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REVIEW OF THE MIL-HDBK-5 PROGRAM

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A summary of the technical accomplishments for the MIL-HDBK-5 Program is presented. Design values (minimum tensile yield and ultimate, compressive yield, shear ultimate, as well as bearing yield and ultimate strengths) for new products, such as, 2090-T83 sheet, 2519-T87 plate, 6013-T6 sheet, 7150-T6151 and T7751 plate, 7150-T61511 and T77511 extrusion, Ti-15V-3Cr-3Al-3Sn sheet, and Ti-10V-2Fe-3Al die forgings, were incorporated into MIL-HDBK-5. Design values for many products in the document were revised due to reanalysis according to current guideline procedures, and/or the availability of additional data. Some design allowables were modified because of specification changes in specification limits so as to achieve compatibility with reference material specifications. Design values for a considerable number of products were upgraded from S basis to A and B basis. Elevated temperature data for various mechanical and physical properties of certain materials were added or revised. Fatigue data in the form of S/N or ε/N curves were included for many materials. Joint design allowables for various (continued on next page)

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19. ABSTRACT (Continued)

fastener systems were added to MIL-HDBK-5. Improved statistical analysis procedures, as well as analytic techniques for analyzing strain control fatigue data, were incorporated. The procedure for analyzing load control data was revised. The Handbook was revised annually incorporating the above-described changes and additions. Four change notice revisions and one complete reissue were published.

PREFACE

This final report was submitted by Battelle, 505 King Avenue, Columbus, Ohio 43201-2693, under Contract F33615-84-C-5009 with the Wright Research and Development Center, Wright-Patterson Air Force Base, Ohio. Mr. C. L. Harmsworth (WRDC/MLSE) was the Air Force Project Engineer. Mr. Paul E. Ruff was the Battelle Project Manager for the MIL-HDBK-5 Program. Other key Battelle personnel were Mr. Richard Rice and Mr. Stephen Ford. This report covers the period July 3, 1984 through September 3, 1989.

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SUMMARY

Military Handbook, MIL-HDBK-5, "Metallic Materials and Elements for Aerospace Structures," contains standardized mechanical property design values and other types of related design information for metallic materials and structural joints (fasteners), as well as other structural elements used in aircraft, missiles, and space vehicles. Department of Defense agencies, the Federal Aviation Administration (FAA), and the National Aeronautics and Space Administration require the use of the data in this Handbook in the design of aerospace vehicles which are purchased or controlled by them. Because of this wide usage, it is imperative that MIL-HDBK-5 be updated regularly. The Air Force, which is responsible for this specification, contracted with Battelle to provide the many diversified services required to update and improve MIL-HDBK-5.

The objective of this program was to provide the planning, coordination, and implementation required to develop and maintain current design allowable data and other related information in MIL-HDBK-5. A review of the MIL-HDBK-5 Program is presented to provide an insight into the technical accomplishments which were achieved during this contract period.

Design data for new products, as well as for other materials (not previously in the Handbook), were incorporated into MIL-HDBK-5. Design values were established for the following recently developed aluminum alloys: 2090, 2519, 6013, and 7150, as well as several new titanium alloy products: Ti-15V-3Cr-3Al-3Sn sheet, and Ti-10V-2Fe-3Al die forgings. The 2090-T83 sheet represented the first aluminum-lithium alloy to be incorporated into the Handbook. Design values for many of the products in the document were revised as a result of reanalysis according to the current guideline procedures, and/or the availability of additional data. In this manner, the reliability of design values was continuously improved due to the use of the latest statistical analysis procedures and/or expansion of the database. Material specifications are revised periodically, and occasionally minimum tensile yield

and ultimate strength limits are changed. As a result, it was necessary to update S* basis design allowables for compatibility with the reference material specifications. A continuing effort was made to obtain sufficient tensile property data representing production material to establish statistically based A** and B*** values. The design values for a considerable number of products were upgraded from an S basis to an A and B basis. A major milestone was achieved with the establishment of A and B values for D357.0-T6 castings. D357.0 was the first casting alloy for which it was feasible to determine A and B values. Elevated temperature data for various mechanical or physical properties for certain materials were added or revised. Fatigue data in the form of S/N or ϵ/N curves were incorporated for many products. The first strain control fatigue data (for A201.0-T7 castings and Alloy 188 annealed bar) were incorporated into MIL-HDBK-5. These curves were determined and presented in accordance with recently developed guidelines for the analysis of strain control data. Joint design allowables for various fastener systems were included in MIL-HDBK-5.

An important activity involved the development of improved statistical analysis procedures for analyzing various types of data. The latest version of statistical analysis techniques, refinements, and more sophisticated procedures were incorporated into the guidelines. A major improvement in analytical capability resulted from the adaptation of the three-parameter Weibull distribution for use in the determination of A and B values. With the incorporation of this procedure, A and B

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*S-Basis.--The S value is the minimum value specified by the governing Federal, Military, or industry specification (as issued by industry standardization groups such as SAE Aerospace Materials Division, ASTM, etc.) for the material. For certain products heat treated by the user (for example, steels hardened and tempered to a designated F_{tu}), the S value may reflect a specified quality-control requirement. Statistical assurance associated with this value is not known.

**A-Basis.--At least 99 percent of the population of values is expected to equal or exceed the A basis mechanical property allowable, with a confidence of 95 percent.

***B-Basis.--At least 90 percent of the population of values is expected to equal or exceed the B basis mechanical property allowable, with a confidence of 95 percent.

values can be computed for both normal and skewed distributions represented by a moderate sample size. Another major effort involved the development of statistical analytical techniques for the analysis of strain control fatigue data and the update of statistical analysis procedures for load control data.

The above accomplishments were achieved with the assistance of MIL-HDBK-5 task groups. An example of the contribution made by the Elevated Temperature Task Group (ETTG) is contained in the Appendix. The ETTG developed statistical analytical techniques for the analysis of strain control fatigue data and updated and improved the analytical technique for load control fatigue data.

The results of these technical efforts were manifested in the publication of five revisions of MIL-HDBK-5. The Handbook was revised annually incorporating the above-described changes and additions. Four change notice revisions and one complete reissue (MIL-HDBK-5E) were made during the contract period. The adaptation of state-of-the-art statistical techniques, the reanalysis of mechanical property data for products incorporated into MIL-HDBK-5 many years ago, expanded databases, and the use of tensile property, specification minimum values from the latest material specification revisions have resulted in design values with improved reliability. Coupled with incorporation of new types of data, such as strain control fatigue data, and design values for recently developed alloys, MIL-HDBK-5 is a greatly improved, up-to-date specification.

INTRODUCTION

Since many aerospace companies manufacture both commercial and military products, the standardization of metallic materials design data which are acceptable to government procurment or certification agencies is very beneficial to those manufacturers as well as governmental agencies. Although the design requirements for military and commercial products may differ greatly, the design values for the strength of materials and elements or other needed material characteristics are often identical. Therefore, Military Handbook, MIL-HDBK-5, "Metallic Materials and Elements for Aerospace Structures", contains standardized mechanical property design values and other related design information for metallic materials, fasteners, and joints, as well as other structural elements used in aircraft, missiles, and space vehicles. The Handbook lists the minimum strength values for those mechanical properties which are widely used in the design of aerospace structures. Information and data for other properties and characteristics, such as fracture toughness, fatigue strength, creep strength, rupture strength, crack growth rate, and resistance to stress corrosion, are also included. The mechanical property design allowables are presented on a statistical or specification basis. Data for other properties are typical. The products included in this document are standardized with regard to composition and processing methods and are described by industry or government specifications. In addition, the Handbook contains some of the more commonly used methods and formulas by which the strengths of various structural elements or components are calculated. The last chapter of the document contains the guidelines for the analysis and presentation of data for MIL-HDBK-5. Department of Defense agencies, the Federal Aviation Administration (FAA), and the National Aeronautics and Space Administration require the use of the data in this Handbook in the design of aerospace vehicles which are purchased or controlled by them.

Although the Air Force has the responsibility for MIL-HDBK-5, the Handbook is a joint effort of the Air Force, Army, Navy, and FAA. As such, the Air Force contracted with Battelle to provide the many services required to update and improve MIL-HDBK-5.

OBJECTIVE

The overall objective was to provide the planning, coordination, and implementation required to develop and maintain current design allowable data and other related information in MIL-HDBK-5. The technical objective was to add, update, and improve design data and other information in MIL-HDBK-5.

ACCOMPLISHMENTS

Details of the functional activities performed by the Contractor for the MIL-HDBK-5 Program have been reported in previous final reports for this program. Therefore, a summary of the technical accomplishments, which were achieved during the contract period, are described. All of these changes and additions to MIL-HDBK-5 were approved by the MIL-HDBK-5 Coordination Group at biannual meetings.

DESIGN ALLOWABLES

Design values for new products, as well as for other materials (not previously in the Handbook), were incorporated into MIL-HDBK-5, as shown in Table 1*. This list includes products from recently developed aluminum alloys: 2090, 2519, 6013, and 7150. Design data were incorporated for 2090-T83 sheet, the first aluminum-lithium product, Ti-15V-3Cr-3Al-3Sn sheet, and Ti-10V-2Fe-3Al die forging.

The design allowables for many products were revised due to reanalysis in accordance with the current guidelines and/or the availability of additional data, as shown in Table 2. This effort included the establishment of design values for additional sizes (thickness ranges). In this manner, the reliability of design values was continuously improved due to the expansion of the database by the addition of data representing recent production material. Also, the procedures for statistically analyzing data have been continuously modified and improved. Many of the products in MIL-HDBK-5 were incorporated into the Handbook many years ago; in some cases, before standard guidelines were established. Therefore, data for such products, were reanalyzed using the current guideline procedures, so that the design values for these products now have improved reliability.

Material specifications, especially AMS standards, are revised periodically and occasionally minimum tensile yield and ultimate

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*Tables are listed at the end of this report.

strength limits are changed. Therefore, it was necessary to update S basis design allowables for these products for compatibility with the reference specifications. Table 3 presents a list of products for which design values were modified due to specification revisions.

A continuing effort was made to obtain sufficient tensile property data representing production material to establish statistically based A and B values. A list of products for which A and B values were incorporated into MIL-HDBK-5 is shown in Table 4. A major milestone was achieved with the establishment of A and B values for D357.0-T6 aluminum alloy castings. D357.0 was the first casting alloy for which it was feasible to determine A and B values.

Table 5 shows the products for which elevated temperature curves for various mechanical or physical properties were added or revised.

Fatigue data in the form of S/N or ϵ/N curves were incorporated for the products shown in Table 6. The first strain control fatigue data (for A201.0-T7 castings and Alloy 188 annealed bar) were added to MIL-HDBK-5. These curves were determined and presented in accordance with new guidelines which were recently incorporated.

In addition, typical data for elastic and physical properties were added or revised. Information regarding stress corrosion, weldability, and fracture toughness was added or modified. Editorial changes and corrections were made to the Handbook.

To ensure that the Handbook is lean, obsolete information and data were deleted. Table 7 contains a list of the alloys or products deleted from the document. Obsolete information was also deleted from the text of various sections.

Fastener systems for which joint design allowables were incorporated are shown in Table 8. In addition, confirmatory data from other suppliers for fastener systems already in the Handbook were evaluated. This information concerning other suppliers of a fastener was added to the appropriate MIL-HDBK-5 design allowable table via a footnote.

GUIDELINES

An important activity in the updating of MIL-HDBK-5 involved the development and improvement of the guidelines (Chapter 9). Improvement of the statistical analysis procedures for analyzing various types of data was an on-going activity. The latest version of statistical analysis procedures, refinements, and more sophisticated techniques were incorporated into the guidelines. Table 9 is a list of significant revisions of the guidelines.

Previously, A and B values were computed for populations with a normal distribution; or, if the distribution was not normal, A and B values were determined nonparametrically. Based upon the definition of an A value, a minimum of 299 observations were required to determine an A value. Frequently the size of the population was insufficient to permit the determination of an A value. The trend in the distribution of populations for metallic aerospace materials, especially the high strength aluminum alloys, was toward skewed populations. Therefore, a distributional form, which could accommodate skewed as well as normal populations, was needed. An evaluation of the various distributional forms was made and the three-parameter Weibull distribution was selected for use. The method for computing A and B values using the three-parameter Weibull distribution was based upon work by Boeing Commercial Airplane Company. The development of the guidelines for the three-parameter Weibull distribution was coordinated closely with Boeing. The adaptation of the three-parameter Weibull distribution provided a technique for computing A and B values for skewed populations containing fewer than 299 observations.

Another major effort involved the development of a statistical procedure for the analysis of strain control fatigue data. This effort was undertaken by the MIL-HDBK-5 Elevated Temperature Task Group. Simultaneously, the statistical procedure for the analysis of load control fatigue data was modified and updated. The resulting guideline procedures for the analyses of fatigue data are shown in the Appendix. The development of these guidelines is an example of the accomplishments that can be achieved from the combined efforts of individuals within a task group.

As can be seen from Table 9, all of the statistical analysis procedures were updated and new techniques were added to enhance the capability for analysis and to improve the reliability of the resulting design values.

SOFTWARE

Changes in analytical procedures necessitated concomitant revision of existing computer programs and the development of new software to perform these computations. Boeing provided the software for the computation of A and B values from the three-parameter Weibull distribution to the MIL-HDBK-5 Program at no cost. Boeing also agreed to furnish a copy of this software to other aerospace companies, upon request, at no cost. Battelle adapted the Boeing software for use on the MIL-HDBK-5 Program.

TASK GROUPS

The Chairman of the MIL-HDBK-5 Coordination Activity utilizes task groups to study specific problems associated with MIL-HDBK-5. There are currently three task groups. The Fastener Task Group (FTG), which functions continuously, reviews proposals by fastener manufacturers for the incorporation of design values for fastener systems and makes recommendations to the MIL-HDBK-5 Coordination Group concerning changes and additions to Chapter 8 (Fasteners) and Chapter 9 (Guidelines). The objective of the Elevated Temperature Task Group (ETTG), which has been functioning for about 13 years, is to delineate the type of properties that engine manufacturers and other aerospace companies concerned with high temperature applications would like included in MIL-HDBK-5. More recently, the ETTG accepted the assignment to develop statistical analytical techniques for the analysis of strain control fatigue data and to update the analytical procedures for the analysis of load control fatigue data. The Appendix is an example of the results of their most recent work. Recently, the Titanium Casting Task Group (TCTG) was formed to determine the feasibility of establishing A and B values for annealed Ti-6Al-4V castings. It is believed that the availability of A and B values in MIL-HDBK-5 for Ti-6Al-4V castings will facilitate the application of this product without the use of a "casting factor" in design. Battelle personnel participated in these three task groups. Mr. Stephen Ford is Chairman of the FTG, Mr. Richard Rice is Chairman of the ETTG, and Mr. Paul Ruff is Secretary of the TCTG.

MIL-HDBK-5 REVISIONS

The changes and additions to MIL-HDBK-5, approved at the 67th through 76th MIL-HDBK-5 Coordination Meetings, were incorporated into the Handbook. The following revisions of MIL-HDBK-5 were published:

Change Notice 2 to MIL-HDBK-5D, dated 1 May 1985
Change Notice 3 to MIL-HDBK-5D, dated 1 May 1986
MIL-HDBK-5E, dated 1 June 1987
Change Notice 1 to MIL-HDBK-5E, dated 1 May 1988
Change Notice 2 to MIL-HDBK-5E, dated 1 May 1989.

CONCLUSIONS

The adaptation of state-of-the-art statistical techniques, the reanalysis of mechanical property data for products incorporated into MIL-HDBK-5 many years ago, expanded databases, and the use of tensile property, specification minimum values from the latest material specification revisions, have resulted in design values with improved reliability. Coupled with the incorporation of new types of data, such as strain control fatigue data and design values for recently developed alloys, MIL-HDBK-5 is a greatly improved up-to-date specification.

TABLE 1. PRODUCTS INCORPORATED INTO MIL-HDBK-5 WITH S BASIS DESIGN VALUES INCLUDING STRESS-STRAIN CURVES

7175-T73511 Extrusion
7175-T7452 Hand Forging
A201-T7 Casting
Ti-15V-3Cr-3Al-3Sn Solution Treated and Aged Sheet
Ti-10V-2Fe-3Al Solution Treated and Aged Die Forging
15-5PH (H1025) Plate
7050-T74511 Extrusion
15-5PH (H935) Casting
7149-T73 Hand Forging
Ti-15V-3Cr-3Al-2Sn Solution Treated Sheet
7049-T73 Bare Plate
6013-T6 Sheet
7150-T6151 Bare Plate
*7150-T7751 Bare Plate
7150-T61511 Extrusion
*7150-T77511 Extrusion
2519-T87 Plate
**2090-T83 Sheet
Ti-6Al-4V Annealed Die Forging
**7050-T7452 Die Forging
**7149-T73 Extrusion
**MP159 Cold Worked and Aged Bar

*Incorporation pending publication of AMS Specification.

**Approved at the 77th MIL-HDBK-5 Meeting (May 1989); will be incorporated into MIL-HDBK-5F.

TABLE 2. PRODUCTS FOR WHICH DESIGN VALUES WERE REVISED DUE TO REANALYSIS IN ACCORDANCE WITH CURRENT GUIDELINES AND/OR AVAILABILITY OF ADDITIONAL DATA

7178-T651 Bare and Clad Plate
7178-T651 7011 Clad Plate
7075-T651 Bare and Clad Plate
7075-T651 7011 Clad Plate
7075-T62 Bare and Clad Plate
7075-T62 7011 Clad Plate
7075-T7351 Bare Plate
2024-T351 Bare and Clad Plate
7050-T7451 Bare Plate
7010-T7451 Bare Plate
7010-T7651 Bare Plate
7475-T7351 Bare Plate
A-286 Solution Treated and Aged (All Products)
*7175-T74 Hand Forging
*7075-T76 Bare and Clad Sheet
*7075-T76 7011 Clad Sheet
*7049-T73 Hand Forging
*Inconel 718 Solution Treated and Aged Sheet
*7075-T7651 Bare Plate
*7050-T73511 Extrusion
*7050-T74511 Extrusion
*7050-T76511 Extrusion
*7050-T7651 Bare Plate
*300 Series Stainless Steel Sheet-Annealed, 1/4H, 1/2H, 3/4H, and FH
*7075-T7351 Bare Plate (Extended Thickness Coverage)
2024-T851 Bare and Clad Plate

*Additional data available.

TABLE 2. (Concluded)

*Inconel 625 Annealed Sheet and Plate
*Inconel 625 Annealed Bar
*Inconel 718 Solution Treated and Aged Bar
Inconel Alloy X-750 Sheet, Plate, Bar, and Forging
*C17200 (Beryllium Copper) Rod and Bar in AT Temper
*C17200 (Beryllium Copper) Rod and Bar in HT Temper
*A357.0-T6 Casting
*A354.0-T6 Casting
*355.0-T6 Casting
*C355.0-T6 Casting
*356.0-T6 Casting
*A356.0-T6 Casting
*359.0-T6 Casting
*520.0-T4 Casting

*Additional data available.

TABLE 3. PRODUCTS FOR WHICH DESIGN VALUES WERE REVISED DUE TO SPECIFICATION CHANGES

15-5PH Bar and Forging
7050-T7451 Bare Plate
Inconel Alloy 600 Bar, Rod, and Forging
Tin, Manganese, and Aluminum Bronze Castings
Aluminum Bronze Bar, Rod, and Forging
Low Alloy Steels
Ti-6Al-4V Annealed Bar
7475-T61 Bare Sheet
7475-T7651 Bare Plate
7475-T651 Bare Plate
2025-T6 Die Forging
7475-T651 Bare Plate
7475-T761 Bare Sheet
7475-T7351 Bare Plate
Ti-6Al-4V Solution Treated and Aged Extrusion
Ti-6Al-6V-2Sn Annealed Extrusion
*Ti-6Al-4Sn-4Zr-2Mo Triplex Annealed Sheet

*Approved at the 77th MIL-HDBK-5 Meeting (May 1989); will be incorporated into MIL-HDBK-5F.

TABLE 4. PRODUCTS FOR WHICH A AND B DESIGN VALUES WERE INCORPORATED INTO MIL-HDBK-5

7050-T7651 Bare Plate
7050-T7451 Bare Plate
7475-T761 Clad Sheet
7475-T61 Clad Sheet
7475-T7351 Bare Plate
7010-T7651 Bare Plate
7175-T74 Die Forging
*7075-T6 Clad Sheet
7050-T7452 Hand Forging
D357.0-T6 Casting
C17200 (Beryllium Copper) Rod and Bar in HT Temper
**Ti-6Al-2Sn-4Zr-2Mo Duplex Annealed Sheet and Bar

*A and B values revised.

**Existing A and B values reaffirmed.

TABLE 5. PRODUCTS FOR WHICH ELEVATED TEMPERATURE CURVES WERE ADDED OR REVISED

A-286 Solution Treated and Aged (All Products)
300 Series Annealed Stainless Steel Sheet
Inconel 625 Bar
Inconel Alloy X-750 Sheet, Plate, and Bar
*Ti-6Al-4V (All Products)
*Ti-5Al-2.5Sn (All Products)
*Ti-13V-11Cr-3Al (All Products)

*Approved at the 77th MIL-HDBK-5 Meeting (May 1989); will be incorporated into MIL-HDBK-5F.

TABLE 6. PRODUCTS FOR WHICH FATIGUE DATA (S/N OR ε/N CURVES) WERE INCORPORATED INTO MIL-HDBK-5

ZK60A-T5 Bar
HK31A-H24 Sheet
Ti-13V-11Cr-3Al Annealed and Solution Treated and Aged Sheet
N-155 Solution Treated and Aged Bar
Ti-6Al-4V Solution Treated and Aged Plate
7049-T73 Die Forging
7475-T61 Bare Sheet
7475-T761 Bare Sheet
AZ31B-F Forged Disk
7050-T74 Die Forging
7050-T74 Bare Plate
7050-T73 Extrusion
7050-T74 Extrusion
7050-T76 Extrusion
7050-T74 Hand Forging
7049-T73 Hand Forging
7149-T73 Hand Forging
*A201.0-T7 Casting
*Alloy 188 Annealed Bar
Ti-6Al-4V Annealed Sheet

*Strain control data.

TABLE 7. OBSOLETE ALLOYS OR PRODUCTS DELETED FROM MIL-HDBK-5

Ti-8Mn
Ti-7Al-Mo
PH14-8Mo Sheet
7011 Clad 7075 Sheet and Plate
7178
17-7PH Bars and Forgings
PH15-7Mo Bars and Forgings
Ti-11Sn-5Zr-2Al-1Mo
712.0 Casting
*5456-H323 Sheet
*5456-H343 Sheet

*Approved at the 77th MIL-HDBK-5 Meeting (May 1989); deletion will be effected in MIL-HDBK-5F.

TABLE 8. FASTENER SYSTEMS FOR WHICH JOINT DESIGN VALUES WERE INCORPORATED INTO MIL-HDBK-5

MIL-B-8831/4 100° Head Steel Sleeve Bolts in Clad 2024-T3 and Clad 7675-T6 Sheet
CR4622 Blind 100° Flush Head Locked Spindle A-286 Rivets in Clad 7075-T6 Sheet
CR4623 Protruding Head Locked Spindle A-286 Rivets in Clad 7075-T6 Sheet
MS14218 Flush Shear Head 2117 Rivets in Clad 2024-T3 Sheet
BRFR Flush Head 7050 Rivets in Clad 2024-T3 and Clad 7075-T6 Sheet
CR3242 100° Flush Head 5056 Blind Rivets in Clad 2024-T3 Sheet (Addition of 1/4 In. Dia.)
CR3243 Protruding Head 5056 Blind Rivets in Clad 2024-T3 Sheet (Addition of 1/4 In. Dia.)
BRFR Flush Head 7050 Rivets in Clad 7075-T6 Sheet
LGPL2SC 100° Flush Shear Head Ti-6Al-4V Lockbolts in Clad 2024-T3 Sheet and Clad 7075-T6 Sheet
CR4522 100° Flush Head Locked Spindle Blind Monel Rivets in Clad 7075-T6 Sheet
CR4523 Protruding Head Locked Spindle Blind Monel Rivets in Clad 7075-T6 Sheet

TABLE 9. GUIDELINE ADDITIONS, IMPROVEMENTS, AND REVISIONS FOR MIL-HDBK-5

Added Three-Parameter Weibull Distribution for Determination of A and B Values
Added Anderson-Darling Test for Normality and Weibullness
Added Treatment of Test Specimen Locations
Added Analytical Procedure for Analyzing Grouped Data for Determination of A and B Values
Revised Procedure for Determination of Acceptability of Sample Population
Revised Two-Sample Anderson-Darling Test Procedure
Revised Procedure for Confirmatory Fastener Data For Other Than Original Supplier
Revised Procedures for Calculating Design Allowables Using Regression Analysis
Revised Procedures for Specifying the Population and Replaced 2-Sample with K-Sample Anderson-Darling Test
Revised Procedures for Determining Adequacy of Regression
Revised Primary Testing Direction for Several Alloy Systems
Developed Improved Procedures for Analyzing K_{Ic} Data
Developed Procedure for Lap Joint Testing as Part of Fastener Qualification to Applicable Specifications
Revised Procedures for the Analysis of Load-Control Fatigue Data and Added Procedure for the Analysis of Strain-Control Fatigue Data
Addition to Permit Use of Dimensionally Discrepant Castings for Test Programs to Determine Derived Property Values
*Addition to Permit Use of Special Test Configurations for Test Programs to Determine Derived Property Values
*Deleted Procedure for the Use of Lot Averages for Determination of A and B Values

*Approved at the 77th MIL-HDBK-5 Meeting (May 1989); will be effected in MIL-HDBK-5F.

APPENDIX

GUIDELINES FOR ANALYSIS OF FATIGUE DATA FOR MIL-HDBK-5

Developed by

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9.3.4 FATIGUE DATA ANALYSIS

9.3.4.1 *Introduction* --Fatigue has been defined as "the process of progressive localized permanent structural change occurring in a material subjected to conditions that produce fluctuating stresses and strains at some point or points, and which may culminate in cracks or complete fracture after a sufficient number of fluctuations."

For many years tests have been performed on specimens having simple geometries in attempts to characterize the fatigue properties of particular materials. Fatigue tests have been conducted for many reasons. Basic fatigue-life information may be desired for design purposes, or to evaluate the differences between materials. The effects of heat treatments, mechanical working or material orientation may also be studied through comparative fatigue testing.

Many types of machines and specimen designs have been used to develop fatigue data. Machine types include mechanical, electromechanical, hydraulic, and ultrasonic. Specimens have been designed for testing in cyclic tension and/or compression, bending, and torsion. Cyclic loading conditions have been produced by rotating bending, axial loading and cantilever bending. In- and out-of-phase biaxial and multiaxial fatigue conditions have also been examined using specially designed specimens. Tests have been conducted in a variety of simulated environments including temperatures ranging from cryogenic to near melting point levels. The fatigue data included in MIL-HDBK-5 are limited to constant-amplitude axial fatigue data on simple laboratory specimens tested according to ASTM E606 [reference 9.3.4.1(a)]. Data obtained under both strain control and load (stress) control are included. Figure 9.3.4.1(a) shows examples of trends for stress-life and strain-life fatigue data. Generally, stress-life data for unnotched specimens are limited to stress levels that produce intermediate-to-long fatigue lives because of unstable cyclic creep and tensile failure that can occur at high stress ratios in load-control testing. This phenomenon is shown in Figure 9.3.4.1(b). Strain-life curves are often focused on strain ranges that produce short-to-intermediate fatigue lives because of strain rate and frequency limitations which require long testing times to generate long-life fatigue data under strain control. However, there is no inherent limit to the life range that can be evaluated in strain-control testing.

For fatigue to occur, a material must undergo cyclic plasticity, at least on a localized level. The relationship between total strain, plastic strain and elastic strain is shown in Figure 9.3.4.1(c). Low-cycle fatigue tests involve relatively high levels of cyclic plasticity. Intermediate-life fatigue tests usually involve plastic strains of the same order as the elastic strains. Long-life fatigue tests normally involve very low levels of cyclic plasticity. These trends are shown in Figure 9.3.4.1(d). In the MIL-HDBK-5 fatigue analysis guidelines, engineering strain is denoted as e and true or local strain is denoted as ε. These symbols are used interchangeably within MIL-HDBK-5 for small strain values.

The limited plasticity involved in intermediate and long-life fatigue tests often results in a similar stress-strain response for both fully reversed strain-control and fully reversed load-control tests. A fatigue test under strain control that produces a stable maximum stress of X, should produce (on the average) a fatigue life that is comparable to that obtained for a sample tested under load control at a maximum stress of X. Strictly speaking, the results are likely to be most comparable in terms of crack initiation life and not total life. If the comparison is made in terms of total life, the load-control results will tend to be more conservative than those generated by strain-control testing. When a specimen cracks in a test under strain control, it will usually display a decrease in maximum tensile load. Under load control, the maximum tensile load will remain constant but stress will increase as the crack grows, resulting in a shorter period of crack growth before the specimen fails.

A number of factors can significantly influence fatigue properties for a particular material--whether the data are developed under load or under strain control. The surface condition

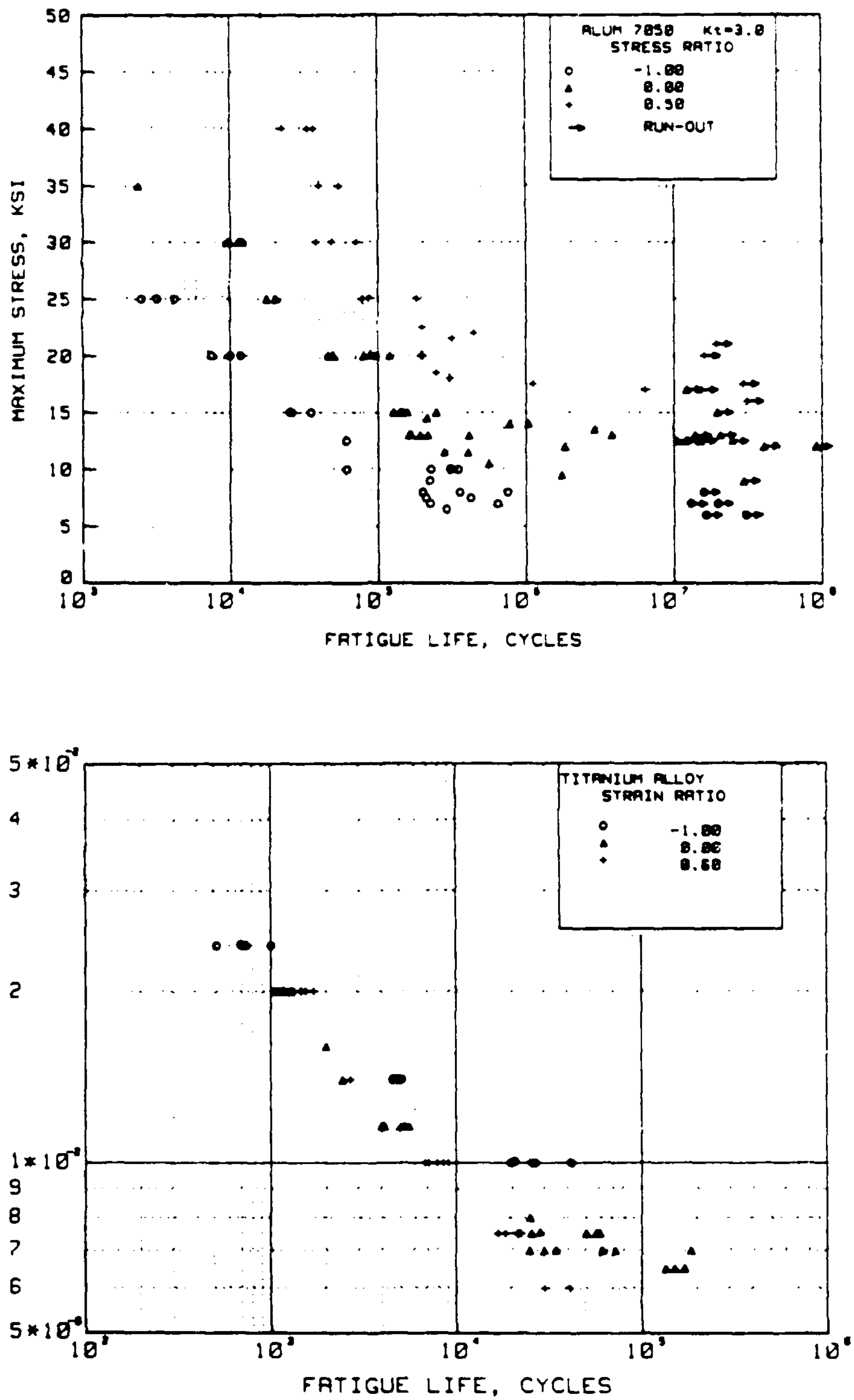

FIGURE 9.3.4.1(a). *Examples of stress-life and strain-life fatigue trends.*

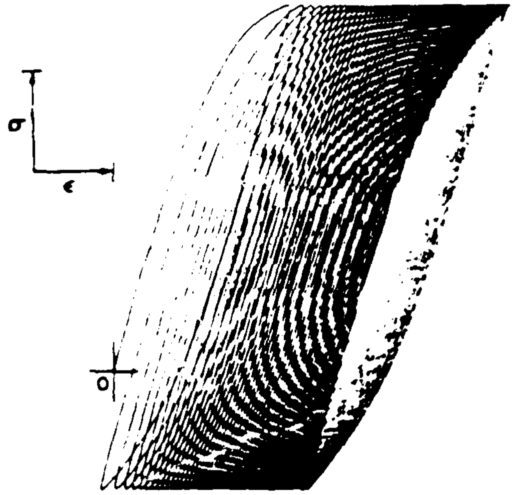

FIGURE 9.3.4.1(b). *Example of cyclic creep phenomenon that can occur in a load control test with a high tensile mean stress [Reference 9.3.4.1(b)].*

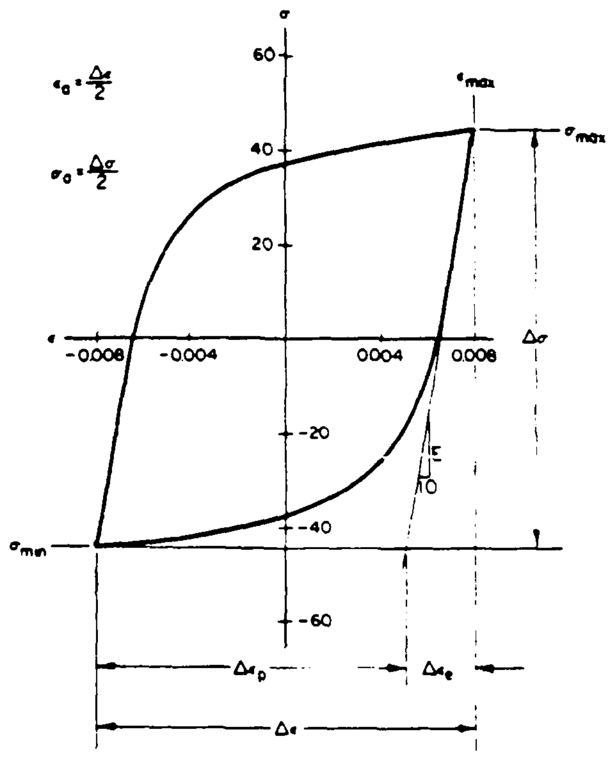

FIGURE 9.3.4.1(c). *A typical hysteresis loop for a material tested in fatigue under strain control illustrating the relationship between stress and strain parameters.*

Supersedes page 9-71 of MIL-HDBK-5E.

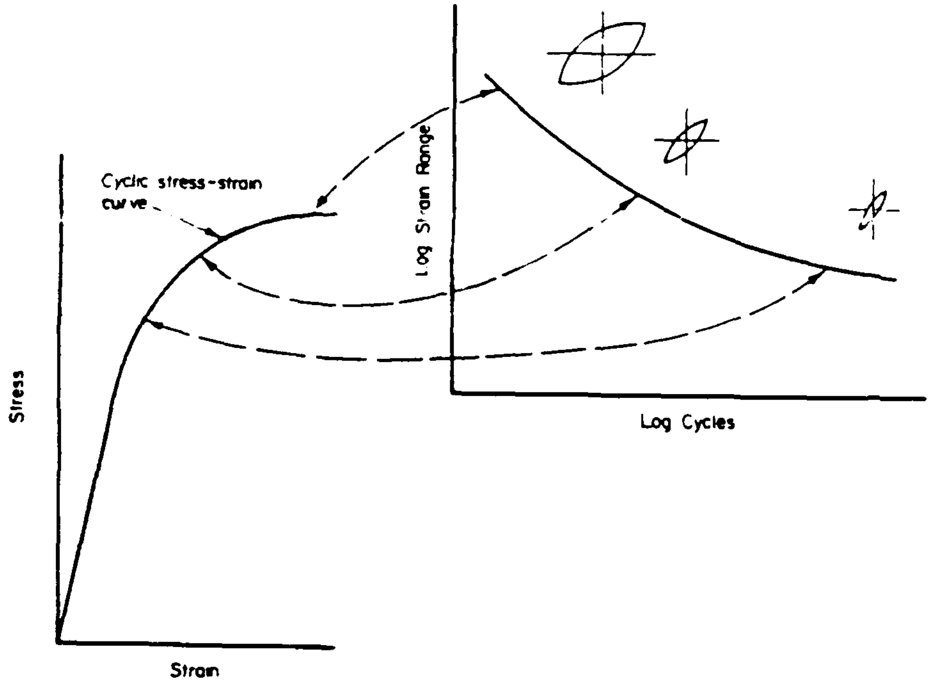

FIGURE 9.3.4.1(d). *An example of a strain-life fatigue curve and the stress-strain response at short, intermediate, and long fatigue lives.*

(such as surface roughness) of the test specimens is an important factor. The methods used for fabricating the specimens are also important--principally because such methods influence the state of surface residual stresses and residual stress profiles. Other factors such as mean stress or strain, specimen geometry (including notch type), heat treatment, environment, frequency and temperature can also be significant variables. In MIL-HDBK-5, fatigue data are always presented in separate displays for different theoretical stress concentration factors. However, data sets may be presented for various combinations of variables if preliminary analyses indicate that the data sets are compatible. In any case, it is very important to fully document both the input data and their resulting illustrations in MIL-HDBK-5 with regard to variables that can influence fatigue.

9.3.4.2 *Disclaimer*.–The selection of the specific procedures and methods that are outlined in this guideline for fatigue data presentation should not be construed as an endorsement of these procedures and methods for life prediction of components. The selection was made for consistency in data presentation only. For the purpose of life prediction, other methods and models are also commonly employed. Depending on the material, component and loading history, other models may be more appropriate for the particular situation. It is beyond the scope of these guidelines to make recommendations with respect to a specific life prediction methodology (e.g., the construction of design allowable fatigue curves).

9.3.4.3 *Terminology*.–The symbols and definitions used in analyzing and reporting fatigue data have been standardized by the American Society for Testing and Materials (ASTM). Most definitions of terms are consistent with those defined by the ASTM. However, some of the symbols and definitions differ and are noted with an asterisk (*). The terms are listed in alphabetical order. In the definitions listed below, the term for load (P) is taken to represent stress (S), strain (ε or e), or any other expression or function of loading. The most common terms relating to fatigue loading are illustrated in Figure 9.3.4.3.

Alternating Load.--See Loading Amplitude.

Confidence Interval.--An interval estimate of a population parameter computed so that the statement "the population parameter lies in this interval" will be true, on the average, in a stated proportion of the times such statements are made.

Confidence Level (or Coefficient).--The stated portion of the time that the confidence interval is expected to include the population parameter.

Confidence Limits*.--The two numeric values that define a confidence interval.

Constant-Amplitude Loading.--A loading in which all of the peak loads are equal and all of the valley loads are equal.

Constant-Life Fatigue Diagram.--A plot (usually on Cartesian coordinates) of a family of curves, each of which is for a single fatigue life, N--relating S, S_{max} and/or S_{min} to the mean stress, S_m. Generally, the constant life fatigue diagram is derived from a family of S-N curves, each of which represents a different stress ratio (A or R) for a 50 percent probability of survival. NOTE--MIL-HDBK-5 no longer presents fatigue data in the form of constant- life diagrams.

Cycle.--Under constant-amplitude loading, the load varies from the minimum to the maximum and then to the minimum load (see Figure 9.3.4.3). The symbol n or N (see definition of fatigue life) is used to indicate the number of cycles.

Discontinued Test.--See Runout.

Fatigue.--The process of progressive localized permanent structural change occurring in a material subjected to conditions that produce fluctuating stresses and strains at some point or points, and which may culminate in cracks or complete fracture after a sufficient number of fluctuations. NOTE--Fluctuations may occur in stress and in time (frequency), as in the case of "random vibration".

Fatigue Life.--N--the number of cycles of stress or strain of a specified character that a given specimen sustains before failure of a specified nature occurs.

Fatigue Limit.--S_f--the limiting value of the median fatigue strength as N becomes very large. NOTE--Certain materials and environments preclude the attainment of a fatigue limit. Values tabulated as "fatigue limits" in the literature are frequently (but not always) values of S_N for 50 percent survival at N cycles of stress in which $S_m = 0$.

Fatigue Loading.--Periodic or non-periodic fluctuating loading applied to a test specimen or experienced by a structure in service. (Also known as cyclic loading.)

Fatigue Notch Factor*.--The fatigue notch factor, K_f (also called fatigue strength reduction factor) is the ratio of the fatigue strength of a specimen with no stress concentration to the fatigue strength of a specimen with a stress concentration at the same number of cycles for the

*Different from ASTM.

Supersedes page 9-73 of MIL-HDBK-5E.

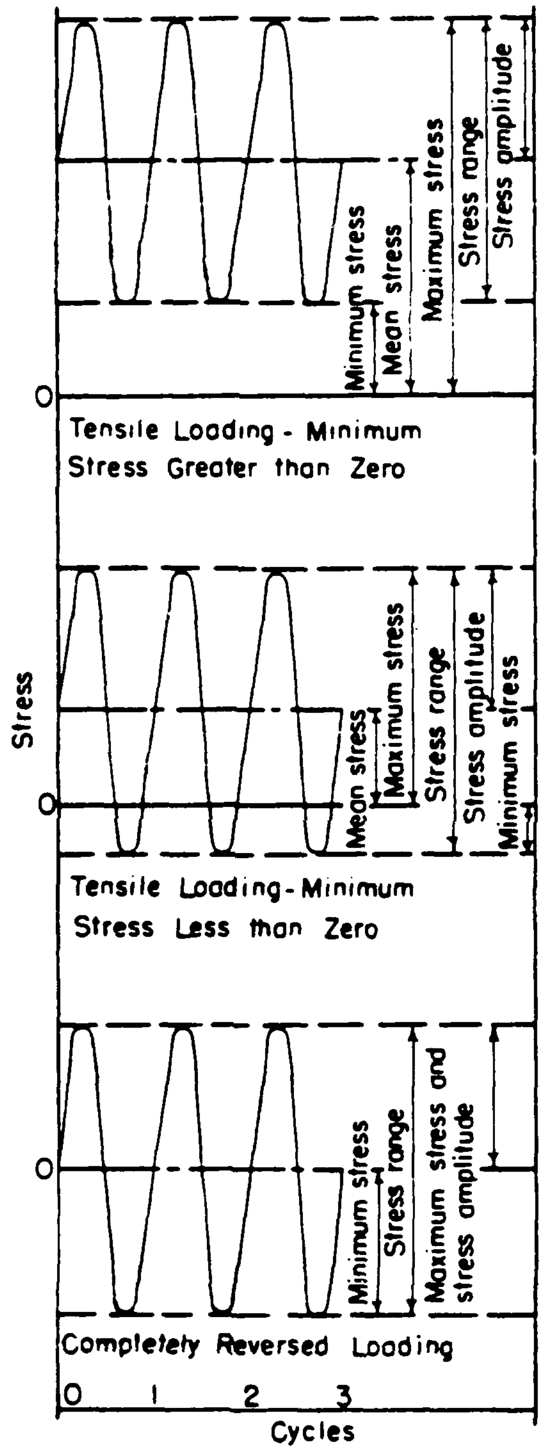

FIGURE 9.3.4.3. *Typical fatigue loadings.*

Supersedes page 9-74 of MIL-HDBK-5E.

same conditions. NOTE--In specifying K_f, it is necessary to specify the geometry, mode of loading, and the values of S_{max}, S_m and N for which it is computed.

Fatigue Notch Sensitivity.--The fatigue notch sensitivity, q, a measure of the degree of agreement between K_f and K_t. NOTE--the definition of fatigue notch sensitivity is $q = (K_f - 1)/(K_t - 1)$.

Hysteresis Diagram.--The stress-strain path during a fatigue cycle.

Interrupted Test*.--Tests which have been stopped before failure because of some mechanical problem, e.g., power failure, load or temperature spikes.

Loading Amplitude.--The loading amplitude, P_a, S_a, or ε_a represents one-half of the range of a cycle (see Figure 9.3.4.3). (Also known as alternating load, alternating stress or alternating strain.)

Loading (Unloading) Rate.--The time rate of change in the monotonically increasing (decreasing) portion of the load-time function.

Load Ratio.--The load ratio, R, A, or R_ε, A_ε, or R_σ, A_σ, is the algebraic ratio of the two loading parameters of a cycle; the two most widely used ratios are

$$R = \frac{\text{minimum load}}{\text{maximum load}} = \frac{P_{min}}{P_{max}}$$

or

$$R_\sigma = \frac{S_{min}}{S_{max}}$$

or

$$R_\varepsilon = \varepsilon_{min}/\varepsilon_{max}$$

and

$$A = \frac{\text{loading amplitude}}{\text{mean load}} = \frac{P_a}{P_m} \text{ or } \frac{S_a}{S_m}$$

*Different from ASTM.

Supersedes page 9-75 of MIL-HDBK-5E

$$A_\varepsilon = \frac{\text{strain amplitude}}{\text{mean strain}} = \frac{\varepsilon_a}{\varepsilon_m} \text{ or } (\varepsilon_{max} - \varepsilon_{min})/(\varepsilon_{max} + \varepsilon_{min})$$

NOTE--load ratios R or R_ε are generally used in MIL-HDBK-5.

Maximum Load.--The maximum load, P_{max}, S_{max}, ε_{max} is the load having the greatest algebraic value.

Mean Load.--The mean load, P_m is the algebraic average of the maximum and minimum loads in constant-amplitude loading:

$$P_m = \frac{P_{max} + P_{min}}{2}, \text{ or}$$

$$S_m = \frac{S_{max} + S_{min}}{2}, \text{ or}$$

$$\varepsilon_m = \frac{\varepsilon_{max} + \varepsilon_{min}}{2},$$

or the integral average of the instantaneous load values.

Median Fatigue Life.--The middlemost of the observed fatigue life values (arranged in order of magnitude) of the individual specimens in a group tested under identical conditions. In the case where an even number of specimens are tested, it is the average of the two middlemost values (based on log lives in MIL-HDBK-5). NOTE 1--The use of the sample median instead of the arithmetic mean (that is, the average) is usually preferred. NOTE 2--In the literature, the abbreviated term "fatigue life" usually has meant the median fatigue life of the group. However, when applied to a collection of data without further qualification, the term "fatigue life" is ambiguous.

Median Fatigue Strength at N Cycles.--An estimate of the stress level at which 50 percent of the population would survive N cycles. NOTE--The estimate of the median fatigue strength is derived from a particular point of the fatigue-life distribution, since there is no test procedure by which a frequency distribution of fatigue strengths at N cycles can be directly observed. That is, one can not perform constant-life tests.

Minimum Load.--The minimum load, P_{min}, S_{min}, or ε_{min} is the load having the least algebraic value.

Outlier*.--An experimental observation which deviates markedly from other observations in the sample. An outlier is often either an extreme value of the variability in the data, or the result of gross deviation in the material or experimental procedure.

*Different from ASTM.

Supersedes page 9-76 of MIL-HDBK-5E.

Peak.--The point at which the first derivative of the load-time history changes from a positive to a negative sign; the point of maximum load in constant-amplitude loading (see Figure 9.3.4.3).

Precision* --The degree of mutual agreement among individual measurements. Relative to a method of test, precision is the degree of mutual agreement among individual measurements made under prescribed like conditions. The lack of precision in a measurement may be characterized as the standard deviation of the errors in measurement.

Range.--Range, ΔP, S_r, $\Delta\varepsilon$, ε_r, $\Delta\sigma$ is the algebraic difference between successive valley and peak loads (positive range or increasing load range) or between successive peak and valley loads (negative range or decreasing load range), see Figure 9.3.4.3. In constant-amplitude loading, for example, the range is given by $\Delta P = P_{max} - P_{min}$.

Residual*.--The difference between the observed fatigue (log) life and the fatigue (log) life estimated from the fatigue model at a particular stress/strain level.

Runout*.--A test that has been terminated prior to failure. Runout tests are usually stopped at an arbitrary life value because of time and economic considerations. NOTE--Runout tests are useful for estimating a pseudo fatigue limit for a fatigue data sample.

Sample.--The number of specimens selected from a population for test purposes. NOTE--The method of selecting the sample determines the population about which statistical inferences or generalization can be made.

Sample Average (Arithmetic Mean).--The sum of all the observed values in a sample divided by the sample size (number). It is a point estimate of the population mean.

Sample Median.--The middle value when all observed values in a sample are arranged in order of magnitude if an odd number of samples are tested. If the sample size is even, it is the average of the two middlemost values. It is a point estimate of the population median, or 50 percentile point.

Sample Standard Deviation*.--The standard deviation of the sample, s, is the square root of the sample variance. It is a point estimate of the standard deviation of a population, a measure of the "spread" of the frequency distribution of a population. NOTE--this value of s provides a statistic that is used in computing interval estimates and several test statistics.

Sample Variance*.--Sample variance, s^2, is the sum of the squares of the differences between each observed value and the sample average divided by the sample size minus one. It is a point estimate of the population variance. NOTE--This value of s^2 provides both an unbiased point estimate of the population variance and a statistic that is used on computing the interval estimates and several test statistics. Some texts define s^2 as "the sum of the squared differences between each observed value and the sample average divided by the sample size", however; this statistic underestimates the population variance, particularly for small sample sizes.

Significant (Statistically significant).--An effect or difference between populations is said to be present if the value of a test statistic is significant, that is, lies outside of predetermined limits. NOTE--An effect that is statistically significant may not have engineering importance.

*Different from ASTM.

Supersedes page 9-77 of MIL-HDBK-5E.

Significance Level.--The stated probability (risk) that a given test of significance will reject the hypothesis that a specified effect is absent when the hypothesis is true

S-N Curve for 50 Percent Survival*.--A curve fitted to the median values of fatigue life at each of several stress levels. It is an estimate of the relationship between applied stress and the number of cycles-to-failure that 50 percent of the population would survive. NOTE 1--This is a special case of the more general definition of S-N curve for P percent survival. NOTE 2--In the literature, the abbreviated term "S-N Curve" usually has meant either the S-N curve drawn through the mean (averages) or through the medians (50 percent values) for the fatigue life values. Since the term "S-N Curve" is ambiguous, it should be used only when described appropriately. NOTE 3--Mean S-N curves (based on log lives) are shown in MIL-HDBK-5.

S-N Diagram.--A plot of stress against the number of cycles to failure. The stress can be S_{max}, S_{min}, or S_a. The diagram indicates the S-N relationship for a specified value of S_m, A, or R and a specified probability of survival. Typically, for N, a log scale (base 10) is used. Generally, for S, a linear scale is used, but a log scale is used occasionally. NOTE--S_{max}-versus-log N diagrams are used commonly in MIL-HDBK-5.

Theoretical Stress Concentration Factor (or Stress Concentration Factor).--This factor, K_t, is the ratio of the nominal stress to the greatest stress in the region of a notch (or other stress concentrator) as determined by the theory of elasticity (or by experimental procedures that give equivalent values). NOTE--The theory of plasticity should not be used to determine K_t.

Tolerance Interval.--An interval computed so that it will include at least a stated percentage of the population with a stated probability.

Tolerance Level.--The stated probability that the tolerance interval includes at least the stated percentage of the population. It is not the same as a confidence level but the term confidence level is frequently associated with tolerance intervals.

Tolerance Limits.--The two statistics that define a tolerance interval. (One value may be "minus infinity" or "plus infinity".)

Transition Fatigue Life*.--The point on a strain-life diagram where the elastic and plastic strains are equal.

Waveform.--The shape of the peak-to-peak variation of a controlled mechanical test variable (for example, load, strain, displacement) as a function of time.

The following symbols are used frequently for the terms covered by the preceding definitions. For stress, the use of S with appropriate lower case subscripts is preferred for general purposes; for mathematical analysis the use of Greek symbols is generally preferred.

Symbol	Term
A	"A" ratio, loading amplitude/mean load, or area
A_ε	Strain "A" ratio, strain amplitude/mean strain
A_i	Model parameter

*Different from ASTM.

Supersedes page 9-78 of MIL-HDBK-5E.

Symbol	Term
D or d	diameter, or Durbin Watson statistic
E	modulus of elasticity in tension or compression
e	engineering strain
ε	true or local strain
ε_{eq}*	equivalent strain
ε_m	mean strain, $(\varepsilon_{max} + \varepsilon_{min})/2$
ε_{max}	maximum strain
ε_{min}	minimum strain
$\Delta\varepsilon$ or ε_r*	strain range, $\varepsilon_{max} - \varepsilon_{min}$
$\Delta\varepsilon_e$	elastic strain range
$\Delta\varepsilon_p$	plastic strain range
K_f	fatigue notch factor, or fatigue strength reduction factor
K_t	theoretical stress concentration factor
μ	Poisson's ratio
N	fatigue life, number of cycles
N_f	fatigue life, cycles to failure
N_i*	fatigue life, cycles to initiation
N_t*	transition fatigue life where plastic and elastic strains are equal
P	load
P_a	load amplitude
P_m	mean load
P_{max}	maximum load
P_{min}	minimum load
q	fatigue notch sensitivity

*Different from ASTM.

Supersedes page 9-79 of MIL-HDBK-5E.

Symbol	Term
R	load (stress) ratio, or residual (observed minus predicted value)
R_ε	strain ratio, $\varepsilon_{min}/\varepsilon_{max}$
S	nominal or engineering stress
s	sample standard deviation
s^2	sample variance
S_a	stress amplitude
S_{eq}*	equivalent stress
S_f	fatigue limit
S_m	mean stress
S_{max}	maximum stress
S_{min}	minimum stress
SR	studentized residual
ΔS (S_r)*	stress range
TUS (S_u)*	tensile ultimate strength
σ	true or local stress; or population standard deviation
σ_x	population standard deviation of x
σ_x^2	population variance of x
$\Delta\sigma$	true or local stress range.

9.3.4.4 *Data Requirements* --Both strain-controlled and load-controlled axial fatigue data can be considered for inclusion in MIL-HDBK-5. Constant-amplitude test data are the primary focus. Well documented, initial and/or periodic overstrain data may also be included. Data obtained under strain control are considered only on unnotched, uniform-gage-length specimens, while both notched and unnotched specimens are considered for load-control test conditions.

Fatigue data generated under load control over a wide range of stress ranges and ratios can be acceptable. However, load-control experiments on unnotched samples can produce ratcheting failures rather than true fatigue failures. This can be a problem with materials that cyclically soften. In the absence of cyclic stress-strain data, the acceptability of short-life data obtained under load control on

*Different from ASTM.

Supersedes page 9-80 of MIL-HDBK-5E.

unnotched specimens can be difficult to evaluate. In most cases, test results obtained under load control that have produced average fatigue lives on unnotched specimens of less than 10^3 cycles should be excluded. Short-life, load-control data generated on notched samples tested at high stress levels may be considered--provided that the nominal stresses are below the onset of plastic strain.

Fatigue data generated under strain control over a wide range of strain ratios and ranges can be acceptable also. High-strain-range tests producing low fatigue lives can be considered assuming that documented bending strains were held within ASTM E606 limits and buckling failures were not produced. Documenting the stress response associated with each test result is important. The stress data that are reported should reflect the material's stable response, including effects of cyclic hardening or softening and of mean stress relaxation--provided such data were obtained at other than R_ϵ = -1. The normal convention is to report the stress values associated with one-half the material's fatigue life to crack initiation. Several criteria are commonly used to define crack initiation in a test under strain control. The primary requirements for inclusion in MIL-HDBK-5 are that the criteria be specific and applied consistently. If multiple sources of data are being considered, the potential problem of inconsistent crack initiation criteria must be addressed before the data are merged.

If strain-control data only are reported with fatigue test results obtained under strain control, these data must be supported by well-documented cyclic stress-strain curves and mean stress relaxation data for that specific material.

For fatigue experiments under load control, data are normally generated at specific stress ratios or mean stress levels. If the stress ratio is held constant, a fatigue curve is generated by performing a series of experiments at prescribed maximum stress levels such that the desired range of fatigue lives is achieved. If mean stress levels are held constant, a range of maximum stress levels is also used, but the stress ratio for each maximum stress level is different. Presentation of the latter type of data in a traditional S_{max}-versus-log N_f display, with individual stress ratio curves, can be cumbersome because of the large number of stress ratios involved. For this reason, constant mean-stress fatigue data should be identified by mean stress level, even though they are plotted on a standard S_{max}-versus-log N_f display. The illustrations should be clearly labeled to properly identify the mean-stress or stress-ratio levels.

To evaluate analytically the effects of stress or strain ratio on the fatigue performance of a particular material, it is recommended that data be available for at least three stress or strain ratios, or alternatively, three mean-stress or strain levels. Similarly, at least three stress or strain levels are recommended to evaluate the effects of mean stress on fatigue performance. In the case of data under strain control, a specific strain ratio or mean strain may not define a mean-stress level uniquely. For R_ϵ = -1.0 (mean strain = 0), the stress ratio is usually very close to R = -1.0 (mean stress = 0)--if it is not, the data should be examined carefully for validity. For strain ratios greater than R_ϵ = -1.0, the stress ratio is usually less than the strain ratio, and the difference is generally greater at the greatest strain ranges. For very large strain ranges in ductile materials, the stable stress ratio will approach R = -1.0 (mean stress = 0), regardless of the strain ratio, R_ϵ. Mean stress relaxation behavior is shown in Figure 9.3.4.4.

There should be at least six non-runout fatigue test results for each condition, and these data should be distributed over at least two orders of magnitude in fatigue life. These requirements are the minimum sample sizes normally required to consider developing a fatigue data display. Meeting the minimum data requirements does not insure an acceptable set of fatigue curves. In cases involving highly scattered data, substantially larger sample sizes may be required to achieve a meaningful description of mean fatigue trends. The statistical procedures used to evaluate the significance of a fatigue data collection are described in 9.3.4.12.

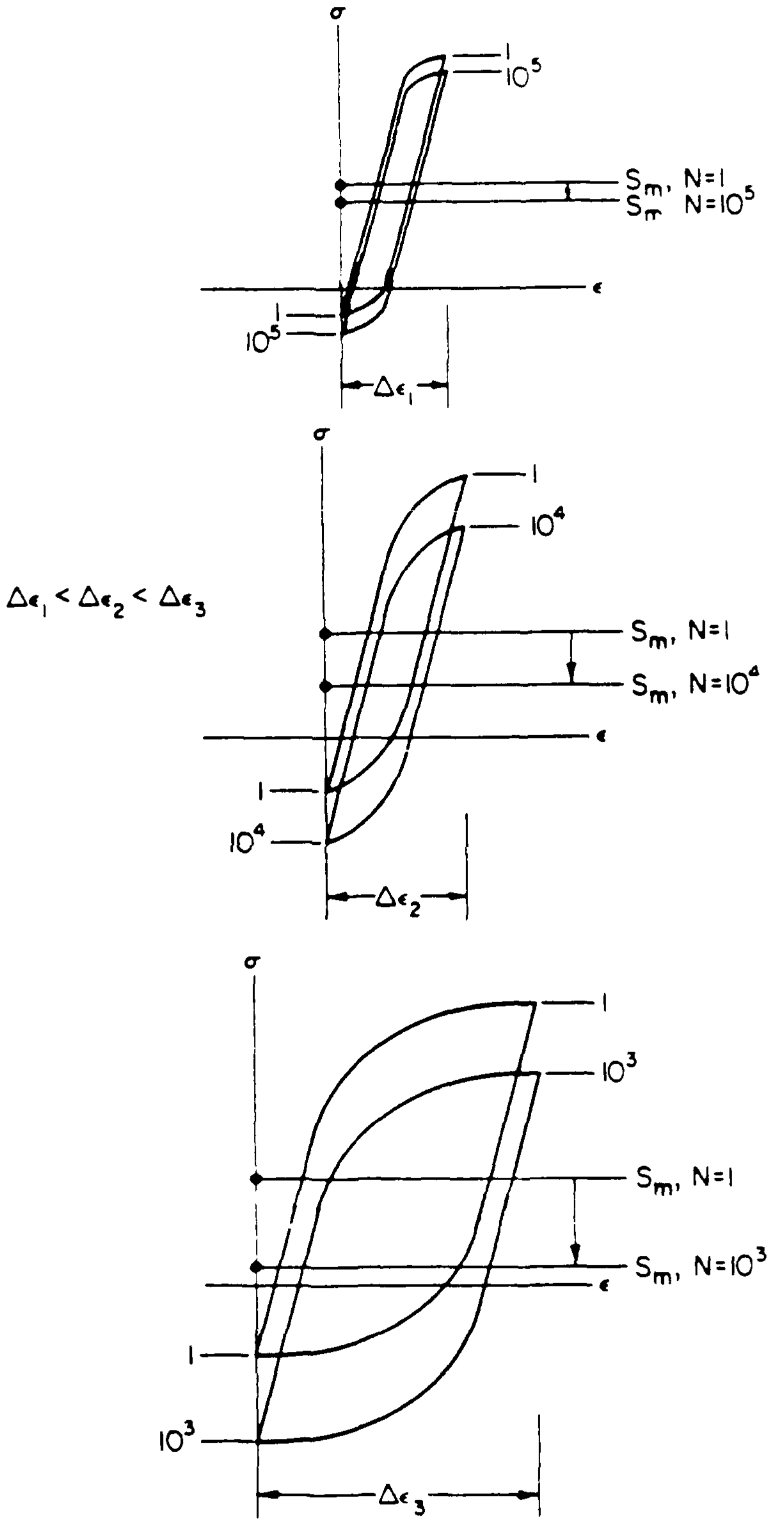

FIGURE 9.3.4.4. *Schematic of stabilized mean stress relaxation for different strain ranges at $R_\epsilon = 0$.*

9.3.4.5 *Fatigue Test Planning and Data Development* --In view of the above data requirements, fatigue data generated for inclusion in MIL-HDBK-5 should be the result of a well planned test program. The following general discussion of fatigue test planning is based in large part on the concepts presented in references 9.3.4.5(a), (b), and (c). Those interested in the detailed aspects of fatigue test planning should refer to these and other sources. The discussion that follows pertains to fatigue testing under either load control or strain control.

Traditionally, fatigue testing under load control has been performed to evaluate the fatigue performance of engineering materials and components subjected to numerous load fluctuations. Notched specimens are often used to evaluate the effect of stress concentrations upon fatigue life in load-control testing. The nominal stresses during load-control testing are generally below the materials yield strength and the resulting fatigue lives are usually greater than 10^4 cycles. Load-control tests with high mean-stress levels may develop unconstrained cyclic plasticity which may lead to rachetting failures [see Figure 9.3.4.1(b)]. Unless cyclic strains are monitored in load-control tests, it is not possible to know exactly when unconstrained cyclic plasticity will develop. In general, however, there are test conditions that should be avoided when operating under load control, as follows:

(1) Unnotched-specimen fatigue tests in which fatigue lives less than 10^3 cycles to failure are expected.

(2) Fatigue tests involving net-section maximum stresses greater than the yield strength or over 95 percent of the typical monotonic ultimate strength of the material.

Strain-controlled fatigue testing has emerged since the mid 1950s because the fatigue damage process was found to be highly dependent upon cumulative plastic deformation. Cycling a material between two strain limits can alter the material's stress-strain response (cyclic hardening or softening) compared to the monotonic response. Fatigue testing under strain control should be considered in cases where constrained inelastic cyclic strains may occur in the actual component. Strain control should also be used for any conditions where unconstrained cyclic plasticity may lead to rachetting failures in load-control testing.

Fatigue data obtained under load control for use in MIL-HDBK-5 should be generated for at least three stress ratios. Fatigue lives ranging from approximately 10^3 to 10^6 cycles are most commonly of interest while the stress ratios chosen should normally span the range from about R = -1.0 to 0.50 or greater.

Fatigue data obtained under strain control are commonly generated at R_ε = -1.0. These data will be considered for MIL-HDBK-5, but generating data for at least two other strain ratios is also desirable.

The stabilized value of mean stress attained in a strain-control test at R_ε greater than -1.0 will be different from that observed at the beginning of the test for materials that undergo cyclic mean stress relaxation. The degree of stress relaxation will depend on strain range and strain ratio, the magnitude being greater at larger strain ranges or larger strain ratios. Complete relaxation to a zero mean stress is the limiting case. When testing at strain ratios greater than -1.0 it is appropriate to limit the strain ranges to values below those at which total cyclic mean stress relaxation occurs.

The amount of cyclic stress relaxation also varies with the anticipated fatigue life. Large-strain-range, low-cycle tests usually exhibit the greatest mean stress relaxation. Because of this behavior, it is usually appropriate to run the positive mean strain experiments at strain ranges less than or equal to the level that produces complete mean stress relaxation.

A given series of fatigue tests conducted under strain control should be targeted to describe the useful life range for the material. The life range explored need only be limited on the low side by the maximum strain ranges that can be performed without specimen buckling problems, and on the high side by the maximum strain rates that are allowable, in combination with the permissible duration of individual tests. Life ranges of 10 to 10^6 cycles are reasonable to explore in strain-control tests with many materials and specimen geometries. Strain-control tests performed for inclusion in MIL-HDBK-5 should normally be conducted with symmetric waveforms, with no hold times at frequencies ranging from 0.10 to 5 Hz--depending on the response of extensometry and recording equipment. It is important to document the strain rates and conformance of the testing techniques with ASTM E606 [reference 9.3.4.1(a)].

Long-life fatigue tests are a special situation in strain-control testing because of the extended test periods that may be required, especially if maximum test frequencies must be kept at or below 1 Hz. For example, a test run at 1 Hz involving one million cycles requires about 11½ days. Decreasing the duration of long-life, strain-control fatigue tests is desirable whenever possible, otherwise a few tests in the 10^5 to 10^7 cycle range can take as much time as the rest of the life curve.

Switching from strain-control testing to load-control testing at a greater frequency at some point in the life of the specimen is becoming a common practice. This switch is typically done when the cyclic response is nominally elastic. Usually the frequency can be increased by a factor of 10 or more but even a factor of 2 or 3 is certainly worthwhile.

When the control mode and/or frequency are changed, certain criteria should be observed. When generating a strain-control fatigue curve, ranging from the short-life regime (10 to 10^3 cycles) to the long-life regime (10^6 to 10^8 cycles), the fatigue tests can be placed in three groups for consideration.

At the short-life end of the curve the material response will typically vary throughout the test. In this regime, a significant amount of inelastic strain may be present, cyclic hardening or softening may occur as well as mean stress shifts. In short, no consistent relationships exist between stress and strain, and therefore no control mode change is recommended in this life regime.

For intermediate life tests, some inelastic strain may be present and, for a period of time, the stress-strain relationship may vary. Generally, however, a stabilized, consistent relationship is eventually achieved. Under these conditions, it may be possible to switch the test mode to load control at a higher frequency.

In the long-life regime, very little inelastic strain will normally be present, and stress-strain stabilization is achieved very rapidly. Here, switching from the strain-control mode to the load-control mode can be accomplished.

The material behavior cited above can only be evaluated by starting all of the tests in the strain-control mode and then switching the mode and frequency when stabilized stress-strain behavior is achieved. An evaluation of the strain rate behavior of the material in the strain-control mode (within the normal response capabilities of the equipment) may be desirable to determine if the stress-strain relationship is likely to change when the frequency is changed.

In summary, do not switch control modes in the low life regime of the fatigue curve. When some inelastic strain is present; switching may be employed if stable stress-strain response can be obtained and a negligible strain rate effect at the test temperature and strain range of interest can be demonstrated (i.e., it can be shown that fatigue life and stress range are not influenced by loading rate). One very good check is to produce overlapping data points in this regime where some tests are run to failure in the strain-control mode while others are switched to high-frequency load-control mode after stabilization is obtained. This is necessary to provide assurance that the switching procedure is not influencing results.

At the very long-life end of the curve, the essentially elastic behavior of the material is most conducive to switching of control modes. The greatest benefit of the increased frequency can also be obtained here. If results have shown that switching is successful at the intermediate strain range level, then the probability of the long-life tests being at least as successful is high. If, however, the material exhibits a measurable inelastic strain and is slow to stabilize even after many cycles, caution should be exercised in making the decision for a control mode change.

When the determination that a test should be switched from strain control to load control has been made, the following sequence is recommended:

(1) Note the maximum and minimum stabilized load levels.

(2) Gradually reduce the strain range to zero. This process should take several cycles (at least 10). If a measurable inelastic strain is present, the strain range reduction should take sufficient cycles so the magnitudes of the maximum and minimum loads are reduced symmetrically.

(3) At this point (strain range at zero) the load may or may not be at zero, depending on the conditions of strain ratio and strain range to which the specimen was exposed. If a residual load is present, the load should be adjusted to zero by carefully changing the strain level.

(4) Next, the test system should be switched to the load-control mode and the test restarted. The strain-control cycling may have been performed using a triangular waveform. The higher frequency testing under load control generally employs a sine wave. The waveshape difference is only of secondary importance, and most machines can easily control a high frequency sine wave. The actual frequency used should be well within the capability of the test equipment so that the load can be accurately measured and controlled. Furthermore, care must be taken to avoid frequency effects, e.g., self heating, and strain rate effects. This is commonly a problem with tests involving a significant amount of inelastic strain.

When reproducing the maximum and minimum stresses that existed under strain-control testing, first introducing the mean load on the specimen and then gradually increasing the load range symmetrically from this point is generally preferred. Whatever procedures are used should be clearly defined and well documented.

The tendency of the load-control results to be slightly more conservative than those generated in strain-control testing is worth repeating. When a specimen develops a fatigue crack, a test that is being conducted under strain-control mode will generally exhibit a reduced tensile load as the crack propagates. Under load-control testing, the load remains constant and the crack will grow faster, resulting in a lesser life. For this reason all data generated by this technique should be so noted and identified on data tables and graphs.

Essentially two steps are involved in the generation of a fatigue curve for a specific stress or strain ratio. First, the general shape of the curve should be determined. Nonreplicated fatigue tests completed at not more than four to six maximum stress levels are usually sufficient to define the basic shape of the curve above the fatigue limit. After the shape of the curve is found from test results, or estimated from fatigue data on similar materials, then the mean curve should be verified through carefully planned replicate fatigue tests.

If the lower maximum stress levels or strain ranges chosen result in nonfailures or runouts*, do not repeat these stress levels while defining the general shape of the fatigue curve. Simply focus on relatively evenly spaced stress or strain levels that generally provide fatigue failures.

In performing these exploratory fatigue tests, obtaining the test specimens from a random sample that adequately represents the material is important. In that context, specimens should be taken from several different lots if possible. Particular care should also be given to minimizing nuisance variables such as test machine effects, frequency effects, surface finish irregularities, residual stress effects, or environmental variations. Unfortunately, variables such as specimen fabrication can influence fatigue results to such an extent that the effect being studied is eclipsed. Composition, thermal-mechanical processing and the origin of the material should be well documented. The same type documentation should apply to the fabrication of the specimens. ASTM E606 provides an example of a machining procedure in Appendix X3 [reference 9.3.4.1(a)]. It is frequently referenced in machining specifications.

In addition, fabricating fatigue specimens also involves many special considerations. For example, simulating a component fabrication process for making the specimens may be desired, e.g., heat treating before or after machining. The specimens may be ground or lathe turned. A mechanical polish or electro-polish may be employed. Special processing such as shot peening, stress relieving, plating or coating may be used. All of these procedures (including their sequence) must be documented.

The formation of surface residual stresses should be recognized as one of the most influential effects of machining, although it is frequently overlooked. Any mechanical removal of material from the specimen can produce residual stresses on the surface. Even when special care is taken to remove material very gradually, residual stresses (either surface or profile) may approach the yield point of the material. Under certain conditions these stresses can have a dramatic effect on the fatigue life of the specimen. Whenever the test environment and strain range are such that these stresses are not dissipated, they can alter the stress on the surface of the specimen. Crack initiation and propagation life will therefore be affected. Machining processes for producing fatigue specimens, therefore, should be evaluated not only on the basis of machining tolerances and surface finish, but also on the magnitude, consistency, and profile of these residual stresses.

Fatigue tests that exhibit little inelastic strain are especially influenced by the procedures employed in specimen preparation. Test results in these intermediate- and long-life regimes can be very confusing and misleading if the residual stresses are not considered. These stresses should at least be measured and documented and in some cases it may be desirable to stress relieve or electro-polish the specimens.

After the general shape of the fatigue curve has been identified (as shown in Figure 9.3.4.5 for three different stress and strain ratios), replicate tests at specific stress or strain levels may be performed to improve the statistical definition of the fatigue curve. Normally, replications at three levels are sufficient, if no fatigue limit is anticipated (or no attempt is to be made to define one).

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* A specific fatigue cycle limit should be chosen as a runout point, and that limit should be used for all further tests on that material, regardless of the stress or strain ratio. For materials that typically display constant amplitude fatigue limits (many steels do) a runout limit as low as 3×10^6 cycles may be satisfactory. Normally, however, a runout limits of 10^7 cycles is preferred, especially for materials that typically do not show a definite fatigue limit (many aluminums do not) and for experiments conducted at reasonably high cyclic frequencies (10^7 cycles is accumulated in less than 4 days of continuous cycling at 30 Hz). Fatigue tests for cast metals are traditionally continued to 2×10^7 cycles as a fatigue limit.

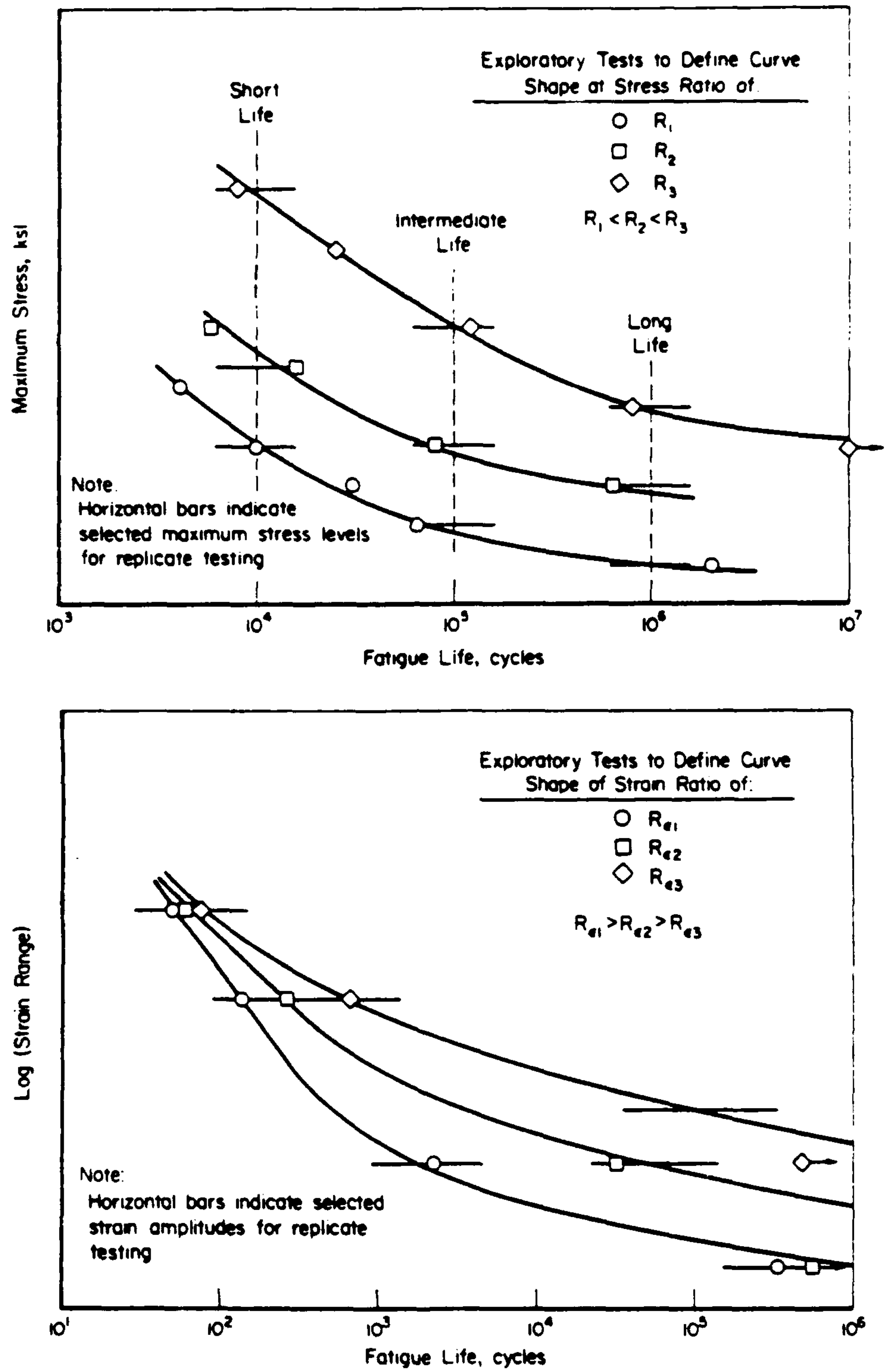

FIGURE 9.3.4.5. *Schematic fatigue data displays (showing the initial exploratory tests as symbols and the strain levels subsequently chosen for replicate fatigue testing as bars; the length of the bars denote observed data variability).*

The replicated stress or strain levels should be selected to represent initial estimates (based on the exploratory experiments) that would be expected to provide average fatigue lives at the extremes of the life interval of interest and at an intermediate fatigue life. For example, if load-control tests are to be performed and the fatigue performance between 10^4 and 10^6 cycles to failure is of concern, select three maximum stress levels for each stress ratio that appear likely to provide average fatigue lives of about 10^4, 10^5, and 10^6 cycles to failure, respectively.

Figure 9.3.4.5 illustrates this maximum stress and strain level selection process. As this figure suggests, specifying the levels with great precision is not necessary (or justified). The use of levels that have been established from exploratory testing may be appropriate. Use the same levels as those used on one of the exploratory tests if it results in a fatigue life near one of the life ranges of interest. The order of fatigue testing at these stress levels should be randomized for each series of replicates.

If further definition of the fatigue curve is desired in the long-life regime, replication at a fourth maximum stress level may be helpful*. To select this stress level, examine the number of runouts obtained at the lowest of the three replicated stress levels. If the number of runouts is less than 50 percent at the lowest stress level, select another, somewhat lower stress level for replication (5 to 10 percent is suggested). Alternately, if the number of runouts at the lowest of the three replicated stress levels is above 50 percent, select a fourth replicated stress level that is somewhat higher (again, 5 to 10 percent is suggested) Using such an approach, defining a fatigue limit stress at the selected runout level in clearly defined statistical terms will, in many cases, be possible.

The amount of replication required at each maximum stress level or strain range is the key remaining issue. Reference 9.3.4.5(a) recommends a minimum of 50 to 75 percent replication for design allowables data. This translates into two to four specimens at each stress or strain level. If the data displays minor variability, two specimens per level may be sufficient. If the data are highly variable even four specimens per level may still not clearly define a statistically significant mean fatigue curve (see 9.3.4.12).

Adding the number of specimens recommended for curve shape definition and the number recommended for replication, the normal minimum number of fatigue tests per curve ranges from 8 to 16. Therefore, the development of fatigue curves for three stress or strain ratios for a fatigue data display in MIL-HDBK-5 might be based on 24 to 48 specimens. If additional stress or strain ratios are to be considered, the number of recommended tests would expand further, although fewer tests may be employed at these R-ratios.

More fatigue specimens are recommended for test in developing a fatigue data display for use in MIL-HDBK-5 than are actually required by current minimum data standards (see 9.3.4.4). This discrepancy exists primarily because the satisfaction of current minimum data standards does not ensure a statistically significant set of fatigue curves. The chance of producing a significant set of fatigue curves is much greater if the recommended fatigue test planning procedure is used and the designed test matrix is carefully completed.

Strain control fatigue data for a particular material must be accompanied by sufficient information to allow the construction of a cyclic stress-strain curve. Normally, such a curve can be constructed from stress-strain pairs recorded from stable hysteresis loops. Pairs obtained from a number of different tests covering a wide range of plastic strain ranges will allow construction of a complete cyclic stress-strain curve. Results from replicated incremental step tests may also be used to construct cyclic stress- strain tests [reference 9.3.4.5(d)].

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*It is assumed here that long-life fatigue tests will be run in load control or started in strain control and switched to load control as discussed earlier.

9.3.4.6 *Data Collection*.--If a set of strain- or load-control data that meet the minimum requirements can be isolated for a material of interest, the data should be processed for analysis Load-control data reports should clearly specify the net section stresses, stress ratios, and associated cycles to failure. Strain-control data reports should clearly specify the strain levels used, the stable stress response values, and the associated cycles to initiation and/or failure, along with a clear and concise definition of the failure criterion. Acceptable definitions of failure in a strain-control fatigue test report include:

(1) total specimen separation

(2) decrease of 50 percent in the maximum or stabilized tensile load value.

Acceptable definitions of crack initiation in a strain-control fatigue test report include:

(1) First significant deviation from the stabilized load range or a stabilized rate-of-change of the load range. Detection reliability is dependent upon the sensitivity of the monitoring equipment and consequently values as small as 1 to 5 percent are used in some cases, while values as great as 10 to 20 percent are used in other cases.

(2) Verifiable results from a calibrated nondestructive inspection device, such as an electrical potential drop system.

The definition of crack initiation or failure used in a particular study must be clearly and quantitatively documented. Other correlative information that is important for load or strain-control test data includes detailed specimen dimensions, fabrication procedures (and their sequence), surface finish, product form, environment, frequency, waveform, surface residual stresses, and temperature. Other useful information includes average material tensile properties, product dimensions, and manufacturer.

All fatigue data that are not listed as invalid by the author of the test report will be prepared for analysis. The identity of different sources should be retained to determine whether combinations of data are appropriate. If all conditions from the different sources are virtually identical, the data should be analyzed together. Data should be identified as invalid if errors in specimen preparation or testing errors are discovered.

Runouts should be designated differently from failure data, since runouts are given special consideration in the regression analysis used to define mean fatigue curves. Runouts are generally defined as tests that have accumulated some predetermined number of cycles and been subsequently stopped to control the total test time. Tests which have been stopped due to distinct problems encountered during testing are termed interrupted tests. Typical problems include power failures, temperature deviations and load spikes. Interrupted tests are generally valid up until the time at which the problem occurred. In this context, interrupted tests are treated the same as runouts in determining the mean fatigue-life trends of a data collection. However, if the interrupted test is stopped long before tests exhibiting typical failures, the information that the interrupted test adds to the sample is minimal, and the data point should be discarded.

Tests which exhibit failures outside of the gage section may, in certain circumstances, be included in the analysis and treated as interrupted tests. Failures occurring just outside the gage section are essentially normal failures and should be included for analysis. In strain-control tests, however, the crack initiation is not sensed by the extensometer. Failures at threads, shoulders or button-heads may be indicative of a problem with the specimen design or test procedure.

Strain-control fatigue data must be accompanied by sufficient information to construct a cyclic stress-strain curve. The cyclic stress-strain curve may be established based on incremental step-strain results or multiple specimen data for which stable stress amplitudes are defined for the complete range of strain ranges. The method used to define the cyclic stress-strain curve must be recorded so that it can be included in the correlative information along with the strain-life fatigue data displays.

9.3.4.7 *Analysis Procedures*.--Once a collection of data is compiled for the material of interest, analysis of that data may begin. An outline of the analysis procedure that is normally followed is given in Figure 9.3.4.7. Each of the elements in the flow chart are discussed in the following sections.

The same basic analysis procedure is used for strain- and load-control data except these data types are normally analyzed separately even if they represent the same material and product form. The only case where load- and strain-control data can be combined is in the situation where some specimens have been switched from strain- to load-control testing. In this case, the load- and strain-control data may be analyzed on an equivalent strain basis. In all other cases load-control data should be analyzed on an equivalent stress basis. Load-control data generated at different stress concentrations should always be analyzed separately.

9.3.4.8 *Fatigue Life Models*.--To clarify the fatigue data trends for a specific stress or strain ratio, a linear regression model can be applied as follows:

$$\log(N_i \text{ or } N_f) = A_1 + A_2 \log(S_{max} \text{ or } \Delta\epsilon) \qquad [9.3.4.8(a)]$$

Note that fatigue life is specified as the dependent variable. The alternative approach, using stress or strain as the dependent variable, is sometimes used, but this procedure will not be employed in developing mean fatigue curves in MIL-HDBK-5. The use of fatigue life, or more specifically, logarithm (base 10) of fatigue life as the dependent variable will be used since stress or strain is the controlled parameter in a fatigue experiment, and the resultant fatigue life is a random variable.

If Equation 9.3.4.8(a) does not adequately describe long-life data trends, a nonlinear model (or a more complicated linear model) may be warranted. For example, long-life, load-control data might be modeled by the non-linear expression

$$\log N_f = A_1 + A_2 \log(S_{max} - A_3) \qquad [9.3.4.8(b)]$$

or by the more complicated equation [reference 9.3.4.8]

$$\log N_f = A_1 + A_2 \log S_{max} + A_3 \sqrt{\log S_{max} + A_4} \;. \qquad [9.3.4.8(c)]$$

These more complex forms should only be employed in instances where they are warranted based on a distinct fatigue limit at long lives and when the simpler linear model proves inadequate.

Standard least squares regression procedures and the procedure for detecting outliers in 9.3.4.11 require that the variance be relatively constant at all fatigue life values. Traditionally, the logarithm of fatigue life is approximated by a normal distribution. However, the variability or scatter of fatigue life is generally not constant, but increases with increasing fatigue life. To ensure the reliable use of the outlier detection procedure, a weighting scheme designed to produce a more uniform distribution of residuals is suggested in 9.3.4.10.

Supersedes page 9-80j of Change Notice 1, MIL-HDBK-5E.

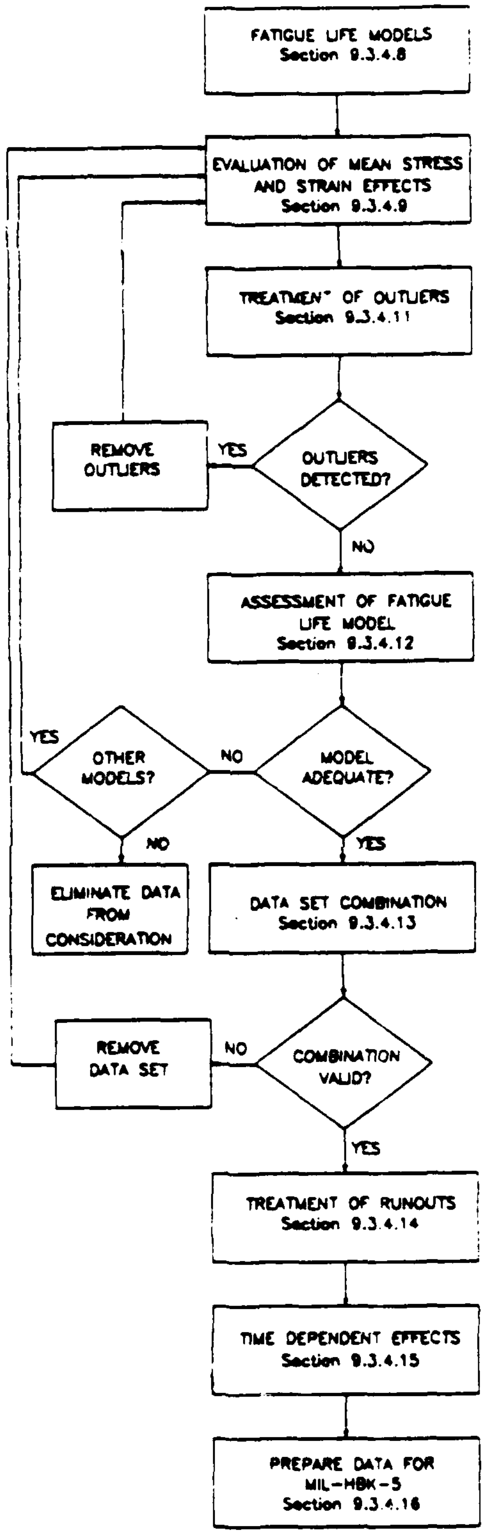

FIGURE 9.3.4.7. *Flow chart of general fatigue analysis procedure.*

9.3.4.9 *Evaluation of Mean Stress and Strain Effects*--Commonly, load-controlled fatigue data generated over a range of stress ratios can be represented by the following equivalent stress--fatigue life formulation.

$$\log N_f = A_1 + A_2 \log (S_{eq} - A_4) \qquad [9.3.4.9(a)]$$

where

$$S_{eq} = (\Delta S)^{A_3} (S_{max})^{1-A_3}$$

$$S_{eq} = S_{max} (1-R)^{A_3}$$

The equivalent stress model (and the related equivalent strain model) are derived from Reference 9.3.4.9(a).

Equation 9.3.4.9(a) is nonlinear in its general form and must, therefore, normally be optimized through use of a nonlinear regression package. However, the above equation can be solved through a linear analysis, if A_3 and A_4 are optimized through an iterative solution. The parameter A_3 normally lies in the range of 0.30 to 0.70, while A_4 represents, in essence, the fatigue limit stress. In cases where the optimum value of A_4 is negative or insignificant, it should be omitted. Unnotched data, especially aluminum alloy data, can frequently be represented without using the nonlinear A_4 term. Parameter optimization is discussed more thoroughly in 9.3.4.10.

If A_4 is zero or set equal to zero, Equation 9.3.4.9(a) becomes linear in log S_{max} and log (1-R), and it can be written as follows:

$$\log N_f = A_1 + A_2 \log S_{max} + B \log (1-R) \qquad [9.3.4.9(b)]$$

where $B = A_2A_3$. Thus, if A_4 is zero, then

$$A_3 = B / A_2$$

Strain-controlled fatigue data generated over a range of strain ratios often can be consolidated by the following equivalent strain formulation:

$$\log N_f = A_1 + A_2 \log (\varepsilon_{eq} - A_4) \qquad [9.3.4.9(c)]$$

where

$$\varepsilon_{eq} = (\Delta\varepsilon)^{A_3} (S_{max}/E)^{1-A_3}$$

Note that Equation 9.3.4.9(c) is very similar in form to Equation 9.3.4.9(a). It is important to note, however, that the maximum stress value used in Equation 9.3.4.9(c) is not a controlled quantity. It is a measured quantity and its magnitude depends primarily on the amount of cyclic softening or hardening that occurs in combination with mean stress relaxation. Although S_{max} can be predicted with reasonable accuracy if the cyclic response of the material is well established, using the stable measured values of S_{max}, when analyzing strain-control data for presentation in MIL-HDBK-5, is preferred.

The equivalent stress and strain approaches are very useful for computing mean fatigue life estimates for conditions intermediate to those for which the test data have been generated. Caution should be used, however, in making life predictions for stress/strain conditions beyond the range of those represented in the data base. Also, when only two stress/strain ratios are used in the equivalence formulation, fatigue life estimates at conditions other than those two ratios (either intermediate or beyond) may be unreliable.

Supersedes page 9-801 of Change Notice 1, MIL-HDBK-5E.

If the basic formulations just described do not realistically represent the data, alternative equivalent stress or strain formulations should be considered. Two formulations [references 9.3.4.9(a) and (b)], in particular, may apply in these specific instances where equivalent stress is defined as:

$$S_{eq} = S_a + A_3 S_m \quad [9.3.4.9(d)]$$

or

$$S_{eq} = S_a + S_m^{A_3} \quad [9.3.4.9(e)]$$

and equivalent strain is defined as:

$$\varepsilon_{eq} = \varepsilon_a + A_3 S_m/E \quad [9.3.4.9(f)]$$

or

$$\varepsilon_{eq} = \varepsilon_a + (S_m/E)^{A_3} \quad [9.3.4.9(g)]$$

where

S_{eq} = equivalent stress
ε_{eq} = equivalent strain
S_a = alternating stress
S_m = mean stress
ε_a = alternating strain
E = elastic modulus (from each test result).

Other data consolidation parameters may also be used provided they do not violate other guideline requirements, and they can be proven adequate. Adequacy may be assessed by employing the procedures described in 9.3.4.12.

To evaluate the adequacy of one equivalent stress or strain formulation compared to another, it is also useful to construct a plot of residuals versus stress or strain identifying individual stress or strain ratios. In this way the usefulness of a given formulation for modeling stress or strain ratio effects is visually apparent.

9.3.4.10 *Estimation of Fatigue-Life Model Parameters*.–The fatigue-life model parameters are estimated to obtain the best-fit S/N or ε/N curve for the data. The procedure used to determine the parameters includes a statistical method for adjusting the fatigue model for the nonconstant variance commonly observed in long-life fatigue data. The motivation for this adjustment is the fact that constant variance is an inherent assumption in least squares regression analysis. To estimate the parameters in Equation 9.3.4.9(a) or Equation 9.3.4.9(c) and adjust the model to incorporate nonuniform variance, the following six-step procedure is performed.

Step 1 - Initial Parameter Estimates. If A_4 is assumed to be zero, then a linear least squares regression analysis is performed to obtain the initial parameter estimates for A_1, A_2, and A_3. If A_4 is to be estimated from the data, a nonlinear least squares regression analysis is performed to obtain the initial parameter estimates for A_1, A_2, A_3, and A_4. Runout observations are ignored in the calculation of the initial parameter estimates and residuals.

To facilitate convergence of the nonlinear least squares fit when A_4 is to be estimated from the data, the following procedure may be used to obtain starting values. Set A_3 equal to 0.5 and calculate equivalent stress (strain) values for each observation. Set A_4 equal to one half the smallest equivalent stress (strain) not associated with a runout. Using these values of A_3 and A_4 as constants, obtain least squares estimates of A_1 and A_2 using a linear regression routine.

Supersedes page 9-80m of Change Notice 1, MIL-HDBK-5E.

Step 2 - Fitting the Variability Model. The magnitude of the residuals from these fatigue-life models typically increases with decreasing stress or strain as illustrated in Figure 9.3.4.10(a). The residuals plotted are the observed log(life) values minus the predicted log(life) values.

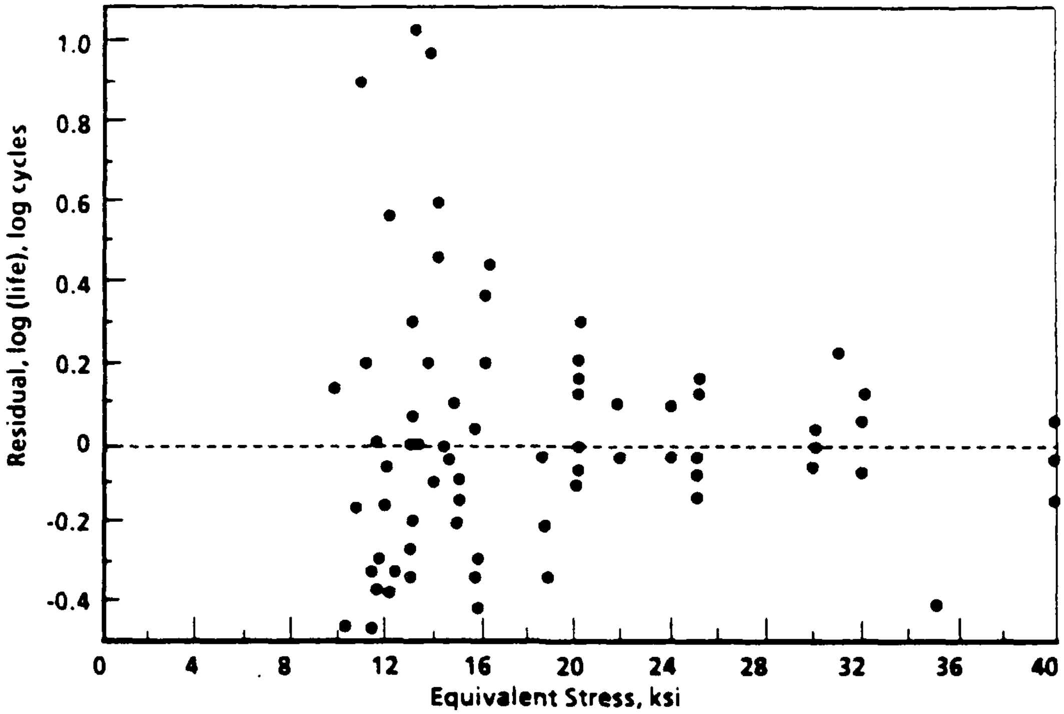

FIGURE 9.3.4.10(a). *Example plot showing increasing magnitude of residuals with decreasing stress/strain levels.*

To evaluate the fatigue-life model for nonuniform variance, it is useful to construct a model to estimate the standard deviation of log(life) as a function of equivalent stress (strain). If there is non-uniform variance, such a model can then be used to perform a weighted regression to estimate the fatigue life model parameters where the weight for each observation is inversely proportional to its estimated variance.

The suggested standard deviation model is

$$\frac{|R|}{\sqrt{2/\pi}} = \sigma_0 + \sigma_1 \left[\frac{1}{S_{eq}} \right] = g(S_{eq})$$

or [9.3.4.10(a)]

$$\frac{|R|}{\sqrt{2/\pi}} = \sigma_0 + \sigma_1 \left[\frac{1}{\varepsilon_{eq}} \right] = h(\varepsilon_{eq})$$

where R (observed log(life) minus predicted log(life)) represents the residuals from the fatigue life model fitted in Step 1. This model assumes that the standard deviation of log(life) is a linear function of the reciprocal of equivalent stress (strain). The absolute values of the residuals are divided by $\sqrt{2/\pi}$ so that $g(S_{eq})$ or $g(\varepsilon_{eq})$ is an estimate of the standard deviation of log(life).

Supersedes page 9-80n of Change Notice 1, MIL-HDBK-5E.

The intercept, σ_0, and the slope σ_1, are first estimated by ordinary least squares. If the least squares estimate of σ_0 is negative, σ_0 should be set to zero and σ_1 should be estimated by performing a least squares regression through the origin (no intercept term). A 90 percent confidence interval for σ_1 should also be obtained. If the lower bound of the confidence interval for σ_1 is positive, there is evidence of nonuniform variance and one should proceed to Step 3A. If the confidence interval for σ_1 contains zero, there is no evidence of nonuniform variance and one should proceed to Step 3B. If the upper bound of the confidence interval for σ_1 is negative, this indicates abnormal behavior requiring further examination of the data set before proceeding with the analysis.

Figure 9.3.4.10(b) is a plot of the absolute values of the residuals from Figure 9.3.4.10(a) versus the reciprocal of equivalent stress. The slope and vertical intercept of the least squares line displayed in this plot are the estimated parameters σ_1 and σ_0.

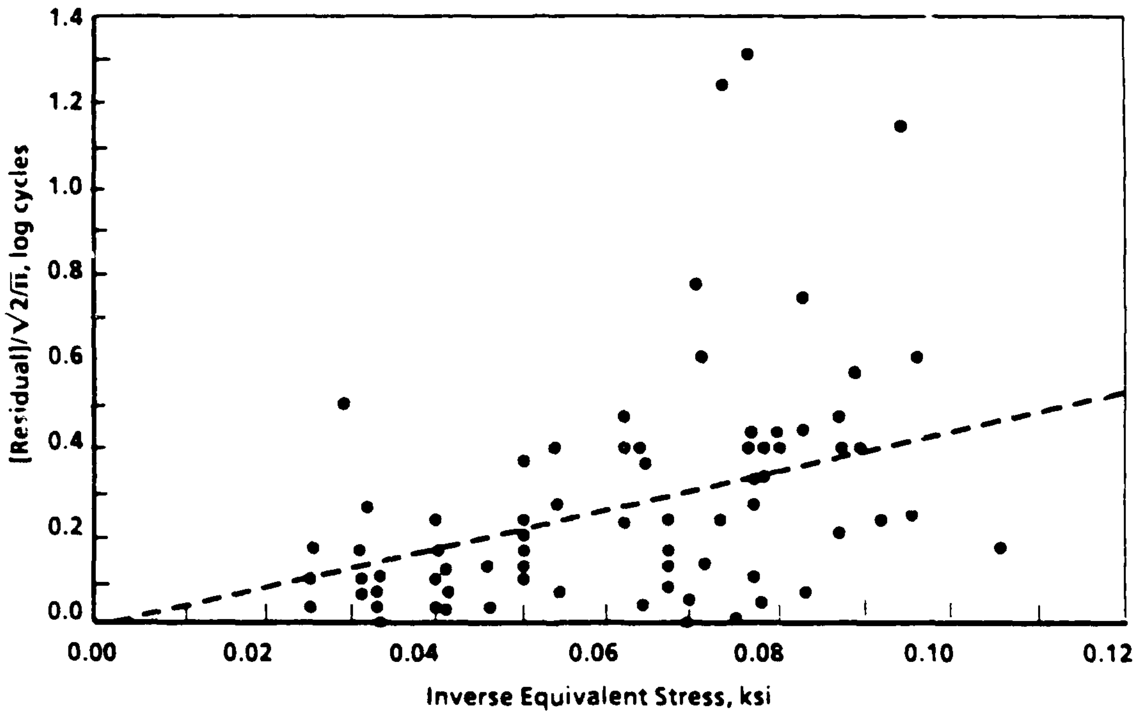

FIGURE 9.3.4.10(b). *Example plot showing the magnitude of the residuals versus the inverse of equivalent stress/strain levels.*

Step 3A - Fitting the Weighted Fatigue Model. Adjust the fatigue model for nonconstant variance by dividing each term in the model by $g(S_{eq})$ or $h(\varepsilon_{eq})$, the estimated standard deviation of the

Supersedes page 9-80o of Change Notice 1, MIL-HDBK-5E.

dependent regression variable. If the four parameter fatigue model is being used, the adjusted model becomes

$$\left[\frac{\log(N)}{g(S_{eq})}\right] = A_1\left[\frac{1}{g(S_{eq})}\right] + A_2\left[\frac{\log(S_{eq} - A_4)}{g(S_{eq})}\right]$$

or [9.3.4.10(b)]

$$\left[\frac{\log(N)}{g(\varepsilon_{eq})}\right] = A_1\left[\frac{1}{g(\varepsilon_{eq})}\right] + A_2\left[\frac{\log(\varepsilon_{eq} - A_4)}{g(\varepsilon_{eq})}\right]$$

where S_{eq} and ε_{eq} are defined in Equations 9.3.4.9(a)and 9.3.4.9(c). Perform a nonlinear least squares regression analysis (no intercept) using the adjusted model to obtain new estimates of A_1, A_2, A_3, and A_4. When performing this regression, all runouts above the minimum S_{eq} or ε_{eq} at which a failure occurred should be included in the analysis and treated as failures. The inclusion of runouts in this step should be determined based on equivalent stress (strain) values using the value of A_3 estimated in Step 1. Assuming that the equivalent stress/strain model is valid, this qualifying stress/strain level allows the use of all runouts above stresses or strains at which failures have been observed. Below this level there is no statistical evidence that discontinued tests would have failed. Therefore, runouts below the minimum S_{eq} or ε_{eq} value at which a failure occurred are not assigned finite life values in estimating the parameters.

It should be noted that the regression analysis performed using the adjusted model [Equation 9.3.4.10(b)] is equivalent to performing a weighted least squares regression analysis using the original fatigue-life model [Equation 9.3.4.9(c)] and weights equal to $1/g^2(S_{eq})$ or $1/g^2(\varepsilon_{eq})$. Also, it may be desirable in certain situations to fit alternative standard deviation models to the residuals from Step 1. In this case, simply redefine $g(S_{eq})$ or $g(\varepsilon_{eq})$ to be equal to the desired model and follow Steps 1 through 3 above. Upon completion of Step 3A, proceed to Step 4.

Step 3B - Fitting the Unweighted Fatigue Model. Using the initial estimate of A_3 obtained in Step 1, calculate equivalent stress (strain) values for all observations including runouts. All runouts above the minimum equivalent stress (strain) at which a failure occurred should be included in the analysis and treated as failures. (See Step 3A for an explanation of this rationale.) Using the same regression techniques employed in Step 1, obtain least squares estimates of the parameters A_1, A_2, A_3, and A_4.

Step 4 - Testing the Significance of Model Parameters. Obtain a 90 percent confidence interval for A_4. If the lower bound of the confidence interval is negative, there is no evidence that A_4 is different from zero. In this case, set A_4 equal to zero and repeat Step 3A or 3B using a linear regression procedure.

Next obtain a 90 percent confidence interval for A2. If the upper bound of the confidence interval is negative, this indicates that the relationship between log(life) and equivalent stress (strain) is significant. If the upper bound of the confidence interval is positive, there is no evidence of a significant relationship between log(life) and equivalent stress (strain) and the data set should be examined further before proceeding with the analysis.

Step 5 - Re-estimating A_1 and A_2. If a weighted least squares analysis was performed in Step 3A, A_1 and A_2 should be re-estimated to include the effect of the new value of A_3 on the calculation of weights and the inclusion of runouts. First, recompute the weights $g(S_{eq})$ or $g(\varepsilon_{eq})$ using the value of A_3 obtained in Step 3A. Then perform a linear regression (no intercept) to obtain updated estimates of A_1 and A_2 in Equation 9.3.4.10(b) treating A_3 as a constant. The inclusion of runouts in this linear regression should be determined based on equivalent stress (strain) values using the value of A_3 obtained in Step 3A.

Step 6 - Estimating the Standard Deviation and Calculating Standardized Residuals. The method for estimating the "standard deviation of log(life)" (SD) depends on whether there is evidence of nonuniform variance in the fatigue life data. If an unweighted regression was performed in Step 3B to obtain the model parameters, SD should be set equal to the root mean square error (RMSE) associated with the fitted and unweighted fatigue life model. In this case, SD may be calculated as

$$SD = RMSE = \sqrt{\sum_{i=1}^{n} R_i^2 / (n-k)} \qquad [9.3.4.10(c)]$$

where k is the number of parameters estimated in Step 3, and

$$R_i = N_i - \log{}^{*}N_i \qquad [9.3.4.10(d)]$$

where R_i is the residual, $\log N_i$ is the logarithm of observed number of cycles, and $\log{}^{*}N_i$ is the logarithm of predicted number of cycles associated with the ith observation.

If a weighted regression was performed in Step 3A to obtain the model parameters, SD should be reported as linear function of the reciprocal of equivalent stress (strain). This function should be obtained by multiplying the fitted standard deviation model ($g(S_{eq})$ or $g(\varepsilon_{eq})$) from Step 2 by the root mean square error (RMSE) associated with

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the fitted and weighted fatigue life model to obtain an updated standard deviation model. In this case, SD may be calculated as

$$SD = RMSE * (\sigma_0 + \sigma_1 / S_{eq})$$

or [9.3.4.10(e)]

$$SD = RMSE * (\sigma_0 + \sigma_1 / \varepsilon_{eq})$$

where

$$RMSE = \sum_{i=1}^{n} WR_i^2 / (n-k) \quad , \qquad [9.3.4.10(f)]$$

k is the number of parameters estimated in Step 3, and

$$WR_i = \frac{\log N_i - \log \hat{N}_i}{g(S_{eq,i} \text{ or } \varepsilon_{eq,i})} \qquad [9.3.4.10(g)]$$

with WR_i denoting the weighted residual and $S_{eq,i}$ ($\varepsilon_{eq,i}$) the equivalent stress (strain) associated with the ith observation.

As a final step associated with the estimation of fatigue life model parameters, standardized residuals should be calculated for use in the judging the appropriateness of the fitted model. Standardized residuals are calculated as

$$SR_i = R_i / SD \qquad [9.3.4.10(h)]$$

where the form of the residual R_i is given in Equation 9.3.4.10(d) and the estimated standard deviation SD is given by either Equation 9.3.4.10(c) or Equation 9.3.4.10(e).

Figure 9.3.4.10(c) is a plot of the standardized residuals for the same data plotted in Figure 9.3.4.10(b) but based on a standard deviation model to correct the nonuniform variance. Note that the pattern of nonconstant variance has been eliminated.

Note - When performing any of the regression analyses described above to estimate the parameters A_1, A_2, A_3, and A_4, the estimate of A_4 should be restricted to be greater than or equal to zero. Some regression programs allow such restrictions as an option. If such an option is not

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available and if the estimate of A_4 is negative, set A_4 equal to zero and refit the model treating A_4 as a constant. Also note that the parameter estimates obtained from the regression analysis of Step 3A or 3B need not necessarily be reported as the final parameter estimates. If the data set includes runout observations, final estimates of the A_1 and A_2 parameters may be calculated using the maximum likelihood techniques presented in Section 9.3.4.14, provided that software for performing this procedure is available.

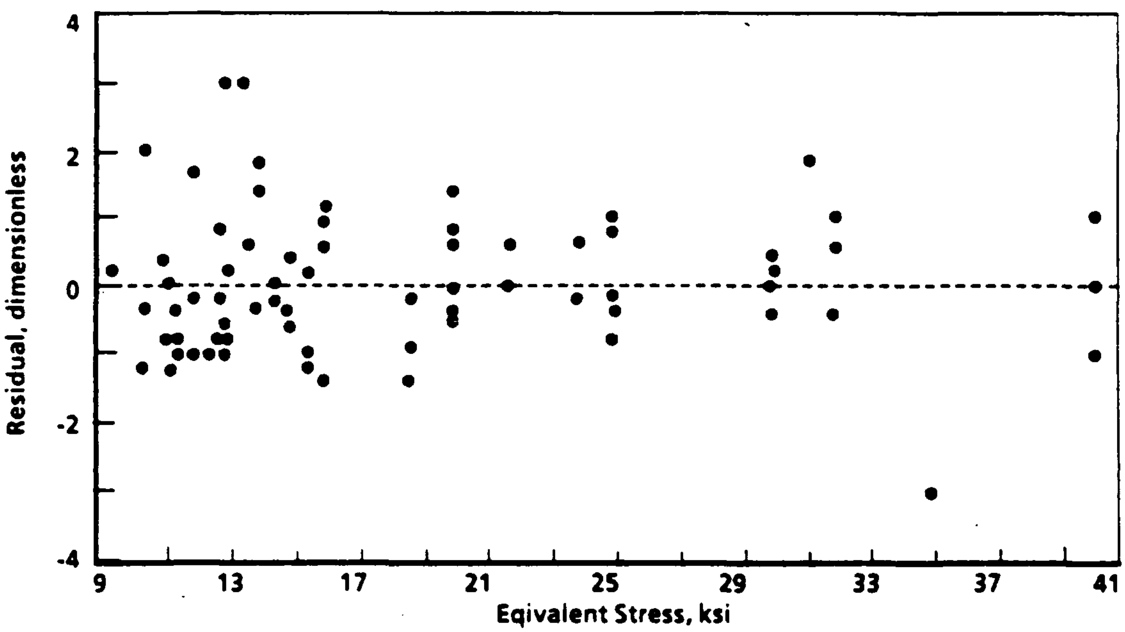

FIGURE 9.3.4.10(c). *Example plot showing constant variance of standardized residuals.*

9.3.4.11 *Treatment of Outliers*.--An outlying observation (or outlier), is one that appears to deviate markedly from other observations in the sample in which it occurs. Outliers may essentially be classified into two groups:

(1) An extreme value of the random variable inherent in the data (in this case fatigue life). If this is true, the value should be retained in future analyses.

(2) An unusual result caused by a gross deviation in material or prescribed experimental procedure or an error in calculating or recording any experimental data.

An outlier of the second type is sometimes correctable by a review of the test sample and/or test records which may provide sufficient evidence for rejection of the observation. An outlying value from a failure that occurred in the fillet of an unnotched fatigue test sample is an example of a potentially rejectable result based on physical evidence alone. The more difficult case is one where an observation is an obvious outlier and no physical reasons can be identified to justify its exclusion.

Assuming uniform variance in the standardized residuals over the complete range in equivalent stress or strain, the problem of identifying certain observations as potential outliers should be addressed as follows. Calculate the studentized residuals,

$$T_i = \frac{SR_i}{(1 - h_i)^{1/2}} \left[\frac{RMSE}{RMSE(i)} \right]$$

for $i = 1, \ldots, n$ where SR_i is the standardized residual from Equation 9.3.4.10(h), RMSE is the root mean square error based on the entire sample as calculated in either Equation 9.3.4.10(c) or Equation 9.3.4.10(f), and RMSE(i) is the root mean square error based on the sample which excludes the ith observation as calculated by either Equation 9.3.4.10(c) or Equation 9.3.4.10(f).

The value h_i is calculated using the formula

$$h_i = \frac{X_{1i}^2 (\Sigma X_{2j}^2) - 2 X_{1i} X_{2i} (\Sigma X_{1j} X_{2j}) + X_{2i}^2 (\Sigma X_{1j}^2)}{(\Sigma X_{1j}^2)(\Sigma X_{2j}^2) - (\Sigma X_{1j} X_{2j}^2)}$$

where X_{1i} is the value of 1/SD for the ith specimen, X_{2i} is the value of $\log(S_{eq}\text{-}A_4)$/SD for the ith specimen and all summations are over $j = 1, \ldots, n$. Note that

$$RMSE^2(i) = \frac{(n - k)RMSE^2 - SR_i^2/(1 - h_i)}{(n - k - 1)}$$

where RMSE is the root mean square error based on the entire sample as calculated in either Equation 9.3.4.10(c) or Equation 9.3.4.10(f) and k is the number of parameters estimated in Step 3 of 9.3.4.10.

Supersedes page 9-80t of Change Notice 1, MIL-HDBK-5E.

It can be shown that each T_i has a central t distribution with n-k-1 degrees of freedom. Applying the Bonferroni inequality [Reference 9.3.4.11] to obtain a conservative critical value leads to the following outlier test. Calculate the maximum absolute studentized residual

$$G = \max |T_i|$$

and declare the data value corresponding to G to be an outlier if

$$G > t(\alpha/2n, n-k-1)$$

where $t(\alpha/2n,n\text{-}k\text{-}1)$ is the upper $\alpha/2n$ percentile point of the central t distribution with n-k-1 degrees of freedom and α represents the significance level of the outlier test. Under the hypothesis that no outliers are present in the data, the probability is less than α that the data value corresponding to G will be falsely declared an outlier.

In applying this test to fatigue life data, a significance level of $\alpha = 0.05$ is used and the test is first applied to the entire sample. If an outlier is detected, the outlying observation is removed from the sample and the entire analysis is repeated on the smaller sample of n-1 observations starting with Step 1 of Section 9.3.4.10. (When a nonlinear least squares fit is performed in Step 1, use the current estimates for A_1, A_2, A_3, and A_4 as starting values rather than following the starting value algorithm.) This process of removing outliers and repeating the analysis continues until no outliers are detected in the remaining sample. For strain-control data, apply the procedure described above replacing S_{eq} with ε_{eq} throughout.

The data analyst may also wish to carry out the outlier test procedure using a significance level of $\alpha = 0.20$ in order to identify additional observations that may warrant investigation. To identify even more suspect observations, a larger significance level may be used. Any data values identified by this procedure should be examined but retained in the data set unless physical evidence justifies their exclusion.

9.3.4.12 *Assessment of the Fatigue Life Model.*--The fit of the fatigue model S/N curve to the data may be assessed in two ways--the adequacy of the equivalent stress/strain model and the adequacy of the fatigue life model. The equivalent stress model lack of fit test and the overall lack of fit test described below provide a reasonable assessment of the fatigue life model.

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When three or more stress (strain) ratios are used, the fit of the equivalent stress (strain) model may be tested by determining the relationship between the standardized residuals from Equation 9.3.4.10(h) and stress (strain) ratio. A difference in the means of the standardized residuals at each stress (strain) ratio indicates that the equivalent stress (strain) model is inadequate. To determine whether or not there is a statistically significant difference in the means of the standardized residuals at each stress (strain) ratio, an analysis of variance should be performed on the standardized residuals using stress (strain) ratio as the treatment variable. A statistical F-test should be used to determine if the effect of stress ratio is significant at the 5 percent level [Reference 9.3.4.12]. The equivalent stress (strain) model should be considered inadequate when the effect of stress (strain) ratio is significant according to the statistical F-test.

The plot of the standardized residuals versus stress ratio shown in Figure 9.3.4.12(a) illustrates such a relationship between the standardized residuals and stress ratio. Since there would be no such relationship if the equivalent stress model were adequate, the plot indicates that the equivalent stress model must have been misspecified in this case. In addition to the lack of fit shown by differences in standardized residual means, other types of lack of fit could exist. Therefore, it would be prudent to examine stress-life plots in addition to performing the statistical test for lack of fit of the equivalent stress model.

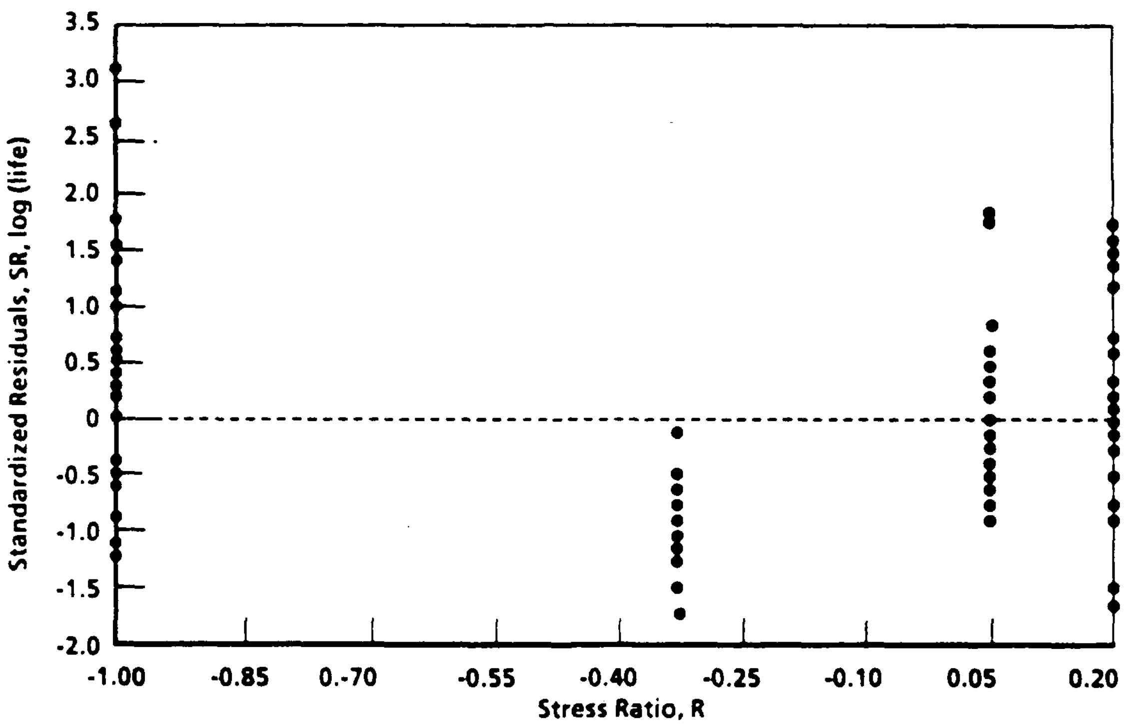

FIGURE 9.3.4.12(a). *Standardized residuals versus stress ratio.*

Supersedes page 9-80v of Change Notice 1, MIL-HDBK-5E.

If the equivalent stress (strain) model is inappropriate, then a new equivalent stress (strain) model should be selected. When a suitable stress (strain) model is not available, an alternative strategy is to present the data with best fit regression lines for each stress (strain) ratio. To be acceptable, each curve must meet minimum data requirements and satisfy significance checks as discussed in Section 9.3.4.10. This approach is less desirable than the equivalent stress (strain) modeling approach because it requires the estimation of fatigue trends using a graphical technique for intermediate conditions where no data exist. It should, therefore, be used only in cases where significant fatigue data collections cannot be handled by standard procedures.

Once an equivalent stress (strain) model has been found that describes the general fatigue data trends for all stress (strain) ratios, an overall test of the fit of the fatigue model should be performed. The stress-life plot shown in Figure 9.3.4.12(b) is characteristic of an overall lack of fit.

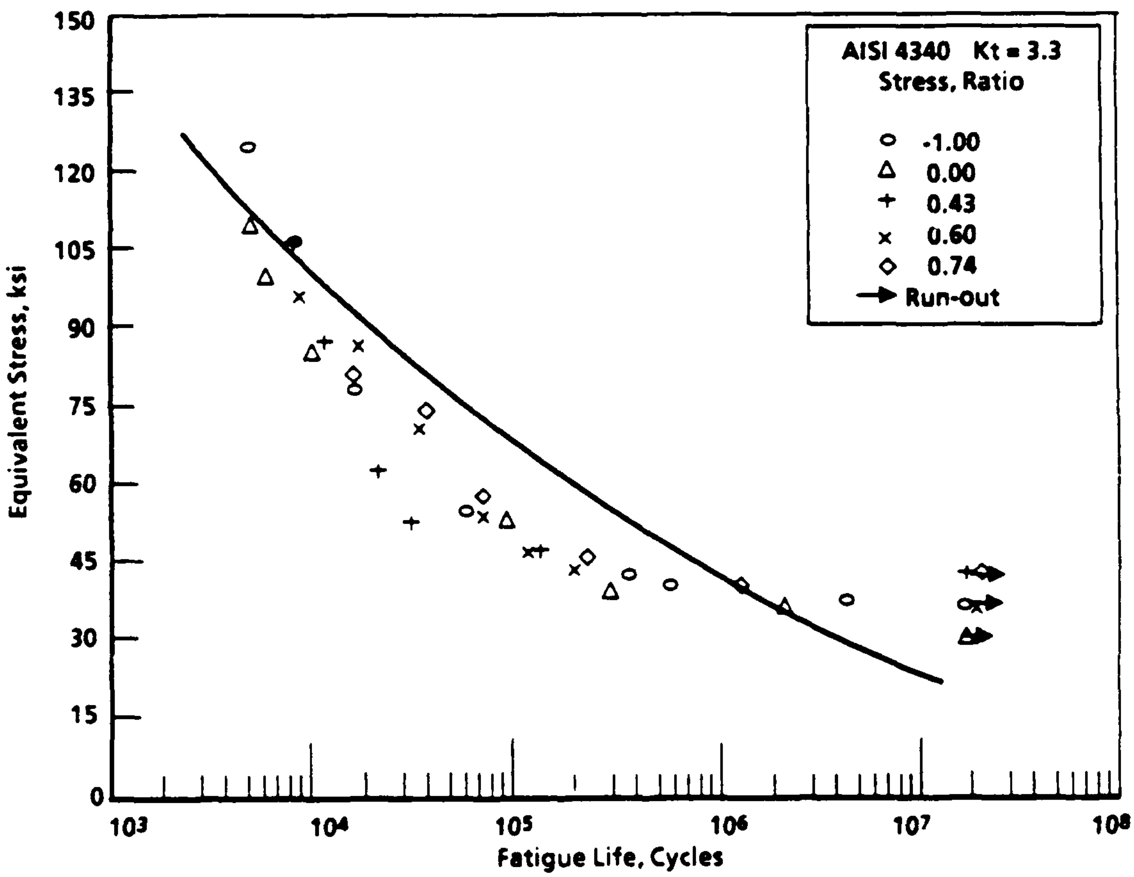

FIGURE 9.3.4.12(b). *Stress-life plot showing lack of fit.*

To identify such a lack of fit, the Durbin-Watson test may be used [reference 9.3.4.12]. The statistic D

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should be computed according to the formula

$$D_i = \frac{\sum_{i=2}^{n} (SR_i - SR_{i-1})^2}{\sum_{i=1}^{n} SR_i^2} \qquad [9.3.4.12(a)]$$

where SR_i is the ith standardized residual [Equation 9.3.4.10(h)] ordered by increasing values of equivalent stress (strain). If

$$D < 2 - 4.73 / n^{0.555} \qquad [9.3.4.12(b)]$$

conclude that there is a significant lack of fit at the 5 percent significance level. This equation was derived from the conservative critical value (d_L) reported in Table A.6 of Montgomery and Peck [reference 9.3.4.12]. When an overall lack of fit is determined from this test, the modeling procedure should be repeated with a more appropriate fatigue model.

9.3.4.13 Data Set Combination.--In many cases, data from different sources, orientations, etc., may need to be combined for analysis. When data set combinations of this sort are performed, the validity of the combination should be tested with the method described below. The test is similar to that used to determine the adequacy of the equivalent stress (strain) model in the previous section.

If there is a relationship between the standardized residuals from Equation 9.3.4.10(h) and the data set from which they were obtained, such as that shown in Figure 9.3.4.13, then the data sets should normally not be combined. To determine whether or not the mean of the standardized residuals is significcantly different for any of the data sets, an analysis of variance should be performed on the standardized residuals using data set as the treatment variable. The analysis of variance F-test should be used to determine if the combined data sets are significantly different at the 5 percent level.

When the data sets are found to be significantly different, at least one of the data sets should normally be removed from the data set combination. In this situation, the data analyst may wish to apply a standard multiple comparison procedure to the standardized residual data to determine which standardized residual means are significantly different from the others. For a discussion of standard multiple comparison procedures, see pages 185-201 of Winer [reference 9.3.4.13].

There may be situations where differences between data sets are found to be statistically significant, yet these differences are so small as to be unimportant from an engineering standpoint. If a particular analysis reveals such a case, exceptions may be taken, if clearly noted and explained in the fatigue data proposal.

9.3.4.14 Treatment of Runouts.--It is difficult to incorporate information from runouts (or interrupted tests) when using the least squares criterion to fit fatigue life models to data since the failure times for these observations are not known. The runouts must be either ignored or treated as failures and neither of these alternatives adequately incorporates the information contained in the runout observations. Both of these approaches tend to produce smaller predicted lives at a given equivalent stress (strain) value than is appropriate. The treatment of runouts presented below is more appropriate but requires that two of the fatigue life model parameters be estimated by maximum likelihood techniques rather than by least squares procedures.

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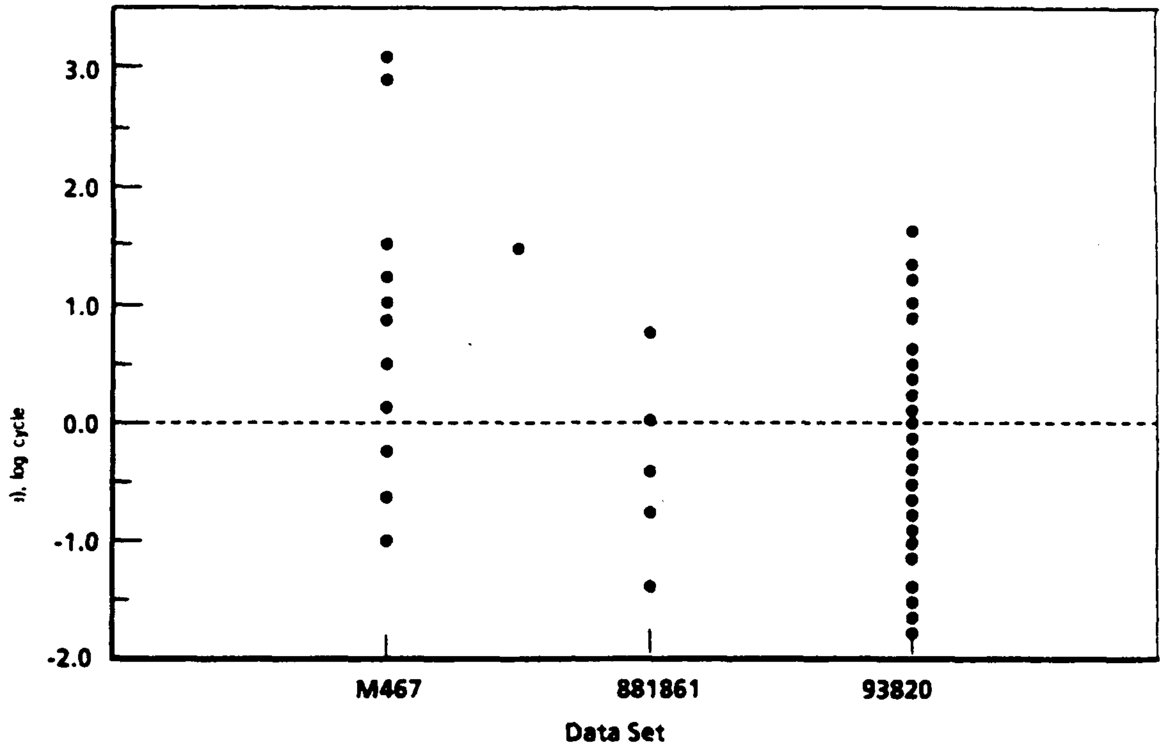

FIGURE 9.3.4.13. *Standardized residual plot showing different mean trends between data sets.*

The maximum likelihood procedure is employed to obtain new estimates for the parameters A_1 and A_2 in Equation 9.3.4.9(a) or 9.3.4.9(c). For the purpose of this analysis, fatigue life (cycles to failure) is assumed to be log normally distributed and the parameters A_3 and A_4 are considered to be constants which are equal to the values obtained using the procedures of Section 9.3.4.10.

The estimated values of A_1 and A_2 obtained previously are used as initial values. The maximum likelihood procedure then determines the values of A_1 and A_2 which maximize the log-likelihood function

$$L(A_1,A_2,\sigma) = \sum_{i=1}^{n} (1 - d_i)[\log (f(w_i)/\sigma)] + d_i \log S(w_i)$$

where

$$f(w) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{w^2}{2}\right]$$

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is the standard normal density function,

$$S(w) = \int_{w}^{\infty} f(t)\,dt$$

is the survival function for the standard normal distribution, d_i is equal to 1 if the ith observation is a runout and zero otherwise, σ is a scale parameter to be estimated, and

$$w_i = \left[\frac{\log(N)}{SD}\right] - A_1\left[\frac{1}{SD}\right] - A_2\left[\frac{\log(S_{eq} - A_4)}{SD}\right]$$

where N is the cycles to failure and SD is the standard deviation for the ith observation as calculated from Equation 9.3.4.10(c) or Equation 9.3.4.10(e).

For more information on the maximum likelihood procedure see reference 9.3.4.14(a). For use in standard data analysis, the maximum likelihood procedure is conveniently implemented in some statistical software packages such as SAS [see reference 9.3.4.14(b)].

When runouts are present, the fitted curve produced by maximum likelihood will generally predict longer average cycles to failure at given equivalent stress (strain) values than the fitted curve produced by least squares. Although it would be desirable to update all of the parameters in the fatigue model with maximum likelihood, algorithms to perform maximum likelihood on nonlinear models are not readily available. For this reason, the least squares estimates of the parameters A_3 and A_4 must be used.

9.3.4.15 Recognition of Time Dependent Effects.--All prior discussion has been based on the assumption that time dependent effects in the fatigue data sample of interest are negligible. When dealing with elevated temperature fatigue properties of materials (or room temperature fatigue properties in a corrosive environment, for example) this assumption may not be realistic. Analysis methods that are approved for use in MIL-HDBK-5 do not account for time dependent effects. Therefore, every effort must be made to identify data that embody significant time dependent effects.

There are no absolute methods presently available for sensing time dependent effects in fatigue data; however, there are some useful approximation techniques. One of the more useful approaches applied to "suspect" data is to include time dependent terms in the regression model. If the terms are significant, there is reason to believe that the population contains time dependent data. Subdividing the data into subsets that do not show time dependent effects may be possible. If this is not possible, the data set should either be rejected or included with a disclaimer restricting usage of the data to predict performance at other frequencies or temperatures.

One other possible indicator of time dependent effects is an abnormal equivalent stress (strain) model. If data for different stress or strain ratios do not fit the customary models (as described in 9.3.4.9), or abnormal optimum parameters are defined, the problem may be caused by time dependent effects. In the case of the primary equivalent stress (strain) formulation equation the exponent normally is between zero and one. If the A_3 exponent approaches or exceeds one, the influence of maximum stress on fatigue life is negligible. This is a very unusual result that usually indicates problems with the data sample. The problem may result from mixed sources, where the data from each source were generated at different stress (strain) ratios. Possible rejection of such data sets is discussed in 9.3.4.13. In the case of the primary equivalent stress model [Equation 9.3.4.9(a)] if the exponent (A_3)

approaches or is less than zero, it indicates the influence of maximum stress on fatigue life is "too strong" This result implies that creep effects are affecting the data.

If data are available for a material at a range of different temperatures it may be possible to analyze these sets separately and make comparisons between best-fit mean trend lines for increasing temperatures. If the different mean trend lines are not consistent, with the higher-temperature curves converging or diverging from the lower-temperature curves, there is probably a significant time dependent effect in the data. The suspect data should either be excluded from the sample set or included with a disclaimer as previously cited. If data are excluded for time dependent effects, the preliminary analyses of those data should be included in the data proposal and reasons for their exclusion should be given.

9.3.4.16 Presentation of Fatigue Analysis Results.--Separate data presentations are made for strain-controlled and load-controlled data. The only case where load-controlled data can be presented with strain-controlled data is when long-life tests have been switched from strain to load control in accordance with recommended procedures (see 9.3.4.5). Separate plots should be constructed for each material, notch concentration (in the case of load-controlled data), temperature, or other documented parameters that have been demonstrated to cause significant variations in fatigue behavior.

Load-controlled data presentations should consist of a family of at least three stress ratio or mean stress curves, with at least six data points per curve covering two orders of magnitude in life. (See exceptions noted in 9.3.4.4). The basic data should be included on each plot, with separate symbols used for each stress ratio or mean stress. Runouts should be identified with an arrow (→). The analytically defined mean S-N curves for each stress ratio or mean stress should also be included on each plot. These curves should not be extrapolated beyond existing data.

The fatigue curve for each stress ratio should be constructed based on the following criteria:

(1) The curve should start at a maximum stress no more than 5 ksi above the greatest maximum stress fatigue data point for that specific stress ratio. Unnotched fatigue curves should not extend above the typical ultimate strength for the material.

(2) The curve should terminate at the least maximum stress value for that specific stress ratio

In addition to the stress-life plot [such as shown in Figure 9.3.4.17(h)] a tabulation of test and material conditions should also be included. At a minimum the following information should be included with an S-N plot:

(1) Material
(2) Product Form, Grain Direction, Thickness, Processing History, Fabrication Sequence
(3) Test Parameters
- Loading
- Test Frequency
- Temperature
- Environment
(4) Average Tensile Properties
(5) Specimen Details
- Notch Description
- Specimen Dimensions
(6) Surface Condition/Surface Residual Stresses/Finish
- Finish
- Residual Stress Data

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(7) Equivalent Stress Equation
  - Life Equation With Parameter Estimates
  - Standard Deviation of log(Life)
  - Adjusted R-squared Statistic
  - Sample Size
(8) Reference Numbers
(9) No. of Heats/Lots.

The following cautionary note should be included with each equivalent stress equation (Caution The equivalent stress model may provide unrealistic life predictions for maximum stresses and stress ratios beyond those represented above.) In calculating the "standard deviation of log(life)" and the adjusted R-squared statistic, all quantities should be computed using the final estimates of the fatigue model parameters and excluding runout observations.

The method for reporting the "standard deviation of log (life)" (SD) depends on whether there is evidence of nonuniform variance in the fatigue life data. If an unweighted fatigue model was fitted to the data, the single SD value from Equation 9.3.4.10(c) should be reported. If a weighted fatigue model was fitted to the data, SD should be reported as the linear function of the reciprocal of equivalent stress (strain) as calculated from Equation 9.3.4.10(e).

If an unweighted fatigue life model was fitted to the data, the adjusted R-squared statistic may be calculated as

$$R^2 = 1 - (RMSE)^2/(RTE)^2 \qquad [9.3.4.16(a)]$$

where

$$RTE = \sqrt{\sum_{i=1}^{n} D_i^2/(n-1)}$$

$$D_i = \log(N_i) - \overline{\log(N)}$$

$$\overline{\log(N)} = \frac{1}{n}\sum_{i=1}^{n}\log(N_i)$$

and RMSE is as calculated in Equation 9.3.4.10(c). If a weighted fatigue life model was fitted to the data, the adjusted R-squared statistic may be calculated as

$$R^2 = 1 - (RMSE)^2/(RTE)^2 \qquad [9.3.4.16(b)]$$

Supersedes page 9-80bb of Change Notice 1, MIL-HDBK-5E where

$$RTE = \sqrt{\sum_{i=1}^{n} WD_i^2/(n-1)}$$

$$WD_i = \frac{\log(N_i) - \overline{\log(N)}}{g(S_{eq,i} \text{ or } \varepsilon_{eq,i})}$$

$$\overline{\log(N)} = \frac{\sum_{i=1}^{n} \log(N_i)/g(S_{eq,i} \text{ or } \varepsilon_{eq,i})}{\sum_{i=1}^{n} (1/g(S_{eq,i} \text{ or } \varepsilon_{eq,i})}$$

and RMSE is as calculated in Equation 9.3.4.10(f).

Strain-controlled data presentations should consist of a plot of log (strain range) versus log (life) and a separate graph displaying the monotonic and cyclic stress-strain response for the material. Normally the fatigue curves should be based on at least six data points for each of three or more strain ratios, and the data should cover at least two orders of magnitude in life. As with the load-controlled data, the individual data points should be included on each plot, with separate symbols used for each strain ratio. If runouts are included in the data, they should be identified with an arrow (→). Data points that are based on tests that were switched from strain to load control should be identified clearly. The mean curves should extend from slightly above the greatest strain value to slightly below the least strain value.

Plotting the strain-life curves for different strain ratios is not as straightforward as plotting stress-life curves. The equivalent strain models cannot be written explicitly in terms of R_ε. Therefore, other information must be used to model the data trends for the various strain ratios. The mean-stress relaxation behavior for each strain ratio must be identified and mathematically defined. In general, the onset of mean stress relaxation occurs at smaller strain amplitudes for larger strain ratios. This behavior is shown in the mean stress relaxation plot of Figure 9.3.4.16(b). The elastic response (dashed lines) predicts much higher mean stresses than those actually observed, suggesting that mean stress relaxation has occurred. The regression line correlating the relaxed mean stresses with strain amplitude intersects the elastic response lines at larger strain amplitudes for smaller strain ratios. The elastic response line for the higher strain ratio ($R_\varepsilon = 0.6$) intersects the mean stress relaxation line at approximately $\Delta\varepsilon/2 = 0.0007$. The elastic response line for the lower strain ratio ($R_\varepsilon = 0.0$) intersects the mean stress relaxation line at approximately $\Delta\varepsilon/2 = 0.002$. This information can be used to construct reasonable mean curves for each strain ratio for which fatigue data are available.

Considering the primary equivalent strain relation [Equation 9.3.4.9(c)]

$$\varepsilon_{eq} = (\Delta\varepsilon)^{A_3} (S_{max}/E)^{1-A_3} \quad ,$$

Supersedes page 9-80cc of Change Notice 1, MIL-HDBK-5E.

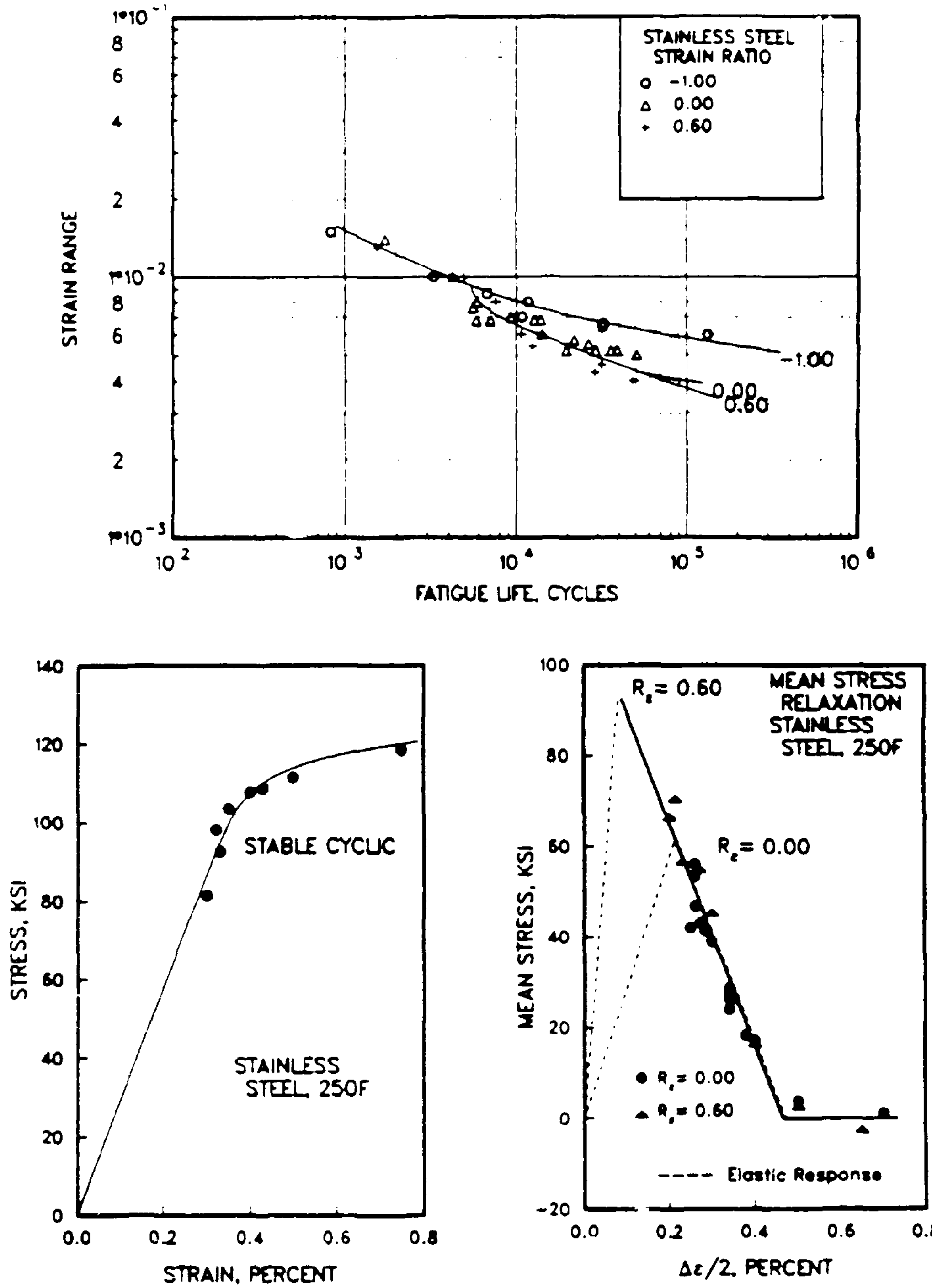

FIGURE XXXX. *Best fit ε/N curve, cyclic stress-strain curve, and mean stress relaxation curves for stainless steels.*

FIGURE 9.3.4.16(b). *Example strain-life, cyclic stress-strain, and mean stress relaxation curves*

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S_{max} can be written as

$$S_{max} = S_m - S_a$$

where S_m is the relaxed mean stress and S_a is the stress amplitude found from the cyclic stress-strain curve. Given the mean stress relaxation data, both S_m and S_a can be estimated for a particular strain amplitude and strain ratio. Once S_{max} is defined, based on S_a and S_m, ε_{eq} can be calculated and a fatigue life can be determined. Through this procedure an approximate mean curve can be constructed for each strain ratio as shown in Figure 9.3.4.16(b).

If the stress amplitude (S_a) and the mean stress relaxation pattern can reasonably be assumed to be independent of strain ratio, the following procedure may be used to construct mean curves for each strain ratio by expressing S_a as a function of the strain range and S_m as a function of the strain range and strain ratio. Using the data corresponding to a strain ratio of $R_\varepsilon = -1$ only, fit the regression equation

$$\log(S_{max}) = \alpha_1 - \beta_1 \log(\Delta\varepsilon/2 - S_{max}/E)$$

In some cases it may be necessary to exclude small plastic strain observations from the regression, because of the scatter (and likely unreliability) in these values. In other words, it is recommended that the cyclic stress strain curve be defined, through a least squares regression treating stress as the dependent variable, with consideration given to a cutoff in cyclic plastic strain. A cutoff of approximately 0.0001 in plastic strain amplitude is often useful.

Assuming that stress amplitude is independent of strain ratio and provided that the estimate of the parameter β_1 is greater than zero, a mean value for stress amplitude can be determined as a function of strain range by solving the formula

$$S_a/\bar{E} + (S_a/k)^{1/n} = \Delta\varepsilon/2 \qquad [9.3.4.16(c)]$$

for S_a where $\bar{E}$ is the average elastic modulus for all specimens tested and

$$n = \beta_1 \text{ and } k = \text{Alog}(\alpha_1).$$

If the estimate of the parameter β_1 is less than or equal to zero, the data set should be examined further before proceeding with the analysis.

Using the data corresponding to all strain ratios other than $R_\varepsilon = -1$, fit the regression equation

$$S_m = \alpha_2 + \beta_2(\Delta\varepsilon/2)$$

using weighted least squares to give higher weight to the observations which exhibit partial mean stress relaxation. If there is no way to directly calculate S_m from the data reported in the data set, an S_m value for use in fitting the above regression equation may be calculated by solving

Supersedes page 9-80e of Change Notice 1, MIL-HDBK-5E.

Equation 9 3 4 16(c) for S_a and subtracting this value from the reported S_{max} value. The weighting function

$$w = (S_m/S^*)(1 - S_m/S^*)^2$$

where

$$S^* = ((1 + R_\varepsilon)/(1 - R_\varepsilon)) E (\Delta\varepsilon/2)$$

appears to work well in general. Assuming that the mean stress relaxation pattern is independent of strain ratio and provided that the estimate of the parameter β_2 is less than zero, a mean value for S_m can be determined as a function of strain range and strain ratio according to the formula

$$S_m = \begin{cases} \beta_3(\Delta\varepsilon/2) & (\Delta\varepsilon/2) \leq \alpha_2/(\beta_3 - \beta_2) \\ \alpha_2 + \beta_2(\Delta\varepsilon/2) & \alpha_2/(\beta_3 - \beta_2) \leq (\Delta\varepsilon/2) \leq -\alpha_2/\beta_2 \\ 0 & -\alpha_2/\beta_2 \leq (\Delta\varepsilon/2) \end{cases}$$

where

$$\beta_3 = ((1 + R_\varepsilon)/(1 - R_\varepsilon)) \bar{E}$$

If the estimate of parameter β_2 is greater than or equal to zero, the data set should be examined further before proceeding with the analysis.

Mean curves determined according to the above procedures exhibit the following characteristics.

(1) At large strain ranges, enough plastic strain is available to relax the mean stress to zero, regardless of the strain ratio. Therefore, all strain ratios result in equivalent predicted fatigue lives.

(2) At strain ranges corresponding to mean stresses represented by the relaxation regression line, strain ratios other than R_ε = -1 (zero mean stress) result in equivalent predicted fatigue lives.

(3) At low strain ranges, the individual strain ratios assume their elastic mean stress response and diverge from each other.

The above procedure is used for plotting the strain-life curves in MIL-HDBK-5 when multipie strain ratios are involved.* The curves generally represent the mean data trends closely.

In addition to the strain-life plot, stress-strain curves and a mean stress relaxation curve should be presented as shown in Figure 9.3.4.16(b). A tabulation of test and material conditions should also be included as shown in Figure 9.3.4.16(c). This information should include:

(1) Material
(2) Product Form, Grain Direction, Thickness, Processing History, Fabrication Sequence

Correlative Information for Figure 9.3.4.16(b)

Product Form:
Die forging, 2-inch thick

Thermal Mechanical Processing History:
Anneal at 1800 F, water quench

Properties:

TUS, ksi	TYS, ksi	E, ksi	Temp., F
155-160	135-140	29,000	250

Stress-Strain Equations:
Monotonic
Proportional Limit = 111 ksi
$\sigma = 289\,(\varepsilon_p)^{0.138}$
Cyclic (Companion Specimens)
Proportional Limit = 92 ksi
$(\Delta\varepsilon/2) = 156\,(\Delta\varepsilon_p/2)^{0.046}$
Mean Stress Relaxation
$\sigma_m = 114.0\text{-}24562(\Delta\varepsilon/2)$

Specimen Details:
Uniform gage test section
0.250-inch diameter
Polished with increasingly finer grits of emery paper to surface roughness of 10 RMS with polishing marks longitudinal.

References: 3.4.5.6.8(a)

Test Parameters:
Strain Rate/Frequency - 180 cpm
Wave Form - Sinusoidal
Temperature - 250 F
Atmosphere - Air

No. of Heats/Lots: 2

Equivalent Strain Equation:
$\log N_f = -6.56 - 4.20 \log(\varepsilon_{eq} - 0.0022)$

$\varepsilon_{eq} = (\Delta\varepsilon)^{0.46}\,(S_{max}/E)^{0.54}$

Standard Deviation of log(Life): 0.123
Adjusted R^2 Statistic: 93%

Sample Size: 33

[Caution: The equivalent strain model may provide unrealistic life predictions for strain ratios and ranges beyond those represented above.]

FIGURE 9.3.4.16(c). *Example of correlative information and analysis results for a strain control fatigue data presentation.*

*In the general case, data generated at different strain ratios will not necessarily follow the same mean stress relaxation pattern. If different patterns for each strain ratio are evident in a particu-lar case it is suggested that a family of mean stress relaxation curves be constructed.

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(3) Test Parameters
  - Strain Rate and/or Frequency
  - Wave Form
  - Temperature
  - Environment
(4) Average Tensile Properties
(5) Stress-Strain Equation
  - Monotonic (if available and appropriate) - Cyclic
(6) Specimen Details
  - Specimen Type
  - Specimen Dimensions
  - Fabrication Sequence
(7) Surface Condition/Surface Residual Stresses/Finish
  - Finish
  - Residual Stress Data
(8) Equivalent Strain Equation
  - Life Equation with Parameter Estimates
  - Standard Deviation of log (life)
  - Adjusted R-squared Statistic
  - Sample Size
(9) Reference Numbers
(10) No. of Heats/Lots.

The following cautionary note should be included with each equivalent strain equation [Caution. The equivalent strain model may provide unrealistic life predictions for strain ratios and ranges beyond those represented above.]

9.3.4.17 Example Problems --

EXAMPLE 1. STRAIN CONTROL

A collection of iron alloy bar strain-controlled fatigue data at 70 F is given in Table 9.3.4.17. The required steps for the analysis of the data set are presented below. The guideline sections relating to each step in the analysis are noted.

Data Requirements (Section 9.3.4.4).--The data set includes three strain ratios (R_ε = -1.0, 0.0, 0.6) each consisting of at least eight nonrunout data points. This satisfies the minimum recommended sample size for analysis. Two runouts ($N_f = 10^6$ and 10^7) at R_ε = -1 are included in the data set.

Data Collection (Section 9.3.4.6).--The specimen design for the test program is reported as uniform gage section with a diameter of 0.20 inches. Failure is defined as complete separation. The tensile properties are presented in the correlative information. No information is available regarding the fabrication sequence for the specimens. Fabrication information is important, although in this case it is not considered sufficient cause to reject the data set for analysis. The test data at the R_ε = -1.0 strain ratio provide information regarding this material's cyclic stress-strain response. The cyclic stress-strain curve constructed from the data is shown in Figure 9.3.4.17(a). The monotonic curve (dashed) is estimated from the reported yield and ultimate strengths.

TABLE 9.3.4.17 *Iron alloy strain-controlled fatigue data at 70 F*

Specimen Number	$\Delta\varepsilon$	S_{max} (ksi)	Cycles to Failure	Strain Ratio
1	0.600	71.1	10223	-1.00
2	0.600	77.8	10396	-1.00
3	0.600	79.2	8180	-1.00
4	0.970	117.2	605	-1.00
5	1.000	110.7	672	-1.00
6	1.000	112.8	642	-1.00
7	1.500	126.9	209	-1.00
8	1.500	127.1	340	-1.00
9	0.600	116.6	3958	0.0
10	0.600	124.2	3895	0.0
11	0.597	118.2	3919	0.0
12	0.600	128.3	4050	0.0
13	0.600	122.6	2470	0.0
14	0.400	106.4	16388	0.0
15	0.393	101.9	22896	0.0
16	0.400	102.1	15388	0.0
17	0.400	93.7	38648	0.0
18	0.400	101.2	11960	0.0
19	0.750	139.4	1099	0.60
20	0.750	137.3	1544	0.60
21	0.750	113.0	966	0.60
22	0.500	124.5	4665	0.60
23	0.500	140.6	4342	0.60
24	0.500	138.4	4240	0.60
25	0.400	158.0	7460	0.60
26	0.400	146.1	11134	0.60
27	0.400	119.1	10876	0.60
28	0.440	65.8	1000000*	-1.00
29	0.330	50.0	1000000*	-1.00

* Did not fail.

Evaluation of Mean Stress and Strain Effects (Section 9.3.4.9).--The data set consists of three strain ratios and therefore an equivalent-strain formulation is used to consolidate the data on the basis of equivalent strain. Equation [9.3.4.9(c)],

$$\log N_f = A_1 + A_2 \log(\varepsilon_{eq} - A_4)$$

where

$$\varepsilon_{eq} = (\Delta\varepsilon)^{A_3} (S_{max}/E)^{1-A_3} ,$$

is the initial model attempted for fitting the data and proves to be adequate throughout the analysis.

Supersedes page 9-80ii of Change Notice 1, MIL-HDBK-5E.

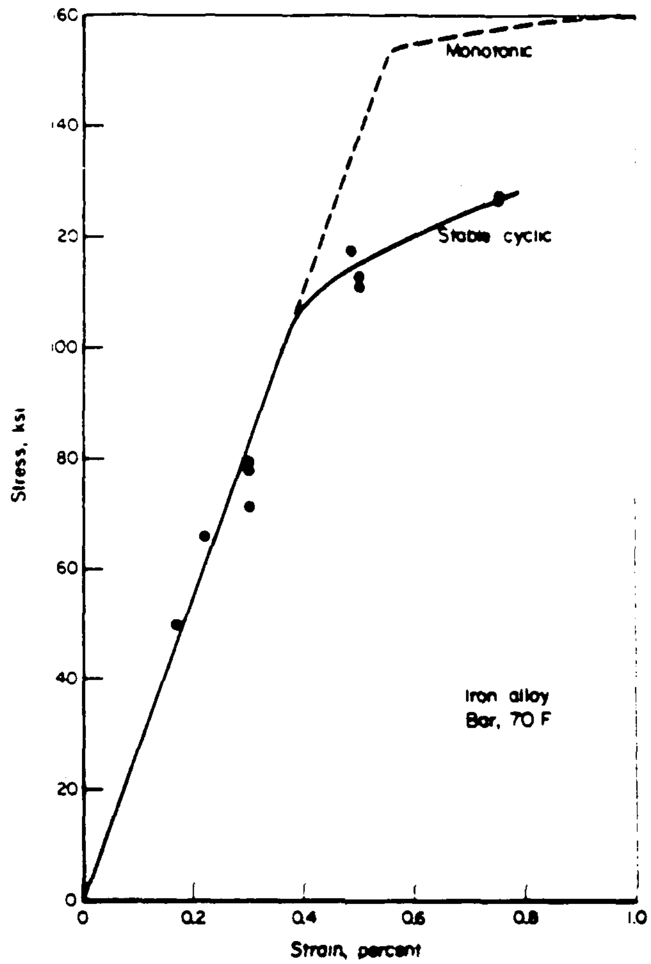

FIGURE 9.3.4.17(a). *Stable cyclic and monotonic stress-strain curves for iron alloy at 70 F.*

Estimation of Fatigue Life Model Parameters - Least Squares (Section 9.3.4.10).--The initial least-squares regression results in the following fatigue-life equation parameters:

$$A_1 = -4.62$$

$$A_2 = -3.28$$

$$A_3 = 0.610$$

$$A_4 = 0.00198.$$

A plot of the residuals for the fatigue model using these parameters is shown in Figure 9.3.4.17(b). These residuals do not exhibit the characteristic pattern of increasing residual magnitudes with decreasing equivalent strain levels shown in Figure 9.3.4.10(a). Rather, the variance appears to be relatively uniform. During Step 2 of the parameter estimation procedure, a negative, but insignificant, estimate of the residual model slope, $\sigma 1$, was obtained. This result indicates the the residuals are already uniformly distributed and a constant variance model can be used. The constant variance model, in effect, does not weight the fatigue life model, so the initial parameter estimates are retained.

Supersedes page 9-80jj of Change Notice 1, MIL-HDBK-5E

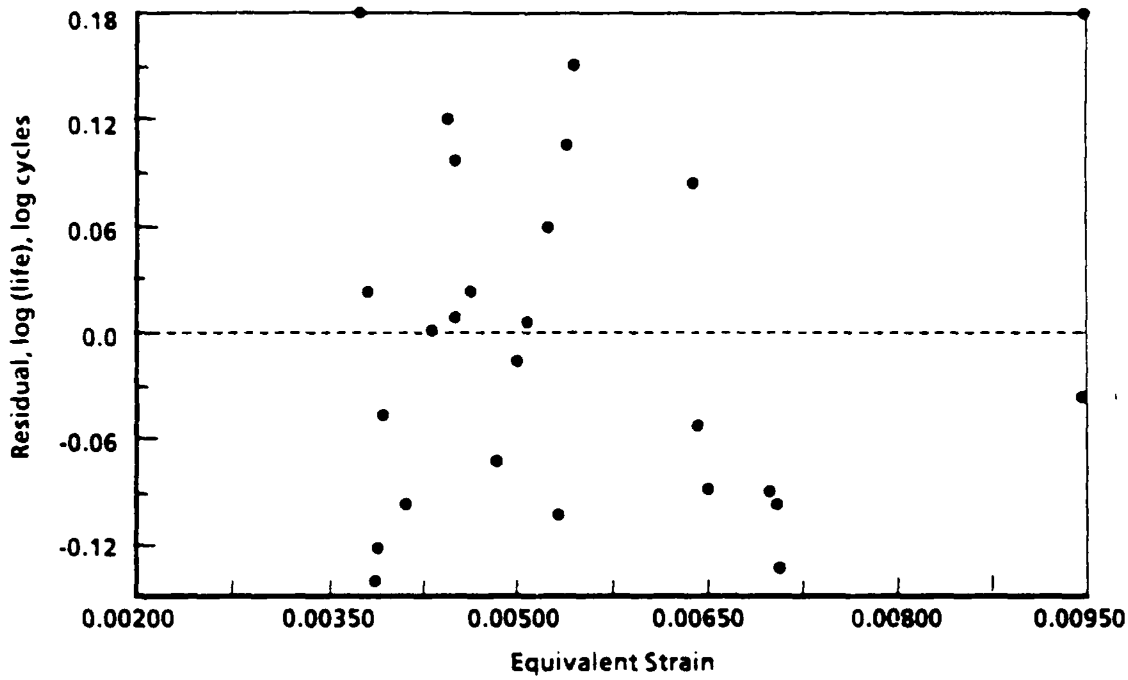

FIGURE 9.3.4.17(b). *Residual plot of fatigue-life model for initial parameter estimates.*

Treatment of Outliers (Section 9.3.4.11).--After the data have been checked for uniformity of variance, they can be screened to determine if any outliers are present. The critical studentized residual at the 5 percent significance level for this sample of 27 observations is found to be 3.53. Any of the observations with the absolute value of the studentized residuals being greater than 3.53 would be considered outliers. The largest studentized residual from the data was 2.09, therefore none of the observations are identified as statistically significant outliers.

Assessment of the Fatigue Life Model (Section 9.3.4.12).--The equivalent strain formulation is MARGINALLY acceptable at the 5 percent level. The lack of fit test for the fatigue-life model results in a Durbin-Watson D statistic of 1.042. The critical value of D for a sample size of 27 is 1.241 [Equation 9.3.4.12(b)].

Since the Durbin-Watson statistic is less than the critical value, the equivalent strain model must be considered questionable in terms of its compensation for effects of strain ratio. However, no other model was found to perform better and a review of the plotted data revealed very low scatter compared to the predicted trends. Therefore, engineering judgement was used, and the proposed model was accepted.

Data Set Combination (Section 9.3.4.13).--All of the data for this analysis came from a single source, therefore, this test is not applicable.

Supersedes page 9-80kk of Change Notice 1, MIL-HDBK-5E.

Treatment of Runouts (Section 9.3.4.14).--The data set being considered includes two runout observations. The parameters A_1 and A_2 are therefore reestimated using the maximum likelihood regression to account for censored life values. The maximum likelihood estimates are.

$$A_1 = -5.07$$

$$A_2 = -3.47$$

$$A_3 = 0.610$$

$$A_4 = 0.00198.$$

The change in parameters A_1 and A_2 shift the predicted lives to greater values than the least squares parameter estimates.

Presentation of Fatigue Analysis Results (Section 9.3.4.16).--The presentation of the strain-life curve and correlative information shown in Figure 9.3.4.17(c) is typical of a MIL-HDBK-5 strain-control fatigue data proposal. Regarding the mean stress relaxation plot, note that a single regression has been performed to represent both the $R_\epsilon = 0.6$ and $R_\epsilon = 0.0$ strain ratios. Although it would be expected that higher strain ratios would result in higher stabilized mean stresses, the limited amount of data precludes performing separate regressions for each strain ratio. It can be seen from the strain-life plot that using the single regression does represent the mean fatigue trends fairly well.

EXAMPLE 2. LOAD CONTROL

A large collection of 300 M alloy die forging fatigue data is presented in Figure 9.3.4.17(d). The required steps for the analysis of the data set are presented below.

Data Requirements (Section 9.3.4.4).--The data set consists of four stress ratios (R = -1.0, -0.33, 0.05, 0.2). Each stress ratio includes at least twenty-three nonrunout observations, easily satisfying the minimum sample size requirement of six tests per stress ratio.

Data Collection (Section 9.3.4.6).--The data shown in Figure 9.3.4.17(d) were compiled from four sources. Each source reports the results of fatigue testing programs conducted within two years of each other (1968-1970).

The failure criteria for all tests is reported as complete separation of the specimen. Those tests which did not fail are identified on the S/N plot with an arrow (→). These runout observations are treated differently in the regression analysis which define the mean fatigue curves (see 9.3.4.14).

Evaluation of Mean Stress and Strain Effects (Section 9.3.4.9).--The collection of data consists of four stress ratios and therefore an equivalent-stress formation was used to consolidate the data. Equation [9.3.4.9(a)].

$$\log N_f = A_1 + A_2 \log (S_{eq} - A_4)$$

where

$$S_{eq} = S_{max} (1 - R)^{A_3} ,$$

is the initial model attempted for fitting the data, and it proved adequate throughout the analysis.

Supersedes page 9-80ll of Change Notice 1, MIL-HDBK-5E

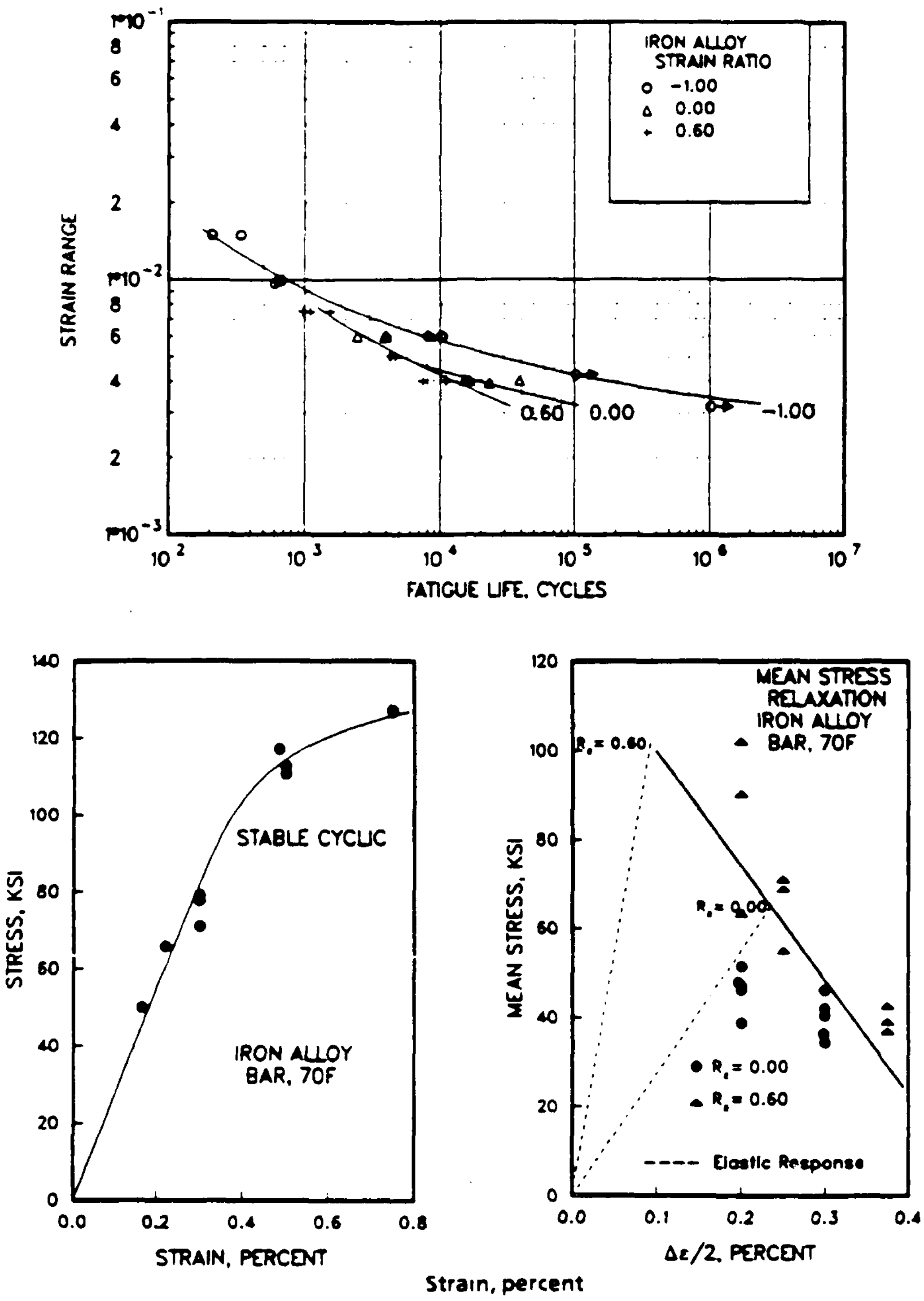

FIGURE 9.3.4.17(c). *ε/N curve and correlative information for iron alloy at 70 F.*

Supersedes page 9-80mm of Change Notice 1, MIL-HDBK-5E.

Correlative Information for Figure 9 3 4 17(c)

Product Form/Thickness:
Bar/1 inch thick

Thermal Mechanical Processing History
Not available

Properties:

TUS, ksi	TYS, ksi	E, ksi	Temp., F
175-180	150-155	27,500	70

Stress-Strain Equations:
Monotonic
Proportional Limit = 150 ksi
$\sigma = 280\,(\varepsilon_p)^{0.12}$
Cyclic (Companion Specimens)
Proportional Limit = 105 ksi (est.)
$(\Delta\sigma/2) = 196\,(\Delta\varepsilon_p/2)^{0.076}$
Mean Stress Relaxation
$\sigma_m = 125 + 25666(\Delta\varepsilon/2)$

Specimen Details:
Uniform gage test section
0.200-inch diameter

References: 3.4.5.6.8(a)

Test Parameters
Strain Rate/Frequency - 180 cpm
Wave Form - Sinusoidal
Temperature - 70 F

No. of Heats/Lots: 4

Equivalent Strain Equation
$\log N = -5.07 - 3.47 \log(\varepsilon_{eq} - 0.00198)$

$\varepsilon_{eq} = (\Delta\varepsilon)^{0.61}\,(S_{max}/E)^{0.39}$

Standard Deviation of log(Life): 0.111

Adjusted R^2 Statistic: 96%

Sample Size: 29

[Caution: The equivalent strain model may provide unrealistic life predictions for strain ratios and ranges beyond those represented above.]

FIGURE 9.3.4.17(c). *ε/N curve and correlative information for iron alloy at 70 F - Continued.*

Estimation of Fatigue Life Model Parameters--Least Squares (Section 9.3.4.10).--The initial least-squares regression (runouts exlcuded) results in the following fatigue-life equation parameters:

$$A_1 = 23.7$$

$$A_2 = -8.41$$

$$A_3 = 0.366$$

$$A_4 = 0.0$$

The fatigue-limit parameter (A_4) of zero seems somewhat inconsistent with the data shown in Figure 9.3.4.17(d). A visual examination of the S/N plot reveals a tendency for the data to asymptotically approach some limiting value. The zero fatigue limit term suggests that some problem may exist within the data collection. A plot of the residuals for the fatigue model using these parameters is shown in Figure 9.3.4.17(e).

Supersedes page 9-80nn of Change Notice 1, MIL-HDBK-5E

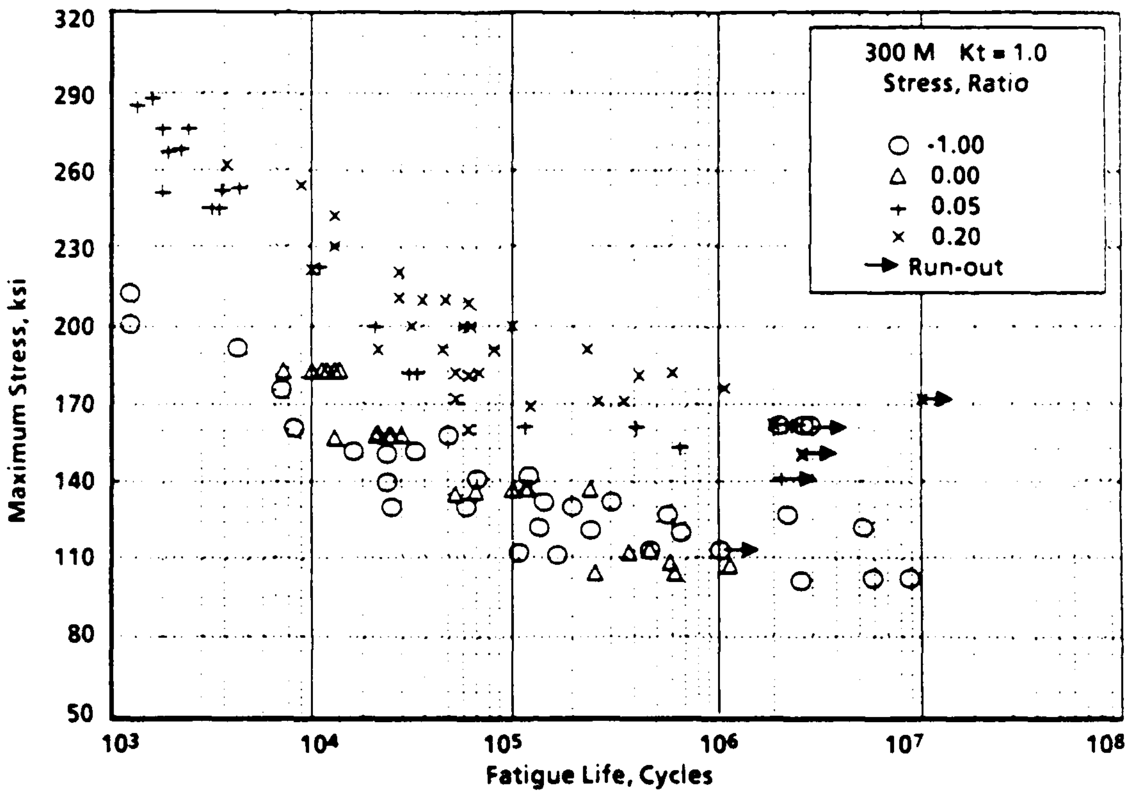

FIGURE 9.3.4.17(d). *S/N plot of unnotched 300M die forging fatigue data, transverse orientation.*

The parameters obtained after the model is adjusted for nonconstant variance are:

$$A_1 = 23.4$$

$$A_2 = -8/38$$

$$A_3 = 0.40$$

$$A_4 = 13.5$$

Note that a fatigue limit term of 13 ksi has now been estimated. However, a check on the significance of the A_4 term revealed that it was clearly insignificant. All of the runouts in the data collection were above this equivalent stress level and, therefore, all runouts were used in the regression procedure. A plot of the residuals after the fatigue life model has been adjusted is shown in Figure 9.3.4.17(f). Note the relative shift in the magnitude of the residuals at the higher and lower S_{eq} values compared to Figure 9.3.4.17(e).

Treatment of Outliers (Section 9.3.4.11).--None of the observations were identified as outliers. The critical studentized residual at the 5 percent significance level for this data set of 114 observations is 3.63. The largest standardized residual was 3.23, resulting from a runout observation.

Supersedes page 9-80oo of Change Notice 1, MIL-HDBK-5E.

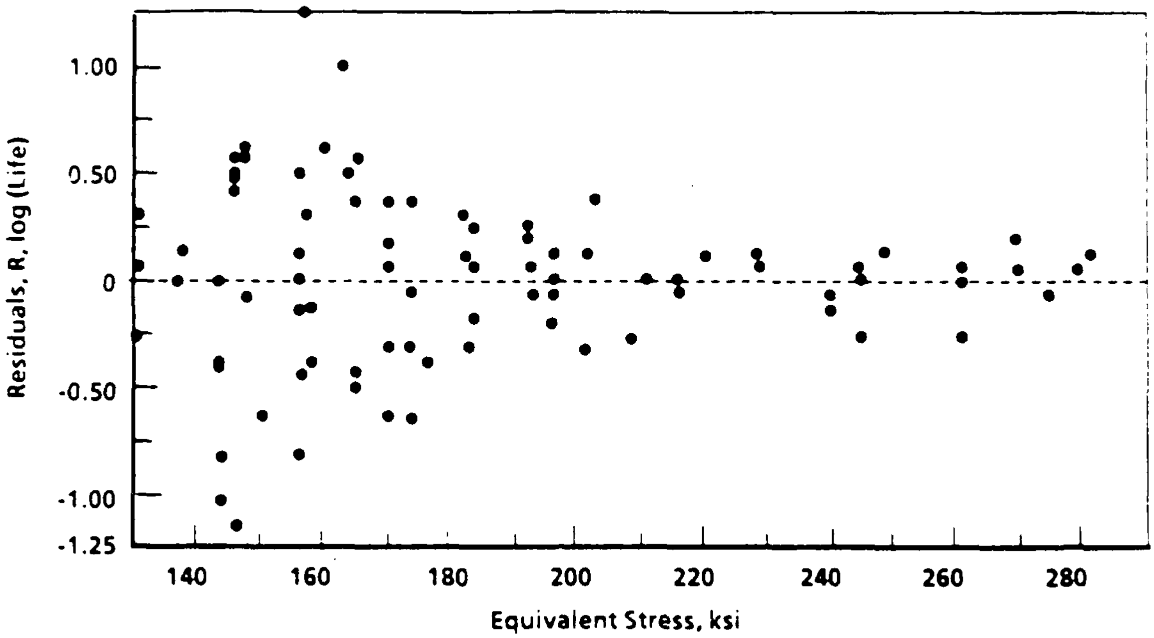

FIGURE 9.3.4.17(e) *Residual plot before model has been adjusted for nonconstant variance*

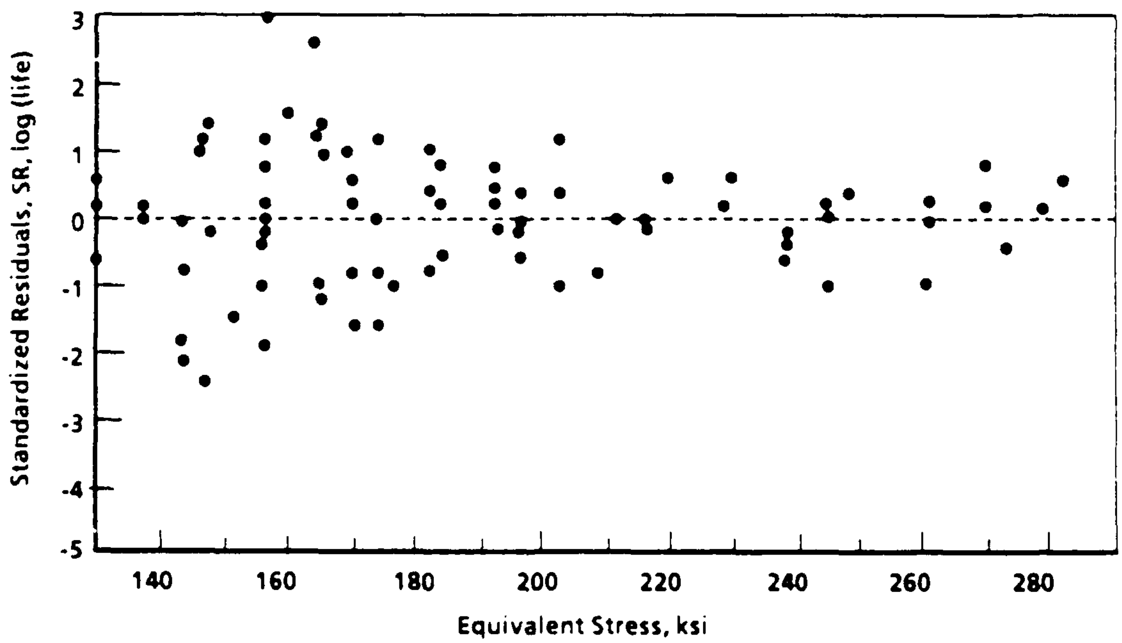

FIGURE 9.3.4.17(f). *Standardized residual plot after model has adjusted for nonconstant variance.*

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Assessment of the Fatigue Life Model (Section 9.3.4.12) --The equivalent stress model is not able to consolidate the R = -0.33 stress ratio with the other stress ratios. The F-test performed on the residuals of the stress ratios proves significant at the 5 percent level for R = -0.33. This indicates that the mean of the residuals for R = -0.33 differs significantly from the mean of the residuals from the other ratios. The plot of stress ratios versus residuals, as shown in Figure 9.3.4.17(g) illustrates that the mean of the residuals for R = -0.33 is significantly different than those for the other stress ratios. A close examination of the original S/N plot shown in Figure 9.3.4.17(d) reveals that the R = -0.33 data tend to overlap the R = -1.0 data. At the lower maximum stress levels (about 100 ksi), the R = -1.00 data actually show longer average fatigue lives than do the R = -0.33 data, when the reverse would be expected. The Durbin-Watson D statistic for determining lack of fit is 1.61 indicating a poor fit of the model to the data. The critical value of D for a sample of 114 observations [Equation 9.4.3.12(b)] is 1.66.

This incompatibility among stress ratios indicates that either a problem exists with the data or with the assumed equivalent stress model. The data sources were re-examined to possibly determine if some difference in specimen preparation or testing procedure among the sources may have caused the inconsistencies. Unfortunately, no significant differences were discovered that would provide sufficient reason to exclude the suspect R = -0.33 data due to testing methods alone. The problem is confounded because all of the R = -0.33 data comes from a single source which does not include other stress ratios. This precludes examining source to source variability.

In situations such as this where a data set for a single source is determined to statistically deviate from the fatigue trends exhibited by the bulk of the data, it should be evaluated for exclusion. Engineering judgement suggests that the R = -0.33 data be excluded from the data collection based on the following:

(1) unrealistic fatigue limit

(2) lack of fit for fatigue life model based upon Durbin-Watson statistic

(3) stress ratio incompatibility.

The modified data collection is now reanalyzed. For the sake of brevity, the details of the analysis procedure for Section 9.3.4.4 (Data Requirements) through 9.3.4.11 (Treatment of Runouts) will be omitted. It is interesting to note, however, that the fatigue limit term (A_4) resulting from the least squares regression with the R = -0.33 data excluded is 94.2 ksi. This result more realistically represents the longer life fatigue trends compared to the previous (insignificant) estimate of 13.5 ksi. With the suspect data removed, the equivalent stress model is determined to be acceptable at the 5 percent level. The Durbin-Watson D statistic also is increased to 2.18 indicating that the model now provides an adequate fit to the data.

Dataset Combination (Section 9.3.4.13).--With the exclusion of the source containing the R = -0.33 data, the remaining data set combination is determined acceptable at the 5 percent level.

Treatment of Runouts (Section 9.3.4.14).--The data collection includes seven runout observations. The maximum likelihood procedure has the effect of essentially shifting these runouts to the fatigue lives at which they most likely would have failed. The resulting fatigue life model parameters should reflect the slight increase in estimated fatigue life over the least squares parameters, particularly in the long life region. In general, the maximum likelihood regression will result in a higher intercept term (A_1) and a steeper (more negative) slope (A_2). The A_3 and A_4 terms are taken as constants to reduce the problem to a linear analysis.

Supersedes page 9-80qq of Change Notice 1, MIL-HDBK-5E

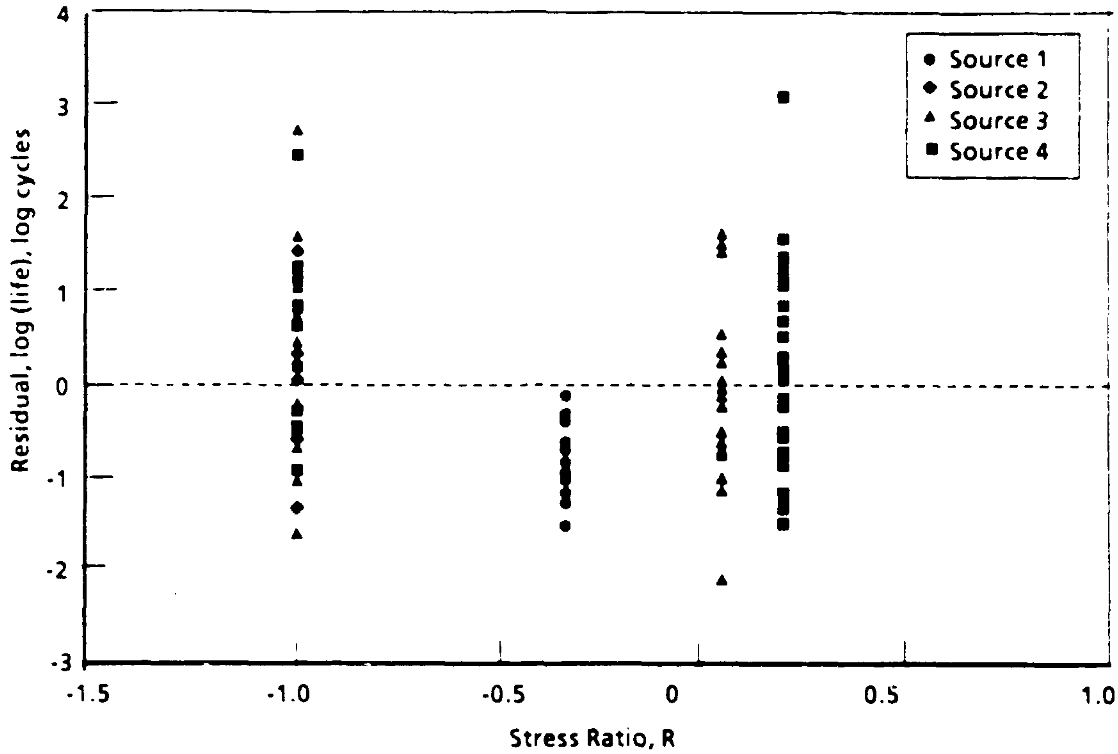

FIGURE 9.3.4.17(g) *Residual plot of stress ratios. Note the low mean value of R = -0.33*

The parameters resulting from the least squares regression are

$$A_1 = 14.54$$

$$A_2 = -5.04$$

$$A_3 = 0.385$$

$$A_4 = 94.2$$

The maximum likelihood parameters conform to the expected trends for A_1 and A_2

$$A_1 = 14.79$$

$$A_2 = -5.16$$

$$A_3 = 0.385$$

$$A_4 = 94.2$$

Note the increase in A_1 and the decrease (more negative slope) in A_2

Supersedes page 9-80rr of Change Notice 1, MIL-HDBK-5E

Presentation of Fatigue Analysis Results (Section 9.3.4.16) --The stress-life curve and correlative information shown in Figure 9.3.4.17(h) is typical of a MIL-HDBK-5 load-control fatigue data proposal.

Supersedes page 9-80ss of Change Notice 1, MIL-HDBK-5E

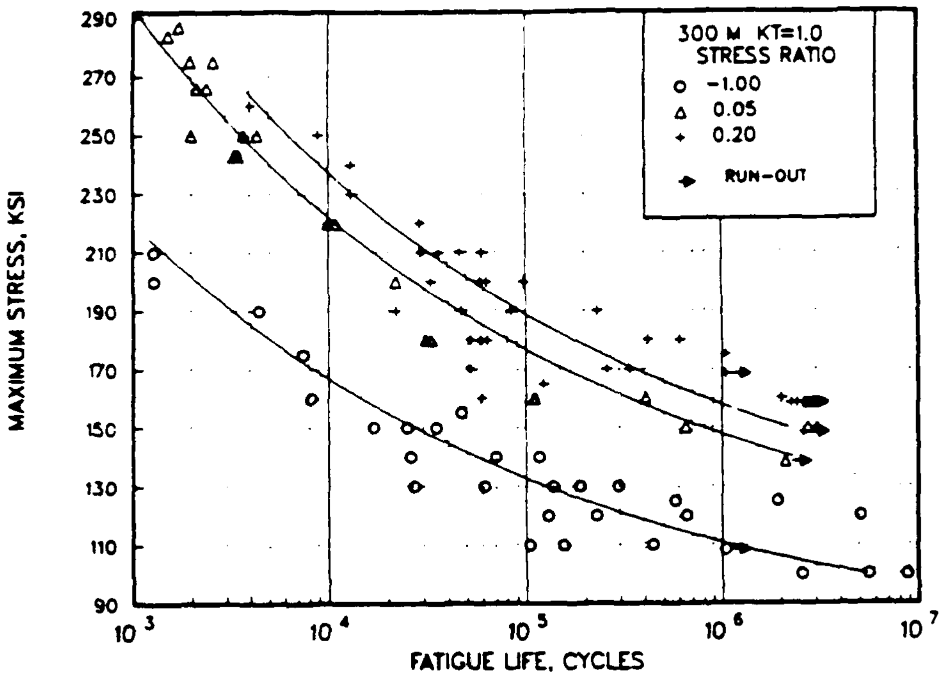

FIGURE X.X.X.X.X. *Best fit S/N curves for unnotched 300M alloy forging* F_{tu} *= 280 ksi, transverse orientation.*

Correlative Information for Figure X.X.X.X.X.

Product Forms: Forged billet, uncertain size
Die forging, 6-1/2 x 20 inches
6-inch RCS billet
1-1/4 x 8-inch forged bar
all CRVM

Properties:

TUS, ksi	TYS, ksi	Temp., F
272-294	226-243	RT

Specimen Details: Unnotched

0.200-0.250-inch diameter

Surface Condition: Heat treat and finish grind to a finish of 63 RMS or better with light grinding parallel to specimen length, stress relieve.

References: 2.3.1.2.8(c), (d), (e)

Test Parameters:

Loading - Axial
Frequency - 1800-2000 cpm
Temperature - RT
Environment - Air

No. of Heats/Lots: 5

Equivalent Stress Equation:

$\log N_f = 14.8 - 5.2 \log (S_{eq} - 94.2)$

$S_{eq} = S_{max}(1-R)^{0.38}$

Standard Deviation of log(Life) = 65.8 1/(S_{eq})

Adjusted R^2 Statistic = 87%

Sample Size = 90

(Caution: The equivalent stress model may provide unrealistic life predictions for stress ratios and maximum stress beyond those represented above.)

FIGURE 9.3.4.17(h). *Example S/N curve and correlative information.*

Supersedes page 9-80tt of Change Notice 1, MIL-HDBK-5E.

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