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## BAYESIAN RELIABILITY DEMONSTRATION: PHASE III - DEVELOPMENT OF TEST PLANS

Hughes Aircraft Company

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Rome Air Development Center  
Air Force Systems Command  
Griffiss Air Force Base, New York

BAYESIAN RELIABILITY DEMONSTRATION:  
PHASE III - DEVELOPMENT OF TEST PLANS

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Hughes Aircraft Company

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FOREWORD

This report was prepared by Hughes Aircraft Company, Systems Effectiveness Department, under Contract F30602-72-C-0067, Job Order No. 55190254. Mr. Anthony Feduccia (RBRS) was the RADC Project Engineer.

This report has been reviewed by the Office of Information, RADC and approved for release to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved.

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## ABSTRACT

This Final Report is the result of a study performed for RADC under Contract F30602-72-C-0067. This study is the third phase of a three phase effort to develop Bayesian Reliability Demonstration Tests (BRDT).

The objectives of this phase were

- i) develop and tabulate BRDT of fixed time and sequential types.
- ii) develop and tabulate tests (called Bayes/Classical in this report) when the producer and consumer cannot agree on a prior distribution.
- iii) develop methods of updating existing prior distributions.
- iv) develop a preliminary military standard for BRDT.
- v) investigate some special problems.
- vi) fit additional prior distributions.

Bayesian fixed times tests, Bayesian/Classical fixed time tests, and sequential Bayesian tests were developed and tabulated. These tests form an essential part of the preliminary military standard which was also developed. Additional fits of the inverted gamma distribution reconfirmed its choice as a prior distribution and further study showed that updates in the prior distribution are easily made. A test based on probability of acceptance is satisfactory to test for shifts in the prior distribution. Tables were developed giving the truncation points for the sequential tests. At this time, no satisfactory solution has been found for placing more than one equipment on test at a time.

ACKNOWLEDGEMENT

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## EVALUATION

This report marks the end of the three-phased RADC Bayesian Reliability Demonstration Program and, at the same time, signals the beginning of a concerted effort to incorporate Bayesian demonstration methods into an official military standard. This Phase III report, together with the previous Phase I and II final reports, represents the most complete and most meaningful work in Bayesian reliability demonstration yet reported. However, the real worth of this work will not be known until the methods are actually applied and it is shown that they are effective in reducing demonstration testing time and costs. Readers are encouraged to exercise these methods and voice their opinion of the results obtained.



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## 0.0 SUMMARY

This study is the third phase of a three phase study program which has resulted in the development and tabulation of Bayesian Reliability Demonstration Tests. Thus, the primary objective of this, the third phase, was the development of a preliminary Bayesian Reliability Test Standard. This has been accomplished and the "Standard" contains three types of tests.

- i) Bayesian fixed time tests (Table 1)
- ii) Bayesian/Classical fixed time tests (Table 2)
- iii) Sequential Bayesian tests (Table 3)

These tests, test procedures and test parameters are thoroughly described both in this final report and in the Standard.

There were also other objectives for this, the third phase:

- i) Fit additional prior distributions.
- ii) Conduct a special problems analysis.
- iii) Develop tests for lack of agreement on the prior distribution between producer and consumer.
- iv) Develop methods of updating existing prior distributions.

All of these objectives are discussed in detail in this final report; hence, only a summary is given here.

Data was gathered on seven (7) additional equipments and seven (7) good fits were obtained to the inverted gamma prior distribution. This makes a total number of fits (inverted gamma prior distribution) in the three phases of twenty-nine (29) out of thirty-seven (37) data sets available.

Three special problems were addressed: Truncating the sequential tests, testing for shifts in the prior distribution and placing more than one equipment on test at a time. The first was solved and the number of failures,  $n_t$ , at which the test is stopped is given for each sequential test. The second was solved by using a test based on  $P(A)$ , the probability of acceptance. The third was not completely solved in a satisfactory manner.

The tests of the "lack of agreement" case were developed and are the Bayes/Classical tests (B/C for short) mentioned above.

Methods of updating existing prior distributions have been developed for two cases: new observed data available, and new predictions available.

## 1.0 INTRODUCTION - STATEMENT OF OBJECTIVES

This report presents the results of the third phase of a three phase study effort to investigate and develop Bayesian reliability demonstration tests (BRDT). The first phase had the objective of developing methods and criteria for fitting prior distributions and to fit some prior distributions. The primary objective of the Phase II effort was to investigate and develop methods for determining BRDT and to fit additional prior distributions.

The objectives of this phase were

- i) develop and tabulate BRDT of fixed time and sequential types.
- ii) develop and tabulate tests (called Bayes/Classical in this report) when the producer and consumer cannot agree on a prior distribution.
- iii) develop methods of updating existing prior distributions.
- iv) develop a preliminary military standard for BRDT.
- v) investigate some special problems.
- vi) fit additional prior distributions.

The objectives are certainly not listed in the order of their importance. In fact, the overriding objective was the development of a preliminary military standard for BRDT, iv) above, and actually the other tasks will furnish inputs to this standard. For example, the fixed time and sequential BRDT, objective i) above, form an essential part of the preliminary standard.

The results of these objectives are discussed in some detail in the remainder of this report.

## 2.0 FITTING PRIOR DISTRIBUTIONS

It has been one of the goals of the three phases of the Bayes study to demonstrate that prior distributions can be fit to actual data. This section describes the field data collected in this, the third phase, and gives the results of the data fits to the inverted gamma prior distribution. Also given in this section is a summary of the equipments and results of fits to prior distributions from the three phases of the study. These results show that it is not only feasible, but relatively simple to fit data to the inverted gamma prior distribution from a wide range of different equipment types.

### 2.1 FITTING PRIOR DISTRIBUTIONS FOR PHASE III

#### 2.1.1 DATA COLLECTION

The field data collected for fitting prior distributions for the Phase III study was the Type 1. data described in the Phase I and II studies. That is, the observed random variable is the number,  $x$ , of failures of a unit occurring in a fixed time  $T$ .  $x$  is a discrete variable, taking on only the values  $0, 1, 2, \dots$ . Observations on  $x$  are obtained by putting  $n$  units on test for time  $T$ , and recording the number of failures for each unit  $\{x_1, \dots, x_n\}$ . Data of Type 2. (the observed random variable is the sample MTBF,  $\bar{\theta}$ ) was also searched for, but none was found of suitable quality to fit to the inverted gamma prior distribution.

Since quality Air Force 66-1 data was previously obtained from Tinker Air Force Base in Oklahoma City, Oklahoma, contact was again made and data retrieved on six different equipments. This data was summarized in Table 2.1.1.1, data sets 2 through 7. Most of these data were based on a fixed time of 4320 hours, with two of them based on 8640 hours. Type 1. data was also collected from Hughes Aircraft Company on PPI consoles, listed in Table 2.1.1.1 as data set number 1. This fixed time data was based on 6278 hours of operation.

Given in Table 2.1.1.1 are the data set numbers assigned for this phase, the equipment name,  $n$  = the number of "identical" equipments,  $\Sigma K$  = the total number of failures observed from all  $n$  equipments, and finally, the source of the data.

#### 2.1.2 THE FITTED PRIOR DISTRIBUTIONS

As in Phases I and II, the inverted gamma distribution family was assumed for the prior distribution. The equation is

$$g(\theta) = \frac{\alpha^\lambda}{\Gamma(\lambda)} \left(\frac{1}{\theta}\right)^{\lambda+1} e^{-(\alpha/\theta)}. \quad (2.1.2.1)$$

TABLE 2.1.1.1 FIELD DATA FOR FITTING PRIOR DISTRIBUTIONS

ASSIGNED DATA SET NUMBER	EQUIPMENT	n	$\Sigma k$	DATA SOURCE
1.	PPI Console	19	23	HAC
2.	Receiver	41	430	AF66-1
3.	Relay Control Assembly	41	122	AF66-1
4.	Recorder-Reproducer	56	459	AF66-1
5.	Amplifier Recorder	51	192	AF66-1
6.	Amplifier Fail Safe	49	144	AF66-1
7.	Recorder Reproducer Section	80	528	AF66-1

A complete description of the method for fitting prior distributions when the family is specified can be found in the Phase I final report, Section 4.2.1 (Ref. 2). The equations for the parameter estimates, as derived in the Phase I report, are as follows:

$$\hat{\alpha} = \frac{\frac{\sum x_i}{n} T}{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 - \frac{\sum x_i}{n}} \quad (2.1.2.2)$$

$$\hat{\lambda} = \frac{\sqrt{\frac{\sum x_i}{n}}}{T}. \quad (2.1.2.3)$$

The  $\chi^2$  goodness-of-fit test was used to test the validity of the assumed inverted gamma prior distribution. The data is divided into C cells, and since 2 unknown parameters ( $\alpha$  and  $\lambda$ ) were estimated, (C-3) degrees of freedom are used.

Table 2.1.2.1 shows the results of the Phase III prior distribution fits. For each data set, corresponding to the data in Table 2.1.1.1, the following information is given.

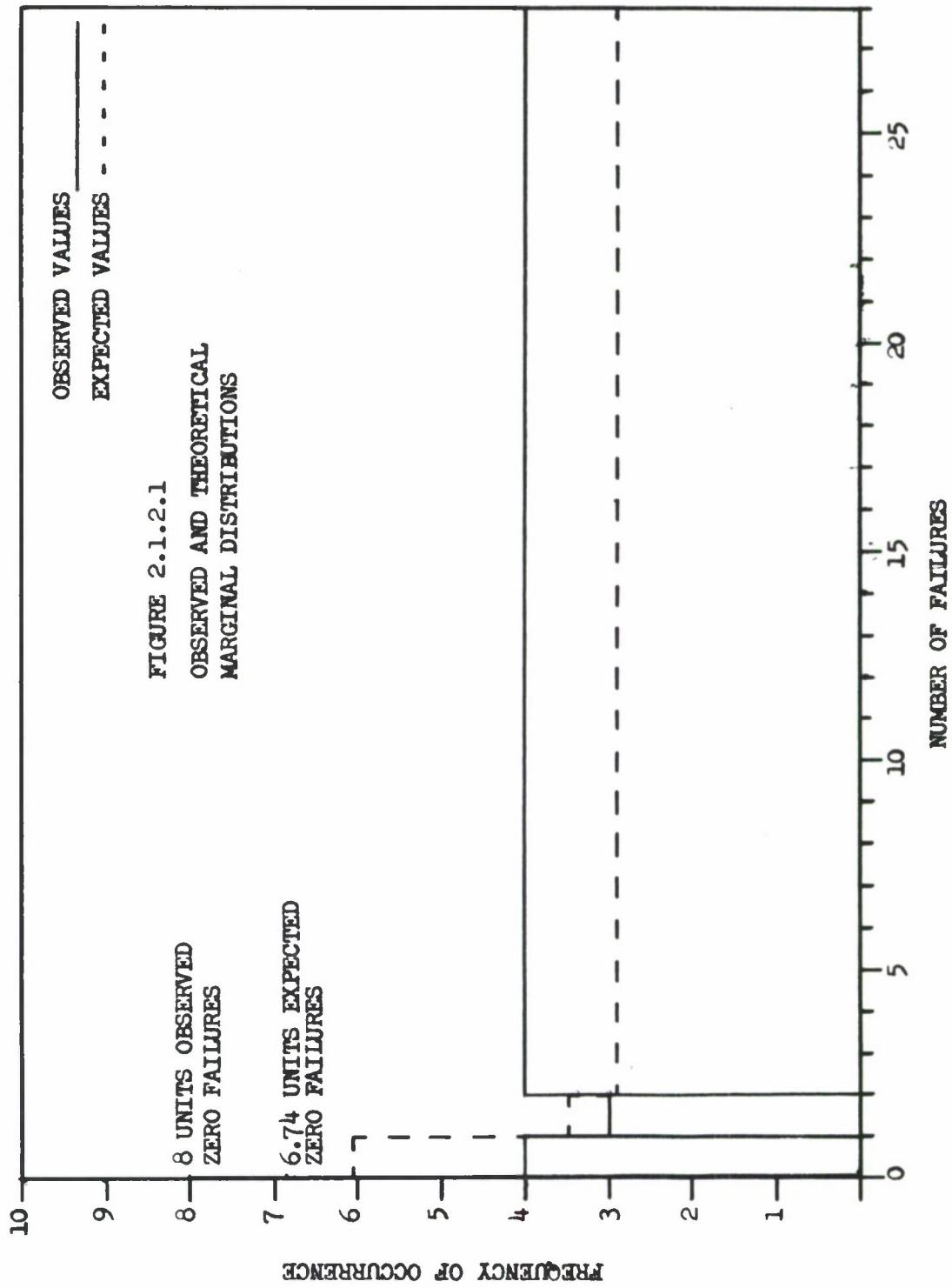
- Sample Mean =  $\frac{\sum x_i}{n}$ .
- Sample Variance =  $\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$ .
- Parameter Estimates  $\hat{\alpha}$  and  $\hat{\lambda}$ .
- Number of  $\chi^2$  cells into which the data has been divided.
- The  $\chi^2$  value computed from the data.
- Whether or not the  $\chi^2$  test passes at the  $P = .99$  and  $P = .90$  significance levels.

Figures 2.1.2.1 through 2.1.2.7 are plots of the observed (solid lines) and theoretical (dotted lines) marginal distributions for the seven data sets. Figures 2.1.2.8 through 2.1.2.14 are the plots of the theoretical prior distributions that were fit from the seven field data sets. These curves exhibit the representative rapid approach to flatness of the left-hand tails which illustrates why Bayes tests are often shorter than Classical tests.

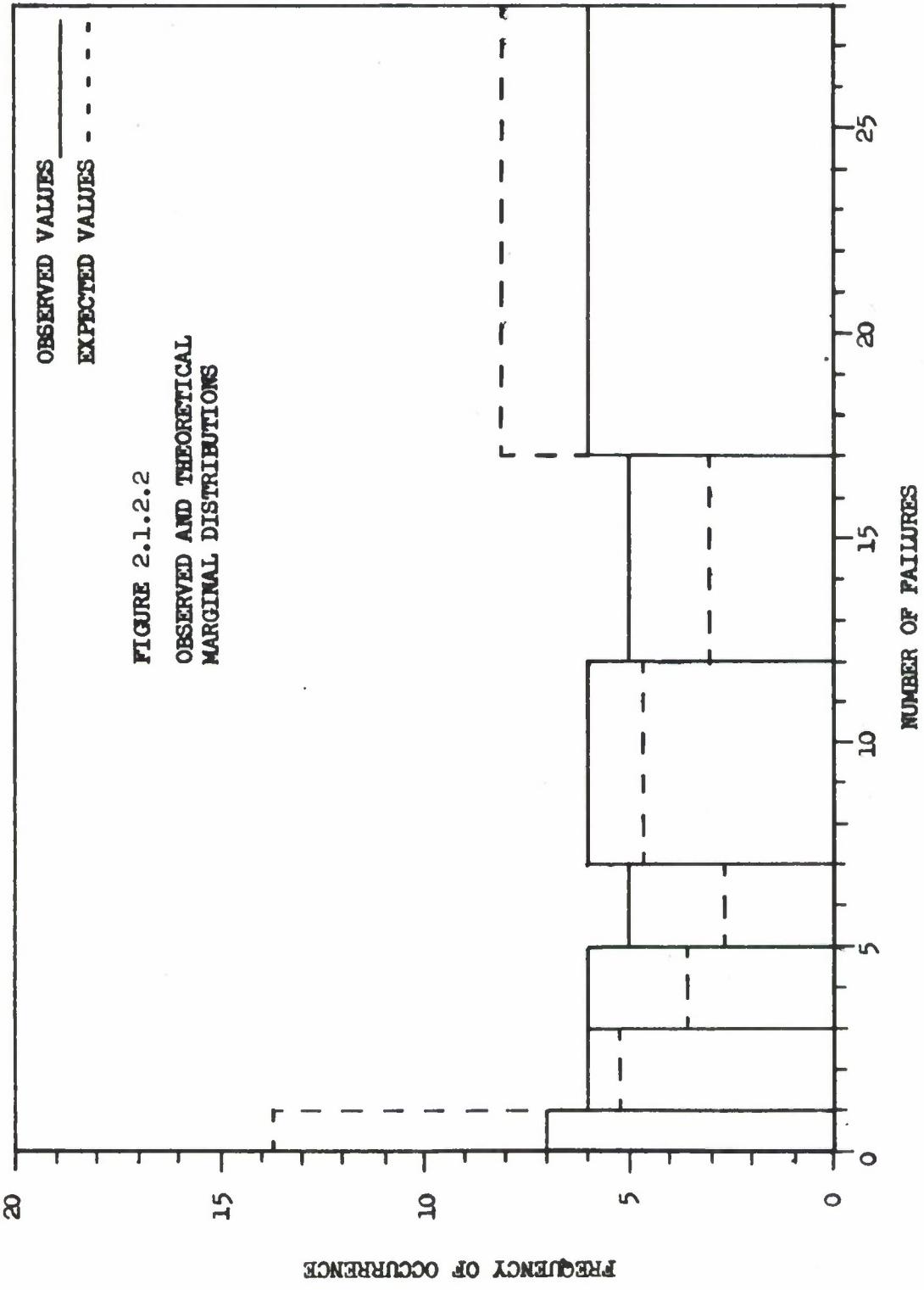
TABLE 2.1.2.1 FIELD DATA FITTED TO INVERTED GAMMA PRIOR DISTRIBUTIONS

DATA SET NO.	SAMPLE MEAN	SAMPLE VARIANCE	PARAMETER $\alpha$	ESTIMATES $\lambda$	NO. OF $\chi^2$ CELLS	$\chi^2$	PASS AT .99 LEVEL	PASS AT .90 LEVEL
1.	1.211	1.731	17699.9	3.4129	4	1.54642	YES	YES
2.	10.49	256.1	384.939	.459605	7	9.3498	YES	NO
3.	2.976	11.87	1518.02	1.02847	4	2.49176	YES	YES
4.	6.411	80.90	385.456	.562625	6	5.51425	YES	YES
5.	3.765	20.70	2000.21	.857262	6	7.22946	YES	NO
6.	2.939	9.434	4095.95	1.37034	5	4.79052	YES	NO
7.	6.600	65.26	1002.32	.753112	9	13.7968	YES	NO

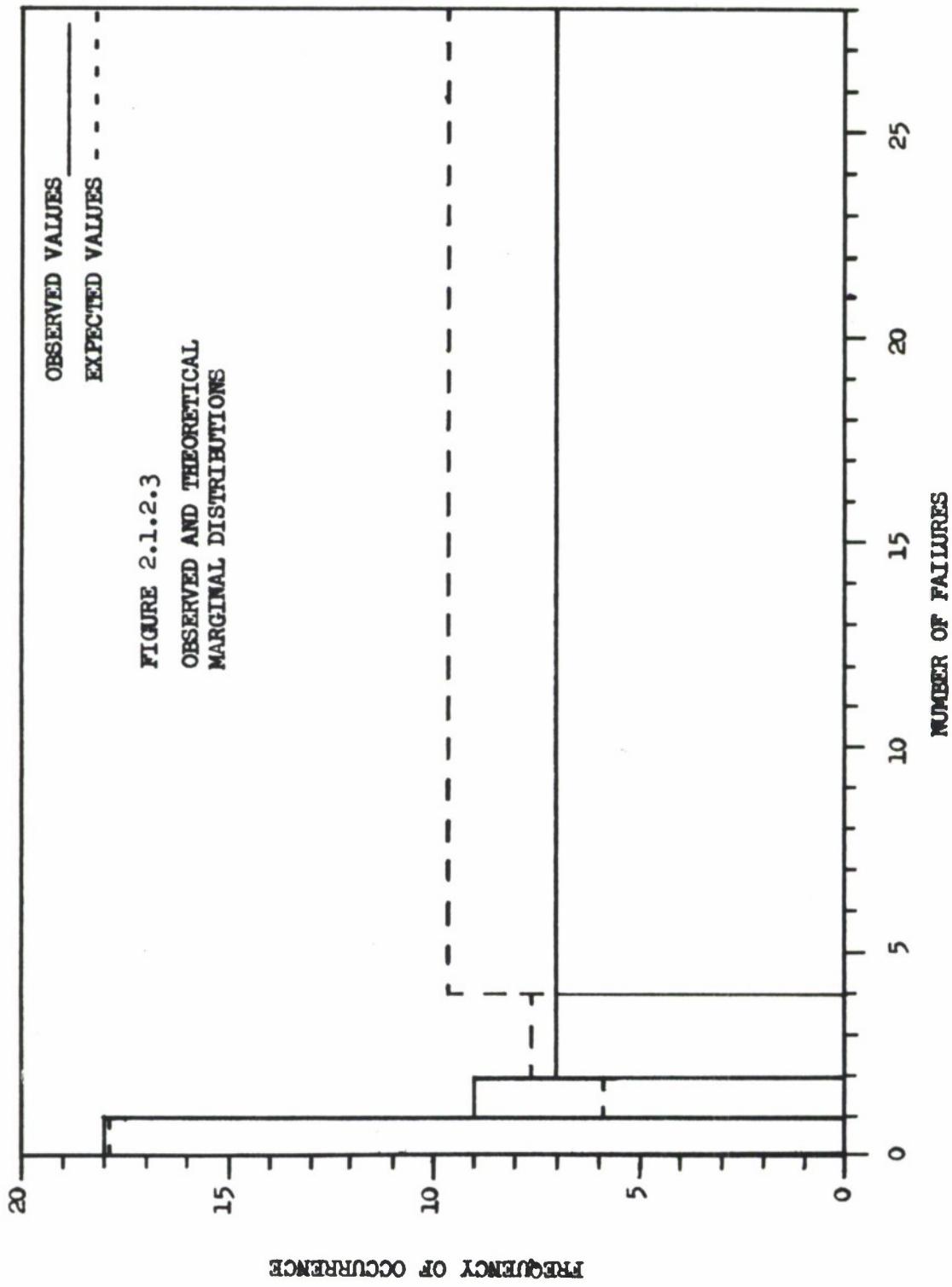
## DATA SET NO. 1



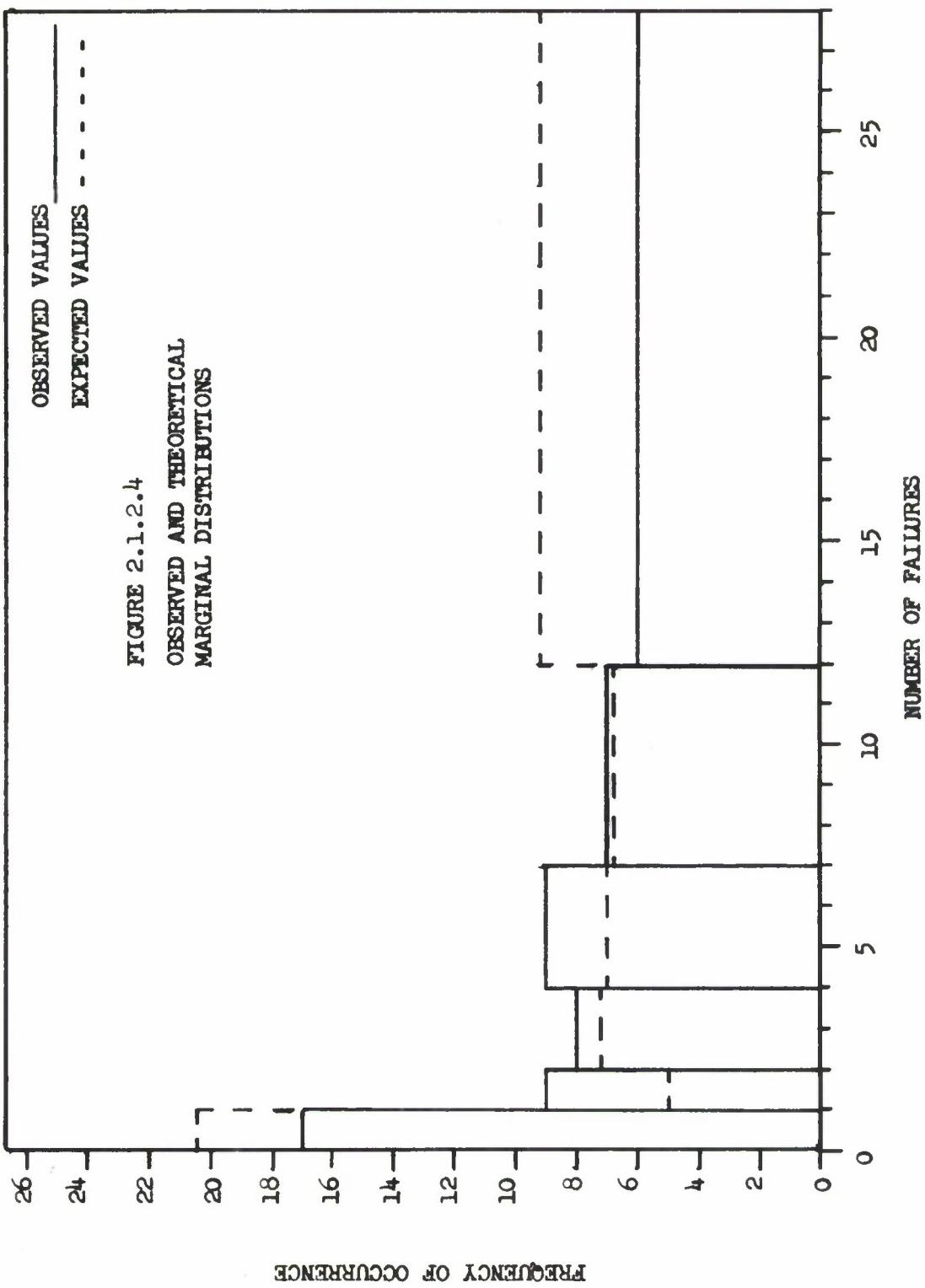
DATA SET NO. 2



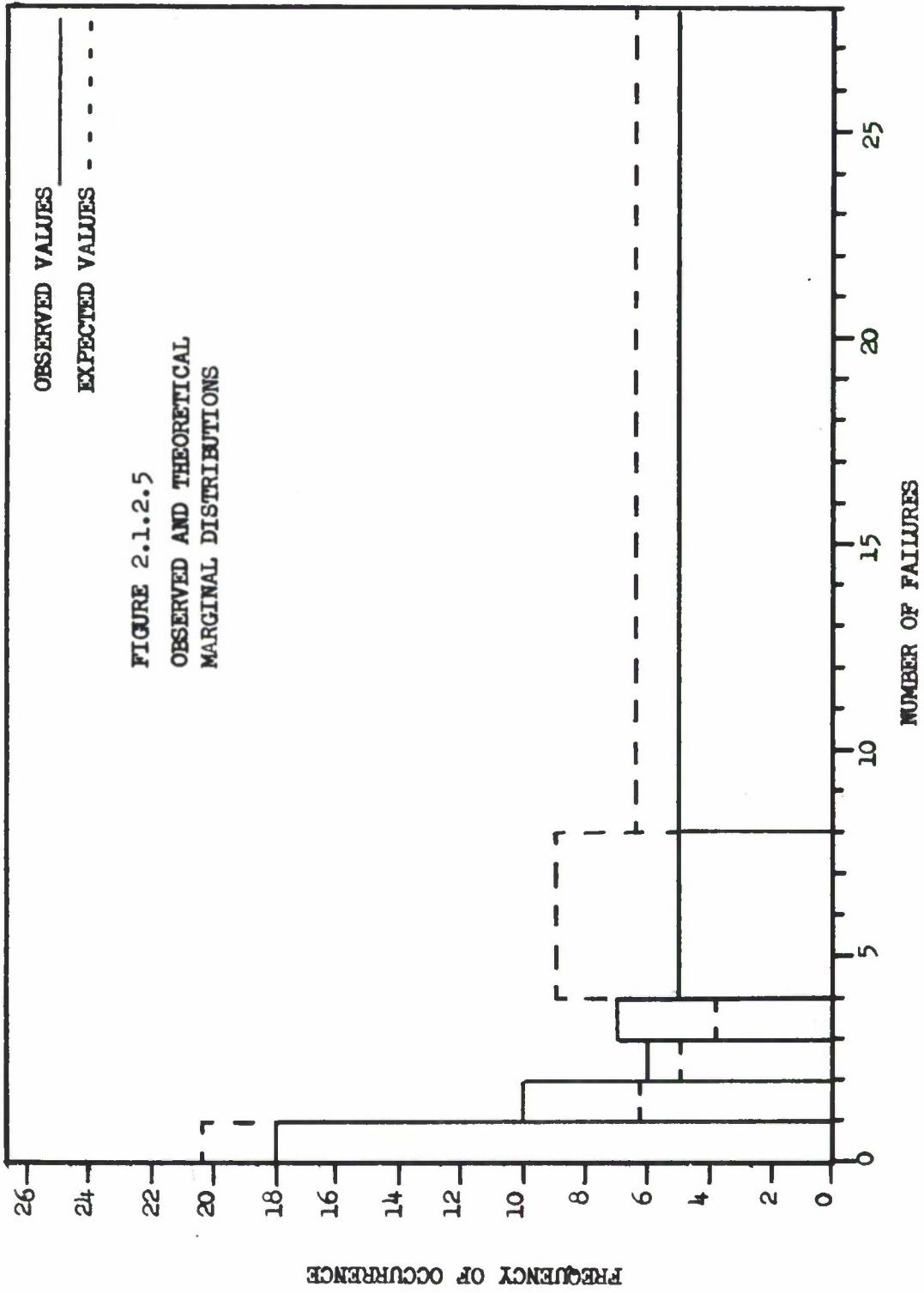
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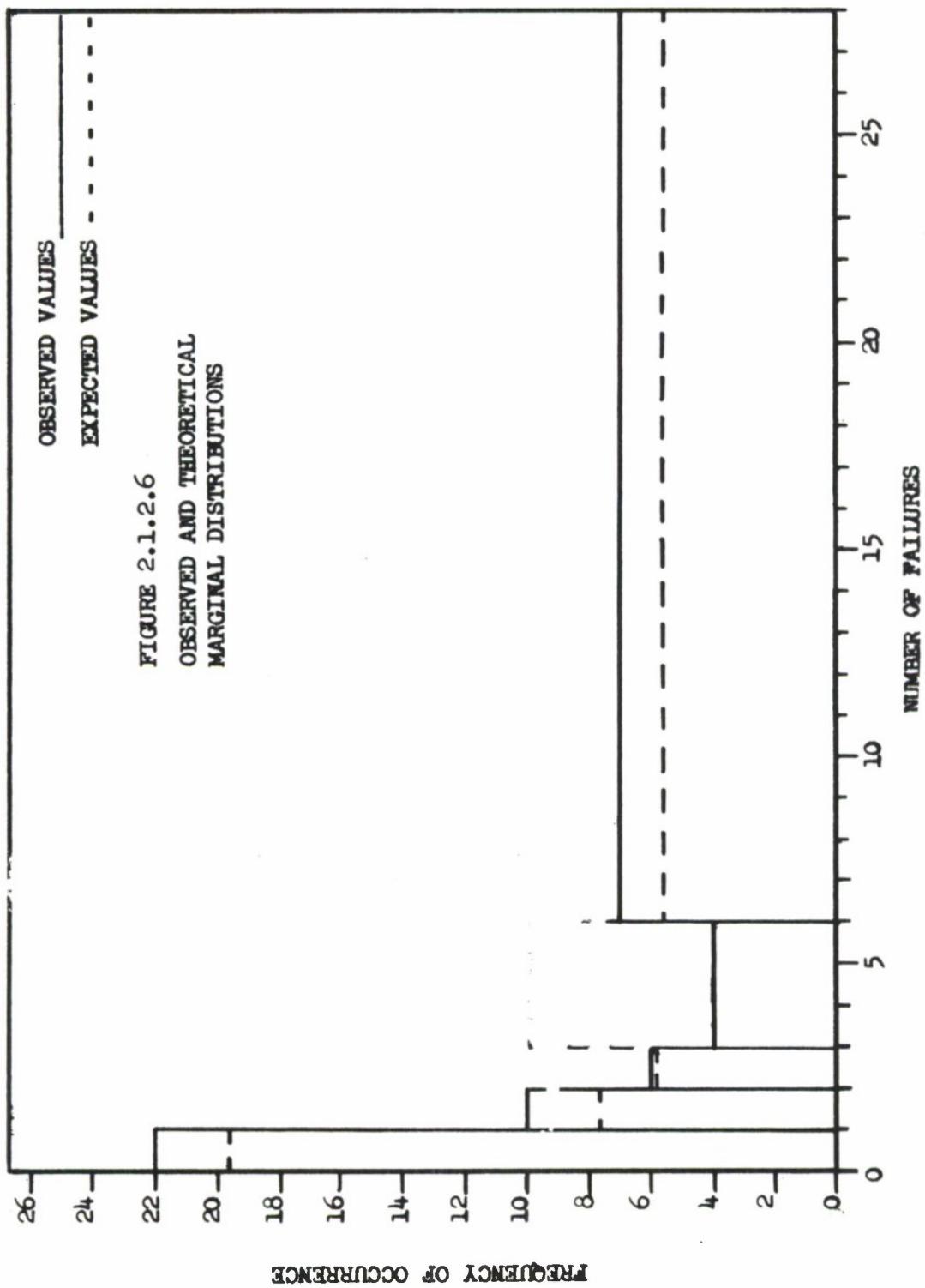
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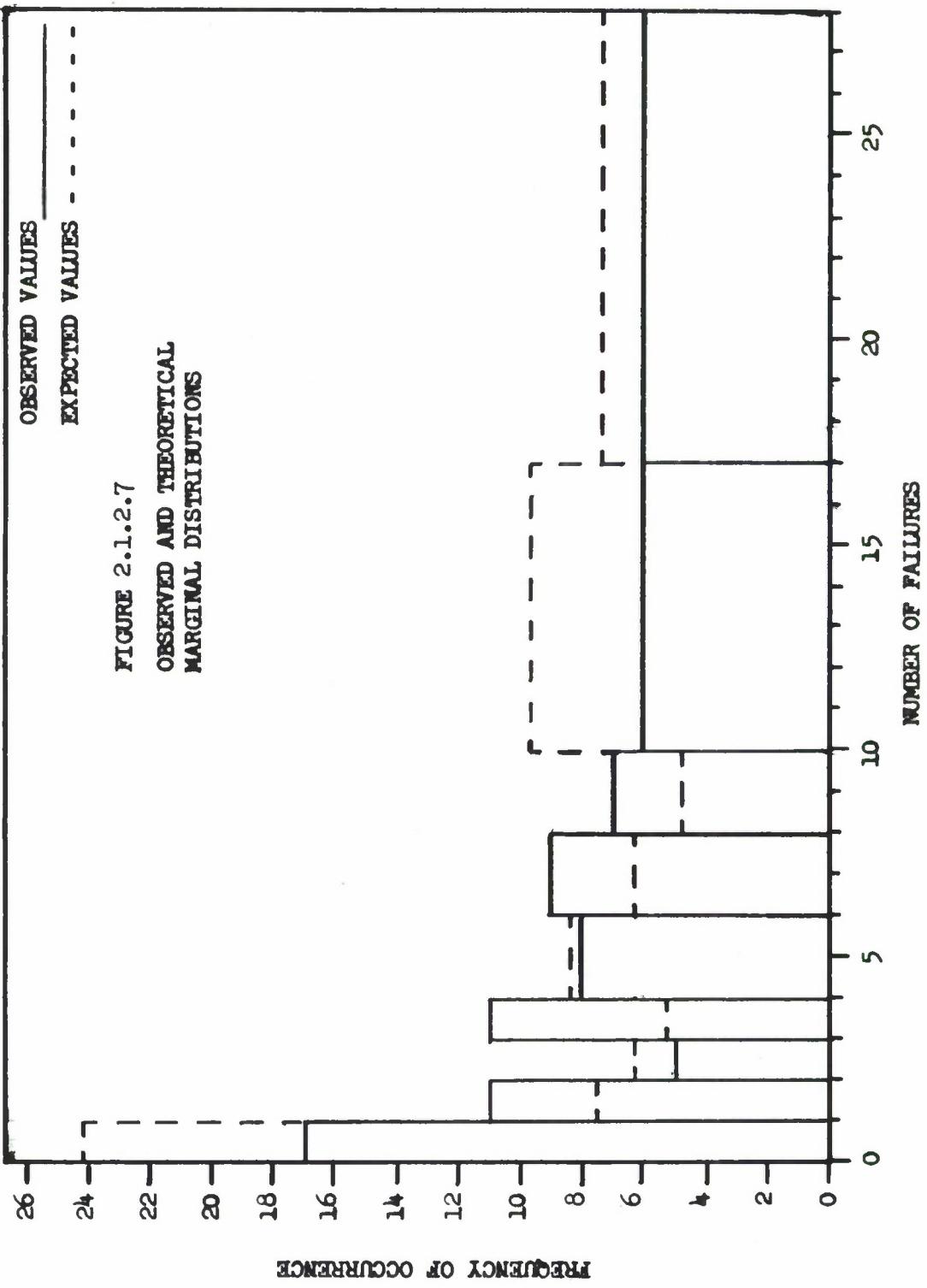
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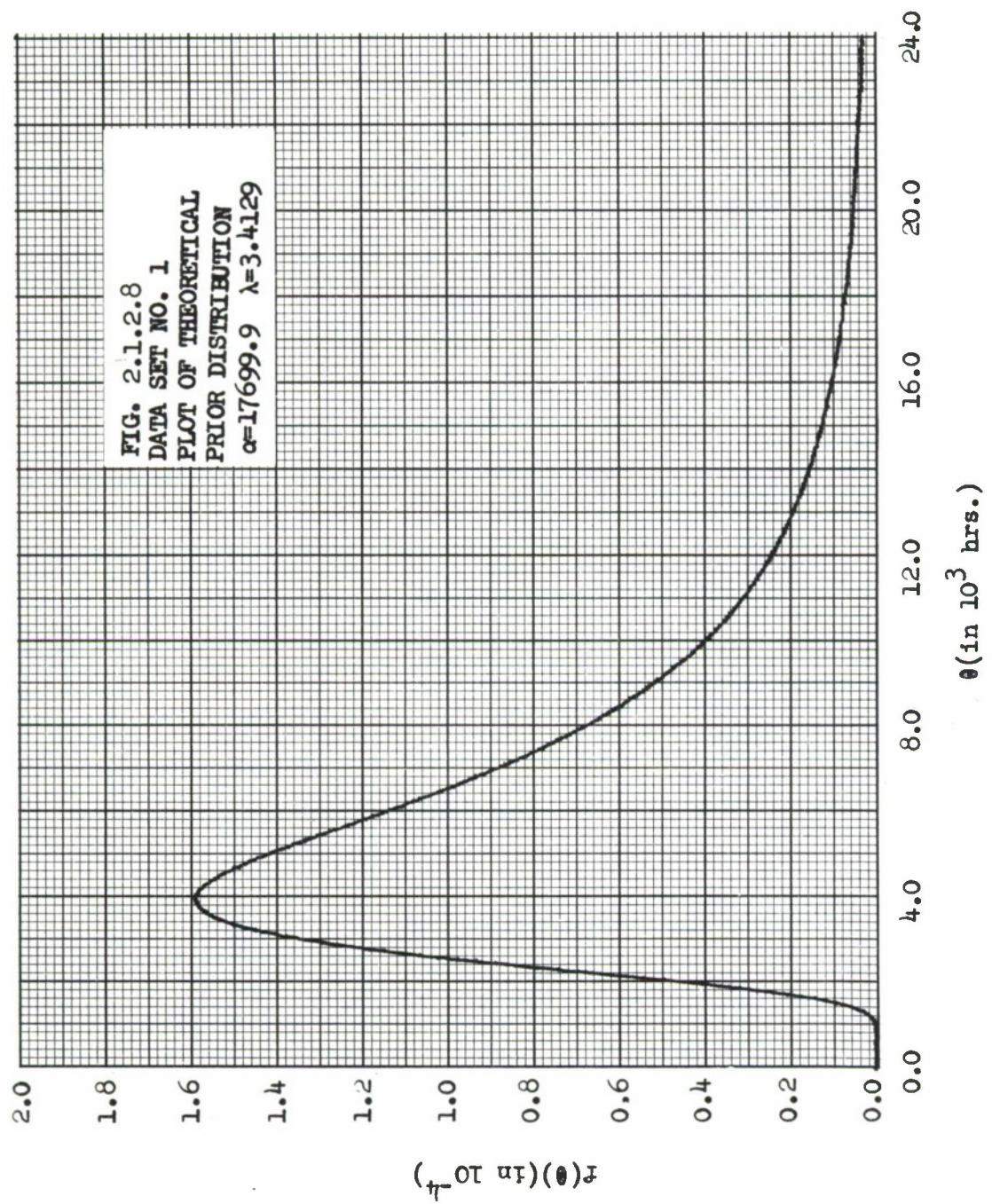


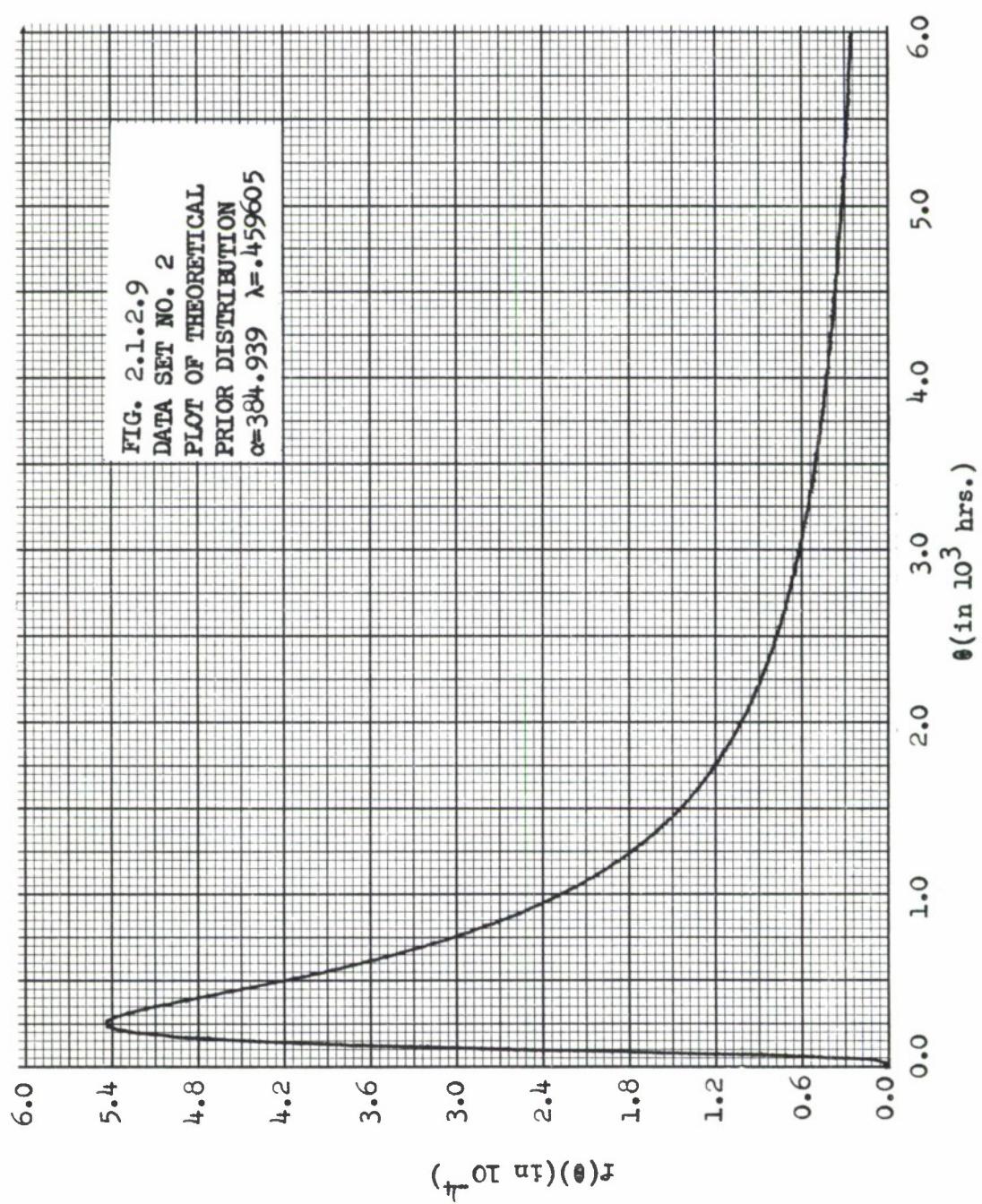
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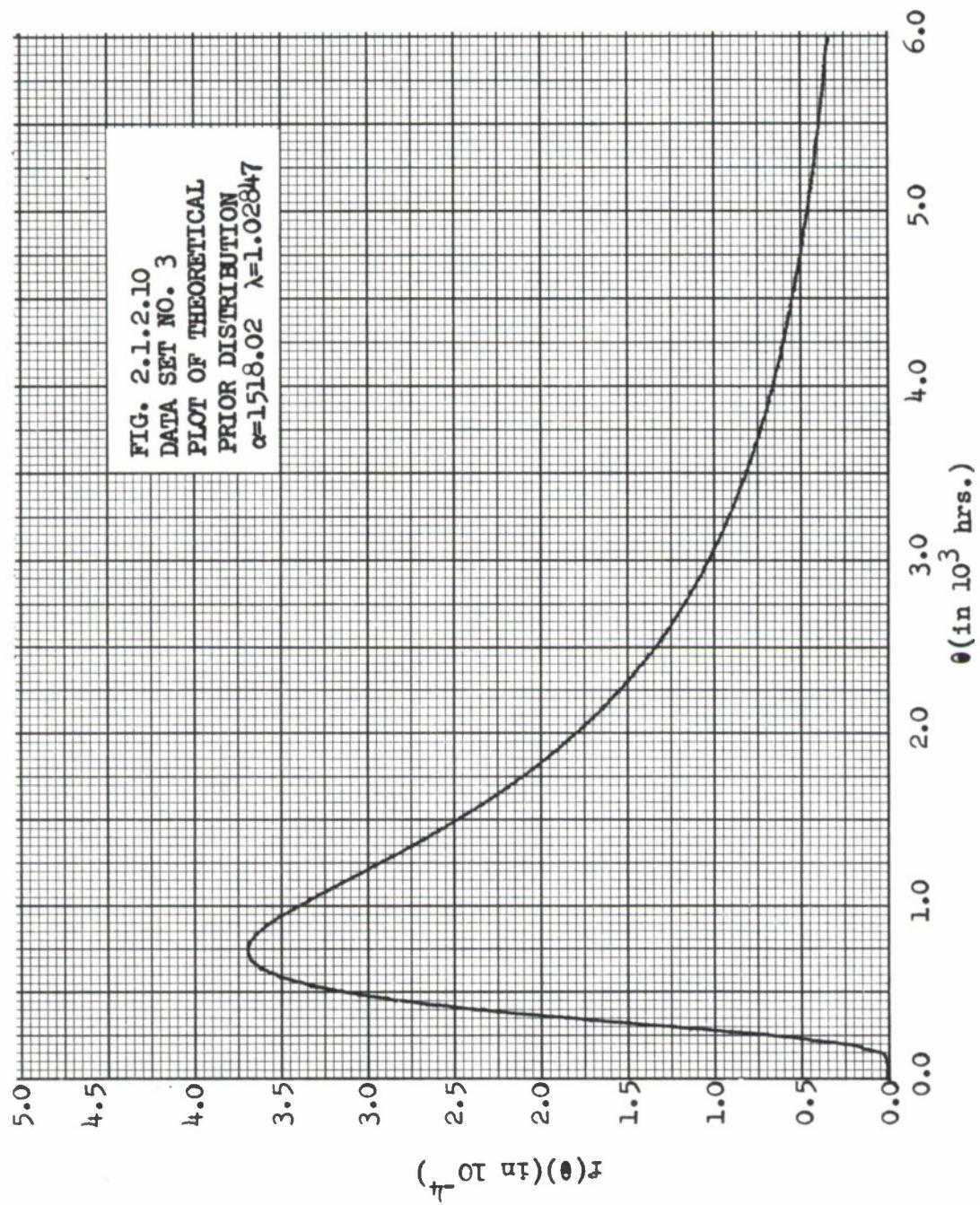


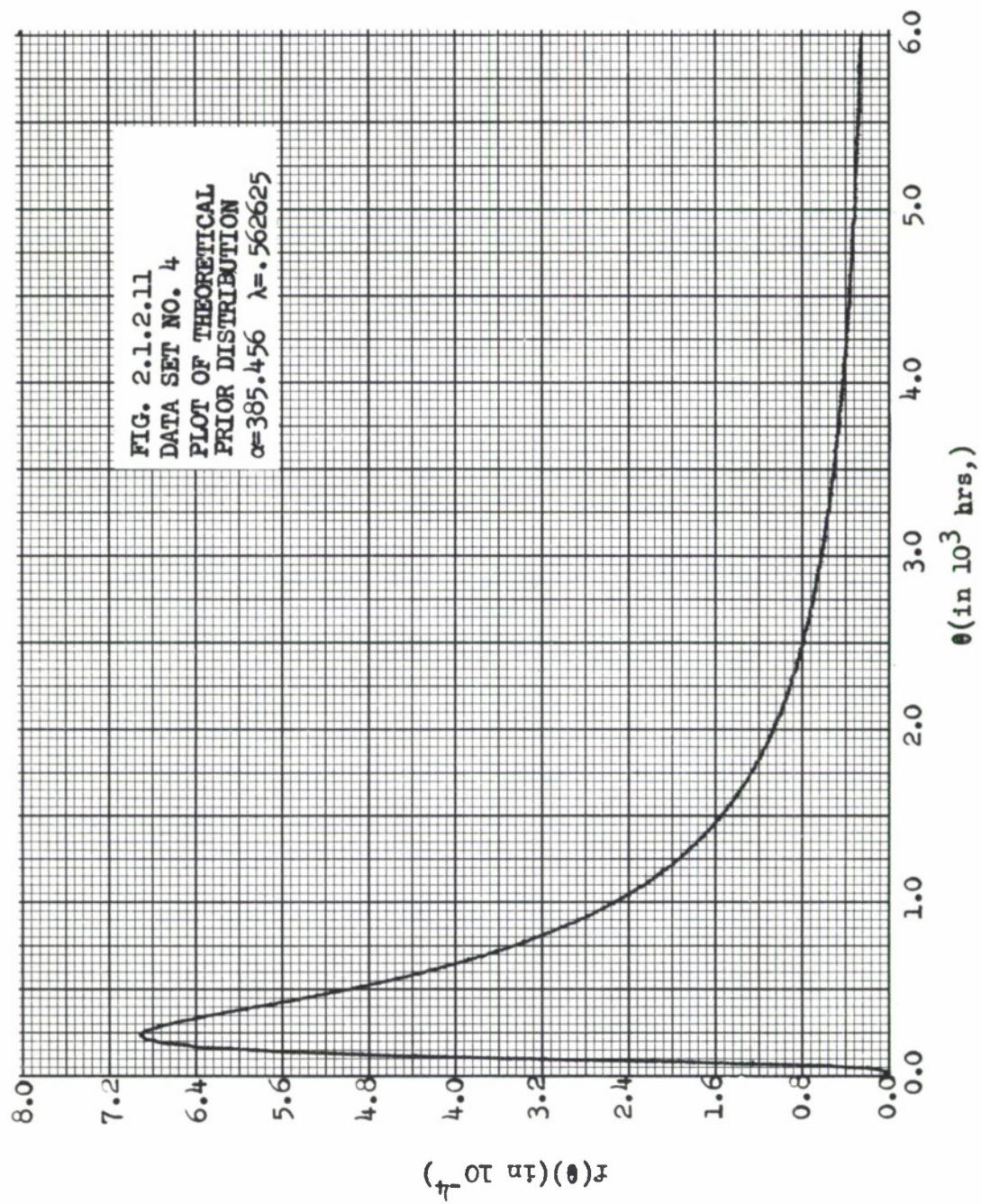
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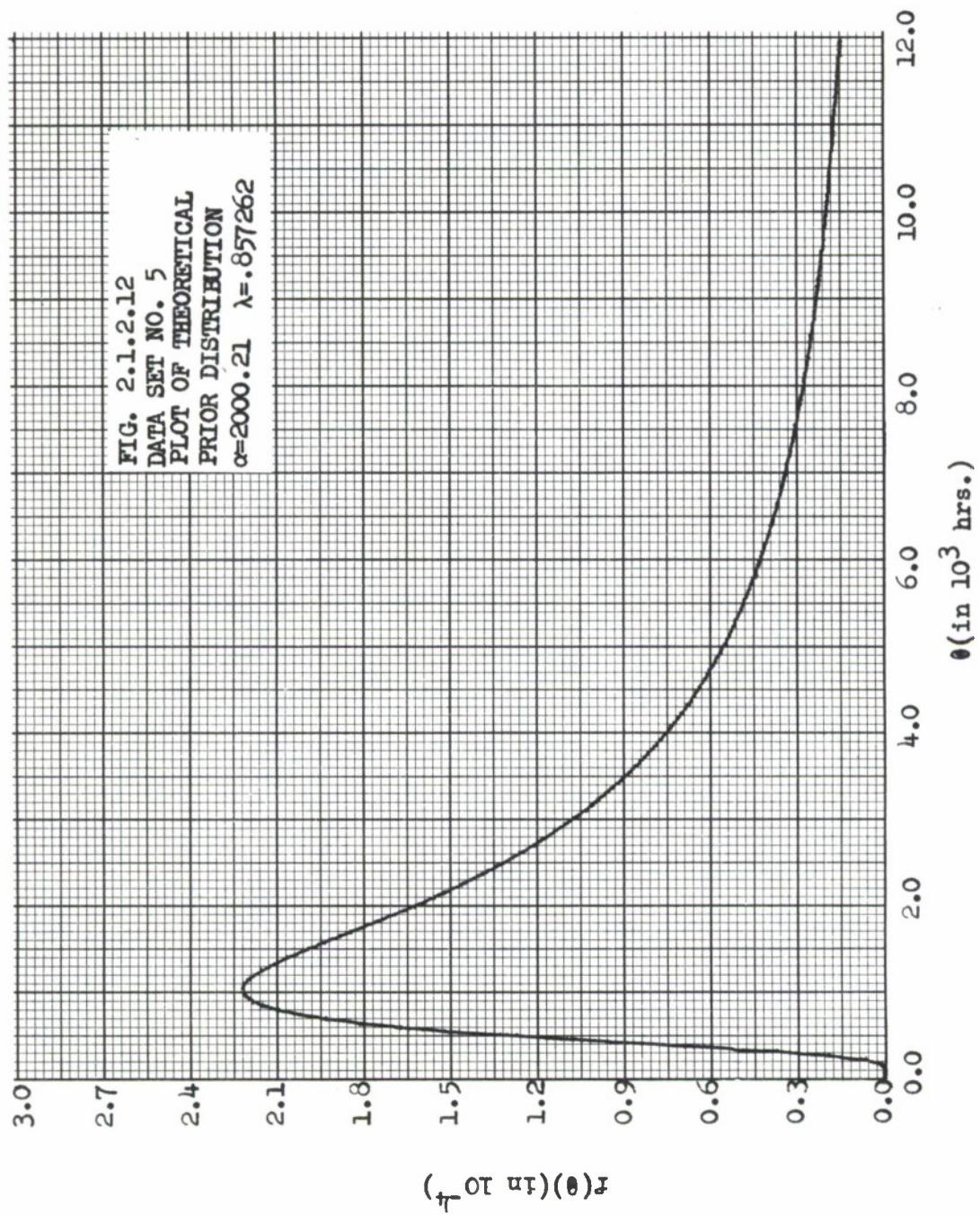


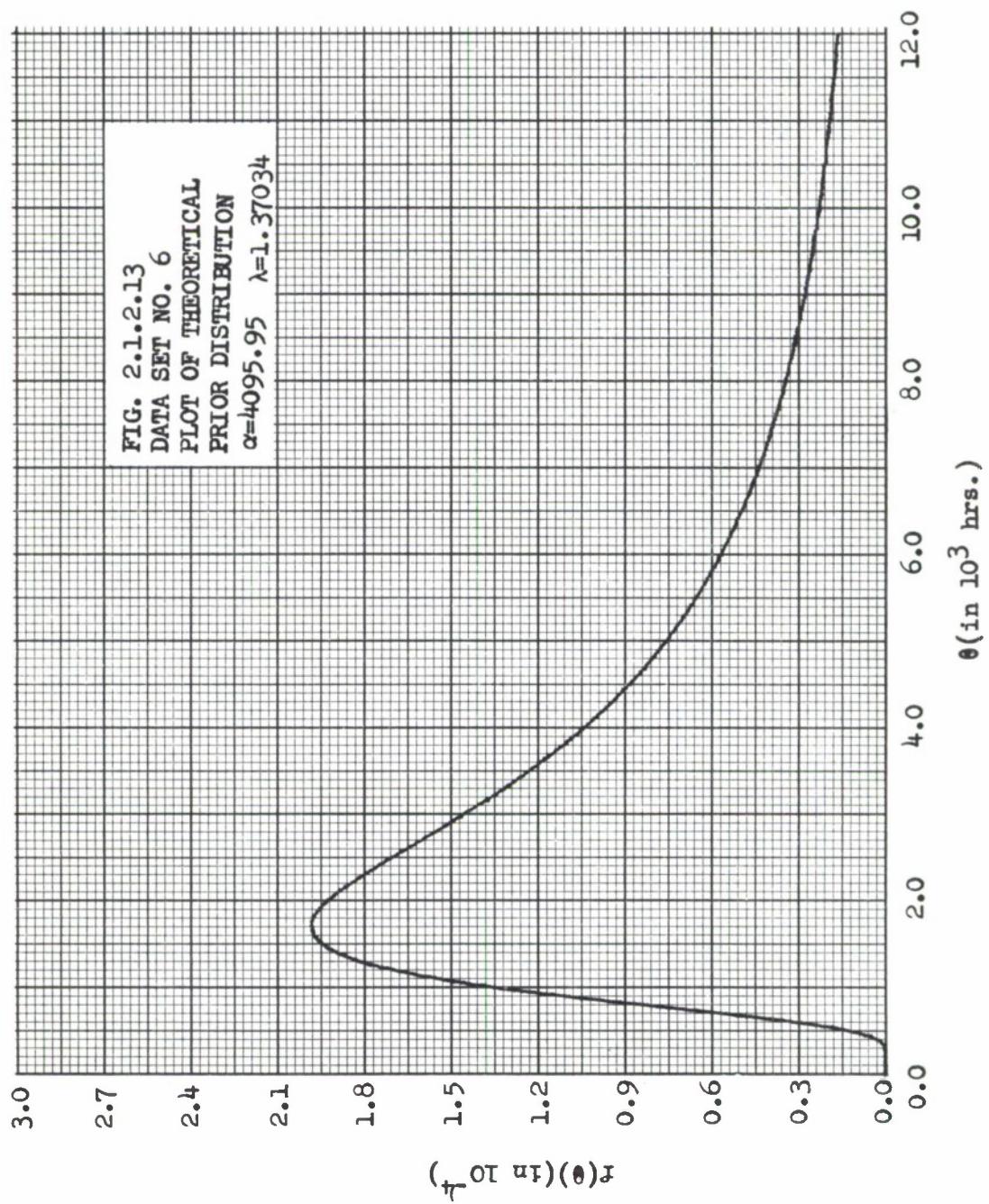


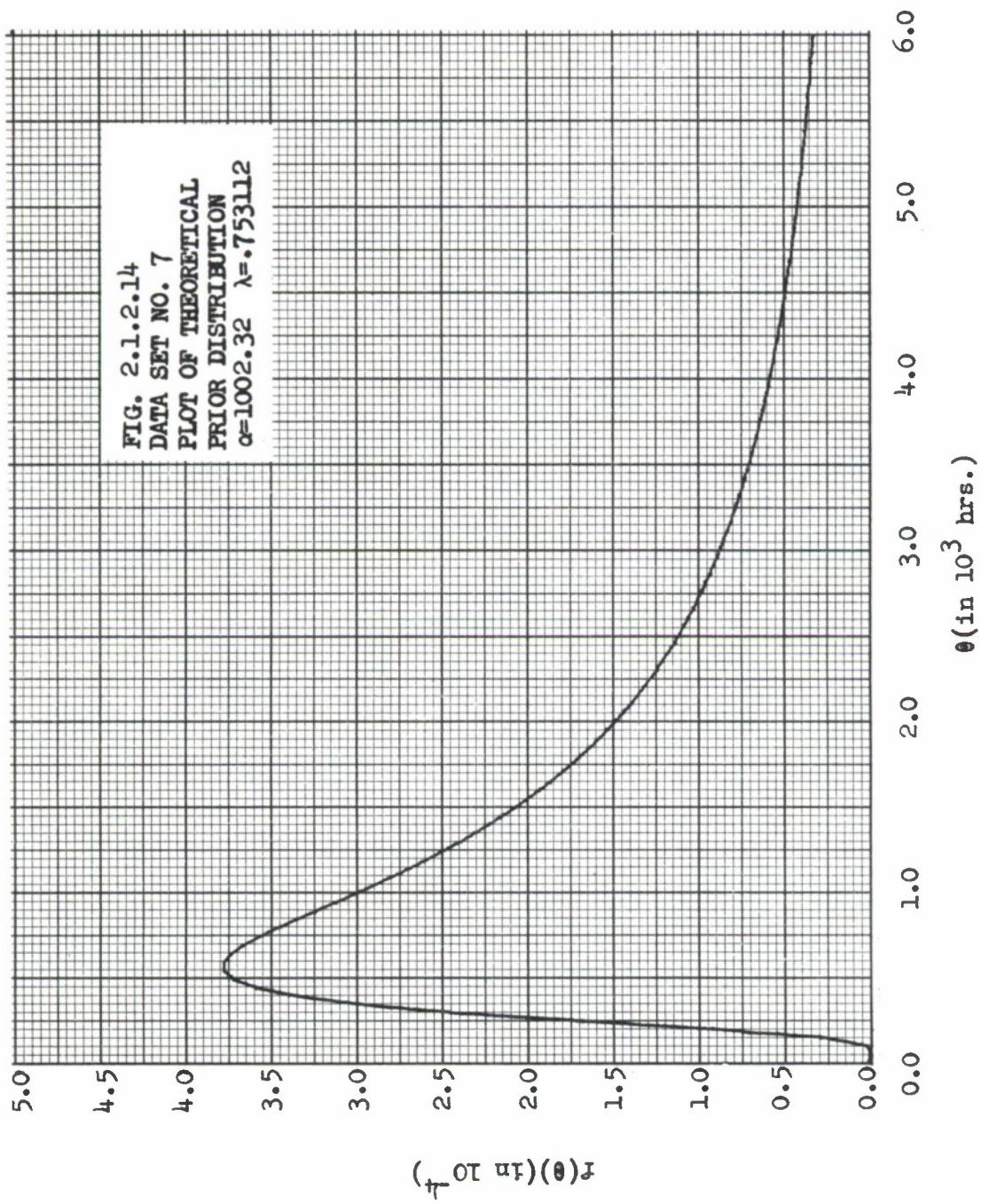












## 2.2 SUMMARY OF PRIOR DISTRIBUTIONS FIT IN PHASES I, II AND III

In the three phases of the Bayes study, field data, all of Type 1., has been collected on thirty-seven (37) different types of equipments. Of these equipments, twenty-nine (29) yielded suitable inverted gamma prior distribution parameter estimates and passed the  $\chi^2$  goodness-of-fit test at the  $P = .99$  significance level or better. It may be of interest to note that entries 2 and 11 in Table 2.2.1 are the same type of equipment operated in different systems. Table 2.2.1 is a complete summary of these equipments including:

- Equipment name - with as much information included as was available.
- System, W.U.C. - The Air Force system designator and Work Unit Code at the time of the data collection.
- Data source.
- Inverted Gamma prior distribution parameter estimates  $\hat{\alpha}$  and  $\hat{\lambda}$ .
- Number of  $\chi^2$  cells into which the data was divided.
- The  $\chi^2$  value computed from the data.
- Whether or not the  $\chi^2$  test was passed at the  $P = .90$  and  $P = .99$  significance levels.

TABLE 2.2.1 SUMMARY OF FIELD DATA FITTED TO INVERTED GAMMA PRIOR DISTRIBUTIONS

	EQUIPMENT	SYSTEM W.U.C.	DATA SOURCE	PARAMETER $\hat{\alpha}$	$\hat{\lambda}$	NO. QF $\chi^2$ CELLS	$\chi^2$	PASS AT .90 LEVEL	PASS AT .99 LEVEL
1.	MTI REFLECTOR SM-225	MPN013 AA500	AF66-1	1279.67	1.24991	4	1.35756	YES	YES
2.	SEARCH HVPS PP-3132	MPN013 AD300	AF66-1	758.591	1.2716	8	6.07651	YES	YES
3.	OSCILLOSCOPE OS-126	MPN013 AE400	AF66-1	3609.57	2.60495	4	1.31484	YES	YES
4.	VIDEO AMPLIFIER AM-472	MPN013 AF200	AF66-1	7584.73	4.03817	4	5.60818	NO	YES
5.	SYNCH POWER SUPPLY SN-315	MPN013 AE600	AF66-1	2251.61	1.73419	5	1.50224	YES	YES
6.	SEARCH INDICATOR IP-128A	MPN013 AF400	AF66-1	1767.69	1.87662	5	5.64293	NO	YES
7.	AMPLIFIER PANEL PN-8-	BC0640 AAABO	AF66-1	27388.4	14.9852	4	4.10793	NO	YES
8.	TRANSMITTER	BC0640 AAATO	AF66-1	1313.34	.990540	8	12.9803	NO	YES
9.	DEMOD AUD AMP	HCO150 AAAFA	AF66-1	1544.37	1.63954	4	5.28692	NO	YES
10.	SYN LO FREQ	HCO150 AAATO	AF66-1	352.147	.922144	5	1.25033	YES	YES
11.	SEARCH HVPS PP-3132	MPN014 AF300	AF66-1	3558.94	3.16857	4	.860676	YES	YES

TABLE 2.2.1 SUMMARY OF FIELD DATA FITTED TO INVERTED GAMMA PRIOR DISTRIBUTIONS (Continued)

	EQUIPMENT	SYSTEM W.U.C.	DATA SOURCE	PARAMETER ESTIMATES		NO. OF $\chi^2$ CELLS	$\chi^2$	PASS AT .90 LEVEL	PASS AT .99 LEVEL
				$\hat{\alpha}$	$\hat{\lambda}$				
12.	TRANSCEIVER C/O 618T-3	MRC108 AAEAG	AF66-1	7256.62	3.85502	6	8.61483	NO	YES
13.	RCVR-TRNS UNIT RT-742	MRC108 AABEB	AF66-1	16929.7	10.7568	6	8.09386	NO	YES
14.	RCVR-TRNS RT-781	MRN012A AAECO	AF66-1	5684.21	6.57895	4	.058616	YES	YES
15.	RECEIVER R-417A	TRC035 AABL0	AF66-1	1227.16	1.15249	4	1.47611	YES	YES
16.	AMP-CONVERTER AM3813	TRC066A AAAB0	AF66-1	1735.01	1.8073	4	.464289	YES	YES
17.	POWER SUPPLY PP3835	TRC066A AAAEO	AF66-1	2777.38	2.12964	4	.799789	YES	YES
18.	AMPLIFIER-DETECTOR AM3819	TRC066A AAAGO	AF66-1	1781.23	1.74882	4	.099587	YES	YES
19.	RADIO RECR GP R761	TSC015 AAFOO	AF66-1	1152.97	1.22656	4	3.52765	NO	YES
20.	LINEAR PWR AMP 208U3	URGCOMM ALOO	AF66-1	1152.10	1.04095	4	1.38745	YES	YES
21.	PWR AMP 10KW 208U10	URGCOMM AMOO0	AF66-1	882.354	1.24464	6	5.40455	YES	YES
22.	AMP-MOD AN1701A	URM003A AAAO	AF66-1	1686.64	1.28008	6	2.89829	YES	YES

TABLE 2.2.1 SUMMARY OF FIELD DATA FITTED TO INVERTED GAMMA PRIOR DISTRIBUTIONS (Continued)

EQUIPMENT	SYSTEM W.U.C.	DATA SOURCE	PARAMETER ESTIMATES		NO. OF $\chi^2$ CELLS	$\chi^2$	PASS AT .90 LEVEL	PASS AT .99 LEVEL
			$\hat{\alpha}$	$\hat{\lambda}$				
23. PPI CONSOLE	---	H.A.C.	17699.9	3.4129	4	1.54642	YES	YES
24. RECEIVER BC-639 RAD	BC0639 AA000	AF66-1	384.939	.459605	7	9.3498	NO	YES
25. RELAY CNTRL ASSY	TRL510 AAHHO	AF66-1	1518.02	1.028447	4	2.49176	YES	YES
26. RECORD-REPRODUCE	TRL510 AA000	AF66-1	385.456	.562625	6	5.51425	YES	YES
27. AMPL RECORD	TRL510 AAAAO	AF66-1	2000.21	.857262	6	7.22946	NO	YES
28. AMPL FAIL SAFE	TRL510 AAADO	AF66-1	4095.95	1.37034	5	4.79052	NO	YES
29. FCRD REPRO SECTION	TRL510 AAA00	AF66-1	1002.32	.753112	9	13.7968	NO	YES

### 3.0 THE PROBABILITY DISTRIBUTIONS USED IN THE BAYESIAN RELIABILITY DEMONSTRATION TESTS (BRDT)

#### 3.1 INTRODUCTION

There are four probability distributions that recur in the development of BRDT of the fixed time and sequential types. They are:

- i) the prior distribution in  $\theta$ ,  $g(\theta)$ .
- ii) the conditional distribution of  $X$ , the number of failures in time  $T$  given  $\theta$ ,  $f(x|\theta)$ .
- iii) the marginal distribution of  $X$ ,  $f(x)$ .
- iv) the posterior distribution of  $\theta|X$ ,  $g(\theta|x)$ .

Under very general conditions, in particular in our case here, the first two distributions listed, the prior distribution and the conditional (sometimes called sampling) distribution uniquely determine the other two. It is mandatory to select a family\* of distributions for the prior distribution on  $\theta$  and the conditional distribution of  $X$  (the number of failures occurring in fixed time  $T$ ) given  $\theta$ . The selection of the two families is discussed in the next two sections.

#### 3.2 THE PRIOR DISTRIBUTION

A basic assumption in a Bayesian analysis is that the parameter(s) of concern is a random variable. Thus, a prior probability distribution (prior distribution for short) is required to describe the behavior of the random variable  $\theta$ : the equipment/system mean time to failure. The family selected for the prior distribution is the inverted gamma family:

$$g(\theta) = \frac{\alpha^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-(\alpha/\theta)}, \quad \alpha, \lambda, \theta > 0$$

= 0 elsewhere.

The parameters  $\alpha, \lambda$  are, respectively, scale and shape parameters. The inverted gamma distribution gets its name from the familiar gamma distribution. That is, if a random variable  $u$  has a gamma distribution, then the random variable  $u^{-1}$  is said to have an inverted gamma distribution. Thus, if we had been working with failure rate ( $1/\theta$ ) instead of  $\theta$ , the gamma distribution would have been selected. Before beginning a discussion of the reasons why the inverted gamma family was selected a minor point should be made. The  $K$ th moment of  $\theta$  is

$$E(\theta^K) = \frac{\alpha^K}{\prod_{i=1}^K (\lambda-i)} \quad \text{provided } \lambda > K.$$

---

\*Here, family indicates a set of distributions indexed by one or more parameters. For example, the family of normal distributions indexed by  $(\mu, \sigma)$ .

Thus, for  $K=1$ ,  $\lambda > 1$ , the mean is  $\alpha/(\lambda-1)$ ; for  $\lambda \leq 1$  the mean does not exist. In fact, all moments up to and including  $[\lambda]$  (the largest integer smaller than  $\lambda$ ) exist and none beyond (larger than)  $[\lambda]$  exist. This minor matter is brought up here because some readers may be inclined against distributions which do not possess moments of all orders. This fact is completely immaterial here; causing only a small problem, discussed later, in the indexing of the plans.

The inverted gamma family has been selected for just three reasons:

- i) the choice is well supported by empirical evidence.
- ii) it is quite flexible.
- iii) it is mathematically tractable.

In each of the three phases of this program, the first two phases and the present Phase III, one of the assigned tasks has been to fit prior distributions to observed equipment/system reliability. Considering the three phases together, data suitable for trying fits was obtained on thirty-eight (38) equipments and the results are summarized in Section 2 of this report. It is enough to note here that twenty-nine (29) of the cases of the inverted gamma distribution fitted the observed data. That is, what is meant by i) above.

Since the inverted gamma distribution is a two parameter distribution: one scale ( $\alpha$ ) and one shape ( $\lambda$ ), it is extremely flexible. That is, the "shapes" it takes are varied enough to graduate a wide variety of data. Actually this flexibility is probably reflected in the fact that twenty-nine (29) good fits were obtained.

Finally, the inverted gamma distribution is mathematically tractable. By this we mean that at worst the solution pairs  $(r^*, T/\theta_0)$  can be found by analytic means on a high speed computer. In the sequential case parameters (e.g.,  $P(A)$ ) were developed by simulation but the technique was inexpensive. In particular, the inverted gamma distribution is closed under Poisson sampling. Sometimes, this property of being "closed" is expressed by saying that the inverted gamma family is conjugate. Whichever word is used, it merely means that if the prior distribution is inverted gamma and the conditioned distribution is Poisson (which is the case as will be discussed in the next section) then the posterior distribution is also of the inverted gamma family. This helps the mathematical tractability a great deal.

### 3.3 THE CONDITIONAL DISTRIBUTION

In some Bayesian applications the conditional (sampling) distribution is determined by the physical process. For example, consider "lots" of a product, submitted for inspection, of finite and constant lot size. It is common nowadays to assume that the (unknown) number defective in each lot is a random variable, say  $p$ . Whatever be the prior distribution selected to graduate  $p$ , it is clear that for fixed  $p$  the sampling distribution is hypergeometric. The case in BRDT is not quite so simple. However, the exponential distribution, which implies the Poisson distribution when  $T$  is

fixed and the number of failures occurring in T is the random variable, is widely accepted at the equipment/system level. Rather than present numerous references on this matter, we point to popular Mil. Std. 781B (Ref. (3)), which is based on the exponential distribution. Thus, the conditional distribution selected for the BRDT plan development is the Poisson.

## 4.0 DEVELOPMENT, DESCRIPTION AND USE OF THE BAYES FIXED SAMPLE TESTS

### 4.1 THE DECISION CRITERIA

In general, fixed sample tests can be of two varieties: those stopped after a fixed test time  $T$  and those stopped after a fixed number of failures. Unfortunately, these latter variety involve an unknown test time so, following the lead of Mil. Std. 781B, we have adopted the fixed time tests for the Bayesian Reliability Demonstration Tests (BRDT). These fixed time tests are characterized by a pair  $(r^*, T)$  such that if less than or equal to  $r^*$  failures are observed in time  $T$  the test is passed.

For such fixed time tests, there are a number of indexing parameters. The exact form the indexing parameters take depends, of course, on the decision criteria selected. The decision criteria investigated and the rationale for the final selection are discussed in some detail in the Phase II report (Ref. (1)). We will briefly review the rationale here. The possible decision criteria were assessed against the following requirements:

- i) the decision criteria should be a unique 1-1 map of the possible criteria values and the space of all possible pairs  $(r^*, T)$ . That is, the selection of a particular value of the decision criteria, whatever be the class of the criteria, should result in a unique pair  $(r^*, T)$  and conversely.
- ii) the decision criteria should be intuitively appealing.
- iii) the decision criteria should be easily usable in practice.
- iv) the decision criteria should be tractable, at most requiring a computer, in determining the unique pair  $(r^*, T)$ .

The Classical criteria (used in Mil. Std. 781B): the producer's risk ( $\alpha$ ), the consumer's risk ( $\beta$ ), the specified MTBF ( $\theta_0$ ) and the minimum acceptable MTBF ( $\theta_1$ ) form an excellent example of decision criteria which satisfy all of the above requirements and hence, probably explains their great popularity. Now any Bayes decision criteria will require use of the prior distribution in some form or other so that, in a sense, no BRDT will be as easy to use as a Classical test. However, in looking over the available BRDT methods\*, it is clear that some are far easier to use than others. For example, any BRDT based on a loss function as a decision criteria is difficult to use because of the lack of agreement (between producer and consumer) on the type of loss function and the intractability of all but the simplest loss functions. Moreover, the use of loss functions would have entailed additional development costs since none were available in the literature. Thus, the decision criteria were reduced to those involving risks of some sort. It should be noted, at this point, that many people refuse to call

\* Only about four or five BRDT were available in the literature.

a method a Bayes method unless it involves both a prior distribution and a loss function. We do not concur in this. It is a minor point of semantics but since the fixed time BRDT do not involve loss functions the point should be noted. The decision criteria finally selected were the posterior risks; these risks were judged the most suitable in terms of the requirements i), ii), iii), and iv), previously mentioned, in this section. These were subsequently modified and the final criteria are presented and discussed below.

The posterior risks are defined as follows

$$\text{Posterior consumer's risk} = P(\theta \leq \theta_1 | \text{acceptance})$$

$$\text{Posterior producer's risk} = P(\theta \geq \theta_0 | \text{rejection})$$

where  $\theta_0 > \theta_1$ . One can find a unique test  $(r^*, T)$  by selecting two positive numbers  $\alpha^*, \beta^*$  (both less than 1/2) and finding the smallest  $T$  (and its associated  $r^*$ ) such that

$$P(\theta \leq \theta_1 | \text{acceptance}) \leq \beta^*, \quad P(\theta \geq \theta_0 | \text{rejection}) \leq \alpha^*.$$

The similarity between these posterior risks and the Classical risks is striking since in the Classical case

$$\text{Consumer's risk} = P(\text{acceptance} | \theta = \theta_1);$$

$$\text{Producer's risk} = P(\text{rejection} | \theta = \theta_0)$$

and the Classical fixed time test is developed by finding the smallest  $T$  (and its associated  $r^*$ ) such that

$$P(\text{acceptance} | \theta = \theta_1) \leq \beta, \quad P(\text{rejection} | \theta = \theta_0) \leq \alpha.$$

The "reversing" of events in the Bayes risks requires determination of the prior distribution on  $\theta$ . The indexing parameters now become evident for the BRDT. In addition to the prior distribution (itself indexed by  $\alpha, \lambda$ , the scale and shape parameters respectively), the four numbers  $\theta_0, \theta_1, \alpha^*, \beta^*$  are the parameters much as in the Classical case. The indexing parameters are discussed in the next section.

#### 4.2 THE INDEXING PARAMETERS

It is the purpose of this section to discuss the quantities finally selected as indexing parameters and the ranges of values provided for them. As will be seen shortly, some useful modifications in the indexing parameters were necessary. There is an "auxiliary" parameter, not appearing explicitly in the posterior risks, of great importance to the user of BRDT. It is the probability of passing the test and is denoted in this report by  $P(A)$ . The probability of acceptance is related to the Classical operating characteristic (O.C.) curve but differs in that for each BRDT,  $P(A)$  is a single number and not a curve:  $P(A)$  is computed simply by summing the marginal distribution

of the number of failures (occurring in fixed time T) from 0 to  $r^*$ . Put another way, it is the average probability of acceptance; the averaging being with respect to the prior distribution. It was mentioned in the Phase II report (TR-71-209) that  $P(A)$  might well be more important to the producer than posterior producer's risk. The posterior producer's risk keeps low ( $\leq \alpha^*$ ) the fraction of rejected equipments which indeed are of acceptable MTBF ( $\theta \geq \theta_0$ ). But this low posterior producer's risk can be of little comfort when, say,  $P(A) = .10$ . That is, facing the test each time, a producer has one chance in ten of passing the test. In fact, when some preliminary BRDT were computed, for reasonable values of the indexing parameters,  $P(A)$  was generally quite low; so low in many cases that a rational producer would not want to use the test plan. For this reason, the posterior producer's risk was replaced in the decision criteria pair (the two risks) with  $P(A)$ . For example, the smallest test time T (and its associated  $r^*$ ) was found such that  $P(A) \geq .80$  and the posterior consumer's risk  $\leq \beta^*$ . The values selected for the various indexing parameters are discussed next. For convenience here, the decision criteria  $(\beta^*, P(A))$  are considered indexing parameters.

#### Prior Distribution

The inverted gamma family of prior distributions has been selected to develop the BRDT. The reasons for this choice are discussed in Section 3.2. The inverted gamma family is indexed by a shape parameter,  $\lambda$ , and a scale parameter,  $\alpha$ . In the previous two study phases, twenty-nine (29) inverted gamma prior distributions were fitted on twenty-nine (29) equipments. In virtually all of these cases  $0 < \lambda < 6$ . The values of  $\lambda$  selected for the BRDT are then  $\lambda = 1/2, 1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5$ .

The steps of size 0.5 are thought to be small enough to provide enough flexibility.

Since the mean of the prior distribution is  $E(\theta) = \alpha/(\lambda-1)$ ,  $\lambda > 1$ , it is clear that for fixed  $\lambda, \alpha$  and the prior mean  $\alpha/(\lambda-1)$  are uniquely related. Moreover, since the predicted MTBF,  $\theta_p$ , will likely be assumed to be equal to the prior mean, it is thought that it would be more convenient for the user if the indexing was done on the prior mean rather than  $\alpha$ . To make the tests even more convenient to use the prior mean has been stated in units of  $\theta_0$ . The values selected are

$$\text{Prior mean} = E(\theta) = \theta_0, 1.1\theta_0, 1.2\theta_0, 1.3\theta_0, 1.4\theta_0, 1.5\theta_0, 1.6\theta_0, 1.7\theta_0, \\ 1.8\theta_0, 1.9\theta_0, 2\theta_0.$$

The above steps provide the flexibility required over the range of likely values of the prior mean. If it turns out that  $\lambda \leq 1$ , the indexing is done on the median of the prior distribution which always exists.

Criteria for  $(r^*, T)$ 

The values of the posterior consumer's risk selected are

$$\beta^* = .05, .10, .15, .20.$$

These were determined by following pretty much what is accepted statistical practice for risks and the values used in Mil. Std. 781B. For  $P(A)$  two values are used

$$P(A) = .80, .90.$$

These are reasonable selections in that they are generally acceptable to the producer and they help keep test time  $T$  low.

Values of  $(\theta_0, \theta_1)$ 

It turns out (as in the Classical case) that if the test time  $T$  is stated in units of  $\theta_0$ , say  $T^* = T/\theta_0$  then for fixed  $(\lambda, \text{prior mean}, \beta^*, P(A))$  all users with the same discrimination ratio  $d = \theta_0/\theta_1$  will have the same test, say,  $(r^*, T^*)$ . The values selected for  $d = \theta_0/\theta_1$  pretty much follow the lead of Mil. Std. 781B and common statistical practice with the exception that large  $d$  (e.g., 3.0) are not required since the prior mean always  $\geq \theta_0$ . That is, to take these tests, it is required that the prior mean  $\geq \theta_0$ . The values of  $d$  are

$$d = \theta_0/\theta_1 = 3/2, 2, 5/2.$$

Thus, a total of  $2400 = 10$  (values of  $\lambda$ )  $\times$  10 (values of prior mean)  $\times$  4 (values of  $\beta^*$ )  $\times$  2 (values of  $P(A)$ )  $\times$  3 (values of discrimination ratio) BRDT of the fixed time variety have been provided.

#### 4.3 USING THE TEST TABLES

Ordinarily, to determine the test time  $T$  and acceptance number  $r^*$ , one would select a value of  $\lambda$ , a value of the prior mean, a value of  $P(A)$ , a value of the discrimination ratio  $\theta_0/\theta_1$ , and a value of  $\beta^*$ . Using the test tables this would determine a pair  $(r^*, T^* = T/\theta_0)$ . Two points should be noted explicitly here. First, since the tests are computer printouts  $r^* = R$ ; second,  $T/T_0 = T/\theta_0$  (again, due to computer print limitations) and to obtain real test time the user must multiply  $T/T_0$  by  $\theta_0$ .

Example. Suppose a user has  $\beta^* = .20$ ,  $P(A) = .80$ ,  $\lambda = 3.0$ , prior mean =  $1.3\theta_0$ , and  $\theta_0 = 300$  hrs.,  $\theta_1 = 200$  hrs. Then the discrimination ratio is  $3/2$ . The test time, from Table 1, is  $(T/T_0)(\theta_0) = .583 (300 \text{ hrs.}) = 175 \text{ hrs.}$  and  $r^* = R = 1$ . Thus, the test is conducted for 175 hrs. and if less than two failures occur the test is passed.

There are three special situations which may occur that cause departure from the "ordinary" use of the tables. These situations and how they are handled are discussed now.

#### Special Test Situation One

It may be, for a given selection of ( $\lambda$ , prior mean,  $P(A)$ ,  $\beta^*$ , discrimination ratio) that *a priori*, that is before the test is run, the  $\beta^*$  risk is satisfied. That is, it may be that

$$P(\theta \leq \theta_1) = \int_0^{\theta_1} g(\theta) d\theta \leq \beta^* \text{ where } g(\theta) \text{ represents the inverted gamma}$$

prior distribution. This is a case in which, strictly speaking, no test is required. However, we are unwilling to permit this situation to occur. At least some small test time is almost always a good policy. Thus if, for a particular fixed combination of ( $\lambda$ , prior mean,  $P(A)$ ,  $\beta^*$ , discrimination ratio), a single asterisk \* appears in the test column, it means special test situation one exists. A single asterisk means that to obtain a test the user is to: reduce  $\beta^*$  until, for the first time, a test is obtained. If this does not occur at or before  $\beta=0.05$ , the user is to reduce (lower) the discrimination ratio by leaving  $\theta_0$  alone and increasing  $\theta_1$  in steps until, for the first time, a test is obtained. If after reaching  $\theta_0/\theta_1 = 1.5$  and  $\beta^*=0.05$  no test is obtained, the user is to take  $T=\theta_0/10$  and  $r^*=0$ . To repeat, in reducing the discrimination ratio (to obtain a test) it is required that only  $\theta_1$  be changed (increased).

Example: Suppose  $\theta_0=500$ ,  $\theta_1=200$ ,  $\beta^*=.20$ ,  $P(A)=.80$ , prior mean=2.0 $\theta_0$  and  $\lambda=1.5$ . Then the discrimination ratio is  $d=5/2$  and the test column for  $P(A)=.80$  and the row prior mean = 2.0 $\theta_0$  in Table 1 contains a single asterisk. Proceeding to  $\beta^*=.15$  a single asterisk remains. Proceeding to  $\beta^*=.10$ , we find  $T/\theta_0 = .537$  and  $r^*=1$ . Thus, the test time is  $.537(500) = 269$  hrs. with less than or equal to one failure allowed.

#### Special Test Situation Two

This test situation is indicated in the test tables by a double asterisk (\*\*). The appearance of this double asterisk indicates the existence of the following situation. The prior distribution is so "bad" relative to the values selected for  $\beta^*$ ,  $\theta_0/\theta_1$ , and  $P(A)$  that no pair  $(r^*, T/\theta_0)$  ( $R$  and  $T/T_0$  in the tables) exists. That is, no pair  $(r^*, T)$  exists that will provide the required  $\beta^*$  and  $P(A)$ . In this situation, first check to see if  $P(A) = .90$ . If so, try  $P(A) = .80$  and if a double asterisk is not present, use the test corresponding to  $P(A) = .80$ . If double asterisks appear in both P(A) columns, a fixed time BRDT cannot be used. Either a Classical test must be used or the special test tables, discussed elsewhere in this report, must be used.

### Special Test Situation Three

This situation is indicated in the test tables with a triple asterisk (\*\*\*) . The triple asterisk indicates the following situation. In some cases the prior distribution is so "good" with respect to the values of the indexing parameters selected that for both  $P(A) = .80$  and  $P(A) = .90$  both posterior consumer's risks are under  $\beta^*$  at  $r^*=0$  (and some  $T/\theta_0$ ). In fact,  $T/\theta_0$  ( $T/T_0$  in the test tables) is a decreasing function of (increasing)  $P(A)$ . In such a situation, naturally,  $P(A) = .80$  is absurd since it requires more test time than (the more desirable)  $P(A) = .90$ . In this situation we have gone ahead and found the pair  $(r^*=0, T/\theta_0)$  such that the posterior consumer's risk equals  $\beta^*$  and then given the  $P(A)$  for this test.

Example: Suppose  $\theta_0 = 60$ ,  $\theta_1 = 30$  ( $d=2.0$ ),  $\beta^* = .05$ ,  $\lambda = 1$  and prior mean =  $1.8\theta_0$ . Then, in Table 1, for  $T/\theta_0 = .250$  and  $r^*=0$ ,  $P(A) = .833$ . Larger  $T/\theta_0$  (larger than .250) will (with  $r^*=0$ ) result in lower  $P(A)$ . Note that in the  $P(A) = .90$  column the test given, in this case, gives the exact  $P(A) = .90$ . However, in the  $2.0\theta_0$  row,  $P(A) = .927$  for the test given and any larger  $T/\theta_0$  will result in  $P(A) < .927$ .

#### 4.4 TREATMENT OF CASES IN WHICH REQUIRED PARAMETER VALUES ARE NOT TABULATED IN THE TEST TABLES

This situation is expected to be a rare situation in view of the fact that in Phase I it was shown that small departures (from the true values) of the prior mean and  $\lambda$  do not materially affect results. Perhaps the most frequent situation might be that a test is desired with a value of the prior mean  $< 1.0\theta_0$ . The answer in this situation is simple: no BRDT is permitted.

If it should turn out that there is such a strong belief in a value of  $\lambda$  and/or prior mean (in excess of  $1.0\theta_0$ ) not tabulated or special permission has been obtained from the contracting agency for a BRDT with a combination of  $\beta^*$ , discrimination ratio and  $P(A)$  not tabulated, then a fixed time BRDT can be developed for this specific combination. The equations are:

$$P(A) = \sum_{x=0}^{x=r^*} \frac{\Gamma(\lambda+x)}{\Gamma(\lambda)x!} \left(\frac{T}{\alpha+T}\right)^x \left(\frac{\alpha}{\alpha+T}\right)^\lambda \quad (1)$$

and

$$\sum_{x=0}^{x=r^*} \left[ \int_0^{\theta_1} \frac{\alpha^\lambda \theta^{-(\lambda+x+1)}}{\Gamma(\lambda) x!} T^x e^{-(\alpha+T)/\theta} d\theta \right] \leq \beta^*, \quad (2)$$

$$\sum_{x=0}^{x=r^*} \frac{\Gamma(\lambda+x)}{\Gamma(\lambda)x!} \left(\frac{T}{\alpha+T}\right)^x \left(\frac{\alpha}{\alpha+T}\right)^\lambda$$

where  $T$  is real test time and  $\Gamma(u)$  is the gamma function of  $u$ . The method of finding the pair  $(r^*, T/\theta_0)$  in terms of these two equations is described below.

#### 4.5 TEST TABULATION PROCEDURE

For each combination of posterior consumer's risk  $\beta^*(.20,.15,.10,.05)$ , discrimination ratio  $(1.5, 2.0, 2.5)$ , prior distribution shape parameter  $(1/2, 1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5)$ , prior distribution mean  $(\theta_0, 1.1\theta_0, 1.2\theta_0, 1.3\theta_0, 1.4\theta_0, 1.5\theta_0, 1.6\theta_0, 1.7\theta_0, 1.8\theta_0, 1.9\theta_0, 2\theta_0)$ , and probability of acceptance  $P(A)(.8,.9)$ , the procedure is as follows:

A test is made to see if the prior probability satisfies the  $\beta^*$  risk requirement. If so, then this test is not tabulated and a special test "situation one" is indicated. If the prior probability does not satisfy the risk requirement, the test  $(r^*, T/\theta_0)$  is tabulated as follows:

For each  $r$ , starting at zero, the  $T$  which yields the required  $P(A)$  is found.

Each  $(r, T)$  combination is used to calculate a posterior consumer's risk, say,  $\hat{\beta}$ . The first time (i.e.; the smallest  $T$ )  $\hat{\beta} \leq \beta^*$ , the test time ( $T$ ) and number of failures ( $r^*$ ) is tabulated.

When the  $\hat{\beta}$  values from one  $r$  to the next first begin to differ by  $10^{-4}$  or less, a test is made to see if  $\hat{\beta}$  is within .01 of  $\beta^*$ . If so, the  $T$  and  $r^*$  are tabulated at that point. If  $(\hat{\beta} - \beta^*) > .01$  the  $\beta^*$  risk is considered unreachable with this set of conditions and special test situation two is indicated.

The other case considered is when  $r^*=0$  and  $\hat{\beta} \leq \beta^*$ . In this case the  $T$  which gives the desired  $\beta^*$  is found and the appropriate  $P(A)$  is computed and tabulated. This is special test situation three.

TABLE 1 BAYES FIXED TIME TESTS

- \* - SEE SECTION 4.3, SPECIAL TEST SITUATION ONE, FOR THIS AND ALL FOLLOWING VALUES OF PRIOR MEAN.
- \*\* - SEE SECTION 4.3, SPECIAL TEST SITUATION TWO, FOR THIS AND ALL PRECEDING VALUES OF PRIOR MEAN.
- \*\*\* - SEE SECTION 4.3, SPECIAL TEST SITUATION THREE

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5 LAMBDA=0.5

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T₀	R	T/T₀
BETA*=.20	1.10THETA ZERO	**		**	
	1.20THETA ZERO	7	2.445	**	
	1.30THETA ZERO	2	0.856	**	
	1.40THETA ZERO	1	0.542	**	
	1.50THETA ZERO	1	0.479	**	
		***P(A)=0.833			
	1.60THETA ZERO	0	0.182	**	
		***P(A)=0.816			
	1.70THETA ZERO	0	0.161	5	1.461
		***P(A)=0.840			
	1.80THETA ZERO	0	0.138	2	0.651
		***P(A)=0.865			
	1.90THETA ZERO	0	0.115	1	0.380
		***P(A)=0.889			
	2.00THETA ZERO	*			
BETA*=.15	1.40THETA ZERO	**		**	
	1.50THETA ZERO	5	2.227	**	
	1.60THETA ZERO	2	1.053	**	
	1.70THETA ZERO	1	0.658	**	
	1.80THETA ZERO	1	0.615	**	
		***P(A)=0.822			
	1.90THETA ZERO	1	0.573	**	
		***P(A)=0.843			
	2.00THETA ZERO	0	0.234	**	
		***P(A)=0.812			
BETA*=.10	1.60THETA ZERO	**		**	
	1.70THETA ZERO	11	5.344	**	
	1.80THETA ZERO	7	3.667	**	
	1.90THETA ZERO	4	2.296	**	
	2.00THETA ZERO	2	1.316	**	
BETA*=.05	2.00THETA ZERO	**		**	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5 | LAMBDA=1.0

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA*=.20	1.00THETA ZERO	8	3.539	**	
	1.10THETA ZERO	2	1.074	**	
	1.20THETA ZERO	1	0.552	**	
		***P(A)=0.841			
	1.30THETA ZERO	0	0.172	2	0.780
		***P(A)=0.840			
	1.40THETA ZERO			0	0.104
		***P(A)=0.903			
	1.50THETA ZERO	*			
BETA*=.15	1.00THETA ZERO	**		**	
	1.10THETA ZERO	12	5.784	**	
	1.20THETA ZERO	4	2.190	**	
	1.30THETA ZERO	2	1.269	**	
	1.40THETA ZERO	1	0.646	9	3.747
		***P(A)=0.840			
	1.50THETA ZERO	0	0.229	2	0.900
		***P(A)=0.819			
	1.60THETA ZERO	0	0.156	1	0.513
		***P(A)=0.876			
	1.70THETA ZERO	*			
BETA*=.10	1.20THETA ZERO	**		**	
	1.30THETA ZERO	9	5.159	**	
	1.40THETA ZERO	4	2.555	**	
	1.50THETA ZERO	2	1.464	**	
	1.60THETA ZERO	1	0.897	11	5.240
	1.70THETA ZERO	1	0.750	5	2.518
		***P(A)=0.849			
	1.80THETA ZERO	0	0.292	2	1.081
		***P(A)=0.810			
	1.90THETA ZERO	0	0.219	1	0.609
		***P(A)=0.858			
	2.00THETA ZERO			0	0.146
		***P(A)=0.905			
BETA*=.05	1.40THETA ZERO	**		**	
	1.50THETA ZERO	13	8.531	**	
	1.60THETA ZERO	7	4.974	**	
	1.70THETA ZERO	4	3.102	**	
	1.80THETA ZERO	3	2.518	**	
	1.90THETA ZERO	2	1.854	13	7.365
	2.00THETA ZERO	2	1.854	9	5.354

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5		LAMBDA=1.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA*=.20	2.00THETA ZERO	**	**
BETA*=.15	2.00THETA ZERO	**	**
BETA*=.10	2.00THETA ZERO	**	**
BETA*=.05	2.00THETA ZERO	**	**

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5

LAMBDA=2.0

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.40THETA ZERO	**		**	
	1.50THETA ZERO	9	4.534	**	
	1.60THETA ZERO	3	1.664	**	
	1.70THETA ZERO	1	0.685	11	4.646
	1.80THETA ZERO	0	0.198	2	0.849
	***P(A)=0.812				
	1.90THETA ZERO	*			
BETA**=.15	1.50THETA ZERO	**		**	
	1.60THETA ZERO	17	9.099	**	
	1.70THETA ZERO	5	2.884	**	
	1.80THETA ZERO	3	1.872	**	
	1.90THETA ZERO	1	0.766	8	3.740
	2.00THETA ZERO	1	0.583	3	1.426
	***P(A)=0.870				
BETA**=.10	1.70THETA ZERO	**		**	
	1.80THETA ZERO	12	7.237	**	
	1.90THETA ZERO	6	3.852	**	
	2.00THETA ZERO	3	2.079	**	
BETA**=.05	1.90THETA ZERO	**		**	
	2.00THETA ZERO	18	12.042	**	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5

LAMBDA=2.5

	PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
		R	T/T₀	R	T/T₀
BETA**=.20	1.20THETA ZERO	**		**	
	1.30THETA ZERO	5	2.689	**	
	1.40THETA ZERO	2	1.073	11	4.792
	***P(A)=0.836				
	1.50THETA ZERO	0	0.177	2	0.868
	***P(A)=0.827				
	1.60THETA ZERO	*			
BETA**=.15	1.30THETA ZERO	**		**	
	1.40THETA ZERO	7	4.035	**	
	1.50THETA ZERO	3	1.895	* <sup>a</sup>	
	1.60THETA ZERO	1	0.775	4	1.904
	1.70THETA ZERO	0	0.156	1	0.504
	***P(A)=0.862				
	1.80THETA ZERO	*			
BETA**=.10	1.40THETA ZERO	**		**	
	1.50THETA ZERO	11	6.779	**	
	1.60THETA ZERO	5	3.310	**	
	1.70THETA ZERO	2	1.474	14	7.469
	1.80THETA ZERO	1	0.872	4	2.141
	1.90THETA ZERO	0	0.229	1	0.563
	***P(A)=0.824				
	2.00THETA ZERO	*			
BETA**=.05	1.50THETA ZERO	**		**	
	1.60THETA ZERO	23	15.115	**	
	1.70THETA ZERO	12	8.380	**	
	1.80THETA ZERO	7	5.188	**	
	1.90THETA ZERO	4	3.161	16	9.573
	2.00THETA ZERO	3	2.526	10	6.203

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5

LAMBDA=3.0

PRIOR MEAN

P(A)=.8

P(A)=.9

R T/T<sub>0</sub>R T/T<sub>0</sub>

BETA*=.20	1.10THETA ZERO	**	**
	1.20THETA ZERO	4	2.245
	1.30THETA ZERO	1	0.583
	$***P(A)=0.844$		
	1.40THETA ZERO	*	

BETA*=.15	1.10THETA ZERO	**	**
	1.20THETA ZERO	18	10.042
	1.30THETA ZERO	4	2.431
	1.40THETA ZERO	1	0.755
	1.50THETA ZERO	0	0.146
	$***P(A)=0.867$		
	1.60THETA ZERO	*	

BETA*=.10	1.20THETA ZERO	**	**
	1.30THETA ZERO	20	12.089
	1.40THETA ZERO	6	3.904
	1.50THETA ZERO	3	2.125
	1.60THETA ZERO	1	0.863
	1.70THETA ZERO	0	0.146
	$***P(A)=0.882$		
	1.80THETA ZERO	*	

BETA*=.05	1.30THETA ZERO	**	**
	1.40THETA ZERO	25	16.286
	1.50THETA ZERO	13	9.057
	1.60THETA ZERO	7	5.201
	1.70THETA ZERO	4	3.180
	1.80THETA ZERO	2	1.746
	1.90THETA ZERO	1	1.024
	2.00THETA ZERO	0	0.208
	$***P(A)=0.859$		

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5

LAMBDA=3.5

	PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
		R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.00THETA ZERO	**		**	
	1.10THETA ZERO	5	2.772	**	
	1.20THETA ZERO	1	0.694	4	1.776
	1.30THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	**		**	
	1.10THETA ZERO	21	11.693	**	
	1.20THETA ZERO	5	3.025	**	
	1.30THETA ZERO	1	0.752	5	2.440
	1.40THETA ZERO	*			
BETA**=.10	1.10THETA ZERO	**		**	
	1.20THETA ZERO	23	13.979	**	
	1.30THETA ZERO	6	3.928	**	
	1.40THETA ZERO	2	1.463	9	4.904
	1.50THETA ZERO	1	0.604	2	1.066
	****P(A)=0.881				
	1.60THETA ZERO	*			
BETA**=.05	1.20THETA ZERO	**		**	
	1.30THETA ZERO	25	16.469	**	
	1.40THETA ZERO	12	8.477	**	
	1.50THETA ZERO	6	4.533	17	10.203
	1.60THETA ZERO	3	2.447	9	5.604
	1.70THETA ZERO	1	0.984	3	1.852
	1.80THETA ZERO	0	0.208	1	0.646
	****P(A)=0.854				
	1.90THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5			LAMBDA=4.0			
PRIOR MEAN			$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>		
BETA**=.20	1.00THETA ZERO	16	8.599	**		
	1.10THETA ZERO	2	1.212	14	6.568	
	1.20THETA ZERO	*				
BETA**=.15	1.00THETA ZERO	**		**		
	1.10THETA ZERO	9	5.294	**		
	1.20THETA ZERO	2	1.322	9	4.503	
	1.30THETA ZERO			0	0.104	
	1.40THETA ZERO	*			***P(A)=0.900	
BETA**=.10	1.10THETA ZERO	**		**		
	1.20THETA ZERO	9	5.776	**		
	1.30THETA ZERO	3	2.101	12	6.604	
	1.40THETA ZERO	1	0.583	2	1.056	
	1.50THETA ZERO	*			***P(A)=0.884	
BETA**=.05	1.20THETA ZERO	**		**		
	1.30THETA ZERO	14	9.771	**		
	1.40THETA ZERO	6	4.484	18	10.854	
	1.50THETA ZERO	3	2.424	8	4.969	
	1.60THETA ZERO	1	0.973	3	1.854	
	1.70THETA ZERO	*				

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5		LAMBDA=4.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA**=.20	1.00THETA ZERO	7	3.910 **
	1.10THETA ZERO	1	0.542 3 1.336
	***P(A)=0.860		
	1.20THETA ZERO	*	
BETA**=.15	1.00THETA ZERO	**	**
	1.10THETA ZERO	4	2.457 **
	1.20THETA ZERO	1	0.542 2 0.947
	***P(A)=0.877		
	1.30THETA ZERO	*	
BETA**=.10	1.00THETA ZERO	**	**
	1.10THETA ZERO	19	11.797 **
	1.20THETA ZERO	5	3.346 17 9.276
	1.30THETA ZERO	1	0.821 4 2.154
	1.40THETA ZERO	*	
BETA**=.05	1.10THETA ZERO	**	**
	1.20THETA ZERO	24	16.292 **
	1.30THETA ZERO	8	5.815 **
	1.40THETA ZERO	3	2.359 11 6.854
	1.50THETA ZERO	1	0.948 3 1.822
	1.60THETA ZERO	*	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=1.5		LAMBDA=5.0			
BETA*=	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T0	R	T/T0
.20	1.00THETA ZERO	4	2.310	**	
	1.10THETA ZERO	*			
.15	1.00THETA ZERO	16	9.359	**	
	1.10THETA ZERO	2	1.302	10	5.083
	1.20THETA ZERO	*			
.10	1.00THETA ZERO	**		**	
	1.10THETA ZERO	11	7.042	**	
	1.20THETA ZERO	3	2.089	9	4.958
	1.30THETA ZERO	0	0.125	1	0.531
	1.40THETA ZERO	*			
***P(A)=0.888					
.05	1.10THETA ZERO	**		**	
	1.20THETA ZERO	14	9.810	**	
	1.30THETA ZERO	5	3.753	18	11.156
	1.40THETA ZERO	2	1.657	5	3.072
	1.50THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=0.5			
PRIOR MEAN		P(A)=.8		P(A)=.9	
	R	T/T0	R	T/T0	
BETA**=.20	1.00THETA ZERO	2	0.658	**	
	1.10THETA ZERO	1	0.367	**	
	***P(A)=0.826				
	1.20THETA ZERO	0	0.137	**	
	***P(A)=0.816				
	1.30THETA ZERO	0	0.113	4	0.900
	***P(A)=0.850				
	1.40THETA ZERO	0	0.094	1	0.280
	***P(A)=0.879				
	1.50THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	**		**	
	1.10THETA ZERO	9	2.850	**	
	1.20THETA ZERO	2	0.790	**	
	1.30THETA ZERO	1	0.503	**	
	1.40THETA ZERO	1	0.438	**	
	***P(A)=0.837				
	1.50THETA ZERO	0	0.176	**	
	***P(A)=0.812				
	1.60THETA ZERO	0	0.156	3	0.842
	***P(A)=0.836				
	1.70THETA ZERO	0	0.133	1	0.340
	***P(A)=0.863				
	1.80THETA ZERO	0	0.109	1	0.340
	***P(A)=0.888				
	1.90THETA ZERO	*			
BETA**=.10	1.20THETA ZERO	**		**	
	1.30THETA ZERO	11	4.086	**	
	1.40THETA ZERO	4	1.691	**	
	1.50THETA ZERO	2	0.987	**	
	1.60THETA ZERO	2	0.987	**	
	1.70THETA ZERO	1	0.658	**	
	1.80THETA ZERO	1	0.563	**	
	***P(A)=0.837				
	1.90THETA ZERO	1	0.523	8	2.586
	***P(A)=0.857				
	2.00THETA ZERO	0	0.219	4	1.385
	***P(A)=0.822				
BETA**=.05	1.50THETA ZERO	**		**	
	1.60THETA ZERO	15	6.805	**	
	1.70THETA ZERO	7	3.463	**	
	1.80THETA ZERO	5	2.672	**	
	1.90THETA ZERO	3	1.771	**	
	2.00THETA ZERO	2	1.316	**	

TABLE 1. BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=1.0

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T0	R	T/T0
BETA**=.20	1.00THETA ZERO	0	0.109	1	0.320
		$***P(A)=0.864$			
	1.10THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	1	0.561	**	
	1.10THETA ZERO	0	0.188	3	0.979
		$***P(A)=0.803$			
	1.20THETA ZERO	0	0.117	1	0.385
		$***P(A)=0.876$			
	1.30THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	6	2.682	**	
	1.10THETA ZERO	3	1.539	**	
	1.20THETA ZERO	1	0.673	11	3.930
	1.30THETA ZERO	1	0.547	4	1.541
		$***P(A)=0.857$			
	1.40THETA ZERO	0	0.180	1	0.449
		$***P(A)=0.844$			
	1.50THETA ZERO			0	0.109
		$***P(A)=0.905$			
	1.60THETA ZERO	*			
BETA**=.05	1.00THETA ZERO	**		**	
	1.10THETA ZERO	18	8.625	**	
	1.20THETA ZERO	7	3.730	**	
	1.30THETA ZERO	4	2.373	**	
	1.40THETA ZERO	2	1.366	**	
	1.50THETA ZERO	2	1.366	9	4.016
	1.60THETA ZERO	1	0.897	4	1.896
	1.70THETA ZERO	1	0.656	2	1.021
		$***P(A)=0.872$			
	1.80THETA ZERO	0	0.250	1	0.577
		$***P(A)=0.833$			
	1.90THETA ZERO	0	0.188	1	0.577
		$***P(A)=0.875$			
	2.00THETA ZERO			0	0.109
		$***P(A)=0.927$			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=1.5			
		PRIOR MEAN		P(A)=.8	P(A)=.9
		R	T/T0	R	T/T0
BETA**=.20	1.50THETA ZERO		**		**
	1.60THETA ZERO	13	4.520	**	
	1.70THETA ZERO	5	1.885	**	
	1.80THETA ZERO	2	0.852	**	
	1.90THETA ZERO	1	0.510	9	2.660
	2.00THETA ZERO	1	0.391	3	0.912
	***P(A)=0.867				
BETA**=.15	1.70THETA ZERO		**		**
	1.80THETA ZERO	12	4.699	**	
	1.90THETA ZERO	5	2.106	**	
	2.00THETA ZERO	3	1.366	**	
BETA**=.10	1.90THETA ZERO		**		**
	2.00THETA ZERO	15	6.512	**	
BETA**=.05	2.00THETA ZERO		**		**

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=2.0			
	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.00THETA ZERO	**		**	
	1.10THETA ZERO	15	5.523	**	
	1.20THETA ZERO	3	1.248	**	
	1.30THETA ZERO	1	0.445	6	1.900
	***P(A)=0.838				
	1.40THETA ZERO	0	0.098	1	0.341
	***P(A)=0.874				
	1.50THETA ZERO	*			
BETA**=.15	1.10THETA ZERO	**		**	
	1.20THETA ZERO	17	6.824	**	
	1.30THETA ZERO	4	1.777	**	
	1.40THETA ZERO	2	1.004	12	4.184
	1.50THETA ZERO	1	0.438	3	1.069
	***P(A)=0.870				
	1.60THETA ZERO	*			
BETA**=.10	1.20THETA ZERO	**		**	
	1.30THETA ZERO	20	8.695	**	
	1.40THETA ZERO	7	3.301	**	
	1.50THETA ZERO	3	1.560	**	
	1.60THETA ZERO	2	1.147	9	3.559
	1.70THETA ZERO	1	0.531	3	1.211
	***P(A)=0.857				
	1.80THETA ZERO	0	0.141	1	0.438
	***P(A)=0.860				
	1.90THETA ZERO	*			
BETA**=.05	1.40THETA ZERO	**		**	
	1.50THETA ZERO	18	9.031	**	
	1.60THETA ZERO	9	4.836	**	
	1.70THETA ZERO	5	2.884	**	
	1.80THETA ZERO	3	1.872	17	7.688
	1.90THETA ZERO	2	1.362	7	3.258
	2.00THETA ZERO	1	0.806	4	1.920

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=2.5		
	PRIOR MEAN	$P(A)=.8$		$P(A)=.9$
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.00THETA ZERO	3	1.263	**
	1.10THETA ZERO	1	0.406	3
		$***P(A)=0.862$		
	1.20THETA ZERO	*		
BETA**=.15	1.00THETA ZERO	18	7.391	**
	1.10THETA ZERO	3	1.389	**
	1.20THETA ZERO	1	0.581	4
	1.30THETA ZERO	*		1.428
BETA**=.10	1.00THETA ZERO	**		**
	1.10THETA ZERO	16	7.227	**
	1.20THETA ZERO	5	2.482	**
	1.30THETA ZERO	2	1.127	9
	1.40THETA ZERO	1	0.469	2
		$***P(A)=0.880$		
	1.50THETA ZERO	*		
BETA**=.05	1.10THETA ZERO	**		**
	1.20THETA ZERO	23	11.336	**
	1.30THETA ZERO	10	5.342	**
	1.40THETA ZERO	5	2.896	**
	1.50THETA ZERO	3	1.895	10
	1.60THETA ZERO	1	0.775	4
	1.70THETA ZERO	0	0.219	1
		$***P(A)=0.814$		
	1.80THETA ZERO	*		

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0

LAMBDA=3.0

		PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
			R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.00THETA ZERO		0	0.141	2	0.655
	1.10THETA ZERO		*			
	1.20THETA ZERO		*			
BETA**=.15	1.00THETA ZERO		3	1.417	**	
	1.10THETA ZERO		0	0.164	2	0.720
	1.20THETA ZERO		*			
BETA**=.10	1.00THETA ZERO	13	6.037	**		
	1.10THETA ZERO	3	1.558	15	5.965	
	1.20THETA ZERO	1	0.647	3	1.199	
	1.30THETA ZERO	*				
BETA**=.05	1.00THETA ZERO	**		**		
	1.10THETA ZERO	18	9.203	**		
	1.20THETA ZERO	7	3.900	**		
	1.30THETA ZERO	3	1.842	11	5.109	
	1.40THETA ZERO	1	0.755	4	1.896	
	1.50THETA ZERO	0	0.156	1	0.499	
	1.60THETA ZERO	*				

\*\*\*P(A)=0.816

\*\*\*P(A)=0.806

\*\*\*P(A)=0.859

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=3.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA*=.20	1.00THETA ZERO	*	
BETA*=.15	1.00THETA ZERO	1      0.438	3      1.089
		***P(A)=0.865	
	1.10THETA ZERO	*	
BETA*=.10	1.00THETA ZERO	4      2.022	**
	1.10THETA ZERO	1      0.636	3      1.197
	1.20THETA ZERO	*	
BETA*=.05	1.00THETA ZERO	22     11.141	**
	1.10THETA ZERO	7      3.877	**
	1.20THETA ZERO	3      1.835	9      4.203
	1.30THETA ZERO	1      0.752	2      0.924
	1.40THETA ZERO	*	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=4.0	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA**=.20	1.00THETA ZERO	*	
BETA**=.15	1.00THETA ZERO	*	
BETA**=.10	1.00THETA ZERO	2	1.102
	1.10THETA ZERO	*	6
			2.438
BETA**=.05	1.00THETA ZERO	11	5.893
	1.10THETA ZERO	4	2.354
	1.20THETA ZERO	1	0.730
	1.30THETA ZERO	*	3
			5.590
			1.391

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=4.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA**=.20	1.00THETA ZERO	*	
BETA**=.15	1.00THETA ZERO	*	
BETA**=.10	1.00THETA ZERO	0 0.172 2 0.789 ***P(A)=0.806	
	1.10THETA ZERO	*	
BETA**=.05	1.00THETA ZERO	6 3.348 19 8.676	
	1.10THETA ZERO	2 1.262 5 2.318	
	1.20THETA ZERO	*	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.0		LAMBDA=5.0	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T0	R
BETA*=.20	1.00THETA ZERO	*	
BETA*=.15	1.00THETA ZERO	*	
BETA*=.10	1.00THETA ZERO	*	
BETA*=.05	1.00THETA ZERO	4	2.311
	1.10THETA ZERO	0	0.188
	1.20THETA ZERO	*	
		***P(A)=0.812	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=.5

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T0	R	T/T0
BETA**=.20	1.00THETA ZERO	0	0.100	5	0.859
		***P(A)=0.833			
	1.10THETA ZERO	0	0.078	1	0.220
BETA**=.15	1.20THETA ZERO	*			
	1.00THETA ZERO	2	0.659	**	
	1.10THETA ZERO	1	0.356	**	
		***P(A)=0.831			
	1.20THETA ZERO	0	0.141	**	
BETA**=.10		***P(A)=0.812			
	1.30THETA ZERO	0	0.119	3	0.684
		***P(A)=0.845			
	1.40THETA ZERO	0	0.097	1	0.280
		***P(A)=0.876			
BETA**=.05	1.50THETA ZERO	*			
	1.00THETA ZERO	**		**	
	1.10THETA ZERO	5	1.633	**	
	1.20THETA ZERO	2	0.790	**	
	1.30THETA ZERO	1	0.503	**	
	1.40THETA ZERO	1	0.462	**	
		***P(A)=0.828			
	1.50THETA ZERO	1	0.425	8	2.044
		***P(A)=0.852			
	1.60THETA ZERO	0	0.175	4	1.108
		***P(A)=0.822			
	1.70THETA ZERO	0	0.156	2	0.614
		***P(A)=0.844			
	1.80THETA ZERO	0	0.131	1	0.360
		***P(A)=0.870			
	1.90THETA ZERO			0	0.106
		***P(A)=0.900			
	2.00THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=1.0			
PRIOR MEAN		P(A)=.8	P(A)=.9		
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>	
BETA**=.20	1.00THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	1	0.561	7	2.078
	1.10THETA ZERO	0	0.162	2	0.660
		***P(A)=0.824			
	1.20THETA ZERO	*			
BETA**=.05	1.00THETA ZERO	5	2.253	**	
	1.10THETA ZERO	3	1.539	**	
	1.20THETA ZERO	2	1.171	9	3.212
	1.30THETA ZERO	1	0.729	4	1.541
	1.40THETA ZERO	0	0.225	2	0.841
		***P(A)=0.812			
	1.50THETA ZERO	0	0.162	1	0.480
		***P(A)=0.865			
	1.60THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=1.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA*=.20	1.20THETA ZERO	**	**
	1.30THETA ZERO	13	3.673 **
	1.40THETA ZERO	3	0.957 **
	1.50THETA ZERO	1	0.402 **
	1.60THETA ZERO	1	0.313 3 0.730
	***P(A)=0.867		
	1.70THETA ZERO	0	0.078 1 0.270
	***P(A)=0.876		
	1.80THETA ZERO	*	
BETA*=.15	1.40THETA ZERO	**	**
	1.50THETA ZERO	6	1.984 **
	1.60THETA ZERO	3	1.093 **
	1.70THETA ZERO	1	0.456 11 2.922
	1.80THETA ZERO	1	0.369 4 1.100
	***P(A)=0.858		
	1.90THETA ZERO	0	0.112 1 0.302
	***P(A)=0.845		
	2.00THETA ZERO	*	
BETA*=.10	1.50THETA ZERO	**	**
	1.60THETA ZERO	15	5.209 **
	1.70THETA ZERO	6	2.248 **
	1.80THETA ZERO	3	1.230 **
	1.90THETA ZERO	2	0.899 13 3.872
	2.00THETA ZERO	1	0.537 7 2.166
BETA*=.05	1.70THETA ZERO	**	**
	1.80THETA ZERO	21	8.184 **
	1.90THETA ZERO	11	4.550 **
	2.00THETA ZERO	7	3.073 **

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=2.0			
	PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
		R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
BETA**=.20	1.00THETA ZERO	2	0.637	**	
		$***P(A)=0.833$			
	1.10THETA ZERO	0	0.100	1	0.268
		$***P(A)=0.840$			
	1.20THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	7	2.358	**	
	1.10THETA ZERO	2	0.789	**	
	1.20THETA ZERO	1	0.350	3	0.855
		$***P(A)=0.870$			
	1.30THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	**		**	
	1.10THETA ZERO	9	3.325	**	
	1.20THETA ZERO	3	1.248	**	
	1.30THETA ZERO	1	0.523	6	1.900
	1.40THETA ZERO	0	0.156	2	0.660
		$***P(A)=0.809$			
	1.50THETA ZERO	*			
BETA**=.05	1.10THETA ZERO	**		**	
	1.20THETA ZERO	18	7.225	**	
	1.30THETA ZERO	7	3.066	**	
	1.40THETA ZERO	4	1.914	16	5.619
	1.50THETA ZERO	2	1.075	9	3.334
	1.60THETA ZERO	1	0.645	4	1.536
	1.70THETA ZERO	0	0.200	2	0.802
		$***P(A)=0.801$			
	1.80THETA ZERO			0	0.100
		$***P(A)=0.900$			
	1.90THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=2.5	
PRIOR MEAN		P(A)=.8	P(A)=.9
	R	T/T <sub>0</sub>	R
BETA**=.20	1.00THETA ZERO	*	
BETA**=.15	1.00THETA ZERO	0	0.125
	1.10THETA ZERO	***P(A)=0.819	
BETA**=.10	1.00THETA ZERO	3	1.262
	1.10THETA ZERO	1	0.533
	1.20THETA ZERO	*	0.968
BETA**=.05	1.00THETA ZERO	15	6.159
	1.10THETA ZERO	6	2.722
	1.20THETA ZERO	3	1.516
	1.30THETA ZERO	1	0.630
	1.40THETA ZERO	0	0.112
	1.50THETA ZERO	*	0.415
		***P(A)=0.878	

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=3.0			
PRIOR MEAN		P(A)=.8	P(A)=.9		
R	T/T0	R	T/T0		
BETA**=.20	1.00THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	0	0.125	1	0.332
		***P(A)=0.834			
	1.10THETA ZERO	*			
BETA**=.05	1.00THETA ZERO	4	1.870	17	6.172
	1.10THETA ZERO	2	1.067	5	1.887
	1.20THETA ZERO	0	0.125	1	0.399
		***P(A)=0.859			
	1.30THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=3.5	
PRIOR MEAN		P(A)=.8 R T/T0	P(A)=.9 R T/T0
BETA*=.20	1.00THETA ZERO	*	
BETA*=.15	1.00THETA ZERO	*	
BETA*=.10	1.00THETA ZERO	*	
BETA*=.05	1.00THETA ZERO 1.10THETA ZERO	2 *	1.045 5 1.878

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=4.0

	PRIOR MEAN	P(A)=.8		P(A)=.9	
		R	T/T0	R	T/T0
BETA*=.20	1.00THETA ZERO	*			
BETA*=.15	1.00THETA ZERO	*			
BETA*=.10	1.00THETA ZERO	*			
BETA*=.05	1.00THETA ZERO	0	0.100	1	0.379
	1.10THETA ZERO	*			

\*\*\*P(A)=0.877

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=4.5			
PRIOR MEAN		$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>	
BETA**=.20	1.00THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	*			
BETA**=.05	1.00THETA ZERO	*			

TABLE 1 BAYES FIXED TIME TESTS

DISCRIMINATION RATIO=2.5		LAMBDA=.0			
PRIOR MEAN		P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>	
BETA**=.20	1.00THETA ZERO	*			
BETA**=.15	1.00THETA ZERO	*			
BETA**=.10	1.00THETA ZERO	*			
BETA**=.05	1.00THETA ZERO	*			

## 5.0 BAYES/CLASSICAL TESTS: DEVELOPMENT, DESCRIPTION AND USE

### 5.1 INTRODUCTION: THE AGREEMENT PROBLEM

There is overwhelming evidence that the central problem in applying the BRDT is the failure of the producer and consumer to agree on a suitable prior distribution. For example, in a survey conducted during the Phase II study this "agreement" problem was noted by many respondents. Here, we will address the most commonly occurring "lack of agreement" situation: the producer has fitted a prior distribution and either because of the "quality" or paucity of the data the consumer is unwilling to accept the use of that prior distribution. The rarely occurring situation in which the producer and consumer each have different prior distributions in mind is not considered here for obvious reasons. One of the obvious reasons is that the equipment/system under discussion is manufactured by the producer and generally he will be the only one in a position to determine the prior distribution.

A question arises in such a situation, as outlined above: must the producer and consumer resort to Classical test methods, e.g., Mil. Std. 781B? The answer, we have determined, is 'no'. Before giving the selected solution to this problem we review, very briefly, Classical fixed time test procedures. The set of four parameters  $(\theta_0, \alpha, \theta_1, \beta)$ , which are respectively, the specified MTBF, the producer's risk, the minimum acceptable MTBF, and the consumer's risk, uniquely determine the test. In various applications one may find the consumer specifying all four numbers or the producer specifying all four numbers or the consumer specifying  $(\theta_0, \alpha)$ . Speaking statistically, it is this latter situation that seems most fair. That is, the consumer "gets to choose" only  $(\theta_1, \beta)$ . In statistical terms, in specifying  $(\theta_1, \beta)$  the consumer is telling the producer that as a result of passing the test  $(r^*, T)$  the lower confidence limit on the true (but unknown)  $\theta$  shall be at least  $\theta_1$  with at least  $1-\beta$  confidence. The producer is now faced with a problem and, fortunately, a solution. The producer knows there are many pairs  $(r^*, T)$  which will satisfy the consumer's requirement; this is the problem. However, the producer also knows what  $\theta$  his "process" can deliver. That is, what value he can produce. He can select this value, say  $\theta_0$ , an accompanying small risk of failing the test at  $\theta_0$ , say  $\alpha$ , and the test is determined uniquely.

Now, the solution to the agreement problem, called Bayes/Classical (hereafter, B/C), tests is simply that the consumer select values he desires for  $\theta_1$  and  $\beta$  in the usual Classical sense and the producer, using his prior distribution, selects a value of  $P(A)$  which is suitable to him and a pair  $(r^*, T)$  is uniquely determined. Thus, the consumer gets the protection he requires in the Classical sense and the producer gets the  $P(A)$  he desires.

### 5.2 THE INDEXING PARAMETERS FOR THE B/C TESTS

The prior inverted gamma distribution is required so  $\alpha, \lambda$  are needed. However, we will again index on the prior mean (prior median when  $\lambda \leq 1$ )

instead of  $\alpha$ . The Classical  $\beta$  and  $\theta_1$  are required also. Actually, for a particular test instance, it would not be necessary to specify a value of  $\theta_0$  since neither  $P(A)$  nor  $(\theta_1, \beta)$  require it. However, for simplicity and ease of indexing the test plans, it again turns out that all users with the same discrimination ratio ( $d = \theta_0/\theta_1$ ) and identical values for the other parameters will have the same test. So, in keeping with Mil. Std. 781B, test time  $T$  will be stated in units of  $\theta_0$ . Thus, the test plan  $T/T_0$  must be multiplied by  $\theta_0$ . Because these B/C tests differ in application from the fixed time BRDT previously discussed, the ranges of the values of the indexing parameters require discussion again.

- i) the values of  $\beta$  provided are the same as in the BRDT, i.e., .05,.10,.15,.20.
- ii) the values of the discrimination ratio  $d = \theta_0/\theta_1$  are the same as in the BRDT, i.e.,  $3/2, 2, 5/2$  except  $d=3$  has been added. The value  $d=3$  was not required in the BRDT because a "single asterisk" (special test situation one) occurred so frequently at  $d=3$ .
- iii) the values of  $\lambda$  provided remain the same as in the BRDT:  $1/2, 1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5$ .
- iv) the case of the largest difference is in the values of the prior mean tabulated. For the BRDT the graduation was  $\theta_0(.1)2\theta_0$  because special test situations occurred for other values and thus tests were not required. However, for the B/C tests additional (higher) values had to be provided to have useful tests. Hence, the prior mean is tabulated:  $\theta_0(.5)4\theta_0$ .

### 5.3 USE OF THE B/C TEST TABLES (TABLE 2)

Again, we point out immediately that for prior mean  $< \theta_0$  these B/C test tables cannot be used. Also, there is no special test situation one: one is never too good for these tests. There is a special test situation two (denoted by a double asterisk as before) which still means: these (B/C) tables cannot be used. Special test situation three (denoted by a triple asterisk) is as before.

Example 1. Suppose  $\theta_0=60$ ,  $\theta_1=20$  ( $d=3.0$ ),  $\beta=.20$ ,  $\lambda=2$ , and prior mean =  $\theta_0$ . Then a double asterisk appears in both columns and no test is permitted.

Example 2. In the Example 1 above, suppose  $\lambda=2.5$ . Then  $T/\theta_0 = 8.625$  and the test time is 518 hrs. with 21 allowed failures for  $P(A) = .80$ . No test is permitted for  $P(A) = .90$ .

Example 3. In Example 1, suppose prior mean =  $3\theta_0$  and  $P(A)$  is desired to be  $.90$ . Then  $T/\theta_0 = 1.839$  so that test time is  $1.839(60) = 111$  hrs. and the number of failures allowed for a pass is 3.

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

\*\* - SEE SECTION 4.3, SPECIAL TEST SITUATION TWO, FOR THIS AND ALL PRECEDING VALUES OF PRIOR MEAN.

\*\*\* - SEE SECTION 4.3, SPECIAL TEST SITUATION THREE.

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/TB	R	T/TB
<b>BETA=.20</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	16	13.583	**	
3.50THETA ZERO	6	6.161	**	
4.00THETA ZERO	3	3.729	**	
<b>BETA=.15</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	23	19.396	**	
3.50THETA ZERO	8	8.094	**	
4.00THETA ZERO	5	5.667	**	
***P(A)=.816				
<b>BETA=.10</b>				
3.00THETA ZERO	**		**	
3.50THETA ZERO	12	11.969	**	
4.00THETA ZERO	6	7.042	**	
<b>BETA=.05</b>				
3.00THETA ZERO	**		**	
3.50THETA ZERO	19	18.758	**	
4.00THETA ZERO	10	11.469	**	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=1.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	13	11.375	**	
2.50THETA ZERO	4	4.563	**	
3.00THETA ZERO	2	2.928	12	10.729
3.50THETA ZERO	2	2.854	6	6.052
	***P(A)=0.842		***P(A)=0.906	
4.00THETA ZERO	1	2.000	4	4.479
	***P(A)=0.824		***P(A)=0.910	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	18	15.677	**	
2.50THETA ZERO	6	6.469	**	
	***P(A)=0.810			
3.00THETA ZERO	3	4.010	15	13.438
	***P(A)=0.812			
3.50THETA ZERO	2	3.146	7	7.271
	***P(A)=0.820			
4.00THETA ZERO	2	3.146	5	5.667
	***P(A)=0.850		***P(A)=0.908	
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	26	22.563	**	
2.50THETA ZERO	8	8.849	**	
3.00THETA ZERO	4	5.474	21	18.844
3.50THETA ZERO	3	4.458	10	10.417
	***P(A)=0.824			
4.00THETA ZERO	2	3.542	6	7.115
	***P(A)=0.823			
<b>BETA=.05</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	12	13.146	**	
3.00THETA ZERO	6	8.042	**	
3.50THETA ZERO	4	6.385	14	14.625
4.00THETA ZERO	3	5.167	9	10.708
	***P(A)=0.821			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=1.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
3.50THETA ZERO		**		**
4.00THETA ZERO	14	12.161	**	
<b>BETA=.15</b>				
3.50THETA ZERO		**		**
4.00THETA ZERO	19	16.469	**	
<b>BETA=.10</b>				
3.50THETA ZERO		**		**
4.00THETA ZERO	26	22.498	**	
<b>BETA=.05</b>				
4.00THETA ZERO		**		**

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5 LAMBDA=2.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/TB	R	T/TB
<b>BETA=.20</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	18	15.052	**	
3.00THETA ZERO	6	6.081	**	
3.50THETA ZERO	4	4.479	13	11.354
	$***P(A) = 0.822$			
4.00THETA ZERO	2	2.867	7	6.859
<b>BETA=.15</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	25	20.891	**	
3.00THETA ZERO	8	8.070	**	
3.50THETA ZERO	5	5.667	17	14.948
	$***P(A) = 0.816$			
4.00THETA ZERO	3	4.010	9	8.896
	$***P(A) = 0.812$			
<b>BETA=.10</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	12	12.063	**	
3.50THETA ZERO	6	7.094	23	20.333
4.00THETA ZERO	4	5.469	12	11.958
<b>BETA=.05</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	17	17.063	**	
3.50THETA ZERO	9	10.578	**	
4.00THETA ZERO	6	8.109	17	17.083

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=2.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	21	17.250	**	
2.50THETA ZERO	6	6.052	28	22.313
	$***P(A)=0.809$			
3.00THETA ZERO	3	3.677	10	9.104
	$***P(A)=0.810$		$***P(A)=0.906$	
3.50THETA ZERO	2	2.854	6	6.052
	$***P(A)=0.818$		$***P(A)=0.913$	
4.00THETA ZERO	2	2.854	4	4.479
	$***P(A)=0.855$		$***P(A)=0.913$	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	29	23.833	**	
2.50THETA ZERO	8	8.042	35	28.099
	$***P(A)=0.809$			
3.00THETA ZERO	4	4.844	12	11.248
	$***P(A)=0.811$			
3.50THETA ZERO	3	4.010	7	7.271
	$***P(A)=0.832$		$***P(A)=0.908$	
4.00THETA ZERO	2	3.146	5	5.667
	$***P(A)=0.829$		$***P(A)=0.914$	
<b>BETA=.10</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	11	11.297	**	
3.00THETA ZERO	5	6.206	16	15.115
3.50THETA ZERO	4	5.333	9	9.458
	$***P(A)=0.831$		$***P(A)=0.908$	
4.00THETA ZERO	3	4.458	6	7.021
	$***P(A)=0.840$		$***P(A)=0.909$	
<b>BETA=.05</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	15	15.481	**	
3.00THETA ZERO	8	9.870	23	21.917
3.50THETA ZERO	5	7.240	12	13.115
4.00THETA ZERO	4	6.083	8	9.833
	$***P(A)=0.831$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5 LAMBDA=3.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	9	8.354	**	
2.50THETA ZERO	4	4.479	12	10.604
		***P(A)=.816		***P(A)=.904
3.00THETA ZERO	2	2.910	6	6.052
		***P(A)=.850		***P(A)=.908
3.50THETA ZERO	2	2.854	4	4.479
		***P(A)=.850		***P(A)=.913
4.00THETA ZERO	1	2.000	3	3.677
		***P(A)=.819		***P(A)=.917
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	13	12.073	**	
2.50THETA ZERO	5	5.667	15	13.557
		***P(A)=.811		
3.00THETA ZERO	3	4.010	7	7.344
		***P(A)=.820		
3.50THETA ZERO	2	3.146	5	5.667
		***P(A)=.823		***P(A)=.914
4.00THETA ZERO	2	3.146	4	4.844
		***P(A)=.860		***P(A)=.924
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	17	15.802	**	
2.50THETA ZERO	7	7.854	19	17.302
		***P(A)=.814		
3.00THETA ZERO	4	5.333	9	9.563
		***P(A)=.819		
3.50THETA ZERO	3	4.458	6	7.021
		***P(A)=.836		***P(A)=.909
4.00THETA ZERO	2	3.542	4	5.417
		***P(A)=.828		

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=3.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	23	21.401	**	
2.50THETA ZERO	10	11.604	26	23.854
3.00THETA ZERO	6	7.917	13	14.031
	***P(A)=0.822			
3.50THETA ZERO	4	6.083	8	9.859
	***P(A)=0.827			
4.00THETA ZERO	3	5.167	6	7.917
	***P(A)=0.831		***P(A)=0.913	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5 LAMBDA=3.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	7	6.823	24	19.406
	$***P(A)=.814$			
2.50THETA ZERO	3	3.677	8	7.583
	$***P(A)=.814$		$***P(A)=.906$	
3.00THETA ZERO	2	2.854	5	5.271
	$***P(A)=.829$		$***P(A)=.916$	
3.50THETA ZERO	1	2.025	3	3.677
	$***P(A)=.908$			
4.00THETA ZERO	1	2.000	3	3.677
	$***P(A)=.836$		$***P(A)=.933$	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	8	8.057	32	26.042
2.50THETA ZERO	4	4.844	10	9.781
	$***P(A)=.817$			
3.00THETA ZERO	3	4.010	6	6.469
	$***P(A)=.845$		$***P(A)=.914$	
3.50THETA ZERO	2	3.146	4	4.844
	$***P(A)=.845$		$***P(A)=.915$	
4.00THETA ZERO	1	2.315	3	4.010
	$***P(A)=.917$			
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	11	11.094	**	
2.50THETA ZERO	5	6.302	13	12.870
3.00THETA ZERO	3	4.589	7	8.057
3.50THETA ZERO	2	3.656	5	6.188
	$***P(A)=.915$			
4.00THETA ZERO	2	3.542	4	5.333
	$***P(A)=.849$		$***P(A)=.923$	
<b>BETA=.05</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	17	17.188	**	
2.50THETA ZERO	7	8.813	17	17.000
3.00THETA ZERO	5	7.000	9	10.505
	$***P(A)=.831$			
3.50THETA ZERO	3	5.354	6	7.979
4.00THETA ZERO	3	5.167	5	7.000
	$***P(A)=.856$		$***P(A)=.917$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=1.5      LAMBDA=4.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/TB	R	T/TB
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	24	19.411	**	
2.00THETA ZERO	5	5.339	15	12.833
2.50THETA ZERO	3	3.677	6	6.094
	$***P(A)=0.833$			
3.00THETA ZERO	2	2.854	4	4.479
	$***P(A)=0.845$		$***P(A)=0.913$	
3.50THETA ZERO	1	2.000	3	3.677
	$***P(A)=0.817$		$***P(A)=0.920$	
4.00THETA ZERO	1	2.000	2	2.854
	$***P(A)=0.848$		$***P(A)=0.911$	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	32	25.927	**	
2.00THETA ZERO	7	7.271	20	17.302
	$***P(A)=0.813$			
2.50THETA ZERO	3	4.039	8	8.042
	$***P(A)=0.909$			
3.00THETA ZERO	2	3.146	5	5.667
	$***P(A)=0.816$		$***P(A)=0.915$	
3.50THETA ZERO	2	3.146	3	4.055
	$***P(A)=0.860$			
4.00THETA ZERO	1	2.250	3	4.010
	$***P(A)=0.820$		$***P(A)=0.929$	
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	9	9.628	26	22.667
2.50THETA ZERO	4	5.349	10	10.490
3.00THETA ZERO	3	4.458	6	7.021
	$***P(A)=0.830$		$***P(A)=0.911$	
3.50THETA ZERO	2	3.542	4	5.333
	$***P(A)=0.827$		$***P(A)=0.908$	
4.00THETA ZERO	2	3.542	3	4.458
	$***P(A)=0.864$		$***P(A)=0.909$	
<b>BETA=.05</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	13	13.948	**	
2.50THETA ZERO	6	8.005	13	13.813
3.00THETA ZERO	4	6.419	8	9.938
3.50THETA ZERO	3	5.167	5	7.016
	$***P(A)=0.832$			
4.00THETA ZERO	2	4.489	4	6.313

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=1.5      LAMBDA=4.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	17	14.375	**	
2.00THETA ZERO	5	5.271	12	10.734
	$***P(A)=0.824$			
2.50THETA ZERO	2	2.867	5	5.271
3.00THETA ZERO	2	2.854	4	4.479
	$***P(A)=0.856$		$***P(A)=0.924$	
3.50THETA ZERO	1	2.000	3	3.677
	$***P(A)=0.826$		$***P(A)=0.929$	
4.00THETA ZERO	1	2.000	2	2.854
	$***P(A)=0.857$		$***P(A)=0.919$	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	22	18.651	**	
2.00THETA ZERO	6	6.469	15	13.563
	$***P(A)=0.816$			
2.50THETA ZERO	3	4.010	7	7.271
	$***P(A)=0.818$		$***P(A)=0.912$	
3.00THETA ZERO	2	3.146	4	4.969
	$***P(A)=0.829$			
3.50THETA ZERO	2	3.146	3	4.010
	$***P(A)=0.871$		$***P(A)=0.913$	
4.00THETA ZERO	1	2.250	2	3.156
	$***P(A)=0.830$			
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	30	25.500	**	
2.00THETA ZERO	8	8.667	19	17.354
	$***P(A)=0.815$			
2.50THETA ZERO	4	5.333	8	8.729
	$***P(A)=0.819$			
3.00THETA ZERO	3	4.458	5	6.323
	$***P(A)=0.844$			
3.50THETA ZERO	2	3.542	4	5.333
	$***P(A)=0.839$		$***P(A)=0.920$	
4.00THETA ZERO	2	3.542	3	4.458
	$***P(A)=0.875$		$***P(A)=0.918$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=1.5      LAMBDA=4.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/Tθ	R	T/Tθ
<b>BETA=.05</b>				
1.5θTHETA ZERO	**		**	
2.0θTHETA ZERO	11	12.344	26	24.000
2.5θTHETA ZERO	6	7.917	11	12.240
	*** $P(A)=0.825$			
3.0θTHETA ZERO	4	6.083	7	9.083
	*** $P(A)=0.838$			
3.5θTHETA ZERO	3	5.167	5	7.380
	*** $P(A)=0.846$			
4.0θTHETA ZERO	2	4.208	4	6.083
	*** $P(A)=0.828$		*** $P(A)=0.920$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=1.5      LAMBDA=5.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	13	11.375	**	
2.00THETA ZERO	4	4.479	10	9.104
		***P(A)=0.813		***P(A)=0.905
2.50THETA ZERO	2	2.854	5	5.271
		***P(A)=0.812		***P(A)=0.911
3.00THETA ZERO	2	2.854	3	3.677
		***P(A)=0.865		***P(A)=0.906
3.50THETA ZERO	1	2.000	2	2.859
		***P(A)=0.833		
4.00THETA ZERO	1	2.000	2	2.854
		***P(A)=0.863		***P(A)=0.925
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	17	14.927	**	
2.00THETA ZERO	5	5.773	12	11.214
2.50THETA ZERO	3	4.010	6	6.469
		***P(A)=0.830		***P(A)=0.909
3.00THETA ZERO	2	3.146	4	4.844
		***P(A)=0.838		***P(A)=0.916
3.50THETA ZERO	1	2.276	3	4.010
				***P(A)=0.921
4.00THETA ZERO	1	2.250	2	3.146
		***P(A)=0.837		***P(A)=0.908
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	24	21.156	**	
2.00THETA ZERO	7	7.854	16	15.177
		***P(A)=0.815		
2.50THETA ZERO	4	5.333	7	7.891
		***P(A)=0.832		
3.00THETA ZERO	2	3.551	5	6.188
				***P(A)=0.916
3.50THETA ZERO	2	3.542	4	5.333
		***P(A)=0.849		***P(A)=0.928
4.00THETA ZERO	1	2.602	3	4.458
				***P(A)=0.926

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=1.5      LAMBDA=5.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	10	11.625	22	21.135
2.50THETA ZERO	5	7.216	10	11.552
3.00THETA ZERO	3	5.221	6	8.016
3.50THETA ZERO	3	5.167	5	7.000
	$***P(A)=.857$		$***P(A)=.923$	
4.00THETA ZERO	2	4.208	4	6.083
	$***P(A)=.837$		$***P(A)=.928$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=0.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	8	5.781	**	
3.00THETA ZERO	3	2.797	**	
3.50THETA ZERO	2	2.141	**	
	$***P(A)=0.814$			
4.00THETA ZERO	1	1.549	11	7.461
<b>BETA=.15</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	11	7.859	**	
3.00THETA ZERO	5	4.250	**	
	$***P(A)=0.810$			
3.50THETA ZERO	3	3.008	**	
	$***P(A)=0.816$			
4.00THETA ZERO	2	2.359	15	10.148
	$***P(A)=0.821$			
<b>BETA=.10</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	16	11.320	**	
3.00THETA ZERO	6	5.281	**	
3.50THETA ZERO	4	4.000	**	
	$***P(A)=0.811$			
4.00THETA ZERO	3	3.344	21	14.188
	$***P(A)=0.822$			
<b>BETA=.05</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	10	8.602	**	
3.50THETA ZERO	6	6.164	**	
4.00THETA ZERO	4	4.832	**	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=1.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/TB	R	T/TB
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	13	8.531	**	
2.00THETA ZERO	3	2.799	27	16.172
2.50THETA ZERO	2	2.141	7	5.195
	<b>***P(A)=.831</b>			
3.00THETA ZERO	1	1.500	4	3.359
	<b>***P(A)=.824</b>		<b>***P(A)=.910</b>	
3.50THETA ZERO	1	1.500	3	2.758
	<b>***P(A)=.854</b>		<b>***P(A)=.920</b>	
4.00THETA ZERO	1	1.500	2	2.141
	<b>***P(A)=.877</b>		<b>***P(A)=.917</b>	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	18	11.758	**	
2.00THETA ZERO	4	3.650	**	
2.50THETA ZERO	2	2.439	9	6.691
3.00THETA ZERO	2	2.359	5	4.250
	<b>***P(A)=.850</b>		<b>***P(A)=.908</b>	
3.50THETA ZERO	1	1.688	3	3.008
	<b>***P(A)=.832</b>		<b>***P(A)=.906</b>	
4.00THETA ZERO	1	1.688	2	2.402
	<b>***P(A)=.857</b>			
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	26	16.922	**	
2.00THETA ZERO	6	5.363	**	
2.50THETA ZERO	3	3.344	12	8.938
	<b>***P(A)=.812</b>			
3.00THETA ZERO	2	2.656	6	5.336
	<b>***P(A)=.823</b>			
3.50THETA ZERO	1	1.963	4	4.000
			<b>***P(A)=.907</b>	
4.00THETA ZERO	1	1.945	3	3.344
	<b>***P(A)=.830</b>		<b>***P(A)=.911</b>	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=1.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.05				
1.50THETA ZERO	**		**	
2.00THETA ZERO	9	7.938	**	
2.50THETA ZERO	5	5.250	18	13.453
	$***P(A) = 0.819$			
3.00THETA ZERO	3	3.875	9	8.031
	$***P(A) = 0.821$			
3.50THETA ZERO	2	3.156	6	6.227
	$***P(A) = 0.819$			
4.00THETA ZERO	2	3.156	4	4.738
	$***P(A) = 0.849$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.0      LAMBDA=1.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	14	9.121	**	
3.50THETA ZERO	6	4.539	**	
	$***P(A)=.806$			
4.00THETA ZERO	4	3.359	18	11.344
	$***P(A)=.820$			
<b>BETA=.15</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	19	12.352	**	
3.50THETA ZERO	8	6.129	**	
4.00THETA ZERO	5	4.250	24	15.172
	$***P(A)=.814$			
<b>BETA=.10</b>				
2.50THETA ZERO	**		**	
3.00THETA ZERO	26	16.867	**	
3.50THETA ZERO	11	8.383	**	
4.00THETA ZERO	6	5.289	31	19.648
<b>BETA=.05</b>				
3.00THETA ZERO	**		**	
3.50THETA ZERO	17	12.898	**	
4.00THETA ZERO	9	7.863	**	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=2.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	12	8.043	**	
2.50THETA ZERO	4	3.418	18	11.313
3.00THETA ZERO	2	2.150	7	5.145
3.50THETA ZERO	2	2.141	5	3.953
	***P(A)=0.844		***P(A)=0.915	
4.00THETA ZERO	1	1.500	3	2.758
	***P(A)=0.817		***P(A)=0.907	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	16	10.707	**	
2.50THETA ZERO	6	4.852	24	15.164
	***P(A)=0.816			
3.00THETA ZERO	3	3.008	9	6.672
	***P(A)=0.812			
3.50THETA ZERO	2	2.359	6	4.852
	***P(A)=0.818		***P(A)=0.912	
4.00THETA ZERO	2	2.359	4	3.633
	***P(A)=0.853		***P(A)=0.912	
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	22	14.711	**	
2.50THETA ZERO	7	5.895	31	19.656
3.00THETA ZERO	4	4.102	12	8.969
3.50THETA ZERO	3	3.344	7	6.000
	***P(A)=0.826			
4.00THETA ZERO	2	2.656	5	4.641
	***P(A)=0.822		***P(A)=0.909	
<b>BETA=.05</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	11	9.219	**	
3.00THETA ZERO	6	6.082	17	12.813
3.50THETA ZERO	4	4.785	10	8.672
4.00THETA ZERO	3	3.875	7	6.859
	***P(A)=0.822			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=2.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	21	12.938	**	
2.00THETA ZERO	5	3.953	17	10.727
	***P(A)=0.817			
2.50THETA ZERO	2	2.168	6	4.551
3.00THETA ZERO	2	2.141	4	3.359
	***P(A)=0.855			
3.50THETA ZERO	1	1.500	3	2.758
	***P(A)=0.830			
4.00THETA ZERO	1	1.500	2	2.141
	***P(A)=0.859			
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	29	17.875	**	
2.00THETA ZERO	6	4.949	23	14.609
2.50THETA ZERO	3	3.808	8	6.148
	***P(A)=0.816			
3.00THETA ZERO	2	2.359	5	4.250
	***P(A)=0.829			
3.50THETA ZERO	1	1.695	3	3.082
4.00THETA ZERO	1	1.688	3	3.008
	***P(A)=0.833			
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	8	6.582	31	19.797
2.50THETA ZERO	4	4.000	18	7.754
	***P(A)=0.814			
3.00THETA ZERO	3	3.344	6	5.266
	***P(A)=0.840			
3.50THETA ZERO	2	2.656	4	4.000
	***P(A)=0.839			
4.00THETA ZERO	2	2.656	3	3.344
	***P(A)=0.872			
	***P(A)=0.911			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=2.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.05</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	12	9.859	**	
2.50THETA ZERO	6	6.188	14	10.977
3.00THETA ZERO	4	4.563	8	7.375
	<b>***P(A)=0.831</b>			
3.50THETA ZERO	3	3.875	5	5.262
	<b>***P(A)=0.842</b>			
4.00THETA ZERO	2	3.156	4	4.758
	<b>***P(A)=0.828</b>			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=3.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	9	6.266	**	
2.00THETA ZERO	3	2.758	9	6.266
	$***P(A)=0.809$		$***P(A)=0.905$	
2.50THETA ZERO	2	2.141	4	3.387
	$***P(A)=0.837$			
3.00THETA ZERO	1	1.500	3	2.758
	$***P(A)=0.819$		$***P(A)=0.917$	
3.50THETA ZERO	1	1.500	2	2.141
	$***P(A)=0.854$		$***P(A)=0.912$	
4.00THETA ZERO	1	1.500	2	2.141
	$***P(A)=0.880$		$***P(A)=0.933$	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	13	9.055	**	
2.00THETA ZERO	4	3.633	11	7.859
	$***P(A)=0.811$			
2.50THETA ZERO	2	2.425	5	4.289
3.00THETA ZERO	2	2.359	4	3.633
	$***P(A)=0.860$		$***P(A)=0.924$	
3.50THETA ZERO	1	1.688	3	3.008
	$***P(A)=0.828$		$***P(A)=0.929$	
4.00THETA ZERO	1	1.688	2	2.359
	$***P(A)=0.857$		$***P(A)=0.919$	
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	17	11.852	**	
2.00THETA ZERO	5	4.656	14	10.102
2.50THETA ZERO	3	3.344	7	5.891
	$***P(A)=0.820$		$***P(A)=0.910$	
3.00THETA ZERO	2	2.656	4	4.063
	$***P(A)=0.828$			
3.50THETA ZERO	2	2.656	3	3.344
	$***P(A)=0.868$		$***P(A)=0.910$	
4.00THETA ZERO	1	1.945	3	3.344
	$***P(A)=0.826$		$***P(A)=0.934$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=3.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	23	16.051	**	
2.00THETA ZERO	8	7.426	20	14.586
2.50THETA ZERO	4	4.676	9	7.969
3.00THETA ZERO	3	3.875	6	5.938
	$***P(A)=0.831$		$***P(A)=0.913$	
3.50THETA ZERO	2	3.156	4	4.742
	$***P(A)=0.822$			
4.00THETA ZERO	2	3.156	3	3.996
	$***P(A)=0.859$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=3.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.20				
1.00THETA ZERO	**		**	
1.50THETA ZERO	7	5.117	24	14.555
	***P(A)=0.814			
2.00THETA ZERO	3	2.758	6	4.559
	***P(A)=0.836			
2.50THETA ZERO	2	2.141	4	3.359
	***P(A)=0.858		***P(A)=0.922	
3.00THETA ZERO	1	1.500	3	2.758
	***P(A)=0.836		***P(A)=0.933	
3.50THETA ZERO	1	1.500	2	2.141
	***P(A)=0.869		***P(A)=0.926	
4.00THETA ZERO			1	1.500
<hr/>				
BETA=.15				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	6.043	32	19.531
2.00THETA ZERO	3	3.059	8	6.031
	***P(A)=0.907			
2.50THETA ZERO	2	2.359	4	3.699
	***P(A)=0.831			
3.00THETA ZERO	1	1.736	3	3.008
	***P(A)=0.917			
3.50THETA ZERO	1	1.688	2	2.359
	***P(A)=0.845		***P(A)=0.910	
4.00THETA ZERO	1	1.688	2	2.359
	***P(A)=0.872		***P(A)=0.932	
<hr/>				
BETA=.10				
1.00THETA ZERO	**		**	
1.50THETA ZERO	11	8.320	**	
2.00THETA ZERO	4	4.045	10	7.824
2.50THETA ZERO	3	3.344	5	4.695
	***P(A)=0.845			
3.00THETA ZERO	2	2.656	4	4.000
	***P(A)=0.849		***P(A)=0.923	
3.50THETA ZERO	1	2.025	3	3.344
	***P(A)=0.926			
4.00THETA ZERO	1	1.945	2	2.656
	***P(A)=0.843		***P(A)=0.913	

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TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=3.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	17	12.891	**	
2.00THETA ZERO	6	6.043	14	11.117
2.50THETA ZERO	4	4.563	7	6.711
	$***P(A) = 0.838$			
3.00THETA ZERO	3	3.875	5	5.250
	$***P(A) = 0.856$		$***P(A) = 0.917$	
3.50THETA ZERO	2	3.156	4	4.563
	$***P(A) = 0.844$		$***P(A) = 0.928$	
4.00THETA ZERO	2	3.156	3	3.875
	$***P(A) = 0.878$		$***P(A) = 0.924$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=4.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	5	4.004	15	9.625
2.00THETA ZERO	2	2.141	5	4.010
	$***P(A) = 0.810$			
2.50THETA ZERO	1	1.521	3	2.758
	$***P(A) = 0.911$			
3.00THETA ZERO	1	1.500	2	2.141
	$***P(A) = 0.848$		$***P(A) = 0.911$	
3.50THETA ZERO	1	1.500	2	2.141
	$***P(A) = 0.879$		$***P(A) = 0.935$	
4.00THETA ZERO			1	1.500
	$***P(A) = 0.902$			
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	7	5.453	28	12.977
	$***P(A) = 0.813$			
2.00THETA ZERO	3	3.008	6	4.875
	$***P(A) = 0.826$			
2.50THETA ZERO	2	2.359	4	3.633
	$***P(A) = 0.847$		$***P(A) = 0.919$	
3.00THETA ZERO	1	1.688	3	3.008
	$***P(A) = 0.820$		$***P(A) = 0.929$	
3.50THETA ZERO	1	1.688	2	2.359
	$***P(A) = 0.856$		$***P(A) = 0.920$	
4.00THETA ZERO	1	1.688	2	2.359
	$***P(A) = 0.882$		$***P(A) = 0.940$	
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	9	7.221	26	17.000
2.00THETA ZERO	4	4.000	8	6.625
	$***P(A) = 0.826$			
2.50THETA ZERO	2	2.755	5	4.641
	$***P(A) = 0.919$			
3.00THETA ZERO	2	2.656	3	3.344
	$***P(A) = 0.864$		$***P(A) = 0.909$	
3.50THETA ZERO	1	1.945	3	3.344
	$***P(A) = 0.823$		$***P(A) = 0.937$	
4.00THETA ZERO	1	1.945	2	2.656
	$***P(A) = 0.854$		$***P(A) = 0.923$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=4.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/Tθ	R	T/Tθ
<hr/>				
BETA=.05				
1.00THETA ZERO	**		**	
1.50THETA ZERO	13	10.461	**	
2.00THETA ZERO	5	5.340	11	9.273
2.50THETA ZERO	3	4.039	6	6.094
3.00THETA ZERO	2	3.307	4	4.734
3.50THETA ZERO	2	3.156	3	4.055
	$***P(A) = 0.859$			
4.00THETA ZERO	1	2.434	3	3.875
	$***P(A) = 0.935$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=2.0 LAMBDA=4.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	5	3.953	12	8.051
	$***P(A)=.824$			
2.00THETA ZERO	2	2.141	5	3.953
	$***P(A)=.822$		$***P(A)=.917$	
2.50THETA ZERO	1	1.500	3	2.758
	$***P(A)=.814$		$***P(A)=.920$	
3.00THETA ZERO	1	1.500	2	2.141
	$***P(A)=.857$		$***P(A)=.919$	
3.50THETA ZERO	1	1.500	2	2.141
	$***P(A)=.887$		$***P(A)=.941$	
4.00THETA ZERO			1	1.500
	$***P(A)=.908$			
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	4.852	15	10.172
	$***P(A)=.816$			
2.00THETA ZERO	3	3.008	6	4.852
	$***P(A)=.840$		$***P(A)=.915$	
2.50THETA ZERO	2	2.359	3	3.037
	$***P(A)=.859$			
3.00THETA ZERO	1	1.688	2	2.367
	$***P(A)=.830$			
3.50THETA ZERO	1	1.688	2	2.359
	$***P(A)=.864$		$***P(A)=.928$	
4.00THETA ZERO			2	2.359
	$***P(A)=.946$			
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	6.500	19	13.016
	$***P(A)=.815$			
2.00THETA ZERO	3	3.369	7	5.891
	$***P(A)=.908$			
2.50THETA ZERO	2	2.656	4	4.000
	$***P(A)=.825$		$***P(A)=.909$	
3.00THETA ZERO	2	2.656	3	3.344
	$***P(A)=.875$		$***P(A)=.918$	
3.50THETA ZERO	1	1.945	2	2.762
	$***P(A)=.833$			
4.00THETA ZERO	1	1.945	2	2.656
	$***P(A)=.863$		$***P(A)=.930$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=4.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA = .05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	11	9.258	26	18.000
2.00THETA ZERO	5	5.250	9	7.918
	$***P(A) = 0.826$			
2.50THETA ZERO	3	3.875	5	5.270
	$***P(A) = 0.830$			
3.00THETA ZERO	2	3.156	4	4.563
	$***P(A) = 0.828$			
3.50THETA ZERO	2	3.156	3	3.875
	$***P(A) = 0.870$			
4.00THETA ZERO	1	2.527	2	3.156

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=5.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.20</b>				
1.00THETA ZERO	32	18.859	**	
1.50THETA ZERO	4	3.359	10	6.828
		***P(A)=.813	***P(A)=.905	
2.00THETA ZERO	2	2.141	4	3.359
		***P(A)=.832	***P(A)=.906	
2.50THETA ZERO	1	1.500	3	2.758
		***P(A)=.821	***P(A)=.928	
3.00THETA ZERO	1	1.500	2	2.141
		***P(A)=.863	***P(A)=.925	
3.50THETA ZERO			1	1.500
			***P(A)=.900	
4.00THETA ZERO			1	1.500
			***P(A)=.913	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	5	4.338	12	8.410
2.00THETA ZERO	2	2.367	5	4.250
			***P(A)=.909	
2.50THETA ZERO	2	2.359	3	3.008
		***P(A)=.867	***P(A)=.911	
3.00THETA ZERO	1	1.688	2	2.359
		***P(A)=.837	***P(A)=.908	
3.50THETA ZERO	1	1.688	2	2.359
		***P(A)=.871	***P(A)=.933	
4.00THETA ZERO			1	1.688
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	7	5.891	16	11.383
		***P(A)=.815		
2.00THETA ZERO	3	3.344	6	5.344
		***P(A)=.815		
2.50THETA ZERO	2	2.656	4	4.000
		***P(A)=.835	***P(A)=.918	
3.00THETA ZERO	1	1.951	3	3.344
			***P(A)=.926	
3.50THETA ZERO	1	1.945	2	2.656
		***P(A)=.840	***P(A)=.914	
4.00THETA ZERO	1	1.945	2	2.656
		***P(A)=.869	***P(A)=.935	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.0 LAMBDA=5.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/Tθ	R	T/Tθ
<hr/>				
BETA=.05				
1.00THETA ZERO	**		**	
1.50THETA ZERO	10	8.719	22	15.852
2.00THETA ZERO	4	4.621	8	7.285
2.50THETA ZERO	3	3.875	5	5.484
	$***P(A)=.842$			
3.00THETA ZERO	2	3.156	4	4.563
	$***P(A)=.837$		$***P(A)=.928$	
3.50THETA ZERO	2	3.156	3	3.875
	$***P(A)=.878$		$***P(A)=.927$	
4.00THETA ZERO	1	2.375	2	3.270
	$***P(A)=.824$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	8	4.625	**	
2.50THETA ZERO	3	2.206	**	
	$***P(A)=.811$			
3.00THETA ZERO	2	1.712	17	8.619
	$***P(A)=.827$			
3.50THETA ZERO	1	1.200	7	4.094
	$***P(A)=.821$		$***P(A)=.903$	
4.00THETA ZERO	1	1.200	4	2.688
	$***P(A)=.843$		$***P(A)=.904$	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	11	6.287	**	
2.50THETA ZERO	4	2.986	**	
	$***P(A)=.808$			
3.00THETA ZERO	2	1.887	23	11.658
	$***P(A)=.809$			
3.50THETA ZERO	1	1.355	9	5.350
4.00THETA ZERO	1	1.350	5	3.438
	$***P(A)=.824$			
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	16	9.056	**	
2.50THETA ZERO	6	4.212	**	
	$***P(A)=.809$			
3.00THETA ZERO	3	2.675	**	
	$***P(A)=.809$			
3.50THETA ZERO	2	2.125	13	7.700
	$***P(A)=.816$			
4.00THETA ZERO	2	2.125	7	4.775
	$***P(A)=.840$			
<b>BETA=.05</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	8	5.781	**	
3.00THETA ZERO	5	4.200	**	
	$***P(A)=.812$			
3.50THETA ZERO	3	3.266	19	11.237
4.00THETA ZERO	2	2.634	10	6.787

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=1.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	4	2.737	**	
2.00THETA ZERO	2	1.712	7	4.156
	***P(A)=0.831			
2.50THETA ZERO	1	1.200	3	2.225
	***P(A)=0.833			
3.00THETA ZERO	1	1.200	2	1.712
	***P(A)=0.866		***P(A)=0.908	
3.50THETA ZERO			1	1.200
	***P(A)=0.900			
4.00THETA ZERO	0	0.644	1	1.200
	***P(A)=0.812		***P(A)=0.909	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	3.881	**	
	***P(A)=0.810			
2.00THETA ZERO	2	1.952	9	5.353
2.50THETA ZERO	1	1.402	4	2.962
3.00THETA ZERO	1	1.350	3	2.406
	***P(A)=0.845		***P(A)=0.917	
3.50THETA ZERO	1	1.350	2	1.887
	***P(A)=0.872		***P(A)=0.916	
4.00THETA ZERO			1	1.350
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	5.309	**	
2.00THETA ZERO	3	2.675	12	7.150
	***P(A)=0.812			
2.50THETA ZERO	2	2.125	6	4.212
	***P(A)=0.833		***P(A)=0.910	
3.00THETA ZERO	1	1.556	4	3.200
	***P(A)=0.817		***P(A)=0.918	
3.50THETA ZERO	1	1.556	3	2.675
	***P(A)=0.847		***P(A)=0.924	
4.00THETA ZERO	1	1.556	2	2.125
	***P(A)=0.871		***P(A)=0.918	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.5      LAMBDA=1.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	12	7.887	**	
2.00THETA ZERO	5	4.200	18	10.762
	$***P(A)=0.819$			
2.50THETA ZERO	3	3.100	8	5.944
	$***P(A)=0.831$			
3.00THETA ZERO	2	2.525	5	4.444
	$***P(A)=0.835$			
3.50THETA ZERO	1	1.962	3	3.116
4.00THETA ZERO	1	1.900	3	3.100
	$***P(A)=0.835$		$***P(A)=0.922$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.5      LAMBDA=1.5

PRIOR MEAN	$P(A)=.8$			$P(A)=.9$		
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>						
2.00THETA ZERO	**			**		
2.50THETA ZERO	11	5.987		**		
3.00THETA ZERO	5	3.162	30		14.256	
	$***P(A)=0.816$					
3.50THETA ZERO	3	2.206	11		5.912	
	$***P(A)=0.823$			$***P(A)=0.904$		
4.00THETA ZERO	2	1.712	6		3.631	
	$***P(A)=0.827$			$***P(A)=0.904$		
<b>BETA=.15</b>						
2.00THETA ZERO	**			**		
2.50THETA ZERO	14	7.600		**		
3.00THETA ZERO	6	3.967		**		
3.50THETA ZERO	4	2.906	14		7.688	
	$***P(A)=0.823$					
4.00THETA ZERO	2	1.894	8		4.825	
	$***P(A)=0.907$					
<b>BETA=.10</b>						
2.00THETA ZERO	**			**		
2.50THETA ZERO	20	10.831		**		
3.00THETA ZERO	8	5.253		**		
3.50THETA ZERO	5	3.712	18		9.925	
	$***P(A)=0.814$					
4.00THETA ZERO	3	2.733	10		6.237	
<b>BETA=.05</b>						
2.50THETA ZERO	**			**		
3.00THETA ZERO	12	7.831		**		
3.50THETA ZERO	7	5.378		**		
4.00THETA ZERO	5	4.434	14		8.787	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=2.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.20				
1.00THETA ZERO	**		**	
1.50THETA ZERO	18	9.031	**	
2.00THETA ZERO	4	2.734	18	9.050
2.50THETA ZERO	2	1.712	6	3.653
	$***P(A)=0.813$			
3.00THETA ZERO	1	1.289	4	2.688
			$***P(A)=0.914$	
3.50THETA ZERO	1	1.288	3	2.286
	$***P(A)=0.838$		$***P(A)=0.923$	
4.00THETA ZERO	1	1.288	2	1.712
	$***P(A)=0.865$		$***P(A)=0.916$	
<hr/>				
BETA=.15				
1.00THETA ZERO	**		**	
1.50THETA ZERO	25	12.534	**	
2.00THETA ZERO	6	3.881	24	12.131
	$***P(A)=0.816$			
2.50THETA ZERO	3	2.406	8	4.922
	$***P(A)=0.824$			
3.00THETA ZERO	2	1.887	5	3.400
	$***P(A)=0.836$		$***P(A)=0.914$	
3.50THETA ZERO	1	1.350	3	2.406
	$***P(A)=0.811$		$***P(A)=0.907$	
4.00THETA ZERO	1	1.350	3	2.406
	$***P(A)=0.841$		$***P(A)=0.930$	
<hr/>				
BETA=.10				
1.50THETA ZERO	**		**	
2.00THETA ZERO	7	4.716	31	15.725
2.50THETA ZERO	4	3.200	10	6.194
	$***P(A)=0.822$			
3.00THETA ZERO	2	2.150	6	4.212
			$***P(A)=0.909$	
3.50THETA ZERO	2	2.125	4	3.200
	$***P(A)=0.845$		$***P(A)=0.910$	
4.00THETA ZERO	1	1.611	3	2.675
			$***P(A)=0.912$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=2.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.05				
1.5 $\theta$ THETA ZERO	**		**	
2.0 $\theta$ THETA ZERO	11	7.375	**	
2.5 $\theta$ THETA ZERO	5	4.241	15	9.394
3.0 $\theta$ THETA ZERO	3	3.120	8	5.906
3.5 $\theta$ THETA ZERO	3	3.100	5	4.234
	***P(A)=.848			
4.0 $\theta$ THETA ZERO	2	2.525	4	3.837
	***P(A)=.835			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=2.5      LAMBDA=2.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	3.631	28	13.387
	$***P(A)=0.809$			
2.00THETA ZERO	2	1.734	6	3.641
2.50THETA ZERO	1	1.211	4	2.688
	$***P(A)=0.921$			
3.00THETA ZERO	1	1.200	2	1.737
	$***P(A)=0.845$			
3.50THETA ZERO	1	1.200	2	1.712
	$***P(A)=0.876$			
4.00THETA ZERO			1	1.200
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	4.825	35	16.800
	$***P(A)=0.859$			
2.00THETA ZERO	3	2.406	8	4.919
	$***P(A)=0.816$			
2.50THETA ZERO	2	1.887	4	2.975
	$***P(A)=0.840$			
3.00THETA ZERO	1	1.350	3	2.406
	$***P(A)=0.818$			
3.50THETA ZERO	1	1.350	2	1.887
	$***P(A)=0.853$			
4.00THETA ZERO	1	1.350	2	1.887
	$***P(A)=0.879$			
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	11	6.778	**	
2.00THETA ZERO	4	3.200	10	6.203
	$***P(A)=0.814$			
2.50THETA ZERO	2	2.169	5	3.759
3.00THETA ZERO	2	2.125	4	3.200
	$***P(A)=0.856$			
3.50THETA ZERO	1	1.556	3	2.675
	$***P(A)=0.821$			
4.00THETA ZERO	1	1.556	2	2.125
	$***P(A)=0.851$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=2.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.05				
1.00THETA ZERO	**		**	
1.50THETA ZERO	15	9.241	**	
2.00THETA ZERO	6	4.950	14	8.781
2.50THETA ZERO	3	3.156	7	5.350
3.00THETA ZERO	2	2.602	5	4.200
			***P(A)=0.916	
3.50THETA ZERO	2	2.525	4	3.650
		***P(A)=0.852	***P(A)=0.927	
4.00THETA ZERO	1	1.938	3	3.100
			***P(A)=0.924	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=3.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	4	2.688	12	6.362
2.00THETA ZERO	2	1.712	4	2.709
2.50THETA ZERO	1	1.200	3	2.206
3.00THETA ZERO	1	1.200	2	1.712
3.50THETA ZERO			1	1.200
4.00THETA ZERO			1	1.200
			***P(A)=.915	
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	5	3.400	15	8.134
2.00THETA ZERO	2	1.940	5	3.431
2.50THETA ZERO	2	1.887	3	2.406
3.00THETA ZERO	1	1.350	2	1.887
3.50THETA ZERO	1	1.350	2	1.887
4.00THETA ZERO			1	1.350
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	7	4.712	19	10.381
2.00THETA ZERO	3	2.675	7	4.712
2.50THETA ZERO	2	2.125	4	3.200
3.00THETA ZERO	1	1.617	3	2.675
3.50THETA ZERO	1	1.556	2	2.125
4.00THETA ZERO	1	1.556	2	2.125
	***P(A)=.934		***P(A)=.934	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=3.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	10	6.962	26	14.313
2.00THETA ZERO	4	3.741	9	6.375
2.50THETA ZERO	3	3.100	5	4.291
	***P(A)=0.844			
3.00THETA ZERO	2	2.525	4	3.650
	***P(A)=0.842		***P(A)=0.923	
3.50THETA ZERO	2	2.525	3	3.100
	***P(A)=0.880		***P(A)=0.924	
4.00THETA ZERO	1	1.900	2	2.619
	***P(A)=0.831			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.5      LAMBDA=3.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	17	8.594	**	
1.50THETA ZERO	3	2.206	8	4.550
		***P(A)=0.814	***P(A)=0.906	
2.00THETA ZERO	2	1.712	4	2.688
		***P(A)=0.858	***P(A)=0.922	
2.50THETA ZERO	1	1.200	2	1.712
		***P(A)=0.846	***P(A)=0.907	
3.00THETA ZERO	1	1.200	2	1.712
		***P(A)=0.882	***P(A)=0.936	
3.50THETA ZERO			1	1.200
				***P(A)=0.907
4.00THETA ZERO	0	0.644	1	1.200
		***P(A)=0.804	***P(A)=0.925	
<b>BETA=.15</b>				
1.00THETA ZERO	23	11.650	**	
1.50THETA ZERO	4	2.906	10	5.869
		***P(A)=0.817		
2.00THETA ZERO	2	1.887	4	2.959
		***P(A)=0.831		
2.50THETA ZERO	1	1.350	3	2.406
		***P(A)=0.818	***P(A)=0.925	
3.00THETA ZERO	1	1.350	2	1.887
		***P(A)=0.859	***P(A)=0.922	
3.50THETA ZERO	1	1.350	2	1.887
		***P(A)=0.888	***P(A)=0.943	
4.00THETA ZERO			1	1.350
				***P(A)=0.909
<b>BETA=.10</b>				
1.00THETA ZERO	30	15.212	**	
1.50THETA ZERO	5	3.781	13	7.722
2.00THETA ZERO	3	2.675	5	3.756
		***P(A)=0.845		
2.50THETA ZERO	2	2.125	3	2.722
		***P(A)=0.860		
3.00THETA ZERO	1	1.556	2	2.133
		***P(A)=0.828		
3.50THETA ZERO	1	1.556	2	2.125
		***P(A)=0.862	***P(A)=0.927	
4.00THETA ZERO	1	1.556	2	2.125
		***P(A)=0.887	***P(A)=0.945	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.5      LAMBDA=3.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.05				
1.00THETA ZERO	**		**	
1.50THETA ZERO	7	5.287	17	10.200
2.00THETA ZERO	4	3.650	7	5.369
	$***P(A) = 0.838$			
2.50THETA ZERO	2	2.612	4	3.700
3.00THETA ZERO	2	2.525	3	3.266
	$***P(A) = 0.862$			
3.50THETA ZERO	1	2.025	3	3.100
	$***P(A) = 0.939$			
4.00THETA ZERO	1	1.900	2	2.525
	$***P(A) = 0.848$			
	$***P(A) = 0.921$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=4.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.20				
1.00THETA ZERO	12	6.434	**	
1.50THETA ZERO	3	2.206	6	3.656
	$***P(A)=0.833$			
2.00THETA ZERO	1	1.217	3	2.206
	$***P(A)=0.911$			
2.50THETA ZERO	1	1.200	2	1.712
	$***P(A)=0.857$		$***P(A)=0.918$	
3.00THETA ZERO			1	1.200
	$***P(A)=0.900$			
3.50THETA ZERO			1	1.200
	$***P(A)=0.915$			
4.00THETA ZERO	0	0.644	1	1.200
	$***P(A)=0.811$		$***P(A)=0.931$	
<hr/>				
BETA=.15				
1.00THETA ZERO	15	8.056	**	
1.50THETA ZERO	3	2.423	8	4.825
	$***P(A)=0.909$			
2.00THETA ZERO	2	1.887	4	2.906
	$***P(A)=0.847$		$***P(A)=0.919$	
2.50THETA ZERO	1	1.350	3	2.406
	$***P(A)=0.831$		$***P(A)=0.936$	
3.00THETA ZERO	1	1.350	2	1.887
	$***P(A)=0.870$		$***P(A)=0.931$	
3.50THETA ZERO			1	1.350
	$***P(A)=0.900$			
4.00THETA ZERO			1	1.350
	$***P(A)=0.917$			
<hr/>				
BETA=.10				
1.00THETA ZERO	21	11.313	**	
1.50THETA ZERO	4	3.209	10	6.294
2.00THETA ZERO	2	2.204	5	3.712
	$***P(A)=0.919$			
2.50THETA ZERO	2	2.125	3	2.675
	$***P(A)=0.874$		$***P(A)=0.917$	
3.00THETA ZERO	1	1.556	2	2.125
	$***P(A)=0.840$		$***P(A)=0.912$	
3.50THETA ZERO	1	1.556	2	2.125
	$***P(A)=0.872$		$***P(A)=0.936$	
4.00THETA ZERO			1	1.556

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=4.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	4.803	13	8.287
2.00THETA ZERO	3	3.231	6	4.875
2.50THETA ZERO	2	2.525	4	3.650
	***P(A)=0.828		***P(A)=0.918	
3.00THETA ZERO	2	2.525	3	3.100
	***P(A)=0.876		***P(A)=0.924	
3.50THETA ZERO	1	1.900	2	2.641
	***P(A)=0.829			
4.00THETA ZERO	1	1.900	2	2.525
	***P(A)=0.859		***P(A)=0.931	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=4.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	9	5.039	37	17.231
1.50THETA ZERO	2	1.720	5	3.162
2.00THETA ZERO	1	1.200	3	2.206
	$***P(A)=0.814$		$***P(A)=0.920$	
2.50THETA ZERO	1	1.200	2	1.712
	$***P(A)=0.865$		$***P(A)=0.925$	
3.00THETA ZERO			1	1.200
	$***P(A)=0.900$			
3.50THETA ZERO			1	1.200
	$***P(A)=0.920$			
4.00THETA ZERO	6	0.644	1	1.200
	$***P(A)=0.817$		$***P(A)=0.936$	
<b>BETA=.15</b>				
1.00THETA ZERO	12	6.741	**	
1.50THETA ZERO	3	2.406	7	4.362
	$***P(A)=0.818$		$***P(A)=0.912$	
2.00THETA ZERO	2	1.887	3	2.430
	$***P(A)=0.859$			
2.50THETA ZERO	1	1.350	2	1.887
	$***P(A)=0.840$		$***P(A)=0.908$	
3.00THETA ZERO	1	1.350	2	1.887
	$***P(A)=0.878$		$***P(A)=0.938$	
3.50THETA ZERO			1	1.350
	$***P(A)=0.904$			
4.00THETA ZERO			1	1.350
	$***P(A)=0.922$			
<b>BETA=.10</b>				
1.00THETA ZERO	16	9.016	**	
1.50THETA ZERO	4	3.200	8	5.237
	$***P(A)=0.819$			
2.00THETA ZERO	2	2.125	4	3.200
	$***P(A)=0.825$		$***P(A)=0.909$	
2.50THETA ZERO	1	1.580	3	2.675
			$***P(A)=0.926$	
3.00THETA ZERO	1	1.556	2	2.125
	$***P(A)=0.849$		$***P(A)=0.920$	
3.50THETA ZERO	1	1.556	2	2.125
	$***P(A)=0.880$		$***P(A)=0.942$	
4.00THETA ZERO			1	1.581
	$***P(A)=0.902$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=2.5      LAMBDA=4.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	23	13.006	**	
1.50THETA ZERO	6	4.750	11	7.344
		***P(A)=0.825		
2.00THETA ZERO	3	3.100	5	4.216
		***P(A)=0.830		
2.50THETA ZERO	2	2.525	4	3.650
		***P(A)=0.840		***P(A)=0.929
3.00THETA ZERO	2	2.525	3	3.100
		***P(A)=0.886		***P(A)=0.932
3.50THETA ZERO	1	1.900	2	2.525
		***P(A)=0.838		***P(A)=0.917
4.00THETA ZERO	1	1.900	2	2.525
		***P(A)=0.867		***P(A)=0.937

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5      LAMBDA=5.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	8	4.550	26	12.563
		***P(A)=0.809		
1.50THETA ZERO	2	1.712	5	3.162
		***P(A)=0.812		***P(A)=0.911
2.00THETA ZERO	1	1.200	3	2.206
		***P(A)=0.821		***P(A)=0.928
2.50THETA ZERO	1	1.200	2	1.712
		***P(A)=0.871		***P(A)=0.931
3.00THETA ZERO			1	1.200
				***P(A)=0.903
3.50THETA ZERO			1	1.200
				***P(A)=0.925
4.00THETA ZERO	0	0.644	1	1.200
		***P(A)=0.821		***P(A)=0.940
<b>BETA=.15</b>				
1.00THETA ZERO	10	5.813	33	16.056
1.50THETA ZERO	3	2.406	6	3.881
		***P(A)=0.830		***P(A)=0.909
2.00THETA ZERO	2	1.887	3	2.406
		***P(A)=0.867		***P(A)=0.911
2.50THETA ZERO	1	1.350	2	1.887
		***P(A)=0.847		***P(A)=0.915
3.00THETA ZERO	1	1.350	2	1.887
		***P(A)=0.884		***P(A)=0.943
3.50THETA ZERO			1	1.350
				***P(A)=0.909
4.00THETA ZERO			1	1.350
				***P(A)=0.926
<b>BETA=.10</b>				
1.00THETA ZERO	13	7.584	**	
1.50THETA ZERO	4	3.200	7	4.734
		***P(A)=0.832		
2.00THETA ZERO	2	2.125	4	3.200
		***P(A)=0.835		***P(A)=0.918
2.50THETA ZERO	1	1.627	3	2.675
		***P(A)=0.886		***P(A)=0.933
3.00THETA ZERO	1	1.556	2	2.125
		***P(A)=0.855		***P(A)=0.926
3.50THETA ZERO	1	1.556	2	2.125
		***P(A)=0.886		***P(A)=0.947
4.00THETA ZERO			1	1.633
				***P(A)=0.907

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=2.5 LAMBDA=5.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	20	11.731	**	
1.50THETA ZERO	5	4.330	10	6.931
2.00THETA ZERO	3	3.100	5	4.387
	$***P(A) = .842$			
2.50THETA ZERO	2	2.525	3	3.153
	$***P(A) = .849$			
3.00THETA ZERO	1	1.952	3	3.100
	$***P(A) = .939$			
3.50THETA ZERO	1	1.900	2	2.525
	$***P(A) = .845$		$***P(A) = .923$	
4.00THETA ZERO	1	1.900	2	2.525
	$***P(A) = .873$		$***P(A) = .942$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	16	6.792	**	
2.00THETA ZERO	3	1.865	**	
2.50THETA ZERO	2	1.427	17	7.182
	***P(A)=.827			
3.00THETA ZERO	1	1.000	6	3.078
	***P(A)=.826			
3.50THETA ZERO	1	1.000	3	1.841
	***P(A)=.851			
4.00THETA ZERO	1	1.000	2	1.445
	***P(A)=.871			
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	23	9.698	**	
2.00THETA ZERO	5	2.833	**	
	***P(A)=.810			
2.50THETA ZERO	2	1.573	23	9.708
	***P(A)=.809			
3.00THETA ZERO	1	1.161	8	4.083
3.50THETA ZERO	1	1.125	4	2.424
	***P(A)=.832			
4.00THETA ZERO	1	1.125	3	2.005
	***P(A)=.854		***P(A)=.907	
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	6	3.521	**	
2.50THETA ZERO	3	2.229	**	
	***P(A)=.809			
3.00THETA ZERO	2	1.771	11	5.594
	***P(A)=.821			
3.50THETA ZERO	1	1.354	6	3.594
4.00THETA ZERO	1	1.297	4	2.667
	***P(A)=.831		***P(A)=.906	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=0.5

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<hr/>				
BETA=.05				
1.50THETA ZERO	**		**	
2.00THETA ZERO	10	5.734	**	
2.50THETA ZERO	5	3.500	**	
	$***P(A) = 0.812$			
3.00THETA ZERO	3	2.583	16	8.115
	$***P(A) = 0.816$			
3.50THETA ZERO	2	2.104	9	5.354
	$***P(A) = 0.817$			
4.00THETA ZERO	2	2.104	6	4.104
	$***P(A) = 0.842$			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=1.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	13	5.688	**	
1.50THETA ZERO	2	1.464	12	5.365
2.00THETA ZERO	1	1.000	4	2.240
		***P(A)=0.824		***P(A)=0.910
2.50THETA ZERO	1	1.000	2	1.427
		***P(A)=0.866		***P(A)=0.908
3.00THETA ZERO			1	1.000
				***P(A)=0.900
3.50THETA ZERO	0	0.536	1	1.000
		***P(A)=0.819		***P(A)=0.915
4.00THETA ZERO	0	0.536	1	1.000
		***P(A)=0.838		***P(A)=0.930
<b>BETA=.15</b>				
1.00THETA ZERO	18	7.839	**	
1.50THETA ZERO	3	2.005	15	6.719
		***P(A)=0.812		
2.00THETA ZERO	2	1.573	5	2.833
		***P(A)=0.850		***P(A)=0.908
2.50THETA ZERO	1	1.125	3	2.005
		***P(A)=0.845		***P(A)=0.917
3.00THETA ZERO	1	1.125	2	1.573
		***P(A)=0.877		***P(A)=0.920
3.50THETA ZERO			1	1.125
				***P(A)=0.900
4.00THETA ZERO	0	0.630	1	1.125
		***P(A)=0.815		***P(A)=0.917
<b>BETA=.10</b>				
1.00THETA ZERO	26	11.281	**	
1.50THETA ZERO	4	2.737	21	9.422
2.00THETA ZERO	2	1.771	6	3.557
		***P(A)=0.823		
2.50THETA ZERO	1	1.297	4	2.667
		***P(A)=0.817		***P(A)=0.918
3.00THETA ZERO	1	1.297	2	1.801
		***P(A)=0.852		
3.50THETA ZERO	1	1.297	2	1.771
		***P(A)=0.879		***P(A)=0.925
4.00THETA ZERO			1	1.297

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
 DISCRIMINATION RATIO=3.0      LAMBDA=1.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	4.821	**	
2.00THETA ZERO	3	2.583	9	5.354
	$***P(A)=0.821$			
2.50THETA ZERO	2	2.104	5	3.703
	$***P(A)=0.835$			
3.00THETA ZERO	1	1.682	3	2.672
3.50THETA ZERO	1	1.583	2	2.102
	$***P(A)=0.844$			
4.00THETA ZERO	1	1.583	2	2.102
	$***P(A)=0.868$		$***P(A)=0.920$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=1.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.20</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	14	6.081	**	
2.50THETA ZERO	5	2.635	30	11.880
	***P(A)=.816			
3.00THETA ZERO	3	1.839	9	4.203
	***P(A)=.831			
3.50THETA ZERO	2	1.427	5	2.688
	***P(A)=.839			
4.00THETA ZERO	1	1.000	4	2.240
	***P(A)=.816		***P(A)=.917	
<b>BETA=.15</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	19	8.234	**	
2.50THETA ZERO	6	3.306	**	
3.00THETA ZERO	3	2.049	12	5.635
3.50THETA ZERO	2	1.573	6	3.237
	***P(A)=.814			
4.00THETA ZERO	2	1.573	4	2.444
	***P(A)=.848			
<b>BETA=.10</b>				
1.50THETA ZERO	**		**	
2.00THETA ZERO	26	11.245	**	
2.50THETA ZERO	8	4.378	**	
3.00THETA ZERO	4	2.686	16	7.547
3.50THETA ZERO	3	2.229	8	4.346
	***P(A)=.821			
4.00THETA ZERO	2	1.771	6	3.510
	***P(A)=.818		***P(A)=.911	
<b>BETA=.05</b>				
2.00THETA ZERO	**		**	
2.50THETA ZERO	12	6.526	**	
3.00THETA ZERO	6	3.966	23	10.901
3.50THETA ZERO	4	3.134	12	6.573
4.00THETA ZERO	3	2.733	8	4.969

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=3.0      LAMBDA=2.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	6	3.040	**	
2.00THETA ZERO	2	1.434	7	3.430
2.50THETA ZERO	1	1.007	4	2.240
			***P(A)=0.914	
3.00THETA ZERO	1	1.000	3	1.839
		***P(A)=0.844	***P(A)=0.927	
3.50THETA ZERO	1	1.000	2	1.427
		***P(A)=0.874	***P(A)=0.924	
4.00THETA ZERO			1	1.000
<b>BETA=.15</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	4.035	**	
2.00THETA ZERO	3	2.005	9	4.448
		***P(A)=0.812		
2.50THETA ZERO	2	1.573	5	2.833
		***P(A)=0.836	***P(A)=0.914	
3.00THETA ZERO	1	1.125	3	2.005
		***P(A)=0.817	***P(A)=0.912	
3.50THETA ZERO	1	1.125	2	1.573
		***P(A)=0.851	***P(A)=0.908	
4.00THETA ZERO	1	1.125	2	1.573
		***P(A)=0.877	***P(A)=0.929	
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	12	6.031	**	
2.00THETA ZERO	4	2.734	12	5.979
2.50THETA ZERO	2	1.792	6	3.510
			***P(A)=0.909	
3.00THETA ZERO	2	1.771	4	2.667
		***P(A)=0.852	***P(A)=0.916	
3.50THETA ZERO	1	1.297	3	2.229
		***P(A)=0.820	***P(A)=0.921	
4.00THETA ZERO	1	1.297	2	1.771
		***P(A)=0.850	***P(A)=0.911	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=2.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	17	8.531	**	
2.00THETA ZERO	6	4.055	17	8.542
2.50THETA ZERO	3	2.600	8	4.922
3.00THETA ZERO	2	2.151	5	3.630
3.50THETA ZERO	2	2.104	4	3.042
	***P(A)=0.848		***P(A)=0.920	
4.00THETA ZERO	1	1.611	3	2.583
	***P(A)=0.919			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=2.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.20</b>				
1.00THETA ZERO	21	8.625	**	
1.50THETA ZERO	3	1.839	10	4.552
		***P(A)=0.810	***P(A)=0.906	
2.00THETA ZERO	2	1.427	4	2.240
		***P(A)=0.855	***P(A)=0.913	
2.50THETA ZERO	1	1.000	2	1.448
		***P(A)=0.845		
3.00THETA ZERO	1	1.000	2	1.427
		***P(A)=0.881	***P(A)=0.932	
3.50THETA ZERO			1	1.000
			***P(A)=0.905	
4.00THETA ZERO	0	0.536	1	1.000
		***P(A)=0.807	***P(A)=0.923	
<b>BETA=.15</b>				
1.00THETA ZERO	29	11.917	**	
1.50THETA ZERO	4	2.422	12	5.620
		***P(A)=0.811		
2.00THETA ZERO	2	1.573	5	2.833
		***P(A)=0.829	***P(A)=0.914	
2.50THETA ZERO	1	1.125	3	2.005
		***P(A)=0.818	***P(A)=0.918	
3.00THETA ZERO	1	1.125	2	1.573
		***P(A)=0.859	***P(A)=0.917	
3.50THETA ZERO	1	1.125	2	1.573
		***P(A)=0.887	***P(A)=0.939	
4.00THETA ZERO			1	1.125
			***P(A)=0.908	
<b>BETA=.10</b>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	5	3.103	16	7.557
2.00THETA ZERO	3	2.229	6	3.510
		***P(A)=0.840	***P(A)=0.909	
2.50THETA ZERO	2	1.771	4	2.667
		***P(A)=0.856	***P(A)=0.923	
3.00THETA ZERO	1	1.297	3	2.229
		***P(A)=0.828	***P(A)=0.931	
3.50THETA ZERO	1	1.297	2	1.771
		***P(A)=0.861	***P(A)=0.923	
4.00THETA ZERO	1	1.297	2	1.771
		***P(A)=0.886	***P(A)=0.941	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=2.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<u>BETA=.05</u>				
1.00THETA ZERO	**		**	
1.50THETA ZERO	8	4.935	23	10.958
2.00THETA ZERO	4	3.042	8	4.917
	***P(A)=0.831			
2.50THETA ZERO	2	2.168	5	3.500
	***P(A)=0.916			
3.00THETA ZERO	2	2.104	3	2.641
	***P(A)=0.859			
3.50THETA ZERO	1	1.695	3	2.583
	***P(A)=0.932			
4.00THETA ZERO	1	1.583	2	2.104
	***P(A)=0.847			
	***P(A)=0.917			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=3.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	9	4.177	**	
1.50THETA ZERO	2	1.455	6	3.026
2.00THETA ZERO	1	1.000	3	1.839
2.50THETA ZERO	1	1.000	2	1.427
3.00THETA ZERO	1	1.000	1	1.000
3.50THETA ZERO	0	0.536	1	1.000
4.00THETA ZERO	0	0.536	1	1.000
	***P(A)=0.819		***P(A)=0.917	
	***P(A)=0.868		***P(A)=0.924	
	***P(A)=0.823		***P(A)=0.936	
<b>BETA=.15</b>				
1.00THETA ZERO	13	6.036	**	
1.50THETA ZERO	3	2.005	7	3.672
2.00THETA ZERO	2	1.573	4	2.422
2.50THETA ZERO	1	1.125	2	1.573
3.00THETA ZERO	1	1.125	2	1.573
3.50THETA ZERO	1	1.164	1	1.164
4.00THETA ZERO	1	1.125	1	1.125
	***P(A)=0.820		***P(A)=0.924	
	***P(A)=0.860		***P(A)=0.907	
	***P(A)=0.844		***P(A)=0.936	
	***P(A)=0.880		***P(A)=0.905	
	***P(A)=0.923			
<b>BETA=.10</b>				
1.00THETA ZERO	17	7.901	**	
1.50THETA ZERO	4	2.667	9	4.781
2.00THETA ZERO	2	1.771	4	2.708
2.50THETA ZERO	1	1.348	3	2.229
3.00THETA ZERO	1	1.297	2	1.771
3.50THETA ZERO	1	1.297	2	1.771
4.00THETA ZERO	1	1.331	1	1.331
	***P(A)=0.819		***P(A)=0.918	
	***P(A)=0.828		***P(A)=0.941	
	***P(A)=0.852		***P(A)=0.904	
	***P(A)=0.882			

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=3.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.05</b>				
1.00THETA ZERO	23	10.701	**	
1.50THETA ZERO	6	3.958	13	7.016
		***P(A)=0.822		
2.00THETA ZERO	3	2.583	6	3.958
		***P(A)=0.831		***P(A)=0.913
2.50THETA ZERO	2	2.104	4	3.042
		***P(A)=0.842		***P(A)=0.923
3.00THETA ZERO	1	1.617	3	2.583
				***P(A)=0.929
3.50THETA ZERO	1	1.583	2	2.104
		***P(A)=0.843		***P(A)=0.915
4.00THETA ZERO	1	1.583	2	2.104
		***P(A)=0.870		***P(A)=0.936

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=3.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.20</b>				
1.00THETA ZERO	7	3.411	24	9.703
		***P(A)=.814		
1.50THETA ZERO	2	1.427	5	2.635
		***P(A)=.829		***P(A)=.916
2.00THETA ZERO	1	1.000	3	1.839
		***P(A)=.836		***P(A)=.933
2.50THETA ZERO	1	1.000	2	1.427
		***P(A)=.882		***P(A)=.936
3.00THETA ZERO			1	1.000
			***P(A)=.911	
3.50THETA ZERO	0	0.536	1	1.000
		***P(A)=.812		***P(A)=.931
4.00THETA ZERO	0	0.536	1	1.000
		***P(A)=.833		***P(A)=.944
<b>BETA=.15</b>				
1.00THETA ZERO	8	4.029	32	13.021
1.50THETA ZERO	3	2.005	6	3.234
		***P(A)=.845		***P(A)=.914
2.00THETA ZERO	1	1.158	3	2.005
				***P(A)=.917
2.50THETA ZERO	1	1.125	2	1.573
		***P(A)=.859		***P(A)=.922
3.00THETA ZERO			1	1.125
			***P(A)=.900	
3.50THETA ZERO			1	1.125
			***P(A)=.916	
4.00THETA ZERO	0	0.630	1	1.125
		***P(A)=.807		***P(A)=.932
<b>BETA=.10</b>				
1.00THETA ZERO	11	5.547	**	
1.50THETA ZERO	3	2.294	7	4.029
2.00THETA ZERO	2	1.771	4	2.667
		***P(A)=.849		***P(A)=.923
2.50THETA ZERO	1	1.297	2	1.777
		***P(A)=.828		
3.00THETA ZERO	1	1.297	2	1.771
		***P(A)=.867		***P(A)=.931
3.50THETA ZERO			1	1.297
			***P(A)=.900	
4.00THETA ZERO			1	1.297
			***P(A)=.915	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=3.5

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.05</b>				
1.00THETA ZERO	17	8.594	**	
1.50THETA ZERO	5	3.500	9	5.253
	***P(A)=0.831			
2.00THETA ZERO	3	2.583	5	3.500
	***P(A)=0.856		***P(A)=0.917	
2.50THETA ZERO	2	2.104	3	2.721
	***P(A)=0.862			
3.00THETA ZERO	1	1.583	2	2.133
	***P(A)=0.824			
3.50THETA ZERO	1	1.583	2	2.104
	***P(A)=0.858		***P(A)=0.929	
4.00THETA ZERO	1	1.583	2	2.104
	***P(A)=0.884		***P(A)=0.946	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=4.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	5	2.669	15	6.417
1.50THETA ZERO	2	1.427	4	2.240
		***P(A)=0.845		***P(A)=0.913
2.00THETA ZERO	1	1.000	2	1.427
		***P(A)=0.848		***P(A)=0.911
2.50THETA ZERO			1	1.000
			***P(A)=0.900	
3.00THETA ZERO			1	1.000
			***P(A)=0.919	
3.50THETA ZERO	0	0.536	1	1.000
		***P(A)=0.819		***P(A)=0.937
4.00THETA ZERO	0	0.536	1	1.000
		***P(A)=0.840		***P(A)=0.949
<b>BETA=.15</b>				
1.00THETA ZERO	7	3.635	20	8.651
		***P(A)=0.813		
1.50THETA ZERO	2	1.573	5	2.833
		***P(A)=0.816		***P(A)=0.915
2.00THETA ZERO	1	1.125	3	2.005
		***P(A)=0.820		***P(A)=0.929
2.50THETA ZERO	1	1.125	2	1.573
		***P(A)=0.870		***P(A)=0.931
3.00THETA ZERO			1	1.125
			***P(A)=0.902	
3.50THETA ZERO			1	1.125
			***P(A)=0.923	
4.00THETA ZERO	0	0.630	1	1.125
		***P(A)=0.815		***P(A)=0.938
<b>BETA=.10</b>				
1.00THETA ZERO	9	4.814	26	11.333
1.50THETA ZERO	3	2.229	6	3.510
		***P(A)=0.830		***P(A)=0.911
2.00THETA ZERO	2	1.771	3	2.229
		***P(A)=0.864		***P(A)=0.909
2.50THETA ZERO	1	1.297	2	1.771
		***P(A)=0.840		***P(A)=0.912
3.00THETA ZERO	1	1.297	2	1.771
		***P(A)=0.878		***P(A)=0.940
3.50THETA ZERO			1	1.328
			***P(A)=0.904	
4.00THETA ZERO			1	1.297
			***P(A)=0.922	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS  
DISCRIMINATION RATIO=3.0 LAMBDA=4.0

PRIOR MEAN	P(A)=.8		P(A)=.9	
	R	T/Tθ	R	T/Tθ
<b>BETA=.05</b>				
1.00THETA ZERO	13	6.974	**	
1.50THETA ZERO	4	3.210	8	4.969
2.00THETA ZERO	2	2.204	4	3.156
2.50THETA ZERO	2	2.104	3	2.583
	***P(A)=0.876		***P(A)=0.924	
3.00THETA ZERO	1	1.583	2	2.104
	***P(A)=0.836		***P(A)=0.914	
3.50THETA ZERO	1	1.583	2	2.104
	***P(A)=0.869		***P(A)=0.937	
4.00THETA ZERO			1	1.583

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=4.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	5	2.635	12	5.367
	$***P(A)=0.824$			
1.50THETA ZERO	2	1.427	4	2.248
	$***P(A)=0.856$		$***P(A)=0.924$	
2.00THETA ZERO	1	1.000	2	1.427
	$***P(A)=0.857$		$***P(A)=0.919$	
2.50THETA ZERO			1	1.000
	$***P(A)=0.900$			
3.00THETA ZERO			1	1.000
	$***P(A)=0.924$			
3.50THETA ZERO	0	0.536	1	1.000
	$***P(A)=0.825$		$***P(A)=0.941$	
4.00THETA ZERO	0	0.536	1	1.000
	$***P(A)=0.844$		$***P(A)=0.953$	
<b>BETA=.15</b>				
1.00THETA ZERO	6	3.234	15	6.781
	$***P(A)=0.816$			
1.50THETA ZERO	2	1.573	4	2.484
	$***P(A)=0.829$			
2.00THETA ZERO	1	1.125	2	1.578
	$***P(A)=0.830$			
2.50THETA ZERO	1	1.125	2	1.573
	$***P(A)=0.878$		$***P(A)=0.938$	
3.00THETA ZERO			1	1.125
	$***P(A)=0.908$			
3.50THETA ZERO			1	1.125
	$***P(A)=0.928$			
4.00THETA ZERO	0	0.630	1	1.125
	$***P(A)=0.820$		$***P(A)=0.943$	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=4.5

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.10</b>				
1.00THETA ZERO	8	4.333	19	8.677
	***P(A)=0.815			
1.50THETA ZERO	3	2.229	5	3.161
	***P(A)=0.844			
2.00THETA ZERO	2	1.771	3	2.229
	***P(A)=0.875		***P(A)=0.918	
2.50THETA ZERO	1	1.297	2	1.771
	***P(A)=0.849		***P(A)=0.928	
3.00THETA ZERO	1	1.297	2	1.771
	***P(A)=0.885		***P(A)=0.946	
3.50THETA ZERO			1	1.297
	***P(A)=0.910			
4.00THETA ZERO			1	1.297
	***P(A)=0.927			
<b>BETA=.05</b>				
1.00THETA ZERO	11	6.172	26	12.000
1.50THETA ZERO	4	3.042	7	4.542
	***P(A)=0.838			
2.00THETA ZERO	2	2.104	4	3.042
	***P(A)=0.828		***P(A)=0.928	
2.50THETA ZERO	2	2.104	3	2.583
	***P(A)=0.886		***P(A)=0.932	
3.00THETA ZERO	1	1.583	2	2.104
	***P(A)=0.845		***P(A)=0.921	
3.50THETA ZERO	1	1.583	2	2.104
	***P(A)=0.877		***P(A)=0.944	
4.00THETA ZERO			1	1.583

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=5.0

PRIOR MEAN	$P(A)=.8$		$P(A)=.9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.20</b>				
1.00THETA ZERO	4	2.240	10	4.552
	***P(A)=0.813		***P(A)=0.905	
1.50THETA ZERO	2	1.427	3	1.839
	***P(A)=0.865		***P(A)=0.906	
2.00THETA ZERO	1	1.000	2	1.427
	***P(A)=0.863		***P(A)=0.925	
2.50THETA ZERO			1	1.000
			***P(A)=0.903	
3.00THETA ZERO	0	0.536	1	1.000
	***P(A)=0.804		***P(A)=0.928	
3.50THETA ZERO	0	0.536	1	1.000
	***P(A)=0.829		***P(A)=0.944	
4.00THETA ZERO	0	0.536	1	1.000
	***P(A)=0.848		***P(A)=0.956	
<b>BETA=.15</b>				
1.00THETA ZERO	5	2.887	12	5.607
1.50THETA ZERO	2	1.573	4	2.422
	***P(A)=0.838		***P(A)=0.916	
2.00THETA ZERO	1	1.125	2	1.573
	***P(A)=0.837		***P(A)=0.908	
2.50THETA ZERO	1	1.125	2	1.573
	***P(A)=0.884		***P(A)=0.943	
3.00THETA ZERO			1	1.125
			***P(A)=0.913	
3.50THETA ZERO	0	0.630	1	1.125
	***P(A)=0.802		***P(A)=0.932	
4.00THETA ZERO	0	0.630	1	1.125
	***P(A)=0.824		***P(A)=0.946	

TABLE 2 BAYES/CLASSICAL FIXED TIME TESTS

DISCRIMINATION RATIO=3.0 LAMBDA=5.0

PRIOR MEAN	$P(A) = .8$		$P(A) = .9$	
	R	T/T <sub>0</sub>	R	T/T <sub>0</sub>
<b>BETA=.10</b>				
1.00THETA ZERO	7	3.927	16	7.589
	$***P(A) = 0.815$			
1.50THETA ZERO	2	1.775	5	3.094
	$***P(A) = 0.916$			
2.00THETA ZERO	1	1.301	3	2.229
	$***P(A) = 0.926$			
2.50THETA ZERO	1	1.297	2	1.771
	$***P(A) = 0.855$		$***P(A) = 0.926$	
3.00THETA ZERO			1	1.297
	$***P(A) = 0.900$			
3.50THETA ZERO			1	1.297
	$***P(A) = 0.914$			
4.00THETA ZERO			1	1.297
	$***P(A) = 0.931$			
<b>BETA=.05</b>				
1.00THETA ZERO	10	5.813	22	10.568
1.50THETA ZERO	3	2.611	6	4.008
2.00THETA ZERO	2	2.104	4	3.042
	$***P(A) = 0.837$		$***P(A) = 0.928$	
2.50THETA ZERO	1	1.626	3	2.583
	$***P(A) = 0.939$			
3.00THETA ZERO	1	1.583	2	2.104
	$***P(A) = 0.852$		$***P(A) = 0.927$	
3.50THETA ZERO	1	1.583	2	2.104
	$***P(A) = 0.883$		$***P(A) = 0.948$	
4.00THETA ZERO			1	1.633
	$***P(A) = 0.905$			

## 6.0 DEVELOPMENT, DESCRIPTION AND USE OF THE BAYES SEQUENTIAL TESTS

### 6.1 THE TEST STATISTIC AND THE DECISION CRITERIA

The sequential test is designed around the sequence of statistics  $p_n$  where

$$P_n = P(\theta \geq \theta_1 | t(n) = \sum_{i=1}^n x_i).$$

That is,  $P_n$  is the (posterior) probability that  $\theta \geq \theta_1$  (minimum acceptable MTBF) given the total operate time  $t(n) = \sum_{i=1}^n x_i$  where  $n$  is the number of failures. The decision criteria are simply:

when, for the first time,

$$P_n \geq 1-\beta^* \text{ stop the test and accept,}$$

or when, for the first time,

$$P_n \leq \beta^* \text{ stop the test and reject.}$$

If, at the  $n=m$  stage

$$\beta^* < P_m < 1-\beta^* \text{ the test is continued.}$$

It turns out that, for each  $n$ , there exists a  $t^*(n)$  such that  $P_n \geq 1-\beta^*$  is necessary and sufficient for  $t(n) \geq t^*(n)$  and there exists a  $t_*(n)$  such that  $P_n \leq \beta^*$  is necessary and sufficient for  $t(n) \leq t_*(n)$ . Thus, it is not necessary to compute  $P_n$  at each step; it is necessary only to "precompute"  $t_*(n)$  and  $t^*(n)$  for each  $n$ . This also is quite simple since  $t_*(n)$  and  $t^*(n)$  are easily derived from tables of the  $\chi^2$  distribution when  $\lambda$  (the prior shape parameter) is an integer or an integer divided by two. When  $t_*(n) \leq 0$  no reject decision is allowed.

### 6.2 THE INDEXING PARAMETERS

The sole indexing parameter for the sequential tests, given in Table 3, is  $\beta^*$ . The choices provided are  $\beta^* = .10, .20, .30, .40$ . However, to use Table 3 it is required to know  $\lambda, \theta_1$  and the prior mean (ordinarily, taken to be equal to the predicted MTBF). The value of  $\lambda$  must be an integer or an integer divided by two. All that is tabled in Table 3 is the percentage points of the  $\chi^2$  distribution corresponding to  $\beta^*$  and  $1-\beta^*$  for  $m$  degrees of freedom. Because the Bayes sequential test statistic  $t(n)$  jumps over the accept/reject limit, they actually provide smaller risks than the Classical tests and hence, risks of  $\beta^* = .30, .40$  are not necessarily to be avoided.

## 6.3 USE OF TABLE 3

### 6.3.1 PRETEST CHECKS

As previously mentioned, the prior mean is to be taken equal to the predicted MTBF. An MTBF is required to use the tests. Before the test begins, one of the following three quantities must be computed

$$C_1 = 2(\lambda-1)(\text{prior mean})/\theta_1, \quad \lambda > 1,$$

$$C_2 = 2(\text{prior median})/1.443\theta_1, \quad \lambda = 1,$$

or

$$C_3 = 2(\text{prior median})/4.397\theta_1, \quad \lambda = 1/2.$$

For  $C_2$  or  $C_3$  the prior median is to be taken equal to the predicted MTBF. Now, select Table 3 with value of  $\beta^*$  desired and enter that table in the  $m = 2\lambda$  row and verify that  $C_i > T_*$ . Here  $(T_*, T^*)$  is notation for the  $\chi^2$  percentage points. If  $C_i$  is less than or equal to  $T_*$ , no Bayes test of any variety is permitted and a Classical test is suggested, e.g., Mil.-Std. 781B. Having verified  $C_i > T_*$  in the  $m=2\lambda$  row, continue as follows.

### 6.3.2 DETERMINATION OF THE ACCEPT/REJECT CRITERIA

First, it is necessary to determine which values of  $m$  apply to this particular test. The relationship between  $n$  and  $m$  is simple:  $2(\lambda+i)$  is the  $m$  value corresponding to  $n=i$ . For example, suppose  $\lambda=4.5$ , then for  $n=1$ , use  $m = 2(4.5+1) = 11$ ; for  $n=2$ , use  $m = 2(4.5+2) = 13$ ; for  $n=3$ , use  $m=15$  and so on.

It remains to determine what the values of  $t_*(n)$  and  $t^*(n)$  are for each  $n$ . This is done by entering the corresponding  $m$  row, that is, the  $m$  that corresponds to  $n$  and determining

#### Accept Limit

$$t^* = \frac{\theta_1 T^*}{2} - [(\lambda-1)(\text{prior mean})], \quad \lambda > 1,$$

$$t^* = \frac{\theta_1 T^*}{2} - \frac{(\text{prior median})}{1.443}, \quad \lambda = 1,$$

or

$$t^* = \frac{\theta_1 T^*}{2} - \frac{(\text{prior median})}{4.397}, \quad \lambda = 1/2.$$

THE REJECT LIMITS  $t_*$  ARE EXACTLY THE SAME EXCEPT  $T^*$  IS TO BE REPLACED BY  $T_*$  IN THE ABOVE EQUATIONS. For example

$$t_* = \frac{\theta_1 T_*}{2} - [(\lambda-1)(\text{prior mean})], \quad \lambda > 1.$$

By way of summary: the test continues as long as  $t_*(n) < t(n) < t^*(n)$ . When  $t(n) \geq t^*(n)$  the test is passed. When  $t(n) \leq t_*(n)$  the test is failed. Sometimes  $t^*(n)$  will be negative until some number  $n_0 > 1$ . This does not mean that  $n_0$  failures must be observed; it merely means that the total operate time for the first  $n_0$  failures must exceed  $t^*(n_0)$  for a pass decision. Thus, if the first failure time exceeds  $t^*(n_0)$  the test is passed.

### 6.3.3 TRUNCATION OF THE TESTS

Truncation of the tests of Table 3 is provided for in Tables 3T1, 3T2, 3T3 and 3T4. These tables give the step,  $n_t$ , at which the test is to be truncated for  $\beta^* = .10, .20, .30$  and  $.40$ , respectively. Finally, the truncation rule is if  $t(n_t) \geq t^*(n_t) + \frac{t_*(n_t)}{2}$  the test is passed. Otherwise, the test

is failed. The truncation points,  $n_t$  have been selected so that for all tests, the probability of over reaching  $n_t$  is  $\leq 0.10$ .

### 6.3.4 THE PROBABILITY OF ACCEPTANCE AND EXPECTED TEST TIME

Tables 3P1, 3P2, 3P3 and 3P4 give (for  $\beta^* = .10, .20, .30$  and  $.40$  respectively) the probability of acceptance,  $P(A)$  and the expected test time,  $\bar{t}$  in multiples of  $\theta_1$ . The tables are indexed on  $\lambda$  and  $(\text{prior mean})/\theta_1$ . To obtain real expected test time,  $\bar{t}$  must be multiplied by  $\theta_1$ . Also given in these tables are numbers  $P_1(A)$ : the probability of acceptance at the very first opportunity, i.e., when  $t^* > 0$  for the first time. Naturally,  $P_1(A) \leq P(A)$ .

EXAMPLE: Suppose  $\lambda=3.5$ , prior mean = 100,  $\theta_1=25$  and  $\beta^*=0.10$  is desired. Table 3,  $\beta=0.10$  is to be used for calculating the accept/reject criteria.

Step 1. Compute  $C_1 = 2(\lambda-1)(\text{prior mean})/\theta_1 = 20$ . Notice (in Table 3) that  $C_1 = 20 > T_* = 2.833$  (in the  $m = 2\lambda = 7$  row) so a sequential test can be used.

Step 2. Determine, for each  $n$ , the corresponding values of  $m$  to be used from Table 3.

$$\begin{aligned} \text{For } n=1, \quad m &= 2(\lambda+1)=9 \\ n=2, \quad m &= 2(\lambda+2)=11 \\ n=3, \quad m &= 2(\lambda+3)=13 \text{ and so on.} \end{aligned}$$

Step 3. The values of  $t^*$  and  $t_*$  are now easy to determine

$$\begin{aligned} t^*(1) &= \frac{\theta_1 T^*}{2} - (\lambda-1)(\text{prior mean}) = \frac{25}{2} (14.684) - (2.5)(100) \\ &= - 66.45 \end{aligned}$$

$$t^*(2) = \frac{\theta_1 T^*}{2} - (\lambda-1)(\text{prior mean}) = \frac{25}{2} (17.275) - (2.5)(100) \\ = -34.06$$

$$t^*(3) = \frac{25}{2} (19.812) - 250 = -2.35$$

$$t^*(4) = \frac{25}{2} (22.307) - 250 = 28.84 \text{ and so on.}$$

Thus, the first time  $t^*$  is positive is at  $n=4$ .

HOWEVER, THIS MAY BE MISLEADING. WHAT IS MEANT IS THAT IF THE ITEM UNDER TEST RUNS 28.84 HRS. OR MORE WITHOUT A FAILURE THE TEST IS PASSED AND NO FAILURE NEED BE OBSERVED. IF THE FIRST FAILURE TIME IS LESS THAN 28.84 BUT THE SUM OF THE FIRST TWO FAILURE TIMES EXCEEDS 28.84 THE TEST IS PASSED. IN SHORT: THE  $n_0$  FOR WHICH  $t^*(n_0)$  FIRST BECOMES POSITIVE IS USED ONLY TO DETERMINE THE TOTAL OPERATE TIME SUCH THAT IF THE SUM OF THE FIRST  $n_0$  FAILURE TIMES EXCEEDS  $t^*(n_0)$  THE TEST IS PASSED. IN THIS CASE  $n_0=4$  AND THAT DOES NOT MEAN 4 FAILURES MUST BE OBSERVED.

The values  $t^*(n)$  are computed in a similar manner only  $T^*$  is used. For this example, Table 3P1 yields  $P(A)=1.00$ ,  $P_1(A)=1.00$  AND  $\bar{t}=1.16$ . Thus, in terms of actual calendar time the test runs, on the average,  $1.16(\theta_1) = 29$  hrs. Table 3T1 yields  $n_t=5$ . In this example  $P(\theta \geq \theta_1)$  was very high, *a priori*, so the reader will find, on calculating  $t^*(n)$ , rejection cannot occur for quite a few values of  $n$ .

TABLE 3 BAYESIAN SEQUENTIAL TESTS

TABLE 3 BAYESIAN SEQUENTIAL TESTS

 $\beta^* = .10$ 

m	$T_*$	$T^*$
1	.016	2.706
2	.211	4.605
3	.584	6.251
4	1.064	7.779
5	1.610	9.236
6	2.204	10.645
7	2.833	12.017
8	3.490	13.362
9	4.168	14.684
10	4.865	15.987
11	5.578	17.275
12	6.304	18.549
13	7.042	19.812
14	7.790	21.064
15	8.547	22.307
16	9.312	23.542
17	10.085	24.769
18	10.865	25.989
19	11.651	27.204
20	12.443	28.412
21	13.240	29.615
22	14.042	30.813
23	14.848	32.007
24	15.659	33.196
25	16.473	34.382
26	17.292	35.563
27	18.114	36.741
28	18.939	37.916
29	19.768	39.088
30	20.599	40.256

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .10$ 

m	T*	T*
31	21.434	41.422
32	22.271	42.585
33	23.110	43.745
34	23.952	44.903
35	24.797	46.059
36	25.643	47.212
37	26.492	48.363
38	27.343	49.513
39	28.196	50.660
40	29.051	51.805
41	29.907	52.949
42	30.765	54.090
43	31.625	55.230
44	32.487	56.369
45	33.350	57.505
46	34.215	58.641
47	35.081	59.774
48	35.949	60.907
49	36.818	62.038
50	37.689	63.167
51	38.560	64.295
52	39.433	65.422
53	40.308	66.548
54	41.183	67.673
55	42.060	68.796
56	42.937	69.919
57	43.816	71.040
58	44.696	72.160
59	45.577	73.279
60	46.459	74.397

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .10$ 

m	T*	T*
61	47.342	75.514
62	48.226	76.630
63	49.111	77.745
64	49.996	78.860
65	50.883	79.973
66	51.771	81.086
67	52.659	82.197
68	53.548	83.308
69	54.438	84.418
70	55.329	85.527
71	56.221	86.635
72	57.113	87.743
73	58.006	88.850
74	58.900	89.956
75	59.795	91.062
76	60.690	92.166
77	61.586	93.270
78	62.483	94.374
79	63.380	95.476
80	64.278	96.578
81	65.177	97.680
82	66.076	98.780
83	66.976	99.881
84	67.876	100.980
85	68.777	102.079
86	69.679	103.177
87	70.581	104.275
88	71.484	105.372
89	72.387	106.469
90	73.291	107.565

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .10$ 

m	T*	T*
91	74.196	108.661
92	75.101	109.756
93	76.006	110.850
94	76.912	111.944
95	77.818	113.038
96	78.725	114.131
97	79.633	115.223
98	80.541	116.315
99	81.449	117.407
100	82.358	118.498
101	83.268	119.589
102	84.177	120.679
103	85.088	121.769
104	85.998	122.858
105	86.909	123.947
106	87.821	125.035
107	88.733	126.123
108	89.645	127.211
109	90.558	128.298
110	91.471	129.385
111	92.385	130.472
112	93.299	131.558
113	94.213	132.643
114	95.128	133.729
115	96.043	134.813
116	96.958	135.898
117	97.874	136.982
118	98.790	138.066
119	99.707	139.149
120	100.624	140.233

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .10$ 

m	T*	T*
121	101.541	141.315
122	102.458	142.398
123	103.376	143.480
124	104.295	144.562
125	105.213	145.643
126	106.132	146.724
127	107.051	147.805
128	107.971	148.885
129	108.891	149.965
130	109.811	151.045
131	110.732	152.125
132	111.652	153.204
133	112.573	154.283
134	113.495	155.361
135	114.416	156.440
136	115.338	157.518
137	116.261	158.595
138	117.183	159.673
139	118.106	160.750
140	119.029	161.827
141	119.953	162.904
142	120.876	163.980
143	121.800	165.056
144	122.724	166.132
145	123.649	167.207
146	124.574	168.283
147	125.499	169.358
148	126.424	170.432
149	127.349	171.507
150	128.275	172.581

TABLE 3 BAYESIAN SEQUENTIAL TESTS

 $\beta^* = .20$ 

m	T*	T*
1	.064	1.642
2	.446	3.219
3	1.005	4.642
4	1.649	5.989
5	2.342	7.289
6	3.070	8.558
7	3.822	9.803
8	4.594	11.030
9	5.380	12.242
10	6.179	13.442
11	6.989	14.631
12	7.807	15.812
13	8.634	16.985
14	9.467	18.151
15	10.307	19.311
16	11.152	20.465
17	12.002	21.615
18	12.857	22.760
19	13.716	23.900
20	14.578	25.038
21	15.445	26.171
22	16.314	27.302
23	17.187	28.429
24	18.062	29.553
25	18.940	30.675
26	19.820	31.795
27	20.703	32.912
28	21.588	34.027
29	22.475	35.139
30	23.364	36.250

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)  
 $\beta^* = .20$

m	T*	T*
31	24.255	37.359
32	25.148	38.466
33	26.042	39.572
34	26.938	40.676
35	27.836	41.778
36	28.735	42.879
37	29.636	43.978
38	30.537	45.076
39	31.441	46.173
40	32.345	47.269
41	33.251	48.363
42	34.157	49.456
43	35.065	50.548
44	35.974	51.639
45	36.884	52.729
46	37.796	53.818
47	38.708	54.906
48	39.621	55.993
49	40.534	57.079
50	41.449	58.164
51	42.365	59.248
52	43.281	60.332
53	44.199	61.414
54	45.117	62.496
55	46.036	63.577
56	46.955	64.658
57	47.876	65.737
58	48.797	66.816
59	49.718	67.895
60	50.641	68.972

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)  
 $\beta^* = .20$

<i>m</i>	T*	T*
61	51.564	70.049
62	52.487	71.125
63	53.412	72.201
64	54.337	73.276
65	55.262	74.351
66	56.188	75.425
67	57.115	76.498
68	58.042	77.571
69	58.970	78.643
70	59.898	79.715
71	60.827	80.786
72	61.756	81.857
73	62.686	82.927
74	63.616	83.997
75	64.547	85.066
76	65.478	86.135
77	66.409	87.203
78	67.342	88.271
79	68.274	89.338
80	69.207	90.405
81	70.140	91.472
82	71.074	92.538
83	72.008	93.604
84	72.943	94.669
85	73.878	95.734
86	74.813	96.799
87	75.749	97.863
88	76.685	98.927
89	77.622	99.991
90	78.558	101.054

TABLE 3 BAYESIAN SEQUENCE TESTS (CONTD.)

 $\beta^* = .20$ 

m	T*	T*
91	79.496	102.117
92	80.433	103.179
93	81.371	104.241
94	82.309	105.303
95	83.248	106.364
96	84.187	107.425
97	85.126	108.486
98	86.065	109.547
99	87.005	110.607
100	87.945	111.667
101	88.886	112.726
102	89.827	113.786
103	90.768	114.845
104	91.709	115.903
105	92.650	116.962
106	93.592	118.020
107	94.534	119.078
108	95.477	120.135
109	96.420	121.192
110	97.362	122.250
111	98.306	123.306
112	99.249	124.363
113	100.193	125.419
114	101.137	126.475
115	102.081	127.531
116	103.025	128.587
117	103.970	129.642
118	104.915	130.697
119	105.860	131.752
120	106.806	132.806

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)  
 $\beta^* = .20$

m	T*	T*
121	107.751	133.861
122	108.697	134.915
123	109.643	135.969
124	110.589	137.022
125	111.536	138.076
126	112.483	139.129
127	113.430	140.182
128	114.377	141.235
129	115.324	142.288
130	116.272	143.340
131	117.219	144.392
132	118.167	145.444
133	119.116	146.496
134	120.064	147.548
135	121.012	148.599
136	121.961	149.651
137	122.910	150.702
138	123.859	151.753
139	124.809	152.803
140	125.758	153.854
141	126.708	154.904
142	127.658	155.954
143	128.608	157.004
144	129.558	158.054
145	130.508	159.104
146	131.459	160.153
147	132.409	161.202
148	133.360	162.251
149	134.311	163.300
150	135.263	164.349

TABLE 3 BAYESIAN SEQUENTIAL TESTS

 $\beta^* = .30$ 

m	T*	T*
1	.148	1.074
2	.713	2.408
3	1.424	3.665
4	2.195	4.878
5	3.000	6.064
6	3.828	7.231
7	4.671	8.383
8	5.527	9.524
9	6.393	10.656
10	7.267	11.781
11	8.148	12.899
12	9.034	14.011
13	9.926	15.119
14	10.822	16.222
15	11.721	17.322
16	12.624	18.418
17	13.531	19.511
18	14.440	20.601
19	15.352	21.689
20	16.266	22.775
21	17.182	23.858
22	18.101	24.939
23	19.021	26.018
24	19.943	27.096
25	20.867	28.172
26	21.792	29.246
27	22.719	30.319
28	23.648	31.391
29	24.577	32.461
30	25.508	33.530

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .30$ 

m	T*	T*
31	26.440	34.598
32	27.373	35.665
33	28.307	36.731
34	29.242	37.795
35	30.178	38.859
36	31.115	39.922
37	32.053	40.984
38	32.992	42.045
39	33.932	43.105
40	34.872	44.165
41	35.813	45.224
42	36.755	46.282
43	37.698	47.339
44	38.641	48.396
45	39.585	49.452
46	40.529	50.507
47	41.474	51.562
48	42.420	52.616
49	43.366	53.670
50	44.313	54.723
51	45.261	55.775
52	46.209	56.827
53	47.157	57.879
54	48.106	58.930
55	49.055	59.981
56	50.005	61.031
57	50.956	62.080
58	51.906	63.129
59	52.858	64.178
60	53.809	65.227

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .30$ 

m	T*	T*
61	54.761	66.275
62	55.714	67.322
63	56.666	68.369
64	57.620	69.416
65	58.573	70.462
66	59.527	71.509
67	60.481	72.554
68	61.436	73.600
69	62.391	74.645
70	63.346	75.689
71	64.302	76.734
72	65.258	77.778
73	66.214	78.822
74	67.170	79.865
75	68.127	80.908
76	69.084	81.951
77	70.042	82.994
78	70.999	84.036
79	71.957	85.078
80	72.915	86.120
81	73.874	87.161
82	74.833	88.203
83	75.792	89.244
84	76.751	90.284
85	77.710	91.325
86	78.670	92.365
87	79.630	93.405
88	80.590	94.445
89	81.551	95.484
90	82.511	96.524

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .30$ 

m	T*	T*
91	83.472	97.563
92	84.433	98.602
93	85.394	99.641
94	86.356	100.679
95	87.318	101.717
96	88.279	102.755
97	89.242	103.793
98	90.204	104.831
99	91.166	105.868
100	92.129	106.906
101	93.092	107.943
102	94.055	108.980
103	95.018	110.017
104	95.982	111.053
105	96.945	112.090
106	97.909	113.126
107	98.873	114.162
108	99.837	115.198
109	100.801	116.233
110	101.766	117.269
111	102.730	118.304
112	103.695	119.340
113	104.660	120.375
114	105.625	121.410
115	106.590	122.444
116	107.556	123.479
117	108.521	124.513
118	109.487	125.548
119	110.453	126.582
120	111.419	127.616

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .30$ 

m	T*	T*
121	112.385	128.650
122	113.351	129.684
123	114.317	130.717
124	115.284	131.751
125	116.251	132.784
126	117.217	133.817
127	118.184	134.850
128	119.151	135.883
129	120.118	136.916
130	121.086	137.949
131	122.053	138.981
132	123.021	140.014
133	123.989	141.046
134	124.956	142.078
135	125.924	143.110
136	126.892	144.142
137	127.860	145.174
138	128.829	146.206
139	129.797	147.237
140	130.766	148.269
141	131.734	149.300
142	132.703	150.331
143	133.672	151.362
144	134.641	152.393
145	135.610	153.424
146	136.579	154.455
147	137.548	155.486
148	138.518	156.516
149	139.487	157.547
150	140.457	158.577

TABLE 3 BAYESIAN SEQUENTIAL TESTS  
 $\beta^* = .40$

m	T*	T*
1	.275	.708
2	1.022	1.833
3	1.869	2.946
4	2.753	4.045
5	3.656	5.132
6	4.570	6.211
7	5.493	7.283
8	6.423	8.351
9	7.357	9.414
10	8.295	10.473
11	9.238	11.530
12	10.182	12.584
13	11.129	13.636
14	12.079	14.685
15	13.030	15.733
16	13.983	16.780
17	14.937	17.824
18	15.893	18.868
19	16.850	19.910
20	17.809	20.951
21	18.768	21.992
22	19.729	23.031
23	20.690	24.069
24	21.653	25.106
25	22.616	26.143
26	23.579	27.179
27	24.544	28.214
28	25.509	29.249
29	26.475	30.283
30	27.442	31.316

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)  
 $\beta^* = .40$

m	T*	T*
31	28.409	32.349
32	29.376	33.381
33	30.344	34.413
34	31.313	35.444
35	32.282	36.475
36	33.252	37.505
37	34.222	38.535
38	35.192	39.564
39	36.163	40.594
40	37.134	41.622
41	38.106	42.651
42	39.077	43.679
43	40.050	44.706
44	41.022	45.734
45	41.995	46.761
46	42.968	47.787
47	43.942	48.814
48	44.915	49.840
49	45.890	50.866
50	46.864	51.892
51	47.838	52.917
52	48.813	53.942
53	49.788	54.967
54	50.764	55.992
55	51.739	57.016
56	52.715	58.040
57	53.691	59.064
58	54.667	60.088
59	55.643	61.112
60	56.620	62.135

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .40$ 

m	T*	T*
61	57.597	63.158
62	58.574	64.181
63	59.551	65.204
64	60.528	66.226
65	61.506	67.249
66	62.484	68.271
67	63.462	69.293
68	64.440	70.315
69	65.418	71.337
70	66.396	72.358
71	67.375	73.380
72	68.353	74.401
73	69.332	75.422
74	70.311	76.443
75	71.290	77.464
76	72.270	78.485
77	73.249	79.505
78	74.229	80.526
79	75.208	81.546
80	76.188	82.566
81	77.168	83.586
82	78.148	84.606
83	79.128	85.626
84	80.108	86.646
85	81.089	87.665
86	82.069	88.685
87	83.050	89.704
88	84.031	90.723
89	85.012	91.742
90	85.993	92.761

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .40$ 

m	T <sub>*</sub>	T*
91	86.974	93.780
92	87.955	94.799
93	88.936	95.818
94	89.918	96.836
95	90.899	97.855
96	91.881	98.873
97	92.862	99.892
98	93.844	100.910
99	94.826	101.928
100	95.808	102.946
101	96.790	103.964
102	97.772	104.982
103	98.754	105.999
104	99.737	107.017
105	100.719	108.035
106	101.701	109.052
107	102.684	110.070
108	103.667	111.087
109	104.649	112.104
110	105.632	113.121
111	106.615	114.138
112	107.598	115.155
113	108.581	116.172
114	109.564	117.189
115	110.547	118.206
116	111.531	119.223
117	112.514	120.239
118	113.498	121.256
119	114.481	122.273
120	115.465	123.289

TABLE 3 BAYESIAN SEQUENTIAL TESTS (CONTD.)

 $\beta^* = .40$ 

m	T*	T*
121	116.448	124.305
122	117.432	125.322
123	118.416	126.338
124	119.399	127.354
125	120.383	128.370
126	121.367	129.386
127	122.351	130.402
128	123.335	131.418
129	124.320	132.434
130	125.304	133.450
131	126.288	134.465
132	127.272	135.481
133	128.257	136.497
134	129.241	137.512
135	130.226	138.528
136	131.210	139.543
137	132.195	140.559
138	133.180	141.574
139	134.164	142.589
140	135.149	143.604
141	136.134	144.619
142	137.119	145.635
143	138.104	146.650
144	139.089	147.665
145	140.074	148.680
146	141.059	149.694
147	142.044	150.709
148	143.029	151.724
149	144.015	152.739
150	145.000	153.753

TABLE 3T1 NUMBER OF FAILURES,  $n_t$ , AT WHICH TEST IS TRUNCATED  
 $\beta^* = .10$

$\lambda$	PRIOR MEAN/ $\theta_1$							
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	25	25	30	30	30	25	25	25
1.0	45	45	45	40	35	15	12	5
1.5	35	50	55	60	65	75	65	60
2.0	70	80	75	70	50	25	10	4
2.5	80	90	70	45	16	3	3	4
3.0	115	80	55	15	4	4	4	5
3.5	150	100	30	5	4	5	6	7
4.0	180	85	12	3	4	6	7	8
4.5	140	55	7	4	5	7	8	10
5.0	150	50	3	5	6	8	10	11

For all tests the probability of ever reaching the truncation point is less than 0.10.

TABLE 3T2 NUMBER OF FAILURES,  $n_t$ , AT WHICH TEST IS TRUNCATED $\beta^* = .20$ 

$\lambda$	PRIOR MEAN/ $\theta_1$							
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	8	8	9	10	10	10	9	8
1.0	13	13	13	11	8	4	4	3
1.5	11	13	15	17	17	16	15	14
2.0	15	20	20	18	13	4	3	3
2.5	25	24	16	8	4	3	4	5
3.0	25	24	9	4	4	5	5	6
3.5	30	25	6	4	5	6	7	8
4.0	40	15	4	4	6	7	8	10
4.5	35	8	4	5	7	8	10	11
5.0	40	8	4	6	8	10	11	13

For all tests the probability of ever reaching the truncation point is less than 0.10.

TABLE 3T3 NUMBER OF FAILURES,  $n_t$ , AT WHICH TEST IS TRUNCATED  
 $\beta^* = .30$

$\lambda$	PRIOR MEAN/ $\theta_1$							
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	5	5	5	5	5	5	5	5
1.0	5	5	5	5	4	3	3	3
1.5	5	5	5	6	6	6	6	5
2.0	6	7	7	7	3	3	3	4
2.5	8	8	7	4	3	4	5	5
3.0	8	8	6	4	4	5	6	7
3.5	10	7	3	4	6	7	8	9
4.0	11	7	4	5	7	8	9	11
4.5	12	4	5	6	8	9	11	13
5.0	12	3	5	7	9	11	13	15

For all tests the probability of ever reaching the truncation point is less than 0.10.

TABLE 3T4 NUMBER OF FAILURES,  $n_t$ , AT WHICH TEST IS TRUNCATED  
 $\beta^* = .40$

$\lambda$	PRIOR MEAN/ $\theta_1$							
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	3	3	3	3	3	3	3	3
1.0	3	3	3	3	3	3	4	4
1.5	3	3	3	3	3	3	3	2
2.0	3	3	3	3	3	3	4	4
2.5	4	4	4	4	4	5	5	6
3.0	4	4	4	4	5	6	7	8
3.5	5	5	4	5	6	8	9	10
4.0	5	4	5	6	8	9	10	12
4.5	5	4	5	7	9	10	12	14
5.0	5	4	6	8	10	12	14	16

For all tests the probability of ever reaching the truncation point is less than 0.10.

TABLE 3P1 PROBABILITY OF ACCEPTANCE ( $P(A)$ ), PROBABILITY OF ACCEPTANCE AT FIRST OPPORTUNITY ( $P_1(A)$ ), EXPECTED TEST TIME ( $\bar{t}$ ) (IN UNITS OF  $\theta_1$ )

$\beta^* = .10$

$\lambda$		PRIOR MEAN/ $\theta_1$							
		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	$P(A)$	.58	.65	.73	.77	.80	.83	.85	.87
	$P_1(A)$	.34	.36	.42	.47	.52	.55	.58	.60
	$\bar{t}$	7.07	7.59	8.29	8.04	7.57	5.76	5.35	4.92
1.0	$P(A)$	.65	.75	.84	.90	.94	.96	.98	.99
	$P_1(A)$	.25	.35	.43	.55	.62	.71	.81	.90
	$\bar{t}$	15.8	13.4	10.3	7.11	5.09	2.65	1.63	.688
1.5	$P(A)$	.29	.41	.53	.63	.70	.75	.81	.85
	$P_1(A)$	.07	.11	.14	.18	.24	.28	.34	.41
	$\bar{t}$	11.3	18.3	22.0	22.3	21.0	20.9	16.1	13.0
2.0	$P(A)$	.46	.59	.74	.83	.89	.94	.98	1.00
	$P_1(A)$	.08	.14	.21	.31	.44	.58	.73	.87
	$\bar{t}$	29.6	31.2	24.0	17.0	9.86	4.64	1.96	1.89
2.5	$P(A)$	.52	.73	.84	.93	.96	1.00	1.00	1.00
	$P_1(A)$	.08	.19	.31	.48	.70	1.00	1.00	1.00
	$\bar{t}$	35.0	29.2	16.8	7.34	2.39	.016	.725	1.35
3.0	$P(A)$	.59	.80	.92	.97	1.00	1.00	1.00	1.00
	$P_1(A)$	.09	.21	.43	.73	.92	.97	1.00	1.00
	$\bar{t}$	47.2	22.9	9.64	2.09	1.41	1.60	.274	.532
3.5	$P(A)$	.66	.84	.96	.99	1.00	1.00	1.00	1.00
	$P_1(A)$	.10	.26	.57	.90	.99	1.00	1.00	1.00
	$\bar{t}$	53.5	22.4	4.51	2.57	1.31	1.16	1.31	1.10
4.0	$P(A)$	.70	.89	.98	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.11	.32	.76	.99	1.00	1.00	1.00	1.00
	$\bar{t}$	60.5	15.9	1.58	.310	.032	.996	.706	.406
4.5	$P(A)$	.71	.90	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.11	.39	.87	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	46.5	8.80	1.61	.655	.134	.807	.253	.870
5.0	$P(A)$	.75	.95	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.12	.51	.98	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	46.8	6.37	.608	1.00	.206	.598	.958	.128

TABLE 3P2 PROBABILITY OF ACCEPTANCE ( $P(A)$ ), PROBABILITY OF ACCEPTANCE AT FIRST OPPORTUNITY ( $P_1(A)$ ), EXPECTED TEST TIME ( $\bar{t}$ ) (IN UNITS OF  $\theta_1$ )

$\beta^* = .20$

		PRIOR MEAN/ $\theta_1$							
		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$\lambda$	$P(A)$	.56	.64	.72	.77	.79	.83	.87	.89
	$P_1(A)$	.40	.45	.50	.54	.57	.63	.67	.69
	$\bar{t}$	2.40	2.38	2.90	2.77	2.71	2.49	2.38	2.14
1.0	$P(A)$	.65	.77	.86	.92	.96	.99	.99	1.00
	$P_1(A)$	.34	.45	.57	.70	.81	.92	.93	.97
	$\bar{t}$	4.75	4.39	3.33	2.15	1.15	.367	1.25	.847
1.5	$P(A)$	.24	.36	.51	.63	.72	.77	.83	.88
	$P_1(A)$	.09	.14	.21	.25	.35	.39	.48	.57
	$\bar{t}$	2.66	4.02	5.62	6.56	6.22	5.44	4.41	3.40
2.0	$P(A)$	.42	.61	.76	.85	.93	.98	1.00	1.00
	$P_1(A)$	.13	.21	.32	.49	.69	.88	.91	.97
	$\bar{t}$	5.92	8.46	7.07	4.61	2.16	.526	1.11	.732
2.5	$P(A)$	.51	.75	.87	.96	.99	1.00	1.00	1.00
	$P_1(A)$	.15	.31	.52	.81	.93	1.00	1.00	1.00
	$\bar{t}$	9.34	8.50	4.04	1.01	1.00	.122	.571	1.00
3.0	$P(A)$	.59	.82	.96	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.17	.39	.74	.94	.99	1.00	1.00	1.00
	$\bar{t}$	10.5	6.75	1.51	.827	.961	1.08	.075	.232
3.5	$P(A)$	.66	.89	.99	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.18	.49	.88	.99	1.00	1.00	1.00	1.00
	$\bar{t}$	11.2	4.99	1.35	1.06	.956	.807	.700	.586
4.0	$P(A)$	.72	.94	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.20	.64	.98	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	13.8	2.56	.449	.075	.880	.519	.151	.897
4.5	$P(A)$	.74	.97	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.22	.81	.98	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	11.3	.875	.929	.307	.836	.214	.706	.069
5.0	$P(A)$	.78	.98	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.27	.84	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	12.7	1.55	.232	.519	.776	1.01	.125	.338

TABLE 3P3 PROBABILITY OF ACCEPTANCE ( $P(A)$ ), PROBABILITY OF ACCEPTANCE AT FIRST OPPORTUNITY ( $P_1(A)$ ), EXPECTED TEST TIME ( $\bar{t}$ ) (IN UNITS OF  $\theta_1$ )

$\beta^* = .30$

$\lambda$		PRIOR MEAN/ $\theta_1$							
		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	$P(A)$	.53	.63	.69	.76	.79	.84	.87	.91
	$P_1(A)$	.43	.50	.56	.61	.63	.70	.75	.78
	$\bar{t}$	1.32	1.42	1.49	1.52	1.48	1.34	1.18	1.06
1.0	$P(A)$	.62	.78	.88	.96	1.00	1.00	1.00	1.00
	$P_1(A)$	.41	.56	.71	.86	1.00	1.00	1.00	1.00
	$\bar{t}$	1.92	1.69	1.24	.644	.021	.884	.512	.152
1.5	$P(A)$	.21	.32	.45	.61	.71	.79	.86	.91
	$P_1(A)$	.13	.19	.27	.34	.45	.53	.63	.75
	$\bar{t}$	1.19	1.50	1.81	2.32	2.21	2.09	1.64	1.04
2.0	$P(A)$	.34	.58	.77	.90	.99	.99	1.00	1.00
	$P_1(A)$	.17	.29	.45	.69	.95	.95	1.00	1.00
	$\bar{t}$	1.91	2.85	2.50	1.46	.177	.804	.314	.922
2.5	$P(A)$	.46	.75	.93	.97	1.00	1.00	1.00	1.00
	$P_1(A)$	.21	.46	.76	.91	1.00	1.00	1.00	1.00
	$\bar{t}$	2.74	2.63	1.10	.937	.081	.457	.809	.059
3.0	$P(A)$	.55	.87	.96	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.27	.59	.88	.97	1.00	1.00	1.00	1.00
	$\bar{t}$	3.17	2.02	1.15	1.03	.005	.111	.209	.300
3.5	$P(A)$	.65	.95	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.30	.81	.99	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	3.46	.852	.207	.059	1.01	.844	.679	.509
4.0	$P(A)$	.73	.96	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.34	.86	.99	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	3.69	1.38	.621	.209	.887	.469	.048	.695
4.5	$P(A)$	.76	.99	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.39	.94	.99	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	3.65	.647	1.01	.344	.759	.086	.480	.865
5.0	$P(A)$	.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	3.40	.114	.300	.469	.623	.765	.897	1.02

TABLE 3P4 PROBABILITY OF ACCEPTANCE ( $P(A)$ ), PROBABILITY OF ACCEPTANCE AT FIRST OPPORTUNITY ( $P_1(A)$ ), EXPECTED TEST TIME ( $\bar{t}$ ) (IN UNITS OF  $\theta_1$ )

$\beta^* = .40$

$\lambda$		PRIOR MEAN/ $\theta_1$							
		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	$P(A)$	.52	.61	.69	.75	.79	.86	.90	.93
	$P_1(A)$	.48	.56	.63	.68	.71	.79	.84	.88
	$\bar{t}$	.835	.853	.836	.812	.749	.681	.554	.429
1.0	$P(A)$	.59	.79	.92	.94	.98	1.00	1.00	1.00
	$P_1(A)$	.50	.67	.84	.89	.95	.99	.99	1.00
	$\bar{t}$	.923	.831	.414	1.07	.708	.339	1.08	.714
1.5	$P(A)$	.20	.30	.42	.55	.72	.82	.91	.98
	$P_1(A)$	.16	.24	.35	.43	.56	.67	.82	.96
	$\bar{t}$	.771	.900	.951	.983	.972	.822	.475	.098
2.0	$P(A)$	.30	.53	.79	.97	.97	1.00	1.00	1.00
	$P_1(A)$	.23	.40	.63	.94	.94	1.00	1.00	1.00
	$\bar{t}$	.942	1.05	.875	.164	.729	.178	.743	.236
2.5	$P(A)$	.42	.79	.92	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.31	.62	.85	.99	.99	1.00	1.00	1.00
	$\bar{t}$	1.11	.979	1.08	2.15	.521	.819	.068	.366
3.0	$P(A)$	.53	.95	.99	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.39	.89	.99	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	1.22	.322	.251	.293	.342	.390	.434	.475
3.5	$P(A)$	.66	.95	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.45	.88	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	1.41	.908	.571	.366	.162	.996	.784	.571
4.0	$P(A)$	.75	.99	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.54	.98	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	1.21	.308	.896	.434	1.02	.553	.089	.658
4.5	$P(A)$	.83	.99	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.65	.97	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	.977	.888	.162	.496	.821	.107	.424	.737
5.0	$P(A)$	.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$P_1(A)$	.77	.99	1.00	1.00	1.00	1.00	1.00	1.00
	$\bar{t}$	.656	.399	.475	.553	.624	.690	.752	.811

## 7.0 METHODS OF UPDATING THE PRIOR DISTRIBUTION

Methods of fitting prior distributions have been considered and described in detail in RADC-TR-71-209 (Ref. (1)) and RADC-TR-69-389 (Ref. (2)). In this section we consider methods of updating an already existing prior distribution. Updating may be required because of design changes and/or use condition changes (on the one hand) or because new failure data has been required. In regard to using the Bayesian Reliability Demonstration Tests (BRDT) of the test specification, the following are requirements:

- i) in addition to having determined a prior distribution, it is required to have a predicted mean time between failure (MTBF) of the subject equipment.
- ii) in the face of new "data", as mentioned above, an updating of the prior distribution is required.

We consider, first, the case of a design or environmental use change. Attention is restricted, as it is throughout, to the inverted gamma (prior) distribution for  $\text{MTBF} = \theta$ :

$$g(\theta) = \frac{\alpha^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\alpha/\theta}, \quad \alpha, \lambda, \theta > 0.$$

The parameters  $\alpha, \lambda$  are parameters of scale and shape respectively. Let  $\theta_1$  denote the random variable MTBF before the "change" with predicted MTBF  $\theta_{p1} = E(\theta_1)$  (the mean value of  $\theta_1$ ). We will let  $\theta_2$  denote the random variable MTBF after the corresponding predicted MTBF  $\theta_{p2} = E(\theta_2)$  (the mean of  $\theta_2$ ). In the Phase II final report (TR-71-209) it was found that for design and environmental changes in equipment, a linear relation between  $\theta_1$  and  $\theta_2$  fitted well. That is,

$$\theta_2 = b\theta_1, \quad b > 0.$$

This linear relation has the advantage, in addition to the empirical evidence of it being satisfactory, that it "preserves" the inverted gamma family. Thus, if  $\theta_1$  has an inverted gamma (prior) distribution with parameters  $(\alpha, \lambda)$ , then  $\theta_2 = b\theta_1$  has an inverted gamma (prior) distribution with parameters  $(b\alpha, \lambda)$ , i.e.,  $\theta_1$  and  $\theta_2$  are inverted gamma distributed with the same shape parameter  $\lambda$  and scale parameters  $\alpha$  and  $b\alpha$  respectively. Since it has been explicitly assumed that the two predicted\* values  $\theta_{p1}$  and  $\theta_{p2}$ , are such that they are equal to the respective prior means,  $E(\theta_1)$  and  $E(\theta_2)$  it must be that  $\theta_{p2}/\theta_{p1} = E(\theta_2)/E(\theta_1)$ . But  $E(\theta_2) = bE(\theta_1)$  so that  $\theta_{p2}/\theta_{p1} = b$ . In updating the prior distribution of  $\theta_1$  to the new distribution (of  $\theta_2$ ) it is necessary

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\*By "predicted" we mean the usual MTBF prediction based on standard engineering analyses and failure rate sources.

only to take the ratio of the two predicted MTBF's  $\theta_{p_2}/\theta_{p_1} = b$  and find that

the new distribution has parameters  $(b\alpha, \lambda)$ . This procedure avoids having to obtain new data to refit the prior distribution. If the prior means do not exist (i.e.,  $\lambda \leq 1$ ) the same procedure can be used with the prior medians.

The second situation considered here is that in which the prior distribution must be re-evaluated because new failure data is available. That is, additional data, over and above the data used for the original fit has become available. The procedure to be used is fairly simple: the new observed data is tested against the prior distribution in use by employing a  $\chi^2$  test. If the test is passed, the new data is combined with the old and the parameters  $(\alpha, \lambda)$  re-estimated. If the test is failed the prior distribution is changed to fit the new data. The methods of fitting distributions is discussed in detail in the Phase I report (RADC-TR-69-389) but we will review the procedure briefly in the context of new data for the case of Type I data, i.e., the observed failure data is the number of failures,  $X$ , occurring in a fixed time  $T$ . Here the random variable  $X$  is such that  $X = 0, 1, \dots$ . The original prior distribution was fitted by determining (moment) estimates of  $\alpha, \lambda$  from the two equations

$$\hat{\alpha} = \bar{x}T / (\sum x_i^2/n - \bar{x}^2 - \bar{x}) \text{ and } \hat{\lambda} = \frac{\hat{\alpha}\bar{x}}{T}, \text{ where}$$

$\bar{x} = \sum x_i/n$ . A  $\chi^2$  test is then used against the marginal distribution of  $X$  (since the parameter estimates are used in place of  $\alpha, \lambda$  the total degrees of freedom will be the number of cells minus three) which is estimated to be

$$f(x) = \frac{\Gamma(\hat{\lambda}+x)}{\Gamma(\hat{\lambda}) x!} \left( \frac{T}{T+\hat{\lambda}} \right)^x \left( \frac{\hat{\lambda}}{T+\hat{\lambda}} \right)^{\hat{\lambda}}.$$

Now, when the new data is available, say, a sample  $x' = (x'_1, \dots, x'_n)$ , a  $\chi^2$  test is run with the new data used against the previously estimated, i.e., original, marginal distribution. Put another way, the  $\chi^2$  is run with the new data using the marginal distribution of  $X$  based on the original estimates  $\hat{\alpha}$  and  $\hat{\lambda}$ . This time, then, since the parameters are not being re-estimated, the degrees of freedom is the number of cells minus one. If the test is passed the old and new data should be combined and the parameters  $\alpha, \lambda$  estimated using all the data and, in this way, a new prior distribution is obtained. If the test is failed, a choice must be made between the original prior distribution and the prior distribution obtained using the new data. Usually, it is the most recent data which will prevail.

## 8.0 SPECIAL PROBLEMS

Three special problems, each relating to the ease of use and breadth of applicability of the military standard for Bayesian reliability demonstration tests, were investigated in this study. The three problems were:

1. To determine truncation times of the sequential tests.
2. To determine a way, if possible, of saving total test time by putting two or more equipments on test at the same test.
3. To find a test for changes or "shifts" in the prior distribution during the repetitive use of a particular Bayes test for the "same" equipment type.

Problem 1 was successfully solved during the development of the sequential Bayes tests. The solution is described in the discussion of the sequential tests (Sec. 6.0 of this report).

The following two subsections discuss the investigations of Problems 2 and 3 respectively.

### 8.1 PLACING MORE THAN ONE EQUIPMENT ON TEST

An investigation was made of whether total test time could be saved by putting two or more equipments on test at the same time. It is not at all obvious that this can be done. Two different equipments have two different MTBF's,  $\theta_1$ , and  $\theta_2$ , arising randomly from the prior distribution  $g(\theta)$ . Any test "pooling" the data from the two equipments will have to make a joint inference about the locations of both  $\theta_1$  and  $\theta_2$ . There is no advantage in using the data separately, say, by computing  $\hat{\theta}_1$  for the first equipment and  $\hat{\theta}_2$  for the second. For in this case, the joint posterior distribution factors as the product of the "separate" posterior distributions:

$$f(\theta_1, \theta_2 | \hat{\theta}_1, \hat{\theta}_2) = f(\theta_1 | \hat{\theta}_1) \cdot f(\theta_2 | \hat{\theta}_2).$$

However, methods could be developed (using a multivariate inverted gamma distribution) which would allow assessment of whether all of  $n$  equipment/unit/systems had  $\theta_i \geq \theta_1$   $i=1, \dots, n$ .

Attempts were made to compute the posterior distribution in the inverted gamma case using "combined" statistics (such as the number of failures in both equipments divided by the total number of equipment hours). Unfortunately, no tractable results could be obtained.

### 8.2 TESTS FOR SHIFTS IN THE PRIOR DISTRIBUTION

One of the most important considerations in using Bayes tests is that the user be confident the prior distribution being used is correct (or at least close enough to the true prior that the true test risks are fairly close to

those expected). In Phase I of this study, methods were developed to use equipment MTBF data to fit prior distributions. These fitting methods can be used in an obvious way to test whether the prior distribution has changed during a series of tests of equipments of the same type. Suppose the prior distribution at the beginning of the series of tests is known to be inverted gamma with parameters  $\alpha_0, \lambda_0$ . Then at any point in the series of tests, the MTBF data generated by the previously tested equipments can be used for a Chi-square test (given in Phase I of this study) of the null hypothesis that  $g(\alpha_0, \lambda_0)$  is the prior distribution. If the Chi-square test is not passed, then one infers that the prior distribution has changed. One then estimates new prior distribution parameters (by one of the methods given in Phase I), and then changes to a test in the handbook indexed by these new parameters.

The test discussed above is an acceptable test for a change in the prior distribution, but it is somewhat difficult to carry out. Another test has been developed, which is much more easily applied, and which serves as a running test for change in the prior distribution in the important situation of repetitive use of a particular Bayes test plan for the same equipment type. The description of this test and the investigation of its properties follow in the paragraphs below.

Consider a particular prior distribution (inverted gamma with parameters fixed at, say,  $\alpha_0, \lambda_0$ ), and a particular fixed time test plan ( $T, r^*$ ). (Although we are focusing on fixed time tests, this discussion carries over completely to sequential tests.) Then  $P(A)$  is fixed. Explicitly, for the fixed time test, we have:

$$P(A) = P(r \leq r^*) = \sum_{r=0}^{r^*} \frac{\Gamma(\lambda_0 + r)}{\Gamma(\lambda_0)r!} \left(\frac{T}{\alpha_0 + T}\right)^r \left(\frac{\alpha_0}{\alpha_0 + T}\right)^{\lambda_0} \quad (1)$$

Now, if there is a shift in the prior distribution to, say, the inverted gamma with parameters  $\alpha_1, \lambda_1$ , then  $P(A)$  generally will change (for the given test plan) to a value  $\hat{P}(A)$ , computed by replacing  $\alpha_0, \lambda_0$  in Formula (1) by  $\alpha_1, \lambda_1$ . This suggests a natural test statistic:  $(\text{number of tests passed}) / (\text{number of tests taken})$ . This statistic is designated by  $\hat{P}(A)$ , since it is an estimate of  $P(A)$ .

The null hypothesis for the test is that the prior parameters are  $\alpha_0, \lambda_0$ . The null hypothesis is rejected if  $|\hat{P}(A) - P(A)|$  is too large. Under the null hypothesis, the distribution of  $n\hat{P}(A)$  is binomial with parameters  $n$  and  $P(A)$ , where  $n$  denotes the number of tests taken. So for any desired significance level  $\alpha$ , the appropriate critical values for the test can be found with a table of the binomial distribution. The power of the test can also be evaluated easily. If the prior parameters have shifted to  $\alpha_1, \lambda_1$ , then the probability of acceptance for each test has shifted to  $P'(A)$ , and the power can be found by noting that the distribution of  $n\hat{P}(A)$  under the shift is binomial with parameters  $n$  and  $P'(A)$ .

When  $n$  is fairly large, the "normal" approximation to the test works fairly well. Let  $p=P(A)$ ,  $q=1-p$ ,  $p'=P'(A)$  and  $q'=1-p'$ . Then  $\hat{P}(A)$  is approximately normally distributed under the null hypothesis with mean  $p$  and variance  $pq/n$ . The approximate test is to reject the null hypothesis if

$$\frac{\hat{P}(A) - p}{\sqrt{pq/n}} < z_{\alpha/2}$$

$$\text{or } > z_{1-\alpha/2}$$

where the  $Z$ 's denote percentage points of the unit normal distribution. Then calculations show that the power of the test (i.e., the probability of rejecting under the alternative hypothesis) is approximately

$$\Phi \left[ \frac{p-p' - \sqrt{pq/n} z_{1-\alpha/2}}{\sqrt{p'q'/n}} \right] + 1 - \Phi \left[ \frac{p-p' + \sqrt{pq/n} z_{1-\alpha/2}}{\sqrt{p'q'/n}} \right],$$

where  $\Phi(\cdot)$  is the cumulative normal distribution function.

To investigate the power of the  $\hat{P}(A)$  test against various alternatives, 4 representative fixed time Bayes tests were considered. The 4 tests considered are given in Table 8.1. (The meanings of the column headings that index the tests are discussed in the section of this report on the Fixed Time tests.)

Figures 8.1 through 8.4 gives  $P(A)$  for a wide class of alternatives for each of Tests 1 through 4, respectively. The alternatives are given by different values of the inverted gamma parameters  $\lambda$  and  $\frac{\alpha}{\theta_0}$  (the shape parameter  $\alpha$  given in multiples of  $\theta_0$ ). For selected values of  $P(A)$ , "contours" are drawn in the  $(\lambda, \frac{\alpha}{\theta_0})$ -plane corresponding to  $P(A)$  (through Equation 1). An

"x" is marked on each graph to denote the  $(\lambda, \frac{\alpha}{\theta_0})$  - pair for which the test

was designed. For example, in Figure 8.1, an "x" is marked on the  $P(A) = .60$  contour at the point corresponding to the prior parameters being tested -  $\lambda=3$ ,  $\alpha/\theta_0=2$ . If one wishes to find the power of the  $\hat{P}(A)$  test against any  $(\lambda, \alpha/\theta_0)$  pair on the (say)  $P(A) = .80$  contour when Test 1 is taken  $n$  times, one need only use the binomial distribution or the normal approximation to find the power of the test of  $P(A) = .60$  against  $P(A) = .80$ .

One objection to the  $\hat{P}(A)$  test may be that the null hypothesis  $P(A)=.60$  does not distinguish between the parameter pairs  $(\lambda=3, \alpha/\theta_0=2)$  and, say,  $(\lambda=5, \alpha/\theta_0=3.4)$ . This objection is overcome by noting that any reasonable

TABLE 8.1 THE FOUR (4) FIXED TIME BAYES TESTS INVESTIGATED

TEST NO.	$\beta^*$	DISCRIM. RATIO	$\lambda$	PRIOR MEAN**	$\frac{\alpha}{\theta_0}$	R	$\frac{T}{\theta_0}$	P(A)
1	.05	2	3	$\theta_0$	2	5	3.539	.5983
2	.10	2	1	$\theta_0$	.6930	2	1.184	.7491
3	.05	2	2	$\theta_0$	1	4	3.402	.4111
4	.10	3	.5	$\theta_0$	.3412	1	.270	.9124

\*\*This column gives the prior median when  $\lambda \leq 1$ .

prior distribution on the prior parameters  $(\lambda, \alpha/\theta_0)$  would assign probability 0 to the whole contour  $P(A) = .60$  in the  $(\lambda, \alpha/\theta_0)$  plane. Hence, if one takes the  $\hat{P}(A)$  test and it results in acceptance of the null hypothesis, there is no reason to think that the true parameters are anything but  $\lambda=3$ ,  $\alpha/\theta_0=2$ .

In practice, acceptance of a null hypothesis in a statistical test really only allows one to infer that the true parameters are in a "close neighborhood" of the null hypothesis. We note in Figures 8.1 through 8.4 that "moderate" departures from the point "x" in the plane entails only moderate changes in both  $\lambda$  and  $\alpha/\theta_0$ . This gives further indication that the  $\hat{P}(A)$  test is a good test for shifts in the prior distribution.

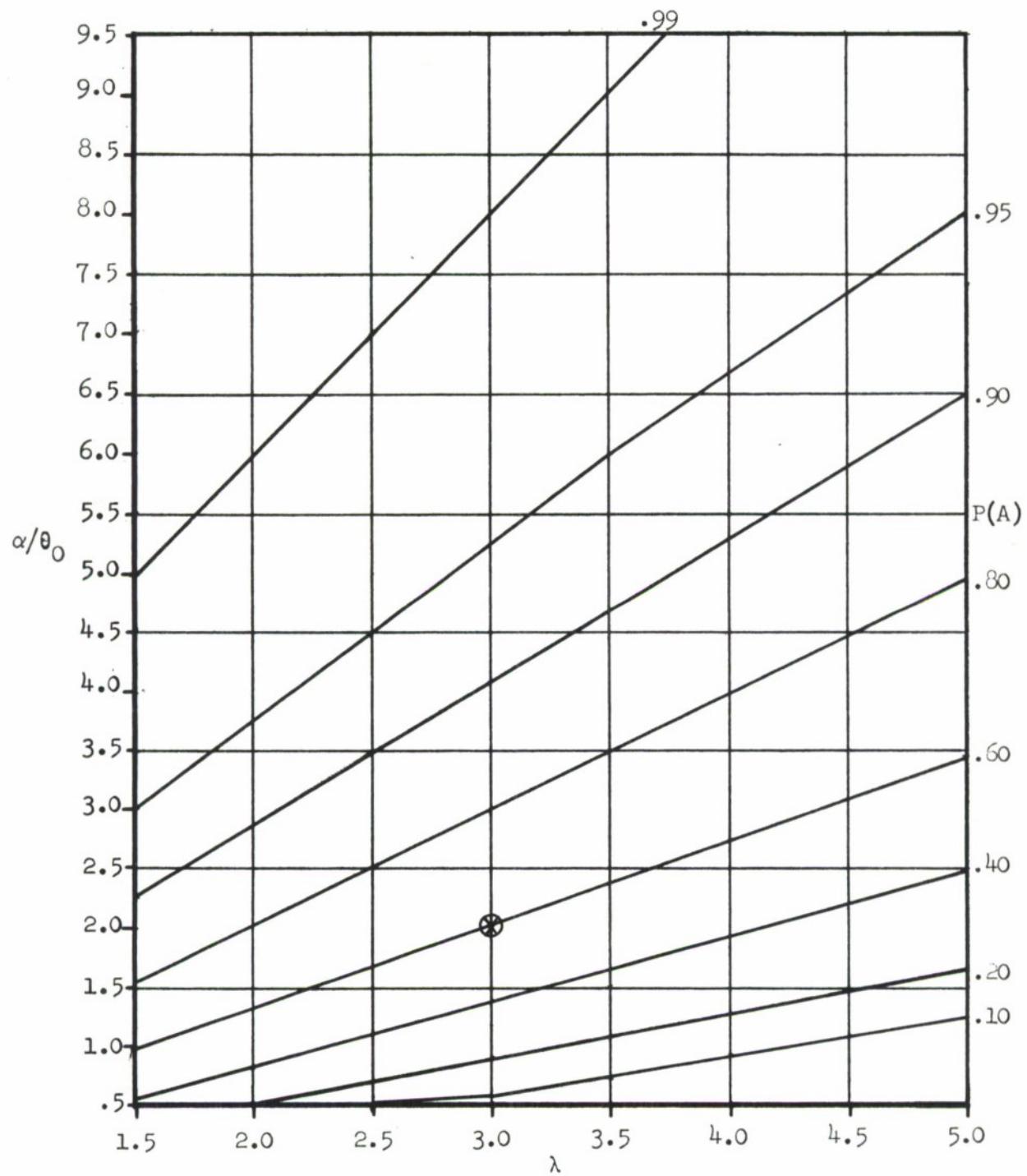
FIGURE 8.1 P(A) AS A FUNCTION OF  $\lambda$  AND  $\alpha/\theta_0$  FOR TEST NO. 1

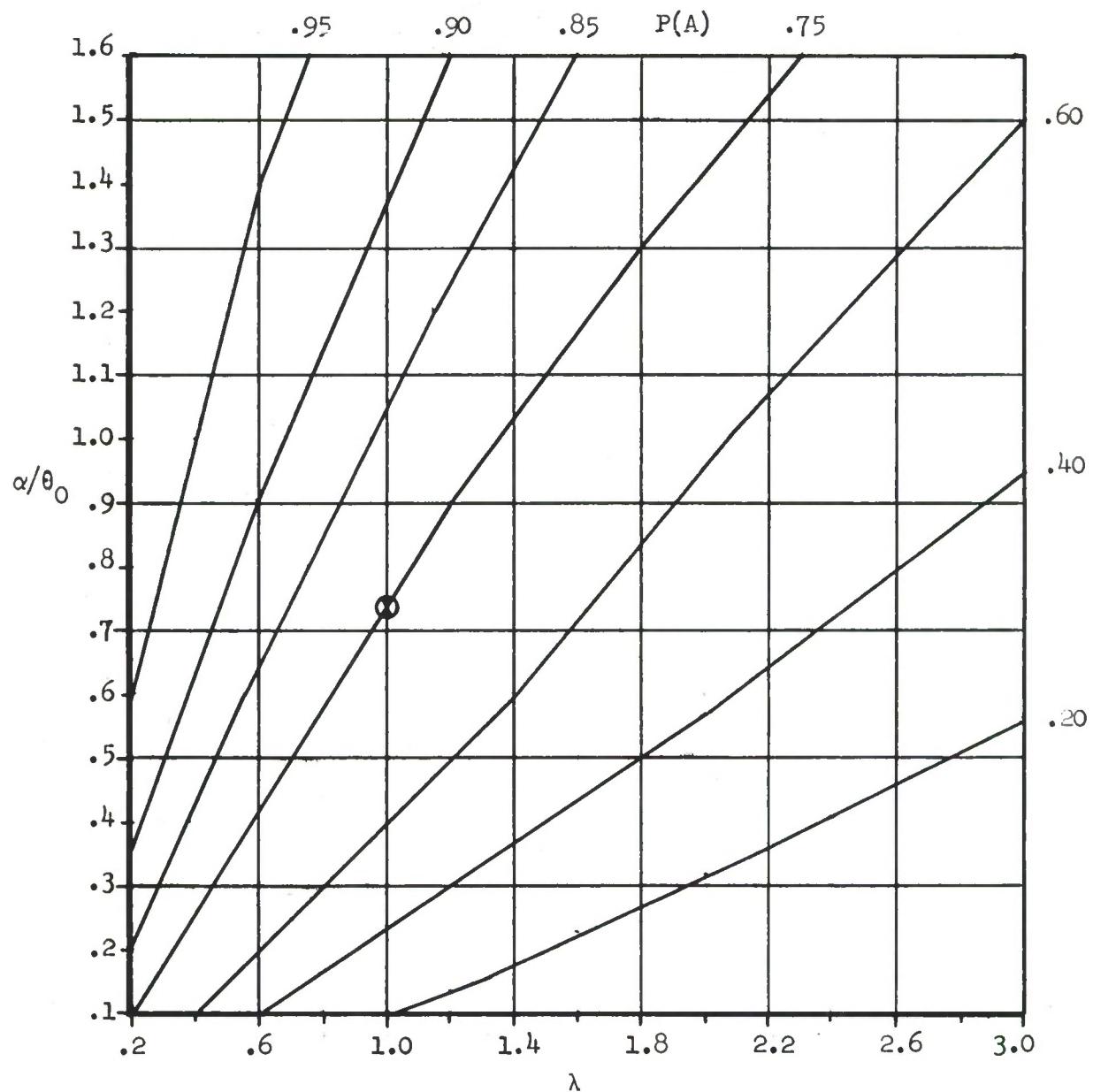
FIGURE 8.2  $P(A)$  AS A FUNCTION OF  $\lambda$  AND  $\alpha/\theta_0$  FOR TEST NO. 2

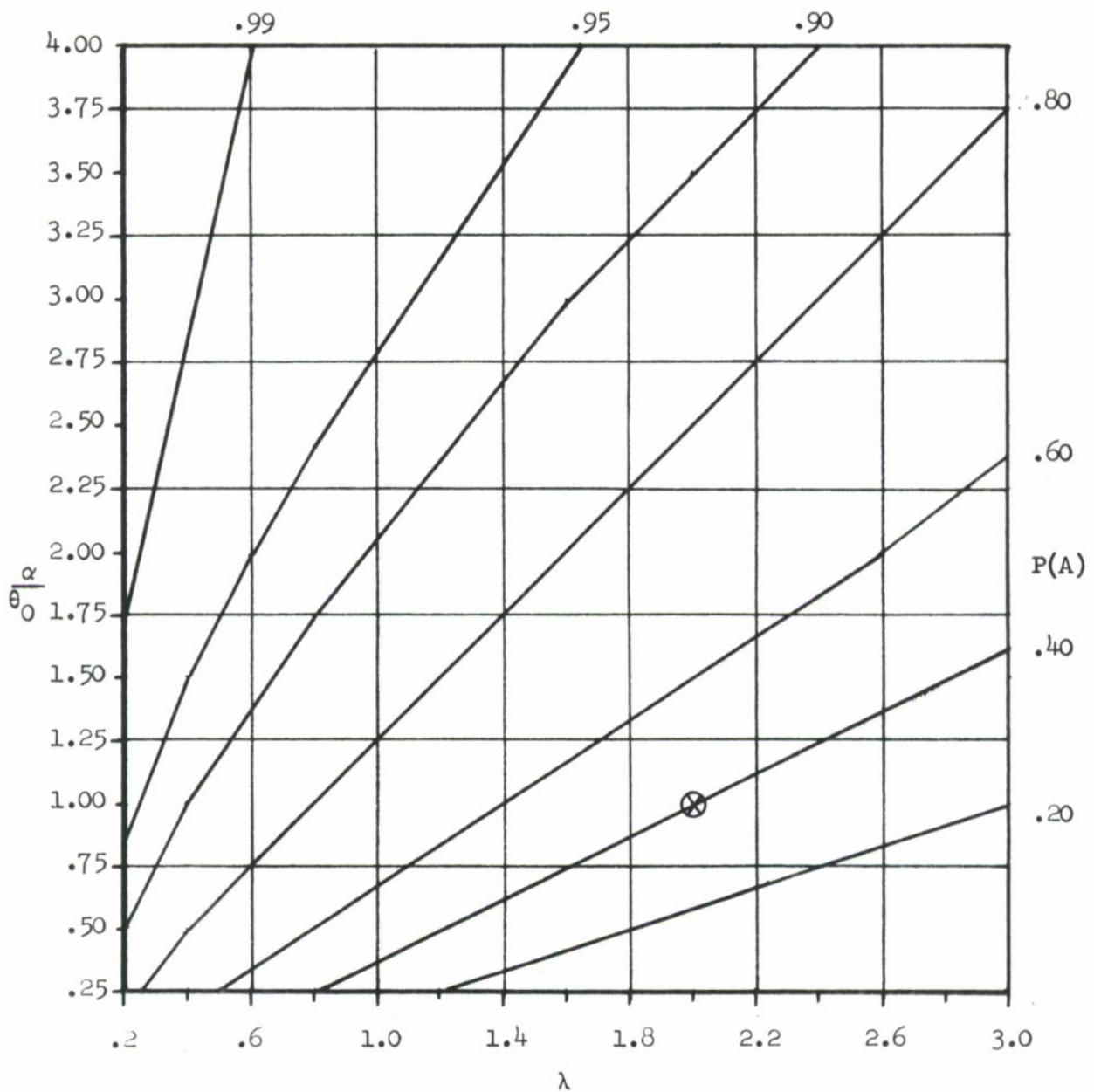
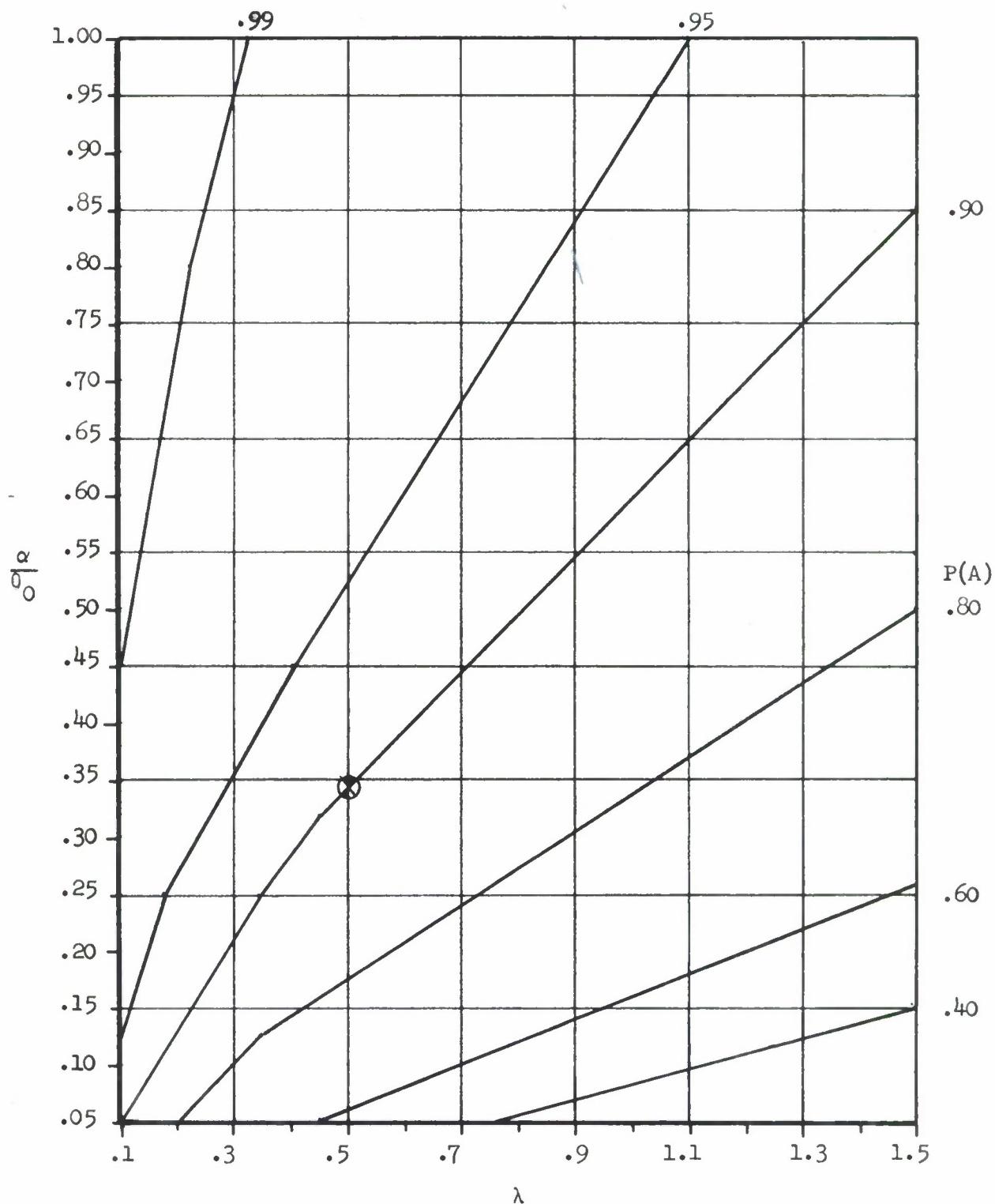
FIGURE 8.3 P(A) AS A FUNCTION OF  $\lambda$  AND  $\alpha/\theta_0$  FOR TEST NO. 3

FIGURE 8.4 P(A) AS A FUNCTION OF  $\lambda$  AND  $\alpha/\theta_0$  FOR TEST NO. 4

## 9.0 RECOMMENDATIONS

It is suggested that the three types of Bayes tests: Bayes fixed time, Bayes/Classical and sequential Bayes, which have been proposed, be implemented in a D.O.D. test standard. A preliminary version of this standard has been prepared. Secondly, it is recommended, depending on comments received, other (additional) values of the indexing parameters (prior mean,  $\lambda$ ,  $\beta^*$ ,  $P(A)$ , etc.) be provided in the test tables.

## 10.0 REFERENCES

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- (3) Reliability Tests: Exponential Distribution, Mil. Std. 781B, Department of Defense, USA.

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13. ABSTRACT This Final Report is the result of a study performed for RADC under Contract F30602-72-C-0067. This study is the third phase of a three phase effort to develop Bayesian Reliability Demonstration Tests (BRDT). The objectives of this phase were: <ul style="list-style-type: none"> <li>i) develop and tabulate BRDT of fixed time and sequential types.</li> <li>ii) develop and tabulate tests (called Bayes/Classical in this report) when the producer and consumer cannot agree on a prior distribution.</li> <li>iii) develop methods of updating existing prior distributions.</li> <li>iv) develop a preliminary military standard for BRDT.</li> <li>v) investigate some special problems</li> <li>vi) fit additional prior distributions.</li> </ul> <p>Bayesian fixed times tests, Bayesian/Classical fixed time tests, and Sequential Bayesian tests were developed and tabulated. These tests form an essential part of the preliminary military standard which was also developed. Additional fits of the inverted gamma distribution reconfirmed its choice as a prior distribution and further study showed that updates in the prior distribution are easily made. A test based on probability of acceptance is satisfactory to test for shifts in the prior distribution. Tables were developed giving the truncation points for the sequential tests. At this time, no satisfactory solution has been found for placing more than one equipment on test at a time.</p>		

14.  KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability Testing Statistical Tests Electronic Equipment						

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