applicability of the exponential distribution to most nonelectronic parts in section 2 of this notebook, these documents should be edequate most of the time. When the exponential distribution is not fustified, then a time truncated, nonreplacement demonstration test is recomended. Alchough they possess interesting statistical properties and are mathematically tractable, failure truncated demonstration teats (which include the complete sample case) are not desirable when time must be limited, and are not recommended here. For information concerning statistical inference for various life distributions under failure truncated sampling, refer to section 3 of this notebook, MIL-HDBK-108, or to Mann, et.al. (1974) or to Lawless (1982).

### 6.3.2 Time Truncated Demonstration Test Plans

### 6.3.2.1 Nonparametric Reliability Demonstration Test

1. When to use

This type of test is applicable to any situation in which reliability (1.e. probability of survival for a preselected time period), median life, or any quantile of the underlying life distribution is specified. This test procedure is valid no matter what form the underlying life distribution assumes (i.e. exponential, Weibull, Gamma, Lognormal, etc.) as long as it is of the continuous type.
2. Conditions for use

The user of this type of test plan must specify (or select from the table of test plans) the producer's risk ( $\alpha$ ), the consumer's risk
( $b$ ), the acceptable reliability ( $R_{0}$ ), the unacceptable reliability $\left(R_{1}\right)$, and the time ( $T$ ) corresponding to the reliability values (i.e. $R_{0}$ is the acceptable probability of aurviving the time $T$, while $R_{1}$ is the unacceptable probability of surviving the time $T$ ).

The test entails placing a predetermined fixed number of parts or equipments on test for $T$ units of time, and recording the number items that fail before time $T$. Failed items are not replaced. The demonstration test is passed (i.e. items are judged to have the acceptable reliability $\mathrm{RO}_{0}$ ) if c or less items fail before tiae T , and the demonstration test is falled if ctlor more items fall before time T. The value of $c i s$ predetermined by the user's apecificatione.

## 3. Method

a. Specify Ro, Rl, T, $\alpha$, B or select them from the table of test plans, table 6.3.2.1.
b. Determine sample size $n$, and pass/fail number $c$ as follows:

Choose the smallest $c$ and the smallest $n$ which satisfy the

$$
\begin{aligned}
& \text { inequalities: } \\
& 1-\alpha \leq \sum_{k=0}^{c}\binom{n}{k}\left(1-R_{0}\right)^{k_{R_{0}}}{ }_{0}^{n-k} \\
& 0 \geq \sum_{k=0}^{c}\binom{n}{k}\left(1-R_{1}\right)^{k_{R}}{ }_{1}^{n-k}
\end{aligned}
$$

This value of $n$ is the sample oize, and $c 18$ the decision criterion; that

## Example

a. An axial blower must survive $T=100$ hours of continuous use with high probability. The acceptable reliability is
$\mathrm{R}_{0}=.95$
and unacceptable reliability is
$\mathbf{R}_{1}=.85$.
A producer's risk of no more than . 10 and a consumer's risk of no more then. 10 are acceptable.
b. From table 6.3.2.1, test plan 9A 18 appropriate. The sample size is $n=60$, and the test is passed if 5 or less fallures occur before time $T$, and the test is failed if 6 or more fallurey occur before time $T$.
3. Method
b. 18, the test 18 passed for $c$ or less failures before time $T$, and the test is failed if $c+1$ or more fallures occur before time $T$.

Alternatively, table 6.3.2.1 cen be used to identify $n$ and $c$.
c. Because the binumial distribution is discrete, the planned risks cannot be achlevad exactly. The exact producer's and consumer's risks are given by one oinus the first summation in b above, and the second summation in b above, respectively. Alternatively, if table 6.3.2.1 18 used, the exact producer's and consumer's risks are given there. The test plans 1A-13A are based on planned values of . 10 for both risks, and the test plans 1B-13B are based on planned values of . 20 for both risks.
d. Once a test plan is defined, it is often necessary to know what the probability of passing the test is as a function of true reliability, that is, the operating characteristic curve is needed. This curve gives the probability of passing the test (1.e. the probability of accepting the parts or equipments) for the entire range of possible values of the reliability,

Example
c. From table 6.3.2.1, test plan 9A, the exact risks are:
$\alpha=.079$
$b=.097$
d. Figure 6.3.2.9A is the operating characteristic curve for test plan 9A. As expected, when true reliability is R0 = .95, the probability of acceptance is 1-.079=. .921, and when true reliability is $\mathrm{R}_{1}=.85$, the probability of acceptance is . 097 . If, for example, true reliability is . 90 , then the probability of acceptance is about. 45 .
3.

> Method
> d. not just at $R_{0}$ and $R_{1}$. The operating characteristic curve is defined by:
> P \{acceptance $\mid$ reliability $=R\}$
> $=\sum_{k=0}^{c}\left(\int_{k}^{n}\right)(1-R)^{k_{R} n-k}$

Example

Figures 6.3.2.1A-6.3.2.13A and 6.3.2.1B-6.3.2.13B are the operating characteristic curves for test plans 1A-13A and 1B-13B, respectively.

TABLE 6.3.2.1. NONPARAMETRIC RELIABILITY DEMONSTRATION TEST PLANS
The planned producer's and consumer's risks are . 10 for test plans 1A-13A, and . 20 for test plans $18-13 B$.

ACCEPT
REJECT
Plan alpha beta RO Rl $n$ equal or less equal cr more

| 1A | . 095 | . 096 | . 50 | . 40 | 168 | 92 | 93 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 A | . 091 | . 099 | . 80 | . 70 | 127 | 31 | 32 |
| 3A | . 087 | . 099 | . 85 | . 75 | 109 | 21 | 22 |
| 4 A | . 086 | . 099 | . 90 | . 80 | 86 | 12 | 13 |
| 5A | . 096 | . 094 | . 91 | . 81 | 79 | 10 | 11 |
| 6A | . 087 | . 093 | . 92 | . 82 | 77 | 9 | 10 |
| 7 A | . 095 | . 098 | . 93 | . 83 | 67 | 7 | 8 |
| 8A | . 088 | . 096 | . 94 | . 134 | 64 | 6 | 7 |
| 9A | . 079 | . 097 | . 95 | . 85 | 60 | 5 | 6 |
| 10A | . 073 | . 092 | . 96 | . 86 | 56 | 4 | 5 |
| 11A | . 063 | . 096 | . 97 | . 87 | 50 | 3 | 4 |
| 12A | . 055 | . 097 | . 98 | . 88 | 43 | 2 | 3 |
| 13A | . 045 | . 099 | . 99 | . 89 | 34 | 1 | 2 |
| 1 B | . 181 | . 194 | . 50 | . 40 | 77 | 42 | 43 |
| 2 B | . 197 | . 190 | . 80 | . 70 | 55 | 13 | 14 |
| 3 B | . 191 | . 183 | . 85 | . 75 | 49 | 9 | 10 |
| 4 B | . 190 | . 180 | . 90 | . 80 | 39 | 5 | 6 |
| 5 B | . 187 | . 199 | . 91 | . 81 | 34 | 4 | 5 |
| 6B | . 157 | . 199 | . 92 | . 82 | 36 | 4 | 5 |
| 7 B | . 183 | . 183 | . 93 | . 83 | 32 | 3 | 4 |
| 8 B | . 145 | . 184 | . 94 | . 84 | 34 | 3 | 4 |
| 9 B | . 163 | . 187 | . 95 | . 85 | 28 | 2 | 3 |
| 10 B | . 117 | . 189 | . 96 | . 86 | 30 | 2 | 3 |
| 1iB | . 151 | . 180 | . 97 | . 87 | 23 | 1 | 2 |
| 12 B | . 083 | . 199 | . 98 | . 88 | 24 | 1 | 2 |
| 13B | . 131 | . 196 | . 99 | . 89 | 14 | 0 | 1 |



Figure 6.3.2.1A. Operating Characteristic Curve for Teat Plan 1A


Figure 6.3.2.2A. Operating Characteristic Curve for Test Plan 2A


Figure 6.3.2.3A. Operating Characteristic Curve for Tess Plan 3A


Figure 6.3.2.4A. Operating Characteristic Curve for Tcst Plan 4A


Figure 6.3.2.5A. Operating Characteristic Curve for Test Plan 5A


Figure 6.3.2.6A. Operating Characteristic Curve for Test Plan 6A


Figure 6.3.2.7A. Operating Characteristic Curve for Tear Plan 7A


Figure 6.3.2.8A. Operating Characteristic Curve for Test Plan 8A


Figure 6.3.2.9A. Operating Characteristic Curve for Test Plan 9A


Figure 6.3.2.10A. Operating Characteristic Curve for Test Plan 10A


Figure 6.3.2.11A. Operating Characteristics Curve for Test Plan 11A


Figure 6.3.2.12A. Operating Characteristic Curve for Test Plan 12A


Figure 6.3.2.18. Operating Charaçeristic Curve for Tout Plan 18


Figure 6.3.2.2B. Operating Characteriatic Curve for Tesc Plan 2B


Figure 6.3.2.3B. Operating Characteristic Curve for Test Plan 38


Figure 6.3.2.4R. Operating Characteristic Curve for Test Plan 4B


Figure 6.3.2.5B. Operating Characteristic Curve for Test Plan 5B


Figure 6.3.2.6B. Operating Characteristic Curve for Tesi Plan 6B

Figure 6.3.2.7B. Operating Characteristic Curve for Test Plan 7B


Figure 6.3.2.8B. Operating Characteristic Curve for Test Plan 8B


Figure 6.3.2.9B. Operating Characteristic Curve for Test Plan 9B


Figure 6.3.2.10B. Operating Characteristic Curve for Test Plan 10B


Figure 6.3.2.11B. Operating Characteristic Curve for Test Plan 11B


Figure 6.3.2.12B. Operating Characteristic Curve for Test Plan 12B 6-32


Figure 6.3.2.13B. Operating Characteristic Curve for Teat Plan 13B

## APPEND IX I

REFERENCES AND BIBLIOGRAPHY

## APPENDIX I REFERENCES AND BIBLIOGRAPHY

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APPENDIX II
STATISTICAL TABLES

TABLE I. MEDIAN RANKS

```
sample size \(=n\)
failure rank \(=j\)
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| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | .5000 | .2929 | .2063 | .1591 | .1294 | .1091 | .0943 | .0830 | .0741 | .0670 |
| 2 |  | .7071 | .5000 | .3864 | .3147 | .2655 | .2295 | .2021 | .1806 | .1632 |
| 3 |  |  | .7937 | .6136 | .5000 | .4218 | .3648 | .3213 | .2871 | .2594 |
| 4 |  |  |  | .8409 | .6853 | .5782 | .5000 | .4404 | .3935 | .3557 |
| 5 |  |  |  |  | .8706 | .7345 | .6352 | .5596 | .5000 | .4519 |
| 6 |  |  |  |  |  | .8909 | .7705 | .0787 | .6065 | .5481 |
| 7 |  |  |  |  |  |  | .9057 | .7979 | .7129 | .6443 |
| 8 |  |  |  |  |  |  | .9170 | .8194 | .7406 |  |
| 9 |  |  |  |  |  |  |  | .9259 | .8368 |  |
| 10 |  |  |  |  |  |  |  | .9330 |  |  |

sample size $=n$
failure size $=j$

| 1 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | .0611 | .0561 | .0519 | .0483 | .0452 | .0424 | .0400 | .0378 | .0358 | .0341 |
| 2 | .1489 | .1368 | .1266 | .1788 | .1101 | .1034 | .0975 | .0922 | .0874 | .0831 |
| 3 | .2366 | .2175 | .2013 | .1873 | .1751 | .1644 | .1550 | .1465 | .1390 | .1322 |
| 4 | .3244 | .2982 | .2760 | .2568 | .2401 | .2254 | .2125 | .2009 | .1905 | .1812 |
| 5 | .4122 | .3789 | .3506 | .3263 | .3051 | .2865 | .2700 | .2553 | .2421 | .2302 |
| 6 | .5000 | .4596 | .4253 | .3958 | .3700 | .3475 | .3275 | .3097 | .2937 | .2793 |
| 7 | .5878 | .5404 | .5000 | .4653 | .4350 | .4085 | .3850 | .3641 | .3453 | .3283 |
| 8 | .6756 | .6211 | .5747 | .5347 | .5000 | .4695 | .4425 | .4184 | .3968 | .3774 |
| 9 | .7634 | .7018 | .6494 | .6042 | .5650 | .5305 | .5000 | .4728 | .4484 | .4264 |
| 10 | .8511 | .7825 | .7240 | .6737 | .6300 | .5915 | .5575 | .5272 | .5000 | .4755 |
| 11 | .9389 | .8632 | .7987 | .7432 | .6949 | .6525 | .6150 | .5816 | .5516 | .5245 |
| 12 |  | .9439 | .0734 | .8127 | .7599 | .7135 | .6725 | .6359 | .6032 | .5736 |
| 13 |  |  | .9481 | .8822 | .8249 | .7746 | .7300 | .6903 | .6547 | .6226 |
| 14 |  |  |  | .9517 | .8899 | .8356 | .7875 | .7447 | .7063 | .6717 |
| 15 |  |  |  |  | .9548 | .8966 | .8450 | .7991 | .7579 | .7207 |
| 16 |  |  |  |  |  | .9576 | .9025 | .8535 | .8095 | .7698 |
| 17 |  |  |  |  |  |  | .9600 | .9078 | .8610 | .8188 |
| 18 |  |  |  |  |  |  |  | .9622 | .9126 | .8678 |
| 19 |  |  |  |  |  |  |  |  | .9642 | .9109 |
| 20 |  |  |  |  |  |  |  |  |  | .9059 |

TABLE II. TABLE of $5 \%$ RANK $S$
sample size = n

| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | . 0500 | . 0253 | . 0170 | . 0127 | . 0102 | . 0085 | . 0074 | . 0065 | . 0057 | . 0051 |
| 2 |  | . 2236 | . 1354 | . 0976 | . 0764 | . 0629 | . 0534 | . 0468 | . 0410 | . 0368 |
| 3 |  |  | . 3684 | . 2486 | . 1893 | . 1532 | . 1287 | . 1111 | . 0978 | . 0873 |
| 4 |  |  |  | . 4729 | . 3426 | . 2713 | . 2253 | . 1929 | . 1688 | . 1500 |
| 5 |  |  |  |  | . 5493 | . 4182 | . 3413 | . 2892 | . 2514 | . 2224 |
| 6 |  |  |  |  | . 54 | . 6070 | . 4793 | . 4003 | . 3449 | . 3035 |
| 7 |  |  |  |  |  |  | . 6318 | . 5293 | . 4504 | . 3934 |
| 8 |  |  |  |  |  |  |  | . 6877 | . 5709 | . 4931 |
| 9 |  |  |  |  |  |  |  |  | . 7169 | . 6058 |
| 10 |  |  |  |  |  |  |  |  |  | . 7411 |
| sample size $=$ n |  |  |  |  |  |  |  |  |  |  |
| $j$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | . 0047 | . 0043 | . 0040 | . 0037 | . 0034 | . 0032 | . 0030 | . 0029 | . 0028 | . 0026 |
| 2 | . 0333 | . 0307 | . 0281 | . 0263 | . 0245 | . 0227 | . 0216 | . 0205 | . 0194 | . 0183 |
| 3 | . 0800 | . 0719 | . 0665 | . 0611 | . 0574 | . 0536 | . 0499 | . 0476 | . 0452 | . 0429 |
| 4 | . 1363 | . 1245 | . 1127 | . 1047 | . 0967 | . 0910 | . 0854 | . 0797 | . 0761 | . 0725 |
| 5 | . 2007 | . 1824 | . 1671 | . 1527 | . 1424 | . 1321 | . 1247 | . 1173 | . 1099 | . 1051 |
| 6 | . 2713 | . 2465 | . 2255 | . 2082 | . 1909 | . 1786 | . 1664 | . 1575 | . 1485 | . 1396 |
| 7 | . 3498 | . 3152 | . 2883 | . 2652 | . 2459 | . 2267 | . 2128 | . 1990 | . 1887 | . 1735 |
| 8 | . 4356 | . 3909 | . 3548 | . 3263 | . 3015 | . 2805 | . 2601 | . 2449 | . 2298 | . 2183 |
| 9 | . 5299 | . 4727 | . 4274 | . 3904 | . 3608 | . 3350 | . 3131 | . 2912 | . 2749 | . 2586 |
| 10 | . 6356 | . 5619 | . 5054 | . 4600 | . 4226 | . 3922 | . 3542 | . 3429 | . 3201 | . 3029 |
| 11 | . 7616 | . 6613 | . 5899 | . 5343 | . 4893 | . 4517 | . 4208 | . 3927 | . 3703 | . 3469 |
| 12 |  | . 7791 | . 6837 | . 6416 | . 5602 | . 5156 | . 4781 | . 4460 | . 4190 | . 3957 |
| 13 |  |  | . 7942 | . 7033 | . 6366 | . 5834 | . 5395 | . 5022 | .4711 | . 4434 |
| 14 |  |  |  | . 8074 | . 7206 | . 6562 | . 6044 | . 5611 | . 5242 | . 4932 |
| 15 |  |  |  |  | . 8190 | . 7360 | . 6738 | . 6233 | . 5809 | . 5444 |
| 16 |  |  |  |  |  | . 8274 | . 7475 | . 6871 | . 6379 | . 5964 |
| 17 |  |  |  |  |  |  | . 8358 | . 7589 | . 7005 | . 6525 |
| 18 |  |  |  |  |  |  |  | . 8441 | . 7704 | . 7138 |
| 19 |  |  |  |  |  |  |  |  | . 8525 | . 7818 |
| ? 0 |  |  |  |  |  |  |  |  |  | . 8609 |

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TABLE III. TABLE OF 95\% RANKS
sample size $=n$

| J | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | . 9500 | . 7766 | . 6316 | . 5271 | . 4507 | . 3930 | . 3482 | . 3123 | . 2831 | . 2589 |
| 2 |  | . 9747 | . 8646 | . 7514 | . 6574 | . 5818 | . 5207 | . 4707 | . 4291 | . 3942 |
| 3 |  |  | . 9830 | . 9024 | . 8107 | . 7287 | . 6587 | . 5997 | . 5496 | . 5069 |
| 4 |  |  |  | . 9873 | . 9236 | . 8468 | . 7747 | . 7108 | . 6551 | . 6056 |
| 5 |  |  |  |  | . 9898 | . 9371 | . 8713 | . 8071 | . 7486 | . 6965 |
| 6 |  |  |  |  |  | . 9915 | . 9466 | . 8889 | . 831.2 | . 7776 |
| 7 |  |  |  |  |  |  | . 9926 | . 9532 | . 9032 | . 8500 |
| 8 |  |  |  |  |  |  |  | . 9935 | . 9590 | . 9127 |
| 9 |  |  |  |  |  |  |  |  | . 9943 | . 9632 |
| 10 |  |  |  |  |  |  |  |  |  | . 9949 |
| sample size - $n$ |  |  |  |  |  |  |  |  |  |  |
| j | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | . 2384 | . 2209 | . 2058 | . 1926 | . 1810 | . 1726 | . 1642 | . 1559 | . 1475 | . 1391 |
| 2 | . 3644 | . 3387 | . 3163 | . 2967 | . 2194 | . 2640 | . 2525 | . 2411 | . 2296 | . 2182 |
| 3 | . 4701 | . 4381 | . 4101 | . 3854 | . 3634 | . 3438 | . 3262 | . 3129 | . 2995 | . 2862 |
| 4 | . 5644 | . 5273 | . 4946 | . 4657 | . 4398 | . 4166 | . 3956 | . 3767 | . 3621 | . 3475 |
| 5 | . 6502 | . 6091 | . 5726 | . 5400 | . 5107 | . 4844 | . 4605 | . 4389 | . 4191 | . 4036 |
| 6 | . 7287 | . 6848 | . 6452 | . 6096 | . 5774 | . 5483 | . 5219 | . 4978 | . 4758 | . 4556 |
| 7 | . 7993 | . 7535 | . 7117 | . 6737 | . 6392 | . 6078 | . 5792 | . 5540 | . 5289 | . 5068 |
| 8 | . 8637 | . 8176 | . 7745 | . 7348 | . 6984 | . 6650 | . 6458 | . 6063 | . 5884 | . 5566 |
| 9 | . 9200 | . 8755 | . 8329 | . 7918 | . 7541 | . 7195 | . 6869 | . 6571 | . 6297 | . 6043 |
| 10 | . 9667 | . 9281 | . 8873 | . 8473 | . 8091 | . 7733 | . 7399 | . 7088 | . 6799 | . 6531 |
| 11 | . 9953 | . 9693 | . 9335 | . 8953 | . 8576 | . 214 | . 7872 | . 7551 | . 7251 | . 6971 |
| 12 |  | . 9957 | . 9719 | . 9389 | . 9033 | . 8679 | . 8336 | . 8010 | . 7702 | . 7413 |
| 13 |  |  | . 9960 | . 9737 | . 9426 | . 9090 | . 8753 | . 8425 | . 8113 | . 7818 |
| 14 |  |  |  | . 9963 | . 9755 | . 9464 | . 9146 | . 8827 | . 8525 | . 8215 |
| 15 |  |  |  |  | . 9966 | . 9773 | . 9501 | . 9203 | . 8901 | . 8604 |
| 16 |  |  |  |  |  | . 9968 | . 9784 | . 9534 | . 9239 | . 8949 |
| 17 |  |  |  |  |  |  | . 9970 | . 9795 | . 9548 | . 9275 |
| 18 |  |  |  |  |  |  |  | . 9971 | . 9806 | .9571 |
| 19 |  |  |  |  |  |  |  |  | . 9972 | . 9817 |
| 20 |  |  |  |  |  |  |  |  |  | . 9974 |



TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION

| $s / X$ | . 5 th | 18t | 2.5 th | 5 th |
| :---: | :---: | :---: | :---: | :---: |
| .01 .02 | $\begin{array}{r} .974522 \\ .949602 \end{array}$ | $\begin{aligned} & .976956 \\ & .954342 \end{aligned}$ | $\begin{aligned} & .980542 \\ & .96137 \end{aligned}$ | $\begin{aligned} & .983637 \\ & .967447 \end{aligned}$ |
| . 03 | . 925235 | . 932184 | . 942484 | . 951434 |
| . 04 | . 901413 | . 91045 | . 923886 | . 9356 |
| . 05 | . 87813 | . 889146 | . 905574 | . 919946 |
| . 06 | . 855381 | . 86827 | . 887551 | . 904476 |
| . 07 | . 833158 | . 847817 | . 869815 | . 889191 |
| . 08 | . 811453 | . 827785 | . 852368 | . 874093 |
| . 09 | . 790261 | . 808169 | . 835208 | . 859184 |
| . 1 | . 769574 | . 788966 | . 818334 | . 844465 |
| . 11 | . 749384 | . 77017 | . 801747 | . 829938 |
| . 12 | . 729683 | . 751778 | . 785445 | . 815604 |
| . 13 | . 710465 | . 733785 | . 769428 | . 801464 |
| . 14 | . 69172 | . 716186 | . 753693 | . 787519 |
| . 15 | . 673442 | . 698976 | . 73824 | . 773769 |
| . 16 | . 655622 | . 68215 | . 723067 | . 760215 |
| . 17 | . 638252 | . 665703 | . 708172 | . 746857 |
| . 18 | . 621324 | . 649629 | . 693553 | . 733697 |
| . 19 | . 60483 | . 6333924 | . 679208 | . 720732 |
| . 2 | . 588761 | . 618581 | . 665135 | . 707965 |
| . 21 | . 57311 | . 603596 | . 651331 | . 695394 |
| . 22 | . 557868 | . 588961 | . 637794 | . 683019 |
| . 23 | . 543026 | . 574672 | . 624522 | . 670839 |
| . 24 | . 528577 | . 560723 | . 611511 | . 658855 |
| . 25 | . 514512 | . 547108 | . 598759 | . 647065 |
| . 26 | . 500823 | . 533821 | . 586263 | . 635468 |
| . 27 | . 487502 | . 520856 | . 57402 | . 624063 |
| . 28 | . 474541 | . 508207 | . 562026 | . 61285 |
| .29 | . 461931 | . 495863 | . 550279 | . 601827 |
| . 3 | . 449665 | . 483834 | . 538775 | . 590992 |
| . 31 | . 437735 | .472097 | . 527512 | . 580345 |
| . 32 | . 426132 | . 460653 | . 516484 | . 569884 |
| .33 | . 414849 | . 449495 | . 50569 | . 559607 |
| . 34 | . 403879 | . 438618 | . 495125 | . 549513 |
| . 35 | . 393213 | . 428015 | .484787 | . 539599 |
| . 36 | . 382844 | . 417681 | . 47467 | . 529865 |
| . 37 | . 372764 | . 407609 | . 464773 | . 520308 |
| . 38 | . 362967 | . 397795 | . 455091 | . 510926 |
| .39 | . 353445 | . 388232 | . 445621 | . 501718 |
| . 4 | .344191 | . 378914 | . 436358 | . 49268 |
| . 41 | . 335198 | . 369837 | . 4273 | . 483812 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $S / \bar{X}$ | $.5 t h$ | $18 t$ | 2.5 th | 5th |
| :---: | :---: | :---: | :---: | :---: |
| .42 | .326459 | .3609 .94 | .418442 | .475111 |
| .43 | .317968 | .35238 | .409782 | .466575 |
| .44 | .309717 | .34399 | .401315 | .458201 |
| .45 | .301701 | .335818 | .393037 | .449987 |
| .46 | .293912 | .327858 | .384946 | .441932 |
| .47 | .286346 | .320107 | .377036 | .434032 |
| .48 | .278994 | .312558 | .369306 | .426286 |
| .49 | .271853 | .305208 | .3617751 | .418691 |
| .5 | .264915 | .298049 | .354368 | .411244 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / X$ | 10 th | 20 th | 30 th | 40 th |
| :---: | :---: | :---: | :---: | :---: |
| .01 .02 | $\begin{aligned} & .987217 \\ & .974502 \end{aligned}$ | $\begin{array}{r} .991569 \\ .983113 \end{array}$ | $\begin{array}{r} .99472 \\ .989369 \end{array}$ | $\begin{array}{r} .997419 \\ .994747 \end{array}$ |
| . 03 | . 961858 | . 9764634 | . 983951 | . 991983 |
| . 04 | . 94929 | . 966135 | . 978468 | . 98913 |
| . 05 | . 936799 | . 957621 | . 972921 | . 986188 |
| . 06 | . 92439 | . 949092 | . 967313 | . 9831.59 |
| . 07 | . 912066 | . 940554 | . 961647 | . 980044 |
| . 08 | . 899829 | . 932009 | . 955925 | . 976846 |
| . 09 | . 887682 | . 92346 | . 95015 | . 973566 |
| . 1 | . 87563 | . 914911 | . 944325 | . 97020.5 |
| . 11 | . 863673 | . 906364 | . 938451 | . 966766 |
| . 12 | . 851815 | . 897823 | . 932531 | . 96325 |
| . 13 | . 840058 | . 889289 | . 926569 | . 959659 |
| . 14 | . 828405 | . 880768 | . 920567 | . 955995 |
| . 15 | . 816858 | . 87226 | . 914527 | . 95226 |
| . 16 | . 805419 | . 86377 | . 908451 | . 948456 |
| .17 | . 794089 | . 8553 | . 902344 | . 944586 |
| . 18 | . 782872 | . 846852 | . 896207 | . 940651 |
| . 19 | . 771768 | . 83843 | . 890042 | . 936653 |
| . 2 | . 760779 | . 830035 | . 883853 | . 932594 |
| . 21 | . 749907 | . 821671 | . 877642 | . 928478 |
| . 22 | . 739154 | . 81334 | . 871412 | . 924305 |
| . 23 | . 728519 | . 805044 | . 865164 | . 920079 |
| . 24 | . 718006 | . 796786 | . 858903 | . 915801 |
| . 25 | . 707614 | . 788568 | . 852629 | . 911473 |
| . 26 | . 697344 | . 780391 | . 846345 | . 907099 |
| . 27 | . 687198 | . 772259 | . 840055 | . 902679 |
| . 28 | . 677176 | . 764173 | . 83376 | . 898217 |
| . 29 | . 667278 | . 756135 | . 827462 | . 893714 |
| . 3 | . 657506 | . 748147 | . 821164 | . 889173 |
| . 31 | . 647859 | . 740211 | . 814868 | . 884596 |
| . 32 | . 638337 | . 732328 | . 808576 | . 879986 |
| . 33 | . 628942 | . 7245 | . 80229 | . 875343 |
| . 34 | . 619673 | . 716728 | . 796013 | . 870671 |
| . 35 | . 61053 | . 709015 | . 789746 | . 865972 |
| . 36 | . 601512 | . 70136 | . 783492 | . 861247 |
| . 37 | . 59262 | . 693767 | . 777251 | . 8.56499 |
| . 38 | . 583854 | . 686235 | . 771027 | . 851729 |
| . 39 |  |  |  | . 846941 |
| . 4 | . 566696 | . 671362 | . 758633 | . 842.35 |
| . 41 | . 558304 | . 664022 | . 752467 | . 837314 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S/ $\bar{X}$ | .5 th | Lst | 2.5 th | Sth |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| .42 | .550034 | .656748 | .746324 | .83248 |
| .43 | .541888 | .649541 | .740205 | .827634 |
| .44 | .533864 | .642401 | .734112 | .822779 |
| .45 | .525961 | .635329 | .728046 | .817916 |
| .46 | .518179 | .628327 | .722009 | .813046 |
| .47 | .510517 | .621393 | .716002 | .808173 |
| .48 | .502973 | .61453 | .710026 | .803296 |
| .49 | .495547 | .607736 | .704083 | .798418 |
| .5 | .488238 | .601014 | .698173 | .793541 |

TAGLE IV. PERCENTILES OF THE LOG-NORMAL DISTKIBUTION (Continued)

| S/ $/ \overline{ }$ | 60 th | 70 th | 80 th | yoth |
| :---: | :---: | :---: | :---: | :---: |
| . 01 | 1.00249 | 1.00521 | 1.0084 | 1.01285 |
| . 02 | 1.00488 | 1.01034 | 1.01677 | 1.02575 |
| . 03 | 1.00717 | 1.0154 | 1.0251 | 1.03872 |
| . 04 | 1.00937 | 1.02037 | 1.0334 | 1.05174 |
| . 05 | 1.01148 | 1.02527 | 1.4165 | 1.0648 |
| .06 | 1.01348 | 1.03008 | 1.04986 | 1.07791 |
| . 07 | 1.01539 | 1.03481 | 1.05802 | 1.09107 |
| . 08 | 1.01719 | 1.03945 | 1.06613 | 1.10425 |
| . 09 | 1.0189 | 1.04401 | 1.07418 | 1.11746 |
| . 1 | 1.0205 | 1.04847 | 1.08216 | 1.13073 |
| .11 | 1.02201 | 1.05285 | 1.09012 | 1.144 |
| . 12 | 1.02341 | 1.05713 | 1.09794 | 1.1573 |
| . 13 | $1.024 / 2$ | 1.06131 | 1.1058 | 1.17061 |
| . 14 | 1.02592 | 1.0654 | 1.11355 | 1.18393 |
| . 15 | 1.02702 | 1.0694 | 1.12122 | 1.19726 |
| .10 | 1.02803 | 1.0733 | 1.12882 | 1.2106 |
| .17 | 1.02893 | 1.0771 | 1.13634 | 1.2239 S |
| . 16 | 1.02973 | 2.0808 | 1.14378 | 1.23726 |
| . 19 | 1.03043 | 1.08439 | 1.25115 | 1.25058 |
| . 2 | 1.03104 | 1.08789 | 1.15843 | 1.26389 |
| . 21 | 1.03154 | 1.09129 | 1.10563 | 1.27717 |
| . 22 | 1.03195 | 1.09498 | 1.17274 | 1.29044 |
| . 23 | 1.03226 | 1.09778 | 1.17970 | 1.30368 |
| . 24 | 1.03249 | 1.10087 | 1.18604 | 1.31684 |
| . 25 | 1.03259 | 1.10385 | 1.14353 | 1.33007 |
| . 20 | 1.03261 | 1.10673 | 1.20027 | 1.34321 |
| . 27 | 1.03254 | 1.20951 | 1.20692 | 1.35631 |
| . 68 | 1.03238 | 1.11219 | 1.21347 | 1.36436 |
| . 29 | 1.03212 | 1.11476 | 1.21992 | 1.36237 |
| . 3 | 1.03178 | 1.11723 | 1.22627 | 1.39532 |
| . 31 | 1.03135 | 1.1190 | 1.23252 | 1.40822 |
| . 32 | 1.03083 | 1.12160 | 1.23067 | 1.42105 |
| . 33 | 1.03022 | 1. 12402 | 1.24471 | 1.43383 |
| . 34 | 1.02953 | 1.12608 | 1.25065 | 1.44653 |
| . 35 | 1.02875 | 1.12804 | 1.25644 | 1.45917 |
| . 36 | 1.02789 | 1.1294 | 1.20222 | 1.47174 |
| . 37 | 1.02695 | 1.13166 | 1.26784 | 1.48423 |
| . 36 | 1.02594 | 1.13332 | 1.27335 | 1.45064 |
| . 34 | 1.02484 | 1.13488 | 1.27870 | 1. Suby 7 |
| . 4 | 1.02 .367 | 1.13634 | 1.28400 | 1.52122 |
| . 41 | 1.02242 | 1.13771 | 1.28925 | 1.53338 |

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TABLE IV. PERCEATILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $8 / X$ | $60 t h$ | $70 t h$ | $80 t h$ | $90 t h$ |
| :---: | :---: | :---: | :---: | :---: |
| .42 | 1.02111 | 1.13898 | 1.29433 | 1.54545 |
| .43 | 1.01972 | 1.14016 | 1.29931 | 1.55743 |
| .44 | 1.01826 | 1.14124 | 1.30417 | 1.56931 |
| .45 | 1.1673 | 1.14224 | 1.30893 | 1.5811 |
| .46 | 1.01514 | 1.14314 | 1.31358 | 1.5928 |
| .47 | 1.01348 | 1.14395 | 1.31811 | 1.60439 |
| .48 | 1.01176 | 1.14467 | 1.32255 | 1.61588 |
| .49 | 1.00998 | 1.1453 | 1.32687 | 1.62726 |
| .5 | 1.00814 | 1.14585 | 1.33108 | 1.63854 |

table iv. percentiles of the log-normal distribution (Continued)

| $s / \bar{x}$ | 95 th | 97.5th | 99 th | 99.5 th |
| :---: | :---: | :---: | :---: | :---: |
| . 01 | 1.01653 | 1.10974 | 1.02348 | 1.02604 |
| . 02 | 1.03323 | 1.03977 | 1.04741 | 1.05265 |
| . 13 | 1.0501 | 1.06007 | 1.07178 | 1.07983 |
| . 04 | 1.06713 | 1.08066 | 1.0966 | 1.1076 |
| . 05 | 1.08431 | 1.10152 | 1.12187 | 1.13594 |
| . 06 | 1.10165 | 1.12265 | 1.14758 | 1.16488 |
| . 07 | 1.11913 | 1.14406 | 1.17375 | 1.1944 |
| . 06 | 1.13677 | 1.16574 | 1.20036 | 1.22452 |
| . 09 | 1.15454 | 1.18769 | 1.22742 | 1.25524 |
| . 1 | 1.17240 | 1.20989 | 1.25493 | 1.28055 |
| . 11 | 1.1905 | 1.23230 | 1.28289 | 1.31848 |
| . 12 | 1.20868 | 1.25504 | 1.3113 | 1.351 |
| . 13 | 1.22698 | 1.27807 | 1.34015 | 1.38414 |
| . 14 | 1.2454 | 1.30129 | 1.36944 | 1.41788 |
| . 15 | 1.26394 | 1.32476 | 1.39918 | 1.45223 |
| . 16 | 1.28258 | 1.34848 | 1.42936 | 1.4872 |
| .17 | 1.30133 | 1.37242 | 1.45998 | 1.52277 |
| . 18 | 1.32019 | 1.3966 | 1.49103 | 1.55895 |
| . 19 | 1.33913 | 1.421 | 1.52251 | 1.54575 |
| . 2 | 1.35817 | 1.44563 | 1.55442 | 1.63315 |
| . 21 | 1.37729 | 1.47047 | 1.58676 | 1.67117 |
| . 22 | 1.3965 | 1.49552 | 1.61952 | 1.70978 |
| . 23 | 1.41577 | 1.52077 | 1.65269 | 1.74901 |
| . 24 | 1.43512 | 1.54623 | 1.68628 | 1.78883 |
| . 25 | 1.45453 | 1.57188 | 1.72027 | 1.82926 |
| . 26 | 1.474 | 1.59771 | 1.75467 | 1.87028 |
| . 27 | 1.49352 | 1.62373 | 1.78946 | 1.91189 |
| . 28 | 1.51309 | 1.64992 | 1.82465 | 1.9541 |
| . 29 | 1.53271 | 1.67628 | 1.86022 | 1.99688 |
| . 3 | 1.55236 | 1.70281 | 1.39617 | 2.04025 |
| . 31 | 1.57204 | 1.72949 | 1.93249 | 2.08419 |
| . 32 | 1.59175 | 1.75632 | 1.96918 | 2.12871 |
| . 33 | 1.61148 | 1.78329 | 2.00624 | 2.17379 |
| . 34 | 1.63122 | 1.81041 | 2.04364 | 2.21942 |
| . 35 | 1.05098 | 1.83765 | 2.08139 | 2.26501 |
| . 36 | 1.67074 | 1.86502 | 2.11946 | 2.31235 |
| . 37 | 1.69051 | 1.8925 | 2.15791 | 2.35962 |
| . 36 | 1.71027 | 1.9201 | 2.19666 | 2.40743 |
| . 34 | 1.73002 | 1.9476 | 2.23572 | 2.45577 |
| . 4 | 1.74975 | 1.9756 | 2.2751 | 2.50462 |
| . 41 | 1.76947 | 2.00344 | 2.31478 | 2.55398 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / X$ | 95 th | 97.5 th | 99th | 99.5 th |
| :---: | :---: | :---: | :---: | :---: |
| . 42 | 1.78916 | 2.03146 2.05952 | $\begin{aligned} & 2.35475 \\ & 2.395 \end{aligned}$ | $\begin{aligned} & 2.60385 \\ & 2.6542 \end{aligned}$ |
| . 43 | 1.80883 | $2.05952$ | $2.395$ | 2.6542 |
| . 44 | 1.82846 | 2.08764 | 2.43554 | 2.70505 |
| . 45 | 1.84805 | 2.11583 | 2.47634 | 2.75637 |
| . 46 | 1.86761 | 2.14408 | 2.51741 | 2.80816 |
| . 47 | 1.88711 | 2.17238 | 2.55873 | 2.86041 |
| . 48 | 1.90657 | 2.20073 | 2.60029 | $2.91311$ |
| . 49 | 1.92597 | 2.22912 | 2.64209 | 2.96626 |
| . 5 | 1.94531 | 2.25754 | 2.68412 | 3.01983 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | . 5 th | 186 | 2.5 ch | 5 th |
| :---: | :---: | :---: | :---: | :---: |
| . 5 | . 264915 | . 298049 | . 354368 | . 411244 |
| . 51 | . 258175 | . 291079 | . 347152 | . 403944 |
| . 52 | . 251628 | . 284292 | . 340102 | . 396788 |
| . 53 | . 245268 | . 217683 | . 333213 | . 389774 |
| . 54 | . 239089 | . 271248 | . 326481 | . 382898 |
| . 55 | . 233086 | . 264983 | . 319904 | . 37626 |
| . 56 | . 227256 | . 258882 | . 313478 | . 369557 |
| . 57 | . 22.591 | . 252942 | . 3072 | . 363085 |
| . 58 | . 216088 | . 247159 | . 301066 | . 356743 |
| . 59 | . 210742 | . 241527 | . 295074 | . 350529 |
| . 6 | . 205548 | . 236044 | . 28922 | . 34444 |
| . 61 | . 200502 | . 230706 | . 283501 | . 338474 |
| . 62 | . 195599 | . 225507 | . 277914 | . 332629 |
| . 63 | . 190836 | . 220445 | . 272450 | . 326901 |
| . 64 | . 186207 | . 215517 | . 267124 | . 32129 |
| . 65 | . 18171 | . 210717 | . 261916 | . 315793 |
| . 66 | . 17734 | . 206043 | . 256827 | . 310407 |
| . 67 | . 173093 | . 201492 | . 251856 | . 305131 |
| . 68 | . 108966 | . 19706 | . 247 | . 299962 |
| . 69 | . 164956 | . 192743 | . 242256 | . 294898 |
| . 7 | . 101057 | $.188539$ | $.237621$ | $.289937$ |
| . 71 | . 157258 | . 184444 | $.233 C 92$ | $.285077$ |
| . 72 | . 153585 | . 180456 | . 228668 | . 280316 |
| . 73 | . 150005 | . 176571 | . 224346 | . 275652 |
| . 74 | . 146525 | .172787 | . 220123 | . 271083 |
| . 75 | . 143141 | .169101 | . 215997 | . 266606 |
| . 76 | . 1.39851 | . 16551 | . 211963 | . 262221 |
| . 77 | . 136653 | . 162011 | . 208026 | . 257924 |
| . 78 | . 133542 | . 158603 | . 204176 | . 253715 |
| . 78 | . 130517 | . 155281 | . 200414 | . 249591 |
| . 6 | . 127575 | . 152045 | . 196738 | . 245551 |
| . 81 | . 124713 | . 148891 | . 193146 | . 241593 |
| . 82 | . 12193 | . 145817 | . 189634 | . 237714 |
| . 83 | . 119222 | . 14282 i | . 186203 | . 233914 |
| . 84 | . 110588 | . 139901 | . $18284{ }^{\circ}$ | . 230191 |
| . 65 | . 114025 | . 137055 | .17957 | . 226542 |
| . 86 | . 111531 | . 134281 | . 176365 | . 222967 |
| . 87 | . 109104 | . 131576 | . 173232 | . 219464 |
| . 88 | . 106742 | . 128939 | . 17017 | . 216031 |
| . 89 | . 104443 | . 126368 | . 167175 | . 212666 |
| . 9 | . 106205 | . 12386 | . 164248 | . 209369 |

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TABLE IV. PERCENTILE'S OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S/ | $.5 t h$ | lsc | 2.5 th | Sth |
| :---: | :---: | :---: | :---: | :--- |
| .91 | .100027 | .121415 | .161385 | .206138 |
| .92 | .097908 | .11903 | .158586 | .20297 |
| .93 | .095842 | .116705 | .155849 | .199866 |
| .94 | .093831 | .114436 | .153172 | .196823 |
| .95 | .091873 | .112223 | .150554 | .19384 |
| .96 | .089966 | .110063 | .147994 | .190916 |
| .97 | .088108 | .107957 | .145489 | .18805 |
| .98 | .086299 | .105901 | .143039 | .18524 |
| .99 | .084530 | .103895 | .140642 | .182485 |
| 1. | .082819 | .101938 | .138297 | .179783 |

TABLE IV. PEKCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{x}$ | 10th | 20 th | 30 th | 40 th |
| :---: | :---: | :---: | :---: | :---: |
| . 5 | . 488238 | . 601014 | . 698173 | . 793541 |
| . 51 | . 481044 | . 594362 | . 092297 | . 788666 |
| . 52 | $.473965$ | $\begin{aligned} & .587781 \\ & .581272 \end{aligned}$ | $.686457$ | $\begin{aligned} & .783794 \\ & .778927 \end{aligned}$ |
| . 54 | . 400146 | . 574834 | . 674888 | . 774066 |
| . 55 | . 453404 | . 568468 | . 069161 | . 769213 |
| . 56 | . 440771 | . 562174 | . 663473 | . 764369 |
| . 57 | . 440247 | . 555951 | .057824 | . 759535 |
| . 58 | . 43383 | . 5498 | . 652217 | . 754712 |
| وל. | . 427519 | . 54372 | . 64665 | . 749902 |
| . 6 | . 421313 | . 537712 | . 641126 | . 745106 |
| . 61 | . 41521 | . 331775 | . 635644 | . 740324 |
| . 62 | . 409209 | . 525908 | . 630205 | . 735559 |
| . 63 | . 403308 | . 520113 | . 024309 | . 73081 |
| . 64 | . 397506 | . 514387 | . 619457 | . 726078 |
| . 65 | . 391803 | . 508732 | . 61415 | . 721366 |
| . 66 | . 386196 | . 503147 | .608887 | . 716673 |
| . 67 | . 380083 | . 497631 | .00366: | . 712 |
| . 60 | . 375265 | . 492183 | . 598496 | .707348 |
| . 69 | . 369939 | . 486805 | . 593368 | . 702719 |
| . 7 | . 364703 | . 481494 | . 588287 | . 698112 |
| .71 | . 359557 | . 476251 | . 583251 | . 693528 |
| . 72 | . 354499 | . $4710 \% 5$ | . 578261 | . 688968 |
| . 73 | . 349528 | . 465965 | . 573317 | . 684433 |
| . 74 | . 344642 | . 460922 | . 568419 | . 679923 |
| . 75 | . 33984 | .455943 | . 563567 | . 075438 |
| . 76 | . 33512 | . 45103 | . 558762 | .670979 |
| . 77 | . $33 \mathrm{C4} 82$ | . 446181 | . 554003 | . 066548 |
| . 78 | . 325924 | . 441395 | . 549289 | . 662143 |
| . 78 | . 321444 | . 436673 | . 544622 | . 057765 |
| . 8 | . 317041 | . 432013 | . 540001 | . 653416 |
| . 81 | . 312715 | . 427415 | . 535426 | . 049094 |
| .82 .83 | $\begin{array}{r} .308462 \\ .304284 \end{array}$ | $\begin{aligned} & .422877 \\ & .418401 \end{aligned}$ | $\begin{aligned} & .530897 \\ & .526413 \end{aligned}$ | $\begin{aligned} & .644802 \\ & .640537 \end{aligned}$ |
| . 84 | . 300177 | . 413984 | . 521975 | . 636303 |
| . 85 | . 296141 | . 409626 | . 517582 | . 632097 |
| . 86 | . 292174 | . 40532.7 | . 513234 | . 627921 |
| . 87 | . 288276 | . 401085 | . 508931 | . 023775 |
| . 88 | . 284445 | . 396901 | . 504672 | . 619658 |
| . 89 | . 28068 | . 392773 | . 500458 | . 615572 |
| . 9 | . 276979 | . 38870 i | . 496289 | . 611516 |



TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $\mathrm{S} / \overline{\mathrm{X}}$ | 10th | 20ch | 30th | 40th |
| :---: | :---: | :---: | :---: | :---: |
| .91 | .273342 | .384684 | .492163 | .607491 |
| .92 | .269767 | .380721 | .48808 | .603495 |
| .93 | .266254 | .376812 | .484042 | .599531 |
| .94 | .2628 | .372956 | .480046 | .595597 |
| .95 | .259406 | .369152 | .476093 | .591694 |
| .96 | .256069 | .3654 | .472182 | .587821 |
| .97 | .25279 | .361699 | .468313 | .583979 |
| .98 | .249566 | .358048 | .464487 | .580168 |
| .99 | .246397 | .354447 | .460701 | .576387 |
| 1. | .243282 | .350895 | .456957 | .572638 |

Table IV. PERCENTILES OF THE LUG-NORMAL DISTRIBUTIUN (Continuea)

| $s / \bar{x}$ | 60 ch | 70 th | 80 th | 90 th |
| :---: | :---: | :---: | :---: | :---: |
| . 5 | 1.09814 | 1.14585 | 1.33108 | 1.63854 |
| . 51 | 1.00624 | 1.14631 | 1.33519 | 1.64972 |
| . 52 | 1.00429 | 1.14669 | 1.33919 | 1.66078 |
| . 53 | 1.00228 | 1.14699 | 1.34309 | 1.67174 |
| . 54 | 1.00022 | 1.1472 | 1.34688 | 1.68258 |
| . 55 | . 998103 | 1.14734 | 1.35057 | 1.69331 |
| . 56 | . 99594 | 1.1474 | 1.35415 | 1.70393 |
| . 57 | . 993731 | 1.14738 | 1.35763 | 1.71443 |
| . 58 | . 991475 | 1.14729 | 1.361 | 1.72482 |
| . 59 | . 989174 | 1.14712 | 1.36428 | 1.73504 |
| . 6 | . 986831 | 1.14688 | 1.30745 | 1.74524 |
| . 61 | . 984446 | 1.14657 | 1.37052 | 1.75528 |
| . 62 | . 982021 | 1.14619 | 1.3735 | 1.1652 |
| . 63 | . 979557 | 1.1 .4574 | 1.37638 | 1.775 |
| . 64 | . 977057 | 1.14523 | 1.37916 | 1.78460 |
| . 65 | . 974522 | 1.14405 | 1.38184 | 1.79424 |
| . 60 | .97195s | 1.14401 | 1.33443 | 1.80368 |
| . 67 | .969351 | 1.14331 | 1.38693 | 1.813 |
| . 68 | . 960716 | 1.14254 | 1.38933 | 1.8222 |
| . 69 | . 904050 | 1.14172 | 1.39165 | 1.83128 |
| . 7 | . 901365 | 1.14084 | 1.39387 | 1.84024 |
| .71 | . 458647 | 1.1399 | 1.39601 | 1.84908 |
| . 72 | . 555904 | 1.13891 | 1.39805 | 1.8578 |
| .73 | . 953136 | 1.13787 | 1.40001 | 1.8664 |
| . 74 | . 950345 | 1.13677 | 1.40189 | 1.87486 |
| . 75 | . 447532 | 1.13562 | 1.40368 | 1.88324 |
| . 76 | . 444699 | 1.13443 | 1.40539 | 1.89140 |
| . 77 | . 941846 | 1.13318 | 1.40702 | 1.8996 |
| . 78 | . 938974 | 1.13189 | 1.40857 | 1.90761 |
| . 79 | . 936086 | 1.13055 | 1.41004 | 1.9155 |
| . 8 | . 933181 | 1.12917 | 1.41143 | 1.42327 |
| .81 | . 930261 | 1.12775 | 1.41274 | 1.93092 |
| . 82 | . 427328 | 1.12629 | 1.41398 | 1.93846 |
| . 83 | . 924381 | 1.12478 | 1.41515 | 1.94588 |
| . 84 | . 921422 | 1.12324 | 1.41625 | 1.95319 |
| . 85 | . 918452 | 1.12106 | 1.41727 | 1.96039 |
| . 86 | . 312472 | 1.12004 | 1.41822 | 1.96747 |
| . 67 | . 912483 | 1.11839 | 1.41911 | 1.97444 |
| . ¢ | . yu9485 | 1.11671 | 1.41993 | 1.9813 |
| . 89 | - 90648 | 1.11499 | 1.42068 | 1.98805 |
| . 9 | . 903468 | 1.11323 | 1.42136 | 1.94468 |

table IV. percentiles of the log-normal distribution (Continued)

| $S / \bar{X}$ | $60 t h$ | $70 t h$ | $80 t h$ | 90 th |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
| .91 | .900451 | 1.11145 | 1.42199 | 2.00121 |
| .92 | .897428 | 1.10964 | 1.42255 | 2.00763 |
| .93 | .894401 | 1.1078 | 1.42305 | 2.01395 |
| .94 | .891371 | 1.10593 | 1.42349 | 2.02015 |
| .95 | .888337 | 1.10404 | 1.42387 | 2.02626 |
| .96 | .885302 | 1.10212 | 1.42419 | 2.03226 |
| .97 | .882265 | 1.10017 | 1.42446 | 2.03815 |
| .98 | .879227 | 1.0982 | 1.42467 | 2.04394 |
| .99 | .876189 | 1.09621 | 1.42482 | 2.04964 |
| 1. | .873151 | 1.09419 | 1.42493 | 2.05523 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S $/ \mathrm{X}$ | 95 th | 97.5 th | 99 th | 99.5 th |
| :---: | :---: | :---: | :---: | :---: |
| $.5$ | $\begin{gathered} 1.94531 \\ 1.9646 \end{gathered}$ | $\begin{aligned} & 2.25754 \\ & 2.28599 \end{aligned}$ | $\begin{aligned} & 2.68412 \\ & 2.72636 \end{aligned}$ | $\begin{aligned} & 3.01983 \\ & 3.07383 \end{aligned}$ |
| . 52 | 1.98381 | 2.31446 | 2.76882 | 3.12824 |
| . 53 | 2.00296 | 2.34295 | 2.81148 | 3.18305 |
| . 54 | 2.02203 | 2.37145 | 2.85433 | 3.23826 |
| . 55 | 2.04103 | 2.39995 | 2.89737 | 3.29385 |
| . 56 | 2.05994 | 2.42845 | 2.94059 | 3.34982 |
| . 57 | 2.07878 | 2.45694 | 2.98397 | 3.40615 |
| . 58 | 2.09752 | 2.48542 | 3.02752 | 3.46284 |
| . 59 | 2.11618 | 2.51389 | 3.07122 | 3.51987 |
| . 6 | 2.13475 | 2.54233 | 3.11506 | 3.57723 |
| . 61 | 2.15322 | 2.57074 | 3.15904 | 3.63492 |
| . 62 | 2.17159 | 2.59912 | 3.20315 | 3.69292 |
| . 63 | 2.18986 | 2.62747 | 3.24738 | 3.75123 |
| . 64 | 2.20803 | 2.65577 | 3.29172 | 3.80983 |
| . 65 | 2.2261 | 2.68402 | 3.33616 | 3.86872 |
| . 66 | 2.24406 | 2.71222 | 3.3807 | 3.92788 |
| . 67 | 2.26191 | 2.74036 | 3.42533 | 3.98731 |
| . 68 | 2.27964 | 2.76845 | 3.47005 | 4.04699 |
| . 69 | 2.29727 | 2.79647 | 3.51 .483 | 4.10592 |
| . 7 | 2.31478 | 2.82442 | 3.55969 | 4.16708 |
| .71 | 2.33217 | 2.85229 | 3.6046 | 4.22747 |
| . 72 | 2.34944 | 2.88009 | 3.64956 | 4.28807 |
| . 73 | 2.3666 | 2.90781 | 3.69457 | 4.34889 |
| . 74 | 2.38363 | 2.93545 | 3.73962 | 4.40989 |
| . 75 | 2.40054 | 2.963 | 3.78471 | 4.47109 |
| . 76 | 2.41733 |  | 3.82981 |  |
| . 77 | 2.43399 | 3.01782 | 3.87494 | 4.59401 |
| . 78 | 2.45052 | 3.04509 | 3.92008 | 4.65572 |
| . 79 | 2.46693 | 3.07226 | 3.96522 | 4.71758 |
| . 8 | 2.48321 | 3.09932 | 4.01036 | 4.77958 |
| . 81 | 2.49936 | 3.12628 | 4.0555 | 4.84172 |
| . 82 | 2.51538 | 3.15313 | 4.10063 | 4.90398 |
| . 83 | 2.53127 | 3.17987 | 4.14574 | 4.96636 |
| . 84 | 2.54703 | 3.20649 | 4.19082 | 5.02885 |
| . 85 | 2.56266 | 3.233 | 4.23588 | 5.09144 |
| . 86 | 2.57815 | 3.25939 | 4.2809 | 5.15412 |
| . 87 | 2.59351 | 3.28566 | 4.32588 | 5.21689 |
| . 80 | 2.60874 | 3.31181 | 4.37082 | 5.27973 |
| . 89 | 2.62384 | 3.33783 | 4.41571 | 5.34265 |
| . 9 | 2.63881 | 3.36372 | 4.46054 | 5.40 .562 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $S / \bar{X}$ | $95 t h$ | $97.5 t h$ | $99 t h$ | $99.5 t h$ |
| :---: | :---: | :---: | :---: | :---: |
| .91 | 2.65364 | 3.38949 | 4.50532 | 5.46865 |
| .92 | 2.66834 | 3.41513 | 4.55003 | 5.53173 |
| .93 | 2.6829 | 3.44064 | 4.59468 | 5.59485 |
| .94 | 2.69733 | 3.46601 | 4.63925 | 5.658 |
| .95 | 2.71163 | 3.49125 | 4.68374 | 5.72118 |
| .96 | 2.72579 | 3.51636 | 4.72816 | 5.78438 |
| .97 | 2.73982 | 3.54132 | 4.77248 | 5.84759 |
| .98 | 2.75372 | 3.56615 | 4.81672 | 5.9108 |
| .99 | 2.76749 | 3.59084 | 4.86087 | 5.97402 |
| 1. | 2.78112 | 3.61539 | 4.90493 | 6.03723 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | . 5 th | 1st | 2.5 th | 5th |
| :---: | :---: | :---: | :---: | :---: |
| 1. | . 082319 | .101938 | . 138297 | . 179783 |
| 1.01 | . 081145 | .100027 | .136003 | . 177134 |
| 1.02 | . 079515 | . 098162 | . 133758 | . 174537 |
| 1.03 | . 077925 | . 096342 | . 131561 | . 17199 |
| 1.04 | . 076376 | . 094564 | . 129412 | . 169492 |
| 1.05 | . 074866 | . 092829 | . 127308 | . 167043 |
| 1.06 | . 073393 | . 091135 | . 125249 | . 16464 |
| 1.07 | . 071958 | . 08948 | . 123233 | . 162283 |
| 1.08 | . 070558 | . 087864 | . 12126 | . 159972 |
| 1.09 | . 0691.93 | . 086286 | . 119329 | . 157704 |
| 1.1 | . 067862 | . 084744 | . 117438 | . 15548 |
| 1.11 | . 066563 | . 083238 | . 115586 | . 153297 |
| 1.12 | . 065296 | . 081767 | .113773 | . 151156 |
| 1.13 | . 06406 | . 080329 | . 111998 | . 149055 |
| 1.14 | . 062855 | . 078924 | . 110259 | . 146993 |
| 1.15 | . 061678 | . 077551 | . 108556 | . 14497 |
| 1.16 | . 06053 | . 076209 | . 106889 | . 142985 |
| 1.17 | . 059409 | . 074898 | . 105255 | . 141036 |
| 1.18 | . 058315 | . 073616 | . 103654 | . 139124 |
| 1.19 | . 057247 | . 072363 | . 102086 | . 137247 |
| 1.2 | . 056204 | . 071137 | . 10055 | . 135404 |
| 1.21 | . 055186 | . 069939 | . 099045 | . 133595 |
| 1.22 | . 054192 | . 068768 | . 097569 | . 131819 |
| 1.23 | . 053221 | . 067622 | . 096124 | . 130076 |
| 1.24 | . 052272 | . 066501 | . 094707 | . 128364 |
| 1.25 | . 051346 | . 065405 | . 093318 | . 126683 |
| 1.26 | . 050441 | . 064332 | . 091957 | . 125032 |
| 1.27 | . 049557 | . 063283 | . 090622 | . 123411 |
| 1.28 | . 048693 | . 062257 | . 089314 | . 121818 |
| 1.29 | . 047849 | . 061252 | . 088031 | . 120255 |
| 1.3 | . 047024 | . 060269 | . 086773 | . 118718 |
| 1.31 | . 046217 | . .059307 | . 085539 | . 11721 |
| 1.32 | . 045429 | . 058366 | . 08433 | . 115727 |
| 1.33 | . 044658 | . 057444 | . 083143 | . 114271 |
| 1.34 | . 043905 | . 056542 | . 08198 | . 11284 |
| 1.35 | . 043168 | . 055658 | . 080838 | . 111434 |
| 1.36 | . 042447 | . 054793 | . 079718 | . 110053 |
| 1.37 | . 041743 | . 053946 | . 07862 | . 108695 |
| 1.38 | . 041053 | . 053116 | . 077542 | .107361 |
| 1.39 | . 040379 | . 052304 | . 076484 | . 106049 |
| 1.4 | . 03972 | . 051508 | . 075446 | . 10476 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $S / X$ | $.5 t h$ | $18 t$ | $2.5 t h$ | 5 th |
| :--- | :--- | :--- | :--- | :--- |
| 1.41 | .039074 | .050728 | .074427 | .103493 |
| 1.42 | .038443 | .049965 | .073428 | .102248 |
| 1.43 | .037825 | .049216 | .072446 | .101024 |
| 1.44 | .03722 | .048483 | .071483 | .09982 |
| 1.45 | .036628 | .047764 | .070537 | .098636 |
| 1.46 | .036049 | .04706 | .069609 | .097472 |
| 1.47 | .035482 | .04637 | .068698 | .096328 |
| 1.48 | .034926 | .045694 | .067803 | .095203 |
| 1.49 | .034383 | .04503 | .066924 | .094096 |
| 1.5 | .03385 | .04438 | .066061 | .093007 |

TABLE IV. PERCENTILES GF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | 10th | 20th | 30 th | 40 th |
| :---: | :---: | :---: | :---: | :---: |
| 1. | . 243282 | . 350895 | . 456957 | . 572638 |
| 1.01 | . 240219 | . 34739 | . 453253 | . 568918 |
| 1.02 | . 237208 | . 343934 | . 44959 | . 56523 |
| 1.03 | . 234248 | . 340525 | . 445967 | . 561572 |
| 1.04 | . 231338 | . 337161 | . $442383{ }^{\circ}$ | . 557944 |
| 1.05 | . 228476 | . 333844 | . 438839 | . 554347 |
| 1.06 | . 225663 | . 330571 | . 435334 | . 55078 |
| 1.07 | . 222896 | . .327343 | .431867 | . 547243 |
| 1.08 | . 220176 | . 324158 | . 428438 | . 543737 |
| 1.09 | . 217501 | . 321017 | . 425047 | . 54026 |
| 1.1 | . 21487 | . 317918 | . 421694 | . 536813 |
| 1.11 | . 212283 | . 314861 | . 418377 | . 533396 |
| 1.12 | . 209739 |  | . 415097 |  |
| 1.13 | . 207237 | $.30887$ | . 411854 | $.52665$ |
| 1.14 | . 204776 | . 305935 | . 408646 | . 52332 |
| 1.15 | . 202355 | . 303039 | . 405474 | . 52002 |
| 1.16 | . 199974 | . 300182 | . 402336 | . 516749 |
| 1.17 | . 197632 | . 297363 | . 399234 | . 513507 |
| 1.18 | . 195327 | . 294582 | . 396166 | . 510293 |
| 1.19 | . 193061 | . 291839 | . 393131 | . 507107 |
| 1.2 | . 19083 | . 289131 | . 390131 | . 50395 |
| 1.21 | 1.88636 | . 28646 | . 387163 | . 500821 |
| 1.22 | . 186478 | . 283825 | . 384229 | . 497719 |
| 1.23 | . 184353 | . 281224 | . 381326 | . 494645 |
| 1.24 | . 1842263 | . 278658 | . 378456 | . 491599 |
| 1.25 | . 180206 | . 276125 | . 375618 | . 48858 |
| 1.26 | . 178182 | . 273626 | . 37281 | . 485587 |
| 1.27 | . 17619 | . 27116 | . 370034 | . 482622 |
| 1.28 | . 174229 | . 268726 | . 367288 | . 479683 |
| 1.29 | . 1723 | . 266325 | . 364573 | . 47677 |
| 1.3 | . 1704 | . 263954 | . 361887 | .473884 |
| 1.31 | . 168531 | . 261615 | . 359231 | . 471023 |
| 1.32 | . 16669 | .259306 | . 356604 | . 468188 |
| 1.33 | . 164879 | . 257027 | . 354006 | . 465379 |
| 1.34 | . 163095 | . 254777 | . 351436 | . 462595 |
| 1.35 | . 161339 | . 252557 | . 348894 | . 459836 |
| 1.36 | . 15961 | . 250365 | . 34638 | . 457102 |
| 1.37 | . 157907 | . 248201 | . 343893 | . 454393 |
| 1.38 | . 15623 | . 246065 | . 341434 | . 451708 |
| 1.39 | . 15458 | . 243957 | . 339001 | . 449047 |
| 1.4 | . 152954 | . 241875 | . 336595 | . 44641 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

|  | $10 t h$ | $20 t h$ | 30 th | 40th |
| :--- | :--- | :--- | :--- | :--- |
| 1.41 | .151353 | .23982 | .334214 | .443797 |
| 1.42 | .149776 | .237791 | .331859 | .441208 |
| 1.43 | .148222 | .235788 | .32953 | .438642 |
| 1.44 | .146692 | .23381 | .327226 | .436098 |
| 1.45 | .145185 | .231857 | .324946 | .433578 |
| 1.46 | .143701 | .229929 | .322691 | .431081 |
| 1.47 | .142238 | .228024 | .320461 | .428606 |
| 1.48 | .140797 | .226144 | .318254 | .426153 |
| 1.49 | .139377 | .224286 | .31607 | .423722 |
| 1.5 | .137979 | .222452 | .31391 | .421313 |

TABLE IV, PERCENIILES OF THE LUG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{x}$ | buth | 70 th | 80 ch | 90th |
| :---: | :---: | :---: | :---: | :---: |
| 1. | . 873151 | 1.09419 | 1.42493 | 2.05523 |
| 1.01 | . 870115 | 1.09216 | 1.42498 | 2.06072 |
| 1.02 | . 86708 | 1.0901 | 1.42498 | 2.06611 |
| 1.03 | . 864046 | 1.08803 | 1.42493 | 2.07141 |
| 1.04 | .861016 | 1.08593 | 1.42483 | 2.07661 |
| 1.05 | . 857989 | 1.08382 | 1.42469 | 2.08172 |
| 1.06 | . 854965 | 1.08169 | 1.4245 | 2.08073 |
| 1.07 | .851946 | 1.07955 | 1.42426 | 2.09165 |
| 1.08 | . 84893 | 1.07739 | 1.42398 | 2.09648 |
| l.uy | . 84542 | 1.07521 | 1.42365 | 2.10122 |
| 1.1 | . 842916 | 1.07303 | 1.42328 | 2.10586 |
| 1.11 | . 839917 | 1.07082 | 1.42288 | 2.11042 |
| 1.12 | . 836424 | 1.06861 | 1.42243 | 2.11489 |
| 1.13 | . 833938 | 1.06638 | 1.42194 | 2.11928 |
| 1.14 | . 830959 | 1.06414 | 1.42141 | 2.12358 |
| 1.15 | . 827987 | 1.06189 | 1.42084 | 2.12779 |
| 1.16 | . 825022 | 1.05963 | 1.42024 | 2.13192 |
| 1.17 | . 822065 | 1.05737 | 1.4196 | 2.13597 |
| 1.10 | . 819117 | 1.05509 | 1.41892 | 2.13994 |
| 1.19 | . 816177 | 1.0528 | 1.41821 | 2.14383 |
| 1.2 | . 813246 | 1.05051 | 1.41747 | 2.14764 |
| 1.21 | . 810324 | 1.04821 | 1.4167 | 2.15137 |
| 1.22 | . 807411 | 1.0459 | 1.41589 | 2.15502 |
| 1.23 | . 804508 | 1.04358 | 1.41505 | 2.1586 |
| 1.24 | .801614 | 1.04126 | 1.41418 | 2.1621 |
| 1.25 | . 79873 | 1.03894 | 1.41328 | 2.16553 |
| 1.26 | . 745857 | 1.03661 | 1.41236 | 2.16889 |
| 1.27 | . 792994 | 1.03427 | 1.4114 | 2.17217 |
| 1.28 | . 790141 | 1.03193 | 1.41042 | 2.17539 |
| 1.29 | . 787299 | 1.02959 | 1.40341 | 2.17853 |
| 1.3 | . 704468 | 1.02724 | 1.40838 | 2.1816 |
| 1.31 | . 781648 | 1.0249 | 1.40731 | 2.18461 |
| 1.32 | . 778839 | 1.02254 | 1.40623 | 2.10755 |
| 1.35 | . 770042 | 1.02019 | 1.40512 | 2.15042 |
| 1.34 | . 77325 | 1.01784 | 1.40397 | 2.19323 |
| 1.35 | . 770481 | 1.01548 | 1.40283 | 2.19597 |
| 1.30 | . 767718 | 1.01312 | 1.40166 | 2.19865 |
| 1.57 | . 704967 | 1.01077 | 1.40046 | 2.20127 |
| 1.38 | . 702228 | 1.00841 | $1.3 y y 24$ | 2.20382 |
| 1.34 | . 759501 | 1.00605 | 1.398 | 2.20632 |
| 1.4 | . 750780 | 1.00369 | 1. 39674 | 2.20875 |



TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S/ $\bar{X}$ | 60th | 70th | 80th | 90th |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| 1.41 | .754083 | 1.00134 | 1.39546 | 2.21113 |
| 1.42 | .751393 | .998978 | 1.39416 | 2.21344 |
| 1.43 | .748715 | .996623 | 1.39285 | 2.2157 |
| 1.44 | .746049 | .99427 | 1.39152 | 2.21791 |
| 1.45 | .743396 | .991917 | 1.39017 | 2.22006 |
| 1.46 | .740755 | .989567 | 1.3888 | 2.22215 |
| 1.47 | .738126 | .987219 | 1.38742 | 2.22419 |
| 1.48 | .735511 | .984874 | 1.38602 | 2.22618 |
| 1.49 | .732907 | .982531 | 1.38461 | 2.22811 |
| 1.5 | .730317 | .98019 | 1.38318 | 2.22999 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | 95 ch | 97.5 th | 99 th | 99.5 th |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 2.78112 | 3.61539 | 4.90493 | 6.03723 |
| 1.01 | 2.79462 | 3.6398 | 4.94888 | 6.10043 |
| 1.02 | 2.80799 | 3.66407 | 4.99273 | 6.16361 |
| 1.03 | 2.82123 | 3.68819 | 5.03647 | 6.22677 |
| 1.04 | 2.83434 | 3.71217 | 5.0801 | 6.2899 |
| 1.05 | 2.84731 | 3.736 | 5.12363 | 6.35299 |
| 1.06 | 2.86016 | 3.75969 | 5.16703 | 6.41605 |
| 1.07 | 2.87288 | 3.78324 | 5.21032 | 6.47906 |
| 1.08 | 2.88547 | 3.80664 | 5.25348 | 6.54201 |
| 1.09 | 2.89793 | 3.82989 | 5.29653 | 6.60492 |
| 1.1 | 2.91027 | 3.85294 | 5.33944 | 6.66776 |
| 1.11 | 2.92247 | 3.87595 | 5.38223 | 6.73053 |
| 1.12 | 2.93456 | 3.89876 | 5.42489 | 6.79324 |
| 1.13 | 2.94651 | 3.92143 | 5.46741 | 6.85587 |
| 1.14 | 2.95834 | 3.94394 | 5.50979 | 6.91843 |
| 1.15 | 2.97005 | 3.96631 | 5.55204 | 6.9809 |
| 1.16 | 2.98163 | 3.98853 | 5.59415 | 7.03426 |
| 1.17 | 2.9931 | 4.0106 | 5.63612 | 7.10558 |
| 1.16 | 3.00443 | 4.03252 | 5.67794 | 7.16778 |
| 1.19 | 3.01565 | 4.0543 | 5.71962 | 7.22988 |
| 1.2 | 3.02075 | 4.07593 | 5.76115 | 7.29167 |
| 1.21 | 3.03773 | 4.09741 | 5.80253 | 7.35376 |
| 1.22 | 3.04859 | 4.11874 | 5.84376 | 7.41554 |
| 1.23 | 3.05934 | 4.13992 | 5.88484 | 7.47721 |
| 1.24 | 3.06996 | 4.16096 | 5.92577 | 7.53876 |
| 1.25 | 3.08047 | 4.18185 | 5.96654 | 7.6002 |
| 1.26 | 3.09087 | $4.2025 y$ | 6.00716 | 7.66151 |
| 1.27 | 3.10125 | 4.22319 | 6.04762 | 7.72269 |
| 1.28 | 3.11132 | 4.24364 | 6.05792 | 7.78375 |
| 1.29 | 3.12138 | 4.26395 | 6.12806 | 7.84467 |
| 1.3 | 3.13132 | 4.28411 | 6.10805 | 7.90546 |
| 1.31 | 3.14116 | 4.30413 | 6.20787 | 7.96612 |
| 1.32 | 3.15088 | 4.324 | 6.24753 | 8.02663 |
| 1.33 | 3.1605 | 4.34373 | 6.28702 | 8.08701 |
| 1.34 | 3.17001 | 4.36331 | e. 32630 | 8.14724 |
| 1.35 | 3.17941 | 4.38276 | 6.36552 | 8.20732 |
| 1.36 | 3.1887 | 4.40206 | 6.40453 | 8.26725 |
| 1.37 | 3.19789 | 4.42122 | 6.44337 | 8.32704 |
| 1.38 | 3.20698 | 4.44023 | 0.48204 | 8.38067 |
| 1.39 | 3.21596 | 4.45911 | 6.52054 | 8.44015 |
| 1.4 | 3.22485 | 4.47785 | 6.55888 | 8.50547 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S/X | 95 th | $97.5 t h$ | $99 t h$ | 94.5 th |
| :---: | :---: | :---: | :---: | :---: |
| 1.41 | 3.23363 | 4.49645 | 6.59705 | 8.56464 |
| 1.42 | 3.24231 | 4.51491 | 6.63506 | 8.62364 |
| 1.43 | 3.25089 | 4.53323 | 6.67289 | 8.68248 |
| 1.44 | 3.25937 | 4.55142 | 6.71056 | 8.74116 |
| 1.45 | 3.26775 | 4.56947 | 6.74806 | 8.79968 |
| 1.46 | 3.27604 | 4.58738 | 6.78539 | 8.85802 |
| 1.47 | 3.28423 | 4.60516 | 6.82255 | 8.9162 |
| 1.48 | 3.29233 | 4.6228 | 6.85954 | 8.97422 |
| 1.49 | 3.30034 | 4.64032 | 6.89636 | 9.03206 |
| 1.5 | 3.30825 | 4.65769 | 6.93302 | 9.08973 |

TABLE IV. PERCENTILES OP THE LOG-NORMAL DISTRIBUTION (Continued)

| $s \sqrt{x}$ | . 5 th | $18 t$ | 2.5 th | Sth |
| :---: | :---: | :---: | :---: | :---: |
| 1.5 | . 03385 | . 04438 | . 066061 | . 093007 |
| 1.51 | . 033329 | . 043743 | . 065213 | . 091936 |
| 1.52 | . 032818 | . 043118 | . 06438 | .090883 |
| 1.53 | . 032318 | . 042505 | . 063562 | . 089847 |
| 1.54 | . 031828 | . 041904 | . 062759 | . 088828 |
| 1.55 | . 031347 | . 041314 | . 06197 | . 087825 |
| 1.56 | . 030877 | . 040736 | . 061194 | . 086838 |
| 1.57 | . 030416 | . 040169 | . 060432 | . 085867 |
| 1.58 | . 029964 | . 039612 | . 059684 | . 084911 |
| 1.59 | . 029522 | . 039066 | . 058948 | . 083971 |
| 1.6 | . 029088 | . 03853 | . 058225 | . 083046 |
| 1.61 | . 028662 | . 038005 | . 057514 | . 082135 |
| 1.62 | . 028246 | . 037489 | . 056816 | . 081238 |
| 1.63 | . 027837 | . 036982 | . 056129 | . 080356 |
| 1.64 | . 027436 | . 036485 | . 055454 | . 079487 |
| 1.65 | . 027043 | . 035998 | . 05479 | . 078632 |
| 1.66 | . 026657 | . 035519 | . 054138 | . 07779 |
| 1.67 | . 026279 | . 035048 | . 053496 | . 076961 |
| 1.68 | . 025909 | . 034587 | . 052865 | . 076145 |
| 1.69 | . 025545 | . 034134 | . 052245 | . 075341 |
| 1.7 | . 025188 | . 033688 | . 051635 | . 074549 |
| 1.71 | . 024838 | . 033251 | . 051034 | . 073769 |
| 1.72 | . 024495 | . 032822 | . 050444 | . 073002 |
| 1.73 | . 024158 | . 0324 | . 049863 | . 072245 |
| 1.74 | . 023827 | . 031986 | . 049292 | . 0715 |
| 1.75 | . 023503 | . 031579 | . 04873 | . 070767 |
| 1.76 | . 023184 | . 031179 | . 048177 | . 070044 |
| 1.77 | . 022871 | . 030787 | . 047633 | . 069331 |
| 1.78 | . 022565 | . 030401 | . 047098 | . 06863 |
| 1.79 | . 022263 | . 030021 | . 046571 | . 067938 |
| 1.8 | . 021967 | . 029649 | . 046053 | . 067257 |
| 1.81 | . 021677 | . 029282 | . 045542 | . 066586 |
| 1.82 | . 021392 | . 028922 | . 04504 | . 065924 |
| 1.83 | . 021112 | . 028568 | . 044546 | . 065272 |
| 1.84 | . 020837 | . 02822 | . 044059 | . 064629 |
| 1.85 | . 020566 | . 027878 | . 04358 | . 063996 |
| 1.86 | . 020301 | . 027542 | . 043108 | . 063372 |
| 1.87 | . 02004 | . 027212 | . 042644 | . 062756 |
| 1.88 | . 019784 | . 026886 | . 042187 | . 062149 |
| 1.89 | . 019532 | . 026567 | . 041737 | $.061551$ |
| 1.9 | . 019285 | . 026252 | . 041293 | . 060962 |

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TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S $/ X$ | . Sth | 18t | 2.Sth | Sth |
| :---: | :---: | :---: | :---: | :---: |
| 1.91 | .019042 | .025943 | .040856 | .06038 |
| 1.92 | .018803 | .025639 | .040426 | .059807 |
| 1.93 | .018569 | .02534 | .040003 | .059242 |
| 1.94 | .018338 | .025045 | .039586 | .058684 |
| 1.95 | .018111 | .024756 | .039175 | .058134 |
| 1.96 | .017889 | .024471 | .03877 | .057592 |
| 1.97 | .017669 | .024191 | .038371 | .057057 |
| 1.98 | .017454 | .023915 | .037978 | .05653 |
| 1.99 | .017242 | .023643 | .03759 | .056009 |
| 2. | .017034 | .023376 | .037209 | .055496 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continuea)

| S/ $\bar{X}$ | 10th | 20th | 30 ch | 40 th |
| :---: | :---: | :---: | :---: | :---: |
| 1.5 | . 137979 | . 222452 | . 31391 | .421313 |
| 1.51 | . 1366 | . 220641 | . 311773 | . 418925 |
| 1.52 | . 135242 | . 218852 | . 309658 | :416559 |
| 1.53 | . 133903 | . 217085 | . 307566 | . 414214 |
| 1.54 | . 132584 | . 21534 | . 305496 | . 41289 |
| 1.55 | . 131284 | . 213616 | . 303448 | . 409586 |
| 1.56 | .130003 | .211913 | . 301421 | . 407304 |
| 1.57 | . 12874 | . 210231 | . 299416 | . 405041 |
| 1.58 | . 127495 | . 20857 | . 297431 | . 402799 |
| $1.5 y$ | $.126267$ | . 206928 | . 295468 | . 400576 |
| 1.6 | . 125058 | . 205307 | . 293524 | .398373 |
| 1.61 | . 123865 | . 203704 | . 291601 | . 39619 |
| 1.62 | . 122689 | . 202121 | . 289698 | . 394026 |
| 1.63 | . 121529 | . 200557 | . 287815 | . 391881 |
| 1.64 | . 120396 | .199012 | . 285951 | . 389755 |
| 1.65 | . 119259 | . 197485 | . 284106 | . 387048 |
| 1.66 | .118147 | . 195976 | . 28228 | . 385559 |
| 1.67 | . 11705 | . 194485 | . 280473 | . 383489 |
| 1.68 | . 115469 | .193011 | . 278684 | . 381437 |
| 1.09 | . 114903 | . 191555 | . 276913 | . 379402 |
| 1.7 | . 113851 | . 190116 | . 275161 | . 377386 |
| 1.71 | .112813 | .188693 | . 273426 | . 375387 |
| 1.72 | . 11179 | . 187287 | . 271709 | . 373405 |
| 1.73 | . 11078 | . 185898 | . 270009 | . 371441 |
| 1.74 | . 109784 | . 184524 | . 268327 | . 369493 |
| 1.75 | . 108802 | . 183167 | . 206661 | . 367563 |
| 1.76 |  |  |  |  |
| 1.77 | $.106876$ | $.180498$ | $.263379$ | $.363752$ |
| 1.78 | . 105432 | .179186 | . 261762 | . 361871 |
| 1.79 | . 105001 | . 177889 | . 260162 | . 360006 |
| 1.8 | .104081 | .176607 | . 258577 | . 358157 |
| 1.81 | . 103174 | . 17534 | . 257009 | . 356324 |
| 1.82 | . 102279 | .174086 | . 255455 | . 354506 |
| 1.83 | . 101390 | . 172847 | . 253917 | . 352704 |
| 1.84 | . 100524 | .171622 | . 252394 | . 350918 |
| 1.85 | . 099663 | . 17041 | . 250886 | . 349146 |
| 1.86 | . 098814 | .169212 | . 249392 | . 34739 |
| 1.87 | . 097975 | . 168027 | . 247913 | . 345648 |
| 1.88 | . 097147 | . 166855 | . 246448 | . 343921 |
| 1.89 | . 09633 | . 165696 | . 244998 | . 342208 |
| 1.9 | . 095523 | . 16455 | . 243561 | . 34051 |

TABLE IV. PERCENTILES OF THE LOG NORMAL DISTRIBUTION (Concinued)

| $s / \sqrt{X}$ | $10 t h$ | $20 t h$ | 30 th | 40th |
| :--- | :--- | :--- | :--- | :--- |
| 1.91 | .094726 | .163416 | .242138 | .338826 |
| 1.92 | .093939 | .162295 | .240729 | .337156 |
| 1.93 | .093163 | .161186 | .239334 | .3355 |
| 1.94 | .092395 | .160089 | .237951 | .333858 |
| 1.95 | .091638 | .159004 | .236582 | .33223 |
| 1.96 | .09089 | .15793 | .235226 | .330614 |
| 1.97 | .090151 | .156868 | .233882 | .329013 |
| 1.98 | .089421 | .155817 | .232551 | .327424 |
| 1.99 | .088701 | .154778 | .231233 | .325849 |
| 2. | .087989 | .153749 | .229927 | .324286 |

TABLE IV. PERCENTILES OF THE LOG-NURMAL DISTEIBUT: ON (COnEinued)

| $s / \bar{X}$ | 6uth | 70ch | 80 ch | y0th |
| :---: | :---: | :---: | :---: | :---: |
| 1.5 | . 730317 | . 98019 | 1.38318 | 2.22999 |
| 1.51 | . 727739 | .977853 | 2.38:74 | 2.23182 |
| 1.52 | . 725174 | . 975519 | 1.30028 | 2.23361 |
| 1.53 | . 722622 | . 973188 | 1.37881 | 2.235 .14 |
| 1.54 | . 720082 | . 970861 | 1.37753 | 2.23702 |
| 1.55 | . 717556 | . 968538 | 1.37584 | 2.23866 |
| 1.56 | . 715042 | . 966219 | 1.37433 | 2.24025 |
| 1.57 | . 71254 | . 963904 | 1.37281 | 2.24178 |
| 1.58 | . 710052 | . 961593 | 1.37128 | 2.24329 |
| 1.59 | .707576 | . 959287 | 1.36974 | 2.24474 |
| 1.0 | . 705113 | . 956985 | 1.36819 | 2.24615 |
| 1.01 | . 702663 | . 954688 | 1.36663 | 2.24751 |
| 1.62 | . 700226 | . 952396 | 1.36506 | 2.24883 |
| 1.03 | . 697801 | . 950108 | 1.36348 | 2.25011 |
| 1.64 | . 645389 | . 947826 | 1.36189 | 2.25135 |
| 1.65 | . 69299 | . 94555 | 1.36029 | 2.25255 |
| 1.66 | . 690603 | . 943278 | 1.35868 | 2.2537 |
| 1.67 | . 688229 | . 941012 | 1.35706 | 2.25482 |
| 1.68 | . 685868 | . 958752 | 1.35544 | 2.2559 |
| 1.69 | . 683519 | . 936497 | 1.35381 | 2.25694 |
| 1.7 | . 681183 | . 934249 | 1.35217 | 2.25794 |
| 1.71 | . 67886 | . 932006 | 1.35052 | 2.2589 |
| 1.72 | . 676549 | . 929769 | 1.34887 | 2.25983 |
| ?.73 | . 67425 | . 927538 | 1.34721 | 2.26072 |
| 1.74 | . 671964 | . 925314 | 1.34555 | 2.26158 |
| 1.75 | . 66969 | . 923095 | 1.34388 | 2.2624 |
| 1.76 | . 007429 | . 920883 | 1.3422 | 2.26318 |
| 1.77 | . 66518 | . 918678 | 1.34052 | 2.26394 |
| 1.78 | . 662943 | . 916478 | 1.33883 | 2.26465 |
| 1.79 | . 660719 | . 914286 | 1.33714 | 2.26534 |
| 1.8 | . 658506 | . 9121 | 1.33544 | 2.26599 |
| 1.81 | . 656306 | . 90992 | 1.33374 | 2.26661 |
| 1.82 | . 654118 | . 907746 | 1.33203 | 2.26721 |
| 1.83 | . 651942 | . 905582 | 1.33032 | 2.26776 |
| 1.84 | . 044778 | . 903423 | 1.32861 | 2.26829 |
| 1.85 | . 647626 | . 901271 | 1.32689 | 2.26879 |
| 1.86 | . 645485 | . 899126 | 1.32517 | 2.26926 |
| 1.87 | . 643357 | . 896987 | 1.32345 | 2.2697 |
| 1.88 | . 64124 | . 894850 | 1.12172 | 2.27012 |
| 1.89 | . 639135 | . 892732 | 1.31999 | 2.8705 |
| 1.9 | . 637042 | . 890615 | 1.31826 | 2.27086 |

LABLE IV. YERCENTILES OF THE LOG-NORMAL UISTRIBUIIION (COntinued)

| $s / \bar{X}$ | 60 th | 70th | 80 th | 90 ch |
| :---: | :---: | :---: | :---: | :---: |
| 1.91 | . 63496 | . 888505 | 1.31652 | 2.27119 |
| 1.92 | . 63289 | . 886402 | 1.31478 | 2.27149 |
| 1.93 | . 630832 | . 884306 | 1.31304 | 2.27177 |
| 1.94 | . 628784 | . 882218 | 1.3113 | 2.27202 |
| 1.95 | . 626749 | . 880137 | 1.30956 | 2.27224 |
| 1.90 | . 624724 | . 878063 | 1.30781 | 2.27244 |
| 1.97 | . 622711 | . 875996 | 1.30606 | 2.27262 |
| 1.98 | . 620709 | . 873936 | 1.30432 | 2.27277 |
| 1.99 | . 018718 | . 871884 | 1.30257 | 2.2729 |
| 2. | . 616738 | . 869889 | 1.30081 | 2.273 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / X$ | 95 th | 97.5 th | 99 th | 99.5 ch |
| :---: | :---: | :---: | :---: | :---: |
| 1.5 | 3.30825 | 4.65769 | 6.93302 | 9.08973 |
| 1.51 | 3.31607 | 4.67494 | 6.9695 | 9.14723 |
| 1.52 | 3.32379 | 4.69205 | 7.00581 | 9.20455 |
| 1.53 | 3.33143 | 4.70904 | 7.04196 | 9.2617 |
| 1.54 | 3.33898 | 4.72589 | 7.07793 | 9.31867 |
| 1.55 | 3.34644 | 4.74262 | 7.11374 | 9.37547 |
| 1.56 | 3.35381 | 4.75922 | 7.14938 | 9.43209 |
| 1.57 | 3.36109 | 4.77569 | 7.18484 | 9.48853 |
| 1.58 | 5.36829 | 4.79203 | 7.22014 | 9.54479 |
| 1.59 | 3.37541 | 4.80825 | 7.25527 | 9.60087 |
| 1.6 | 3.38244 | 4.82434 | 7.29024 | 9.65677 |
| 1.61 | 3.38939 | 4.84031 | 7.32503 | 9.71249 |
| 1.62 | 3.39625 | 4.85616 | 7.35966 | 9.76802 |
| 1.63 | 3.40303 | 4.87188 | 7.39412 | 9.82338 |
| 1.64 | 3.40973 | 4.88748 | 7.42841 | 9.87855 |
| 1.65 | 3.41636 | 4.90298 | 7.46253 | 9.93353 |
| 1.66 | 3.4229 | 4.91832 | 7.49649 | 9.98834 |
| 1.67 | 3.42937 | 4.93356 | 7.53028 | 10.043 |
| 1.68 | 3.43575 | 4.94869 | 7.56391 | 10.0974 |
| 1.69 | 3.44206 | 4.96369 | 7.59737 | 10.1516 |
| 1.7 | 3.4483 | 4.97858 | 7.63067 | 10.2057 |
| 1.71 | 3.45446 | 4.99335 | 7.6638 | 10.2596 |
| 1.72 | 3.46055 | 5.00801 | 7.69677 | 10.3133 |
| 1.73 | 3.46656 | 5.02255 | 7.72957 | 10.3668 |
| 1.74 | 3.4725 | 5.03698 | 7.76221 | 10.420 J |
| 1.75 | 3.47837 | 5.0513 | 7.79469 | 10.4732 |
| 1.76 | 3.48417 | 5.06551 | 7.82701 | 10.5262 |
| 1.77 | 3.48989 | 5.0796 | 7.85916 | 10.5789 |
| 1.78 | 3.49555 | 5.09359 | 7.89115 | 10.6315 |
| 1.79 | 3.50114 | 5.10746 | 7.92299 | 10.6839 |
| 1.8 | 3.50666 | 5.12123 | 7.95466 | 10.7361 |
| 1.81 | 3.51211 | 5.13489 | 7.98618 | 10.7881 |
| 1.82 | 3.5175 | 5.14844 | 8.01753 | 10.8399 |
| 1.83 | 3.52282 | 5.16188 | 8.04873 | 10.8915 |
| 1.84 | 3.52807 | 5.17522 | 8.07977 | 10.943 |
| 1.85 | 3.53326 | 5.18846 | 8.11065 | 10.9942 |
| 1.86 | 3.53839 | 5.20159 | 8.14138 | 11.0453 |
| 1.87 | 3.54345 | 5.21462 | 8.17195 | 11.0962 |
| 1.88 | 3. 54845 | 5.22755 | 8.20236 | 11.1469 |
| 1.89 | 3.55339 | 5.24037 | 8.23262 | 11.1974 |
| 1.9 | 3.55827 | 5.2531 | 8.26273 | 11.2477 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| S/ $\bar{X}$ | $95 t h$ | $97.5 t h$ | $99 t h$ | 9.5 th |
| :--- | :--- | :--- | :--- | :--- |
| 1.91 | 3.56309 | 5.26573 | 8.29268 | 11.2979 |
| 1.92 | 3.56785 | 5.27825 | 8.32249 | 11.3478 |
| 1.93 | 3.57254 | 5.29068 | 8.35214 | 11.3976 |
| 1.94 | 3.57718 | 5.30301 | 8.38163 | 11.4472 |
| 1.95 | 3.58177 | 5.31525 | 8.41098 | 11.4966 |
| 1.96 | 3.58629 | 5.32738 | 8.44018 | 11.5458 |
| 1.97 | 3.59076 | 5.33943 | 8.46923 | 11.5948 |
| 1.98 | 3.59517 | 5.35138 | 8.49813 | 11.6437 |
| 1.99 | 3.59953 | 5.36323 | 8.52688 | 11.6923 |
| 2. | 3.60383 | 5.37499 | 8.55549 | 11.7408 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (COntinued)

| $s / \bar{x}$ | . 5 ch | lst | 2.5 th | 5 th |
| :---: | :---: | :---: | :---: | :---: |
| 2. | . 017034 | . 023376 | . 037209 | . 055496 |
| 2.01 | . 016829 | . 023113 | .036833 | . 05499 |
| 2.02 | . 016628 | . 022855 | . 036462 | . 05449 |
| 2.03 | . 01643 | . 0226 | . 036097 | . 053997 |
| 2.04 | . 010235 | . 022349 | . 035737 | . 053511 |
| 2.05 | . 016044 | . 022103 | . 035382 | . 053031 |
| 2.06 | . 015855 | . 02186 | . 035033 | . 052558 |
| 2.07 | . 01567 | . 02162 | . 034688 | . 05209 |
| 2.08 | . 015487 | . 021385 | . 034348 | - ©51629 |
| 2.09 | . 015308 | . 021153 | . 034013 | . 051174 |
| <.1 | . 015131 | . 020924 | .033683 | . 050725 |
| 2.11 | . 014957 | . 0207 | . 033357 | . 050282 |
| 2.12 | . 014786 | . 020478 | . 033036 | . 049844 |
| 2.13 | . 014618 | . 02026 | . 032719 | . 049413 |
| 2. 14 | . 014452 | . 020045 | . 032407 | . 048986 |
| 2.15 | . 014289 | . 019833 | . 032099 | . 048565 |
| 2.16 | . 014129 | . 019625 | . 031795 | . 04815 |
| 2.17 | . 013971 | . 019419 | . 031496 | . 04774 |
| 2.10 | . 013815 | . 019217 | . 0312 | . 047335 |
| 2.19 | . 013662 | . 019017 | . 030909 | . 046935 |
| 2.2 | .013511 | .018821 | . 030621 | . 04654 |
| 2.21 | . 013363 | . 018627 | . 030338 | . 04615 |
| 2.22 | . 013217 | . 018436 | . 030058 | . 045765 |
| 2.23 | . 013073 | . 018248 | . 029782 | . 045384 |
| 2.24 | . 012931 | . 018063 | . 02951 | . 045009 |
| 2.25 | . 012791 | . 01788 | . 029241 | . 044638 |
| 4.26 | . 012654 | . 0177 | . 028976 | . 044272 |
| 2.27 | . 012518 | . 017523 | .028714 | . 04391 |
| 2.28 | . 012385 | . 017346 | . 028456 | . 043552 |
| 2.29 | . 012254 | . 017175 | . 028201 | . 043199 |
| 2.3 | . 012124 | . 017005 | . 027949 | . 04285 |
| 2.31 | . 011996 | . 016838 | . 027701 | . 042506 |
| 2.32 | . 011071 | . 016672 | . 027456 | . 042165 |
| 2.33 | . 011747 | . 016509 | . 027214 | . 041829 |
| 2.34 | . 011625 | . 016348 | . 026975 | . 041496 |
| 2.35 | . 011505 | . 01619 | . 026739 | . 041168 |
| 2.36 | . 011386 | . 016034 | . 026507 | . 040843 |
| 2.37 | . 011269 | . 015879 | . 026277 | . 040523 |
| 2.38 | . 011154 | . 015727 | . 02605 | . 040206 |
| 2.34 | . 01104 | .015577 | . 025826 | . 039892 |
| 2.4 | . 010929 | . 015429 | . 025605 | . 039583 |

Table iv. plrcentiles of the log-normal distribution (Continuea)

| S/X | .5 th | $18 t$ | 2.5 th | Sth |
| :--- | :--- | :--- | :--- | :--- |
| 2.41 | .010818 | .015283 | .025386 | .039277 |
| 2.42 | .010709 | .015139 | .025171 | .038974 |
| 2.43 | .010602 | .014997 | .024958 | .038676 |
| 2.44 | .010497 | .014857 | .024747 | .03838 |
| 2.45 | .010392 | .014719 | .02454 | .038088 |
| 2.46 | .01029 | .014582 | .024334 | .037799 |
| 2.47 | .010188 | .014448 | .024132 | .037514 |
| 2.48 | .010088 | .014315 | .023931 | .037232 |
| 2.49 | .00999 | .014184 | .023734 | .036953 |
| 2.5 | .009892 | .014054 | .023538 | .036677 |

TABLE IV. PEKCENTILES OF THE LOG-NORMAL DISIRIBUTION (Continued)

| $S / \bar{\chi}$ | 10ch | 20 ch | 30th | 40 ch |
| :---: | :---: | :---: | :---: | :---: |
| 2. | . 087989 | . 153744 | . 229927 | . 324286 |
| 2.01 | . 087286 | . 152732 | . 228633 | . 322736 |
| 2.02 | . 086591 | . 151725 | . 227351 | . 321199 |
| 2.13 | . 085905 | . 150729 | . 226081 | . 319674 |
| $2.04$ | $.085227$ | $.149743$ |  | . 316162 |
| $2.05$ | $.084557$ | . 148767 | $.223577$ | . 316662 |
| 2.06 | . 083896 | .147802 | . 222342 | . 315174 |
| 2.07 | . 083242 | . 146847 | . 221118 | . 313698 |
| 2.08 | . 082590 | .145301 | . 219906 | . 312234 |
| 2.04 | . 081957 | . 144966 | . 218704 | . 310782 |
| 2.1 | . 081327 | . 14404 | . 217514 | . 309341 |
| 2.11 | . 080703 | . 143123 | . 216334 | . 307912 |
| $\angle .12$ | . 080087 | . 142216 | . 215165 | . 306494 |
| 2.13 | . 079478 | . 141318 | . 214000 | . 305088 |
| 2.14 | . 078877 | . 140429 | . 212858 | . 303692 |
| 2.15 | . 078282 | . 139549 | . 211721 | . 302308 |
| 2.16 | . 077694 | . 138678 | . 210593 | . 300934 |
| 2.17 | . 077113 | . 137816 | . 209475 | . 299572 |
| <.10 | . 070539 | . 136962 | . 208368 | . 29822 |
| 2.19 | . 075971 | . 136117 | . 20727 | . 296874 |
| 2.2 | . 075409 | . 135281 | . 206182 | . 295548 |
| 2.21 | . 074854 | . 134452 | . 205103 | . 294227 |
| 2.22 | . 074306 | . 133632 | . 204034 | . 294917 |
| 2.23 | . 073763 | . 13282 | . 202974 | . 291017 |
| 2.24 | . 073227 | .132016 | . 201924 | . 290327 |
| 2.25 | . 072697 | . 13122 | . 200883 | . 289640 |
| 2.26 | . 072172 | . 130431 | . 19985 | . 287776 |
| 2.27 | . 071654 | . 12965 | . 198827 | . 286516 |
| 2.28 | . 071141 | . 128877 | . 197812 | . 285265 |
| 2.29 | . 070634 | . 128112 | . 196807 | . 284023 |
| 2.3 | . 070132 | . 127353 | . 19581 | . 282791 |
| 2.31 | . 069636 | . 126602 | . 194821 | . 281569 |
| 2.32 | . 009145 | . 125858 | . 193841 | . 280355 |
| 2.33 | . 06866 | . 125121 | . 192869 | . 279151 |
| 2.34 | . 06818 | . 124392 | . 191905 | . 277956 |
| 2.35 | . 007705 | . 123669 | . 19095 | . 27677 |
| 2.36 | . 067236 | . 122953 | . 190002 | . 275593 |
| 2.37 | . 006771 | . 122244 | . 189063 | . 274424 |
| $2.30$ | $.006311$ | $.121541$ | $.108131$ | . 273265 |
| 2.39 | . 065857 | . 120845 | . 187207 | . 272114 |
| 2.4 | . 065407 | . 120155 | . 186291 | .270971 |

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table IV. percentiles of the log-normal distribution (Continued)

| $s / \bar{X}$ | 10th | 20th | 30th | 40th |
| :--- | :--- | :--- | :--- | :--- |
| 2.41 | .064962 | .119472 | .185383 | .269837 |
| 2.42 | .064521 | .118796 | .184482 | .268711 |
| 2.43 | .064085 | .118125 | .183588 | .267594 |
| 2.44 | .063654 | .117461 | .182702 | .266484 |
| 2.45 | .063228 | .116802 | .181823 | .265383 |
| 2.46 | .062385 | .11615 | .180952 | .26429 |
| 2.47 | .062388 | .115504 | .180087 | .263205 |
| 2.48 | .061974 | .114864 | .179229 | .262128 |
| 2.49 | .061565 | .114229 | .178379 | .261058 |
| 2.5 | .06116 | .1136 | .177535 | .259996 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | 60th | 70th | 80th | 90th |
| :---: | :---: | :---: | :---: | :---: |
| 2. | . 616738 | . 869839 | 1.30081 | 2.273 |
| 2.01 | . 614769 | . 867802 | 1.29906 | 2.27308 |
| 2.02 | . 612811 | . 865772 | 1.29731 | 2.27314 |
| 2.03 | . 610865 | . 863749 | 1.29556 | 2.27318 |
| 2.04 | . 608928 | . 861734 | 1.2938 | 2.27319 |
| 2.05 | . 607003 | . 859726 | 1.29205 | 2.27318 |
| 2.06 | . 605088 | . 857725 | 1.29029 | 2.27316 |
| 2.07 | . 603184 | . 855732 | 1.28854 | 2.27311 |
| 2.08 | . 601291 | . 853746 | 1.28678 | 2.27303 |
| 2.09 | . 599408 | . 851768 | 1.28503 | 2.27294 |
| 2.1 | . 597536 | . 849796 | 1.28327 | 2.27283 |
| 2.11 | . 595674 | . 847833 | 1.28152 | 2.2727 |
| 2.12 | . 593822 | . 845877 | 1.27977 | 2.27255 |
| 2.13 | . 59.1981 | . 843928 | 1.27801 | 2.27238 |
| 2.14 | . 59015 | . 841986 | 1.27626 | 2.2722 |
| 2.15 | . 588329 | . 840052 | 1.27451 | 2.27199 |
| 2.16 | . 586518 | . 838126 | 1.27275 | 2.27177 |
| 2.17 | . 584717 | . 836206 | 1.271 | 2.27153 |
| 2.18 | . 582926 | . 834294 | 1.26325 | 2.27127 |
| 2.19 | . 581145 | . 83239 | 1.2675 | 2.27099 |
| 2.2 | . 579373 | . 830492 | 1.26576 | 2.2707 |
| 2.21 | . 577612 | . 828603 | 1.26401 | 2.27039 |
| 2.22 | . 57596 | . 82672 | 1.26226 | 2.27006 |
| 2.23 | . 574118 | . 824845 | 1.26052 | 2.26972 |
| 2.24 | . 572386 | . 822977 | 1.25878 | 2.26936 |
| 2.25 | . 570663 | . 821116 | 1.25704 | 2.26898 |
| $2.25$ | . 568949 | . 819263 | 1.2553 | 2.26859 |
| 2.27 | . 567245 | . 817417 | 1.25356 | 2.26819 |
| 2.28 | . 56555 | . 815578 | 1.25182 | 2.26777 |
| 2.29 | . 563865 | . 813746 | 1.25009 | 2.26734 |
| 2.3 | . 562189 | . 811922 | 1.24835 | 2.26089 |
| 2.31 | . 560522 | . 810105 | 1.24662 | 2.26642 |
| 2.32 | . 558864 | . 808295 | 1.24489 | 2.26595 |
| 2.33 | . 557215 | . 8064.2 | 1.24317 | 2.26546 |
| 2.34 | . 555575 | . 804696 | 1.24144 | 2.26495 |
| 2.35 | . 553944 | . 802907 | 1.23972 | 2.26444 |
| 2.36 | . 552322 | . 801126 | 1.238 | 2.26391 |
| 2.37 | . 550708 | . 799352 | 1.23628 | 2.26336 |
| 2.38 | . 549104 | . 797584 | 1.23457 | 2.26281 |
| 2.39 | . 547508 | . 795824 | 1.23285 | 2.26224 |
| 2.4 | . 54592 | . 794071 | 1.23114 | 2.26166 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (COntinued)

| S/X | 60th | 70th | 80th | 90th |
| :--- | :--- | :--- | :--- | :--- |
| 2.41 | .544342 | .792324 | 1.22943 | 2.26107 |
| 2.42 | .542712 | .790585 | 1.22773 | 2.26047 |
| 2.43 | .54121 | .788853 | 1.22602 | 2.26985 |
| 2.44 | .539657 | .787127 | 1.22432 | 2.25923 |
| 2.45 | .534112 | .785409 | 1.22262 | 2.25859 |
| 2.46 | .536575 | .783697 | 1.22093 | 2.25794 |
| 2.47 | .535046 | .781993 | 1.21924 | 2.25728 |
| 2.46 | .533526 | .780295 | 1.21754 | 2.25661 |
| 2.49 | .532014 | .778604 | 1.21586 | 2.25593 |
| 2.5 | .53051 | .776919 | 1.21417 | 2.25524 |

table iv. percentiles of the log-normal distribution (Continued)

| S/X | 95 th | 97.5 th | 99 th | 99.5 th |
| :---: | :---: | :---: | :---: | :---: |
| 2. | 3.00383 | 5.37499 | 8.55549 | 11.7408 |
| 2.01 | 3.60808 | 5.38666 | 8.58395 | 11.7891 |
| 2.02 | 3.61227 | 5.39824 | 8.61226 | 11.8372 |
| 2.03 | 3.61641 | 5.40973 | 8.64043 | 11.8852 |
| 2.04 | 3.6203 | 5.42113 | 8.66845 | 11.9329 |
| 2.05 | 3.62454 | 5.43244 | 8.69633 | 11.9805 |
| 2.06 | 3.02853 | 5.44366 | 8.72407 | 12.0279 |
| 2.07 | 3.63246 | 5.45479 | 8.75167 | 12.0751 |
| 2.08 | 3.63634 | 5.46583 | 8.77912 | 12.1221 |
| 2.09 | 3.04018 | 5.47679 | 8.80643 | 12.169 |
| 2.1 | 3.64397 | 5.48767 | 8.83361 | 12.2157 |
| 2.11 | 3.6477 | 5.49846 | 8.86064 | 12.2622 |
| 2.12 | 3.65139 | 5.50916 | 8.88754 | 12.3085 |
| 2.13 | 3.65503 | 5.51978 | 8.91429 | 12.3546 |
| 2.14 | 3.05863 | 5.53032 | 8.94091 | 12.4006 |
| 2.15 | 3.66218 | 5.54077 | 8.9674 | 12.4464 |
| 2.16 | 3.00568 | 5.55114 | 8.99374 | 12.492 |
| 2.17 | 3.66914 | 5.56144 | 9.01996 | 12.5374 |
| 2.16 | 3.67255 | 5.57165 | 9.04604 | 12.5827 |
| 2.19 | 3.67591 | 5.58178 | 9.07198 | 12.6278 |
| 2.2 | 3.67924 | 5.59184 | 9.09779 | 12.6727 |
| 2.21 | 3.68252 | 5.60181 | 9.12347 | 12.7174 |
| 2.22 | 3.68575 | 5.61171 | 9.14902 | 12.762 |
| 2.23 | 3.68895 | 5.62153 | 9.17444 | 12.8064 |
| 2.24 | 3.6921 | 5.63127 | 9.19972 | 12.8506 |
| 2.25 | 3.69521 | 564094 | 9.22488 | 12.8947 |
| $2.26$ | $3.69828$ | 5.65053 | 9.24991 | 12.9386 |
| 2.27 | 3.70131 | 5.66005 | 9.27481 | 12.9823 |
| 2.28 | 3.70429 | 5.6695 | 9.29959 | 13.0258 |
| 2.29 | 3.70724 | 5.67887 | 9.32423 | 13.0692 |
| 2.3 | 3.71015 | 5.68817 | 9.34875 | 13.1124 |
| 2.31 | 3.71302 | 5.69734 | 9.37315 | 13.1554 |
| $2.32$ | $3.71585$ | 5.70655 | 9.39742 | 13.1983 |
| 2.33 | 3.71864 | 5.71563 | 9.42157 | 13.241 |
| 2.34 | 3.7214 | 5.72464 | 9.4456 | 13.2836 |
| 2.35 | 3.72412 | 5.73359 | 9.4695 | 13.326 |
| 2.36 | 3.7268 | 5.74246 | 4.49328 | 13.3682 |
| 2.37 | 3.72944 | 5.75127 | 9.51694 | 13.4102 |
| 2.38 | 3.73205 | 5.76 | 9.54048 | 13.4521 |
| 2.34 | 3.73462 | 5.76867 | 9.5639 | 13.4938 |
| 2.4 | 3.73716 | 5.77728 | 9.5872 | 13.5354 |

TABLE IV. PERCENTILES OF THE LOG-NORMAL DISTRIBUTION (Continued)

| $s / \bar{X}$ | 95 th | 97.5 th | 99 ch | 99.5 ch |
| :---: | :---: | :---: | :---: | :---: |
| 2.41 | 3.73966 | 5.78581 | 9.61038 | 13.5768 |
| 2.42 | 3.74213 | 5.79428 | 9.63345 | $13.618 i$ |
| 2.43 | 3.74456 | 5.80269 | 9.6564 | 13.6592 |
| 2.44 | 3.74696 | 5.81103 | 9.67923 | 13.7001 |
| 2.45 | 3.74932 | 5.8193 | 9.70195 | 13.7408 |
| 2.46 | 3.75165 | 5.82752 | 9.72455 | 13.7815 |
| 2.47 | 3.75395 | 5.83567 | 9.74704 | 13.8219 |
| 2.48 | 3.75622 | 5.84375 | 9.76941 | 13.8622 |
| 2.49 | 3.75845 | 5.85178 | 9.79167 | 13.9023 |
| 2.5 | 3.76066 | 5.85974 | 9.81382 | 13.9423 |

*TABLE $V$. PERCENTAGE POINTS, $b_{\gamma}$, SUCH THAT $P\left[\hat{c} / c<b_{\gamma}\right]=\gamma$

| $Y$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | 0.02 | 0.05 | 0.10 | 0.25 | 0.40 | 0.50 | 0.60 |
| 5 | 0.604 | 0.683 | 0.766 | 0.951 | 1.116 | 1.238 | 1.378 |
| 6 | 0.623 | 0.697 | 0.778 | 0.937 | 1.080 | 1.188 | 1.304 |
| 7 | 0.639 | 0.709 | 0.785 | 0.930 | 1.059 | 1.155 | 1.256 |
| 8 | 0.653 | 0.720 | 0.792 | 0.926 | 1.045 | 1.131 | 1.223 |
| 9 | 0.665 | 0.729 | 0.797 | 0.925 | 1.035 | 1.114 | 1.198 |
| 10 | 0.676 | 0.738 | 0.802 | 0.924 | 1.028 | 1.101 | 1.179 |
| 11 | 0.686 | 0.745 | 0.807 | 0.924 | 1.022 | 1.090 | 1.163 |
| 12 | 0.695 | 0.752 | 0.811 | C. 924 | 1.017 | 1.082 | 1.151 |
| 13 | 0.703 | 0.759 | 0.815 | 0.924 | 1.014 | 1.075 | 1.140 |
| 14 | 0.710 | 0.764 | 0.819 | 0.925 | 1.011 | 1.069 | 1.132 |
| 15 | 0.716 | 0.770 | 0.823 | 0.925 | 1.008 | 1.064 | 1.124 |
| 16 | 0.723 | 0.775 | 0.826 | 0.926 | 1.006 | 1.059 | 1.117 |
| 17 | 0.728 | 0.779 | 0.829 | 0.927 | 1.004 | 1.056 | 1.111 |
| 18 | 0.734 | 0.784 | 0.832 | 0.927 | 1.003 | 1.052 | 1.106 |
| 19 | 0.739 | 0.788 | 0.835 | 0.928 | 1.001 | 1.049 | 1.101 |
| 20 | 0.743 | 0.791 | 0.838 | 0.929 | 1.000 | 1.047 | 1.097 |
| 22 | 0.752 | 0.798 | 0.843 | 0.930 | 0.998 | 1.042 | 1.090 |
| 24 | 0.759 | 0.805 | 0.848 | 0.932 | 0.997 | 1.038 | 1.084 |
| 26 | 0.766 | 0.810 | 0.852 | 0.933 | 0.995 | 1.035 | 1.079 |
| 28 | 0.772 | 0.815 | 0.856 | 0.934 | 0.994 | 1.033 | 1.074 |
| 30 | 0.778 | 0.820 | 0.860 | 0.935 | 0.993 | 1.030 | 1.070 |
| 32 | 0.783 | 0.824 | 0.863 | 0.937 | 0.993 | 1.028 | 1.067 |
| 34 | 0.788 | 0.828 | 0.866 | 0.938 | 0.992 | 1.027 | 1.064 |
| 36 | 0.793 | 0.832 | 0.869 | 0.939 | 0.992 | 1.025 | 1.061 |
| 38 | 0.797 | 0.835 | 0.872 | 0.940 | 0.991 | 1.024 | 1.059 |
| 40 | 0.801 | 0.839 | 0.875 | 0.940 | 0.991 | 1.023 | 1.056 |
| 42 | 0.804 | 0.842 | 0.877 | 0.941 | 0.990 | 1.022 | 1.054 |
| 44 | 0.808 | 0.845 | 0.880 | 0.942 | 0.990 | 1.021 | 1.052 |
| 46 | 0.811 | 0.847 | 0.882 | 0.943 | 0.990 | 1.020 | 1.051 |
| 48 | 0.814 | 0.850 | 0.884 | 0.944 | 0.990 | 1.019 | 1.049 |
| 50 | 0.817 | 0.852 | 0.886 | 0.944 | 0.989 | 1.018 | 1.048 |
| 52 | 0.820 | 0.854 | 0.888 | 0.945 | 0.989 | 1.017 | 1.046 |
| 54 | 0.822 | 0.857 | 0.890 | 0.946 | 0.989 | 1.017 | 1.045 |
| 56 | 0.825 | 0.859 | 0.891 | 0.946 | 0.989 | 1.016 | 1.044 |
| 58 | 0.827 | 0.861 | 0.893 | 0.947 | 0.989 | 1.015 | 1.043 |
| 60 | 0.830 | 0.863 | 0.894 | 0.948 | 0.989 | 1.015 | 1.041 |
| 62 | 0.832 | 0.864 | 0.896 | 0.948 | 0.989 | 1.014 | 1.040 |
| 64 | 0.834 | 0.866 | 0.897 | 0.949 | 0.989 | 1.014 | 1.040 |

*Reproduced from "Inferences on the Parameters of the Weibull Distribution," by Darrel R. Thoman, Lee J. Bain, and Charles E. Antle, Technometrics, Vol. 11 No. 3, (1969), pp. 445-460.

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# table v. percentace points, $b_{\gamma}$, such that <br> $P\left[\hat{c} / c<b_{\gamma}\right]=\gamma$ (Continued) 

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| N |  | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 |
| 66 | 0.836 | 0.868 | 0.899 | 0.949 | 0.988 | 1.014 | 1.039 |
| 68 | 0.838 | 0.869 | 0.900 | 0.950 | 0.988 | 1.013 | 1.038 |
| 70 | 0.840 | 0.871 | 0.901 | 0.950 | 0.988 | 1.013 | 1.037 |
| 72 | 0.841 | 0.872 | 0.903 | 0.951 | 0.988 | 1.012 | 1.036 |
| 74 | 0.843 | 0.874 | 0.904 | 0.951 | 0.988 | 1.012 | 1.036 |
| 76 | 0.845 | 0.875 | 0.905 | 0.952 | 0.988 | 1.012 | 1.035 |
| 78 | 0.846 | 0.876 | 0.906 | 0.952 | 0.988 | 1.011 | 1.034 |
| 80 | 0.848 | 0.878 | 0.907 | 0.952 | 0.988 | 1.011 | 1.034 |
| 85 | 0.852 | 0.881 | 0.910 | 0.953 | 0.988 | 1.011 | 1.032 |
| 90 | 0.855 | 0.883 | 0.912 | 0.954 | 0.988 | 1.010 | 1.031 |
| 95 | 0.858 | 0.886 | 0.914 | 0.955 | 0.988 | 1.009 | 1.030 |
| 100 | 0.861 | 0.888 | 0.916 | 0.956 | 0.988 | 1.009 | 1.029 |
| 110 | 0.866 | 0.893 | 0.920 | 0.958 | 0.988 | 1.008 | 1.027 |
| 120 | 0.871 | 0.897 | 0.923 | 0.959 | 0.988 | 1.007 | 1.025 |

table v. percentage points, by, such that
$P\left\{\hat{c} / c<o_{Y}\right\}=\gamma$ (Continued)

| $\gamma$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.98 |
| 5 | 1.557 | 1.671 | 1.812 | 2.001 | 2.277 | 2.779 | 3.518 |
| 0 | 1.453 | 1.543 | 1.662 | 1.812 | 2.030 | 2.436 | 3.067 |
| 7 | 1.386 | 1.461 | 1.561 | 1.688 | 1.861 | 2.183 | 2.640 |
| $\varepsilon$ | 1.338 | 1.404 | 1.491 | 1.602 | 1.747 | 2.015 | 2.377 |
| 9 | 1.303 | 1.361 | 1.439 | 1.538 | 1.065 | 1.890 | 2.199 |
| 10 | 1.275 | 1.328 | 1.399 | 1.489 | 1.602 | 1.807 | 2.070 |
| 11 | 1.253 | 1.302 | 1.367 | 1.450 | 1.553 | 1.738 | 1.972 |
| 12 | 1.234 | 1.281 | 1.341 | 1.418 | 1.513 | 1.682 | 1.894 |
| 13 | 1.219 | 1.263 | 1.319 | 1.391 | 1.480 | 1.636 | 1.830 |
| 14 | 1.206 | 1.248 | 1.300 | 1.369 | 1.452 | 1.597 | 1.777 |
| 15 | 1.195 | 1.234 | 1.284 | 1.349 | 1.427 | 1.564 | 1.732 |
| 16 | 1.185 | 1.223 | 1.270 | 1.332 | 1.406 | 1.535 | 1.693 |
| 17 | 1.176 | 1.213 | 1.258 | 1.317 | 1.388 | 1.510 | 1.660 |
| 18 | 1.168 | 1.204 | 1.247 | 1.303 | 1.371 | 1.487 | 1.630 |
| 19 | 1.102 | 1.196 | 1.237 | 1.291 | 1.356 | 1.467 | 1.603 |
| 20 | 1.155 | 1.188 | 1.228 | 1.281 | 1.343 | 1.449 | 1.579 |
| 32 | 1.144 | 1.176 | 1.213 | 1.262 | 1.320 | 1.418 | 1.538 |
| 24 | 1.135 | 1.165 | 1.200 | 1.246 | 1.301 | 1.392 | 1.504 |
| 26 | 1.128 | 1.150 | 1.189 | 1.232 | 1.284 | 1.370 | 1.475 |
| 28 | 1.121 | 2.148 | 1.180 | 1.220 | 1.269 | 1.351 | 1.450 |
| 30 | 1.115 | 1.141 | 1.171 | 1.210 | 1.257 | 1.334 | 1.429 |
| 32 | 1.110 | 1.135 | 1.164 | 1.201 | 1.246 | 1.319 | 1.409 |
| 34 | 1.105 | 1.129 | 1.157 | 1.193 | 1.230 | 1.306 | 1.392 |
| 36 | 1.101 | 1.125 | 1.151 | 1.186 | 1.227 | 1.294 | 1.377 |
| 38 | 1.097 | 1.120 | 1.146 | 1.179 | 1.219 | 1.283 | 1.363 |
| 40 | 1.094 | 1.116 | 1.141 | 1.173 | 1.211 | 1.273 | 1.351 |
| 42 | 1.091 | 1.112 | 1.137 | 1.167 | 1.204 | 1.265 | 1.339 |
| 44 | 1.088 | 1.109 | 1.132 | 1.162 | 1.198 | 1.256 | 1.329 |
| 46 | 1.085 | 1.100 | 1.129 | 1.158 | 1.192 | 1.249 | 1.319 |
| 48 | 1.083 | 1.103 | 1.125 | 1.153 | 1.187 | 1.242 | 1.310 |
| 50 | 1.081 | 1.100 | 1.122 | 1.149 | 1.182 | 1.235 | 1.301 |
| 52 | 1.078 | 1.090 | 1.119 | 1.145 | 1.177 | 1.229 | 1.294 |
| 54 | 1.076 | 1.095 | 1.116 | 1.142 | 1.173 | 1.224 | 1.286 |
| 56 | 1.075 | 1.093 | 1.113 | 1.139 | 1.169 | 1.218 | 1.280 |
| 58 | 1.073 | 1.091 | 1.111 | 1.135 | 1.165 | 1.213 | 1.273 |
| 60 | 1.071 | 1.089 | 1.108 | 1.133 | 1.162 | 1.208 | 1.267 |
| 02 | 1.070 | 1.087 | 1.106 | 1.130 | 1.158 | 1.204 | 1.262 |
| 04 | 1.068 | 1.086 | 1.104 | 1.127 | 1.155 | 1.200 | 1.256 |
| 66 | 1.067 | 1.084 | 1.102 | 1.125 | 1.152 | 1.196 | 1.251 |
| 68 | 1.066 | 1.083 | 1.100 | 1.122 | 1.149 | 1.192 | 1.246 |
| 70 | 1.064 | 1.081 | 1.098 | 1.120 | 1.146 | 1.188 | 1.242 |
| 72 | 1.063 | 1.080 | 1.097 | 1.118 | 1.144 | 1.185 | 1.237 |

# table $V$. percentage points, $b_{\gamma}$, such that <br> $P\left\{\bar{c} / c<b_{Y}\right\}=Y$ (Continued) 

| r |  |  |  |  |  |  |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| N |  | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 |
| 74 | 1.062 | 1.078 | 1.095 | 1.116 | 1.141 | 1.182 | 1.233 |
| 76 | 1.061 | 1.077 | 1.093 | 1.114 | 1.139 | 1.179 | 1.229 |
| 78 | 1.060 | 1.076 | 1.092 | 1.112 | 1.136 | 1.176 | 1.225 |
| 80 | 1.059 | 1.075 | 1.090 | 1.110 | 1.134 | 1.173 | 1.222 |
| 85 | 1.057 | 1.072 | 1.087 | 1.106 | 1.129 | 1.166 | 1.213 |
| 90 | 1.055 | 1.069 | 1.084 | 1.102 | 1.124 | 1.160 | 1.206 |
| 95 | 1.053 | 1.067 | 1.081 | 1.099 | 1.120 | 1.155 | 1.199 |
| 100 | 1.051 | 1.065 | 1.079 | 1.096 | 1.116 | 1.150 | 1.192 |
| 110 | 1.048 | 1.061 | 1.074 | 1.090 | 1.110 | 1.141 | 1.181 |
| 120 | 1.046 | 1.058 | 1.070 | 1.086 | 1.104 | 1.133 | 1.171 |

*table vi. percentage points, $\ell_{Y}$, such that
$P\left[\hat{c} \ln (\hat{b} / b)<\ell_{\gamma}\right]=\gamma$
$\left.\begin{array}{cccccccc}\hline & & & & & & & \\ \mathrm{N} & 0.02 & 0.05 & 0.10 & 0.25 & 0.40 & 0.50 & 0.60 \\ \hline & & -1.631 & -1.247 & -0.888 & -0.444 & -0.241 & -0.056\end{array}\right] 0.085$
*Reproduced from "Inferences on the Parameters of the Weibull Distribution," by Darrel R. Thoman, Lee J. Bain, and Charles E. Antle, Technometrics, Vol. 11, No. 3, (1969), pp. 445-460.

TABLE VI. PERCENTAGE POINTS, $\ell_{\gamma}$, SUCH THAT $P\left[\widehat{c} \ln (\hat{b} / b)<\ell_{\gamma}\right]-\gamma$ (Continued)

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma$ |  |  |  |  |  |  |
| N |  | 0.02 | 0.05 | 0.10 | 0.25 | 0.40 | 0.50 |

table vi. percentage points, $\ell_{\gamma}$, such that $P\left[\hat{c} \ln (\hat{b} / b)<\ell_{\gamma}\right]=\gamma($ Continued $)$

|  | $\gamma$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| N | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.98 |
|  | 0.254 | 0.349 | 0.452 | 0.587 | 0.772 | 0.107 | 1.582 |
| 6 | 0.221 | 0.302 | 0.404 | 0.516 | 0.666 | 0.939 | 1.291 |
| 7 | 0.200 | 0.272 | 0.362 | 0.465 | 0.598 | 0.829 | 1.120 |
| 8 | 0.185 | 0.251 | 0.331 | 0.427 | 0.547 | 0.751 | 1.003 |
| 9 | 0.174 | 0.235 | 0.307 | 0.397 | 0.507 | 0.691 | 0.917 |
| 10 | 0.165 | 0.222 | 0.288 | 0.372 | 0.475 | 0.644 | 0.851 |
| 11 | 0.157 | 0.211 | 0.273 | 0.351 | 0.448 | 0.605 | 0.797 |
| 12 | 0.150 | 0.202 | 0.260 | 0.334 | 0.425 | 0.572 | 0.752 |
| 13 | 0.145 | 0.194 | 0.249 | 0.319 | 0.406 | 0.544 | 0.714 |
| 14 | 0.140 | 0.187 | 0.239 | 0.306 | 0.389 | 0.520 | 0.681 |
| 15 | 0.135 | 0.180 | 0.230 | 0.294 | 0.374 | 0.499 | 0.653 |
| 16 | 0.131 | 0.175 | 0.223 | 0.284 | 0.360 | 0.480 | 0.627 |
| 17 | 0.128 | 0.170 | 0.216 | 0.274 | 0.348 | 0.463 | 0.605 |
| 18 | 0.124 | 0.165 | 0.209 | 0.266 | 0.338 | 0.447 | 0.584 |
| 19 | 0.121 | 0.161 | 0.204 | 0.258 | 0.328 | 0.433 | 0.566 |
| 20 | 0.118 | 0.157 | 0.199 | 0.251 | 0.318 | 0.421 | 0.549 |
| 22 | 0.113 | 0.150 | 0.189 | 0.239 | 0.302 | 0.398 | 0.519 |
| 24 | 0.109 | 0.144 | 0.181 | 0.228 | 0.288 | 0.379 | 0.494 |
| 26 | 0.105 | 0.138 | 0.174 | 0.219 | 0.276 | 0.362 | 0.472 |
| 28 | 0.102 | 0.134 | 0.168 | 0.210 | 0.265 | 0.347 | 0.453 |
| 30 | 0.098 | 0.129 | 0.163 | 0.203 | 0.256 | 0.334 | 0.435 |
| 32 | 0.095 | 0.125 | 0.158 | 0.197 | 0.247 | 0.323 | 0.420 |
| 34 | 0.093 | 0.122 | 0.153 | 0.191 | 0.239 | 0.312 | 0.406 |
| 36 | 0.090 | 0.118 | 0.149 | 0.185 | 0.232 | 0.302 | 0.393 |
| 38 | 0.088 | 0.115 | 0.145 | 0.180 | 0.226 | 0.293 | 0.382 |
| 40 | 0.086 | 0.113 | 0.142 | 0.175 | 0.220 | 0.285 | 0.371 |
| 42 | 0.084 | 0.110 | 0.139 | 0.171 | 0.214 | 0.278 | 0.361 |
| 44 | 0.082 | 0.108 | 0.136 | 0.167 | 0.209 | 0.271 | 0.352 |
| 46 | 0.080 | 0.105 | 0.133 | 0.164 | 0.204 | 0.264 | 0.344 |
| 48 | 0.079 | 0.103 | 0.130 | 0.160 | 0.199 | 0.258 | 0.336 |
| 50 | 0.077 | 0.101 | 0.128 | 0.157 | 0.195 | 0.253 | 0.328 |
| 52 | 0.076 | 0.099 | 0.126 | 0.154 | 0.191 | 0.247 | 0.321 |
| 54 | 0.074 | 0.097 | 0.123 | 0.151 | 0.187 | 0.243 | 0.315 |
| 26 | 0.073 | 0.096 | 0.121 | 0.148 | 0.184 | 0.238 | 0.309 |
| 58 | 0.072 | 0.094 | 0.119 | 0.146 | 0.181 | 0.233 | 0.303 |
| 60 | 0.071 | 0.092 | 0.117 | 0.143 | 0.177 | 0.229 | 0.297 |
| 62 | 0.070 | 0.091 | 0.116 | 0.141 | 0.174 | 0.225 | 0.292 |
| 64 | 0.068 | 0.089 | 0.114 | 0.139 | 0.171 | 0.221 | 0.287 |
| 66 | 0.067 | 0.088 | 0.112 | 0.137 | 0.169 | 0.218 | 0.282 |
| 68 | 0.066 | 0.087 | 0.111 | 0.135 | 0.166 | 0.214 | 0.278 |

table vi. percentage points, $\ell_{\gamma}$, suce that $P\left[\hat{c} \ln (\hat{b} / b)<\ell_{Y}\right]=\gamma$ (Continued)

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N |  | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 |
| 70 | 0.065 | 0.085 | 0.109 | 0.133 | 0.164 | 0.211 | 0.274 |
| 72 | 0.064 | 0.084 | 0.108 | 0.131 | 0.161 | 0.208 | 0.269 |
| 74 | 0.064 | 0.083 | 0.107 | 0.129 | 0.159 | 0.205 | 0.266 |
| 76 | 0.063 | 0.082 | 0.105 | 0.128 | 0.157 | 0.202 | 0.262 |
| 78 | 0.062 | 0.081 | 0.104 | 0.126 | 0.155 | 0.199 | 0.258 |
| 80 | 0.061 | 0.080 | 0.103 | 0.125 | 0.153 | 0.197 | 0.255 |
| 85 | 0.059 | 0.077 | 0.100 | 0.121 | 0.148 | 0.190 | 0.246 |
| 90 | 0.057 | 0.075 | 0.097 | 0.118 | 0.143 | 0.185 | 0.239 |
| 95 | 0.056 | 0.073 | 0.095 | 0.115 | 0.139 | 0.179 | 0.232 |
| 100 | 0.054 | 0.071 | 0.093 | 0.112 | 0.136 | 0.175 | 0.226 |
| 110 | 0.051 | 0.067 | 0.089 | 0.107 | 0.129 | 0.166 | 0.215 |
| 120 | 0.049 | 0.064 | 0.085 | 0.103 | 0.123 | 0.159 | 0.205 |


*Reproduced from "An Exact Asymptotically Efficient Confidence Bound for
Reliability in the Case of the Weibull Distribution," by M. V. Johns, Jr. and
G. J. Lieberman, Technometrics, Vol. 8, No. 1, (1966), pp. 135-175.
[ABLE VII. TABLE OF COEFFICIENTS FOK COMPUTING $z_{a}$ aND $z_{b}$ (Crucinued)

| $n=15$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $i$ | $a_{i}$ | $\mathrm{b}_{\mathrm{i}}$ | $a_{i}$ | $\mathrm{b}_{\mathrm{i}}$ | $\mathbf{a}_{1}$ | $\mathrm{b}_{i}$ | $a_{i}$ | $\mathrm{b}_{\mathrm{i}}$ |
| 1 | . 02383 | -. 044619 | .00065 .00683 | -. $07776=$ | .07407 .06265 | $=.13306-$ $-.13055-$ | .47594 .41146 | -.31963 -.29297 |
| 3 | . 03452 | -. 04890 | . 01367 | - . 07754 - | . 04717 | . . 12274 | 1.88740 | . 61260 |
| 4 | . 039641 | - . 048432 | . 02120 | -. 07379 - | . 02811 | -. 11056 |  |  |
| 6 | . 04590 | -. .04392 | . 03871 | - . 05989 | . 02125 | - . 07332 |  |  |
| 7 | . 05525 | -. 03996 | . 04899 | -. .04941 | 1.19616 | . 06438 |  |  |
| 8 | . 06084 | - . 03461 | . 06059 | - . 03610 |  |  |  |  |
| 9 | . 40677 | - . 02757 | . 07386 | -. .01931 |  |  |  |  |
| 10 | . 07315 | - . 01835 | . 00930 | . 00194 |  |  |  |  |
| 11 | . 08016 | - . 00619 | . 61672 | . 53884 |  |  |  |  |
| 12 | . 08808 | . 01025 |  |  |  |  |  |  |
| 13 | . 49740 | . 03359 |  |  |  |  |  |  |
| 14 | . 10911 | . 06982 |  |  |  |  |  |  |
| 15 | . 14734 | . 29532 |  |  |  |  |  |  |

$n=20$

| $p=r / n$ |  | 1 | 15/20 |  | 10/20 |  | 5/20 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $i$ | $\mathrm{a}_{1}$ | $b_{1}$ | $a_{i}$ | $\mathrm{b}_{\mathrm{i}}$ | $\mathbf{a}_{i}$ | $\mathrm{b}_{\mathrm{i}}$ | $\mathrm{a}_{\boldsymbol{i}}$ | $b_{i}$ |
| 1 | . 01682 | -. .03402 | . 00120 | -. 05593 - | . 04527 | - . 09198 - | . 24498 | -. 19315 |
| 2 | . 02007 | - . 03564 | . 00456 | - . 05745 - | . 04032 | - . 09230 - | . 22587 | -. 18644 |
| 3 | . 02312 | -. .03645 | . 00816 | -. .05754 | . 03371 | - . 09010 - | . 19843 | -. 17383 |
| 4 | . 02607 | -. 03667 | . 01201 | - . 05657 - | . 02574 | - . 08597 - | . 16426 | -. 15659 |
| 5 | . 02898 | -. .03039 | . 01612 | -. .05468 - | . 01650 | -. .08013 | 1.83354 | -. 71001 |
| 6 | . 03189 | -. . 03563 | . 02053 | - . $05190-$ | . 00596 | - . 07264 |  |  |
| 7 | . 03482 | -. 03441 | . 02526 | -. 04822 | . 00595 | -. .06345 |  |  |
| 8 | . 03780 | -. . 03269 | . 03037 | -. 04361 | . 01935 | -. .05246 |  |  |
| $y$ | . 04080 | -. 03045 | . 03591 | - . 03798 | . 03444 | -. 03948 |  |  |
| 10 | . 04401 | - . 02762 | . 04196 | - . 03119 | 1.10777 | -. 66851 |  |  |
| 11 | . 04729 | -. 02411 | . 04862 | -. 02306 |  |  |  |  |
| 12 | . 05074 | -. 01980 | . 05600 | -. 01336 |  |  |  |  |
| 13 | . 05434 | -. .01451 | . 06428 | -. 00172 |  |  |  |  |
| 14 | . 05830 | - . 00748 | . 07369 | . 01235 |  |  |  |  |
| 15 | . 06257 | . 00016 | . 56132 | . 52087 |  |  |  |  |
| 10 | . 06730 | . 01054 |  |  |  |  |  |  |
| 17 | . 07268 | . 02419 |  |  |  |  |  |  |
| 18 | . 07906 | . 04312 |  |  |  |  |  |  |
| 19 | . 08714 | . 07197 |  |  |  |  |  |  |
| 20 | . 11608 | . 25640 |  |  |  |  |  |  |

TABLE VII. TABLE OF COEFFICIENTS FOR COMPUTING $z_{a}$ AND $2_{b}$ (Continued)

| $n=30$ |  |  |  |  |  |  |  | 7/30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $a_{1}$ | $b_{1}$ | $a_{1}$ | $b_{1}$ | $a_{1}$ | $b_{1}$ | ${ }^{1} 1$ | $b_{1}$ |
| 1 | . 01046 | -. 02215 | . 00105 | -. 03780 | . 03092 | -. 06072 | . 18830 | 13857 |
| 2 | . 01201 | - . 02314 | . 00042 | -. 03893 | . 02916 | -. 06164 | .18110 | . 13688 |
| 3 | . 01345 | -. . 02379 | . 00198 | - . 03944 | . 02675 | -. .06151 | . 17012 | . 13259 |
| 4 | . 01483 | -. 02419 | . 00363 | - . 03951 | . 02385 | - . 06063 | . 15631 | . 12611 |
| 5 | . 01618 | -. 02440 | . 00536 | - . 03923 | . 02052 | - . 05914 | . 14008 | . 11862 |
| 6 | . 01751 | -. 02445 | . 00718 | - . 03863 | . 01679 | - . 05710 | . 12169 | . 10937 |
| 7 | . 01882 | -. 02434 | . 00910 | - . 03775 | . 01267 | - . 05453 | 1.95751 | . 76245 |
| 8 | . 02013 | - . 02409 | . 01111 | -. 03658 | . 00816 | - . 05146 |  |  |
| 9 | . 02145 | -. .02369 | . 01323 | -. .03513 | . 00325 | - . 04788 |  |  |
| 10 | . 02277 | - . 02315 | . 01546 | - . .03339 | . 00209 | -. .04379 |  |  |
| 11 | . 02411 | -. 02246 | . 01781 | - . 03137 | . 00787 | -. 03915 |  |  |
| 12 | . 02546 | -. .22162 | . 02030 | - . 02903 | . 01413 | -. . 03393 |  |  |
| 13 | . 02684 | -. 02062 | . 02294 | -. 02636 | . 02092 | - . 02811 |  |  |
| 14 | . 02824 | - . 01944 | . 02574 | - . 02334 | . 02827 | -. 02161 |  |  |
| 15 | . 02969 | -. . 01807 | . 02873 | - . 01992 | 1.09870 | -. 68119 |  |  |
| 16 | . 03117 | -. .01648 | . 03192 | -. 01607 |  |  |  |  |
| 17 | . 03270 | -. 01466 | . 03535 | -. 01173 |  |  |  |  |
| 18 | . 03428 | -. . 01257 | . 03904 | - . 00683 |  |  |  |  |
| 19 | . 03593 | -. .01017 | . 04305 | . 00129 |  |  |  |  |
| 20 | . 03767 | -. 00740 | . 04743 | . 00500 |  |  |  |  |
| 21 | . 03949 | -. 00420 | . 05225 | . 01218 |  |  |  |  |
| 22 | . 04143 | -. 00047 | . 56902 | . 52514 |  |  |  |  |
| 23 | . 04350 | . 00391 |  |  |  |  |  |  |
| 24 | . 04574 | . 00913 |  |  |  |  |  |  |
| 25 | . 04821 | . 01545 |  |  |  |  |  |  |
| 25 | . 05096 | . 22331 |  |  |  |  |  |  |
| 27 | . 05412 | . 03343 |  |  |  |  |  |  |
| 28 | . 05789 | . 04719 |  |  |  |  |  |  |
| 29 | . 06270 | . 06779 |  |  |  |  |  |  |
| 30 | . 08227 | . 20535 |  |  |  |  |  |  |


| $p=r / n$ | 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| 1 | $.00588-.01296-.00067-.02197-.01880-.03594-.10798-.08046$ |  |
| :--- | :--- | :--- |
| 2 | $.00648-.01344-.00016-.02259-.01837-.03662-.0666-.08072$ |  |
| 3 | $.00705-.01380-.00038-.02299-.01772-.03694-.0414-.08013$ |  |
| 4 | $.00758-.01407$ | $.00093-.02325-.01691-.03701-.10078-.07896$ |
| 5 | $.00810-.01428-.00150-.02340-.01599-.03689-.09675-.07731$ |  |
| 6 | $.00861-.01444-.00209-.02345-.01496-.03661-.09213-.07527$ |  |
| 7 | $.00911-.01456$ | $.00270-.02342-.01382-.03619-.08699-.07287$ |
| 8 | $.00960-.01463-.00333-.02332-.01260-.03563-.08135-.07013$ |  |

TABLE VII. TABLE OP PORFPICIENTS FOR COMPUTING $z_{a}$ AND $z_{b}$ (Continued)

| $p=r / n \quad 37 / 50^{n=50}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | ${ }^{a_{1}}$ | $b_{1}$ | $\mathrm{a}_{1}$ | $b_{1}$ | $a_{1} \quad b_{1}$ | $a_{1} \quad b_{1}$ |
| 9 | . 01008 | -. 01467 | . 00398 | -. 02315 - | . 01129 - . 03495 - | . $07523-.06709$ |
| 10 | . 01057 | -. 01467 | . 00464 | -. 02291 - | . $000989-.03415-$ | . 06865 - . 06373 |
| 11 | . 01105 | -. 01463 | . 00532 | -. 02261 - | . 00840 - . 03324 - | . 06162 - . 06008 |
| 12 | . 01152 | -. 01457 | . 00603 | - . 02224 - | . 00683 - .03221 | 1.98227 . 80675 |
| 13 | . 01200 | -. . 01447 | . 00675 | - . 02182 | . 00517 - . 03107 |  |
| 14 | . 01248 | -. 01434 | . 00750 | - . 02133 - | . 00342 - . 02982 |  |
| 15 | . 01296 | -. .01418 | . 00827 | -. 02073 | . 00158 - . 02846 |  |
| 16 | . 01344 | -. 01399 | . 00907 | -. 02018 | . 00035 - . 02698 |  |
| 17 | . 01393 | -. 01376 | . 00989 | -. 01951 | . 00238 - . 02538 |  |
| 18 | . 01442 | -. .01351 | . 01073 | -. . 01877 | . 00451 - . 02366 |  |
| 19 | . 01491 | -. 01321 | . 01161 | -. 01796 | . $00675-.02181$ |  |
| 20 | . 01540 | -. 01289 | . 01252 | -. 01709 | . $00910-.01984$ |  |
| 21 | . 01591 | -. . 01252 | . 01345 | -. .01614 | . 01156 - . 01772 |  |
| 22 | . 01642 | -. 01212 | . 01443 | -. 01512 | . 01415 - . 01546 |  |
| 23 | . 01693 | -. .01168 | . 01544 | -. .01401 | . 01687 - . 01305 |  |
| 24 | . 01745 | -. 01119 | . 01648 | -. .01282 | . 01973 - . 01047 |  |
| 25 | . 01798 | -. 01067 | . 01757 | - . .01154 | 1.09038 .69008 |  |
| 26 | . 01852 | -. 01009 | . 01871 | - . 01017 |  |  |
| 27 | . 01907 | -. 00946 | . 01989 | -. 00869 |  |  |
| 28 | . 01964 | - . 00877 | . 02113 | - . 00709 |  |  |
| 29 | . 02021 | -. 00803 | . 02243 | -. 00538 |  |  |
| 30 | . 02080 | -. 00722 | . 02379 | -. .00354 |  |  |
| 31 | . 02140 | -. .00634 | . 02521 | -. 00154 |  |  |
| 32 | . 02202 | -. 000538 | . 02672 | - . 00060 |  |  |
| 33 | . 02266 | -. .00434 | . 02831 | - . 00203 |  |  |
| 34 | . 02332 | - . 00319 | . 02999 | -. 00545 |  |  |
| 35 | . 02401 | -. 00194 | . 03178 | . 00820 |  |  |
| 36 | . 02471 | - . 00057 | . 03369 | . 01119 |  |  |
| 37 | . 02545 | . 00095 | . 53455 | . 51043 |  |  |
| 38 | . 02623 | . 00263 |  |  |  |  |
| 39 | . 02704 | . 00450 |  |  |  |  |
| 40 | . 02789 | . 00659 |  |  |  |  |
| 41 | . 02880 | . 00894 |  |  |  |  |
| 42 | . 02977 | . 01163 |  |  |  |  |
| 43 | . 03082 | . 01473 |  |  |  |  |
| 44 | . 03196 | . 01834 |  |  |  |  |
| 45 | . 03322 | . 02266 |  |  |  |  |
| 46 | . 03464 | . 02793 |  |  |  |  |
| 47 | . 03627 | . 03462 |  |  |  |  |
| 48 | . 03823 | . 04360 |  |  |  |  |
| 49 | . 04074 | . 05685 |  |  |  |  |
| 50 | . 05269 | . 15062 |  |  |  |  |

table vil. table of coefficients for computing $i_{i}$ and $z_{b}$ (Continued)

| $n=100$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{a}_{1}$ | $\mathrm{b}_{1}$ | $a_{1}$ | $b_{i}$ | $\mathrm{a}_{1}$ | $\mathrm{b}_{\mathrm{i}}$ | $\mathrm{a}_{1}$ | $\mathrm{b}_{1}$ |
| 1 | . 00277 | . 00632 - | . 00020 | -. 01056 - | . 00944 | -. 01768 | . 04999 | . 03820 |
| 3 | $\begin{aligned} & .00294 \\ & .00310 \end{aligned}$ | $.00648=$ | $\begin{aligned} & .00013 \\ & .00000 \end{aligned}$ | $\begin{aligned} & -.01079- \\ & -.01096- \end{aligned}$ | $\begin{aligned} & .00940 \\ & .00931 \end{aligned}$ | $\begin{array}{r} -.01797 \\ -.01817 \end{array}$ | $\begin{array}{r} .05003 \\ .04981 \end{array}$ | $\begin{array}{r} .03854 \\ -.03868 \end{array}$ |
| 4 | . 00325 | . 00673 | . 00013 | . 01109 | . 00918 | . 01831 | . 04249 | . . 03869 |
| 5 | . 00339 | . 00682 | . 00027 | . 01121 | . 00902 | . 01841 | . 04889 | . .03861 |
| 6 | . 00353 | . 00690 | . 00040 | . 01130 | . 00885 | . 01847 | . 04824 | . 03844 |
| 7 | . 00367 | . 00698 | . 00054 | -. 01137 | . 00865 | . 01850 | . 04750 | . 03820 |
| $\stackrel{1}{6}$ | . 00380 | . 00704 | . 00068 | . 01143 | . 00844 | .01851 | . 04668 | . 03789 |
| 9 | . 00393 | . .00710 | . 00082 | . 11477 | . 00821 | . 01848 | . 04577 | -. 03754 |
|  | . 00400 | - . 00715 | . 00097 | - . 01150 | . 00797 | -. 01844 | . 04479 | -. 03713 |
| $11$ | . 00419 | - . 00719 | . 00111 | - .01152- | . 00771 | - . 01838 | . 04375 | - . 03667 |
| 12 | . 00432 | - . 00723 | . 00126 | -. 01153 - | . 00745 | - . 01829 | . 04264 | - . U3626 |
| 13 | . 00445 | - . 00726 | . 00141 | - . 01153 - | . 00710 | - . 01819 | . 04146 | - . 03562 |
| 14 | . 04457 | -. 00728 | . 00157 | - . 011152 - | . 00687 | -. 01808 | . 04023 | -. 03503 |
| 15 | . 00469 | $-.00730$ | . 00172 | - . 011150 - | . 006557 | -. .01794 - | . 03894 | -.03440 -.03374 |
| 16 | . 00482 | -. 000732 | . 0002028 | -. 01147 | . 000625 | -. .017762- | . .03619 | - . . 03334 |
| 18 | . 04506 | - . 00733 | . 00220 | -. 01138 | . 00558 | -. 01744 | . 03473 | -. 03229 |
| 19 | .00519 | - . 00733 | . 00236 | - . 01133 | . 00523 | -. 01724 | . 03322 | -. .03151 |
|  | . 00531 | - . 00733 | . 00253 | -. 01127 | . 00487 | -. 01703 | . 03105 | -. 03069 |
| 21 | . 00543 | - . 00732 | . 00270 | -. 01120 | . 00450 | -. .01681 | . 03003 | -. 02984 |
| 22 | . 00555 | - . 00731 | . 00287 | - . 01112 | . 00412 | - . 01650 | . 02836 | -. . 22895 |
| 23 | . 00567 | - . 00730 | . 00304 | - . 01103 | . 00372 | - . 01631 | . 02663 | - . 02803 |
| 24 | . 00579 | - . 00728 | . 00322 | -. 01094 | . 00332 | -. 01604 - | . 02485 | . 02707 |
| 25 | . 00591 | - . 00726 | . 00389 | - . 01084 - | . 00290 | - . 01576 | 1.97141 | . 83495 |
| 20 | . 00603 | - . 00723 | . 00357 | -. 01073 | . 00248 | - . 01546 |  |  |
| 27 | . 00615 | - . 00720 | . 00376 | -. 01001 - | . 00204 | -. 01515 |  |  |
| 28 | . 00627 | - . 00716 | . 00394 | - . 01049 - | . 00159 | - . 01482 |  |  |
| 29 | . 00640 | - . 00712 | . 00413 | -. 01036 - | . 00113 | -. 01448 |  |  |
| 30 | . 00652 | - . 00708 | . 00432 | - . 01022 | . 00065 | -. 01412 |  |  |
| 31 | . 00664 | - .00703 | . 00452 | - . 01007 | . 00017 | -. 01375 |  |  |
| 32 | . 00676 | -. 00698 | . 00472 | -. 00932 | . 00033 | - . 01337 |  |  |
| 33 | . 00688 | - . 00692 | . 00492 | - . 00970 | . 00084 | - . 01297 |  |  |
| 34 | . 00701 | - . 00686 | . 00512 | - . 00459 | . 00137 | - . 01255 |  |  |
| 35 | . 00713 | - . 00680 | . 00533 | -. 00941 | . 00190 | - . 01212 |  |  |
| 36 | . 00725 | - . 00673 | . 00544 | - . 00922 | . 00245 | -. 01167 |  |  |
| 37 | . 00738 | - . 00665 | . 60575 | -. 00903 | . 00301 | -. 01121 |  |  |
| 38 | . 00750 | -. 00658 | . 00597 | -. 00882 | . 00359 | -. 01.073 |  |  |
| 39 | . 00763 | -. 006550 | . 00619 | -. 00861 | . 00418 | -. 01023 |  |  |
| 40 | . 00775 | - .00611 | . 00642 | - . 00839 | .00479 | -.00971 |  |  |

TABLL VII. TABLE OF LOEFFIXIENTS FOK COMPUTING $z_{a}$ AND $z_{b}$ (Continued)

| $n=100$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| i | ${ }^{\text {a }}$ | $b_{i}$ | ${ }^{\mathbf{a}} \mathbf{i}$ | $b_{i}$ | $a_{i}$ | $b_{i}$ | ${ }^{\text {a }}$ | $b_{i}$ |
| 41 | . 00788 | -. 00632 | . 00664 | -. 00816 | . 00541 | -. 00918 |  |  |
| 42 | . 00801 | -. 00622 | . 00688 | -. 00792 | . 00695 | -. .00863 |  |  |
| 43 | . 00813 | -. 00612 | . 00711 | -. 00767 | . 00670 | -. 00806 |  |  |
| 44 | . 00826 | - . 00601 | . 00736 | -. 00741 | . 00737 | -. .00747 |  |  |
| 45 | . 00839 | -. 005590 | . 00760 | -. 00714 | . 00805 | -. 00687 |  |  |
| 46 | . 00852 | - . 00579 | . 00785 | -. 00686 | . 00875 | - . 00624 |  |  |
| 47 | . 00866 | -. .00506 | . 00811 | -. 00657 | . 00948 | -. 00559 |  |  |
| 48 | . 00879 | - . 00554 | . 00837 | -. 00627 | . 01022 | - . 00492 |  |  |
| 49 | . 00892 | - . 00540 | . 00863 | -. 00595 | . 01097 | - . 00422 |  |  |
| 50 | . 00906 | - . 00526 | . 00890 | - . 00563 | 1.08321 | . 69595 |  |  |
| 21 | . 00919 | -. 00512 | . 00918 | - 20529 |  |  |  |  |
| 52 | . 00933 | -. 00497 | . 00946 | -. 00494 |  |  |  |  |
| 53 | . 00947 | - . 00481 | . 00975 | -. 00458 |  |  |  |  |
| 54 | . 00961 | -. 00464 | . 41005 | -. 00420 |  |  |  |  |
| 55 | . 00975 | -. .00447 | . 01035 | -. 00381 |  |  |  |  |
| 50 | . 00990 | -. 00429 | . 01065 | -. 00340 |  |  |  |  |
| 57 | . 01004 | -. 00410 | . 01097 | -. 00298 |  |  |  |  |
| 58 | . 01019 | -. .00390 | . 01129 | - . 00255 |  |  |  |  |
| 59 | . 01034 | -. 00370 | . 01162 | - . 00209 |  |  |  |  |
| 60 | . 01049 | -. 00348 | . 01196 | -. 00102 |  |  |  |  |
| 61 | . 01064 | - . 00326 | . 01231 | - . 00113 |  |  |  |  |
| 62 | . 01080 | -. 00303 | . 01266 | -. 00062 |  |  |  |  |
|  | $.01095$ | $\text { -. . } 00278$ | $.01303$ | $-.00009$ |  |  |  |  |
| 64 | $.01111$ | -. .00253 | $.01341$ | $.00046$ |  |  |  |  |
| 65 | .01127 | -. 00226 | . 01379 | . 00104 |  |  |  |  |
| 06 | . 011144 | - . 00199 | . 01419 | . 00163 |  |  |  |  |
| 67 | . 01160 | -. 00170 | . 01460 | . 00226 |  |  |  |  |
| 68 | . 01177 | -. 00139 | . 01502 | . 00291 |  |  |  |  |
| 69 | . 01195 | -. 00107 | . 01546 | . 00359 |  |  |  |  |
| 70 | . 01212 | -. 00074 | . 01591 | . 00429 |  |  |  |  |
| 71 | . 01230 | -. 00039 | .01637 | . 00504 |  |  |  |  |
| 72 | . 01240 | -. 00002 | . 01685 | . 00581 |  |  |  |  |
| 73 | . 01267 | . 00037 | . 01735 | . 00663 |  |  |  |  |
| 74 | . 01287 | . 00077 | . 01786 | . 00748 |  |  |  |  |
| 75 | . 01306 | . 00120 | . 44752 | . 49223 |  |  |  |  |
| 76 | . 01326 | .00165 |  |  |  |  |  |  |
| 77 | . 01347 | . 00213 |  |  |  |  |  |  |

jABLE VII. TABLE OF COEFFICIENTS FOR COMPUTING $z_{a}$ AND $z_{b}$ (Continued)

| $n=100$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p=r / n$ | 1 |  |  |  | 80/100 |  | 25/100 |  |
| 1 | $\mathbf{a}_{i}$ | $\mathrm{b}_{i}$ | ${ }^{\mathbf{i}}$ | $b_{i}$ | ${ }^{\mathbf{e}}$ | $b_{i}$ | ${ }^{\text {a }}$ | $b_{i}$ |
| 78 | . 01368 | . 00264 |  |  |  |  |  |  |
| 79 | . 01390 | $.00317$ |  |  |  |  |  |  |
| 80 | . 01412 | $.00374$ |  |  |  |  |  |  |
| 81 | . 01436 | . 00435 |  |  |  |  |  |  |
| 82 | .01459 | . 00500 |  |  |  |  |  |  |
| 83 | . 01484 | . 00569 |  |  |  |  |  |  |
| 84 | . 01510 | . 00643 |  |  |  |  |  |  |
| 85 | . 01537 | . 00723 |  |  |  |  |  |  |
| 86 | . 01565 | . 00810 |  |  |  |  |  |  |
| 87 | . 01594 | . 00904 |  |  |  |  |  |  |
| 88 | . 01625 | . 01006 |  |  |  |  |  |  |
| 89 | . 01657 | . 01119 |  |  |  |  |  |  |
| 90 | . 01692 | . 01243 |  |  |  |  |  |  |
| 91 | $.01728$ | . 01382 |  |  |  |  |  |  |
| 92 | . 01768 | . 01538 |  |  |  |  |  |  |
| 93 | . 01810 | . 01716 |  |  |  |  |  |  |
| 94 | . 01857 | . 01922 |  |  |  |  |  |  |
| 95 | . 01908 | . 02164 |  |  |  |  |  |  |
| 96 | . 01967 | . 02458 |  |  |  |  |  |  |
| 97 | $.02034$ | $.02826$ |  |  |  |  |  |  |
| 98 | . 02115 | . 03314 |  |  |  |  |  |  |
| 99 | . 02220 | . 01027 |  |  |  |  |  |  |
| ius | . 02829 | . 09493 |  |  |  |  |  |  |

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BGUNDS BOR R( $\left.\mathrm{E}_{0}\right)$ *


TABLE VIII. TABLE OF EXACT LONER CONFIDENCE BOUNDS POR R( $t_{0}$ ) (Continued)

|  | $\begin{aligned} & n=10 \\ & p=r / n=2 / 10 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $r=0.99$ |
| $2 a^{\prime 2}{ }^{\text {b }}$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(2 / z_{b}\right)$ | $L^{*}\left(2_{a} / z_{b}\right)$ | $\underline{L}\left(2_{a} / z_{b}\right)$ | $L^{*}\left(2_{a} / 2_{b}\right)$ |
| 3.8 | . 934 | . 875 | . 804 | . 740 | . 609 |
| 3.9 | . 937 | . 879 | . 807 | . 745 | . 611 |
| 4.0 | . 939 | . 880 | . 809 | . 748 | . 613 |
| 4.1 | . 941 | . 883 | . 810 | . 752 | . 616 |
| 4.2 | . 944 | . 887 | . 813 | . 753 | . 617 |
| 4.3 | . 946 | . 890 | . 816 | . 754 | . 617 |
| 4.4 | . 948 | . 892 | . 818 | . 755 | . 618 |
| 4.5 | . 950 | . 894 | . 820 | . 756 | . 622 |
| 4.6 | . 951 | . 896 | . 823 | . 759 | . 623 |
| 4.7 | . 953 | . 899 | . 825 | . 762 | . 623 |
| 4.8 | . 955 | . 900 | . 826 | . 764 | . 623 |
| 4.9 | . 957 | . 901 | . 830 | . 766 | . 626 |
| 5.0 | . 959 | . 902 | . 833 | . 769 | . 628 |

[^5]TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS POR R(EO) (Continued)

|  | $\gamma=0.50$ | $\begin{aligned} & n=10 \\ & p=r / n \cdot s / 10 \end{aligned}$ |  |  | $\gamma=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\gamma=0.75$ | $Y=0.90$ | $\gamma=0.95$ |  |
| $2 / 2$ | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | L* $\left(Z_{a} / Z_{b}\right)$ | $L *\left(2_{e} / 2_{b}\right)$ |  |
| . 0 | . 464 | . 341 | . 244 | . 193 | . 095 |
| . 1 | . 490 | . 374 | . 281 | . 234 | . 126 |
| . 2 | . 516 | . 403 | . 315 | . 270 | . 153 |
| . 3 | . 543 | . 434 | . 350 | . 304 | . 180 |
| . 4 | . 568 | . 466 | . 381 | . 334 | . 217 |
| . 5 | . 593 | . 493 | . 410 | . 357 | . 236 |
| . 6 | . 617 | . 518 | . 436 | . 386 | . 269 |
| . 7 | . 640 | . 542 | . 462 | . 413 | . 300 |
| . 8 | . 663 | . 568 | . 484 | . 430 | . 327 |
| . 9 | . 684 | . 589 | . 508 | . 455 | . 352 |
| 1.0 | . 703 | . 611 | . 531 | . 475 | . 378 |
| 1.1 | . 720 | . 633 | . 543 | . 495 | . 404 |
| 1.2 | . 738 | . 653 | . 564 | . 513 | . 426 |
| 1.3 | . 766 | . 674 | . 581 | . 526 | . 440 |
| 1.4 | . 771 | . 692 | . 599 | . 540 | . 453 |
| 1.5 | . 789 | . 707 | . 616 | . 557 | . 467 |
| 1.6 | . 805 | . 719 | . 631 | . 573 | . 486 |
| 1.7 | . 817 | . 731 | . 643 | . 589 | . 495 |
| 1.8 | . 829 | . 745 | . 657 | . 603 | . 505 |
| 1.9 | . 840 | . 759 | . 670 | . 618 | . 518 |
| 2.0 | . 850 | . 773 | . 683 | . 630 | . 530 |
| 2.1 | . 859 | . 784 | . 696 | . 642 | . 538 |
| 2.2 | . 868 | . 794 | . 707 | . 651 | . 551 |
| 2.3 | . 877 | . 804 | . 720 | . 662 | . 561 |
| 2.4 | . 885 | . 815 | . 733 | . 671 | . 568 |
| 2.5 | . 892 | . 824 | . 742 | . 678 | . 576 |
| 2.6 | . 899 | . 832 | . 750 | . 687 | . 592 |
| 2.7 | . 906 | . 843 | . 760 | . 696 | . 606 |
| 2.8 | . 912 | . 850 | . 771 | . 707 | . 616 |
| 2.9 | . 919 | . 858 | . 780 | . 718 | . 627 |
| 3.0 | . 924 | . 864 | . 789 | . 724 | . 634 |
| 3.1 | . 929 | . 870 | . 797 | . 730 | . 640 |
| 3.2 | . 934 | . 878 | . 806 | . 736 | . 646 |
| 3.3 | . 939 | . 885 | . 813 | . 743 | . 652 |
| 3.4 | . 942 | . 891 | . 820 | . 751 | . 657 |
| 3.5 | . 946 | . 896 | . 826 | . 756 | . 663 |
| 3.6 | . 950 | . 901 | . 834 | . 762 | . 669 |
| 3.7 | . 953 | . 906 | . 841 | . 767 | . 675 |
| 3.8 | . 956 | . 911 | . 846 | . 775 | . 681 |
| $L *\left(2_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |

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table vili. table of exact lower confidence bounds for r(to) (Continued)

|  | $\begin{aligned} & n=10 \\ & p=r i n=5 / 10 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=0.50$ | $Y=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $Y=0.99$ |
| $2 / 2{ }_{6}$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L^{*}\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| 3.9 | . 960 | . 916 | . 850 | . 783 | . 687 |
| 4.0 | . 963 | . 921 | . 855 | . 789 | . 693 |
| 4.1 | . 965 | . 925 | . 862 | . 795 | . 699 |
| 4.2 | . 968 | . 929 | . 867 | . 801 | . 704 |
| 4.3 | . 970 | . 932 | . 872 | . 806 | . 710 |
| 4.4 | . 972 | . 935 | . 877 | . 812 | . 716 |
| 4.5 | . 974 | . 939 | . 883 | . 818 | . 721 |
| 4.6 | . 975 | . 942 | . 887 | . 823 | . 727 |
| 4.7 | . 977 | . 944 | . 891 | . 828 | . 732 |
| 4.8 | . 978 | . 947 | . 896 | . 834 | . 738 |
| 4.9 | . 980 | . 950 | . 899 | . 839 | . 744 |
| 5.0 | . 981 | .953 | . 903 | . 843 | . 750 |

[^6]table vili. table of exact lower confidbnce bounds for r( $\left.t_{0}\right)$ (Continued)

|  | $\gamma=0.50$ | $\begin{array}{r} n \\ \mathrm{p} \\ Y=0.75 \end{array}$ | $\begin{aligned} & n=7 / 10 \\ & Y=0.90 \end{aligned}$ | $\gamma=0.95$ | $\gamma=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2 / 2$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ |
| . 0 | . 415 | . 332 | . 260 | . 215 | . 141 |
| . 1 | . 448 | . 362 | . 290 | . 245 | . 163 |
| . 2 | . 480 | . 396 | . 323 | . 275 | . 191 |
| . 3 | . 509 | . 423 | . 351 | . 302 | . 214 |
| . 4 | . 537 | . 453 | . 380 | . 328 | . 243 |
| . 5 | . 567 | . 482 | . 402 | . 357 | . 271 |
| . 6 | . 595 | . 509 | . 426 | . 387 | . 304 |
| . 7 | . 620 | . 535 | . 452 | . 408 | . 322 |
| . 8 | . 645 | . 560 | . 475 | . 425 | . 340 |
| . 9 | . 668 | . 582 | . 493 | . 447 | . 359 |
| 1.0 | . 691 | . 607 | . 514 | . 469 | . 377 |
| 1.1 | . 712 | . 629 | . 537 | .491 | .402 |
| 1.2 | . 730 | . 650 | . 555 | . 508 | . 420 |
| 1.3 | . 747 | . 669 | . 578 | . 522 | . 435 |
| 1.4 | . 767 | . 687 | . 595 | . 537 | .453 |
| 1.5 | . 784 | . 704 | . 612 | . 554 | . 467 |
| 1.6 | . 800 | . 722 | . 630 | . 570 | . 485 |
| 1.7 | . 814 | . 736 | . 645 | . 583 | . 500 |
| 1.8 | . 827 | . 752 | . 659 | . 599 | . 514 |
| 1.9 | . 840 | . 765 | . 672 | . 615 | . 529 |
| 2.0 | . 852 | . 779 | . 687 | . 628 | . 543 |
| 2.1 | . 862 | . 791 | . 702 | . 645 | . 556 |
| 2.2 | . 873 | . 804 | . 714 | . 660 | . 567 |
| 2.3 | . 883 | . 813 | . 727 | . 674 | . 576 |
| 2.4 | . 892 | . 824 | . 741 | . 684 | . 583 |
| 2.5 | . 900 | . 835 | . 752 | . 697 | . 590 |
| 2.6 | . 908 | . 845 | . 763 | . 710 | . 597 |
| 2.7 | . 915 | . 855 | . 773 | . 717 | . 604 |
| 2.8 | . 921 | . 863 | . 782 | . 727 | . 615 |
| 2.9 | . 927 | . 871 | . 792 | . 738 | . 626 |
| 3.0 | . 932 | . 879 | . 802 | . 748 | . 634 |
| 3.1 | . 937 | . 886 | . 811 | . 758 | . 640 |
| 3.2 | . 942 | . 892 | . 819 | . 768 | . 651 |
| 3.3 | . 947 | . 899 | . 830 | . 778 | . 662 |
| 3.4 | . 951 | . 905 | . 837 | . 787 | . 674 |
| 3.5 | . 955 | . 910 | . 845 | . 795 | . 684 |
| 3.6 | . 958 | . 916 | . 853 | . 804 | . 698 |
| 3.7 | . 952 | . 921 | . 860 | . 813 | . 702 |
| 3.8 | . 965 | . 926 | . 866 | . 821 | . 709 |

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R(to) (Continued)

|  | $\begin{aligned} & n=10 \\ & p=r / n=7 / 10 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $\gamma=0.99$ |
| $2 / 2 b$ | $L *\left(2_{a} / Z_{b}\right)$ | $\mathrm{L} *\left(\mathrm{Z}_{8} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ |
| 3.9 | . 968 | . 930 | . 873 | . 827 | . 715 |
| 4.0 | . 970 | . 934 | . 878 | . 836 | . 722 |
| 4.1 | . 973 | . 939 | . 883 | . 845 | . 730 |
| 4.2 | . 975 | . 943 | . 888 | . 851 | . 737 |
| 4.3 | . 977 | . 946 | . 893 | . 856 | . 744 |
| 4.4 | . 979 | . 949 | . 898 | . 861 | . 750 |
| 4.5 | . 980 | . 952 | . 904 | . 866 | . 755 |
| 4.6 | . 982 | . 955 | . 908 | . 870 | . 759 |
| 4.7 | . 983 | . 958 | . 911 | . 875 | . 764 |
| 4.8 | . 985 | . 960 | . 915 | . 880 | . 770 |
| 4.9 | . 986 | . 963 | . 919 | . 884 | . 778 |
| 5.0 | . 987 | . 965 | . 924 | . 889 | . 785 |

[^7]TABLE VIII. TABLE OF EXAGT LOWEE CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\gamma=0.50$ | $\begin{aligned} & n=10 \\ & p=r / n=1 \end{aligned}$ |  |  | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\gamma=0.75$ | $r=0.90$ | $Y=0.95$ |  |
| $2{ }^{12}$ | $L *\left(2 / Z_{b}\right)$ | $L *\left(2 / z_{b}\right)$ | $L *\left(2_{0} / 2_{0}\right)$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(2_{a} / z_{b}\right)$ |
| . 0 | . 393 | . 315 | . 255 | . 212 | . 153 |
| . 1 | . 428 | . 345 | . 283 | . 238 | . 176 |
| . 2 | . 458 | . 376 | . 311 | . 264 | . 202 |
| . 3 | . 492 | . 409 | . 336 | . 291 | . 226 |
| . 4 | . 522 | . 442 | . 363 | . 319 | . 247 |
| . 5 | . 552 | . 474 | . 389 | . 345 | . 273 |
| . 6 | .581 | . 502 | . 416 | . 371 | . 298 |
| . 7 | . 607 | . 527 | . 439 | . 396 | . 317 |
| . 8 | . 634 | . 553 | . 464 | . 421 | . 334 |
| . 9 | . 658 | . 579 | . 490 | . 442 | . 352 |
| 1.0 | . 682 | . 605 | . 512 | . 463 | . 369 |
| 1.1 | . 705 | . 627 | . 535 | . 485 | . 395 |
| 1.2 | . 726 | . 649 | . 559 | . 505 | . 421 |
| 1.3 | . 747 | . 670 | . 582 | . 525 | . 445 |
| 1.4 | . 768 | . 690 | . 605 | . 546 | . 462 |
| 1.5 | . 786 | . 707 | . 621 | . 567 | . 478 |
| 1.6 | . 800 | . 725 | . 644 | . 587 | . 494 |
| 1.7 | . 816 | . 742 | . 662 | . 603 | . 506 |
| 1.8 | . 830 | . 758 | . 680 | . 622 | . 522 |
| 1.9 | . 843 | . 772 | . 697 | . 636 | . 540 |
| 2.0 | . 855 | . 787 | . 710 | . 654 | . 559 |
| 2.1 | . 868 | . 801 | . 727 | . 668 | . 573 |
| 2.2 | . 879 | . 814 | . 743 | . 684 | . 587 |
| 2.5 | . 889 | . 826 | . 758 | . 700 | . 600 |
| 2.4 | . 898 | . 837 | . 772 | . 713 | . 613 |
| 2.5 | . 907 | . 847 | . 783 | . 725 | . 626 |
| 2.6 | . 915 | . 856 | . 795 | . 738 | . 639 |
| 2.7 | . 921 | . 865 | . 806 | . 751 | . 651 |
| 2.8 | . 928 | . 874 | . 816 | . 762 | . 665 |
| 2.9 | . 934 | . 883 | . 826 | . 775 | . 677 |
| 3.0 | . 939 | . 890 | . 835 | . 787 | . 689 |
| 3.1 | . 944 | . 898 | . 846 | . 800 | . 698 |
| 3.2 | . 949 | . 904 | . 853 | . 811 | . 708 |
| 3.3 | . 954 | . 911 | . 861 | . 820 | . 719 |
| 3.4 | . 957 | . 917 | . 868 | . 827 | . 729 |
| 3.5 | . 961 | . 922 | . 875 | . 837 | . 739 |
| 3.6 | . 964 | . 927 | . 883 | . 846 | . 748 |
| 3.7 | . 968 | . 933 | . 889 | . 854 | . 757 |
| 3.8 | . 970 | . 937 | . 895 | . 862 | . 766 |
| $L^{*}\left(\mathcal{L}_{a} / L_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |

table vili. table of exact lowbr confidence bounds for r(to) (Continued)

|  | $\begin{aligned} & n=10 \\ & p=r / n=1 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $Y=0.95$ | $Y=0.99$ |
| $2_{a} / 2{ }^{2}$ | $L *\left(2 / 2{ }_{\text {a }}\right.$ ) | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | L * $\left(2 / z_{b}\right)$ |
| 3.9 | . 973 | . 941 | . 901 | . 869 | . 775 |
| 4.0 | . 975 | . 945 | . 906 | . 875 | . 784 |
| 4.1 | . 977 | . 949 | . 912 | . 880 | . 792 |
| 4.2 | . 979 | . 952 | . 917 | . 885 | . 800 |
| 4.3 | . 981 | . 956 | . 922 | . 892 | . 808 |
| 4.4 | . 982 | . 959 | . 926 | . 898 | . 815 |
| 4.5 | . 984 | . 962 | . 931 | . 903 | . 822 |
| 4.6 | . 985 | . 964 | . 935 | . 907 | . 827 |
| 4.7 | . 987 | . 967 | . 939 | . 912 | . 833 |
| 4.8 | . 988 | . 969 | . 943 | . 916 | . 838 |
| 4.9 | . 989 | . 972 | . 946 | . 921 | . 843 |
| 5.0 | . 990 | . 974 | . 950 | . 925 | . 848 |

[^8]TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\gamma=0.50$ | $\begin{array}{r} \mathrm{p} \\ \mathrm{p} \\ \mathrm{p}=0.75 \end{array}$ | $\begin{aligned} & n=3 / 15 \\ & Y=0.90 \end{aligned}$ | $\gamma=0.95$ | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $z_{a} / z_{b}$ | $L^{*}\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{b} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ |
| . 0 | . 646 | . 443 | . 210 | .090 | . 001 |
| . 1 | . 660 | . 475 | . 242 | . 118 | . 006 |
| . 2 | . 673 | . 508 | . 276 | . 160 | .017 |
| . 3 | . 688 | . 534 | . 318 | . 194 | . 037 |
| . 4 | . 702 | . 561 | . 357 | . 240 | . 064 |
| . 5 | . 716 | . 583 | . 392 | . 285 | . 101 |
| . 6 | . 732 | . 607 | . 433 | . 331 | . 147 |
| . 7 | . 742 | . 627 | . 478 | . 382 | . 202 |
| . 8 | . 755 | . 647 | . 520 | . 428 | . 257 |
| . 9 | . 766 | . 665 | . 553 | . 465 | . 300 |
| 1.0 | . 776 | . 681 | . 584 | . 495 | . 348 |
| 1.1 | . 787 | . 698 | . 610 | . 533 | . 395 |
| 1.2 | . 799 | . 715 | . 631 | . 559 | . 451 |
| 1.3 | . 812 | . 733 | . 651 | . 591 | . 491 |
| 1.4 | . 821 | . 749 | . 671 | . 619 | . 522 |
| 1.5 | . 832 | . 762 | . 691 | . 642 | . 547 |
| 1.6 | . 840 | . 773 | . 707 | . 658 | . 566 |
| 1.7 | . 849 | . 785 | . 725 | . 673 | . 587 |
| 1.8 | . 857 | . 797 | . 736 | . 684 | . 604 |
| 1.9 | . 866 | . 806 | . 744 | . 697 | . 609 |
| 2.0 | . 873 | . 814 | . 751 | . 707 | . 620 |
| 2.1 | . 879 | . 823 | . 761 | . 718 | . 642 |
| 2.2 | . 886 | . 829 | . 768 | . 729 | . 654 |
| 2.3 | . 893 | . 839 | . 774 | . 739 | . 661 |
| 2.4 | . 900 | . 846 | . 782 | . 749 | . 676 |
| 2.5 | . 905 | . 853 | . 791 | . 757 | . 688 |
| 2.6 | . 911 | . 859 | . 798 | . 763 | . 699 |
| 2.7 | . 914 | . 864 | . 805 | . 772 | . 704 |
| 2.8 | . 919 | . 869 | . 814 | . 776 | . 707 |
| 2.9 | . 923 | . 874 | . 819 | . 784 | . 709 |
| 3.0 | . 927 | . 879 | . 824 | . 790 | . 711 |
| 3.1 | . 932 | . 883 | . 829 | . 794 | . 717 |
| 3.2 | . 936 | . 887 | . 832 | . 799 | . 720 |
| 3.3 | . 939 | . 891 | . 836 | . 802 | . 723 |
| 3.4 | . 942 | . 896 | . 842 | . 803 | . 726 |
| 3.5 | . 945 | . 899 | . 849 | . 810 | . 733 |
| 3.6 | . 948 | . 902 | . 852 | . 819 | . 735 |
| 3.7 | . 951 | . 906 | . 857 | . 822 | . 736 |
| 3.6 | . 953 | . 910 | . 860 | . 825 | . 738 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R(TO) (Continued)

|  | $n=15$ <br> $p=r / n=3 / 15$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $Y=0.90$ | $Y=C .95$ | $\gamma=0.99$ |
| $Z_{a} / Z_{b}$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ |
| 3.9 | .955 | .914 | .863 | .828 | .741 |
| 4.0 | .957 | .917 | .865 | .832 | .743 |
| 4.1 | .960 | .919 | .868 | .835 | .745 |
| 4.2 | .962 | .921 | .871 | .837 | .745 |
| 4.3 | .964 | .924 | .875 | .841 | .746 |
| 4.4 | .966 | .927 | .878 | .843 | .748 |
| 4.5 | .968 | .929 | .880 | .846 | .749 |
| 4.6 | .969 | .932 | .884 | .849 | .752 |
| 4.7 | .971 | .934 | .888 | .853 | .757 |
| 4.8 | .972 | .936 | .890 | .857 | .763 |
| 4.9 | .974 | .938 | .892 | .860 | .768 |
| 5.0 | .975 | .939 | 895 | .862 | .769 |

[^9]table vilit. table of exact lowta confidence bounds for reto (Continued)

|  | $\gamma=0.50$ | $\begin{aligned} & n=15 \\ & p=r / n=7 / 15 \end{aligned}$ |  | $\gamma=0.95$ | $\gamma=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\gamma=0.75$ | $\gamma=0.90$ |  |  |
| $2^{2} z_{b}$ | $L *\left(2 / z_{b}\right)$ | $L *\left(2 / L_{b}\right)$ | $L^{*}\left(z_{a} / z_{b}\right)$ | L* $\left(2_{i} / z_{b}\right)$ | $\underline{L *}\left(z_{a} / z_{b}\right)$ |
| . 0 | .443 | . 343 | . 247 | . 205 | . 111 |
| . 1 | . 471 | . 373 | . 284 | . 245 | . 149 |
| . 2 | . 502 | . 404 | . 320 | . 280 | . 197 |
| . 3 | . 530 | . 438 | . 356 | . 317 | . 243 |
| . 4 | . 560 | . 472 | . 391 | . 351 | . 285 |
| . 5 | . 586 | . 500 | . 427 | . 385 | . 318 |
| . 6 | . 615 | . 528 | . 457 | . 417 | . 344 |
| . 7 | . 639 | . 556 | . 486 | . 446 | . 381 |
| . 8 | . 661 | . 582 | . 514 | . 470 | . 403 |
| . 9 | . 684 | . 608 | . 542 | . 496 | . 421 |
| 1.0 | . 705 | . 630 | . 562 | . 521 | . 444 |
| 1.1 | . 725 | . 653 | . 587 | . 548 | . 466 |
| 1.2 | . 743 | . 672 | . 606 | . 567 | . 487 |
| 1.3 | . 760 | . 690 | . 626 | . 585 | . 502 |
| 1.4 | . 776 | . 710 | . 646 | . 604 | . 516 |
| 1.5 | . 794 | . 729 | . 664 | . 624 | . 530 |
| 1.6 | . 809 | . 743 | . 677 | . 642 | . 543 |
| 1.7 | . 823 | . 759 | . 690 | . 658 | . 564 |
| 1.8 | . 835 | . 773 | . 706 | . 673 | . 579 |
| 1.9 | . 846 | . 786 | . 719 | . 689 | . 592 |
| 2.0 | . 857 | . 799 | . 732 | . 702 | . 610 |
| 2.1 | . 868 | . 810 | . 744 | . 715 | . 627 |
| 2.2 | . 877 | . 822 | . 757 | . 725 | . 642 |
| 2.3 | . 886 | . 832 | . 768 | . 735 | . 658 |
| 2.4 | . 895 | . 841 | . 179 | . 744 | . 670 |
| 2.5 | . 902 | . 851 | . 788 | . 755 | . 680 |
| 2.6 | . 910 | . 859 | . 799 | . 764 | . 689 |
| 2.7 | . 916 | . 866 | . 810 | . 774 | . 697 |
| 2.8 | . 922 | . 873 | . 819 | . 783 | . 707 |
| 2.9 | . 928 | . 881 | . 827 | . 791 | . 719 |
| 3.0 | . 933 | . 889 | . 836 | . 798 | . 731 |
| 3.1 | . 938 | . 895 | . 544 | . 806 | . 743 |
| 3.2 | . 943 | . 901 | . 851 | . 815 | . 754 |
| 3.3 | . 948 | . 906 | . 857 | . 822 | . 760 |
| 3.4 | . 952 | . 911 | . 864 | . 828 | . 765 |
| 3.5 | . 955 | . 916 | . 870 | . 836 | . 710 |
| 3.6 | . 958 | . 920 | . 876 | . 843 | . 775 |
| 3.7 | . 962 | . 925 | . 882 | . 851 | . 780 |
| 3.8 | . 965 | . 929 | . 888 | . 857 | . 784 |
| $\underline{L}\left(2_{a} / z_{b}\right)$ is the exact lower confidence bound for $\mathbb{R}\left(t_{0}\right)$ |  |  |  |  |  |

table vili. table of exact lower confidence bounds for r(to) (Concinued)

|  | $\begin{aligned} & n=15 \\ & p=r / n-7 / 15 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $r=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $\gamma=0.99$ |
| $2_{a} / 2$ | $L *\left(2_{a} / z_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ |
| 3.9 | . 967 | . 933 | . 894 | . 862 | . 789 |
| 4.0 | . 969 | . 937 | . 898 | . 867 | . 795 |
| 4.1 | . 972 | . 940 | . 902 | . 872 | . 801 |
| 4.2 | . 974 | . 944 | . 907 | . 876 | . 808 |
| 4.3 | . 976 | . 947 | . 911 | . 882 | . 814 |
| 4.4 | . 978 | . 949 | . 915 | . 886 | . 821 |
| 4.5 | . 979 | . 952 | . 919 | . 891 | . 827 |
| 4.6 | . 981 | . 955 | . 922 | . 896 | . 834 |
| 4.7 | . 982 | . 958 | . 926 | . 900 | . 840 |
| 4.8 | . 984 | . 960 | . 929 | . 905 | . 845 |
| 4.9 | . 985 | . 963 | . 932 | . 908 | . 850 |
| 5.0 | . 986 | . 965 | . 935 | . 912 | . 852 |

[^10]TABLE VIII. TABLE OF EXACT LOWER CONTIDENCE BOUNDS FOR R( $\mathrm{t}_{0}$ ) (Continued)

|  | $Y=0.50$ | $\begin{array}{r} p= \\ p=0.75 \end{array}$ | $\begin{aligned} & =11 / 15 \\ & Y=0.90 \end{aligned}$ | $Y=0.95$ | $\gamma=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2 / 2_{b}$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L^{\star}\left(Z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(2 / 2{ }_{b}\right)$ |
| . 0 | . 401 | . 328 | . 269 | . 233 | . 179 |
| . 1 | . 433 | . 362 | . 304 | . 262 | . 207 |
| . 2 | . 467 | . 395 | . 335 | . 297 | . 233 |
| . 3 | . 499 | . 427 | . 369 | . 325 | . 264 |
| . 4 | . 530 | . 458 | . 400 | . 355 | . 295 |
| . 5 | . 559 | . 488 | . 430 | . 383 | . 318 |
| . 6 | . 587 | . 516 | . 457 | . 412 | . 340 |
| . 7 | . 614 | . 544 | . 485 | . 440 | . 363 |
| . 8 | . 641 | . 573 | . 512 | . 468 | . 388 |
| . 9 | . 664 | . 598 | . 535 | . 492 | . 413 |
| 1.0 | . 688 | . 624 | . 557 | . 518 | . 434 |
| 1.1 | . 712 | . 646 | . 578 | . 539 | . 450 |
| 1.2 | . 732 | . 667 | . 599 | . 359 | . 469 |
| 1.3 | . 752 | . 685 | . 622 | . 580 | . 490 |
| 1.4 | . 772 | . 705 | . 642 | . 601 | . 510 |
| 1.5 | . 790 | . 724 | . 659 | . 620 | . 527 |
| 1.6 | . 807 | . 742 | . 676 | . 639 | . 539 |
| 1.7 | . 823 | . 759 | . 691 | . 657 | . 560 |
| 1.8 | . 837 | . 775 | . 707 | . 672 | . 580 |
| 1.9 | . 851 | . 790 | . 721 | . 687 | . 599 |
| 2.0 | . 862 | . 805 | . 736 | . 700 | . 618 |
| 2.1 | . 873 | . 818 | . 752 | . 716 | . 634 |
| 2.2 | . 883 | . 830 | . 765 | . 729 | . 645 |
| 2.3 | . 892 | . 842 | . 779 | . 742 | . 663 |
| 2.4 | . 901 | . 852 | . 792 | . 755 | . 675 |
| 2.5 | . 909 | . 861 | . 803 | . 768 | . 687 |
| 2.6 | . 917 | . 870 | . 813 | . 779 | . 702 |
| 2.7 | . 924 | . 879 | . 823 | . 790 | . 712 |
| 2.8 | . 930 | . 887 | . 832 | . 801 | . 722 |
| 2.9 | . 936 | . 896 | . 842 | . 812 | . 734 |
| 3.0 | . 941 | . 903 | . 852 | . 822 | . 746 |
| 3.1 | . 946 | . 910 | . 861 | . 829 | . 758 |
| 3.2 | . 951 | . 916 | . 869 | . 837 | . 769 |
| 3.3 | . 955 | . 922 | . 877 | . 845 | . 779 |
| 3.4 | . 958 | . 927 | . 883 | . 853 | . 789 |
| 3.5 | . 962 | . 932 | . 890 | . 861 | . 798 |
| 3.6 | . 965 | . 936 | . 896 | . 869 | . 806 |
| 3.7 | . 968 | . 941 | . 901 | . 875 | . 813 |
| 3.8 | . 971 | . 945 | . 907 | . 882 | . 819 |

table vili. table of exact lower confidence bounds for r(to) (Continued)

|  | $\begin{aligned} & n=15 \\ & p=r / n=11 / 15 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $r=0.75$ | $\gamma \cdot 0.90$ | $\gamma=0.95$ | $r=0.99$ |
| $2 a^{12}$ | L* ( $z_{a} / z_{b}$ ) | L* $\mathrm{z}_{\mathrm{a}} / z_{b}$ ) | L* ( $z_{a} / z_{b}$ ) | L* $z_{\text {a }} / z_{6}$ ) | L* $\left(z_{a} / z_{b}\right)$ |
| 3.9 | . 973 | . 949 | . 912 | . 888 | . 825 |
| 4.0 | . 976 | . 953 | . 917 | . 893 | . 831 |
| 4.1 | . 978 | . 956 | . 922 | . 898 | . 837 |
| 4.2 | . 980 | . 959 | . 927 | . 903 | . 846 |
| 4.3 | . 981 | . 962 | . 932 | . 907 | . 854 |
| 4.4 | . 983 | . 965 | . 936 | . 911 | . 862 |
| 4.5 | . 984 | . 967 | . 939 | . 916 | . 870 |
| 4.6 | . 985 | . 969 | . 942 | . 920 | . 877 |
| 4.7 | . 987 | . 971 | . 946 | . 923 | . 882 |
| 4.8 | . 988 | . 973 | . 949 | . 928 | . 887 |
| 4.9 | . 989 | . 975 | . 952 | . 932 | . 892 |
| 5.0 | . 990 | . 977 | . 955 | . 936 | . 896 |

[^11]table vili. table of exact lowre confidence bounds for $\mathfrak{R}\left(t_{0}\right)$ (Continued)

|  | $Y=0.50$ | $\gamma=0.75^{\frac{\mathbf{p}}{}}$ | $\begin{aligned} \mathrm{n} & =1 \\ \gamma & =0.90 \end{aligned}$ | $Y=0.95$ | $\boldsymbol{Y}=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2_{a} / z_{b}$ | $L *\left(Z_{e} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | L* $Z_{a} / Z_{b}$ ) | $L *\left(z_{a} / z_{b}\right)$ |
| . 0 | . 388 | . 321 | . 271 | . 234 | . 180 |
| . 1 | . 422 | . 355 | . 302 | . 261 | . 207 |
| . 2 | . 455 | . 388 | . 334 | . 286 | . 235 |
| . 3 | . 488 | . 419 | . 365 | . 317 | . 267 |
| . 4 | . 521 | . 452 | . 393 | . 345 | . 291 |
| . 5 | . 552 | . 482 | . 422 | . 376 | . 314 |
| . 6 | . 580 | . 513 | . 450 | . 403 | . 336 |
| . 7 | . 610 | . 540 | . 478 | . 430 | . 362 |
| . 8 | . 638 | . 568 | . 505 | . 461 | . 390 |
| . 9 | . 664 | . 595 | . 531 | . 489 | .411 |
| 1.0 | . 688 | . 620 | . 556 | . 515 | . 428 |
| 1.1 | . 712 | . 645 | . 581 | . 539 | . 454 |
| 1.2 | . 734 | . 668 | . 602 | . 565 | . 471 |
| 1.3 | . 754 | . 690 | . 623 | . 588 | . 495 |
| 1.4 | . 773 | . 711 | . 643 | . 612 | . 518 |
| 1.5 | . 791 | . 731 | . 663 | . 631 | . 539 |
| 1.6 | . 808 | . 748 | . 683 | . 653 | . 555 |
| 1.7 | . 823 | . 765 | . 701 | . 670 | . 571 |
| 1.8 | . 836 | . 781 | . 720 | . 687 | . 587 |
| 1.7 | . 851 | . 797 | . 738 | . 704 | . 605 |
| 2.0 | . 864 | . 813 | . 755 | . 719 | . 623 |
| 2.1 | . 875 | . 826 | . 770 | . 732 | . 640 |
| 2.2 | . 885 | . 839 | . 784 | . 747 | . 658 |
| 2.3 | . 895 | . 850 | . 798 | . 762 | . 673 |
| 2.4 | . 904 | . 861 | . 809 | . 776 | . 688 |
| 2.5 | . 912 | . 871 | . 823 | . 787 | . 703 |
| 2.6 | . 920 | . 880 | . 835 | . 799 | . 718 |
| 2.7 | . 927 | . 889 | . 845 | . 810 | . 732 |
| 2.8 | . 933 | . 897 | . 855 | . 821 | . 745 |
| 2.9 | . 939 | . 906 | . 865 | . 831 | . 758 |
| 3.0 | . 945 | . 912 | . 873 | . 840 | . 770 |
| 3.1 | . 950 | . 919 | . 882 | . 849 | . 782 |
| 3.2 | . 954 | . 925 | . 890 | . 857 | . 793 |
| 3.3 | . 958 | . 931 | . 897 | . 865 | . 802 |
| 3.4 | . 962 | . 936 | . 904 | . 874 | . 812 |
| 3.5 | . 966 | . 940 | . 910 | . 881 | . 822 |
| 3.6 | . 968 | . 945 | . 916 | . 888 | . 830 |
| 3.7 | . 971 | . 949 | . 922 | . 895 | . 836 |
| 3.8 | . 974 | . 953 | . 927 | . 900 | . 844 |
| $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R(to) (Contiaued)

|  | $\begin{aligned} & n=15 \\ & p=r / n=1 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $Y=0.90$ | $\gamma=0.95$ | $\gamma=0.99$ |
| $Z_{a} / Z_{b}$ | $L *\left(2_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{i} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ |
| 3.9 | . 976 | . 957 | . 932 | . 906 | . 854 |
| 4.0 | . 978 | . 960 | . 936 | . 911 | . 863 |
| 4.1 | . 980 | . 963 | . 941 | . 916 | . 871 |
| 4.2 | . 982 | . 966 | . 945 | . 921 | . 878 |
| 4.3 | . 984 | . 969 | . 949 | . 926 | . 884 |
| 4.4 | . 985 | . 971 | . 952 | . 931 | . 887 |
| 4.5 | . 986 | . 973 | . 956 | . 935 | . 890 |
| 4.6 | . 988 | . 975 | . 959 | . 939 | . 893 |
| 4.7 | . 989 | . 977 | . 962 | . 942 | . 896 |
| 4.8 | . 990 | . 979 | . 964 | . 945 | . 899 |
| 4.9 | . 991 | . 980 | . 966 | . 949 | . 902 |
| 5.0 | . 992 | . 982 | . 968 | . 952 | . 907 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

TABLE VIII. TAELE OF EXACT LOWER CONFIDENCE bOUNDS FOK R( $t_{0}$ ) (Continued)

|  | $Y=0.50$ | $\begin{array}{r} \mathrm{n} \\ \mathrm{p} \\ \mathrm{r}=0.75 \end{array}$ | $\begin{aligned} & n=5 / 20 \\ & Y=0.90 \end{aligned}$ | $Y=0.95$ | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2 a_{b}$ | $L *\left(Z_{a} / i_{b}\right)$ | $L *\left(Z_{8} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(2_{a} / L_{b}\right)$ |
| . 0 | . 523 | . 350 | . 188 | . 105 | . 008 |
| . 1 | . 544 | . 387 | . 228 | . 140 | . 020 |
| . 2 | . 569 | . 423 | . 271 | . 186 | .037 |
| . 3 | . 596 | . 459 | . 305 | . 231 | . 068 |
| . 4 | . 620 | . 491 | .343 | . 274 | . 118 |
| . 5 | . 642 | . 524 | . 385 | . 317 | . 164 |
| - 0 | . 663 | . 552 | . 427 | . 360 | . 229 |
| . 7 | . 683 | . 582 | . 465 | . 401 | . 287 |
| . 8 | . 704 | . 610 | . 501 | . 439 | . 331 |
| . 9 | . 720 | . 640 | . 539 | .477 | . 380 |
| 1.0 | . 735 | . 605 | . 570 | . 519 | . 430 |
| 1.1 | . 754 | . 687 | . 601 | . 552 | . 464 |
| 1.2 | . 768 | . 706 | .631 | . 583 | . 484 |
| 1.3 | . 783 | . 724 | . 656 | . 614 | . 519 |
| 1.4 | . 796 | . 742 | . 679 | . 637 | . 538 |
| 1.5 | . 809 | . 758 | . 698 | . 659 | . 559 |
| 1.0 | . 824 | . 775 | . 718 | . 674 | . 587 |
| 1.7 | . 835 | . 790 | . 735 | . 697 | . 609 |
| 1.8 | . 840 | . 802 | . 746 | . 713 | . 628 |
| 1.9 | . 358 | . 813 | . 761 | . 728 | . 644 |
| 2.0 | . 868 | . 823 | . 773 | . 741 | . 665 |
| 2.1 | . 878 | . 834 | . 785 | . 753 | . 677 |
| 2.2 | . 887 | . 842 | . 797 | . 762 | . 687 |
| 2.3 | . 895 | . 850 | . 807 | . 773 | . 695 |
| 2.4 | . 903 | . 857 | . 817 | . 781 | . 702 |
| 2.5 | . 909 | . 865 | . 824 | .791 | . 713 |
| 2.6 | . 916 | . 873 | . 831 | . 798 | . 720 |
| 2.7 | . 922 | . 879 | . 837 | . 803 | . 726 |
| 2.8 | . 928 | . 885 | . 843 | . 809 | . 733 |
| 2.4 | . 933 | . 891 | . 851 | . 815 | . 745 |
| 3.0 | . 938 | . 896 | . 857 | . 823 | . 752 |
| 3.1 | . 942 | . 902 | . 865 | . 826 | . 755 |
| 3.2 | . 946 | . 907 | . 870 | . 833 | . 763 |
| 3.3 | . 950 | . 912 | . 875 | . 830 | . 768 |
| 3.4 | . 953 | . 917 | . 880 | . 844 | . 773 |
| 3.5 | . 956 | . 921 | . 885 | . 850 | . 779 |
| 3.0 | . 959 | . 925 | . 889 | . 854 | . 784 |
| 3.7 | . 962 | . 929 | . 893 | . 858 | . 788 |
| 3.8 | . 965 | . 933 | . 898 | . 862 | . 793 |

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R(to) (Continued)

|  | $\begin{aligned} & n=20 \\ & p=r / n=5 / 20 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=0.50$ | $Y=0.75$ | $Y=0.90$ | $Y=0.95$ | $\gamma=0.99$ |
| $2 / z_{b}$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| 3.9 | . 967 | . 936 | . 902 | . 866 | . 797 |
| 4.0 | . 969 | . 939 | . 905 | . 870 | . 802 |
| 4.1 | . 971 | . 942 | . 909 | . 873 | . 806 |
| 4.2 | . 973 | . 945 | . 912 | . 878 | . 810 |
| 4.3 | . 975 | . 948 | . 915 | . 880 | . 815 |
| 4.4 | . 977 | . 951 | . 919 | . 883 | . 819 |
| 4.5 | . 978 | . 954 | . 922 | . 886 | . 824 |
| 4.6 | . 980 | . 957 | . 924 | . 889 | . 828 |
| 4.7 | . 981 | . 959 | . 927 | . 893 | . 832 |
| 4.8 | . 983 | . 962 | . 931 | . 898 | . 836 |
| 4.9 | . 984 | . 963 | . 933 | . 902 | . 839 |
| 5.0 | . 985 | . 965 | . 936 | . 904 | . 843 |

[^12]TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (COntinued)

|  | $\gamma=0.50$ | $r=0.75$ | $\begin{aligned} & =10 / 20 \\ & \gamma=0.90 \end{aligned}$ | $\gamma=0.95$ | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $z_{a} / z_{b}$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| . 0 | . 415 | . 333 | . 257 | . 224 | . 141 |
| . 1 | . 446 | . 369 | . 294 | .263 | . 180 |
| . 2 | . 479 | . 405 | . 332 | . 297 | . 222 |
| . 3 | . 509 | . 441 | . 365 | . 332 | . 264 |
| . 4 | . 542 | . 472 | . 402 | . 367 | . 290 |
| . 5 | . 573 | . 502 | . 436 | . 401 | . 336 |
| . 6 | . 604 | . 530 | . 468 | . 432 | . 370 |
| . 7 | . 632 | . 559 | . 497 | . 461 | . 404 |
| . 8 | . 657 | . 588 | . 530 | . 492 | . 432 |
| . 9 | . 681 | . 614 | . 559 | . 520 | . 453 |
| 1.0 | . 704 | . 637 | . 584 | . 546 | . 474 |
| 1.1 | . 725 | . 659 | . 608 | . 571 | . 495 |
| 1.2 | . 746 | . 680 | . 633 | . 591 | . 516 |
| 1.3 | . 764 | . 701 | . 652 | . 614 | . 534 |
| 1.4 | . 780 | . 721. | . 672 | . 637 | . 546 |
| 1.5 | . 796 | . 740 | . 691 | . 657 | . 564 |
| 1.6 | . 811 | . 758 | . 707 | . 673 | . 588 |
| 1.7 | . 824 | . 773 | . 721 | . 692 | . 607 |
| 1.8 | . 837 | . 790 | . 737 | . 710 | . 628 |
| 1.9 | . 850 | . 804 | . 751 | . 723 | . 646 |
| 2.0 | . 862 | . 817 | . 764 | . 737 | . 660 |
| 2.1 | . 872 | . 829 | . 777 | . 750 | . 671 |
| 2.2 | . 882 | . 840 | . 789 | . 761 | . 681 |
| 2.3 | . 892 | . 850 | . 801 | . 772 | . 692 |
| 2.4 | . 900 | . 860 | . 812 | . 782 | . 702 |
| 2.5 | . 909 | . 869 | . 823 | . 793 | . 712 |
| 2.6 | . 916 | . 878 | . 832 | . 804 | . 722 |
| 2.7 | . 922 | . 887 | . 842 | . 815 | . 731 |
| 2.8 | . 929 | . 894 | . 851 | . 821 | . 740 |
| 2.9 | . 935 | . 901 | . 859 | . 830 | . 749 |
| 3.0 | . 940 | . 907 | . 858 | . 839 | . 758 |
| 3.1 | . 945 | . 913 | . 875 | . 848 | . 766 |
| 3.2 | . 950 | . 918 | . 883 | . 855 | . 774 |
| 3.3 | . 954 | . 924 | . 890 | . 862 | . 782 |
| 3.4 | . 958 | . 929 | . 896 | . 869 | . 792 |
| 3.5 | . 962 | . 933 | . 902 | . 875 | . 801 |
| 3.6 | . 965 | . 938 | . 908 | . 881 | . 809 |
| 3.7 | . 968 | . 942 | . 913 | . 887 | . 817 |
| 3.8 | . 970 | . 946 | . 918 | . 892 | . 825 |

L* $\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
table vili. table of exact loner confidence bounds for r(to) (Continued)

|  | $\begin{aligned} & n=20 \\ & p=r / n=10 / 20 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $Y=0.90$ | $\gamma=0.95$ | $r=0.99$ |
| $2{ }^{2} / 2{ }_{b}$ | $L *\left(z_{a} / z_{b}\right)$ | $\underline{L}\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L^{*}\left(2_{a} / z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ |
| 3.9 | . 973 | . 949 | . 923 | . 898 | . 833 |
| 4.0 | . 975 | . 953 | . 928 | . 903 | . 840 |
| 4.1 | . 977 | . 956 | . 932 | . 908 | . 848 |
| 4.2 | . 979 | . 959 | . 936 | . 912 | . 854 |
| 4.3 | . 981 | . 961 | . 939 | . 916 | . 860 |
| 4.4 | . 982 | . 964 | . 943 | . 920 | . 864 |
| 4.5 | . 984 | . 967 | . 946 | . 925 | . 868 |
| 4.6 | . 985 | . 969 | . 949 | . 929 | . 871 |
| 4.7 | . 987 | . 971 | . 952 | . 933 | . 875 |
| 4.8 | . 988 | . 973 | . 955 | . 936 | . 879 |
| 4.9 | . 989 | . 975 | . 957 | . 940 | . 882 |
| 5.0 | . 990 | . 976 | . 960 | . 943 | . 886 |

[^13]table viil. table of bxact lower confidence bounds for r $\left(t_{0}\right)$ (Continued)


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TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\begin{aligned} & n=20 \\ & p=r / n=15 / 20 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $Y=0.99$ |
| $z_{a} / z_{b}$ | $L *\left(Z_{a} / 2_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| 3.9 | . 975 | . 956 | . 932 | . 914 | . 865 |
| 4.0 | . 977 | . 960 | . 936 | . 919 | . 872 |
| 4.1 | . 979 | . 963 | . 939 | . 923 | . 878 |
| 4.2 | . 981 | . 966 | . 943 | . 928 | . 884 |
| 4.3 | . 983 | . 958 | . 947 | . 931 | . 890 |
| 4.4 | . 985 | . 971 | . 950 | . 935 | . 895 |
| 4.5 | . 986 | . 973 | . 954 | . 939 | . 900 |
| 4.6 | . 987 | . 975 | . 957 | . 942 | 904 |
| 4.7 | . 988 | . 977 | . 959 | . 945 | . 967 |
| 4.8 | . 989 | . 979 | . 962 | . 948 | . 910 |
| 4.9 | . 990 | . 980 | . 964 | . 951 | . 915 |
| 5.0 | . 991 | . 982 | . 967 | . 954 | . 919 |

[^14]TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $\mathrm{t}_{0}$ ) (Continued)

|  | $\begin{aligned} & n=20 \\ & p=r / n=l \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $Y=0.95$ | $\gamma=0.99$ |
| $2_{a} / 2{ }^{2}$ | $L *\left(z_{a} / z_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ | L* $\left(2_{a} / 2_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / z_{b}\right)$ |
| . 0 | . 380 | . 328 | . 278 | . 249 | . 202 |
| . 1 | . 416 | . 363 | . 309 | . 283 | . 228 |
| . 2 | . 451 | . 395 | . 338 | . 313 | . 256 |
| . 3 | . 487 | . 429 | . 370 | . 348 | . 281 |
| . 4 | . 519 | . 462 | . 403 | . 379 | . 312 |
| . 5 | . 553 | . 491 | . 433 | . 406 | . 338 |
| . 6 | . 586 | . 521 | . 464 | . 434 | . 364 |
| . 7 | . 013 | . 548 | . 494 | . 461 | . 388 |
| . 8 | . 642 | . 575 | . 525 | . 490 | . 413 |
| .9 | . 568 | . 603 | . 552 | . 514 | . 436 |
| 1.0 | . 693 | . 629 | . 579 | . 541 | . 462 |
| 1.1 | . 717 | . 654 | . 605 | . 568 | . 486 |
| 1.2 | . 740 | . 679 | . 628 | . 593 | . 510 |
| 1. 3 | . 761 | . 700 | . 649 | . 615 | . 533 |
| 1.4 | . 781 | . 722 | . 670 | . 636 | . 557 |
| 1.5 | . 799 | . 742 | . 692 | . 657 | . 579 |
| 1.6 | . 816 | . 761 | . 713 | . 680 | . 600 |
| 1.7 | . 831 | . 779 | . 732 | . 700 | . 621 |
| 1.8 | . 846 | . 795 | . 748 | . 721 | . 640 |
| 1.9 | . 859 | . 810 | . 763 | . 737 | . 661 |
| 2.0 | . 871 | . 824 | . 778 | . 755 | . 681 |
| 2.1 | . 883 | . 837 | . 792 | . 768 | . 700 |
| 2.2 | . 893 | . 849 | . 806 | . 785 | . 719 |
| 2.3 | . 902 | . 861 | . 819 | . 798 | . 732 |
| 2.4 | . 911 | . 872 | . 830 | . 811 | . 745 |
| 2.5 | . 919 | . 881 | . 842 | . 823 | . 758 |
| 2.6 | . 926 | . 890 | . 852 | . 834 | . 772 |
| 2.7 | . 933 | . 898 | . 863 | . 844 | . 785 |
| 2.8 | . 939 | . 906 | . 873 | . 854 | . 796 |
| 2.9 | . 944 | . 914 | . 881 | . 862 | . 807 |
| 3.0 | . 949 | . 920 | . 889 | . 872 | . 817 |
| 3.1 | . 954 | . 926 | . 896 | . 881 | . 827 |
| 3.2 | . 958 | . 932 | . 904 | . 888 | . 836 |
| 3.3 | . 962 | . 938 | . 911 | . 895 | . 844 |
| 3.4 | . 965 | . 943 | . 917 | . 901 | . 853 |
| 3.5 | . 969 | . 947 | . 923 | . 907 | . 861 |
| 3.6 | . 971 | . 951 | . 928 | . 914 | . 869 |
| 3.7 | . 974 | . 955 | . 933 | . 920 | . 877 |
| 3.8 | . 976 | . 959 | . 938 | . 925 | .895 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R(*)$

TABLE IIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R(to) (Continued)

|  | $\begin{aligned} & n=20 \\ & p=z / n=1 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=0.50$ | $\gamma=0.75$ | $Y=0.90$ | $Y=0.95$ | $Y=0.99$ |
| $2 a^{12} b$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / z_{b}\right)$ | L* ( $z_{a} / z_{b}$ ) | L* ( $z_{a} / z_{b}$ ) |
| 3.9 | . 979 | . 962 | . 942 | . 930 | . 893 |
| 4.0 | . 981 | . 965 | . 947 | . 935 | . 898 |
| 4.1 | . 982 | . 968 | . 951 | . 939 | . 903 |
| 4.2 | . 984 | . 971 | . 954 | . 943 | . 908 |
| 4.3 | . 985 | . 973 | . 957 | . 947 | . 913 |
| 4.4 | . 987 | . 975 | . 960 | . 950 | . 918 |
| 4.5 | . 988 | . 977 | . 963 | . 953 | . 923 |
| 4.6 | . 989 | . 979 | . 966 | . 956 | . 927 |
| 4.7 | . 990 | . 981 | . 968 | . 959 | . 931 |
| 4.8 | . 991 | . 982 | . 970 | . 962 | . 935 |
| 4.9 | . 992 | . 984 | . 972 | . 964 | . 938 |
| 5.0 | . 993 | . 985 | . 974 | . 967 | . 942 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR $R\left(t_{0}\right)$ (Continued)

|  | $\begin{aligned} & n=30 \\ & p=r / n=7 / 30 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $\gamma=0.75$ | $\gamma=0.90$ | $Y=0.95$ | $\gamma=0.99$ |
| $2_{a} / 2{ }_{b}$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(2 / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| . 0 | . 485 | . 335 | . 198 | . 104 | . 015 |
| . 1 | . 512 | . 372 | . 238 | . 136 | . 033 |
| . 2 | . 542 | . 408 | . 282 | . 179 | . 060 |
| . 3 | . 567 | . 442 | . 323 | . 220 | . 098 |
| . 4 | . 592 | . 477 | . 367 | . 268 | . 143 |
| . 5 | . 615 | . 513 | . 408 | . 322 | . 188 |
| . 6 | .637 | . 543 | . 449 | . 361 | . 237 |
| . 7 | . 061 | . 576 | . 486 | . 409 | . 289 |
| . 8 | . 603 | . 604 | . 520 | . 456 | . 346 |
| . 9 | . 705 | . 630 | . 555 | . 501 | . 397 |
| 1.1 | . 723 | . 656 | . 590 | . 544 | . 442 |
| 1.- | . 742 | . 681 | . 619 | . 577 | . 482 |
| 1.2 | . 759 | . 704 | . 643 | . 604 | . 515 |
| 1.3 | . 776 | . 726 | . 669 | . 631 | . 552 |
| 1.4 | . 793 | . 745 | . 688 | . 655 | . 589 |
| 1.5 | . 809 | . 761 | . 711 | .676 | . 608 |
| 1.0 | . 823 | . 777 | . 729 | . 694 | . 633 |
| 1.7 | . 836 | . 794 | . 746 | . 716 | . 659 |
| 1.8 | . 847 | . 807 | . 761. | . 735 | . 669 |
| 1.9 | . 858 | . 819 | . 774 | . 752 | .691 |
| 2.0 | . 869 | .831 | . 787 | . 765 | . 712 |
| 2.1 | . 878 | . 841 | .799 | . 777 | . 732 |
| 2.2 | .887 | . 851 | . 810 | . 787 | . 741 |
| 2.3 | . 895 | . 860 | . 820 | . 798 | . 750 |
| 2.4 | . 903 | . 869 | . 829 | . 809 | . 762 |
| 2.5 | . 911 | . 877 | . 839 | . 820 | . 773 |
| 2.6 | . 918 | . 885 | . 846 | . 829 | . 783 |
| 2.7 | . 925 | . 892 | . 855 | . 835 | . 791 |
| 2.6 | . 930 | . 898 | . 862 | . 843 | . 800 |
| 2.9 | . 935 | . 903 | . 868 | . 851 | . 809 |
| 3.0 | . 940 | . 909 | . 674 | . 857 | . 816 |
| 3.1 | . 945 | . 914 | . 881 | . 864 | . 824 |
| 3.2 | . 448 | . 920 | . 886 | . 869 | . 831 |
| 3.3 | . 952 | . 925 | . 892 | . 874 | . 838 |
| 3.4 | . 450 | . 929 | . 896 | . 878 | . 842 |
| 3.5 | . 959 | .933 | . 901 | . 882 | . 846 |
| 3.0 | . 962 | . 938 | . 907 | . 886 | . 849 |
| 3.7 | . 965 | . 941 | . 911 | . 891 | . 855 |
| 2.0 | . 968 | . 945 | . 916 | . 895 | . 860 |
| $L *\left(L_{a} / \iota_{b}\right)$ is the exact lower coniidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |



TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $r=0.50$ | $\begin{gathered} n \\ p=0.75 \end{gathered}$ | $\begin{aligned} & =15 / 30 \\ & Y=0.90 \end{aligned}$ | $Y=0.95$ | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2 / 2{ }_{3}$ | $L *\left(z_{a} / z_{b}\right)$ | L* $2_{a} / 2_{b}$ ) | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(2 / 2{ }^{\text {a }}\right.$ ) | $L *\left(z_{a} / z_{b}\right)$ |
| . 0 | . 399 | . 329 | . 269 | . 230 | . 169 |
| . 1 | . 432 | . 369 | . 303 | . 270 | . 212 |
| . 2 | . 464 | . 402 | . 341 | . 309 | . 252 |
| . 3 | . 496 | . 437 | . 375 | . 348 | . 292 |
| . 4 | . 527 | . 470 | . 412 | . 384 | . 330 |
| . 5 | . 560 | . 501 | . 445 | . 421 | . 368 |
| . 6 | . 591 | . 534 | . 481 | . 456 | . 403 |
| . 7 | . 619 | . 563 | . 513 | . 487 | . 434 |
| . 8 | . 646 | . 593 | . 541 | . 514 | . 463 |
| . 9 | . 673 | . 618 | . 571 | . 542 | . 490 |
| 1.0 | . 696 | . 643 | . 597 | . 567 | . 518 |
| 1.1 | . 719 | . 667 | . 620 | . 595 | . 540 |
| 1.2 | . 740 | . 689 | . 645 | . 619 | . 562 |
| 1.3 | . 760 | . 712 | . 664 | . 640 | . 582 |
| 1.4 | . 778 | . 730 | . 685 | . 660 | . 603 |
| 1.5 | . 796 | . 748 | . 706 | . 677 | . 624 |
| 1.6 | . 812 | . 766 | . 723 | . 693 | . 643 |
| 1.7 | . 826 | . 782 | . 740 | . 711 | . 660 |
| 1.8 | . 840 | . 798 | . 756 | . 729 | . 676 |
| 1.9 | . 853 | . 811 | . 771 | . 746 | . 688 |
| 2.0 | . 865 | . 825 | . 784 | . 759 | . 701 |
| 2.1 | . 877 | . 837 | . 797 | . 773 | . 721 |
| 2.2 | . 886 | . 849 | . 810 | . 787 | . 735 |
| 2.3 | . 895 | . 859 | . 820 | . 800 | . 749 |
| 2.4 | . 903 | . 871 | . 832 | . 811 | . 760 |
| 2.5 | . 910 | . 879 | . 841 | . 824 | . 769 |
| 2.6 | . 918 | . 887 | . 852 | . 834 | . 779 |
| 2.7 | . 925 | . 895 | . 861 | . 845 | . 789 |
| 2.8 | . 931 | . 903 | . 870 | . 853 | . 798 |
| 2.9 | . 936 | . 910 | . 878 | . 861 | . 805 |
| 3.0 | . 941 | . 517 | . 885 | . 869 | . 813 |
| 3.1 | . 946 | . 922 | . 891 | . 875 | . 821 |
| 3.2 | . 950 | . 928 | . 898 | . 883 | . 827 |
| 3.3 | . 955 | . 933 | . 904 | . 889 | . 834 |
| 3.4 | . 959 | . 938 | . 910 | . 895 | . 840 |
| 3.5 | . 962 | . 942 | . 916 | . 900 | . 847 |
| 3.6 | . 965 | . 947 | . 922 | . 906 | . 854 |
| 3.7 | . 968 | . 951 | . 927 | . 911 | . 861 |
| 3.8 | . 971 | . 954 | . 932 | . 917 | . 868 |

TABLE VIII. TABLE OP EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\gamma=0.50$ | $\begin{aligned} & a=30 \\ & p=r / a=15 / 30 \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $Y=0.75$ | $Y=0.90$ | $\gamma=0.95$ | $\gamma=0.99$ |
| $2_{a} / z_{b}$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| 3.9 | . 974 | . 958 | . 936 | . 921 | . 873 |
| 4.0 | . 976 | . 961 | . 940 | . 926 | . 877 |
| 4.1 | . 978 | . 964 | . 943 | . 930 | . 883 |
| 4.2 | . 980 | . 966 | . 947 | . 934 | . 889 |
| 4.3 | . 981 | . 969 | . 950 | . 938 | . 895 |
| 4.4 | . 983 | . 971 | . 953 | . 942 | . 899 |
| 4.5 | . 984 | . 973 | . 956 | . 945 | . 904 |
| 4.6 | . 986 | . 975 | . 959 | . 949 | . 908 |
| 4.7 | . 987 | . 977 | . 961 | . 952 | . 912 |
| 4.8 | . 988 | . 979 | . 964 | . 954 | . 916 |
| 4.9 | . 989 | . 980 | . 966 | . 957 | . 920 |
| 5.0 | . 990 | . 982 | . 968 | . 959 | . 924 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

TABLE VIII. TABLE OF EXACT LUWER CUNFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\gamma=0.50$ | $\begin{aligned} n & = \\ p & =0.75 \end{aligned}$ | $\begin{aligned} & =22 / 30 \\ & \gamma=0.90 \end{aligned}$ | $Y=0.95$ | $Y=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Z_{a} / Z_{b}$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / L_{b}\right)$ | $L *\left(2_{a} / L_{b}\right)$ | $L *\left(2_{a} / L_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ |
| . 0 | . 382 | . 333 | . 281 | . 257 | . 206 |
| . 1 | . 416 | . 366 | . 318 | . 290 | . 243 |
| . 2 | . 453 | . 400 | . 353 | . 325 | . 277 |
| . 3 | . 489 | . 433 | . 385 | . 359 | . 307 |
| . 4 | . 521 | . 465 | . 419 | . 392 | . 341 |
| . 5 | . 553 | . 497 | . 449 | . 424 | .373 |
| . 0 | . 583 | . 528 | . 479 | . 455 | . 406 |
| . 7 | . 612 | . 560 | . 512 | . 486 | . 438 |
| . 8 | . 040 | . 589 | . 540 | . 517 | . 465 |
| . 4 | . 666 | . 615 | . 566 | . 544 | . 486 |
| 1.4 | . 091 | . 641 | . 593 | . 509 | . 511 |
| 1.1 | . 710 | . 665 | . 618 | . 592 | . 534 |
| 1.2 | . 737 | . 687 | . 642 | . 615 | . 556 |
| 1.3 | . 758 | . 709 | . 664 | . 637 | . 577 |
| 1.4 | . 777 | . 729 | . 685 | . 059 | . 602 |
| 1.5 | . 795 | . 748 | . 705 | . 677 | .623 |
| 1.0 | . 812 | . 760 | . 722 | . 097 | . 041 |
| 1.7 | . 827 | . 782 | . 741 | . 717 | . 656 |
| 1.6 | . 842 | . 798 | . 758 | . 734 | . 675 |
| 1.9 | . 855 | . 813 | . 774 | . 749 | . 692 |
| 2.0 | . 867 | . 826 | . 788 | . 765 | . 706 |
| 2.1 | . 878 | . 840 | . 803 | . 778 | . 718 |
| 2.2 | . 88 | . 852 | . 816 | . 793 | . 732 |
| 2.3 | . 897 | . 864 | . 828 | . 807 | . 746 |
| 2.4 | . 906 | . 875 | . 839 | . 818 | . 759 |
| 2.5 | . 914 | . 884 | .851 | . 829 | . 771 |
| 2.0 | . 921 | . 894 | . 862 | . 839 | . 783 |
| 2.7 | . 928 | . 901 | . 871 | . 850 | . 795 |
| 2.8 | . 435 | .909 | . 880 | . 859 | . 806 |
| 2.9 | . 940 | . 916 | . 888 | . 867 | . 810 |
| 3.0 | . 945 | . 923 | . 846 | . 876 | . 826 |
| 3.1 | . 950 | . 929 | . 902 | . 884 | . 836 |
| 3.2 | . 955 | . 934 | . 909 | . 891 | . 845 |
| 3.3 | . 958 | . 940 | . 915 | . 698 | . 853 |
| 3.4 | . 902 | . 444 | . 921 | . 905 | . 861 |
| 3.5 | . 966 | . 949 | . 927 | . 911 | . 868 |
| 3.0 | . 909 | . 953 | . 932 | . 917 | . 874 |
| 3.7 | . 971 | . 956 | . 937 | . 922 | . 881 |
| 3.0 | . 974 | . 960 | . 941 | .927 | . 888 |
| $L *\left(L_{a} / L_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |

TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $Y=0.50$ | $\begin{gathered} \mathrm{p} \\ \mathrm{p}=0.75 \end{gathered}$ | $\begin{aligned} & =22 / 30 \\ & Y=0.90 \end{aligned}$ | $Y=0.95$ | $\gamma=0.99$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $z_{a} / 2{ }_{b}$ | L* $\left.\mathrm{Z}_{a} / z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L{ }^{*}\left(\mathbf{Z}_{a} / Z_{b}\right)$ | $L *\left(2_{a} / 2_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ |
| 3.9 | . 976 | . 963 | . 945 | . 931 | . 892 |
| 4.0 | . 979 | . 966 | . 948 | . 935 | . 897 |
| 4.1 | . 981 | . 969 | . 952 | . 939 | . 902 |
| 4.2 | . 982 | . 971 | . 956 | . 943 | . 907 |
| 4.3 | . 984 | . 973 | . 959 | . 947 | . 912 |
| 4.4 | . 985 | . 975 | . 961 | . 950 | . 916 |
| 4.5 | . 987 | . 977 | . 964 | . 953 | . 921 |
| 4.6 | . 988 | . 979 | . 967 | . 956 | . 926 |
| 4.7 | . 989 | . 981 | . 969 | . 959 | . 930 |
| 4.8 | . 990 | . 982 | . 971 | . 962 | . 934 |
| 4.9 | . 991 | . 984 | . 973 | . 964 | . 938 |
| 5.0 | . 992 | . 985 | . 975 | . 966 | . 942 |

[^15]TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE bOUNDS FOR R( $t_{0}$ ) (Continued)


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TABLE VIII. TABLE OF EXACT LOWER CONFIDENCE BOUNDS POR R( $\mathrm{t}_{0}$ ) (Continued)

|  | $\begin{aligned} & n=30 \\ & p=r / n=1 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ | $r=0.75$ | $\gamma=0.90$ | $\gamma=0.95$ | $\gamma=0.99$ |
| $2_{a} / z_{b}$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(Z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / Z_{b}\right)$ | $L *\left(z_{a} / Z_{b}\right)$ | $L *\left(2_{a} / z_{b}\right)$ |
| 3.9 | . 978 | . 966 | . 952 | . 942 | . 914 |
| 4.0 | . 980 | . 969 | . 955 | . 946 | . 920 |
| 4.1 | . 982 | . 971 | . 958 | . 950 | . 925 |
| 4.2 | . 984 | . 974 | . 961 | . 954 | . 929 |
| 4.3 | . 985 | . 976 | . 964 | . 957 | . 934 |
| 4.4 | . 987 | . 978 | . 967 | . 960 | . 938 |
| 4.5 | . 988 | . 980 | . 969 | . 963 | . 942 |
| 4.6 | . 989 | . 981 | . 972 | . 966 | . 945 |
| 4.7 | . 990 | . 983 | . 974 | . 968 | . 949 |
| 4.8 | . 991 | . 984 | . 976 | . 971 | . 952 |
| 4.9 | . 992 | . 986 | . 978 | . 973 | . 955 |
| 5.0 | . 992 | . 987 | . 979 | . 975 | . 958 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

## table vili. table of exact and asymptotic lower confidence

 BOUNDS FOK $R\left(t_{0}\right)$ (Continued)|  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

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TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOR $R\left(t_{0}\right)$ (Continued)

| $\begin{aligned} & \mathrm{n}=50 \\ & \mathrm{p}=\mathrm{r} / \mathrm{n}=12 / 50 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma=0.50$ |  |  | $\gamma=0.75$ |  | $\gamma=0.90$ |  | $\gamma=0.95$ |  | $\gamma=0.99$ |  |
| $\frac{z_{a}}{z_{b}}$ | $L \times\left(\frac{2_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ | $L *\left(\frac{2_{a}}{\sum_{b}}\right)$ | $A *\left(\frac{z_{a}}{z_{b}}\right)$ | $L *\left(\frac{L_{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right.$ | $L \star\left(\frac{2_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right.$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ |
| 3.4 | . 961 | . 967 | . 942 | . 950 | . 923 | . 927 | . 909 | . 909 | . 873 | . 863 |
| 3.5 | . 964 | . 970 | . 946 | . 954 | . 928 | . 932 | . 914 | . 914 | . 878 | . 868 |
| 3.6 | . 967 | . 973 | . 950 | . 957 | . 932 | . 936 | . 919 | . 919 | . 883 | . 873 |
| 3.7 | . 970 | . 976 | . 953 | . 961 | . 936 | . 940 | . 922 | . 923 | . 888 | . 878 |
| 3.8 | . 972 | . 978 | . 957 | . 964 | . 940 | . 944 | . 925 | . 927 | . 892 | . 882 |
| 3.9 | . 975 | . 980 | . 960 | . 967 | . 943 | . 947 | . 930 | . 931 | . 898 | . 887 |
| 4.0 | . 977 | . 982 | . 962 | . 969 | . 946 | . 951 | . 934 | . 935 | . 903 | . 891 |
| 4.1 | . 979 | . 984 | . 965 | . 972 | . 949 | . 954 | . 937 | . 938 | . 908 | . 895 |
| 4.2 | . 981 | . 985 | . 967 | . 974 | . 952 | . 957 | . 940 | . 942 | . 912 | . 899 |
| 4.3 | . 982 | . 987 | . 969 | . 976 | . 955 | . 959 | . 944 | . 945 | . 917 | . 902 |
| 4.4 | . 984 | . 988 | . 971 | . 978 | . 957 | . 962 | . 947 | . 948 | . 921 | . 906 |
| 4.5 | . 985 | . 989 | . 973 | . 980 | . 960 | . 964 | . 949 | . 951 | . 924 | . 909 |
| 4.6 | . 986 | . 990 | . 975 | . 981 | . 962 | . 967 | . 951 | . 953 | . 926 | . 913 |
| 4.7 | . 987 | . 991 | . 977 | . 983 | . 965 | . 969 | . 954 | . 956 | . 928 | . 916 |
| 4.8 | . 988 | . 992 | . 979 | . 984 | . 967 | . 971 | . 957 | . 958 | . 931 | . 919 |
| 4.9 | . 989 | . 993 | . 980 | . 985 | . 969 | . 973 | . 959 | . 960 | . 934 | . 922 |
| 5.0 | . 990 | . 993 | . 982 | . 988 | . 971 | . 974 | . 961 | . 962 | . 937 | . 924 |

[^16]
## TABLE VTII. TABLE OR EXACT AND ASYMPTOTIC LOWPR CONFIDBNCE BOUNLS FOR R(C) (Cantinued)

| $\begin{aligned} & n=50 \\ & p=r / n=25 / 50 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{z^{a}}{z_{b}}$ | $L *\left(\frac{L^{a}}{z_{b}}\right)$ | $L_{A} \star \frac{/ z_{a}}{z_{b}}$ | $2 *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ | $L *\left(\frac{Z_{a}}{Z_{b}}\right)$ | $I_{A} *\left(\frac{z_{a}}{z_{b}}\right.$ | $L *\left(\frac{Z_{a}}{Z_{b}}\right)$ | $A\left(\frac{Z_{a}}{Z_{b}}\right)$ |
| . 0 | . 383 | . 368 | . 333 | . 313 | . 286 | . 264 | . $258{ }^{1}$ | . 236 | . 209 | . 186 |
| . 1 | . 419 | . 405 | . 368 | . 351 | . 325 | . 303 | . 297 | . 275 | . 249 | . 224 |
| . 2 | . 456 | . 441 | . 406 | . 389 | . 363 | . 342 | . 336 | . 315 | . 288 | . 263 |
| . 3 | . 488 | . 477 | . 444 | . 427 | . 399 | . 382 | . 377 | . 354 | . 328 | . 303 |
| .4 | . 522 | . 512 | . 477 | . 464 | . 436 | . 420 | . 414 | . 393 | . 368 | . 342 |
| . 5 | . 555 | . 545 | . 509 | . 499 | . 471 | . 457 | . 451 | . 430 | . 408 | . 381 |
| . 6 | . 586 | . 578 | . 543 | . 534 | . 507 | . 492 | . 483 | .466 | . 441 | . 417 |
| . 7 | . 615 | . 600 | . 573 | . 566 | . 540 | . 526 | . 515 | . 500 | . 475 | . 452 |
| . 8 | . 643 | . 638 | . 603 | . 597 | . 569 | . 557 | . 546 | . 533 | . 502 | . 484 |
| . 9 | . 670 | . 666 | . 630 | . 626 | . 596 | . 587 | . 575 | . 563 | . 534 | . 515 |
| 1.0 | . 695 | . 692 | . 658 | . 653 | . 622 | . 615 | . 601 | . 591 | . 560 | . 544 |
| 1.1 | . 718 | . 717 | . 682 | . 679 | . 647 | . 641 | . 62.6 | . 618 | . 584 | . 570 |
| 1.2 | . 741 | . 740 | . 705 | . 703 | . 672 | . 666 | . 651 | . 542 | . 605 | . 595 |
| 1.3 | . 761 | .761 | . 727 | . 725 | . 693 | . 689 | . 673 | . 666 | . 627 | . 618 |
| 1.4 | . 780 | . 781 | . 746 | . 746 | . 714 | . 710 | . 692 | . 687 | . 649 | . 640 |
| 1.5 | . 798 | . 800 | . 766 | . 766 | . 732 | . 730 | . 711 | . 107 | . 669 | . 660 |
| 1.6 | . 816 | . 817 | . 783 | . 784 | . 750 | . 749 | . 728 | . 726 | . 688 | . 679 |
| 1.7 | . 830 | . 833 | . 799 | . 800 | . 767 | . 767 | . 745 | . 744 | . 706 | . 697 |
| 1.8 | . 845 | . 848 | . 815 | . 816 | . 784 | . 783 | . 761 | . 761 | . 724 | . 714 |
| 1.9 | . 857 | . 801 | . 829 | . 831 | . 800 | . 798 | . 776 | . 776 | . 741 | . 729 |
| 2.0 | . 869 | . 873 | . 842 | . 844 | . 813 | . 812 | . 789 | .792 | . 757 | . 744 |
| 2.1 | . 880 | . 885 | . 854 | . 856 | . 827 | . 825 | . 804 | . 804 | . 770 | . 758 |
| 2.2 | . 890 | . 895 | . 865 | . 868 | . 839 | . 838 | . 816 | . 817 | . 781 | . 771 |
| 2.3 | . 900 | . 905 | . 875 | . 878 | . 850 | . 849 | . 828 | . 829 | . 793 | . 784 |
| 2.4 | . 908 | . 913 | . 885 | . 888 | . 861 | . 860 | . 839 | . 840 | . 803 | . 796 |
| 2.5 | . 916 | . 921 | . 893 | . 897 | . 871 | . 870 | . 849 | . 850 | . 814 | . 807 |
| 2.6 | . 923 | . 928 | . 901 | . 906 | . 879 | . 879 | . 359 | . 860 | . 823 | . 817 |
| 2.7 | . 930 | . 935 | . 909 | . 913 | . 888 | . 888 | . 867 | . 869 | . 834 | . 827 |
| 2.8 | . 936 | . 941 | . 916 | . 920 | . 896 | . 896 | . 875 | . 878 | . 843 | . 836 |
| 2.9 | . 942 | . 946 | . 922 | . 927 | . 903 | . 903 | . 884 | . 886 | . 851 | . 845 |
| 3.0 | . 947 | . 951 | . 928 | . 933 | . 910 | . 910 | . 891 | . 893 | . 859 | . 854 |
| 3.1 | . 951 | . 956 | . 934 | . 938 | . 916 | . 917 | . 898 | . 900 | . 867 | . 862 |
| 3.2 | . 956 | . 960 | . 939 | . 943 | . 922 | . 923 | . 905 | . 907 | . 874 | . 869 |
| 3.3 | . 960 | . 964 | . 944 | . 948 | . 927 | . 928 | . 911 | . 913 | . 880 | . 876 |

$L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
$L_{A}{ }^{*}\left(Z_{a} Z_{b}\right)$ is the asymptotic lower confidence bound for $R\left(t_{0}\right)$
[ABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\begin{aligned} & n=50 \\ & p=r / n=25 / 50 \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ |  | $\gamma=0.75$ |  | $Y=0.90$ |  | $r=0.95$ |  | $Y=0.99$ |  |
| $\frac{a}{Z_{b}}$ | $L *\left(\frac{L_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{L_{a}}{Z_{b}}\right)$ | $L *\left(\frac{z^{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{L_{a}}{Z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{L_{a}}{Z_{b}}\right.$ | $L *\left(\frac{L^{a}}{2_{b}}\right)$ | $L_{A} * *\left(\frac{a}{L_{b}}\right.$ | $L *\left(\frac{z_{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ |
| 3.4 | . 963 | . 967 | . 948 | . 952 | . 932 | . 933 | . 917 | . 919 | . 887 | . 883 |
| 3.2 | . 966 | . 970 | . 952 | . 956 | . 937 | . 938 | . 923 | . 924 | . 894 | . 889 |
| 3.6 | . 969 | . 973 | . 456 | . 960 | . 941 | . 943 | . 928 | . 929 | . 901 | . 895 |
| 3.7 | . 472 | . 976 | . 959 | . 963 | . 946 | . 947 | . 933 | . 934 | . 907 | . 901 |
| 3.8 | . 975 | . 478 | . 963 | . 966 | . 949 | . 951 | . 937 | . 938 | . 913 | . 907 |
| 3.9 | . 977 | . 980 | . 966 | . 969 | . 953 | . 954 | . 941 | . 942 | .918 | . 912 |
| 4.0 | . 979 | . 982 | . 469 | . 972 | . 956 | . 958 | . 945 | . 946 | . 922 | . 917 |
| 4.1 | . 981 | . 984 | . 971 | . 974 | . 959 | . 961 | . 949 | . 950 | . 927 | . 921 |
| 4.2 | . 983 | . 985 | . 973 | . 976 | . 962 | . 964 | . 953 | . 953 | . 931 | . 925 |
| 4.3 | . 984 | . 987 | . 975 | . 978 | . 965 | . 966 | . 956 | . 956 | . 935 | . 930 |
| 4.4 | . 986 | . 988 | . 977 | . 980 | . 967 | . 969 | . 959 | . 959 | . 940 | . 933 |
| 4.5 | . $¢ 87$ | . 989 | . 979 | . 482 | . 969 | . 971 | . 962 | . 962 | . 943 | . 937 |
| 4.6 | . 988 | . 990 | .981 | . 483 | . 972 | . 973 | . 964 | . 965 | . 947 | . 941 |
| 4.7 | 9\%. | . 991 | . 982 | . 985 | . 973 | . 975 | . 967 | . 967 | . 950 | . 944 |
| 4.8 | . 990 | . 992 | . 984 | . 986 | . 975 | . 977 | . 969 | . 969 | . 953 | . 947 |
| 4.9 | . 991 | . 993 | . 985 | . 987 | . 977 | . 979 | . 971 | . 971 | . 955 | . 950 |
| 5.0 | . 992 | . 993 | . 986 | . 988 | . 979 | . 980 | . 973 | . 973 | . 958 | . 953 |

[^17]table vili. table of exact and asympotic lower confidence BOUNDS FOR R( $t_{0}$ ) (Continued)

|  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOK $R\left(t_{0}\right)$ (Continued)

|  | $\begin{aligned} & \mathrm{n}=50 \\ & \mathrm{P}=\mathrm{r} / \mathrm{n}=37 / 50 \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ |  | $\gamma=0.75$ |  | $\gamma=0.90$ |  | $\gamma=0.95$ |  | $\gamma=0.99$ |  |
|  | $\mathrm{L} *\left(\frac{L^{2}}{L_{b}}\right)$ | $L_{A} *\left(\frac{Z^{\prime}}{Z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{L^{\prime}}{L_{\text {b }}}\right)$ | $L *\left(\frac{z_{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{z^{a}}{z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} * *\left(\frac{Z^{\prime}}{Z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | - $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ |
| 3.4 | . 965 | . 967 | . 952 | . 954 | . 937 | . 938 | . 927 | . 926 | . 900 | . 897 |
| 3.5 | . 908 | . 970 | . 956 | . 958 | . 942 | . 943 | . 932 | . 931 | . 905 | . 903 |
| 3.6 | . 971 | . 973 | . 960 | . 962 | . 946 | .947 | . 937 | . 936 | . 911 | . 909 |
| 3.7 | . 974 | .976 | . 963 | . 965 | . 950 | . 951 | . 942 | . 941 | . 916 | . 915 |
| 3.8 | . 976 | . 978 | . 966 | . 968 | . 954 | . 955 | . 946 | . 945 | . 921 | . 920 |
| 3.9 | . 978 | . 480 | . 969 | . 971 | . 957 | . 958 | . 950 | . 949 | . 926 | . 925 |
| 4.0 | . 980 | . 982 | . 972 | . 973 | . 961 | . 962 | . 954 | . 953 | . 930 | . 930 |
| 4.1 | . 982 | . 984 | . 974 | . 975 | . 963 | . 965 | . 957 | . 956 | . 934 | . 934 |
| 4.2 | . 984 | . 985 | . 976 | . 977 | . 966 | . 967 | . 960 | . 959 | . 938 | . 939 |
| 4.3 | . 485 | .987 | . 978 | . 979 | . 969 | . 970 | . 963 | . 962 | . 942 | . 942 |
| 4.4 | . 987 | . 988 | . 980 | . 981 | . 971 | . 972 | . 966 | . 965 | . 945 | . 946 |
| 4.5 | . 988 | . 989 | . 982 | . 983 | . 973 | . 974 | . 968 | . 968 | . 949 | . 950 |
| 4.6 | . 989 | . 990 | . 983 | . 984 | . 975 | . 976 | . 971 | . 970 | . 952 | . 953 |
| 4.7 | . | .991 | . 985 | . 986 | . 977 | .978 | . 973 | . 972 | . 955 | . 956 |
| 4.8 | . 991 | . 992 | . 986 | . 987 | . 979 | . 980 | . 975 | . 974 | . 957 | . 959 |
| 4.9 | . 992 | .993 | . 987 | .988 | . 981 | . 981 | . 977 | . 976 | . 960 | . 961 |
| 5.0 | . 992 | . 993 | . 988 | . 989 | . 982 | . 983 | . 979 | . 978 | . 953 | . 964 |

$L^{*}\left(L_{a} / L_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
$L_{i}{ }^{*}\left(\alpha_{a} / \%_{b}\right)$ is the asymptentic lower confidence bound for $R\left(t_{0}\right)$

TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\gamma=0.50$ |  | $\begin{aligned} \mathrm{a} & =50 \\ \mathrm{p} & =5 / \mathrm{n}=1 \\ 0.75 \quad Y & =0.90 \end{aligned}$ |  |  |  | $\gamma=0.95$ |  | $Y=0.99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{z^{a}}{z_{b}}$ | $L *\left(\frac{2_{a}}{2_{b}}\right)$ | $L_{A} \pm\left(\frac{z^{\frac{a}{e}}}{z_{b}}\right)$ | $L *\left(\frac{L_{z}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z^{\prime}}{\frac{a}{z}}\right.$ | $L *\left(\frac{z^{2}}{z_{b}}\right)$ | $L_{A} * *\left(\frac{Z^{\prime}}{Z_{b}}\right)$ |  | $L_{A}{ }^{*}\left(\frac{z^{\prime}}{z_{b}}\right)$ | $L *\left(\frac{z_{a}}{z_{a}} z_{b}\right.$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ |
| . 0 | . 372 | . 368 | . 336 | . 331 | . 304 | . 298 | . 284 | . 279 | . 263 | . 243 |
| . 1 | . 408 | . 405 | . 372 | . 367 | . 339 | . 333 | . 320 | . 312 | . 294 | . 275 |
| . 2 | . 444 | . 441 | . 408 | . 402 | . 374 | . 367 | . 356 | . 346 | . 325 | . $30 \%$ |
| . 3 | . 480 | . 477 | . 443 | . 438 | . 407 | . 402 | . 390 | . 380 | . 356 | . 339 |
| . 4 | . 514 | . 312 | . 477 | . 472 | . 442 | . 435 | . 422 | . 413 | . 388 | . 371 |
| . 5 | . 547 | . 545 | . 510 | . 506 | . 474 | . 468 | . 456 | . 446 | .417 | . 403 |
| . 6 | . 579 | . 578 | . 542 | . 538 | . 506 | . 501 | . 487 | . 478 | . 448 | .433 |
| . 7 | . 610 | . 609 | . 572 | . 569 | . 537 | . 532 | . 517 | . 508 | . 476 | . 464 |
| . 8 | . 639 | . 638 | . 603 | . 599 | . 586 | . 561 | . 548 | . 538 | . 507 | . 493 |
| . 9 | . 667 | . 666 | . 630 | . 627 | . 595 | . 590 | . 577 | . 567 | . 535 | . 521 |
| 1.0 | . 693 | . 692 | . 657 | . 654 | . 621 | . 617 | . 603 | . 594 | . 562 | . 548 |
| 1.1 | . 718 | . 717 | . 683 | . 680 | . 647 | . 643 | . 628 | . 620 | . 590 | . 574 |
| 1.2 | . 741 | . 740 | . 707 | . 704 | .661 | . 668 | . 652 | . 645 | . 6.17 | . 599 |
| 1.3 | . 762 | . 761 | . 728 | . 726 | . 695 | . 691 | . 675 | . 669 | . 641 | . 623 |
| 1.4 | . 782 | . 781 | . 749 | . 747 | . 718 | . 713 | . 698 | . 691 | . 663 | . 646 |
| 1.5 | . 800 | . 800 | . 769 | . 767 | . 738 | . 734 | . 719 | . 712 | . 082 | . 668 |
| 1.6 | . 818 | . 817 | . 787 | . 786 | . 757 | . 753 | . 738 | . 732 | . 701 | . 689 |
| 1.7 | . 834 | . 833 | . 804 | . 803 | . 775 | . 772 | . 757 | . 751 | . 720 | . 708 |
| 1.8 | . 848 | . 848 | . 819 | . 819 | . 791 | . 789 | . 774 | . 769 | . 739 | . 727 |
| 1.9 | . 861 | . 861 | . 834 | . 833 | . 806 | . 804 | . 791 | . 785 | . 756 | . 744 |
| 2.0 | . 874 | . 873 | . 848 | . 847 | . 821 | . 819 | . 907 | . 801 | . 773 | . 761 |
| 2.1 | . 885 | . 885 | . 860 | . 860 | . 834 | . 833 | . 822 | . 815 | . 789 | . 776 |
| 2.2 | . 895 | . 895 | . 872 | . 871 | . 847 | . 846 | . 835 | . 828 | . 803 | . 791 |
| 2.3 | . 905 | . 905 | . 882 | . 882 | . 859 | . 858 | . 847 | . 841 | . 817 | . 805 |
| 2.4 | . 914 | . 913 | . 892 | . 892 | . 870 | . 869 | . 858 | . 853 | . 829 | . 818 |
| 2.5 | . $\mathrm{C}_{2}$ | . 921 | . 901 | . 901 | . 880 | . 879 | . 869 | . 864 | . 840 | . 830 |
| 2.6 | . 029 | . 928 | .910 | . 910 | . 890 | . 889 | . 878 | . 874 | . 851 | . 842 |
| 2.7 | . 935 | . 935 | . 917 | . 917 | . 899 | . 897 | . 887 | . 883 | . 860 | . 852 |
| 2.8 | . 941 | . 941 | . 924 | . 924 | . 907 | . 905 | . 396 | . 892 | . 870 | . 862 |
| 2.9 | . 946 | . 926 | . 931 | . 931 | . 914 | . 913 | . 904 | . 900 | 878 | . 872 |
| 3.0 | . 951 | .931 | . 937 | . 937 | . 921 | . 920 | . 911 | . 908 | . 886 | . 881 |
| 3.1 | . 956 | . 956 | . 942 | . 942 | . 927 | . 926 | . 918 | . 915 | . 834 | . 859 |
| 3.2 | . 960 | . 960 | . 947 | . 94.7 | . 933 | . 932 | . 924 | . 921 | . 901 | . 897 |
| 3.3 | . 904 | . 964 | . 952 | . 952 | . 939 | . 938 | . 930 | . 927 | . 908 | . 904 |
| LA $\left(Z_{d} / Z_{b}\right)$ is the exact luwer confidence bound for $R\left(r_{g}\right)$ |  |  |  |  |  |  |  |  |  |  |
| $L_{A}^{*}\left(Z_{a} / Z_{b}\right)$ is the asymptotic lower confidence bound for $R\left(t_{0}\right)$ |  |  |  |  |  |  |  |  |  |  |



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table vill. table of exact and asymptotic lówer confidence BUUHDS FOK $R\left(t_{0}\right)$ (Continued)

|  | $Y=0.50$ |  | $\begin{aligned} n= & 50 \\ p= & r / n=1 \\ Y & =0.90 \end{aligned}$ |  |  |  | $r=0.95$ |  | $Y=0.99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $L^{*} *\left(\frac{L_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{Z_{a}}{Z_{b}}\right)$ | $L *\left(\frac{2_{a}}{2_{b}}\right)$ | $L_{A} \star\left(\frac{Z_{a}}{Z_{b}}\right)$ | $L *\left(\frac{Z^{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{2_{a}}{2_{b}}\right.$ | * $\left(\frac{2}{2}{ }_{2}{ }_{b}\right)$ | $L_{A} * * / \frac{2}{z_{b}}$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{L_{a}}{L_{b}}\right)$ |
| 3.4 | . 967 | . 967 | . 956 | . 956 | . 944 | . 943 | . 935 | . 933 | . 914 | . 910 |
| 3.5 | . 470 | . 970 | . 960 | . 960 | . 948 | . 947 | . 940 | . 938 | . 920 | . 917 |
| 3.6 | . 973 | . 973 | . 963 | . 963 | . 952 | . 552 | . 945 | . 943 | . 925 | . 922 |
| 3.7 | . 470 | . 976 | . 966 | . 966 | . 956 | . 956 | . 949 | . 947 | . 930 | . 928 |
| 3.8 | . 978 | . 978 | . 969 | . 909 | . 960 | . 959 | . 953 | . 951 | . 935 | . 933 |
| 3.9 | . 480 | . 980 | . 972 | . 972 | . 963 | . 962 | . 956 | . 955 | . 940 | . 938 |
| 4.0 | . 982 | . 982 | . 974 | . 975 | . 966 | . 965 | . 960 | . 959 | . 944 | . 942 |
| 4.1 | . 983 | . 984 | . 977 | . 977 | . 969 | . 968 | . 963 | . 962 | . 948 | . 946 |
| 4.2 | . 985 | . 985 | . 979 | . 979 | . 971 | . 971 | . 966 | . 965 | . 951 | . 950 |
| 4.3 | . 986 | .987 | . 981 | . 981 | . 974 | . 973 | . 909 | . 908 | . 955 | . 954 |
| 4.4 | . 988 | . 988 | . 982 | . 982 | . 976 | . 975 | . 971 | . 970 | . 958 | . 957 |
| 4.3 | . 964 | . 989 | . 984 | . 484 | . 978 | . 977 | . 973 | . 972 | . 960 | . 960 |
| 4.6 | . 990 | . 990 | . 985 | . 985 | . 980 | . 979 | . 976 | . 975 | . 963 | . 963 |
| 4.7 | . 941 | .991 | . 980 | . 907 | . 481 | . 981 | .976 | . 977 | . 965 | . 905 |
| 4.8 | . 992 | . 992 | . 988 | . 988 | . 983 | . 983 | . 979 | . 978 | . 968 | . 968 |
| 4.9 | . 443 | . 993 | . 989 | . 989 | . 984 | . 984 | . 981 | . 980 | . 970 | .970 |
| 5.0 | .993 | . 993 | . 990 | . 990 | . 986 | . 985 | . 982 | . 982 | . 972 | . 972 |

[^18]table vili. table of exact and asymptotic lower confidence BOUNDS FOR R( $t_{0}$ ) (Continued)

|  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONEIDENCE BOUNDS FOR R( $5_{0}$ ) (Continued)

|  | $\begin{aligned} & n=100 \\ & p=r / n=25 / 100 \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ |  | $\gamma=0.75$ |  | $\gamma=0.90$ |  | $Y=0.95$ |  | $\gamma=0.99$ |  |
| $\frac{z_{a}}{z_{b}}$ | $L *\left(\frac{z^{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z^{2}}{z_{b}}{ }^{\text {b }}\right.$ |  | $\mathrm{L}_{\mathrm{A}} *\left(\frac{2}{2}{ }^{2}\right.$ | $L \star\left(\frac{z^{2}}{z_{b}}\right)$ | $L_{A}{ }^{*}\left(\frac{a}{z_{b}}\right.$ |  | $L_{\text {A }}$ * | $L \star\left(\frac{z_{a}}{z_{b}}\right.$ | $L_{A}{ }^{*}\left(\frac{z^{a}}{z_{b}}\right)$ |
| 3.4 | . 965 | . 967 | . 953 | . 956 | . 940 | . 942 | . 931 | . 933 | . 918 | . 909 |
| 3.5 | . 968 | . 970 | . 956 | . 959 | . 944 | . 947 | . 935 | . 937 | . 923 | . 914 |
| 3.6 | . 971 | . 973 | . 960 | . 963 | . 948 | . 950 | . 939 | . 941 | . 928 | . 919 |
| 3.7 | . 973 | . 976 | . 963 | . 966 | . 951 | . 954 | . 944 | . 945 | . 932 | . 924 |
| 3.8 | . 976 | . 978 | . 965 | . 969 | . 955 | . 957 | . 947 | . 949 | . 936 | . 928 |
| 3.9 | . 978 | . 980 | . 908 | . 971 | . 958 | . 960 | . 951 | . 952 | . 940 | . 932 |
| 4.0 | . 980 | . 982 | . 971 | . 974 | . 961 | . 963 | . 954 | . 955 | . 943 | . 935 |
| 4.1 | . 982 | . 984 | . 973 | . 976 | . 964 | . 966 | . 957 | . 958 | . 947 | . 939 |
| 4.2 | . 983 | . 985 | . 976 | . 978 | . 966 | . 968 | . 960 | . 961 | . 950 | . 942 |
| 4.3 | . 985 | . 987 | . 977 | . 980 | . 968 | . 971 | . 963 | . 964 | . 953 | . 945 |
| 4.4 | . 986 | . 988 | . 979 | . 981 | . 971 | . 973 | . 965 | . 966 | . 956 | . 948 |
| 4.5 | . 987 | . 989 | . 981 | . 983 | . 973 | . 975 | . 967 | . 968 | . 958 | . 951 |
| 4.6 | . 989 | . 990 | . 982 | . 984 | . 975 | . 977 | . 969 | . 970 | . 961 | . 954 |
| 4.7 | . 990 | . 991 | . 984 | . 986 | . 976 | . 978 | . 971 | . 972 | . 963 | . 956 |
| 4.8 | . 991 | . 992 | . 985 | . 987 | . 978 | . 980 | . 973 | . 974 | . 985 | . 959 |
| 4.9 | . 991 | . 993 | . 986 | . 988 | . 979 | . 981 | . 975 | . 976 | . 967 | . 961 |
| 5.0 | . 992 | . 993 | . 987 | . 989 | . 981 | . 983 | . 977 | . 978 | . 969 | . 963 |

[^19]TABLE VIII, TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS POR $R\left(t_{0}\right)$ (Continued)


TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOR $R\left(t_{0}\right)$ (Continue 1 )


[^20]
## TABLE VIII. TAbLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOK R( $\left.\varepsilon_{0}\right)$ (Continued)

|  |  |  |  |  | $\begin{aligned} & 100 \\ & r / n= \end{aligned}$ | $75 / 100$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=$ | 0.50 | $\gamma=$ | 0.75 |  | 0.90 | $\gamma=$ | 0.95 |  | 2.99 |
| $\frac{i_{a}}{U_{b}}$ | $L \star\left(\frac{{ }^{2}}{2_{b}}\right)$ | $L_{A}{ }^{*}\left(\frac{2_{a}}{2_{b}}\right)$ | $L *\left(\frac{L_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{L_{a}}{\frac{2}{c}}\right)$ | $L *\left(\frac{z^{a}}{z_{b}}\right)$ | $L_{A} * *\left(\frac{L^{e}}{L_{b}}\right.$ | $L \star\left(\frac{2_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{L^{a}}{\frac{a}{L_{b}}}\right)$ | $L *\left(\frac{2^{a}}{z_{b}}\right)$ | $A_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ |
| . 0 | . 374 | . 368 | . 347 | . 339 | . 320 | . 314 | . 305 | . 298 | . 273 | .270 |
| . 1 | . 410 | . 405 | . 383 | . 376 | . 357 | . 350 | . 341 | . 335 | . 307 | . 306 |
| . 2 | . 445 | . 441 | . 419 | .412 | . 393 | . 386 | . 377 | . 371 | . 343 | . 342 |
| . 3 | . 480 | . 477 | . 455 | . 448 | . 427 | . 422 | . 414 | . 400 | . 383 | . 377 |
| - 4 | . 515 | . 512 | . 489 | . 483. | . 461 | . 457 | . 448 | . 441 | . 421 | . 411 |
| . 5 | . 548 | . 545 | . 523 | . 517 | . 494 | . 491 | . 481 | . 475 | . 452 | . 445 |
| . 6 | . 579 | . 578 | . 555 | . 550 | . 526 | . 523 | . 513 | . 508 | . 482 | . 477 |
| . 7 | . 609 | . 609 | . 585 | . 581 | . 557 | . 555 | . 543 | . 539 | . 513 | . 508 |
| . 8 | . 039 | . 638 | . 613 | . 611 | . 587 | . 585 | . 572 | . 569 | . 541 | . 538 |
| . 9 | . 006 | . 666 | . 641 | . 639 | . 615 | . 613 | . 599 | . 597 | . 568 | . 566 |
| 1.0 | . 092 | . 692 | .668 | . 665 | . 642 | . 640 | . 625 | . 624 | . 594 | . 593 |
| 1.1 | . 716 | . 717 | . 692 | . 691 | . 666 | . 665 | . 650 | . 650 | . 619 | . 619 |
| 1.2 | . 739 | . 740 | . 715 | . 714 | . 690 | . 689 | . 675 | . 674 | . 642 | . 043 |
| 1.3 | . 701 | . 761 | . 737 | . 736 | . 712 | . 712 | . 695 | . 696 | . 666 | . 066 |
| 1.4 | . 780 | . 781 | . 758 | . 757 | . 732 | . 733 | . 718 | . 718 | . 688 | . 688 |
| 1.5 | .798 | . 800 | . 777 | . 776 | . 753 | . 753 | . 738 | .738 | . 709 | . 708 |
| 1.6 | . 816 | . 817 | . 794 | . 794 | . 771 | . 772 | . 757 | . 757 | . 728 | . 727 |
| 1.7 | .831 | . 833 | . 811 | . 811 | . 788 | . 789 | . 775 | .775 | . 747 | . 746 |
| 1.8 | . 846 | . 848 | . 826 | . 826 | . 804 | .805 | . 790 | . 791 | . 764 | . 763 |
| 1.4 | . 859 | . 861 | . 840 | . 841 | . 819 | . 820 | . 807 | . 806 | . 779 | . 779 |
| 2.0 | . 872 | . 873 | . 853 | . 854 | . 833 | . 834 | . 821 | . 821 | . 793 | . 794 |
| 2.1 | . 883 | .885 | . 865 | . 866 | . 846 | . 847 | . 834 | . 834 | . 808 | . 808 |
| 2.2 | . 893 | . 895 | . 876 | . 877 | . 858 | . 859 | . 847 | . 847 | . 821 | . 821 |
| 2.3 | . 903 | . 905 | . 887 | . 888 | . 809 | . 670 | . 858 | .858 | . 835 | . 834 |
| 2.4 | . 411 | . 913 | . 896 | . 897 | . 880 | . 880 | . 868 | . 869 | . 846 | . 845 |
| 2.3 | . 919 | . 421 | . 905 | . 906 | . 889 | . 890 | . 878 | . 879 | . 856 | . 856 |
| 2.6 | . 927 | . 928 | . 913 | . 914 | . 898 | . 899 | . 887 | . 888 | . 885 | . 866 |
| 2.7 | . 933 | . 935 | . 920 | . 921 | . 906 | . 907 | . 896 | . 897 | . 875 | . 876 |
| 2.8 | . 939 | . 941 | . 427 | . 928 | . 913 | . 914 | . 904 | . 905 | . 885 | . 884 |
| 2.9 | . 945 | . 946 | . 933 | . 934 | . 920 | . 921 | .911 | . 912 | . 894 | . 893 |
| 3.0 | . 950 | . 951 | . 939 | . 940 | . 927 | . 928 | . 918 | . 919 | . 902 | . 900 |
| 3.1 | . 956 | . 956 | . 944 | . 445 | . 932 | . 934 | . 924 | . 925 | . 910 | . 907 |
| 2.2 | . 959 | . 960 | . 949 | . 950 | . 938 | . 939 | . 929 | . 931 | . 916 | . 914 |
| 3.3 | . 962 | . 964 | . 953 | . 954 | . 943 | . 944 | . 935 | . 937 | . 922 | . 920 |
| $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ <br> $L_{A} *\left(L_{a} / Z_{b}\right)$ is the asymptotic lower confidence Dound for $R\left(t_{0}\right)$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

TABLE VIII. TABLE OF EXACT AND ASYMPTOTIC LOWER CONFIDENCE BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\begin{aligned} & n=100 \\ & p=r / n=75 / 100 \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.50$ |  | $\gamma=0.75$ |  |  | 0.90 | $\gamma=0.95$ |  | $\boldsymbol{\gamma}=0.99$ |  |
| $\frac{c_{a}}{c_{b}}$ | $L *\left(\frac{Z^{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{c_{a}}{c_{b}}\right)$ | $L *\left(\frac{z_{a}}{Z_{b}}\right)$ | $I_{A} *\left(\frac{Z_{a}}{L_{b}}\right)$ | $L \star\left(\frac{2_{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{z^{\prime}}{z_{b}}\right.$ | $L *\left(\frac{z_{a}}{Z_{b}}\right)$ | $\mathrm{L}_{A} *\left(\frac{2}{2}\right.$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $A^{*}\left(\frac{z_{a}}{2_{b}}\right)$ |
| 3.4 | . 966 | . 967 | . 957 | . 958 | . 947 | . 949 | . 940 | .942 | . 928 | . 926 |
| 3.5 | . 909 | 970 | . 961 | . 962 | . 952 | . 953 | . 944 | . 946 | . 933 | . 931 |
| 3.0 | . 972 | . 973 | . 964 | . 965 | . 955 | . 957 | . 949 | . 950 | . 938 | . 936 |
| 3.7 | . 975 | . 976 | . 967 | . 968 | . 959 | . 960 | . 953 | . 954 | . 942 | . 941 |
| 3.8 | . 977 | . 975 | . 970 | . 971 | . 962 | . 963 | . 957 | . 958 | . 946 | . 945 |
| 3.9 | . 979 | . 980 | . 973 | . 974 | . 965 | . 966 | . 960 | . 961 | . 950 | . 949 |
| 4.0 | . 981 | . 982 | . 975 | . 976 | . 968 | . 969 | . 963 | . 964 | . 954 | . 953 |
| 4.1 | .983 | . 984 | . 977 | . 978 | . 971 | . 972 | . 966 | . 967 | . 957 | . 956 |
| 4.2 | . 984 | . 985 | . 979 | . 980 | . 973 | . 974 | . 969 | . 970 | . 960 | . 959 |
| 4.3 | . 986 | . 987 | . 981 | . 982 | . 975 | . 976 | . 971 | . 972 | . 963 | . 962 |
| 4.4 | . 987 | . 988 | . 983 | . 983 | . 977 | . 978 | . 978 | . 974 | . 966 | . 465 |
| 4.5 | . 988 | . 989 | . 984 | . 985 | . 979 | . 980 | . 975 | . 976 | . 968 | . 968 |
| 4.6 | . 989 | . 990 | . 986 | . 986 | . 981 | . 982 | . 977 | . 978 | . 971 | . 970 |
| 4.7 | . 990 | . 991 | . 987 | . 987 | . 983 | . 983 | . 979 | . 980 | . 973 | . 972 |
| 4.8 | . 991 | . 992 | . 988 | . 989 | . 984 | . 985 | . 981 | . 982 | . 975 | . 974 |
| 4.9 | . 992 | . 993 | . 989 | . 990 | . 985 | . 986 | . 982 | . 983 | . 976 | . 976 |
| 5.0 | . 993 | . 993 | . 990 | . 991 | . 987 | . 987 | . 984 | . 984 | . 978 | . 978 |

[^21]
## table viII. table of exact and asymptotic lower confidence BOUNDS FOR $R\left(t_{0}\right)$ (Continued)

| $n=100$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma=0.50$ |  | $\gamma=0.75$ |  | $\gamma=0.90$ |  | $\gamma=0.95$ |  | $Y=0.99$ |  |
| $\frac{2}{2} \frac{a}{2_{b}}$ | $L *\left(\frac{z^{2}}{z_{b}}\right)_{z_{b}} L_{A} *\left(\frac{z^{2}}{z_{b}}\right)$ | $L *\left(\frac{Z^{\frac{a}{a}}}{\frac{Z_{b}}{}}\right)$ | $L_{A} *\binom{Z_{2}}{\frac{Z_{b}}{}}$ | $L *\left(\begin{array}{c}2^{a} \\ \frac{a}{2} \\ b_{b}\end{array}\right)$ | $L_{A} *\left(\frac{z^{\prime}}{\frac{a}{2}}{ }_{2}\right.$ | $L *\left(\frac{z^{2}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z_{a}}{z_{b}}\right)$ | $L *\left(\frac{L^{2}}{z_{b}}\right)$ | ${ }^{*}\left(\frac{z^{a}}{z_{b}}\right)$ |
| . 0 | .371 .368 | . 347 | . 342 | . 325 | . 318 | . 304 | . 304 | . 282 | . 279 |
| . 1 | . 408 . 405 | . 382 | . 378 | . 361 | . 354 | . 340 | . 339 | . 314 | . 312 |
| . 2 | .445 .441 | .419 | . 414 | . 394 | . 389 | . 377 | . 374 | . 348 | . 346 |
| . 3 | . 480 . 477 | . 455 | . 449 | . 428 | . 424 | . 412 | . 409 | . 381 | . 380 |
| . 4 | . 514 . 512 | . 489 | . 481 | . 461 | . 458 | . 447 | .443 | .415 | . 413 |
| . 5 | . 547 . 545 | . 522 | . 517 | . 494 | . 492 | . 479 | . 476 | . 448 | . 446 |
| . 6 | . 579 . 578 | . 554 | . 550 | . 526 | . 524 | . 512 | . 508 | . 481 | . 478 |
| . 7 | . 610.609 | . 585 | . 581 | . 558 | . 555 | . 544 | . 539 | . 510 | . 508 |
| . 8 | . 639.638 | . 614 | . 611 | . 587 | . 585 | . 572 | . 569 | . 543 | . 538 |
| . 9 | . 666.666 | . 642 | . 639 | . 615 | . 613 | . 601 | . 598 | . 573 | . 567 |
| 1.0 | .693 . 692 | . 669 | . 666 | . 642 | . 641 | . 529 | . 625 | . 602 | . 594 |
| 1.1 | .717 . 717 | . 694 | . 691 | . 667 | . 666 | . 653 | . 651 | . 626 | . 620 |
| 1.2 | .741 . 740 | . 718 | . 715 | . 692 | . 690 | . 677 | . 675 | . 650 | . 645 |
| 1.3 | . 762.761 | . 740 | . 737 | . 715 | . 713 | . 700 | . 698 | . 672 | . 669 |
| 1.4 | . 782.781 | . 760 | . 758 | . 736 | . 735 | . 721 | . 720 | . 693 | . 691 |
| 1.5 | . 800 . 800 | . 779 | . 777 | . 756 | . 755 | . 742 | . 741 | . 713 | . 712 |
| 1.6 | .817 . 817 | . 797 | . 795 | . 774 | . 774 | . 761 | . 760 | . 732 | . 732 |
| 1.7 | .833 . 833 | . 814 | . 812 | . 792 | . 791 | . 779 | . 778 | . 750 | . 751 |
| 1.8 | . 848 . 848 | . 829 | . 828 | . 808 | . 808 | . 795 | . 795 | . 767 | . 769 |
| 1.9 | . 861 . 861 | . 843 | . 842 | . 823 | . 823 | . 810 | . 810 | . 784 | . 785 |
| 2.0 | . 873.873 | . 856 | . 855 | . 837 | . 837 | . 824 | . 825 | . 799 | . 801 |
| 2.1 | . 885 . 885 | . 868 | . 868 | . 850 | . 850 | . 837 | . 839 | .8i3 | . 815 |
| 2.2 | . 895.895 | . 879 | . 879 | . 862 | . 862 | . 850 | . 851 | . 827 | . 828 |
| 2.3 | . 905.905 | . 890 | .889 | . 873 | . 873 | . 862 | . 863 | . 840 | . 841 |
| 2.4 | . 913 . 913 | . 899 | . 899 | . 884 | . 884 | . 873 | . 874 | . 852 | . 853 |
| 2.5 | . 921 . 921 | . 908 | . 907 | . 893 | . 893 | . 883 | . 884 | . 862 | . 864 |
| 2.6 | . 929.928 | . 916 | . 916 | . 902 | . 902 | . 892 | . 893 | . 871 | . 874 |
| 2.7 | . 935.935 | . 923 | . 923 | . 910 | . 910 | . 901 | . 902 | . 881 | . 883 |
| 2.8 | . 941.941 | . 930 | . 930 | . 918 | . 918 | . 909 | . 909 | . 889 | . 892 |
| 2.9 | . 946 . 946 | . 936 | . 936 | . 925 | . 924 | . 916 | . 917 | . 898 | . 900 |
| 3.0 | . 951.951 | . 942 | . 941 | . 931 | . 931 | . 923 | . 924 | . 905 | . 908 |
| 3.1 | . 996.956 | . 947 | . 947 | . 937 | . 937 | . 929 | . 930 | . 912 | . 915 |
| 3.2 | . 960 . 960 | . 952 | . 951 | . 942 | . 942 | . 935 | . 935 | . 919 | . 921 |
| 3.3 | . 964 . 964 | . 956 | . 956 | . 947 | . 947 | . 941 | . 941 | . 925 | . 927 |
| $\begin{aligned} & L^{*}\left(z_{a} / z_{b}\right) \text { is the exact lower confidence bound for } R\left(t_{0}\right) \\ & L_{A}^{*}\left(z_{a} / Z_{b}\right) \text { is the asymptotic lower confidence bound for } R\left(t_{0}\right) \end{aligned}$ |  |  |  |  |  |  |  |  |  |

table vili. table or exact and asymptotic lowrr confidence BOUNDS FOR R( $t_{0}$ ) (Continued)

|  | $\begin{aligned} & n=100 \\ & p=r / n=1 \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{z^{a}}{z_{b}}$ | $L *\left(\frac{2^{2}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{Z^{a}}{Z_{b}}\right)$ | $L *\left(\frac{Z^{a}}{2_{b}}\right)$ | $L_{A} *\left(\frac{Z_{a}}{Z_{b}}\right)$ | $L *\left(\begin{array}{c}2^{a} \\ \frac{Z_{b}}{} \\ b_{b}\end{array}\right)$ | $L_{A} *\binom{Z_{a}}{\frac{Z_{b}}{}}$ | $L *\left(\frac{z_{a}}{z_{b}}\right)$ | $L_{A} *\left(\frac{z^{\prime}}{z_{b}}\right)$ | $L *\left(\frac{Z^{a}}{Z_{b}}\right)$ | $L_{A} *\left(\frac{2^{\frac{a}{e}}}{\frac{Z_{b}}{}}\right)$ |
| 3.4 | . 967 | . 367 | . 960 | . 960 | . 951 | . 951 | . 945 | . 946 | . 931 | . 933 |
| 3.5 | . 970 | . 970 | . 963 | . 963 | . 955 | . 955 | . 950 | . 950 | . 936 | . 938 |
| 3.6 | . 973 | . 973 | . 966 | . 966 | . 959 | . 959 | . 954 | . 954 | . 941 | . 943 |
| 3.7 | . 976 | . 976 | . 969 | . 969 | . 963 | . 963 | . 958 | . 958 | . 945 | . 947 |
| 3.8 | . 978 | . 978 | . 972 | . 972 | . 966 | . 966 | . 961 | . 961 | . 950 | . 951 |
| 3.9 | . 980 | . 980 | . 975 | . 975 | . 969 | . 969 | . 964 | . 965 | . 953 | . 95.5 |
| 4.0 | . 982 | . 982 | . 977 | . 977 | . 972 | . 971 | . 967 | . 967 | . 957 | . 959 |
| 4.1 | . 984 | . 984 | . 979 | . 979 | . 974 | . 974 | . 970 | . 970 | . 960 | . 962 |
| 4.2 | . 985 | . 985 | . 981 | . 981 | . 976 | . 976 | . 972 | . 973 | . 963 | . 965 |
| 4.3 | . 987 | . 987 | . 983 | . 983 | . 978 | . 978 | . 974 | . 975 | . 966 | . 968 |
| 4.4 | . 988 | . 988 | . 984 | . 984 | . 980 | . 980 | . 977 | . 977 | . 969 | . 970 |
| 4.5 | . 989 | . 989 | . 986 | . 986 | . 982 | . 982 | . 979 | . 979 | . 971 | . 972 |
| 4.6 | . 990 | . 990 | . 987 | . 987 | . 983 | . 983 | . 980 | . 981 | . 973 | . 975 |
| 4.7 | . 991 | . 991 | . 988 | . 988 | . 985 | . 985 | . 982 | . 982 | . 975 | . 977 |
| 4.8 | . 992 | . 992 | . 989 | . 989 | . 986 | . 986 | . 983 | . 984 | . 977 | . 978 |
| 4.9 | . 993 | . 993 | . 990 | . 990 | . 987 | . 987 | . 985 | . 985 | . 979 | . 980 |
| 5.0 | . 993 | . 993 | . 991 | . 991 | . 988 | . 988 | . 986 | . 986 | . 981 | . 982 |

$L *\left(Z_{a} / Z_{b}\right) 18$ the exact lower confidence bound for $R\left(t_{0}\right)$
$L_{A} *\left(Z_{a} / Z_{b}\right)$ is the asymptotic lower conficence bound for $R\left(L_{C}\right)$

TABLE IX. FACTORS FOR TESTS FOR INCREASING HAZARD RATES

| N/ Y | 0.02 | 0.05 | 0.10 | 0.25 | 0.40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 1.656 | 1.464 | 1.305 | 1.052 | 0.896 0.926 |
| 6 | 1.605 | 1.435 | 1.285 | 1.067 1.075 | 0.926 0.944 |
| 7 | 1.565 | 1.410 | 1.274 | 1.075 | 0.957 |
| 8 | 1.531 | 1.389 | 1.263 1.255 | 1.0801 | 0.966 |
| 9 | 1. 504 | 1.372 1.355 | 1.255 1.247 | 1.082 | 0.973 |
| 10 | 1.479 | 1.355 | 1.247 1.239 | 1.082 | 0.978 |
| 11 | 1.458 | 1.342 | 1.239 1.233 | 1.082 | 0.983 |
| 12 | 1.439 | 1.330 | 1.233 1.227 | 1.082 | 0.986 |
| 13 | 1.422 | 1.318 | 1.227 | 1.082 | 0.989 |
| 14 | 1.408 | 1.309 | 1.221 1.215 | 1.081 | 0.992 |
| 15 | 1.397 | 1.299 | 1.215 | 1.081 | 0.994 |
| 16 | 1.383 | 1.290 | 1.211 | 1.079 | 0.996 |
| 17 | 1.374 | 1.284 1.276 | 1.206 1.202 | 1.079 | 0.997 |
| 18 | 1.362 | 1.276 | 1.198 | 1.078 | 0.999 |
| 19 | 1.353 | 1.269 1.264 | 1.198 1.193 | 1.076 | 1.000 |
| 20 | 1.346 | 1.264 1.253 | 1.186 | 1.075 | 1.002 |
| 22 | 1.330 | 1.253 1.242 | 1.186 1.179 | 1.073 | 1.003 |
| 24 | 1.318 | 1.242 1.234 | 1.1774 | 1.072 | 1.005 |
| 26 | 1.305 | 1.234 1.227 | 1.168 | 1.071 | 1.006 |
| 28 | 1.295 | 1.227 | 1.168 | 1.070 | 1.007 |
| 30 | 1.285 | 1.220 | 1.163 | 1.067 | 1.007 |
| 32 | 1.277 | 1.214 | 1.159 1.155 | 1.066 | 1.008 |
| 34 | 1.269 | 1.208 | 1.155 | 1.065 | 1.008 |
| 36 | 1.261 | 1.202 | 1.151 1.147 | 1.064 | 1.009 |
| 38 | 1.255 | 1.198 | 1.147 1.143 | 1.064 | 1.009 |
| 40 | 1.248 | 1.192 | 1.143 1.140 | 1.063 | 1.010 |
| 42 | 1.244 | 1.188 | 1.140 1.136 | 1.062 | 1.010 |
| 44 | 1.238 | 1.183 1.181 | 1.136 1.134 | 1.060 | 1.010 |
| 46 | 1.233 | 1.181 | 1.134 1.131 | 1.059 | 1.010 |
| 48 50 | 1.228 1.224 | $\frac{1}{1.176}$ | 1.131 1.129 | 1.059 | 1.011 |
| 50 | 1.224 1.220 | 1.171 | 1.126 | 1.058 | 1.011 |
| 54 | 1.216 | 1.167 | 1.124 | 1.057 | 1.011 |
| 56 | 1.212 | 1.164 | 1.122 | 1.057 | 1.011 |
| 58 | 1.209 | 1.161 | 1.120 | 1.056 | 1.011 |
| 60 | 1.205 | 1.159 | 1.118 | 1.055 | 1.011 |
| 62 | 1.202 | 1.157 | 1.116 | 1.054 | 1.011 |
| 64 | 1.199 | 1.155 | 1.115 | 1.054 | 1.012 |
| 66 | 1.196 | 1.152 | 1.112 | 1.054 | 1.012 |
| 68 | 1.193 | 1.151 | 1.111 | 1.053 | 1.012 |
| 70 | 1.190 | 1.148 | 1.110 | 1.053 | 1.012 |
| 72 | 1.189 | 1.147 | 1.107 | 1.052 | 1.012 |
| 74 | 1.186 | 1.144 | 1.106 1.105 | 1.050 | 1.012 |
| 76 | 1.183 | 1.143 1.142 | 1.104 | 1.050 | 1.012 |
| 78 | 1.182 1.179 | 1.142 1.139 | 1.104 1.102 | 1.050 | 1.012 |
| 80 90 | 1.179 1.170 | 1.139 1.132 | 1.096 | 1.048 | 1.012 |
| 100 | 1.161 | 1.126 | 1.092 | 1.046 | 1.012 |
| 110 | 1.155 | 1.120 | 1.087 | 1.044 | 1.012 |
| 120 | 1.148 | 1.115 | 1.083 | 1.043 | 1.012 |

table ix. factors for tests for increasing hazard rates (Continued)

| $\mathrm{N} / \mathrm{Y}$ | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 0.808 | 0.726 | 0.642 | 0.598 | 0.552 |
| 6 | 0.842 | 0.767 | 0.688 | 0.648 | 0.602 |
| 7 | 0.866 | 0.796 | 0.722 | 0.684 | 0.641 |
| 8 | 0.884 | 0.818 | 0.747 | 0.712 | 0.671 |
| 9 | $0.898$ | 0.835 | 0.767 | 0.735 | 0.695 |
| 10 | 0.908 | 0.848 | 0.784 | 0.753 | 0.715 |
| 11 | 0.917 | 0.860 | 0.798 | 0.768 | 0.732 |
| 12 | 0.924 | 0.869 | 0.810 | 0.781 | 0.746 |
| 13 | 0.930 | 0.877 | 0.820 | 0.792 | 0.758 |
| 14 | 0.935 | 0.883 | 0.829 | 0.801 | 0.769 |
| 15 | 0.940 | 0.890 | 0.837 | 0.810 | 0.779 |
| 16 | 0.944 | 0.895 | 0.844 | 0.818 | 0.787 |
| 17 | 0.947 | 0.900 | 0.850 | 0.824 | 0.795 |
| 18 | 0.950 | 0.904 | 0.856 | 0.830 | 0.802 |
| 19 | 0.953 | 0.908 | 0.860 | 0.836 | 0.808 |
| 20 | 0.955 | 0.912 | 0.866 | 0.842 | 0.814 |
| 22 | 0.960 | 0.917 | 0.874 | 0.850 | 0.824 |
| 24 | 0.963 | 0.922 | 0.881 | 0.858 | 0.833 |
| 26 | 0.966 | 0.927 | 0.886 | 0.865 | 0.841 |
| 28 | 0.968 | 0.931 | 0.892 | 0.871 | 0.847 |
| 30 | 0.971 | 0.934 | 0.897 | 0.876 | 0.854 |
| 32 | 0.973 | 0.937 | 0.901 | 0.881 | 0.859 |
| 34 | 0.974 | 0.940 | 0.905 | 0.886 | 0.864 |
| 36 | 0.976 | 0.942 | 0.908 | 0.889 | 0.869 |
| 38 | 0.976 | 0.944 | 0.912 | 0.893 | 0.873 |
| 40 | 0.978 | 0.947 | 0.914 | 0.896 | 0.876 |
| 42 | 0.978 | 0.949 | 0.916 | 0.899 | 0.880 |
| 44 | 0.979 | 0.950 | 0.919 | 0.902 | 0.883 |
| 46 | 0.980 | 0.951 | 0.922 | 0.904 | 0.886 |
| 48 | 0.981 | 0.953 | 0.923 | 0.907 | 0.889 |
| 50 | 0.982 | 0.954 | 0.925 | 0.909 | 0.891 |
| 52 | 0.983 | 0.956 | 0.928 | 0.911 | 0.894 |
| 54 | 0.983 | 0.957 | 0.929 | 0.913 | 0.896 |
| 56 | 0.984 | 0.958 | 0.930 | 0.915 | 0.898 |
| 58 | 0.985 | 0.959 | 0.932 | 0.916 | 0.900 |
| $60$ | $0.985$ | 0.961 | 0.934 | 0.918 | 0.902 |
| $62$ | 0.986 | 0.962 | 0.934 | 0.920 | 0.904 |
| 64 | 0.986 | 0.962 | 0.936 | 0.921 | 0.906 |
| 66 | 0.986 | 0.962 | 0.937 | 0.922 | 0.907 |
| $68$ | 0.987 | 0.963 | 0.938 | 0.923 | 0.909 |
| 70 | 0.987 | 0.964 | 0.940 | 0.925 | 0.911 |
| $72$ | $0.988$ | 0.965 | 0.941 | 0.926 | 0.912 |
| 74 | 0.988 | 0.965 | 0.942 | 0.928 | 0.913 |
| 76 | 0.988 | 0.966 | 0.942 | 0.928 | 0.915 |
| 78 | 0.989 | 0.967 | 0.943 | 0.929 | 0.916 |
| 80 | 0.989 | 0.967 | 0.944 | 0.930 | 0.917 |
| 90 | 0.990 | 0.970 | 0.948 | 0.935 | 0.922 |
| 100 | 0.991 | 0.972 | 0.951 | 0,939 | 0.927 |
| 110 | 0.992 | 0.974 | 0.954 | 0.942 | 0.931 |
| 120 | 0.993 | 0.976 | 0.956 | 0.945 | 0.934 |

TABLE IX. FACTORS FOR TESTS FOR INCREASING RAZARD RATES (Continued)

| $N / Y$ | 0.85 | 0.90 | 0.95 | 0.98 |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 0.500 | 0.439 | 0.360 | 0.284 |
| 6 | 0.552 | 0.493 | 0.410 | 0.326 |
| 7 | 0.592 | 0.537 | 0.458 | 0.379 |
| 8 | 0.624 | 0.572 | 0.496 | 0.421 |
| 9 | 0.650 | 0.601 | 0.527 | 0.455 |
| 10 | 0.672 | 0.624 | 0.553 | 0.483 |
| 11 | 0.690 | 0.644 | 0.575 | 0.507 |
| 12 | 0.705 | 0.661 | 0.594 | 0.528 |
| 13 | 0.719 | 0.676 | 0.611 | 0.546 |
| 14 | 0.730 | 0.689 | 0.626 | 0.563 |
| 15 | 0.741 | 0.701 | 0.639 | 0.577 |
| 16 | 0.751 | 0.711 | 0.651 | 0.591 |
| 17 | 0.759 | 0.720 | 0.662 | 0.602 |
| 18 | 0.767 | 0.729 | 0.672 | 0.613 |
| 19 | 0.774 | 0.737 | 0.682 | 0.624 |
| 20 | 0.781 | 0.745 | 0.690 | 0.633 |
| 22 | 0.792 | 0.758 | 0.705 | 0.650 |
| 24 | 0.802 | 0.769 | 0.718 | 0.665 |
| 26 | 0.812 | 0.779 | 0.730 | 0.678 |
| 28 | 0.820 | 0.788 | 0.740 | 0.690 |
| 30 | 0.826 | 0.796 | 0.750 | 0.700 |
| 32 | 0.833 | 0.802 | 0.758 | 0.710 |
| 34 | 0.838 | 0.809 | 0.766 | 0.718 |
| 36 | 0.843 | 0.815 | 0.773 | 0.726 |
| 38 | 0.848 | 0.820 | 0.779 | 0.734 |
| 40 | 0.852 | 0.826 | 0.786 | 0.740 |
| 42 | 0.857 | 0.830 | 0.790 | 0.747 |
| 42 | 0.860 | 0.835 | 0.796 | 0.752 |
| 46 | 0.864 | 0.839 | 0.801 | 0.758 |
| 48 | 0.867 | 0.842 | 0.805 | 0.763 |
| 50 | 0.870 | 0.846 | 0.810 | 0.769 |
| 52 | 0.873 | 0.850 | 0.814 | 0.773 |
| 54 | 0.876 | 0.852 | 0.817 | 0.778 |
| 56 | 0.878 | 0.855 | 0.821 | 0.781 0.786 |
| 58 | 0.881 | 0.858 | 0.824 | 0.786 0.789 |
| 60 | 0.883 | 0.860 | 0.828 0.830 | 0.789 0.792 |
| 62 | 0.885 0.887 | 0.864 0.866 | 0.830 0.833 | 0.796 |
| 64 | 0.887 0.889 | 0.866 0.868 | 0.836 | 0.799 |
| 68 | 0.891 | 0.870 | 0.839 | 0.802 |
| 70 | 0.893 | 0.873 | 0.842 | 0.805 |
| 72 | 0.894 | 0.874 | 0.844 | 0.808 |
| 74 | 0.896 | 0.876 | 0.846 | $0.8 i 1$ |
| 76 | 0.898 | 0.878 | 0.848 | 0.814 |
| 78 | 0.899 | 0.880 | 0.850 | 0.816 |
| 80 | 0.901 | 0.882 | 0.852 | 0.818 |
| 90 | 0.907 | 0.890 | 0.862 | 0.829 |
| 100 | 0.912 | 0.896 | 0.870 | 0.839 |
| 110 | 0.917 | 0.901 | 0.876 | 0.847 |
| 120 | 0.921 | 0.906 | 0.883 | 0.854 |



TABLE XI. UNBIASING FACTORS FOR THE M.L.E. OF C

| n | 5 | $\bigcirc$ | 7 | $\checkmark$ | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B(n)$ | . 669 | . 752 | . 792 | . 820 | . 842 | . 859 | . 872 | . 883 | . 893 | . 901 | . 908 | . 914 |
| n | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 |
| $B(n)$ | . 423 | . 931 | . 938 | . 943 | . 947 | . 951 | . 955 | . 958 | . 960 | . 962 | . 964 | . 966 |
| n | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 |
| $B(n)$ | . 968 | . 970 | . 971 | . 972 | . 973 | . 974 | . 975 | . 976 | . 977 | . 978 | . 979 | . 980 |
| n | 06 | 68 | 70 | 72 | 74 | 76 | 78 | 80 | 85 | 90 | 100 | 120 |
| $B(n)$ | . 980 | . 981 | . 981 | . 982 | . 982 | . 983 | . 983 | . 984 | . 985 | . 986 | . 987 | . 990 |
| This table is reproduced from "Inferences on the Parameters of the Weibull Distribution" by D. R. Thoman, L. J. Bain, and C. E. Antle, Techometrics, Vol. 11, No. 3, August 1964, pp. 445-460. |  |  |  |  |  |  |  |  |  |  |  |  |

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[^0]:    *NOTE: A listing under this column indicates that a Weibull analysis for one or more part types and environments under this part class is included in Sections 5.3-5.5.

[^1]:    *This entire section has been reprinted (with minor corrections) from RADC-TR-7S-22. A revised statistical methods section rontaining additional advanced methods was done and is included as Appendix III of the Final Report describing the study and investigation (RADC-TR-85-66).

[^2]:    NOTE: Low total part operating hours, develop failure data with caution.

[^3]:    NOTE: Low total part operating hours, develop failure data with caution

[^4]:    ‘. ${ }^{3}$ S Smpl: Size limitations. In general sampe size ( $N$ ) (or average ..mpi. :i\%, mi armential demontitration tests) is the dependent variable ot

[^5]:    $L *\left(Z_{a} / Z_{b}\right)$ 1\% the sxact lower confidence bound for $R\left(t_{0}\right)$

[^6]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^7]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^8]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^9]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^10]:    $L *\left(Z_{a} / Z_{b}\right)$ if the exact lower confidence bound for $R\left(t_{0}\right)$

[^11]:    $L *\left(z_{a} / Z_{b}\right)$ is the exact lower confidence borind for $R\left(t_{0}\right)$

[^12]:    $L^{*}\left(Z_{a} / Z_{b}\right)$ is the exact ower confidence bound for $R\left(t_{0}\right)$

[^13]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^14]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{j}\right)$

[^15]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$

[^16]:    $L^{*}\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
    $L_{A}{ }^{*}\left(Z_{a} / Z_{b}\right)$ is the asymptotic lower confidence bound for $R\left(L_{0}\right)$

[^17]:    $L *\left(2_{a} / L_{b}\right)$ is the exact lower confidence bound for $K\left(r_{0}\right)$ $L_{A} *\left(L_{a} / Z_{D}\right)$ is the asymptotic lower confidence bound for $R\left(r_{0}\right)$

[^18]:    $L *\left(L_{a} / L_{b}\right)$ is the exact lower coriridence bound for $R\left(c_{u}\right)$
    $L_{d}{ }^{*}\left(L_{a} / \iota_{b}\right)$ is $t: a s y m p t o t i c ~ l o w e r ~ c o n f i d e n c e ~ b u n d ~ f o r ~ k\left(t_{0}\right)$

[^19]:    $L *\left(z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
    $L_{A}{ }^{*}\left(Z_{a} / Z_{b}\right)$ is the asymptotic lower confidence bound for $R\left(t_{0}\right)$

[^20]:    $L *\left(Z_{a} / Z_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$
    $L_{A} *\left(Z_{a} / Z_{b}\right)$ is the asymptotic lower confidence bound for $K\left(\tau_{0}\right)$

[^21]:    $L *\left(Z_{a} / U_{b}\right)$ is the exact lower confidence bound for $R\left(t_{0}\right)$ $L_{A} *\left(Z_{a} / L_{b}\right) 18$ the asymptotic lower coufidence bound for $R\left(t_{0}\right)$

