



U. S. Air Force

**Cost Risk and Uncertainty Analysis
Handbook**



U.S. AIR FORCE

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Executive Summary

The Air Force Cost Risk and Uncertainty Analysis Handbook (CRUH) serves as a reference for approved methods, practices, and reporting requirements needed to produce a realistic, defensible cost uncertainty analysis. It provides detailed guidance and definitions useful for cost analysts. This Executive Summary provides an overview of AF cost uncertainty analysis expectations, but is not a replacement for the full CRUH content. The Executive Summary is intended to consolidate the main points of the handbook and to provide a quick overview of the guidelines and expectations of a uncertainty analysis. The intended audience of the CRUH begins with the junior analyst and extends to seasoned experts.

The cost uncertainty analysis process is alternatively viewed as too complicated, mysterious, unreliable or irrelevant. The goal of the CRUH is to define and clearly present straightforward, well-defined processes that are repeatable, defensible, acceptable, and easily understood. The CRUH aims to create a more common understanding of this critical cost estimating activity. The guidance in this handbook should be treated as the core instruction and common frame of reference rather than an absolute treatment of the discipline.

The Difference Between Risk and Uncertainty

There is an important distinction between the terms risk and uncertainty. Risk is the chance of loss or injury. Uncertainty is the indefiniteness about the outcome of a situation. In a situation that includes favorable and unfavorable events, risk is the probability an unfavorable event occurs. Uncertainty is assessed in cost estimate models for the purpose of estimating the risk (probability) that a specific funding level will be exceeded.

Cost uncertainty analysis is a process of quantifying the uncertainties associated with elements of the cost model such as cost estimating relationships (CERs), technical parameters that drive CERs, labor and other rates, and the schedule. The probability (risk) that a specific cost target will be exceeded is derived from the total uncertainty of the estimate.

Why Cost Uncertainty Analysis?

The primary objective of analysis is to synthesize data, analyze it, convert it into information, and then present the information in a way such as to aid a decision maker in making sound, reasoned decisions. In the DoD environment, as in other environments, cost data is necessarily limited and accurate cost estimating is a significant challenge. Given this environment of limited data and substantial uncertainty associated with predicting the future, for best decision support it is imperative that analysts quantify the confidence (i.e., uncertainty) of their estimates. *The amount of uncertainty around an estimate is information that helps the decision maker.*

The knowledge required to perform a well done cost uncertainty analysis will serve to sharpen the analytical skill set of each analyst. There is, in fact, a complementary nature between the abilities to estimate uncertainty and the ability to estimate in general. Knowledge of the tools needed to quantify uncertainty in an estimate also serves to help improve the original estimate. Overall, learning the theoretical and practical aspects of cost uncertainty analysis will improve both data analysis and cost estimation.

The Uncertainty Analysis Process

The starting point for developing the point estimate cost is the Program Manager's approved definition of the program. Acquisition Category (ACAT) 1 programs require a Cost Analysis

Requirements Description (CARD) to define the “program of record.” Programs below ACAT 1 may use alternative methods to document the Program Manager’s approved description of the elements to be estimated.

Cost analysts try to develop the best cost estimate possible from the available information. The most common approach is to develop a “most likely” estimate for each element in the estimate. Because every assumption that drives a cost estimate represents a point within a range of possible values, an estimate of this type is called the “point estimate.” No matter how much effort is applied to the lower elements in the estimate, the parent (or total) levels in the point estimate do not reflect a most likely value. The objective of the cost uncertainty analysis is to estimate the uncertainty of the point estimate and provide a basis for assessing its uncertainty or variability for a specific budget. *Because the point estimate is based on assumptions with associated uncertainty, the analyst must consider risk and uncertainty from the very outset of the program or estimate.*

NOTE: This handbook uses the term “Technical Baseline Point Estimate” (TBE) to refer to the cost estimate that is the arithmetic sum of the program’s elements. For the purpose of this handbook, the TBE is based on the CARD or equivalent documentation (with adjustments for inconsistencies and changes where data external to the CARD is more appropriate or accurate) that defines the program. This makes the TBE directly traceable back to the Program Manager’s approved definition of the project.

This Cost Risk and Uncertainty Handbook describes the following methods of uncertainty analysis:

Inputs-based simulation

In the “Inputs-Based” simulation approach, the uncertainty is applied to every element contributing to the cost estimate.

Outputs-based simulation

An alternative to the Inputs-Based Simulation method is to apply uncertainty directly to the results (cost model outputs). The “Outputs-Based” approach relies on historical data to estimate the overall uncertainty at output levels of indenture within the estimate. The Outputs-Based method is appropriate when the data or resources are not available to perform the Inputs-Based method.

Scenario-based method (SBM)

SBM is an analytical approach (not a simulation) for quantifying a program’s cost risks and deriving recommended levels of cost reserve. It utilizes elicited/anticipated scenarios as the basis for deriving and defending a program’s cost and cost reserve recommendations. These scenarios do not have to represent worst cases; rather, they should reflect a set of conditions a Program Manager or decision-maker would want to consider, in the event any or all of those conditions should occur. The SBM is appropriate when sufficient data or resources are not available to perform either the Inputs-Based Method or Outputs-Based Method.

The remainder of this Executive Summary focuses on the key elements of the Inputs-Based Method, these key elements may be applied, in general, to the other methods. Regardless of the method used, always remember: Uncertainty analysis is not a substitute for quality estimating. An analyst must not attempt to use a large distribution around an unreliable point estimate in

hopes of correcting or capturing the right estimate. That is unacceptable and calls into question the credibility of the work.

The basic steps of a cost uncertainty analysis are:

1. Develop the point estimate (analogy, parametric, engineering build-up, etc.)
2. Specify the uncertainty around the point estimate
3. Measure and, if required, apply appropriate correlation techniques
4. Run the simulation or evaluate the scenarios
5. Analyze the results and repeat previous steps as necessary
6. Calculate, allocate, and phase risk dollars
7. Tell the risk story – show the uncertainty distribution and recommend the funding level

1. Develop the point estimate

Before embarking on a cost uncertainty analysis, the analyst should ensure that the technical baseline estimate (TBE) is as complete as possible. All of the careful consideration paid to distribution shapes, distribution bounds and uncertainty distribution correlation *will not* make up for excluding some of the elements required in the estimate or using estimating relationships that are inappropriate for the system in question. So, before delving into risk mechanics, complete the TBE as well as time and resources allow; it is vital to the process.

NOTE: the TBE should not include an adjustment for risk. The inclusion of margin or growth is acceptable if that is the likely outcome, however, margin or growth should not be included in the TBE in an effort to “account for” risk. Risk, or more specifically, uncertainty is addressed in the next step.

2. Specify the uncertainty

Rigorous uncertainty assessments will produce realistic probabilistic cost ranges necessary to communicate confidence levels associated with program budget options.

a. Minimum scope of uncertainty assessed

Cost analysts must assess the uncertainty for all cost estimating methodologies unless the cost method is absolutely certain. *At a minimum, cost estimates should address: the uncertainty associated with cost estimating relationships (CERs), CER inputs, CER technical/schedule factor adjustments, schedules, through-puts, discrete risks, all factors including growth factors, requirements creep factors, and if applicable, congressional withhold factors.* Remember, this list is not all inclusive because there is likely to be uncertainty around every input.

b. Anchor cost driver uncertainty assessments to the most likely estimate

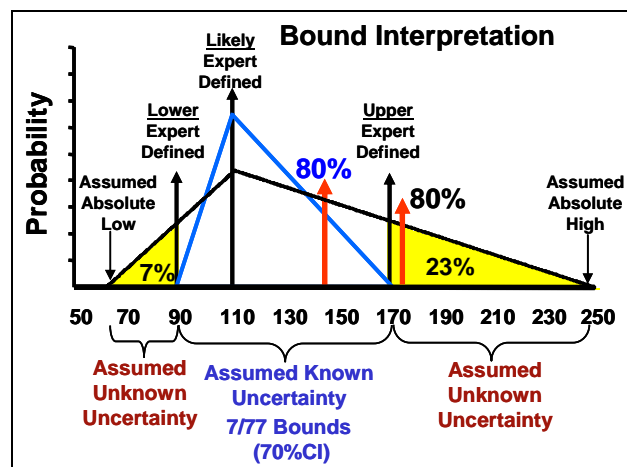
Anchoring uncertainty analysis to realistic estimates produces realistic probability cost distributions. *Therefore, all cost driver (technical, rates, adjustments, etc) assessments in the estimating structure should represent the most likely estimate or the peak (mode) of the assumed distribution.* Depending on the situation, a CER result may represent the mean, median or mode of the CER uncertainty distribution. Therefore, CER results should be anchored to the point in the distribution consistent with how the uncertainty for the CER was defined. In all cases, all uncertainty distributions in cost estimates should be truncated at zero.

c. Subjective uncertainty bound realism

All subjective (expert opinion) uncertainty distribution bounds must be defensible and should pass the test of similar historical data. All uncertainty assessments must avoid optimistic upper and lower distribution bounds. *Sound defense must accompany any subjective distributions with upper and lower bounds narrower than the defaults defined in this Handbook.*

d. Subjective uncertainty bound interpretation

Subjective uncertainty distribution bounds should be requested at and should be interpreted as capturing seventy percent of the total uncertainty. For symmetrical distributions this translates to the lower bound representing the fifteenth percentile and the upper bound represents the eighty-fifth percentile. For skewed distributions, the thirty percent of uncertainty should be apportioned to the upper and lower bound according to the ratio of the skewed distribution. For this purpose, skew is defined as $(\text{mode}-\text{low})/(\text{high}-\text{low})$, where the mode, high and low are the expert opinion.



3. Assess and ensure appropriate application of correlation

Correlation must be applied to all related elements within a cost model when functional correlation is not present. Functional or implicit correlation is defined as the correlation that occurs between cost elements due to the algebraic structure of the cost estimating framework. For instance, if the costs of Data and the costs of Systems Engineering and Program Management (SEPM) are estimated using factors related to the cost of the Prime Mission Equipment (PME), then Data and SEPM will be positively (functionally) correlated. In this situation, as PME changes in the simulation, the costs of Data and SEPM will change in the same direction accordingly; therefore, they are already positively correlated and no further correlation need be applied. In the absence of functional correlation, and better information, this Handbook recommends minimum correlation values of 0.50 (5 elements), 0.25 (10 elements) or 0.10 (20+ elements)

4. Run the analysis, simulation or scenario

5. Analyze the results and repeat previous steps as necessary

Modeling uncertainty is an iterative effort. For each iteration of uncertainty analysis it is useful to examine the results. One metric for assessing uncertainty is the coefficient of variation (CV), (a measure of dispersion defined as the standard deviation divided by the mean), of the top-line

of each phase of the estimate. A higher value indicates a wider dispersion or flatter s-curve. Often a small CV can be an indication of very optimistic ranges or a lack of correlation. Conversely, high CVs can be indicative of a high-risk program or may be an indication of unusually broad distributions. Appropriate values for CV are a function of commodity and phase. For instance, early in the project 35-45% is typical for space systems and software intensive projects; 25-35% is typical for aircraft and similar complexity hardware; and 10-20% is typical of large electronic system procurements.

6. Calculate, allocate and phase the results

A consequence of the uncertainty analysis process is that the lower level WBS element results do not sum to the parent result for a specific confidence level. While this is the mathematically correct way to display uncertainty analysis results, it is often necessary to force WBS elements to sum for budgeting and other reasons. It is not only important to know “where” (what WBS elements) the risk dollars should be allocated, it is equally important to know “when” (what FY) the risk dollars should be made available. It is important to ensure that as the confidence level increases, the time period over which the dollars are spent is reasonable. For example, adding a lot more budget dollars but keeping the schedule the same is not, generally, reasonable.

Once the results have been calculated and allocated, the next step is to determine the proper phasing. Phasing cost estimates and uncertainty results is necessary in order to arrive at annualized values and to properly account for inflation. Often, technical difficulties manifest as cost growth resulting from schedule slips. With this in mind, schedule slips must be considered within the scope of the uncertainty analysis and the resulting allocation and phasing.

7. Communicate the cost estimate

Presenting the risk story to senior leadership or to a review agency must entail a clear communication of the following information:

- Assumptions about the TBE.
- General approach of how the uncertainty was defined and, in the case of the simulation method, how and why the bounds, distributions and correlation were chosen.
- Then-Year (TY) dollar risk allocated results in graphical format. Report the CV and show graphically the location of the TBE, protect scenario (see section 7.3.2), median and proposed funding level on the Cumulative Distribution Function (CDF).
- TY dollars, allocated and phased result.
- Identification of the cost drivers that have the most impact on the cost estimate.
- Identification of the most important contributors to the cost estimate uncertainty and any risk mitigation initiatives captured by the estimate.

NOTE: This Handbook does not dictate a confidence level to which Air Force programs should be funded.

Appendix A contains detailed definitions of the terms and their underlying mathematics. **Appendix B** contains further details to enrich the content of the main body. The final section, **Appendix C** contains complete descriptions of the examples used throughout the main body, in @Risk, ACE and Crystal Ball.

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1.0 INTRODUCTION

1.1 PURPOSE

The purpose of this Cost Risk and Uncertainty Handbook (CRUH) is to describe acceptable analytical techniques to characterize the uncertainty in a cost estimate and to calculate the cost risk. The handbook provides a comprehensive list of cost uncertainty sources, and specific guidance on how to model them for each of the methods described. The goal of the handbook is to define and clearly present simple, well-defined cost risk analysis processes that are repeatable, defensible and easily understood.

Throughout this handbook, certain technical terms are used and distinctions concerning their use and interpretation are made. Many publications contain conflicting definitions for each of these terms. Appendix A contains the most commonly accepted definitions for the terms used in this document.

1.2 THE DIFFERENCE BETWEEN UNCERTAINTY AND RISK

There is an important distinction between the terms risk and uncertainty. Risk is the chance of loss or injury. Uncertainty is the indefiniteness about the outcome of a situation. In a situation that includes favorable and unfavorable events, risk is the probability an unfavorable event occurs. Uncertainty is assessed in cost estimate models for the purpose of estimating the risk (probability) that a specific funding level will be exceeded.

Cost uncertainty analysis is a process of quantifying the uncertainties associated with elements of the cost model such as cost estimating relationships (CERs), technical parameters that drive CERs, labor and other rates, and the schedule. The probability (risk) that a specific cost target will be exceeded is derived from the total uncertainty of the estimate.

1.3 THE REQUIREMENT FOR COST RISK ANALYSIS

Cost analysts do their utmost to develop the best cost estimate possible from the available information. The most common approach is to develop a “most likely” estimate for each element in the estimate. Because every assumption that drives the cost estimate represents a point within a range of possible values, an estimate of this type is called the “point estimate.” **Appendix B.1.4** contains details on the definition of the point estimate and related terms. It turns out that no matter how much effort is applied to the lower level elements in the estimate, the parent (or total) levels in the point estimate rarely reflect a most likely value¹. Moreover, even at the lowest estimating level, most likely values of CER inputs (cost drivers) will not, in general, produce most likely costs. The objective of a cost risk analysis is to estimate the uncertainty of the point estimate and provide a basis for assessing the probability of a cost overrun for a specific budget.

¹ MYTH: The point estimate total is the most likely estimate for the total.

GUIDANCE: The point estimate total is the sum of lower level cost estimates where, in general, the inputs to the cost methods are “most likely.” However, since there is uncertainty associated with many aspects of each estimated cost element, the uncertainty of these elements are rarely symmetrical and their sum generally never equals the most likely.

1.4 THE POINT ESTIMATE

1.4.1 Defining the Technical Baseline Point Estimate

The basis for every risk analysis method presented in this handbook is the point estimate. This section provides general guidance on what should and should not be captured in the point estimate.

The starting point for developing the point estimate cost is the Program Manager's approved definition of the program. Acquisition Category (ACAT) 1 programs require a Cost Analysis Requirements Description (CARD) to define the "program of record." Programs below ACAT 1 may use alternative methods to document the Program Manager's approved description of the elements to be estimated.

This handbook uses the term "Technical Baseline Point Estimate" (TBE) to refer to the cost estimate that is the arithmetic sum of the program's elements. For the purpose of this handbook, the TBE is based on the CARD or equivalent documentation (with adjustments for inconsistencies and changes where data external to the CARD is more appropriate or accurate) that defines the program. This makes the TBE directly traceable back to the Program Manager's approved definition of the project.

The TBE is a point estimate. It represents one possible estimate based on a given set of program characteristics. The creditability of any estimate is based on a realistic and complete technical, schedule and programmatic baseline. However, even when the baseline is sound, many of the technical and schedule components may remain uncertain. The TBE serves as the reference point on which the cost risk analysis is anchored. The definition of what is included and what is excluded from this anchor point should be clearly defined for each program cost estimate, especially any departures from the CARD (or equivalent). This definition influences the magnitude of the point estimate and heavily influences how the analyst chooses to define the uncertainty (distribution shape and bounds) of each element within the cost estimate. Ideally, the TBE should be derived from the most likely WBS structure and most likely (mode) technical, schedule and programmatic inputs. This is consistent with the idea that the TBE inputs should reflect the best assessment of what will actually happen. Typical contents of a TBE include:

- Work Breakdown Structure (WBS) to identify all the elements requiring a cost estimate;
- Technical parameters that properly define the system to be estimated;
- Rates and factors such as labor rates, head counts, fee, overhead, operating hours, or man-hour rates, and other programmatic cost drivers;
- A schedule that properly reflects how the project will unfold.

The TBE **should not** include any extra dollars inserted into individual cost elements to address some measure of estimate uncertainty. Nor should elements like Engineering Change Orders (ECOs) contain adjustments in an effort to capture uncertainty of other cost elements. Elements like ECOs should be estimated in the same way as the other elements of cost – without additive "pads" or "margins". The confidence of every cost estimating method should be addressed in the uncertainty analysis, not the TBE. Cost drivers such as weight, code count, volume, power, rates, etc. should reflect the CARD (or equivalent) and not some "upper bound." The potential for a cost driver to be something other than a most likely value is handled in the uncertainty analysis process that comes later.

Before embarking on the cost risk analysis, the analyst should ensure that the TBE is as complete as possible². No amount of agonizing over distribution shapes, distribution bounds or risk distribution correlation will make up for excluding some of the elements required in the estimate or using estimating relationships that are completely inappropriate for the system in question. So, before delving into risk mechanics, complete the TBE as well as is possible, as time and resources allow.

This handbook uses a simplified missile TBE WBS as shown in **Figure 1-1**. This example will be used throughout the handbook to illustrate cost risk analysis methods, processes and reports.

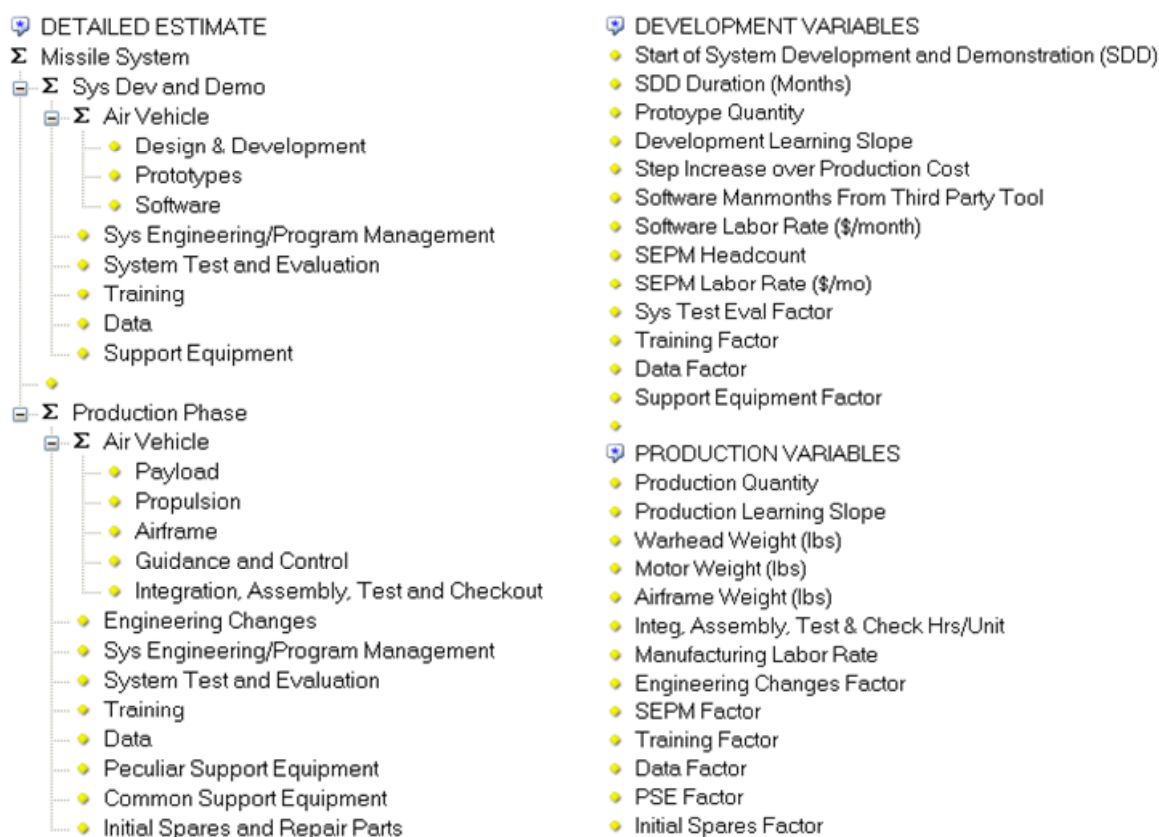


Figure 1-1 Technical Baseline Point Estimate WBS Example

1.4.2 Role of Sensitivity Analysis

A sensitivity analysis is a systematic approach used to identify the cost impacts of potential changes to one or more of an estimate's major input parameters. The objective is to vary input parameters over a range of probable values and recalculate the estimate to determine how sensitive outcomes are to changes in the selected parameters. **Table 1-1** illustrates this.

² MYTH: Greater detail and/or greater modeling complexity will lead to greater accuracy

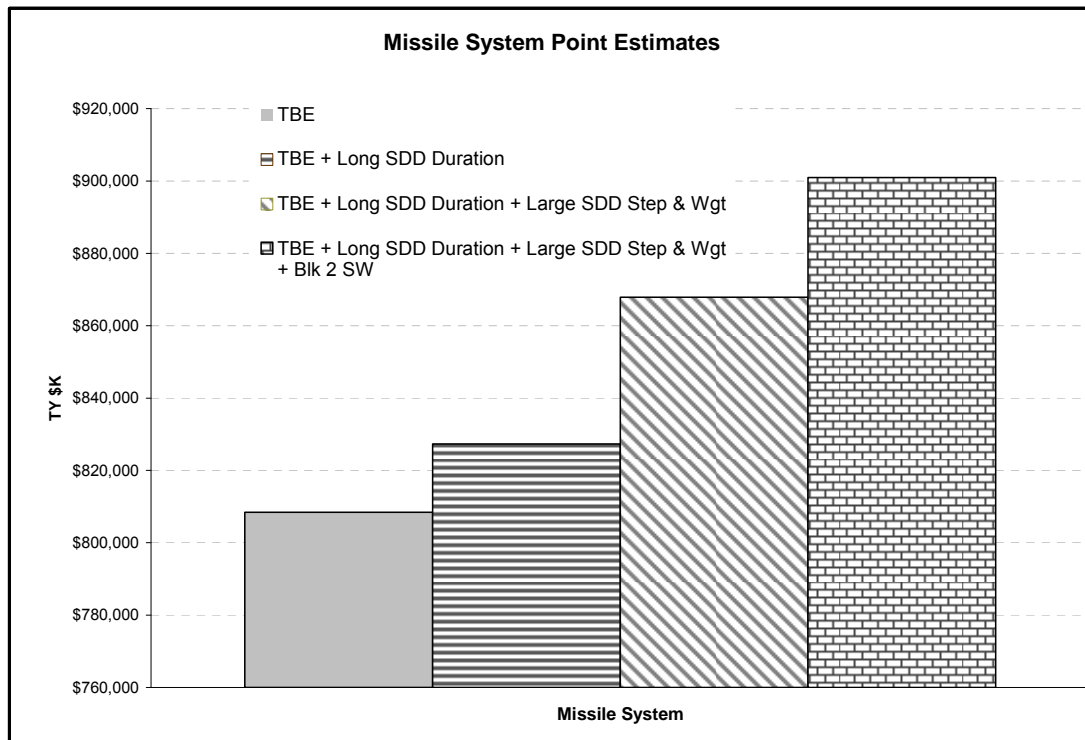
GUIDANCE: Estimate at a level of detail consistent with the maturity of the project and the trade-off studies that are required. Effort to estimate at lower levels than required may be counter productive.

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Table 1-1 Missile Estimate Sensitivity Analysis Example

	TBE Program of Record	TBE plus Long SDD Duration	TBE plus Long SDD Duration plus Large SDD Step & Weight	TBE plus Long SDD Duration plus Large SDD Step & Weight plus Block 2 Software
DEVELOPMENT VARIABLES				
SDD Duration (Months)	60	70	70	70
Step Increase over Production Cost	1.8	1.8	2.2	2.2
SW Man-months From PRICE/SEER/COCOMO/etc	6,400	6,400	6,400	8,400
PRODUCTION VARIABLES				
Warhead Weight (lbs)	12	12	20	20
Motor Weight (lbs)	200	200	225	225
Airframe Weight (lbs)	330	330	400	400
Integ, Assembly, Test & Check Hrs/Unit	120	120	150	150
Manufacturing Labor Rate (BY\$ 2007/hr)	\$90.00	\$90.00	\$95.00	\$95.00

A sensitivity analysis chart is shown in **Figure 1-2**. The example illustrates the cumulative effects of the changes shown in **Table 1-1** on the total missile cost. A sensitivity analysis, while useful, is not sufficient to quantify the program cost risk. Each of the results shown in **Figure 1-2** is a point estimate and the analyst still needs to estimate the uncertainty associated with each of these point estimates.

**Figure 1-2** Example of a Sensitivity Analysis Chart

1.5 ACCEPTABLE METHODS TO CALCULATE COST ESTIMATE UNCERTAINTY

This section identifies and describes acceptable methods for defining or calculating the amount of uncertainty in WBS cost estimates.

1.5.1 Overview of Cost Risk Analysis Methods

This handbook will describe three methods for estimating the uncertainty of a cost estimate: Inputs-Based Simulation, Outputs-Based Simulation, and the Scenario Based Method.

Figure 1-3 illustrates the key elements of the three methods. Guidance on how to allocate, time phase, and report the uncertainty is also presented in this handbook.

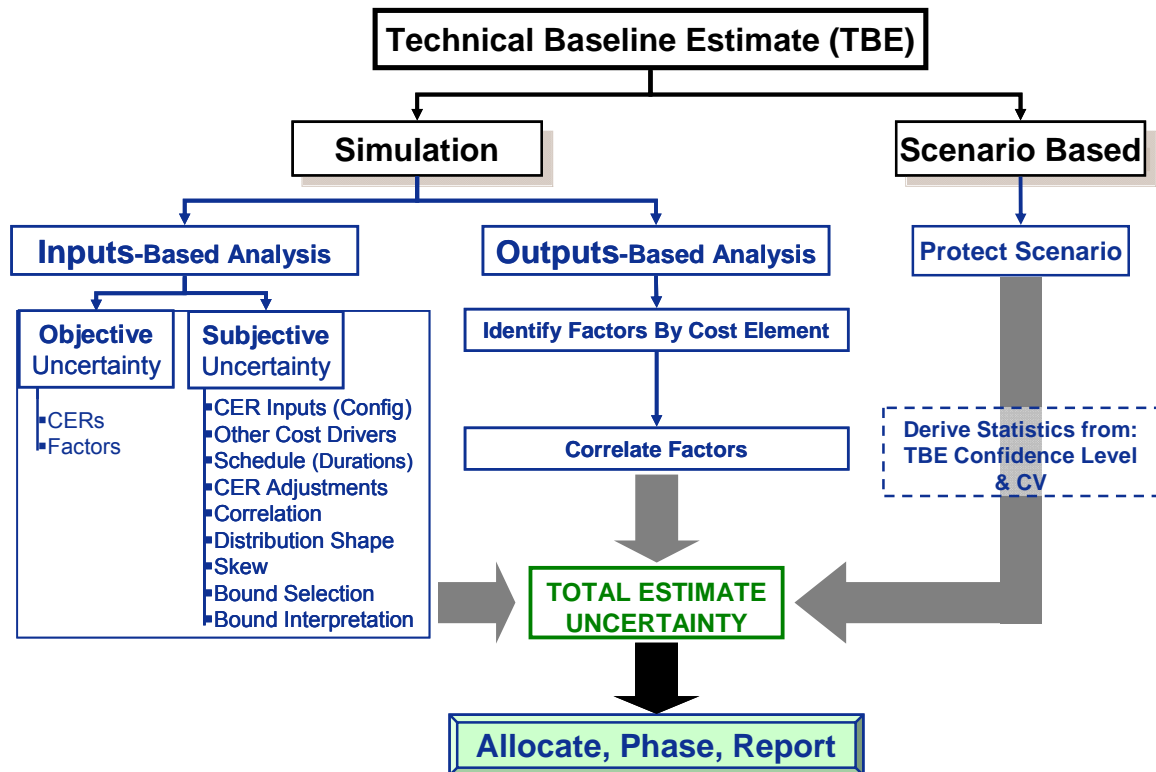


Figure 1-3 Methods to Calculate Total Estimate Uncertainty

“Derive statistics” under the Scenario Based method in **Figure 1-3** is in an optional extension of this method and is described in **B.7.3**.

1.5.2 Simulation Based Cost Risk Analysis

With several powerful commercial and Government tools from which to choose, simulation has become the most popular method of modeling cost uncertainty (for details see **A.9**).

The simulation tools most commonly used in the DoD cost analysis community are Crystal Ball (CB), @Risk, and the Automated Cost Estimator (ACE). CB and @Risk are general purpose commercial risk modeling tools. ACE is a DoD-funded tool designed expressly for the Government cost analyst or Government contractors. This handbook does not recommend any specific tool, but does acknowledge that these three are in common use and widely accepted. Examples in this handbook will be solved in all three of these tools. It is the responsibility of the

analyst to determine which tool to use in accordance with the analyst's organizational policy and tool availability.

The general process for developing the uncertainty analysis in any of these tools is summarized in **Figure 1-4**.

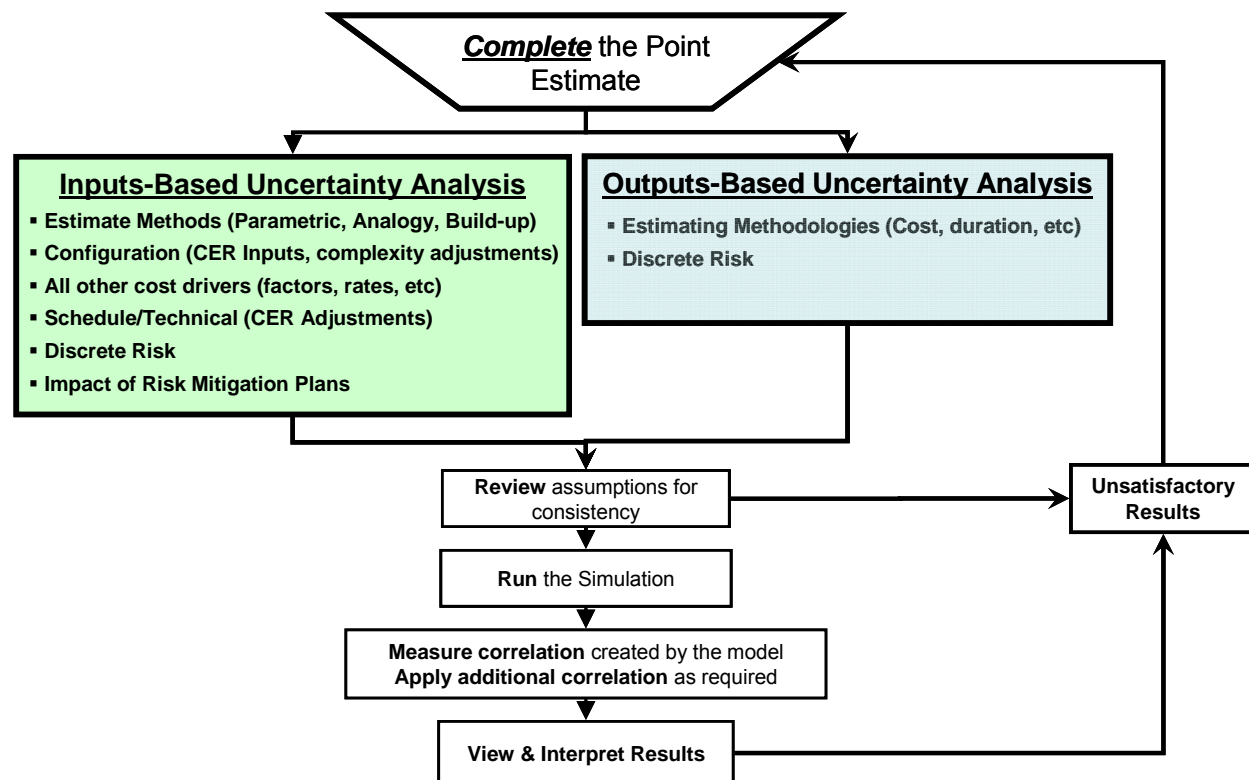


Figure 1-4 Overview of Simulation Method

The process is iterative as indicated by the feedback loop passing through the “unsatisfactory results” block. Pausing to review for consistency in how the risk assumptions have been applied and to compare the statistical results with metrics suggested in **Section 3.3** will improve the overall result. In the “Inputs-Based” simulation approach, the uncertainty is applied to every element contributing to the cost estimate. The “Outputs-Based” approach relies on historical data to estimate the overall uncertainty at output levels within the estimate.

1.5.3 Scenario-Based Method (SBM)

SBM is derived from work presented at **Reference 43, provided in Attachment 1**. SB... provides Program Managers and decision-makers an assessment of the amount of cost reserve needed to protect a program from cost overruns associated with a scenario in which specific adverse events occur. It is an analytical approach (not a simulation) for quantifying a program's cost risks and deriving recommended levels of cost reserve. Besides “High Cost” scenarios, “Low Cost” scenarios can be defined to help establish the amount of overall cost variance. In the Low Cost scenario, optimistic or favorable (but possible) events occur.

This method emphasizes the development of written scenarios as the basis for deriving and defending a program's cost and cost reserve recommendations. It is similar to a “sensitivity analysis,” but with one difference. Instead of arbitrarily varying one or more variables to

measure the sensitivity (or change) in cost, SBM involves specifying a well-defined set of technical and programmatic conditions that collectively affect a number of cost-related variables and associated work breakdown structure elements in a way that increases (or decreases) cost relative to the TBE. The credibility of SBM-derived estimates comes from an assertion that experts are better able to bound the cost uncertainty by envisioning coherent top-to-bottom scenarios than by estimating the multitude of uncertainties and correlations required for the detailed simulation method.

1.6 MODELING COST ESTIMATE UNCERTAINTY ANALYTICALLY

There are a variety of other analytical methods available to model the point estimate uncertainty. However, they are not recommended for other than simple cost estimates (such as summing of ten to twenty throughput costs). Therefore, use of one of the three acceptable methods described in 1.5 is preferred. For more information on other analytical methods, see B.1.6.

2.0 INPUTS-BASED SIMULATION METHOD

2.1 OVERVIEW

2.1.1 Uncertainty to be Captured

The “Inputs-Based” simulation method requires the analyst to assign statistical distributions to model the uncertainty of the following elements of a cost model:

- Estimate Methods³
- Configuration (CER Inputs, complexity factors for analogies, engineering judgment)
- All other cost drivers (factors, rates, overhead, fee, etc.)
- Schedule/Technical (CER Adjustments)
- Correlation
- Discrete Risks
- Impact of Risk Mitigation Plans

Uncertainty is expressed in a simulation by specifying the shape and bounds of the uncertainty distribution for the cost methods and cost drivers (input variables) where the value is not certain. For any given item, this may be determined via one of four approaches:

- Objective
- Subjective
- Risk Score Mapping
- Third Party Tools (aka Feeder Models)

³ MYTH: A wide dispersion on a specific cost estimating method reflects poorly on the analyst or the project

GUIDANCE: A wide dispersion on many cost estimating relationships is a fact and usually has more to do with the uncertainty of the project rather than a measure of the cost analyst’s ability or any other factor.

This handbook will provide guidance on how to adequately model these uncertainties and how to properly apply correlation between these uncertainties.

2.1.2 Uncertainty That Might Not be Captured

There are many other potential sources of cost estimate uncertainty. If defensible methods are available, the analyst should also model the following (specific guidance not contained in this handbook):

- Acquisition strategies
- Requirements creep
- Different contracting options/strategies
- Macro industry trends
- Anything you “know” will affect the project that you can model

No standard, approved modeling methods are provided in this handbook for these uncertainties. Inclusion of these uncertainties should be done with great care to reduce the likelihood of double counting or overly pessimistic assessments, and to maintain the credibility and fidelity of the analysis.

2.1.3 Uncertainty That Should Not be Captured

Special consideration should be given to uncontrollable events that can impact the cost of a program. In most cases, these events should not be included in the uncertainty assessment. If at all, they could be the subject of a separate sensitivity analysis and discussion. Events such as hurricanes, earthquakes, bankruptcies, etc. should generally be excluded from explicit uncertainty modeling.

2.2 ESTIMATE METHODS

2.2.1 Overview

Once the analyst has identified the scope, schedule and defined the program WBS (see WBS example in **Figure 1-1**), there is the task of populating that WBS with cost estimating methods. Some of the most popular include:

- Parametric CERs (cost as a function of technical parameters)
- Factor methods (cost as a factor of another cost)
- Build-up methods (resource quantity times the cost per unit resource, also known as “bottom-up”)
- Third Party tools
- Throughputs (analogies, quotes, expert opinion, etc.)

Figure 2-1 illustrates how these cost estimating methods might be used in a cost model. While there are many other estimating methods, this handbook will provide focused guidance on how to apply uncertainty to the above methods.

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WBS Description	Estimate Method
Missile System	
SDD Phase	
Air Vehicle (AV_PMP)	
Design & Development	[Throughput] 25000
Prototypes	[Factor] ProdT1 * Step
Software	[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$
Sys Engineering/Program Management	[Build-up] SDD_SEPM_LOE * SDD_SEPM_LabRate * SDD_MonthsDur
System Test and Evaluation	[Factor] SDD_STE_Fac * AV_PMP
Training	[Factor] SDD_Trng_Fac * AV_PMP
Data	[Factor] SDD_Data_Fac * AV_PMP
Support Equipment	[Factor] SDD_SptEquip_Fac * AV_PMP
Production Phase	
Air Vehicle (AV_Prod)	
Payload	[Parametric CER: OLS Linear] $30.15 + 1.049 * \text{WarheadWt}$
Propulsion	[Parametric CER: OLS Loglinear] $1.618 * \text{MotorWt}^{0.6848}$
Airframe	[Parametric CER: MUPE Triad] $256.2 + 0.05682 * \text{AirFrameWt}^{1.374}$
Guidance and Control	[Throughput] 700
Integration, Assembly, Test and Checkout	[Build-up] IACO_HsPerUnit * MfgLaborRate\$ * ProdQty
Engineering Changes	[Factor] ECO_Fac * AV_Prod\$
Sys Engineering/Program Management	[Factor] SEPM_Fac * AV_Prod\$
System Test and Evaluation	[Throughput] 250 times 4 years of production
Training	[Factor] Trng_Fac * AV_Prod\$
Data	[Factor] Data_Fac * AV_Prod\$
Peculiar Support Equipment	[Factor] PSE_Fac * AV_Prod\$
Common Support Equipment	[Discrete] CSE\$
Initial Spares and Repair Parts	[Factor] InitSpares_Fac * AV_Prod\$

Figure 2-1 Cost Estimating Method Examples

2.2.2 Uncertainty for Parametric CERs

2.2.2.1 Overview

Parametric CERs are derived from statistical regression analysis (described in A.3). CERs derived in this manner provides an objective assessment of the CER uncertainty. The most common regression methods include (described in detail in A.3):

- Ordinary Least Squares (OLS), Unit Space. OLS solves for linear relationships and minimizes an additive error term. An example of an OLS derived CER is: $\text{Cost} = 30.15 + 1.049 * \text{WarheadWt}$
- Ordinary Least Squares, Log Space. OLS solves for linear relationships and minimizes an additive error term, but in log space (regression performed on the log of the data) An example of a Log Linear OLS derived CER is: $\text{Cost} = 1.618 * \text{MotorWt}^{0.6848}$
- Minimum Unbiased Percentage Error (MUPE)⁴. Derives CERs with multiplicative error terms directly in unit space. An example of a CER that cannot be derived using OLS but can be derived with MUPE is: $\text{Cost} = 256.2 + 0.05682 * \text{AirFrameWt}^{1.374}$
- Zero Bias Minimum Percent Error (ZMPE)⁵. ZMPE is another method that can derive CERs with multiplicative error terms directly in unit space. Like MUPE, it can be used to derive any CER functional form, including those that OLS cannot derive.

⁴ Also known as Iteratively Reweighted Least Squares methods (IRLS).

Methods employing multiplicative error terms are preferred (see **A.3.2.2**). The most appropriate method to estimate the bounds of the CER uncertainty is to calculate a prediction interval. Most statistical software packages will do this calculation for OLS CERs. For details on how to calculate simple CER prediction intervals manually, see **B.2.2.3**.

Unfortunately, analysts may not always have the data or tools available for a detailed analysis of the CER uncertainty. **Table 2-1** identifies some alternatives when this is the case.

Table 2-1 Alternatives for Estimating Parametric CER Uncertainty Distribution Bounds

Available Data	Distribution Shape	Bounds
Source Data, Statistical Package	Consistent with Regression Assumptions	Use Stat Package to calculate Prediction Interval (PI)
Number of Observations, Degrees of Freedom, SEE, Average and Standard Deviation of Driver Variable	Normal or Lognormal	Refer to Appendix B for methods to calculate PI
Standard Error of Estimate	Lognormal	Adjust for Sample Size and data relevance, see Figure 2-2 for calculation
Expert Opinion	Lognormal	Use Handbook default low, medium or high dispersion

2.2.2.2 Using Lognormal to Describe Parametric CER Uncertainty

The most popular distribution shapes used in cost risk analysis include:

- Normal
- Lognormal
- Beta
- Triangular
- Uniform
- Weibull (Rayleigh and Exponential distributions can be created from the Weibull)

In the interest of simplifying the cost risk analysis process, the following approach is recommended:

- Regardless of the parametric CER form or regression method used to create it, the uncertainty of the CER may be modeled with a lognormal distribution.
- In the absence of better information, the result of the CER will be treated as the median (50% value).
- The dispersion of the lognormal distribution will be defined by the CER standard error adjusted for sample size and the position the estimate falls within the dataset used to derive the CER (described in 2.2.2.3).

⁵ Also known as General-Error Regression Methods (GERM), which allow cost estimators to specify any error model and associated constraints, such as zero bias minimum percent error.

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For details on why the lognormal distribution was chosen as the handbook default for parametric CERs, see **B.2.2.2.2**.

2.2.2.3 Adjusting the Standard Error for Sample Size and Relevance

In general, the parameter most commonly reported with a CER is the standard error of the estimate (SEE, see **A.8.1**)⁶ in unit space (also known as the standard deviation in some simulation tools). **Use of the SEE alone to define the distribution is generally not sufficient.** **Table 2-2** should be used to select a factor to account for the *sample size* and the independent variable value's (used in the estimate) *position* relative to the mean of those used to create the OLS-derived CER⁷. The derivation of these multipliers and a more detailed table is contained in **A.8.3** and published in **Reference 13**. **Should the analyst not have a basis for determining sample size or distance from the data center, the recommended default factor is 1.15.**

Table 2-2 Factors to Adjust CER SEE Sample Size and Sample Relevance

# of Stdev estimate variable is from CER data center	Sample Size		
	Small 5	Medium 15	Large 25
0.00 Very Similar	1.10	1.03	1.02
0.75 Similar	1.15	1.05	1.03
1.50 Dissimilar	1.28	1.10	1.06
2.25 Different	1.49	1.18	1.11
3.00 Very Different	1.73	1.29	1.18

The parameters required to fully define the lognormal distribution in any of the available simulation tools will be a function of the tool itself and the preference of the analyst. **Figure 2-2** provides the mathematics to derive all the parameters necessary to specify the lognormal distribution in any of the simulation tools. The adjustment value in **Figure 2-2** was obtained from the more detailed **Table A-1**. Note that applying the adjustment to the log space SEE yields slightly different answers than if the adjustment is applied to the unit space SEE.

	Range Name	Value	Equation
Parametric CER			$UC1 = 1.618 * MotorWt ^ 0.6848$
Parametric CER Result	CERResult	\$60.915	$UC1 = 1.618 * 200 ^ 0.6848$; where MotorWt = 200
Parametric CER Result Probability	CERProb	50.0%	CER result is the median of the log normal distribution
Adjust for Sample Size & Relevance	SEEAdjust	1.267	10 data points, data is different from project to be estimated
If statistics package gives SEE in Unit Space:			
Given: SEE Unit Space	SEEUnitSpace	\$11.75	Statistical Package result (\$9.27) multiplied by SEEAdjust
Calculate:			
SEE Log Space	SEELogSpace	0.1879	$SQRT(LN((1+SQRT(1+4*(SEEUnitSpace/CERResult)^2))/2))$
Mean Log Space	MeanLogSpace	4.1095	$LN(CERResult)$
Mean Unit Space	MeanUnitSpace	\$62.00	$CERResult * EXP((SEELogSpace^2)/2)$
If statistics package gives SEE in Log Space:			
Given: SEE Log Space	SEELogSpace	0.1897	Statistical Package result (0.1497) multiplied by SEEAdjust
Calculate:			
SEE Unit Space	SEEUnitSpace	\$11.87	$((EXP(SEELogSpace^2)-1)*MeanUnitSpace^2)^0.5$
Mean Log Space	MeanLogSpace	4.1095	$LN(CERResult)$
Mean Unit Space	MeanUnitSpace	\$62.02	$EXP(MeanLogSpace + SEELogSpace^2/2)$

Figure 2-2 Template to Derive Alternative Lognormal Distribution Parameters

⁶ Templates to derive alternative parameters from given information are contained in **B.2.5.2**.

⁷ This approach is derived from OLS mathematics, however, in the absence of better information it may be used on any CER.

2.3 UNCERTAINTY FOR NON STATISTICAL CER METHODS AND PARAMETRIC CER INPUTS

2.3.1 Overview

Parametric CERs derived from statistical analysis have uncertainty associated with the CER itself. Non-statistical CERs are also known as “build-up” or “bottom-up” methods. Examples are LaborRate*Hours or UnitCost*Quantity. These methods do not have uncertainty assigned to the estimating equation itself. However, the inputs to both parametric and non-statistical methods should have uncertainty assigned. If there is no objective, statistical basis for selecting the uncertainty distribution shape and bounds for the CER input (such as using a discrete labor rate rather than an average), the analyst must resort to subjective assessments.

2.3.2 Distribution Shapes for Subjective Uncertainty

This section applies to every element in the cost estimate where objective uncertainty distributions are not available. There are a large number of possible distribution shapes defined in the literature and available through a variety of tools. In an effort to promote consistency across program estimates, analysts are encouraged to limit their analysis to the distributions defined in **Table 2-3**.

Table 2-3 Recommended Subjective Uncertainty Distributions

Shape	Typical Applications	CER Result	Remarks
Normal	Linear or non-linear CERs with additive error, mechanical tolerances. All MUPE generated CERS, Univariate methods	Mean, median, mode	Equal probability of overrun or underrun
Lognormal	Log-linear CERs that transform to linear in log space ($y = a * x^b$) Labor rates, labor rate adjustments, factor methods	Median ⁸	The uncertain variable can increase without limits, but can not fall below zero, is positively skewed, with most of the values near the mode
Triangular	Engineering data or analogy estimates (throughputs), labor rates, labor rate adjustments, factor methods	Mode	Popular because they are easy to understand and communicate - use when likelihood decreases with distance from PE
Uniform	Engineering data or analogy estimates (throughputs). Labor rates, labor rate adjustments, factor methods	Unknown	Used when every value across the range of the distribution has an equal likelihood of occurrence
Beta	Engineering data or analogy estimates (throughputs)	Mode	Complicated to explain and to apply consistently across different tools
Weibull	Objective relationship to reliability modeling.	Mode	Popular because of the wide variety of shapes that can be defined, including the Raleigh and Exponential distribution

⁸ This is recommended as the default point estimate interpretation only because OLS appears to be the most common method used within the community to generate CERs. Those using more sophisticated methods (e.g. MUPE, ZMPE) will recommend the appropriate distribution shape and define how the CER result (the point estimate) is interpreted consistent with their method.

2.3.3 Distribution Bounds for Subjective Distributions

In addition to shape, the distributions are characterized by parameters describing their dispersion and skewness. Subjective dispersion parameters are commonly the low and high bounds of the distribution. The low and high bounds for subjective uncertainties are often obtained from experts⁹. **Reference 6** concluded that experts rarely identify 60% of the possible uncertainty range and never did better than 70% (approximately one sigma). The impact of interpreting the lower/upper bound to be the 15%/85% (i.e. 70% of the total range) on a normal and triangular distribution is illustrated in **Figure 2-3** and **Figure 2-4**. The narrower distribution illustrates the distribution shape if the expert bounds are taken as “absolute,” which is rarely the case (for normal, “absolute” is interpreted as 3 standard deviations, or 99.9% of the possible range). The broader shape is the one that is ultimately modeled in the simulation.

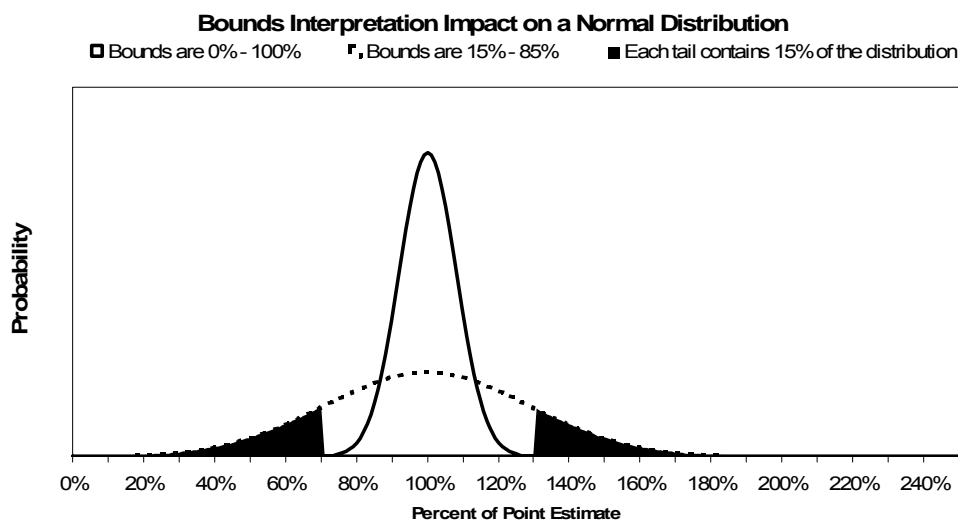


Figure 2-3 Impact of bound interpretation on a normal distribution

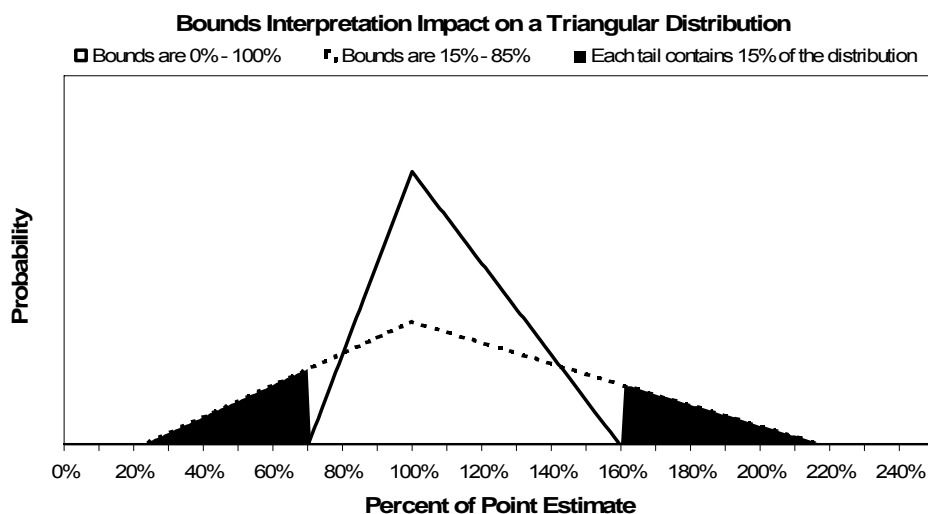


Figure 2-4 Impact of bound interpretation on a triangular distribution

⁹ MYTH: When you ask an expert to give the high and low bounds, they provide the absolute high/low.

GUIDANCE: These bounds are generally closer to the 15/85 range (**Reference 6**).

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Reference 12 suggests that the 30% additional uncertainty should be applied symmetrically. An acceptable refinement is to allocate the additional uncertainty based upon the skew defined by the expert as suggested in **Reference 23**. The steps in performing this refinement are:

1. Obtain expert opinion for Low, Mode, and High
2. Calculate the skew from the expert opinion: $(\text{Mode}-\text{Low})/(\text{High}-\text{Low})$
3. Set total uncertainty captured by the expert.
4. Calculate the bound interpretations: Low Bound Interpretation = Skew * (1-Total Uncertainty Captured) and High Bound Interpretation = Total Uncertainty Captured - Low Bound Interpretation. An example is illustrated in **Figure 2-5**.

Template to adjust upper lower bound interpretation for skew.						
Step 1: Obtain Expert Opinion for Low, Likely and High	Sym-metrical	Prod to SDD Step	SEPM Staff	Warhead Weight		
Low	75	1.5	22	11		Enter low bound
Mode (most likely)	100	1.8	25	12		Enter mode
High	125	3.0	27	20		Enter high bound
Step 2: Calculate skew from Expert Opinion						
Skew	0.50	0.20	0.60	0.11		$(\text{Mode}-\text{Low})/(\text{High}-\text{Low})$
Step 3: Set total uncertainty captured by Expert, default is 70%						
Total uncertainty captured by expert:	70%	70%	70%	70%		Default is 70%
Step 4: Calculate the bound interpretations						
Low Bound Interpretation	15%	6%	18%	3%		$\text{Round}(\text{Skew} * (1-\text{TotUncertCaptured}),2)$
High Bound Interpretation	85%	76%	88%	73%		$\text{TotUncertCaptured} + \text{Low Bound Interp}$

Figure 2-5 Template to Calculate Low/High Bound Interpretation

Figure 2-6 summarizes the combination of CER uncertainty and CER input uncertainty.

WBS Elements	Point Estimate	Cost Estimating Relationship	Form	Adj SE	Low	Low Intrap	High	High Intrap	Comment
Payload	\$11,416	$30.15 + 1.049 * \text{WarheadWt}$	Normal		82.11%	10	117.90%	90	Regression Result
Propulsion	\$16,271	$1.618 * \text{MotorWt}^{0.6848}$	LogNormal		81.30%	10	123.01%	90	Regression Result
Airframe	\$112,250	$256.2 + 0.05682 * \text{AirFrameWt}^{1.374}$	LogNormal	0.1897					Regression Result
Guidance and Control	\$186,979	700	Triangular		85%	8	140%	78	Expert Opinion
Warhead Weight (lbs)	12.0	WarheadWt	Triangular		11	3	20	73	
Motor Weight (lbs)	200.0	MotorWt	Triangular		190	5	250	75	
Airframe Weight (lbs)	330.0	AirFrameWt	LogNormal				120%	80	

Figure 2-6 Combining CER and Input Uncertainties

2.4 ELICITATION OF SUBJECTIVE BOUNDS FROM SUBJECT MATTER EXPERTS

One of the most important tasks in cost risk analysis is to identify the bounds of independent variables in CERs and all the other cost driver elements in the model for which there is no objective approach. The analyst generally has to resort to expert judgment, such as that possessed by engineers, managers, and other knowledgeable people. This process is called elicitation. It can be difficult to do and subject to numerous biases. These biases may be categorized as “Motivational” and “Cognitive.”

Table 2-4 Motivational and Cognitive Bias

Motivational Bias	Cognitive Bias
Social pressure (face to face)	Inconsistency (opinion changes over time)
Impression (not face to face)	Anchoring
Group Think	Relating to irrelevant analogies
Wishful thinking	Underestimation
Career goals	Human Nature
Misunderstanding	
Project Advocacy	
Competitive Pressures	

Best practices for elicitation include:

- Use multiple experts.
- Ask for an upper and lower value. Encourage brainstorming for reasons why the range could be larger, especially in the upper direction.
- Encourage a dialog to identify the value that has a one in five chance of being lower or the value that has one in five chance of being exceeded. Such a dialog makes the participants determine not only the bounds but also their interpretation.
- Seek the most likely value near the end of the discussion.
- Use **Table 2-3** to select a distribution shape.
- In the absence of better information, treat bounds as the 15/85 percent interpretation (see **Section 2.3.3** for details). For skewed distributions, consider skewing the bound interpretations as shown in **Figure 2-5**.
- Crosscheck and when appropriate, challenge experts' inputs against historical experience. Characterize adjustments with meaningful project comparisons derived from relevant historical data (i.e., challenge a software uncertainty assessment of +/- 50% with several relevant, real life examples where 300% or more growth was experienced). Ideally, prepare for elicitation discussions by having on hand meaningful cross checks based upon well known, real-life examples.

2.5 DEFAULT SUBJECTIVE DISTRIBUTION BOUNDS

When specific bounds are not available from Subject Matter Experts (SME), subjective designations of low¹⁰, medium, and high can be used. In an effort to standardize subjective assumptions, the bounds in **Table 2-5** should be used.

¹⁰ MYTH: The absolute bounds described by a low dispersion are in the plus or minus 5-10% range.

GUIDANCE: The low dispersion bounds are generally more like +/- 20% at the 15/85% probability level for parametric CERs.

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Table 2-5 Default Bounds for Subjective Distributions

Distribution	Point Estimate Interpretation	Point Estimate and Probability	Mean	15%	85%
Lognormal Low	Median	1.0 (50%)	1.011	0.856	1.168
Lognormal Med	Median	1.0 (50%)	1.032	0.772	1.296
Lognormal High	Median	1.0 (50%)	1.063	0.696	1.437
Lognormal EHigh**	Median	1.0 (50%)	1.107	0.627	1.594
Normal Low	Mean	1.0 (50%)	1.000	0.845	1.155
Normal Med	Mean	1.0 (50%)	1.000	0.741	1.259
Normal High	Mean	1.0 (50%)	1.002	0.640	1.363
Normal EHigh	Mean	1.0 (50%)	1.015	0.555	1.470
Weibull Low	Mode	1.0 (25%)	1.158	0.956	1.370
Weibull Med	Mode	1.0 (20%)	1.393	0.956	1.855
Weibull High	Mode	1.0 (15%)	2.104	1.000	3.277
Triangle Low Left	Mode	1.0 (75%)	0.878	0.695	1.041
Triangle Low	Mode	1.0 (50%)	1.000	0.834	1.166
Triangle Low Right	Mode	1.0 (25%)	1.122	0.959	1.305
Triangle Med Left	Mode	1.0 (75%)	0.796	0.492	1.069
Triangle Med	Mode	1.0 (50%)	1.000	0.723	1.277
Triangle Med Right	Mode	1.0 (25%)	1.204	0.931	1.508
Triangle High Left*	Mode	1.0 (75%)	0.745	0.347	1.103
Triangle High	Mode	1.0 (50%)	1.000	0.612	1.388
Triangle High Right	Mode	1.0 (25%)	1.286	0.903	1.711
Triangle EHigh Left*	Mode	1.0 (75%)	0.745	0.300	1.150
Triangle EHigh	Mode	1.0 (50%)	1.004	0.509	1.500
Triangle EHigh Right	Mode	1.0 (25%)	1.367	0.876	1.914
Uniform Low Left	Mode	1.0 (75%)	0.870	0.688	1.052
Uniform Low	Mode	1.0 (50%)	1.000	0.818	1.182
Uniform Low Right	Mode	1.0 (25%)	1.130	0.948	1.312
Uniform Med Left	Mode	1.0 (75%)	0.784	0.480	1.087
Uniform Med	Mode	1.0 (50%)	1.000	0.697	1.303
Uniform Med Right	Mode	1.0 (25%)	1.217	0.913	1.520
Uniform High Left	Mode	1.0 (75%)	0.697	0.273	1.121
Uniform High	Mode	1.0 (50%)	1.000	0.576	1.424
Uniform High Right	Mode	1.0 (25%)	1.303	0.879	1.728
Uniform EHigh Left	Mode	1.0 (75%)	0.695	0.2085	1.181
Uniform EHigh	Mode	1.0 (50%)	1.000	0.454	1.546
Uniform EHigh Right	Mode	1.0 (25%)	1.390	0.844	1.935
Beta Low Left	Mode	1.0 (61%)	0.939	0.775	1.099
Beta Low	Mode	1.0 (50%)	1.000	0.838	1.162
Beta Low Right	Mode	1.0 (39%)	1.061	0.901	1.225
Beta Med Left	Mode	1.0 (63%)	0.883	0.605	1.152
Beta Med	Mode	1.0 (50%)	1.000	0.726	1.274
Beta Med Right	Mode	1.0 (37%)	1.117	0.848	1.396
Beta High Left	Mode	1.0 (66%)	0.808	0.412	1.186
Beta High	Mode	1.0 (50%)	1.000	0.605	1.395
Beta High Right	Mode	1.0 (33%)	1.202	0.816	1.606

** EHigh = Extreme High

* To match these parameters, tools must be set to truncate the distribution at zero.

The basis for these defaults may be found in **B.2.5.1**. There is sufficient information in the table to model the distributions in any of the available tools. See **B.2.5.2** for the mathematics of converting a given set of distribution parameters to those that may be more convenient to use in the selected risk tool.

2.6 GUIDANCE ON OBJECTIVE OR SUBJECTIVE DISTRIBUTION BOUNDARY LIMITS

In cost estimating, there is a logical lower bound to uncertainty distributions. Employing distributions that have values less than zero is discouraged. Put another way, the lower limit for all distributions in whatever tool is used **should be truncated at zero unless there is compelling evidence that negative values are a reasonably expected outcome and the model will perform properly**. This is recommended **despite appearing to be** an arbitrary reduction in the variance of the selected distribution. On the contrary, by establishing the lower limit of the distribution to be zero, nonsensical situations of negative dollars in cost elements or negative weight, power, volume, etc. in technical parameters are avoided. Be aware that truncating the lower limit of the distribution at zero moves the mean of the distribution to the right, making it a more conservative estimate. Therefore, in situations when negative tails would represent nonsensical outcomes, the analyst is encouraged to select a distribution, such as lognormal, that does not require truncation to simplify the explanation of the model.

Similarly, if an uncertainty distribution is assigned to a cost improvement (learning) curve slope, unless a case can be made for “reverse learning,” setting the absolute upper bound at 100% may be reasonable.

2.7 DISCRETE UNCERTAINTIES

In this context, discrete uncertainty is defined as a specific dollar cost impact that has a specified probability of occurrence. In general, this analysis is appropriate to account for known “problems” that have some probability of occurring. For example: if “A” happens then the project will incur an additional cost of \$100M. If it does not happen, the additional cost to the project is zero. To continue with the example, there is a 40% probability that “A” will occur.

One of two approaches is recommended:

- If there are only a few discrete uncertainties, their cost impact on the TBE should be included in the form of supplemental “what-if” cases regardless of their probability of occurrence. Funding at the “expected value” (probability of occurrence times the cost impact) is not recommended. Doing so does not capture sufficient funds to pay for the event should it occur, yet allocates unnecessary funds to the project if the event does not occur.
- If there are many discrete uncertainties, then they should be listed and uncertainty assigned using the “yes/no” criteria, which is modeled using the Discrete distribution in @Risk, a logical statement in ACE, and the “yes/no” or Bernoulli distribution in Crystal Ball (see **Appendix C**). In this case, the TBE will include the expected values for these events. However, the objective is to be able to derive a budget at the top level that provides some measure of confidence that the discrete uncertainties can be covered.
- In both cases, uncertainty should be applied to the cost impact as well.

It is not recommended that the discrete uncertainty be captured simply by multiplying the probability of occurrence times the cost impact. **Figure 2-7** illustrates that to account for all the discrete uncertainties (100% probability of occurrence), the program would have to budget \$347. The expected value (cost x probability) is \$113.3. If the discrete uncertainties are independent, the variance can be calculated by summing $P*(1-P)*PE^2$ for each element where PE is the point estimate and P is the probability of occurrence. As **Figure 2-7** demonstrates, all three tools (@Risk, ACE, and Crystal Ball) reproduce the standard deviation at the total level. However, the tools also make it possible to introduce correlation amongst the discrete uncertainties. It is also possible to model constraints such as “if A occurs, then B would cost x, else B would cost y” and many others. Using the tools in this manner, the analyst can calculate the discrete risk budget required to match a specific confidence level. In the case illustrated in **Figure 2-7**, the expected value evaluates to the 52% confidence level. To achieve a 60% confidence level, the budget would need to be \$123.5.

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	Point Estimate	Probability of Occurrence	Expected Value	Theoretical Variance			
Total	347		113.3				
Discrete Uncertainty 1	21	60%	12.6	105.84			
Discrete Uncertainty 2	34	60%	20.4	277.44			
Discrete Uncertainty 3	54	10%	5.4	262.44			
Discrete Uncertainty 4	13	30%	3.9	35.49			
Discrete Uncertainty 5	45	10%	4.5	182.25			
Discrete Uncertainty 6	23	50%	11.5	132.25			
Discrete Uncertainty 7	42	50%	21	441.00			
Discrete Uncertainty 8	49	10%	4.9	216.09			
Discrete Uncertainty 9	39	40%	15.6	365.04			
Discrete Uncertainty 10	27	50%	13.5	182.25			
			Variance	2,200.09	CB	@Risk	ACE
			Standard Deviation	46.91	47.04	46.32	46.74

Figure 2-7 Discrete Uncertainty Example

2.8 UNCERTAINTY VIA THIRD-PARTY TOOLS

Often when an overall program estimate is assembled, the estimated cost for some elements may be obtained via separate and distinct cost models (third-party tools). In such situations it is recommended that the uncertainty distribution inherent in those models be carried forward into the aggregate model.

Commercial parametric models generally have an embedded simulation-based risk model and there may be a requirement to carry forward its resulting distribution into an overall model. An alternative is to apply a subjective distribution. More elaborate methods are possible, but are outside the scope of this handbook¹¹. Simply multiplying a third-party tool's point estimate result by a "management reserve factor" is discouraged. The third-party cost result should be treated as a random variable with uncertainty just like all the other WBS cost estimates.

When a given cost element is modeled using a separate model, apply uncertainty to each of its elements and inputs as appropriate. Follow the instructions of that model's user manual **applying the guidance discussed in this handbook**. Popular parametric models provide sufficient results such as mean, standard deviation, ten-percentile cost, ninety-percentile cost, and distribution graphs that allow the analyst to specify uncertainty in the aggregate model.

In the absence of compelling evidence to the contrary, the analyst should extract the median and the 80% value from the product's simulation results. The median then becomes the point estimate and the 80% value provides a second point that is sufficient for all the simulation tools to generate a lognormal distribution to model the product's uncertainty result¹². The 80% value must represent a defensible set of conditions. Once the third-party tool's resultant distribution is

¹¹ For instance, some analysts have written the interface necessary to run a 3rd party model through Crystal Ball, @Risk or ACE.

¹² Analysts should determine if the third party tool's uncertainty analysis includes the assumption that the tool's estimating equations are "perfect," meaning that their method has no uncertainty (only the inputs). If so, the analyst should consider increasing the CV of distributions derived from third party tools to account for the missing variance.

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entered into the host cost model, assign appropriate correlations with other elements. **Figure 2-8** illustrates how this would be applied, with a point estimate (median) of 6400 staff months and the high of 150% derived from the third party tool's 80% value.

Software		ThirdPartyToolSWManMonths * SWLaborRate\$		\$76,800			
		Form	Low	Low Intrap	High	High Intrap	
SW Manmonth (From 3rd Party Tool)	ThirdPartyToolSWManMonths	6400	Lognormal		150%	80	
SW Labor Rate (\$/month)	SWLaborRate	12	Uniform	95%	0	131%	100

Figure 2-8 Applying Uncertainty to Third Party Tools

2.9 SPECIFIC TECHNICAL/SCHEDULE UNCERTAINTY CONSIDERATIONS

CERs are developed from historical data where some technical characteristic such as weight or power is used to estimate cost. By doing so, it is assumed that the CER is applicable to a project that is using similar technology and facing similar schedule challenges as those represented in the historical data. When this is not the case, the cost estimate should be adjusted to account for the different technical and schedule challenges faced by the project in question. The best way to account for these issues is to develop a cost model that is directly influenced by technical and schedule considerations. Unfortunately, this is not always possible or practical. In the absence of better information, and when direct modeling is not feasible, an acceptable approach is to multiply the CER by a triangular distribution to simulate the effect of extraordinary technical and/or schedule difficulties.

Table 2-6 provides default bounds for the triangular distribution that at least one project office was able to defend (see **Reference 41**). The categories are indicated as None, Low, Medium, High, or Very High. Low can be considered to be “Insignificant Challenge,” Medium as “Moderate Challenge,” High as “Substantial Challenge,” and Very High as an “Extreme Challenge.” Each project office is encouraged to develop its own defensible values for **Table 2-6**. Typically, the supporting documentation is inherently based on opinions and judgments as opposed to calculations.

Table 2-6 Default Schedule/Technical Absolute Upper Bound Values

Schedule/Technical Challenge Beyond Source	Absolute Bound	
	Lower	Upper
None	1.0	1.0
Low	0.9	1.1
Medium	0.9	1.5
High	0.9	2.0
Very High	0.9	3.0

Figure 2-9 illustrates the mechanics of applying these factors.

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WBS Elements

Payload	\$11,416	$(30.15 + 1.049 * \text{WarheadWt}) * \text{PenaltyPayload}$
Propulsion	\$16,271	$1.618 * \text{MotorWt}^{0.6848} * \text{PenaltyProp}$
Airframe	\$112,250	$(256.2 + 0.05682 * \text{AirFrameWt}^{1.374}) * \text{PenaltyAirFrame}$
Guidance and Control	\$186,979	$700 * \text{PenaltyGuidance}$
Integration, Assembly, Test and Checkout	\$6,480	$\text{IACO_HsPerUnit} * \text{MfgLaborRate} * \text{ProdQty} * \text{PenaltyIATC}$

Schedule/Technical Penalty Factors			Absolute Bounds		Correlation Matrix				
			Low	High					
Payload	1.0	Triangular	0.9	1.5	1.000	0.250	0.250	0.250	0.500
Propulsion	1.0	Triangular	0.9	1.5		1.000	0.063	0.063	0.125
Airframe	1.0	Triangular	0.9	1.1			1.000	0.063	0.125
Guidance and Control	1.0	Triangular	0.9	1.5				1.000	0.125
Integration, Assembly, Test and Checkout	1.0	Triangular	0.9	2.0					1.000

Figure 2-9 Example Application of Schedule/Technical Multipliers

These multipliers should only be used in cases where better information is unavailable. In any case, a compelling argument is required for any method that is used.

The correlation matrix is applied to capture the tendency for the multiplier uncertainties to “move together.” Correlation is discussed in **Section 3.2**.

Figure 2-10 illustrates the impact of applying these correlated multipliers to the missile example (7.5% at the 60% confidence level).

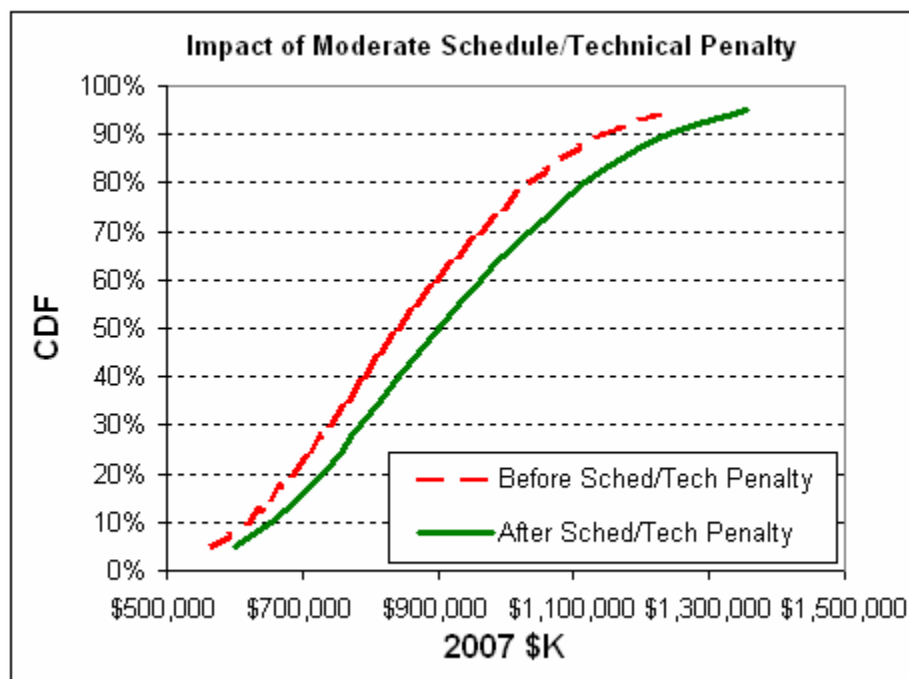


Figure 2-10 Impact of Applying Schedule/Technical Penalty Factors

3.0 COMPLETE THE SIMULATION

3.1 RUN THE SIMULATION

The purpose of the simulation is to combine all the uncertainties specified in the model to estimate the total uncertainty at the parent levels. **Figure 3-1** and **Figure 3-2** illustrate how the simulation process combines uncertainties within the model.

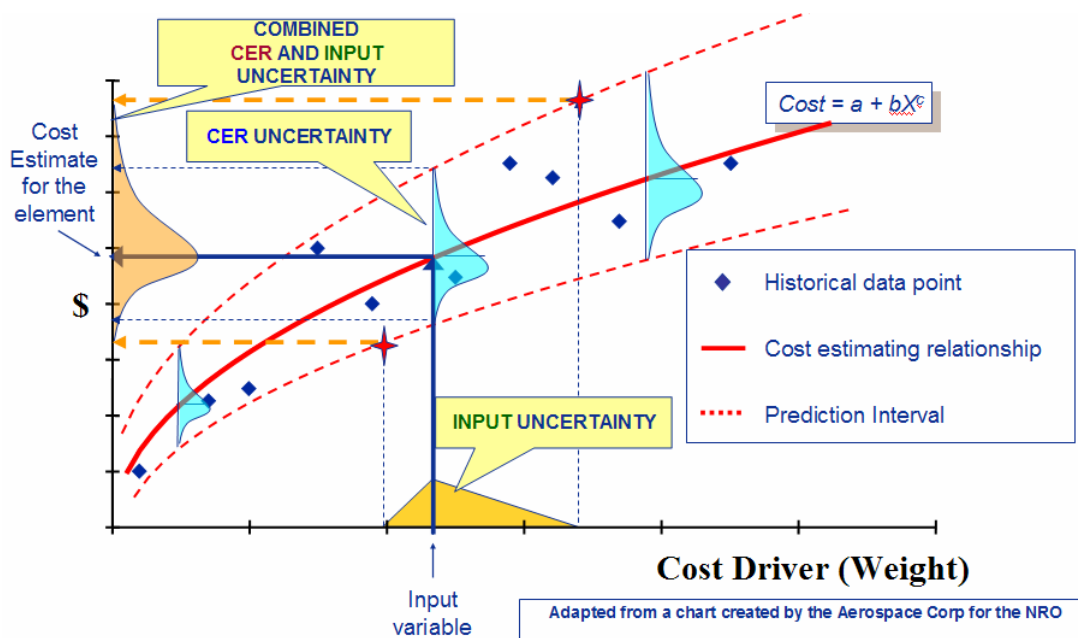


Figure 3-1 Combining Methodology and Methodology Input Uncertainties

With suitable adjustments to capture correlation, schedule and technical considerations, the uncertainty associated with all the elements are combined to arrive at the uncertainty for the total estimate.

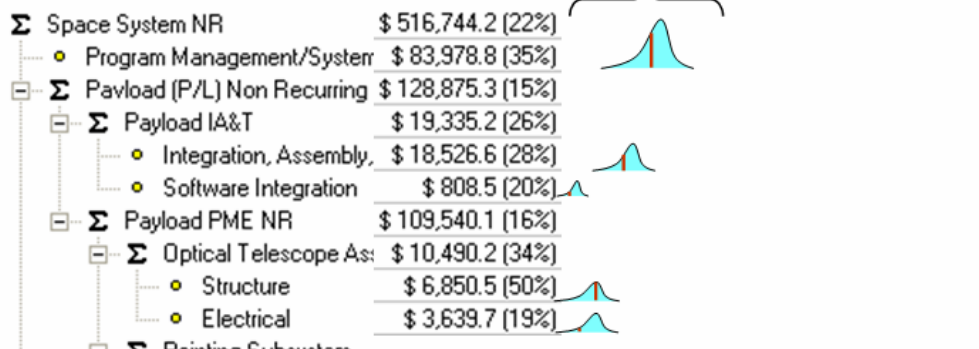


Figure 3-2 Generating the Statistics of the WBS Parent levels

3.2 MEASURE AND THEN CORRECT CORRELATION

3.2.1 Overview of Correlation

The risk analysis is not complete until there is an assessment of correlation. If correlation is ignored, the variance at the total levels in the estimate will be understated, in most cases dramatically. The results of the first simulation run can be used to measure the correlation already present in the model due to functional relationships. All the tools permit the user to add additional correlation to model the correlation between cost methods and between the inputs to cost methods (positively, negatively, or both). For example, when the cost of element A increases due to risk, the cost of element B should also increase, and perhaps element F should decrease. This interrelationship between risk impacts is commonly known as "dependency" or "correlation."

When applying inputs-based simulation, correlation is introduced through the functional relationships in the cost model itself. For instance, if the cost of Training is modeled by using a factor times the cost of the Prime Mission Equipment (PME) cost, then by definition Training will be correlated in risk simulation, meaning as PME increases, so will the cost of Training. These relationships are termed functional dependency or functional correlation. When possible it is best to capture as much of the dependency between elements within the estimating methods themselves. If uncertainty is applied to the factor (or other methods) used to establish a functional relationship between elements, then the correlation between the elements will not be perfect (correlation coefficient of 1.0).

For many cost elements, however, the correlation between elements will not be adequately captured by the estimating relationships (in particular, elements modeled independent of any others). In these situations, correlation values are used to quantify the "strength of the linear relationship" between elements (without regard for whether the underlying relationship is, in fact, linear). Modern risk analysis tools will then utilize this information, together with the specified distributions as well as functional relationships (CERs) to derive parent level WBS element distributions (the statistical sum of the "correlated" child distributions).

There are many approaches to modeling correlation within risk tools that put a huge burden on the analyst to not only define pairwise correlations, but to also ensure that the cross correlation matrix is consistent. Inputs to these models are often based on subjective judgments since there is often little empirical basis for derivation of these correlation values (see **Reference 26, 28**). If a consistent matrix is not specified, then complex math modeling algorithms are applied to transform the specified matrix into a consistent matrix that may vary significantly from the original analyst-specified matrix.

When distributions are summed in a typical WBS structure, correlation will affect the spread of the parent distribution, but not its mean. Most estimates, however, contain many elements that are functionally related through linear and non-linear methods. This often causes uncertainty distributions to be multiplied, divided, exponentiated, etc. For this reason, correlation applied across functionally related uncertainty distributions will have an impact not only on the spread of the parent, but the mean as well. This is why applying functional relationships (rather than simply adding throughputs) within a model wherever possible is so important: it can have a significant impact on the mean of the ultimate uncertainty distribution.

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Before the analyst embarks on assigning correlation, effort should be made to “measure” the correlation present due to functional relationships. When using outputs-based simulation, any measured correlation will be incidental and not due to functional relationships.

3.2.2 How to Measure Correlation

Tools such as Crystal Ball and @Risk report the correlation entered and the correlation actually used (if corrected for inconsistency). Users can also download to a spreadsheet the results for each element for each iteration of the simulation. Once these data are properly sorted in the worksheet, the analyst may use Excel’s CORREL function to calculate pairwise correlation. ACE contains a correlation report that will show the Pearson Product-Moment correlation between user-selected elements of the estimate.

Crystal Ball and @Risk employ “Spearman Rank” correlation and CORREL measures Pearson Product-Moment (see A.10.4 for details). While there have been several papers denouncing rank order correlation as inappropriate for cost analysis (see **References 18 and 21**), it is rare to see notable differences in results between the two approaches (see **References 30, 35, 37, 38, 40**). As an example, the resulting correlation matrices from the missile example as generated by Crystal Ball (which uses Spearman Rank) and ACE (which uses Pearson Product-Moment) are almost identical as illustrated in **Figure 3-3**.

=CORREL(\$D46:\$D10045,E46:E10045)

	C	D	E	F	G	H	I	J	K	L	M
		Missile System	SDD Phase	Air Vehicle	Design & Developr	Prototype	Software	Sys Eng/ P M	Sys Test and Eval	Training	Data
	CB Resultant Pearson Product Moment Correlation										
Missile System	1.00	0.72	0.69	0.40	0.69	0.61	0.47	0.70	0.71	0.71	
SDD Phase		1.00	0.99	0.38	0.46	0.96	0.44	0.92	0.95	0.95	
Air Vehicle			1.00	0.36	0.43	0.98	0.37	0.86	0.95	0.95	
Design & Dev				1.00	0.30	0.22	0.28	0.36	0.38	0.38	
Prototypes					1.00	0.31	0.37	0.45	0.47	0.47	
Software						1.00	0.32	0.83	0.92	0.92	
Sys Eng/ P M							1.00	0.40	0.41	0.40	
Sys Test and Eval								1.00	0.84	0.85	
Training									1.00	0.92	
Data										1.00	

	Missile System	SDD Phase	Air Vehicle	Design & Developr	Prototype	Software	Sys Eng/ P M	Sys Test and Eval	Training	Data
	ACE Resultant Pearson Product Moment Correlation									
Missile System	1.00	0.72	0.68	0.42	0.69	0.60	0.45	0.70	0.71	0.71
SDD Phase		1.00	0.99	0.40	0.45	0.96	0.41	0.92	0.95	0.95
Air Vehicle			1.00	0.37	0.42	0.98	0.34	0.85	0.95	0.95
Design & Dev				1.00	0.31	0.24	0.27	0.39	0.39	0.39
Prototypes					1.00	0.29	0.35	0.45	0.45	0.45
Software						1.00	0.29	0.82	0.92	0.92
Sys Eng/ P M							1.00	0.37	0.38	0.38
Sys Test and Eval								1.00	0.84	0.84
Training									1.00	0.92
Data										1.00

Figure 3-3 Measured Pearson Product Moment Correlations

3.2.3 Metrics for Assessing Correlation Adequacy

In the absence of objective data, analysts are encouraged to make subjective correlation assessments using the following steps:

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1. Measure the correlation present in the simulation because of functional correlation and identify those elements with less than 0.25 correlation.
2. Determine if specific elements should “move together,” that is, be correlated either negatively or positively.
3. Assign additional correlation using a correlation value between -1 and +1. **Table 3-1** provides guidance on default correlation values. Perfect correlation of +/-1.0 is discouraged.
4. In general, all elements’ measured correlation should be at a minimum plus or minus 0.50 (5 elements), 0.25 (10 elements) or 0.10 (20+ elements)¹³ correlation if no information to the contrary is available.

Table 3-1 Default Correlation Factors

Strength	Positive	Negative
None	0.00	0.00
Weak	0.25	-0.25
Medium	0.50	-0.50
Strong	0.90	-0.90
Perfect	1.00	-1.00

3.2.4 Applied Correlation Example

To demonstrate the impact of applying correlation within the model, the correlation matrices shown in **Figure 3-4** were used in the example uncertainty analysis.

	WBS/CES Description	49	60	61	62	63	67	78	81	82	84	86	87	88	97	99	100	101	102	103	104	
49	Design & Development	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
60	Payload		1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
61	Propulsion			1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
62	Airframe				1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
63	Guidance and Control					1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
67	System Test and Evaluation						1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
78	SDD Duration (Months)							1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
81	Step Increase over								1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
82	Software Manmonth From									1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
84	SEPM Headcount										1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
86	Sys Test Eval Factor											1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
87	Training Factor												1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
88	Data Factor													1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
97	Integ. Assembly, Test &														1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250
99	Engineering Changes Factor															1.000	0.250	0.250	0.250	0.250	0.250	0.250
100	SEPM Factor																1.000	0.250	0.250	0.250	0.250	0.250
101	Training Factor																	1.000	0.250	0.250	0.250	0.250
102	Data Factor																		1.000	0.250	0.250	0.250
103	PSE Factor																			1.000	0.250	0.250
104	Initial Spares Factor																				1.000	0.250

	WBS/CES Description	83	85	94	95	96		
	WBS/CES Description			94	95	96		
83	Software Labor Rate	1.000	0.640	94	Warhead Weight (lbs)	1.000	0.700	0.700
85	SEPM Labor Rate (\$/mo)		1.000	95	Motor Weight (lbs)		1.000	0.490
				96	Airframe Weight (lbs)			1.000

Figure 3-4 Example of Applied Correlation Matrices

Analysts should understand that applying correlation in any of the tools augments, rather than replaces, the functional correlation that is already present. For instance, the upper table in

¹³ These factors were derived from charts in **Reference 22, 49** that identify the correlation required to capture 80% of the impact on total variance that would otherwise be lost.

Figure 3-5 shows the functional correlation across the production elements in the missile example before any additional correlation is applied. The lower table illustrates the impact on correlation when the matrices in **Figure 3-4** were applied. The impact on the missile risk results is also shown.

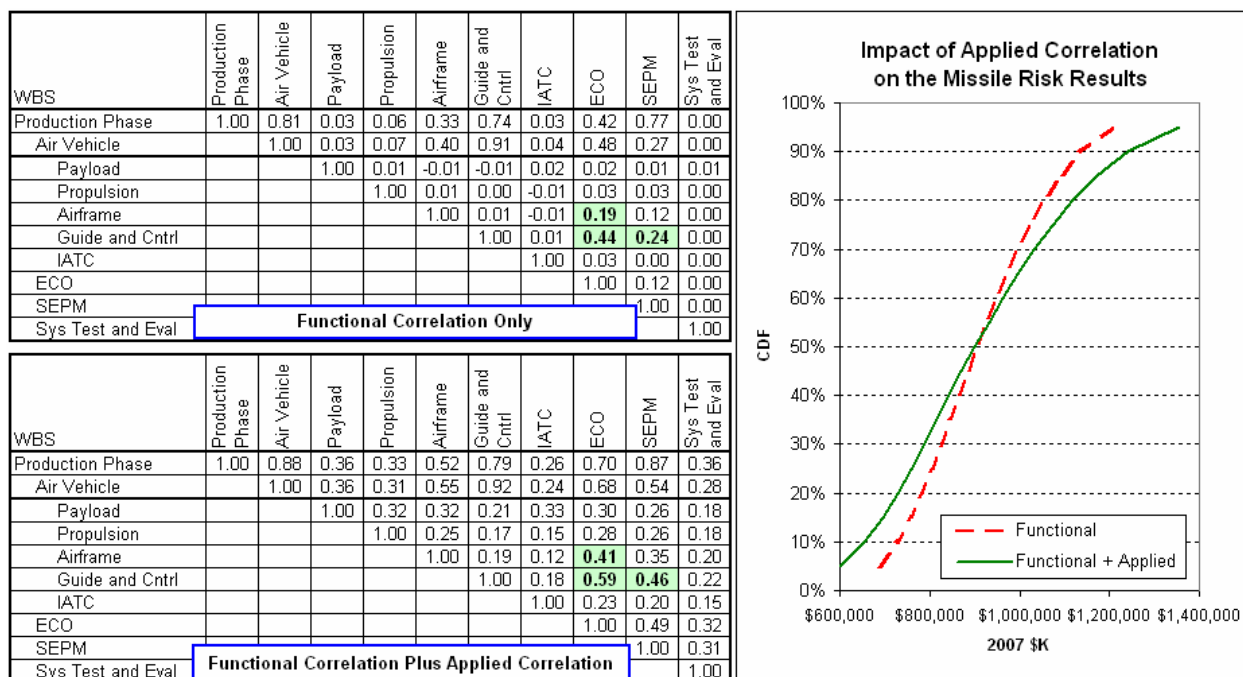


Figure 3-5 Impact of Adding Correlation On Top of Functional Correlation

3.3 REVIEW AND INTERPRET RESULTS

If the tool’s guidelines are followed, the analyst will obtain the same results regardless of which tool is used. **Figure 3-6** illustrates the same results from three different tools in the form of a cumulative distribution function (CDF) for the missile model.

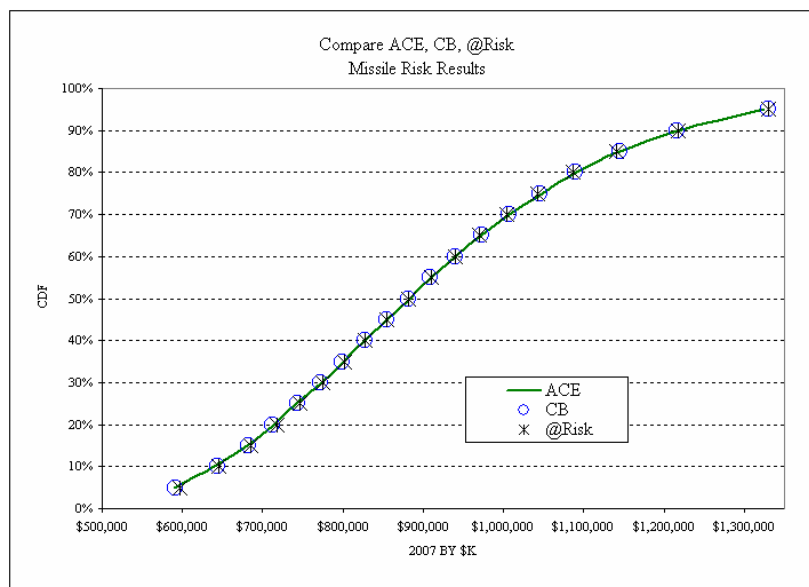


Figure 3-6 Compare Missile Risk Analysis Results from Several Tools

Modeling uncertainty will typically involve several iterations as the analysis progresses. For each iteration of risk analysis, it is useful to examine the coefficient of variation (CV) (a measure of dispersion defined as the standard deviation divided by the mean) of the top-line of each phase of the estimate. Examining lower level elements is desirable; however, the range in acceptable answers is much broader. In general, analysts are likely to be able to compile meaningful ranges of acceptable CV for the overall estimate (by phase) by commodity.

CV is a statistic provided by all the common tools. A higher value indicates a wider dispersion or a flatter s-curve. CVs near 0.15 are indicative of a program with low or modest risks. CVs at 0.35 or above are indicative of a high risk program. Often a small CV of less than 0.15 is an indication of very optimistic ranges or a lack of correlation. CVs larger than 0.35 may be an indication of unusually broad distributions. However, these rules-of-thumb are very commodity dependant and a function of where the program is in the life cycle. For instance, a CV of 50% would not be unexpected for long range planning estimates. Space programs, as another example, at an early stage of development often exhibit a CV of 0.40 or greater. Other observed metrics at the early stages of a project include¹⁴:

- 0.35-0.45 typical for space systems and software intensive projects
- 0.25-0.35 typical for aircraft and similar complexity hardware
- 0.10-0.20 typical for large electronic system procurements; typical for spacecraft follow-on procurement (with no major changes)

While these represent commonly observed values suitable as general guidance, research is required to establish definitive metrics for specific commodities at specific times in their life cycle. **Figure 3-7** shows the CV results for the missile example.

¹⁴ The Coefficient of Variation (CV) metrics were derived from a study of Selected Acquisition Report (SAR) data on completed programs. The results are consistent with observed rules-of-thumb but further study in this area is required for higher fidelity and commodity specific metrics.

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CORREL(CV, 80/50) = 0.989

WBS	Point Estimate	Mean	Std Dev	CV	80% / 50%	50.00%	80.00%
Missile System	\$ 696,110 (15%)	\$929,619	\$231,798	25%	24%	\$900,216	\$1,116,877
Sys Dev and Demo	\$ 164,898 (26%)	\$219,377	\$81,542	37%	35%	\$203,186	\$274,008
Air Vehicle	\$ 111,549 (32%)	\$142,599	\$54,857	39%	37%	\$131,461	\$179,863
Design & Development	\$ 25,000 (25%)	\$30,106	\$6,509	22%	24%	\$29,217	\$36,110
Prototypes	\$ 9,749 (20%)	\$15,058	\$6,044	40%	43%	\$14,012	\$20,028
Software	\$ 76,800 (41%)	\$97,435	\$50,709	52%	51%	\$86,356	\$130,791
Sys Engineering/Program Management	\$ 21,000 (24%)	\$24,799	\$4,958	20%	19%	\$24,383	\$28,963
System Test and Evaluation	\$ 22,310 (23%)	\$37,757	\$21,091	56%	58%	\$32,666	\$51,654
Training	\$ 5,577 (25%)	\$8,120	\$3,680	45%	45%	\$7,284	\$10,545
Data	\$ 2,231 (25%)	\$3,250	\$1,480	46%	44%	\$2,933	\$4,224
Support Equipment	\$ 2,231 (32%)	\$2,852	\$1,097	39%	37%	\$2,629	\$3,597
Production Phase	\$ 531,212 (16%)	\$710,242	\$181,997	26%	25%	\$686,779	\$860,709
Air Vehicle	\$ 333,396 (15%)	\$411,798	\$74,435	18%	17%	\$404,621	\$473,733
Payload	\$ 11,416 (14%)	\$14,590	\$3,006	21%	19%	\$14,305	\$17,024
Propulsion	\$ 16,271 (17%)	\$20,496	\$4,499	22%	20%	\$19,955	\$24,033
Airframe	\$ 112,250 (49%)	\$116,277	\$26,776	23%	21%	\$113,067	\$137,251
Guidance and Control	\$ 186,979 (15%)	\$251,304	\$61,745	25%	24%	\$244,185	\$302,759
Integration, Assembly, Test and C	\$ 6,480 (9%)	\$9,130	\$2,163	24%	25%	\$8,781	\$10,934
Engineering Changes	\$ 16,670 (24%)	\$23,665	\$9,092	38%	39%	\$22,411	\$31,065
Sys Engineering/Program Management	\$ 93,351 (32%)	\$149,022	\$94,298	63%	70%	\$134,459	\$228,247
System Test and Evaluation	\$ 1,000 (40%)	\$1,046	\$135	13%	13%	\$1,036	\$1,169
Training	\$ 33,340 (13%)	\$50,098	\$16,003	32%	32%	\$47,630	\$62,910
Data	\$ 6,668 (13%)	\$9,309	\$2,400	26%	25%	\$8,985	\$11,273
Peculiar Support Equipment	\$ 6,668 (12%)	\$9,314	\$2,424	26%	25%	\$8,983	\$11,261
Common Support Equipment	\$ 113 (52%)	\$113	\$47	41%	36%	\$112	\$152
Initial Spares and Repair Parts	\$ 40,007 (12%)	\$55,878	\$14,520	26%	25%	\$53,942	\$67,407

Figure 3-7 Summary of Missile Risk Analysis Results

If the CV is unavailable, the analyst (or reviewer) can estimate the CV by calculating the percentage difference between the 80% and the 50% confidence level. In the case of the missile example, the percent difference correlates extremely well (see “CORREL” in **Figure 3-7**) with the CV. In the absence of CV, this can be a good approximation.

Another indicator of the quality of the risk assessment is the confidence level of the point estimate. In **Figure 3-7**, the confidence level of the point estimate is reported in parentheses. The point estimate generally falls in the 15% to 30% confidence range. When the point estimate confidence level is very low (<15%) this is often an indication that the CV may also be very low (i.e. insufficient uncertainty). When the point estimate confidence level is greater than 35%, this is often an indication that the point estimate may already be padded with some amount of uncertainty.

3.4 OTHER INFLUENCES ON SIMULATION ACCURACY

3.4.1 Random Seed and Random Number Generators

The random seed is a number that initializes the selection of numbers by a random number generator. Given the same seed, a random number generator will generate the same series of random numbers each time a simulation is run. Both Crystal Ball and @Risk, by default, pick a different random seed each time the simulation runs. To avoid this, an initial random seed may be set by the user. However, if the location of various assumptions is changed on the worksheet, answers will still vary. ACE assigns a random seed to every uncertainty assumption. When the assumption is moved, the random seed moves with it and therefore the random draw sequence is preserved.

Changing the random seed (either manually or by allowing the tool to do so) will cause the percentile results to vary on the order of 0.5%. Consequently, it is not possible to get precise matches across tools since each uses a different random number generator.

3.4.2 Simulation Sampling Method

Some tools allow the user to choose either Monte Carlo or Latin Hypercube (see **Appendix A.9** for details). Latin Hypercube is recommended because it draws random numbers more evenly and it will generally require fewer iterations to obtain the same level of accuracy. When using Latin Hypercube, the sample size should be equal to the number of iterations. @Risk and ACE do not have a user setting for sample size; both fix the sample size to the number of iterations.

3.4.3 Iterations

The number of iterations required to achieve reasonable accuracy is a function of how many uncertainty elements are in the model and how extensive the correlation matrix is.

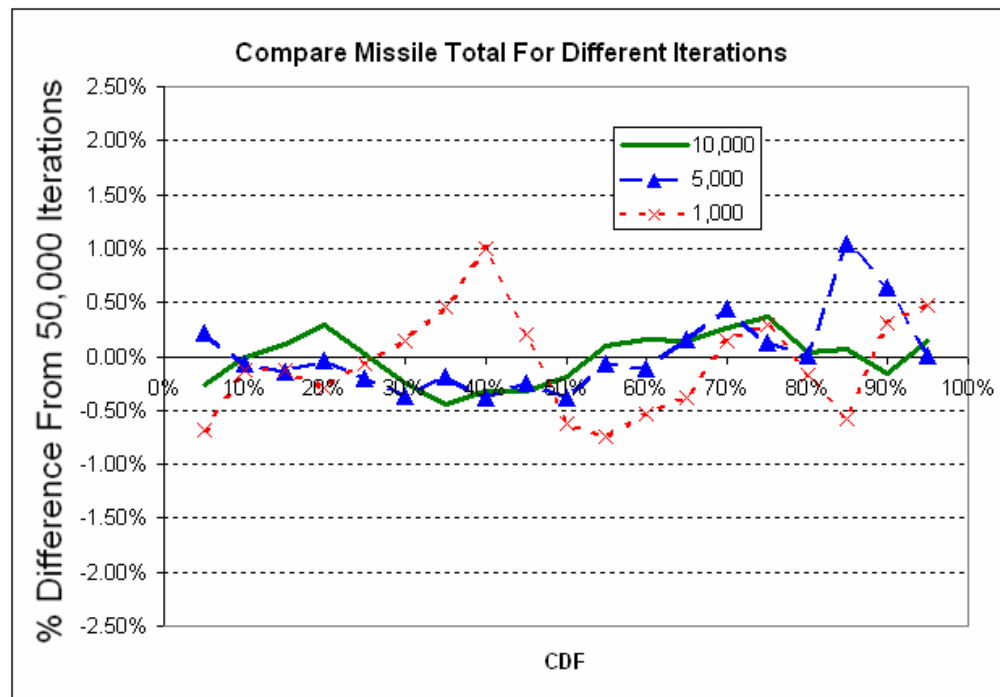


Figure 3-8 Compare Missile Results for Different Iterations

Figure 3-8 illustrates how the missile total cost at different percentiles vary from the 50,000 iteration result for different iterations. The example missile model is a relatively small model (32 uncertainty elements, largest correlation matrix is 19 by 19). Running this model at 10,000 iterations is not a problem. For larger models, however, 10,000 iterations may be problematic regardless of the tool used. However, it has been observed that in larger models, fewer iterations are required for stability. For instance, **Figure 3-9** compares iteration results for a model that has almost 1,000 uncertainty distributions and several large correlation matrices (four 112x112 plus several other smaller ones). In contrast to the smaller missile model, values over the range of interest (20th percentile to 80th percentile) appear to converge with as few as 500 iterations.

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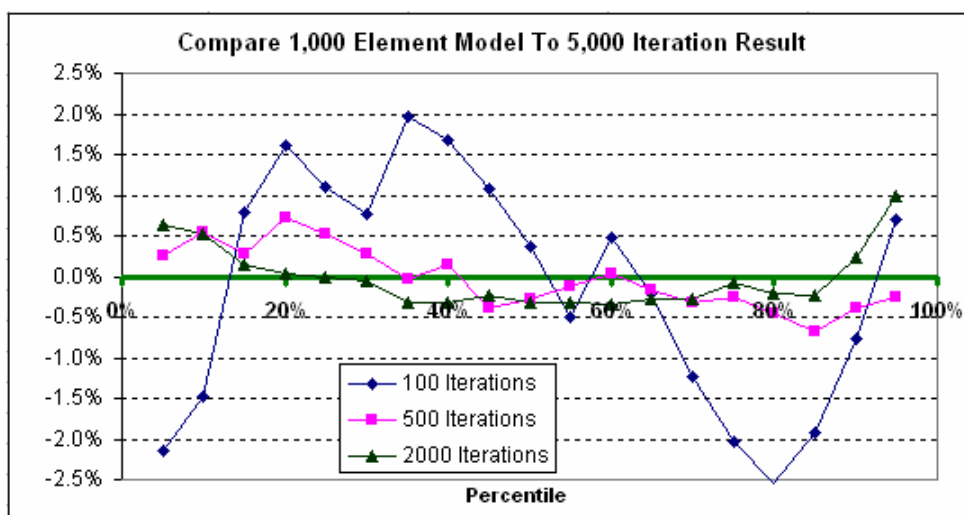


Figure 3-9 Compare 1,000 Element Model Results for Different Iterations

The overall guidance is that relatively few iterations (100-500) may be used as the model is built, while 1,000 to 10,000 iterations should be used for final milestone estimates.

3.5 SPECIAL CASES

3.5.1 Common Elements that are Regarded as Certain

The most obvious portions of the estimate that should be considered certain are sunk costs (see 3.5.2). In many situations, other items may be considered certain for the purposes of conducting cost risk analysis. Elements of cost not paid with program dollars such as launch costs and O&S costs can be treated as certain when they are provided to the estimator by the organization responsible for those costs. Items that are unlikely to change, such as fee, some throughputs, or firm fixed price quotes, can be treated as certain. Finally, elements with inconsequentially low cost need not receive uncertainty treatment, unless there are many of these elements.

While not certain, quantities are generally not assigned uncertainty distributions. Instead, various potential quantity options are treated as separate alternatives.

3.5.2 Sunk Costs

Costs that have been incurred and cannot be recovered are called sunk costs. Further, for many acquisition decisions, funds that have been authorized and obligated in prior years are often deemed sunk though they have not been quite expended. Sunk costs are often part of a life cycle cost model because current and prior years are part of a system's total cost. Prior years' costs (and often current year's costs) should not have uncertainty distributions associated with them. Risk dollars (discussed in Section 4.0) should not be allocated to those years where the estimating method has been replaced by sunk costs.

3.5.3 Funding Level Impact on Schedule

Cost models should not ignore the fact that there will be a point where the magnitude of the dollars estimated will have an impact on the schedule. For instance, a 5 year program where the point estimate is \$100M, it is likely impossible to spend the 80% result of \$140M in the same 5 years. Consequently, the cost model should provide for the percentile cost results to be mapped

to an appropriate schedule. Effort must be made to tie cost, risk and schedule together in a coherent way rather than treating them as independent variables.

3.5.4 Engineering Change Orders (ECOs)

There are many points of view regarding the validity of the ECO line in a cost estimate. Some argue that since CERs are developed from completed programs and/or since uncertainty is assigned to the configuration, ECO lines are redundant and to include them is “padding” the estimate. Others argue that historical projects’ end points are a new project's starting point; therefore, ECOs are inevitable and to exclude them weakens the estimate. ECOs are the result of controlled, approved changes to the requirement or the design. It is extremely rare for a project to proceed through the acquisition cycles without a single ECO. In any case, the ECO cost element is not meant to be catchall for potential system cost growth and it is therefore not acceptable to use it as a wedge for additional risk dollars. It is, however, appropriate to assign uncertainty to the method by which the ECO costs are estimated.

In the absence of a better approach, this handbook recommends that ECOs should be an element in the program WBS and that uncertainty should be applied in a manner consistent with the method by which the element is estimated.

3.5.5 Inflation

Inflation history and projections are updated at least once per year by OMB. **Figure 3-10** illustrates the inflation factor an analyst would use to inflate from 1997 (when an estimate may have been done) to 2006 (when the program may have been executed). The x axis in the table identifies the year of the tables used to generate the inflation factor. As the years progress, projected inflation is replaced with “actual” inflation as reported in the published tables.

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BY to TY Inflation from 1997 to 2006														
Using this FY OSD Table -->	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	Ave	Stdev	CV
BY to BY Aircraft Procurement - 3010	1.217	1.223	1.174	1.150	1.138	1.149	1.139	1.114	1.111	1.132	1.147	1.154	0.037	0.032
BY to TY Aircraft Procurement - 3010	1.260	1.293	1.228	1.191	1.172	1.187	1.168	1.139	1.133	1.159	1.176	1.191	0.050	0.042
BY to TY Missile Procurement - 3020	1.261	1.271	1.217	1.188	1.173	1.181	1.163	1.140	1.131	1.153	1.169	1.186	0.046	0.039
BY to TY Other Procurement - 3080	1.266	1.282	1.221	1.194	1.179	1.193	1.178	1.150	1.120	1.142	1.158	1.190	0.050	0.042
BY to TY RDT&E - 3600	1.239	1.248	1.192	1.163	1.150	1.162	1.150	1.125	1.121	1.146	1.158	1.169	0.042	0.036
BY to TY Non Pay, Non POL - 3400	1.239	1.248	1.190	1.163	1.150	1.161	1.150	1.124	1.120	1.144	1.166	1.169	0.042	0.036

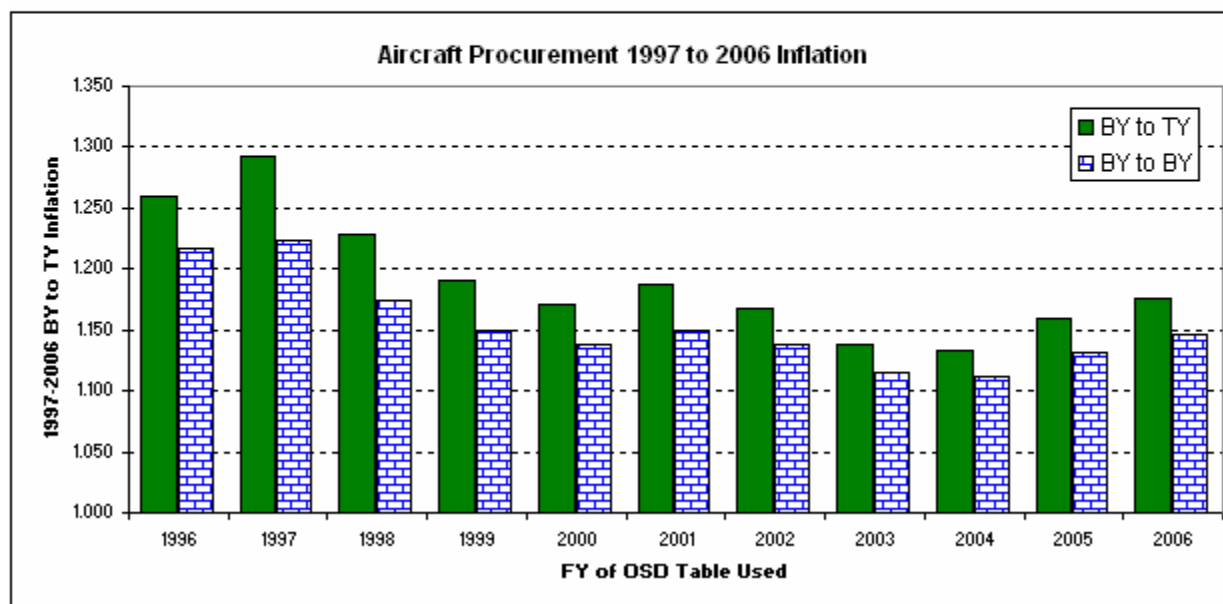


Figure 3-10 Aircraft Procurement Inflation Factor Change Over Past 10 Years

While **Figure 3-10** confirms there is indeed uncertainty in the inflation factor used back in 1997, in this particular case the correct adjustment (had the analyst been privy to the future) would have been to reduce the inflation factor. Another argument would be to observe that the Aircraft Procurement BY to TY factor varied between 1.13 and 1.29 depending on the year the calculation was done. Something else to note is that the CV (3 to 4%) is arguably quite small compared to the other uncertainties in the estimate. Adding a multiplier to account for published inflation uncertainty—analogue to the schedule/technical multiplier suggested in **Section 2.9**—may be attempting a level of accuracy that is not possible in uncertainty analysis. This handbook does not require or recommend that uncertainty to be placed on published inflation tables.

There is another aspect of inflation uncertainty that does warrant attention. If the Program Office has suitable data to demonstrate that the inflation rate experienced and projected for a specific commodity is significantly different from the published inflation, then that difference should be in the TBE and uncertainty analysis. Further, uncertainty should be applied to the inflation adjustment.

3.5.6 Defining Uncertainty for Learning-Adjusted Methodologies

Learning-adjustment uncertainty should not be ignored. However, it is very easy to over-specify risk on an estimate that contains learning. For example, consider a CER is given by a Cumulative Average Cost learning curve:

Equation 1

$$\text{Cost} = \text{T1} * \text{Qty}^{(b+1)} * \epsilon$$

where T1 is the theoretical first unit cost and b is the quantity slope.

If T1 and b are established by regression analysis with quantity as an independent variable (QAIV), the statistics of the regression analysis can be used to objectively define the CER uncertainty. Assigning uncertainty to the learning slope (i.e., $\ln(b)/\ln(2)$) as well, is an example of possible excessive risk specification since it was already contained in the CER's specification. However, if T1 and the learning slope are derived separately or are simple throughputs based upon expert opinion rather than regression analysis, then treating them as two separate "inputs" and assigning uncertainty distributions to each would be appropriate. If uncertainty distributions are defined on T1 and b, then do not specify uncertainty for the learning curve equation as well. Specify uncertainty on either the learning curve equation, or on the T1 and b pair, but **not** both.

3.5.7 Accounting for Risk Mitigation Plans

The word "risk" is used as an adjective for many activities and products associated with Air Force acquisition. All of these need not require overt treatment in the cost risk analysis process. Risk Mitigation Plans are typically part of the program of record and will not, in and of themselves, require treatment. Risk Analysis Matrices, Maxwell Matrices, or products from a Risk IPT¹⁵ may only indirectly influence choice of risk distributions and their bounds. A key challenge to using these products directly in a cost risk analysis is that often they do not address topics in a manner consistent with the cost estimate's WBS. Many analysts, however, find them useful to review as a crosscheck to determine if they have completely specified program uncertainties. These products can be used to help determine risk scores when risk score mapping is used (described in **Section 7.0**). They can also help determine penalty factors (described in **Section 2.9**).

The inclusion of risk mitigation plans often introduces additional cost elements into the cost estimate. This has the effect of "adding to" the initial cost estimate. However, the presence of these risk mitigation plans and the fact that they are to be funded should have an impact on the uncertainties assigned to the WBS elements that they address. For example, spending money to mitigate the cost impact of a schedule/technical risk may cause the WBS element to be treated with a low schedule/technical penalty factor rather than a high one (see **Section 2.9**).

4.0 HOW TO "BUY" ADDITIONAL CERTAINTY

The previous sections fully described how to model uncertainty but do not yet answer the question of how to "buy" additional certainty for the program. In general, certainty can be "bought" by:

- Taking a less risky approach to the program in terms of cost, schedule or technology
- Investing in risk reduction efforts
- Using proven technologies or advancing technology improvements
- Adding additional funding

¹⁵ MYTH: Project risk classification charts (Maxwell matrices) address and encompass cost uncertainty analysis.

GUIDANCE: These charts contain no cost information but rather provide a guideline on the maturity of the design, manufacturing, etc. The analyst has to combine that information with his or her knowledge of estimating to translate risk information into its cost impact.

The first three bullets focus on reduction of risk to increase certainty. The last bullet and the following section, focus on the use of additional funding to increase confidence or certainty.

4.1 USING THE TBE TO DEFINE RISK DOLLARS

For the purposes of convenience and to serve as a reference terminology for future calculations described in this handbook, we define “risk dollars”¹⁶ as simply the amount of funds needed to bring the TBE value up to a selected confidence level. To calculate risk dollars, a desired confidence level must be selected. This level¹⁷ may be mandated or it may be simply an organizational practice. Section 7.0 will discuss this selection further but examples in this section will assume a 60% probability level.

In the case of the simulation approach, the dollar value at the 60% mark on the CDF minus the TBE value is the amount of risk dollars. The TBE will typically be at a confidence level less than 50% as the majority of cost elements will be right skewed. **Figure 4-1** depicts the steps.

1. Determine the TBE dollars.
2. Run the simulation thereby obtaining a CDF.
3. Select a confidence level.
4. Determine the CDF x-axis value at the point of the desired confidence level.
5. Compute risk dollars as the difference between the desired confidence level and the TBE dollars.

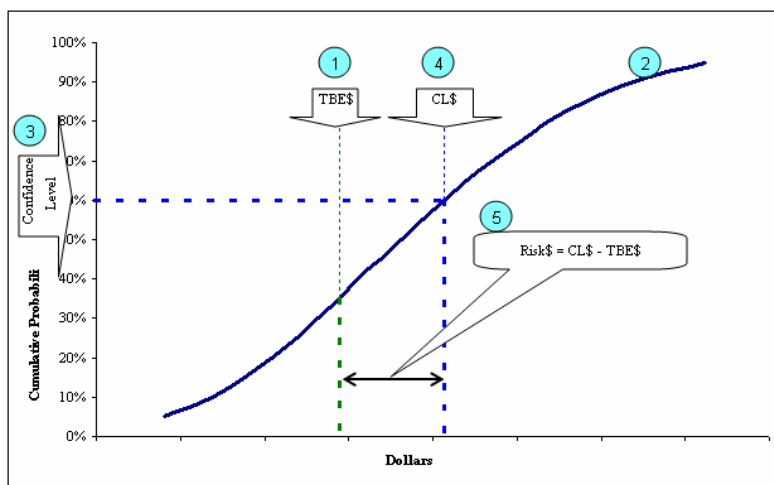


Figure 4-1 Defining “Risk Dollars” Based on TBE

¹⁶ While the term “risk dollars” is useful to describe a set of calculations among cost analysts, use of the term with other audiences can often lead to confusion and unintended interpretations. Instead, simply discuss resulting values in terms of their desired confidence level.

¹⁷ MYTH: Probability level is a measure of the probability of success of the project.

GUIDANCE: Budgeting at the 80% probably level provides a higher likelihood of not overrunning the budget than budgeting, say, at the 50% probability level. However, meeting budget expectations is only one of many factors that lead to program success.

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4.2 EXAMPLES

In this example, assume that the risk dollars are allocated from the SDD and Production levels. Therefore, the SDD TBE and risk statistics are required in order to compute the risk dollars in the SDD. If, however, it was determined that risk dollars should be allocated from a higher or lower level, then a risk allocation process would be needed to establish the risk adjusted value at the SDD level.

In this example, to compute the risk adjusted dollars for the SDD Phase of the missile example, the simulation statistics results are required for that element. **Figure 4-2** is a sample statistics report from ACE RISK. The TBE cost is the value shown on the SDD Phase row in the Point Estimate column. (Note that the probability level of each point estimate value is reported in the parenthesis.) The risk adjusted value is the value shown on the SDD Phase row in the column corresponding to the selected confidence level. The BY risk dollars are \$56,868K (\$221,766K – \$164,898K). The same value can be obtained from the Crystal Ball or @Risk standard output reports. To obtain the TY risk dollars requires three additional steps: allocating risk dollars, phasing the allocated risk dollars and then inflating the risk dollars in each year.. These steps are covered in the next sections.

	WBS/CES	Point Estimate	Mean	Std Dev	CV	50.0% Level	55.0% Level	60.0% Level	65.0% Level	70.0% Level
47	Sys Dev and Demo	\$ 164,898 (26%)	\$ 219,377	\$ 81,542	0.372	\$ 203,186	\$ 211,902	\$ 221,766	\$ 231,741	\$ 243,877
48	Air Vehicle	\$ 111,549 (32%)	\$ 142,599	\$ 54,857	0.385	\$ 131,461	\$ 137,297	\$ 144,041	\$ 151,303	\$ 159,345
49	Design & Development	\$ 25,000 (25%)	\$ 30,106	\$ 6,509	0.216	\$ 29,217	\$ 30,179	\$ 31,196	\$ 32,280	\$ 33,444
50	Prototypes	\$ 9,749 (20%)	\$ 15,058	\$ 6,044	0.401	\$ 14,012	\$ 14,823	\$ 15,674	\$ 16,621	\$ 17,630
51	Software	\$ 76,800 (41%)	\$ 97,435	\$ 50,709	0.520	\$ 86,356	\$ 91,827	\$ 97,452	\$ 104,155	\$ 111,863
52	Sys Engineering/Program Man	\$ 21,000 (24%)	\$ 24,799	\$ 4,958	0.200	\$ 24,383	\$ 25,060	\$ 25,732	\$ 26,407	\$ 27,117
53	System Test and Evaluation	\$ 22,310 (23%)	\$ 37,757	\$ 21,091	0.559	\$ 32,666	\$ 35,007	\$ 37,562	\$ 40,280	\$ 43,165
54	Training	\$ 5,577 (25%)	\$ 8,120	\$ 3,680	0.453	\$ 7,284	\$ 7,686	\$ 8,112	\$ 8,627	\$ 9,160
55	Data	\$ 2,231 (25%)	\$ 3,250	\$ 1,480	0.455	\$ 2,933	\$ 3,085	\$ 3,267	\$ 3,462	\$ 3,666
56	Support Equipment	\$ 2,231 (32%)	\$ 2,852	\$ 1,097	0.385	\$ 2,629	\$ 2,746	\$ 2,881	\$ 3,026	\$ 3,187

Figure 4-2 Risk Statistics Report for the Missile SDD

Another way of computing risk dollars is described in **B.4**.

5.0 HOW TO ALLOCATE AND TIME PHASE RISK DOLLARS

5.1 WHY IT IS NECESSARY TO ALLOCATE AND PHASE RISK DOLLARS

A consequence of the risk analysis process is that the lower level WBS element results do not sum to the parent result for a specific confidence level. While this is the mathematically correct way to display risk analysis results, it is often necessary to force WBS elements to sum at a specific confidence level for budgeting and other reasons.

Once it is known “where” (what WBS elements) the risk dollars should be allocated, it is equally important to know “when” (what FY) the risk dollars should be made available. Phasing risk dollars does not have an impact on base year (BY) estimate totals, but can have a large impact on the then-year (TY) estimate for a given year.

5.2 HOW TO ALLOCATE RISK DOLLARS

The first step in the risk allocation process is to determine at what level in the WBS the risk dollars are to be managed. That is, to select the WBS elements that you either cannot or choose not to allow the assigned budget to change. Some project offices choose set budgets at the Appropriation level. Others may choose to set the budgets at a lower or perhaps higher level.

Once the WBS levels from which cost risk will be managed are set, select the desired confidence level for each one. The difference between the TBE and the risk result at the selected confidence level are the risk dollars to be allocated to lower level elements. The preferred approach is to follow a defensible and repeatable allocation scheme that allocates risk costs to all the lower level elements in a consistent manner. There are at least three objectives for any allocation scheme:

- Derive a scheme whereby the relative adjustment from the statistically correct result is minimized (for example, if the result it requested for the 60% confidence level, then all the “sub” elements should be as close as possible to the 60% level).
- After the adjustment is complete, sub elements should sum to the parent (this is not the case with the statistical results).
- The allocation should be influenced by the statistical results and the correlation at the lower levels. That is, the allocation should be influenced by some measure of the lower level element’s uncertainty.

A simple allocation scheme that is consistent with these objectives is to make the adjustment directly to the mathematically correct (statistical) risk results (not to the point estimate) for each lower level element. In this case, the magnitudes of the adjustments are modest and allocated results tend to remain near the correct risk statistical results. This process follows these steps:

1. Select the level in the WBS from which risk dollars will be allocated. In general this represents the level at which dollars are managed, meaning at this level there is no latitude for passing dollars saved on one element to another element at the same level. This is often the Appropriation levels.
2. Calculate the TBE in BY dollars.
3. Generate the risk statistics in BY dollars.
4. Allocate risk in BY dollars.
 - a. From the risk simulation results for a selected confidence level, sum the children and compute the difference between the sum of the children and the parent value. This difference represents the dollars to be “allocated” to the children to cause them to sum to the parent.
 - b. Beginning with the parent from which you are starting, prorate the difference to its children. Use the standard deviation (not the point estimates) as the basis for prorating. Repeat for the remainder of the WBS structure in this manner.
5. Levels above the level at which you are allocating risk are merely the sum of their children.

An alternative allocation approach is presented in Appendix **A.15.3**.

5.3 HOW TO TIME-PHASE ALLOCATED RISK DOLLARS

Once the risk dollars have been calculated and allocated, the next step is to determine how the risk dollars should be time-phased. Phasing cost estimates is necessary in order to arrive at annualized values and to properly account for inflation. Several approaches are offered.

- **Prorate consistent with point estimate:** If the estimated program phase is several years in the future, simply time-phase the allocated risk dollars in the same manner as the point estimate. This approach is particularly well suited for a production phase that is estimated during early development since it will typically be time-phased by production schedule.
- **“Backload”:** If the estimate is near-term or if the estimated phase is already in progress, it is advisable to “backload” the risk dollars into the later years of the phase. That is, model a time-phasing distribution such that the resulting distribution of risk dollars is small in the near-term and large in the far-term when the issues of uncertainty are more likely to manifest themselves into specific efforts the program must fund. One approach to modeling this is to place the risk dollars on a separate WBS row (this would be done in the absence of allocating the risk dollars to all the WBS elements) and use a beta curve with 40% spent in 60% time.
- **Specific time:** The analyst may consider time-phasing the risk dollars after a specific “risky” event. An example might be after the critical design review or the first flight test of a missile system. If a program is underway and the budget in the near-term years is known to be “fixed,” then the analyst can append the dollars to years beyond the current time-phased TBE. This is particularly well-suited for situations where the uncertainty issues, when manifest, will require additional schedule (i.e. longer program duration) to address.
- **Algorithm at Lowest Levels:** The analyst may consider developing phasing methods at the lowest level that are influenced by the confidence level requested. Specifically, the method can cause the schedule to contract or expand with the confidence level requested.

5.4 EXAMPLE OF ALLOCATING AND TIME-PHASING RISK DOLLARS

Figure 5-1 illustrates the risk allocation process using the missile example. In this example, allocated BY results are derived by setting the SDD and Production elements to the 60% confidence level. In this case, risk dollars are to be managed at the program phase level which is the second level of the WBS. A consequence of this action is that the estimate at the total level will not be at the 60% level, but equal to the total of the SDD and Production 60% results. As the level from which risk dollars is allocated moves further down the WBS, the greater will be the difference between the total of the allocated result and the statistical result. As **Figure 5-1** illustrates, in the case of the missile model, the difference is less than 1% when allocation is from the second level. The difference will get larger as the selected confidence level moves further from the mean.

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	B	C	D	E	G	H	I	J	K	L	M
		Point Estimate	Std Dev	60% Confidence Level From Risk Model	Difference Between Parent and Sum of Children	Sum of Children's Std Dev	Prorated Difference to Each Child	Difference Between Parent (Incl its Additional Proration) and Sum of Children	Prorated Difference to Each Child	Sum of 60% From Risk Model plus Proration	Difference from Statistical Result
2											
3	Missile System	\$696,110	\$231,798	\$963,145						\$956,398	-0.7%
4	Sys Dev & Demo Phase	\$164,898	\$81,542	\$221,766	\$171	\$87,163				\$221,766	0.0%
5	Air Vehicle	\$111,549	\$54,857	\$144,041		\$63,262	\$108	(\$173)		\$144,149	0.1%
6	Design & Development	\$25,000	\$6,509	\$31,196					(\$18)	\$31,178	-0.1%
7	Prototypes	\$9,749	\$6,044	\$15,674					(\$17)	\$15,657	-0.1%
8	Software	\$76,800	\$50,709	\$97,452					(\$139)	\$97,313	-0.1%
9	Sys Engineering/Program Mgmt	\$21,000	\$4,958	\$25,732			\$10			\$25,742	0.0%
10	System Test and Evaluation	\$22,310	\$21,091	\$37,562			\$41			\$37,603	0.1%
11	Training	\$5,577	\$3,680	\$8,112			\$7			\$8,119	0.1%
12	Data	\$2,231	\$1,480	\$3,267			\$3			\$3,270	0.1%
13	Support Equipment	\$2,231	\$1,097	\$2,881			\$2			\$2,883	0.1%
14											
15	Production Phase	\$531,212	\$181,997	\$734,632	(\$4,959)	\$213,354				\$734,632	0.0%
16	Air Vehicle	\$333,396	\$74,435	\$424,253		\$98,189	(\$1,730)	(\$3,774)		\$422,523	-0.4%
17	Propulsion	\$11,416	\$3,006	\$15,065					(\$116)	\$14,949	-0.8%
18	Payload	\$16,271	\$4,499	\$21,116					(\$173)	\$20,943	-0.8%
19	Airframe	\$112,250	\$26,776	\$119,916					(\$1,029)	\$118,887	-0.9%
20	Guidance and Control	\$186,979	\$61,745	\$260,818					(\$2,373)	\$258,445	-0.9%
21	Integration, Assy, Test & Checkout	\$6,480	\$2,163	\$9,382					(\$83)	\$9,299	-0.9%
22	Engineering Changes	\$16,670	\$9,092	\$24,799			(\$211)			\$24,588	-0.9%
23	Sys Engineering/Program Mgmt	\$93,351	\$94,298	\$160,801			(\$2,192)			\$158,609	-1.4%
24	System Test and Evaluation	\$1,000	\$135	\$1,074			(\$3)			\$1,071	-0.3%
25	Training	\$33,340	\$16,003	\$51,664			(\$372)			\$51,292	-0.7%
26	Data	\$6,668	\$2,400	\$9,613			(\$56)			\$9,557	-0.6%
27	Peculiar Support Equipment	\$6,668	\$2,424	\$9,611			(\$56)			\$9,555	-0.6%
28	Common Support Equipment	\$113	\$47	\$124			(\$1)			\$123	-0.9%
29	Initial Spares and Repair Parts	\$40,007	\$14,520	\$57,652			(\$337)			\$57,315	-0.6%
30	G4: =E4-(E5+SUM(E9:E13))										
31	H5: =SUM(D6:D8)										
32	I5: =G\$4*D5/\$H\$4										
33	J5: =E5+I5-SUM(E6:E8)										
34	K6: =J\$5*D6/\$H\$5										
35	L6: =E6+K6										

Figure 5-1 Allocating Risk Dollars from the Second Level WBS in the Missile Example

5.5 HOW TO CALCULATE TY RISK RESULTS

To this point, all the costs have been calculated in BY dollars. Converting point estimate BY calculations to TY dollars is well established. However, there is no standard way to convert BY risk adjusted results to TY dollars. A standard approach is elusive due to the sensitivity of schedule to uncertainty and the degree to which it is modeled. The recommended method for converting risk adjusted BY results to TY is to:

1. Calculate the point estimate BY results
2. Calculate the point estimate TY results
3. Derive the point estimate TY/BY **factors** at the lowest levels in the WBS
4. Calculate the allocated, risk adjusted BY result
5. Multiply the allocated, risk adjusted BY result by the factors

Figure 5-2 illustrates this method as applied to the missile example. While more refined methods addressing the additional inflation a program with a stretched schedule would incur are not discouraged, this simple approach is deemed acceptable.

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	Point Estimate BY Totals (1)	Point Estimate TY Totals (2)	TY/BY (2) / (1) (3)	Allocated BY Result (4)	Allocated TY Result (4) * (3) (5)
Missile System	\$696,110	\$808,161		\$956,398	\$1,110,959
Sys Dev and Demo	\$164,898	\$175,472		\$221,766	\$235,981
Air Vehicle	\$111,549	\$118,496		\$144,149	\$153,155
Design & Development	\$25,000	\$26,536	1.0614	\$31,178	\$33,094
Prototypes	\$9,749	\$10,440	1.0709	\$15,657	\$16,768
Software	\$76,800	\$81,520	1.0615	\$97,313	\$103,294
Sys Engineering/Program Management	\$21,000	\$22,613	1.0768	\$25,742	\$27,719
System Test and Evaluation	\$22,310	\$23,699	1.0623	\$37,603	\$39,945
Training	\$5,577	\$5,925	1.0623	\$8,119	\$8,625
Data	\$2,231	\$2,370	1.0623	\$3,270	\$3,474
Support Equipment	\$2,231	\$2,370	1.0623	\$2,883	\$3,063
Production Phase	\$531,212	\$632,689		\$734,632	\$874,978
Air Vehicle	\$333,396	\$397,096		\$422,523	\$503,255
Payload	\$11,416	\$13,596	1.1910	\$14,949	\$17,804
Propulsion	\$16,271	\$19,379	1.1910	\$20,943	\$24,943
Airframe	\$112,250	\$133,691	1.1910	\$118,887	\$141,595
Guidance and Control	\$186,979	\$222,694	1.1910	\$258,445	\$307,811
Integration, Assembly, Test and Checkout	\$6,480	\$7,736	1.1938	\$9,299	\$11,101
Engineering Changes	\$16,670	\$19,855	1.1911	\$24,588	\$29,286
Sys Engineering/Program Management	\$93,351	\$111,187	1.1911	\$158,609	\$188,914
System Test and Evaluation	\$1,000	\$1,172	1.1720	\$1,071	\$1,255
Training	\$33,340	\$39,710	1.1911	\$51,292	\$61,093
Data	\$6,668	\$7,942	1.1911	\$9,557	\$11,383
Peculiar Support Equipment	\$6,668	\$7,942	1.1911	\$9,555	\$11,380
Common Support Equipment	\$113	\$135	1.1915	\$123	\$146
Initial Spares and Repair Parts	\$40,007	\$47,651	1.1911	\$57,315	\$68,265

Figure 5-2 Developing TY Risk Allocated Results

Figure 5-3 illustrates the impact of the allocated risk phasing selection. In this particular case, although the totals on a yearly basis are quite different, the total risk adjusted TY results are very similar.

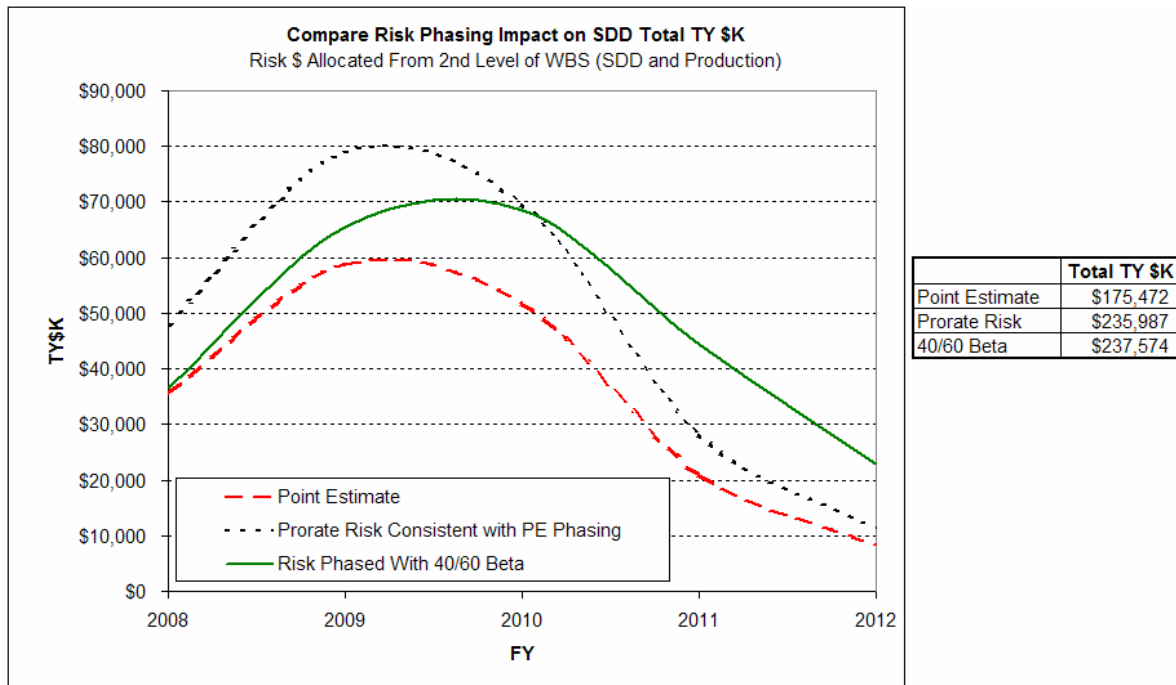


Figure 5-3 Time Phased Risk Allocated TY Dollars

6.0 HOW TO PRESENT THE “RISK STORY”

Presenting the risk story to senior leadership or to a review Agency must entail a clear communication of the following information in a series of charts similar to those shown:

- The nature of the TBE. This chart (**Figure 6-1**) discusses the characterization of the TBE. This is a summary of the content of **Section 1.4.1**.
- General approach of how the uncertainty was defined and, in the case of the simulation method, how the bounds and distributions were chosen. This chart (**Figure 6-2**) is a synopsis of all the activity described in **Section 2.0**.
- Present the TY CDF as an S-curve with FY costs on the X axis (**Figure 6-3**). Show the location of the TBE, protect scenario, median and proposed funding level on the CDF. The chart subtitle should report the CV.
- Risk adjusted, TY costs by year (**Figure 6-4**).
- Identify the cost drivers that have the most impact on the cost estimate.
- Identify the most important contributors to the cost estimate uncertainty and any risk mitigation initiatives captured by the estimate. There are many ways to develop this analysis. A common method is to identify those WBS elements with the largest variance. This approach may be misleading if the impact of correlation is not included.

Note that these charts present risk results without mention of “risk dollars.” These charts will need to be repeated for each Appropriation on which uncertainty analysis was performed.

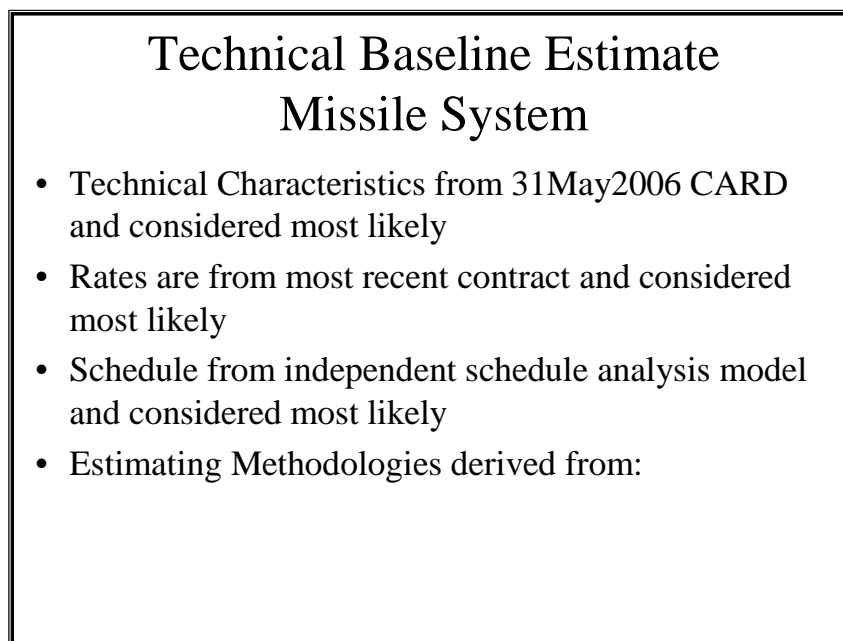


Figure 6-1 Sample Presentation Chart (1 of 4) The Nature of the TBE

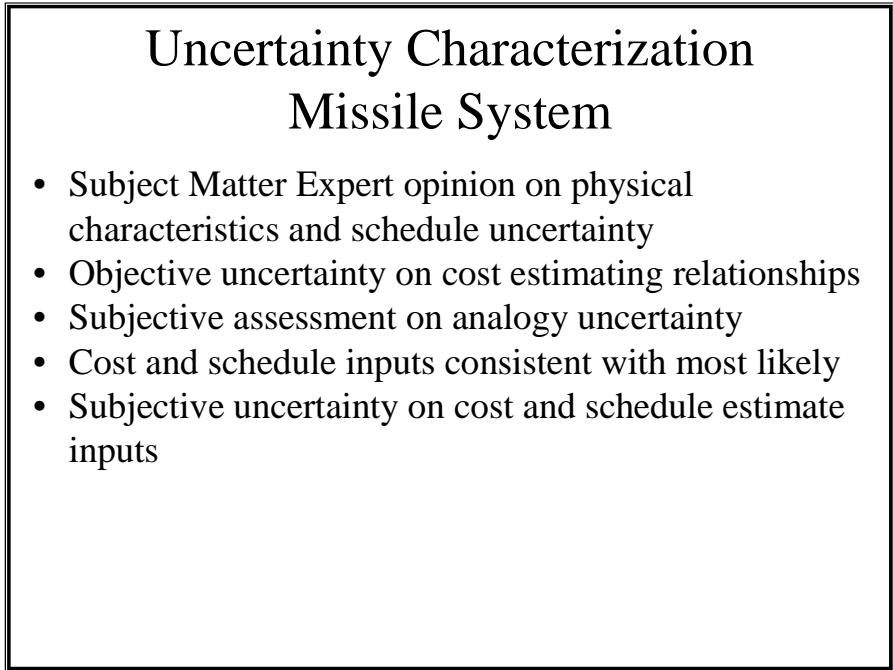


Figure 6-2 Sample Presentation Chart (2 of 4) General Uncertainty Approach

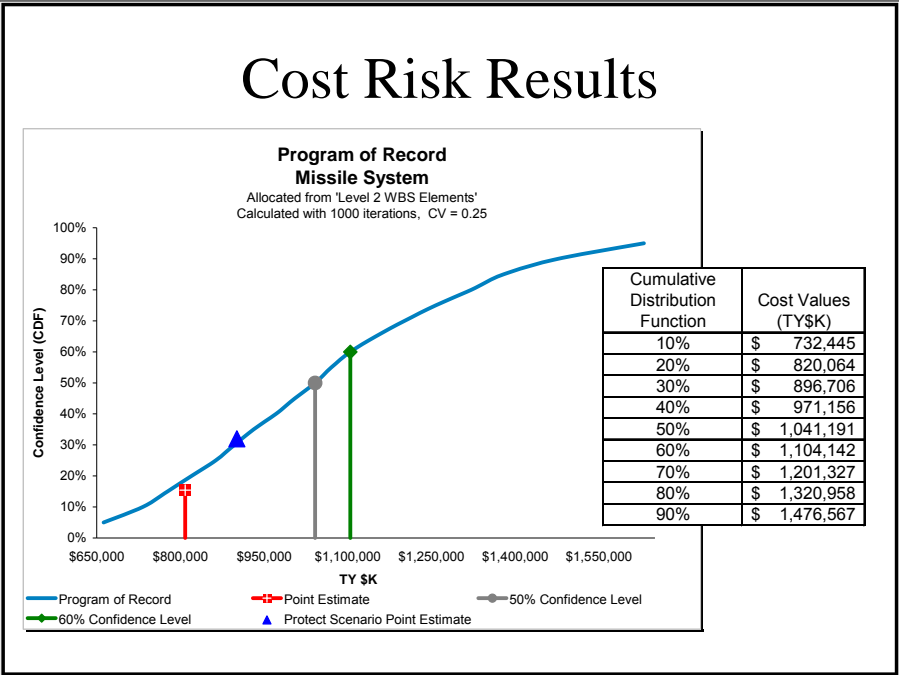


Figure 6-3 Sample Presentation Chart (3 of 4) Cost Risk Results

TY\$ Phased Allocated Result Missile System						
Cost Element	POM FY 2008	POM FY 2009	POM FY 2010	POM FY 2011	POM FY 2012	TOTAL
Sys Dev and Demo	\$47,102	\$78,879	\$71,651	\$27,692	\$10,656	\$235,980
Air Vehicle	\$30,531	\$53,901	\$49,063	\$16,106	\$3,555	\$153,156
Design & Development	\$7,408	\$12,156	\$8,759	\$3,908	\$863	\$33,094
Prototypes		\$3,803	\$12,965			\$16,768
Software	\$23,123	\$37,942	\$27,339	\$12,198	\$2,692	\$103,294
Sys Engineering/Program Management	\$5,322	\$5,419	\$5,533	\$5,652	\$5,792	\$27,718
System Test and Evaluation	\$8,154	\$14,178	\$12,362	\$4,302	\$949	\$39,945
Training	\$1,761	\$3,061	\$2,669	\$929	\$205	\$8,624
Data	\$709	\$1,233	\$1,075	\$374	\$83	\$3,473
Support Equipment	\$625	\$1,087	\$948	\$330	\$73	\$3,063

Figure 6-4 Sample Presentation Chart (4 of 4) Phased Results

7.0 ALTERNATIVES TO THE INPUTS-BASED SIMULATION METHOD

7.1 OVERVIEW

As discussed in **Section 1.5**, there are two acceptable alternatives to the Inputs-Based Simulation Method. Considerations that would lead an analyst to choose to apply Outputs-Based Simulation or the Scenario Based method include: available data, available resources, available schedule, the complexity of the estimate, and the consequences of “less precise results.” This handbook recommends the Inputs-Based simulation method for conducting cost risk analysis. A sound rationale for diverting from this approach is required if either of the following methods are chosen as the primary method.

7.2 OUTPUTS-BASED SIMULATION METHOD

An alternative to the Inputs-Based Simulation method is to apply uncertainty directly to the results (cost model outputs). By use of uncertainty distributions on the outputs, the aggregate uncertainty of both the methodology and the inputs is addressed. The material in **Section 2.1** regarding the scope of what is to be included or excluded in cost risk analysis applies to the outputs-based method as well.

Upon completion of the point estimate, the analyst will examine the WBS and determine the level at which to apply uncertainty. Application at every child element is recommended though circumstance may lead to application at parent levels instead. For example, in a model where a parent element is the sum of a large number of low-cost child elements, it may be appropriate to simply treat uncertainty at the level of that parent. The simulation model is set up such that the distributions are defined with a most likely value of “1” (or in the case of lognormal, the median) to be multiplied by each element’s point estimate. Each simulation pass will draw a sample of

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the distribution and multiply the drawn value times the point estimate value – the simulation need not execute the entire cost model upon each draw. In **Figure 7-1**, outputs for several WBS elements are multiplied by subjectively selected lognormal distributions (see **Table 2-5**).

WBS Description	Simulation	Point Estimate (PE)	Output Uncertainty Median	Distribution Form	85% High	Subjective Uncertainty
Payload	PE * Uncertainty	\$11,416	1	Lognormal	1.168	Low
Propulsion	PE * Uncertainty	\$16,271	1	Lognormal	1.168	Low
Airframe	PE * Uncertainty	\$112,250	1	Lognormal	1.296	Medium
Guidance and Control	PE * Uncertainty	\$186,979	1	Lognormal	1.296	Medium

Figure 7-1 Example Application of Outputs-Based Simulation Set-up

Since the objective is to model combined effects in one distribution, the shape and bounds of the distribution will often by necessity be subjective unless the bounds were derived from a data set or from a more detailed series of simulations (see **Reference 49**). If these bounds are subjectively derived, all of the previous section's guidance on subjective distributions applies.

The subjective selection of uncertainty can often be enhanced by use of risk score mapping and is often used in outputs-based simulations. Risk score mapping is a technique consisting of a risk scoring matrix and a map of uncertainty distribution bounds against risk scores. **Figure 7-2** depicts this method conceptually. The risk scoring matrix at the top of the figure consists of uncertainty-causing categories by row. By column, the attributes of those categories are listed that are deemed low risk, high risk, etc. Separate matrices may be developed for different types of cost elements. The matrices are used to elicit judgments from technical personnel as to the technical and schedule risk associated with the particular cost element.

The columns are quantified with assigned scores of increasing value from low to high risk. The average score from the matrix across the categories is the overall risk score for that cost element. The categories may be weighted if desired. The risk scores are converted to distribution bounds as shown in the bottom of the figure.

Appendix A.15 provides reference material on some of the risk score mapping methods currently in use within the Air Force. These methods have merit in formalizing the assignment of subjective risk and providing a mechanism for eliciting participation from the technical functionals in judging risk. However, at the present time the handbook is not advocating a particular implementation of this approach.

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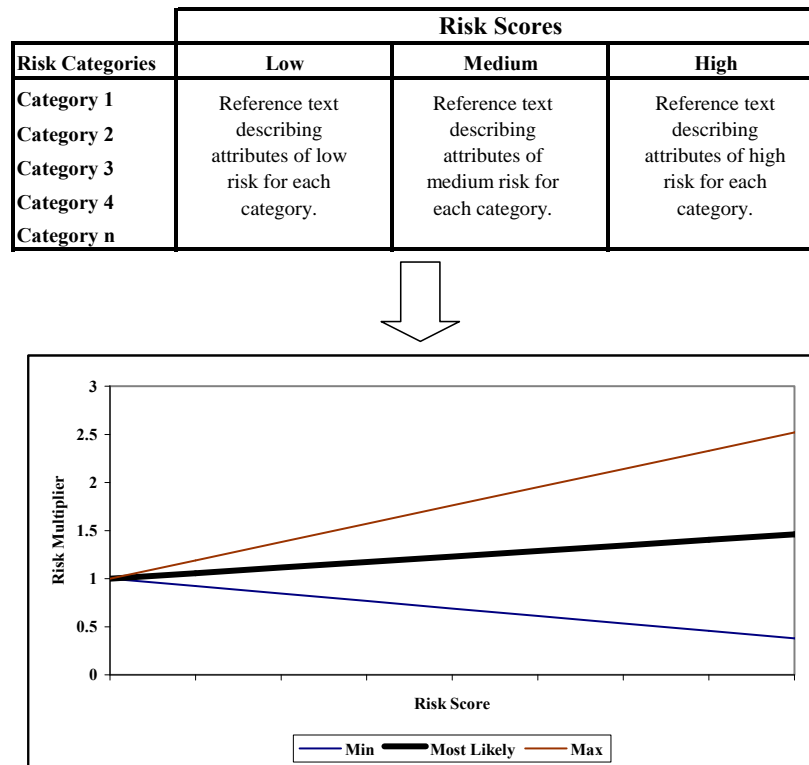


Figure 7-2 Risk Score Mapping Concept

7.3 SCENARIO BASED METHOD

7.3.1 Overview

The Scenario Based Method (SBM) (**Reference 43/Attachment 1**) postulates on specified scenarios that, if they occurred, would result in costs higher than the level planned or budgeted. These scenarios do not have to represent worst cases; rather, they should reflect a set of conditions a Program Manager or decision-maker would want to budget for, should any or all of those conditions occur.

The process of defining scenarios is a good practice. It builds the supportive rationale and provides a traceable and defensible analytical basis behind a “derived” measure of cost risk; this is often lacking in traditional simulation approaches. Visibility, traceability, defensibility, and the cost impacts of specifically identified risks are principal strengths of the SBM.

7.3.2 Approach & Assumptions

The first step (see **Figure 7-3**) is to start with a point estimate (PE) which, for the purposes of this handbook, is the program’s TBE. Next, the analyst must define a protect scenario (PS). The key to a “good PS” is one that identifies, not an extreme worst case, but a scenario that captures the impacts of the major known risks to the program – those events the Program Manager or decision-maker must monitor and guard the costs of the program against. Thus, the PS is not arbitrary. It should reflect the above, as well as provide a possible program cost that, in the opinion of the engineering and analysis team, has an acceptable chance of not being exceeded.

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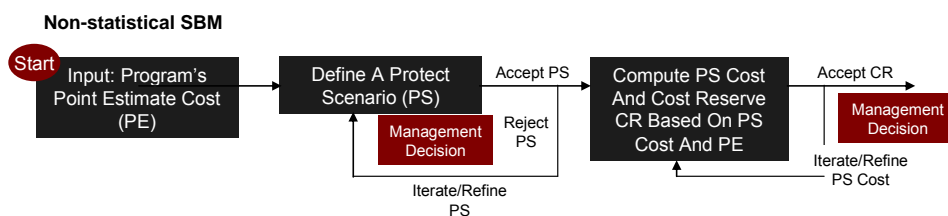


Figure 7-3 A Nonstatistical Scenario-Based Method

Once the protect scenario has been defined and agreed upon, its cost is then determined (a second point estimate also referred to as PS). The next step is to compute the amount of cost reserve dollars (CR) needed to protect the program's cost against the identified risk. This step of the process defines cost reserve as the difference between the PS and PE.

The process of defining scenarios is a valuable exercise in identifying technical and cost estimation risks inherent to the program. Scenario definition encourages a discussion on program risks that otherwise might not be held.

For details on how to apply statistics to estimate the probability levels of SBM scenarios, please see **Appendix B.3**.

8.0 A DISCUSSION ON HOW TO SELECT A FUNDING LEVEL

This handbook does not dictate a confidence level to which Air Force programs should be funded. At the time of this writing there is no official policy guidance on the topic. In the absence of such guidance it is not uncommon for programs to seek to budget at the 50% confidence level. However, an argument for funding to the 60% confidence level is provided.

Consider four types of programs, each having varying top-line uncertainty and call them Low, Medium, High, and Very High Dispersion programs. For the purposes of this discussion assume the top-line distribution of each is lognormal. And further assume the Low Dispersion program has a standard error in log space of 0.15; the Medium Dispersion program has a standard error in log space of 0.25; the High Dispersion program is 0.35; and the Very High Dispersion program is 0.45.

Table 8-1 presents various statistics for the four program types. As cost uncertainty distributions, these would represent the most likely estimate with the mean estimate to its right. The rightmost column shows the 60% confidence level value as a percent of the median value. For example the Medium Dispersion curve shows that multiplying the most likely estimate by 1.065% would provide a 60% confidence level estimate.

Table 8-1 Program Probabilities

	SE in Log Space	Unit Space Stdev	Mean	Median (50/50)	Probability of Mean	Mean/ 50/50	60% Value/ 50/50
Low Dispersion	0.150	0.153	1.011	1.000	53%	1.011	1.039
Med Dispersion	0.250	0.262	1.032	1.000	55%	1.032	1.065
High Dispersion	0.350	0.384	1.063	1.000	57%	1.063	1.093
Very High Dispersion	0.450	0.524	1.107	1.000	59%	1.107	1.121

Now consider a portfolio of comparably sized programs. **Table 8-2** presents analysis of portfolios sized with five, ten, or twenty programs with High Dispersion. The table shows

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assumptions of programs funded at probabilities of 50%, 60%, 70%, and 80%. The third column shows the overall portfolio confidence level of each case with the programs uncorrelated. The fourth column shows the same but with the programs correlated at 25%. Note the results in each case where the constituent programs were funded at 60%. The portfolio probability is near 60% as well. And note that if the portfolio is comprised of ten or more programs the expected result approximately doubles that of a portfolio of programs funded to 50%.

Table 8-2 Portfolio Probabilities

# Projects	Project Probability	Portfolio Probability	
		No Correlation	0.25 Correlation
5	50%	38%	40%
5	60%	61%	59%
5	70%	80%	78%
5	80%	94%	92%
10	50%	32%	36%
10	60%	62%	61%
10	70%	87%	83%
10	80%	98%	96%
20	50%	24%	32%
20	60%	65%	61%
20	70%	94%	86%
20	80%	99%	98%

For uncertainty distributions that approximate lognormal, the median is always lower than the mean and for that reason funding programs at 50% results in weak portfolio probabilities. Funding programs at 60% (generally near or above the mean) brings portfolio expectations to above 50%. Funding programs to 70% or above certainly raises the probability of portfolio success, but naturally requires a higher level of funding. Note from **Table 8-1** that a high dispersion program can be raised from 50% to 60% confidence with only 9% additional funding. 9% additional funding can increase the confidence level of each program 10% but can more than double the confidence level of a portfolio of programs. Another study, **Reference 30**, conducted a similar investigation and the conclusions are very similar.

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APPENDIX A DEFINITIONS

This appendix presents definitions for technical terms used throughout this handbook. Many publications contain definitions of each of these terms, often in conflict with one another. Therefore, this appendix elaborates on many of these terms mathematically and graphically to clarify their use in this handbook.

A.1 UNCERTAINTY

A.1.1 Overview

Uncertainty is defined as a potential deficiency in any phase or activity of the cost estimating process that is due to the lack of knowledge or due to random variations in the cost estimating process. The analyst's challenge is to adequately capture and model the complete uncertainty associated with the cost estimate.

A.1.2 Objective Uncertainty

Objective uncertainty associated with cost model parameters is a measure of source data variability. If the cost estimating relationship (CER) (defined further in **A.3.1**) or the input(s) to the CER are derived from statistical analysis of relevant historical data, the uncertainty associated with the cost estimate can often be characterized "objectively," meaning derived using a repeatable, proven process. The basis for the uncertainty calculation is a function of how the estimate is derived. In the sections that follow, the most common methods for deriving CERs are introduced with their associated "objective" uncertainty.

A.1.3 Distribution Boundaries for Objective Distributions

If statistical methods have been used to generate the CER, the analyst should have the necessary information to replicate the uncertainty in the risk model. Ideally, the statistics will provide the bounds for a specific confidence level. More often than not, however, the analyst will be provided with other data such as the standard deviation for a specific position within the data set. As illustrated in **Figure A-1** the size of the uncertainty distribution will increase (standard deviation gets larger) as the point estimate moves towards (and beyond) the data boundaries. The minimum information required is the CER result and the standard deviation for that result. Analysts should be aware, however, that if log-linear CERs are employed, care must be taken to adjust the CER result to reflect the mean in unit space if the log-space standard deviation is provided.

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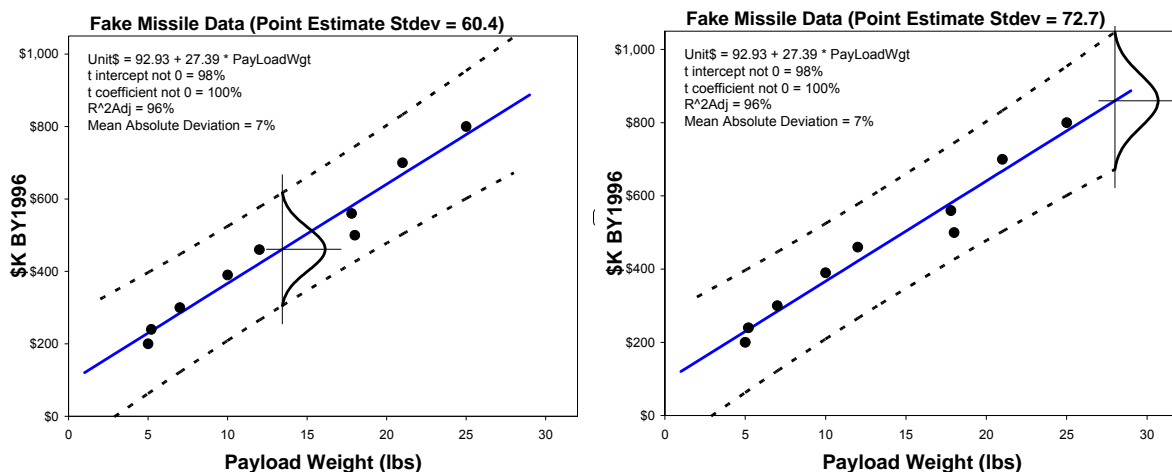


Figure A-1 Objective bounds a function of distance from data center

A.1.4 Subjective Uncertainty

In the context of cost estimating, many decisions that heavily influence the cost risk analysis will be subjective in nature, meaning they are based more on “expert opinion” than rigorous statistical analysis. Uncertainty is characterized as “subjective” when there is a lack of information to characterize it objectively. Subjective uncertainties have long been criticized for their lack of rigor and have relatively poor standing in fields driven by empirical study, where matters of precision and repeatability are considered paramount. Nevertheless, subjective uncertainty plays a large role in cost estimates due to limitations in time, resources and relevant data.

A.1.5 Cost Estimating Uncertainty

Cost estimating uncertainty is the uncertainty associated with the estimating method that is employed, that is the CER. This deals with the fact that our estimating methods, data, and tools are neither totally precise nor totally accurate, although frequently precision is used as a substitute for accuracy. If the CER is merely the sum of labor and material costs, the analyst must assess the cost uncertainty to capture rate, technical, configuration, and schedule uncertainty. When the CER is parametric or a single point analogy, there is technical, configuration, and schedule uncertainty embedded in the underlying data used to create the method. For parametric CERs, the uncertainty is also a function of where the point estimate will fall in the data range (proximity to the center of mass results in less uncertainty). Several other subjective factors may influence how the analyst should adjust (subjectively) the objective uncertainty associated with the CER, such as: deficiencies in the quality of the data due to variations in contractors’ accounting practices; assumptions made to normalize the data; and other cost estimating influences not captured by the CER.

A.1.6 Configuration Uncertainty

Configuration uncertainty is the variation in the fundamental technical cost drivers of a WBS element. Configuration uncertainty is a form of technical uncertainty. From the cost estimator’s perspective, configuration uncertainty falls into two categories: uncertainty in input parameters that are captured by the cost model and uncertainty in parameters or basic configuration features

that cannot be addressed without modifying the structure of the cost model. An example of the first type of uncertainty event would be design changes that resulted in reducing the weight of an antenna by 12 lb. An example of the second kind of uncertainty event would be a requirements change that required a switch from a mechanically-steered antenna to a phased array antenna.

A.1.7 Technical Uncertainty

Ideally, to generate a parametric CER, the analyst will select and use relevant historical data that were successful in meeting similar technical challenges to the program being estimated. By doing so, the technical uncertainty is at least somewhat captured by the analysis of that data. However, if the program to be estimated is facing unusual technical challenges, then an adjustment to the CER uncertainty distribution is required.

A.1.8 Schedule Uncertainty

Schedule uncertainty is the variation in the possible key dates associated with a WBS item. The elemental units of a program's schedule are called activities. A WBS item is normally comprised of several schedule activities. The duration of the individual schedule activities is driven by three factors: the technical difficulty of the work to be performed, the qualifications of the people performing the work, and the availability of an adequate number of people/resources to do the work. In other words, schedule uncertainty is driven by technical uncertainty. Schedule activities influence each other through precedence relationships (e.g., Activity C cannot begin until Activities A and B are complete). Schedule precedence relationships often cut across WBS items. Because of these interrelationships, schedule slips in one WBS item can impact the duration of activities in other WBS items, and can actually increase the labor hours in the impacted items. Programs that have a high degree of technical interrelationship between activities as well as a high degree of concurrence have inherently high schedule uncertainty. In other words, the topology of the program schedule has a strong impact on schedule uncertainty.

A.2 RISK

Risk is the possibility of incurring loss or misfortune. In the context of cost estimating, risk is the possibility the program will not be able to complete on budget. Too often analysts will use the terms risk and uncertainty interchangeably. In this handbook, "risk" is acknowledged as a metric that is the result of an acceptable "uncertainty" analysis. If the complete uncertainty of the estimate is known, the analyst can advise management what the risk will be to not complete a program within a specified budget.

A.3 REGRESSION METHODS

A.3.1 Overview

Regression analysis is a statistical technique used to find relationships between variables for the purpose of predicting future values. In cost estimating, regression analysis is used to develop cost estimating relationships (CERs) between a dependent variable (cost) and one or more independent variables (cost drivers such as weight, power, volume, etc) from completed project data. By a statistical relationship it is meant that the observed variation of the dependent variable (cost) across similar projects can be explained or predicted by one or more independent variables (technical, performance, programmatic, etc). The objective is to find the functional relationship that most accurately estimates the cost of a particular element in a project work breakdown structure. Additionally, the regression method should yield an objective assessment of the

accuracy of the CER. This assessment provides an objective basis for characterizing the uncertainty of the CER itself.

There are various techniques available to perform regression analysis. In order to correctly assign the appropriate uncertainty distribution to a given CER, the analyst needs to know how the CER was generated. The objective uncertainty distribution is a function of the method employed.

Several of the most popular methods are described in the following sections along with guidance on how to capture their uncertainty in a simulation based cost risk model.

A.3.2 Ordinary Least Squares (OLS)

A.3.2.1 Description:

Ordinary least squares is one of the most popular methods to employ because it is easy, the theory is well known, the CERs tend to be easy to understand and it yields very meaningful, descriptive statistics to characterize its significance and accuracy. It is a mathematical optimization technique used to find a "best linear fit" to a set of data. The object is to minimize the sum of the squared errors (SSE), the sum of the squared difference between the fitted line (i.e. CER) and the source data. The goal of the OLS method is to find the linear equation such that all the sum of the square of vertical deviations are collectively as small as possible. Expressed mathematically, the best fitting line is derived by solving for the coefficients (i.e., β_0 , β_1 , ..., β_k) in the following equation such that SSE is minimized:

Equation 2

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \varepsilon \quad \text{where } \varepsilon \text{ is a random error term}$$

$$\text{Minimize } SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

where: n = the total number of data points in the sample

Y_i = the i^{th} observed value of the dependent variable (i.e., cost)

\hat{Y}_i = the i^{th} estimated value

In the context of cost estimating, Y is generally cost, man-hours or some other effort or resource. X is generally some technical or performance characteristic or metric that helps to explain the variation in cost across a number of projects. More than one X parameter might be found to be statistically significant in explaining cost variation.

A.3.2.2 Unit Space OLS Interpretation and Error

The result of a linear OLS derived CER is interpreted to be the mean of a normal distribution (that is, the error is assumed to be normally distributed about the CER line). Since a normal distribution is symmetrical, the CER result can also be characterized as the median or the mode.

In OLS, the cost (Y) variation is assumed to be independent of the magnitude of the cost. In other words, the error is assumed to be a fixed, additive value. It is given by

Equation 3

$$\varepsilon_i = Y_i - f(\mathbf{x}_i, \boldsymbol{\beta})$$

This may be mathematically correct, but it is not reasonable in cost estimating and is a big reason why many analysts will not use the OLS method. For instance, if the CER predicts cost as a function of weight and the valid cost range is \$500 to \$1500, a fixed \$100 (a result consistent with OLS) average error over that range is not appropriate. It may be reasonable at the mid range of the cost, but not at the low or high end. In cost estimating, it may generally be assumed that the error is proportional rather than fixed. In this case, an average error of 10% (rather than \$100) would be used by most analysts to model the error of the OLS CER. A common rule of thumb is that the errors are believed to be proportional to the magnitude of the result (the dependent variable) if it ranges over more than one order of magnitude. This technique can be avoided if the CER is generated using MUPE or ZMPE rather than OLS because those methods can be used to derive a linear CER with a proportional error term directly. However, for small datasets where OLS is the only method available, the uncertainty of a linear OLS derived CER may be modeled as a normal distribution where the standard deviation (or bounds) is modeled as a percent of the CER result (which is always the mean of the distribution).

A.3.2.3 Log Space OLS Interpretation and Error

CERs of the form $\text{Cost} = a \cdot \text{Var1}^b \cdot \text{Var2}^c \dots \cdot \varepsilon$ can be transformed into linear forms in log space. In unit space, the CER result is closer to the median (not the mean) of a lognormal distribution. A key result of this approach is that the error term is multiplicative, that is it is proportional to result of the CER. A log-error model is where the error term is believed to follow a lognormal distribution. This is a very common and intuitive assumption because the error in cost is usually skewed upward and bounded below by zero. Log-linear models are in a very common and distinct class of non-linear relationships that are rendered linear when transformed to log-space.

If the error term (ε_i) is assumed to follow a normal distribution with a mean of 0 and variance σ^2 in log space (i.e., $\varepsilon_i \sim \text{LN}(0, \sigma^2)$), then the error can be measured by the following:

Equation 4

$$\ln(\varepsilon_i) = \ln(Y_i) - \ln(f(\mathbf{x}_i, \boldsymbol{\beta}))$$

where “ln” stands for natural logarithmic function. The objective is then to minimize the sum of squared e_i s (i.e., $(\sum \ln(\varepsilon_i))^2$) in log space. In this case, the errors are assumed to be normally distributed around the CER line (a straight line) in log space. When transformed back to unit space, the mathematics show that:

- The unit space CER error term follows a lognormal distribution
- The CER result, while being the mean of a normal distribution in log space, is closer to the median in unit space. If adjusting the CER result to be closer to the mean of the lognormal distribution is desired, please see the section A.3.2.4.

In summary, if the transformed function is linear in log space, then OLS can be applied in log space to derive a solution for $\boldsymbol{\beta}$. If this function cannot be linearized in log space, then apply a non-linear regression technique to derive a solution. See the pros and cons of log-error models in **Reference 42**.

A.3.2.4 Adjusting a Log Space OLS CER to reflect the Mean

Although a least squares optimization in log space produces an unbiased estimator in log space, the estimator is no longer unbiased when transformed back to unit space (see **Reference 1 and 8**). The unit space CER by direct translation tends to underestimate the mean value of the original population. Two distinct methods for adjusting the OLS log-linear CER result to reflect the mean are presented. As stated in **B.2.2.3.3**, neither of these adjustments is necessary to model the lognormal uncertainty.

The most accurate approach is to make use of a correction factor based upon statistics of the CER. A theoretical correction factor was first introduced by Neyman and Scott in 1960 and again by Goldberger in 1968 to adjust the CER result to reflect the mean in unit space for the log-linear CERs (see **Reference 1 and 4**). Tecolote Research, Inc. later developed a simple multiplicative factor commonly known as the PING Factor (PF) to approximate the magnitude of the bias. The simplified form:

Equation 5

$$PF = \exp\left(\left(1 - \frac{p}{n}\right) \frac{s^2}{2}\right)$$

Where:

p = the total number of coefficients to be estimated

s = standard error of estimate in log space

n = the sample size

This simplified PING Factor can be applied to log-linear CERs to correct the downward bias in unit space. For a more accurate form of the PING Factor and its detailed derivations, see **References 8 and 13**.

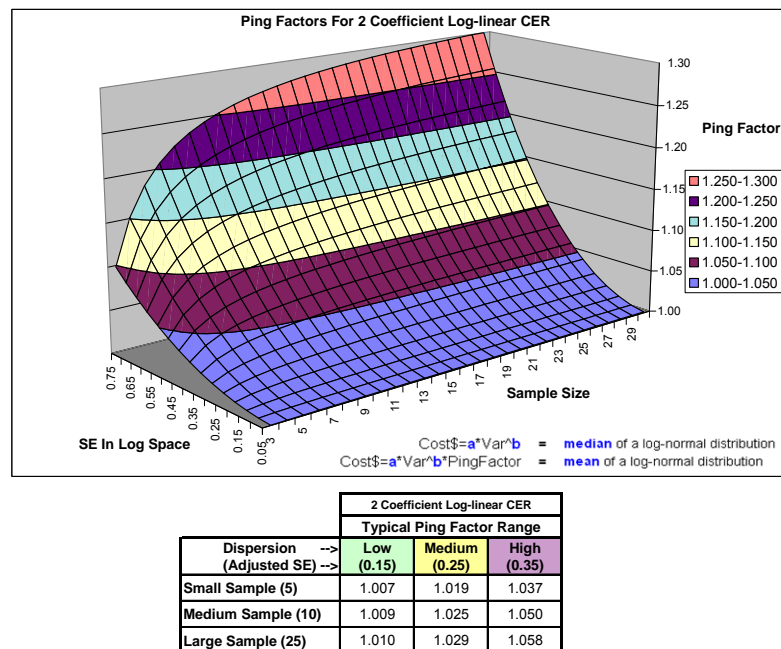


Figure A-2 Ping Factors for 2 Coefficient Non Linear OLS CERs

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For example, consider the following CER to estimate propulsion costs:

$$Y(\$K) = 1.618 * \text{MotorWt}^{0.6848}$$

If this CER was generated from 10 sample points using log-linear regression, then the distribution describing the CER uncertainty is lognormal with a log space SE of 0.25. If the motor weight is 200 lbs, the CER yields \$60.91 K. This is the median of the lognormal distribution. If the mean is desired, then the CER result must be multiplied by the appropriate PING factor, in this case 1.025. The mean of the lognormal is therefore \$62.43 K.

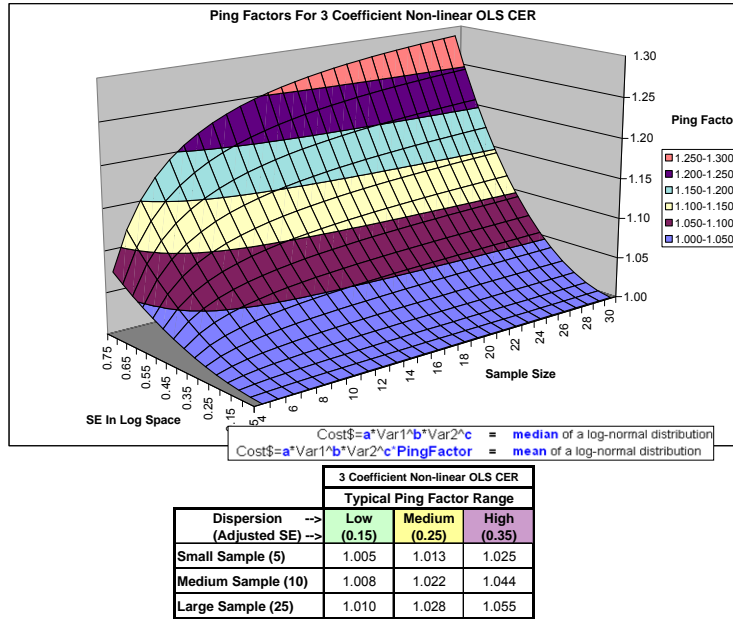


Figure A-3 Ping Factors for 3 Coefficient Non Linear OLS CERs

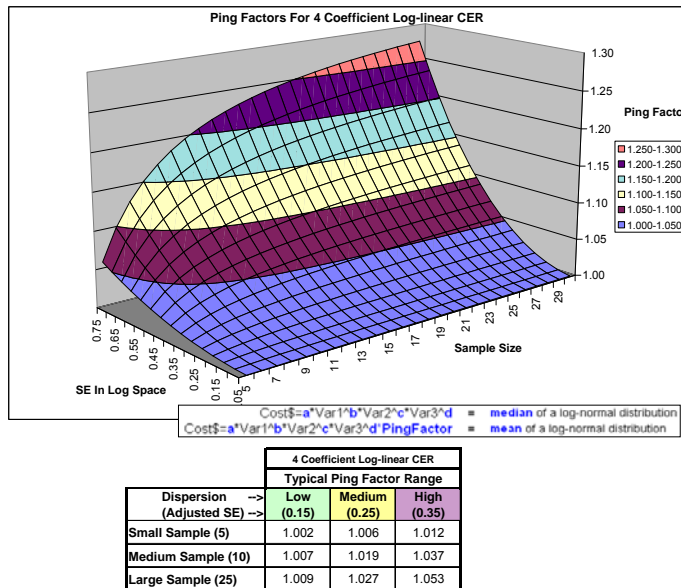


Figure A-4 PING Factors for 4 Coefficient Non Linear OLS CERs

As indicated by **Equation 5**, there are two terms involved in the PING Factor: the first one is for adjusting the downward bias between the mean and the median (a transformation bias); the other is used to adjust the upward bias for estimating the median (a sampling bias). It can be concluded that:

- At a given sample size, PF increases with se.
- At a given standard error, PF increases with the sample size.
- PF is always greater than 1 for $p > n$ and $se > 0$.

A.3.3 Other Multiplicative Error Term Regression Techniques

Multiplicative error terms are preferred in the cost analysis field because experience tells us that the error of an individual observation (e.g., cost) is generally proportional to the magnitude of the observation (not a constant). The general specification for a CER with a multiplicative error is stated as

Equation 6

$$Y_i = f(\mathbf{x}_i, \boldsymbol{\beta}) \varepsilon_i \quad \text{for } i = 1, \dots, n$$

where:

n = sample size

Y_i = observed cost of the i^{th} data point, $i = 1$ to n

$f(\mathbf{x}_i, \boldsymbol{\beta})$ = the value of the hypothesized equation at the i^{th} data point

$\boldsymbol{\beta}$ = vector of coefficients to be estimated by the regression equation

\mathbf{x}_i = vector of cost driver variables at the i^{th} data point

ε_i = error term with mean of 1 and variance σ^2

Minimization algorithms can be explored for modeling CERs with multiplicative errors based upon a generalized error term definition:

Equation 7

$$e_i = \frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta})}{f(\mathbf{x}_i, \boldsymbol{\beta})}$$

where e_i is normally distributed with a mean of 0 and variance σ^2 .

This error term expresses the error of estimation as a percentage of the estimate. The percentage error represents the percent error of the residual about the regression function and the optimization objective is to find the coefficient vector $\boldsymbol{\beta}$ that minimizes the sum of squared e_i s.

A.3.3.1 Minimum Unbiased Percentage Error (MUPE) Method

A refinement to MPE was proposed by Tecolote Research (**Reference 11 and 14**) to solve for the function in the numerator separately from the function in the denominator through an iterative technique.

Equation 8

$$\text{Minimize } \sum_{i=1}^n \left(\frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta}_k)}{f(\mathbf{x}_i, \boldsymbol{\beta}_{k-1})} \right)^2 = \sum_{i=1}^n \left(\frac{y_i - f_k(\mathbf{x}_i)}{f_{k-1}(\mathbf{x}_i)} \right)^2$$

where k is the iteration number and the other terms are as defined previously.

This optimization technique is called the Minimum-Unbiased-Percentage Error (MUPE) method; it is also referred to as Iteratively Reweighted Least Squares (IRLS) (**Reference 3, 5**). As shown in the equation above, the weighting factor of each residual in the current iteration is equal to the reciprocal of the predicted value from the previous iteration. The final solution is derived when the change in the estimated coefficients ($\boldsymbol{\beta}$ vector) between the current iteration and the last iteration is within the analyst-specified tolerance limit. No transformation or adjustment (to correct the bias in unit space) is needed to fit a MUPE CER. Goodness-of-fit measures (or asymptotic goodness-of-fit measures) can be applied to judge the quality of the model under the “normality” assumption (i.e., $\varepsilon_i \sim N(1, \sigma^2)$). The MUPE CER has no “positive” sample bias; it has zero proportional error for all points in the database. It is an unbiased estimator of the model mean if the function is linear. Also, it produces consistent estimates of the parameters. For a more detailed discussion of the MUPE technique, see **Reference 13 and 14**.

A.3.3.2 MUPE Interpretation and Error

The result of a MUPE derived CER is interpreted to be the mean of a normal distribution (that is, the error is assumed to be normally distributed about the CER line) regardless of the functional form of the CER. Since MUPE CERs estimates the mean in unit space, the roll-up elements in an all-MUPE cost estimate will be the expected value of the sum of the elements below. This is not true of CERs developed using uncorrected (see **A.3.2.4**) OLS in log space (and may or may not be true for other regression types).

Note that the standard error of the estimate (also commonly termed multiplicative error) can be used as an estimate of the standard deviation (σ) of the error term. For instance, if the SEE is 20% for a MUPE CER, it can be interpreted that the CER has plus/minus 20% estimating error (for one standard deviation) at the center of the database.

A more rigorous estimate of the MUPE prediction interval is possible for linear MUPE CERs (**Reference 48**). The general specification for a MUPE CER is stated as

$$Y_i = f(\mathbf{X}_i, \boldsymbol{\beta}) \varepsilon_i \quad \text{for } i = 1, \dots, n \quad (1)$$

where:

n = sample size

Y_i = observed cost of the i^{th} data point, $i = 1$ to n

$f(\mathbf{X}_i, \boldsymbol{\beta})$ = the value of the hypothesized equation at the i^{th} data point

$\boldsymbol{\beta}$ = vector of coefficients to be estimated by the regression equation

\mathbf{X}_i = vector of cost driver variables at the i^{th} data point

ε_i = error term with mean of 1 and variance σ^2

Statistical inferences can be made for the regression equation if the normality assumption is further applied to the error term (ε).

If the hypothesized equation is a simple linear function:

$$f(\mathbf{X}_i, \boldsymbol{\beta}) = \alpha + \beta X_i \quad \text{for } i = 1, \dots, n$$

The $(1-\alpha)\%$ prediction interval (PI) for a future observation Y when X is at x_0 (i.e., \hat{y}_0) is then given by

Equation 9

$$\begin{aligned} & \hat{y}(\text{when } X = x_0) \pm t_{(\alpha/2, df)} * Se * \left(\hat{y}_0^2 + \frac{1}{\sum w_i} + \frac{(x_0 - \bar{x}_w)^2}{SS_{wxx}} \right)^{0.5} \\ & = \hat{y}_0 \pm t_{(\alpha/2, df)} * Se * \left(\hat{y}_0^2 + \frac{1}{\sum w_i} + \frac{(x_0 - \bar{x}_w)^2}{(\sum w_i) S_{wx}^2} \right)^{0.5} \end{aligned}$$

where:

w_i is the weighting factor for the i^{th} data point and $w_i = 1/(f_i * f_i)$

f_i denotes the predicted value of the i^{th} data point

Se is CER's standard error of estimate

$$\bar{x}_w = \frac{\sum_{i=1}^n w_i (x_i)}{\sum_{i=1}^n w_i}$$

$$SS_{wxx} = \sum_{i=1}^n w_i (x_i - \bar{x}_w)^2$$

$$S_{wx} = \sqrt{\frac{\sum_{i=1}^n w_i (x_i - \bar{x}_w)^2}{\sum_{i=1}^n w_i}}$$

df is the degrees of freedom

$t_{(\alpha/2, df)}$ is the upper $\alpha/2$ cut-off point for a t-distribution with "df" degrees of freedom

S_{wx} is the weighted sample standard deviation of the independent variable x. It is the sample standard deviation of the x variable evaluated in the fit space.

A.3.3.3 ZPB/MPE Method (or ZMPE Method)

There is another alternative method (see **Reference 17**) to reduce the positive bias for MPE CERs and yet maintain the same objective function. Mathematically, it is stated as follows:

Equation 10

$$\text{Minimize } \sum_{i=1}^n \left(\frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta})}{f(\mathbf{x}_i, \boldsymbol{\beta})} \right)^2 = \sum_{i=1}^n e_i^2$$

$$\text{Subject to } \sum_{i=1}^n e_i = 0$$

This alternative method is called the “Constrained Minimum Percent Error” solution. It is also referred to as the MPE method under the Zero-Percentage Bias constraint, i.e., the ZPB/MPE or ZMPE method by Book and Lao, 1997 (see **Reference 17**).

A.3.3.4 ZPB/MPE (or ZMPE Method) Interpretation and Error

ZMPE was explored by SMC in 1993 as a possible alternative technique in the development of spacecraft CERs. However, this method was not selected, as the technique does not yield the traditional statistical properties suitable for characterizing the meaning of the CER result and its error that are customary to with OLS and LOLS regressions. However, ZMPE has been selected as a technique of choice by the NRO Cost Group and many other organizations.

A.3.3.5 Iterative Regression Techniques

The Gauss Newton method has been observed to have convergence problems on many datasets. For this reason there has been considerable interest in other methods. There are several different non-linear optimization techniques that might be used to fit non-linear functional forms to data. Well known techniques include Quasi-Newton, Conjugate Gradient, Downhill Simplex, and Marquart’s methods. Non-linear regression techniques do not typically result in the normal types of statistics expected when performing OLS or MUPE. For example, rather than reporting traditional t-statistics, a non-linear regression will report approximate prediction ranges for each coefficient.

A.3.3.6 Iterative Regression Interpretation and Error

There have been various attempts to generate suitable statistics from iterative regression analysis. The Bootstrap method (see **Reference 34**) is one approach. However, the reference did not provide an error assessment of the Bootstrap prediction intervals. The only claim is that the Bootstrap PIs were “close” to the PIs generated by the OLS method for a simple linear CER with 10 data points.

A.3.3.7 Error Term Summary

Regardless what method is used to generate the CER, it is very important that the user of the CER is aware of the CER result meaning and how the error should be modeled. Generally, lognormal distributions should be used as a default approach. If normal distributions for cost or effort are selected, they should be supported by appropriate supporting evidence.

For relatively small data sets, all CER best fit methods tend to underestimate the level of underlying dispersion (as measured by the standard error) in the population as a whole. The result is a regression equation that is likely to be substantially different from the underlying relationships between the cost and its driver variables and (on the average) an underestimate of dispersion. Although there is a statistical bias (underestimating the underlying dispersion), in any given case, the CER dispersion can also be greater than the underlying dispersion.

The best way to assess the likelihood of an abnormally low (or high) SE is by comparing many CERs for similar products. Specifically, if the CERs have different size data sets (and degrees of freedom), the SE value for each CER should be plotted against its DOF. CERs with low DOF that are far away from the average should be used carefully. If they represent costly products and hence play a significant role in determining the overall system cost variance, then remedial action may be appropriate in the form of subjective, expert opinion increases (or decreases) to the CER SE.

A.4 PROBABILITY

A.4.1 Overview

Probability is the relative frequency of an outcome of a repeatable, observable experiment. Probability is measured on a scale between 0 and 1. Probability is assigned to each outcome of an experiment based on its relative frequency where 1 represents always and 0 represents never.

A.4.2 Probability Distribution

A probability distribution is a mathematical formula that describes how the relative frequency of occurrence is assigned to the real numbers in the range of a random variable. The distribution may be described by either a density function $p(x)$ or a cumulative probability function $F(x)$. These functions are two different presentations of the same data. In Figure A-5, the dark, curved line represents the statistical distribution underlying the sample data shown in the table at the left. This type of curve is also called a Probability Density Function (PDF).

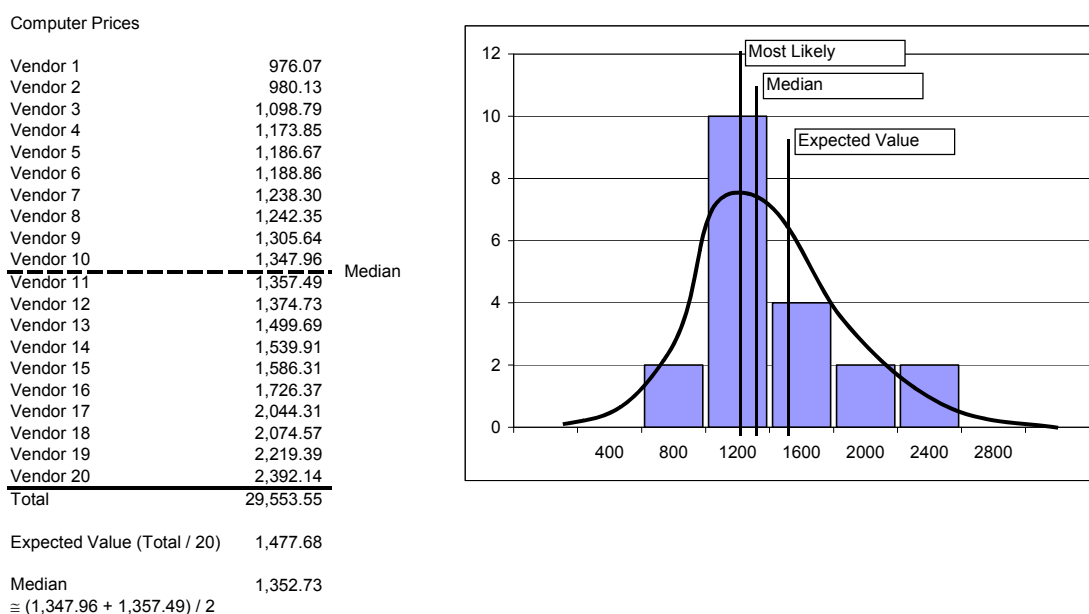


Figure A-5 Distribution Example.

A.4.3 Probability Density Function (PDF)

A continuous PDF is the "smoothed out" version of a histogram. The area under any PDF is equal to 1. A PDF identifies the probabilities associated with specific values or intervals of values of the random variable (see Probability Distribution).” If there is a finite probability associated with a specific value x , then the PDF will have a “spike” at that value of x .

A.4.4 Cumulative Distribution Function (CDF)

The CDF is a mathematical curve that for any given possible value of an item, identifies the probability that the actual value will be less than or equal to the given value. When shown graphically, the CDF is an S-shaped curve. The term S-curve is used synonymously with CDF. In mathematical terms, the definition of the cumulative distribution function $F(x)$ of a random variable X is given by the following:

Equation 11

$F(x)$ = probability of obtaining a value less than or equal to x

$$= P(X \leq x) = \int_{-\infty}^x f(x) dx$$

The value of a cumulative distribution function is bounded between 0 and 1, with 0.5 indicating the median of the population (see the graph below).

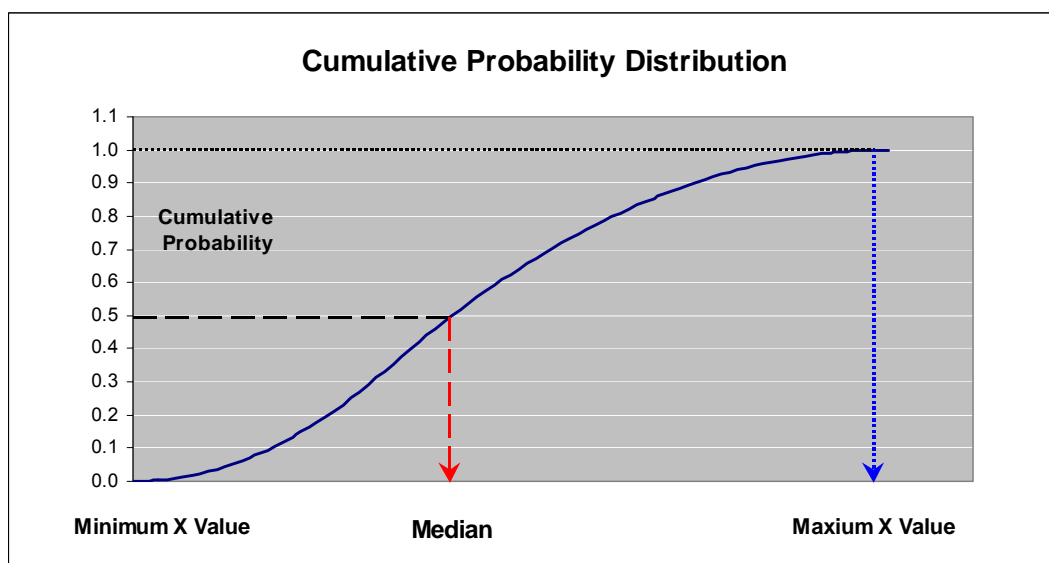


Figure A-6 Cumulative Probability Distribution

A.5 MEASURES OF CENTRAL TENDENCY

A.5.1 Expected Value, Average or Mean

The expected value is the arithmetic average or mean of the distribution of possible values for a variable. For a given set of n values (y_1, y_2, \dots, y_n), the mean (\bar{y}) is defined to be the arithmetic average of these n values. In mathematical notations, it is given by

Equation 12

$$\bar{y} = \frac{\sum_i y_i}{n}$$

The arithmetic mean is a composite measure and has the following characteristics:

- The most widely known and used average.
- It is an artificial value, since it may not coincide with any actual value.
- It is affected by the value of every item, but may be unduly affected by extreme values especially in small data sets.

Figure A-5 shows how the expected value is simply the sum of all values divided by the number of values. Expected values have an important mathematical property: the sum of the expected values of a set of variables is equal to the expected value of the sum of the set of variables. In other words, when summing the expected values of a number of WBS items, the result will be the expected value of the sum of the WBS items. This is not true for percentiles or most likely values.

A.5.2 Median

The median is the point in a distribution where half the observed values will be lower and half will be higher (the 50th percentile). In other words, this is the point where the actual cost is just as likely to be higher as it is to be lower. For a finite number of observations, if the sample size is odd, the median is the middle value. If the sample size is even, the median is the average of the middle two values. The sum of the medians of a number of WBS items is not equal to the median of the sum of the values, except in the unusual cases in which the distributions of all the WBS items are symmetrical.

A.5.3 Most Likely Value (Mode)

The mode is the most probable single value for a variable (the peak of the distribution, see **Figure A-5**). The output of the primary estimating methodology (i.e. the point estimate) for a WBS item is typically interpreted as the most likely value. The sum of the most likely values of a number of WBS items is not equal to the most likely value of the sum of the values, except in the unusual case in which the distributions of all the WBS items are symmetric¹⁸.

¹⁸ MYTH: Percentiles add

GUIDANCE: They don't. Only means add. Consider the following illustration.

Imagine one die...

- How many possible outcomes can you get from one roll? **6**
- How many ways can you roll a six? **1**
- How many ways can you roll a number less than or equal to five? **6 – 1 = 5**
- What is the likelihood that you will roll a number less than or equal to five? **5 / 6 = 83%**
- In other words, our 83%-tile confidence level outcome is five or less
- If we add two 83%-tile outcomes, do we have an 83%-tile confidence level sum? **NO...**
- Imagine two dice...
 - How many possible outcomes can you get from one roll? **6 X 6 = 36**
 - How many ways can you roll a number greater than ten (i.e. five plus five)? **3 (specifically 5-6, 6-5, or 6-6)**
 - How many ways can you roll a number less than or equal to ten? **36 – 3 = 33**

A.6 MEASURES OF SHAPE

Measures of shape provide information about the symmetry and peakedness of the data set.

A.6.1 Skewness

A distribution is said to be skewed if one of its two tails is longer than the other. For example, if there is a long tail to the right of the distribution, then it is said to be positively skewed (or skewed right). This means that the distribution has a long tail in the positive direction. Similarly, if there is a long tail to the left, then the distribution is said to be negatively skewed (or skewed left). If the distribution is symmetrical, then the distribution has no skew. For example, the normal distribution has a skewness value of 0 since it is a symmetric distribution.

For a random variable Y, the measure of skewness is a parameter that describes asymmetry in the probability distribution of Y. It is defined by

Equation 13

$$\text{Skewness}(Y) = E(Y - \mu)^3 / (\sigma)^3$$

As for a sample data set of n values (y_1, y_2, \dots, y_n), the formula to compute the skewness factor is given below:

$$\text{Skewness} = \frac{n}{(n-1)(n-2)} \sum_{i=1}^n (y_i - \bar{y})^3 / (\text{Stdev})^3$$

As a general rule, the mean is larger than the median in positively skewed distributions and less than the median in negatively skewed distributions. Although counter examples can be found, they are rare in real data.

A.7 MEASURES OF DISPERSION

A.7.1 Variance

To calculate the variance, first calculate the arithmetic mean and then for each data point, find the difference between the point and the mean. Next, square all these differences and sum them. This sum is then divided by the number of items in the data set (if the data is from a sample, the sum is divided by the number of items minus one). The variance is a measure of the average squared distance of each value from the mean, but it is not expressed in the units of measure of the mean or the original data. The measure of variance is greatly affected by extreme values.

A.7.2 Standard Deviation

The standard deviation is one of the most widely used statistics for measuring the spread, or dispersion, of values in a population of data. For a given set of n values (y_1, y_2, \dots, y_n), the standard deviation (Stdev or S) is defined by

- What is the likelihood that you will roll a number less than or equal to ten? **33 / 36 = 92%**

In other words, the sum of two 83%-tile outcomes (two rolls of 5 or less add to 10 or less) is a 92%-tile outcome, not an 83%-tile outcome.

Equation 14

$$S = \begin{cases} \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}} & \text{if data is from a sample} \quad - \text{Std. Dev. (Sample)} \\ \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}} & \text{if data is from a population} \quad - \text{RMS (Population)} \end{cases}$$

In essence, the standard deviation measures the average distance of each value from the mean. It is also greatly affected by extreme values. The standard deviation is the square root of the variance. However, unlike the variance, the standard deviation is in the same unit of measure as the data, and this is a primary reason for its popularity.

A.7.3 Inter-quartile Range

The inter-quartile range is the length of the interval that contains the middle fifty percent of the values in an ordered data set. The ordered data is broken into four roughly equal groups. The first quartile separates the lowest valued quarter from the second quarter. The second quartile (the median) separates the second quarter from the third quarter. The third quartile separates the third quarter from the last quarter. The interquartile range consists of the difference between the first quartile and the third quartile, which covers the middle fifty percent of the values in the data set

A.7.4 Coefficient of Variation

The *coefficient of variation* (CV) of a distribution is defined as the ratio of the standard deviation to its mean (i.e., Stdev/Mean). It is a relative measure of dispersion because it expresses the standard deviation as a percentage of the mean. CV is fast becoming one of the more recognized metrics to characterize the spread in a CDF (S-Curve).

A.8 MEASURES OF ESTIMATE ACCURACY**A.8.1 Standard Error of the Estimate****A.8.1.1 Additive Error SEE**

For CERs with additive error terms, the standard deviation of the dependent variable (Y) is assumed to be the same across the entire range of the data, regardless of the value of the independent variables. The standard error of estimate (SEE) in a regression analysis is an estimate of the standard deviation about the regression line. It provides a measure of “average” distance of the sample data from the regression equation. Mathematically, it is equal to the square root of the mean squared error (MSE):

Equation 15

$$SEE = \left(\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 / (n - p) \right)^{0.5} = \sqrt{SSE / dfe} = \sqrt{MSE}$$

Where dfe is the number of degrees of freedom of the sum of squares error and p is the total number of estimated parameters (coefficients in the equation). The SEE measure is typically stated in absolute terms (i.e., if Y is in dollars then the SEE is in dollars as well).

A.8.1.2 Log-Error SEE

If the regression model is fit in log space, the SEE measure is given by

Equation 16

$$SEE = \left(\sum_{i=1}^n [\ln(Y_i) - \ln(\hat{Y}_i)]^2 / (n - p) \right)^{0.5} = \sqrt{SSE / dfe} = \sqrt{MSE}$$

The Log-Error SEE is not stated with the units (e.g. dollars) of Y .

A.8.1.3 MUPE SEE

As for MUPE equations, the SEE measure is typically stated in percentage terms and is provided below.

Equation 17

$$SEE = \left(\frac{1}{n - p} \sum_{i=1}^n \left\{ (Y_i - \hat{Y}_i) / \hat{Y}_i \right\}^2 \right)^{0.5} = \sqrt{SSE / dfe} = \sqrt{MSE}$$

Note: Since SEE measures the average amount of variation between the sample data and the regression equation, the smaller the value of SEE, the tighter the equation (i.e., the more precise the prediction). SEE is also commonly denoted by SE.

A.8.2 Confidence Interval and Confidence Level

A *confidence interval* is a statistic that defines the interval of a distribution parameter from a set of data. For example, when estimating the mean of a normal distribution, the sample mean is a point estimate of (or best guess) for the value of the mean. However, this estimate is almost surely not what the final outcome will be. A confidence interval identifies a range around the estimate. The width of that range is a function of the *confidence level*. The confidence level associated with the interval (e.g., 80%, 90%, or 95%) is the proportion of times in which the interval will contain the true value of the unknown outcome.

A.8.3 Prediction Intervals

The proper measure of the quality of the estimate is the Prediction Interval. Although the regression equation provides a future prediction when the predictor variables are present, this point estimate is almost surely not exactly right. A prediction interval provides a range of values in which the actual value of a variable can be expected to fall with a certain degree of confidence. For example, “There is a 90% probability that the TT&C Non-recurring cost will be between \$10M and \$20M.” The formula for the prediction interval of a parametric CER is based on the standard error of the CER, the CER sample size (i.e., the number of data points in the CER), the desired confidence level, and the distance from the center of the CER’s independent variables to the location (of the independent variable) of the point being estimated. The prediction interval is wider for CERs with larger standard error of estimate.

A prediction interval can be thought of as a range defined by the point estimate plus or minus some number of adjusted standard errors (standard errors adjusted for prediction). This adjusted

standard error is a function of the standard error of the regression, the size of the database used in the CER development, and the “distance” of the estimating point from the center of the database as exemplified by the means of all the independent (driver) variables.

In the simple case of one independent variable [X], the adjusted standard error is specifically defined by the following:

Equation 18

$$Adj. SE = SE * \sqrt{1 + \frac{1}{n} + \frac{\left(\frac{Xe - \bar{x}}{S_X}\right)^2}{n}}$$

Where:

Adj. SE	=	adjusted standard error for prediction interval
SE	=	standard error (linear case must be normalized for base year, adjustments, and buy quantity)
Xe	=	the value of the independent variable used in calculating the estimate
\bar{x}	=	the mean of the independent variable in database
S _X	=	uncorrected sample standard deviation of the independent variable
n	=	the number of data points

In a simple linear CER where $Y = \beta_0 + \beta_1 X + \varepsilon$, a 95% prediction interval when $X = x_0$ is given by

Equation 19

$$\hat{y}_0 \pm t_{0.025, (n-2)} * se * \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS_X}}$$

Where $SS_X = \sum_{i=1}^n (x_i - \bar{x})^2$, \hat{y}_0 is the estimated cost from the CER when $X = x_0$, and $t_{0.025, (n-2)}$ is the upper 2.5% cut-off point of Student's t distribution with (n-2) degrees of freedom.

As one moves away from the mean of the CER dataset, the *adjusted standard error* is always greater than the SEE. Thus, using the SEE as a quantifier for uncertainty underestimates the true error unless the point of interest is at the mean of the data. For estimating points not near the dataset mean, the difference can be significant. This is especially true when the CER is used beyond the range of the data used in developing the CER.

If not all statistics of the CER are available, the Adjusted Standard Error can be calculated with the following equation based on a distance assessment of the primary independent variable.

Equation 20

$$Adj. SE = SE * \sqrt{1 + \frac{1}{n} + \frac{\left(\frac{\text{distance}}{\text{sample std}}\right)^2}{n}}$$

where:

distance = distance between the point estimate independent variable value and the center of the independent variable data used to generate the CER

sample std = uncorrected sample standard deviation of the primary independent variable

Note that this distance assessment need only be characterized in terms of a number of standard deviations from the center. For example, if the distance is assessed as approximately 2 sample standard deviations of the driver variable, then the ratio (of “distance” to “sample std”) becomes 2. For simplicity, the following default values address the assessment of this ratio based upon the similarities between the systems:

$$\frac{\text{distance}}{\text{sample std}} = \begin{cases} 0.25 & \text{Very Similar} \\ 0.75 & \text{Similar} \\ 1.50 & \text{Somewhat Different} \\ 3.00 & \text{Very Different} \end{cases}$$

For example, if the system being estimated is deemed very similar to the database from which the CER was developed, this qualitative assessment might translate into a quantitative assessment of the ratio with a value of 0.25. Similarly, if the system being estimated is deemed very different from the database from which the CER was developed, this qualitative assessment might translate into a quantitative assessment of the ratio with a value of 3.0. Using these default values the adjusted standard error can then be calculated.

In addition, if no statistical information is available, then the Adjusted Standard Error can be chosen subjectively based on a subjective distance assessment of the primary independent variable and the relative sample size. **Table A-1** provides a list of multipliers that can be used to estimate the Adjusted Standard Error.

Table A-1 Standard Error Adjustment Factors

SE Multiplier	Number of Data Points in Sample							
	5	10	15	20	25	30		
Distance / StdDev for Independent Variable	0.00	1.095	1.049	1.033	1.025	1.020	1.017	Very Similar
	0.25	1.101	1.052	1.035	1.026	1.021	1.018	
	0.50	1.118	1.061	1.041	1.031	1.025	1.021	
	0.75	1.146	1.075	1.051	1.038	1.031	1.026	Similar
	1.00	1.183	1.095	1.065	1.049	1.039	1.033	
	1.25	1.230	1.121	1.082	1.062	1.050	1.042	
	1.50	1.285	1.151	1.103	1.078	1.063	1.053	Dissimilar
	1.75	1.346	1.186	1.127	1.097	1.078	1.066	
	2.00	1.414	1.225	1.155	1.118	1.095	1.080	
	2.25	1.487	1.267	1.185	1.142	1.115	1.096	Different
	2.50	1.565	1.313	1.218	1.167	1.136	1.114	
	2.75	1.647	1.362	1.253	1.195	1.159	1.134	
3.00	1.732	1.414	1.291	1.225	1.183	1.155	Very Different	

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Figure A-7 illustrates the use of the standard error adjustment factor. The correct prediction interval high value was calculated to be 123.01% (as described in **B.2.2.3**) of the point estimate at the 90% interpretation. This is compared with the lognormal based upon the reported SEE in log space (0.1413). The regression SE underestimates the standard deviation and 95% value. If, however, the SEE multiplied by the adjustment consistent with 10 data points and an assessment that the source data is “different” for the project to be estimated (1.267), the results are more conservative than those obtained using SE alone. The adjustment factor approach tends to over compensate, but is a reasonable, conservative and simple approach when statistical tools and/or source data is unavailable for a more precise estimate.

WBS/CES Description	BASELINE	Equation / Throughput	Distribution Form	Adj SE	High or High %	High Interpretation
Propulsion Calculated PI	60.915 (50%) *	$1.618 * Motor/Wt ^ 0.6848$	LogNormal		123.01%	90
Propulsion SE Log Space	60.915 (50%) *	$1.618 * Motor/Wt ^ 0.6848$	LogNormal	0.1413		
Propulsion Adjusted SE Log Space	60.915 (50%) *	$1.618 * Motor/Wt ^ 0.6848$	LogNormal	0.1790		

WBS/CES	Point Estimate	Mean	Std Dev	CV	5.0% Level	25.0% Level	50.0% Level	75.0% Level	95.0% Level
Propulsion Calculated PI	60.91 (50%)	61.72	10.04	0.16	46.70	54.63	60.92	67.93	79.46
Propulsion SE Log Space	60.91 (50%)	61.53	8.74	0.14	48.28	55.38	60.91	67.01	76.86
Propulsion Adjusted SE Log Space	60.91 (50%)	61.90	11.17	0.18	45.38	53.99	60.92	68.73	81.77

Figure A-7 Compare Calculated Prediction Interval with Estimated Approach

A.9 RISK SIMULATION METHODS

A.9.1 Monte Carlo

Monte Carlo simulation is a method for assessing the overall uncertainty inherent in a model. It calculates the model iteratively using randomly selected values from the error distribution for each of the model components (WBS element CER, input variable, throughputs, etc.), and then using the set of results from all the iterations to estimate the distribution of the overall model results.

The Monte Carlo method has been successfully used in scientific applications for at least 60 years. It is a problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables. Credit for inventing the Monte Carlo method often goes to Stanislaw Ulam, a Polish born mathematician who worked for John von Neumann on the United States’ Manhattan Project during World War II. Ulam is primarily known for designing the hydrogen bomb with Edward Teller in 1951. He invented the Monte Carlo method in 1946 while pondering the probabilities of winning a card game of solitaire. Ulam and Metropolis published the first paper on the Monte Carlo method in 1949 (**Reference 1**).

A.9.2 Latin Hypercube

Latin Hypercube sampling (also known as stratified sampling) has been shown to require fewer model iterations to approximate the desired variable distribution to the same level of accuracy

than the Monte Carlo method. The Latin Hypercube technique ensures that the entire range of each variable is sampled. It works as follows:

- The distribution is divided into segments of equal probability
- The distribution of samples over these segments is proportional to the probabilities of falling in the segments
- Each sample is drawn from its segment by uniform random sampling

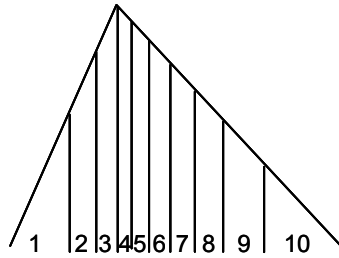


Figure A-8 Segments of Equal Probability

In **Figure A-8**, a triangular cumulative distribution function has been divided into 10 intervals of equal probability (i.e. the area of each interval is the same) and a sample is randomly selected from each interval. Once a sample is taken from a particular interval, this interval is not sampled again in the sampling process until all segments have been sampled. Samples derived by Latin Hypercube method can more accurately reflect the original input distribution with fewer samples when compared to the clustered samples drawn using the Monte Carlo method.

A.10 CORRELATION

A.10.1 Overview

Correlation is the term used to describe the degree to which variables are related or associated. Correlation between any two random variables does not prove or disprove a cause-and-effect relationship between them.

An important consideration in risk analysis is to adequately account for the relationships between the cost elements during a risk simulation. This interrelationship between the WBS elements is commonly known as "dependency" or "correlation." For example, if something causes the cost of WBS element A to increase, it may be appropriate that the cost of WBS element B also increases (positive correlation), and perhaps the cost of WBS element F should decrease (negative correlation). For inputs-based analyses, correlation between random input variables should be included where significant.

Correlation does not change the cost distribution at the lowest levels of the WBS, where correlations are defined. It does have a significant impact on the parent level of the WBS. Correlation can be defined only across those elements that have uncertainty modeled. The specification of correlation within an uncertainty assessment will magnify the uncertainty impact at the aggregate level as child WBS elements are forced to move "together." If the child element uncertainty distributions are left to be sampled independently of one another, then the high sample on one distribution can be canceled by a simultaneous low on another element and they will tend to cancel each other out to some extent. This is a significant reason for very steep S-curves in estimates with many elements. Positive correlation causes elements to move in the

same direction (tending to magnify the risk effect), while negative correlation causes elements to move opposite to each other (tending to cancel each other).

Implementing positive correlation across the uncertainty estimates will result in broader dispersion (increased variance) of the uncertainty result at the aggregate or parent levels in the WBS. The impact can be significant. There have been many papers on the topic, such as **References 18, 21, 22, 26, 28, 30, 33, and 39**. **Reference 33** provides reasons why the correlation between weight variables should not be calculated from the same data set used to develop a CER.

A.10.2 Functional Correlation (Implicit)

Correlation of the risk distributions in a cost model will often be already captured through the functional relationships within the cost model. For instance, if both the costs of Data and SEPM are modeled by using certain factors times the cost of the Prime Mission Equipment (PMP), then Data and SEPM will be positively correlated in the risk analysis. In this situation, as PMP changes in the risk simulation, the costs of Data and SEPM will change in the same direction accordingly; therefore, they are positively correlated.

A.10.3 “User-Defined” Correlation (Explicit)

User-Defined correlations are those specified by the user and implemented within a model. Before specifying any additional correlations among the WBS elements, it is recommended that the user measure the correlations already present in the cost risk model. Correlations (or dependencies) between the uncertainties of estimates for the WBS elements are determined by the structure of the project. These correlations should not be estimated by the cost-vs.-cost correlations in the historical database from which the CERs are derived. In other words, strong correlations between cost elements in a database should not be mistaken as evidence that residuals or percentage errors of the CERs derived from the same database are correlated. See **Reference 28** for details.

A.10.4 MEASURING CORRELATION

A.10.4.1 Overview

The two primary methods are Pearson’s Product Moment and Spearman Rank order. The appropriateness of these two types in the context of cost estimating is discussed in **References 18, 21, 22, 30**. Crystal Ball and @Risk employ Spearman Rank and ACE employs a variation on the Pearson Product Moment method recommended in **Reference 18**.

A.10.4.2 PEARSON’S CORRELATION COEFFICIENT

Pearson's correlation coefficient between two sets of numbers is a measure of the linear association between these two sets. It measures the degree to which two sets of data move together in a linear manner. A high positive correlation indicates a strong direct linear movement and a high negative correlation represents a strong inverse relationship. The correlation ranges between -1.0 and $+1.0$, where -1 indicates a perfect inverse relationship, 0 indicates no correlation, and $+1$ indicates a perfect positive relationship. In probability theory and statistics, Pearson’s correlation coefficient indicates both strength and direction of the linear relationship between two variables. Pearson’s correlation is important in cost-risk analysis because it appears explicitly in the formula for the total-cost standard deviation and therefore impacts the spread of the total-cost distribution.

By definition, Pearson's correlation coefficient (Pearson's r) calculated between two sets of numbers $\{x_i\}$ and $\{y_i\}$ is given by

Equation 21

$$r_{xy} = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2} \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

Where \bar{x} and \bar{y} are the means of $\{x_i\}$ and $\{y_i\}$, respectively, and n is the sample size.

A.10.4.3 SPEARMAN'S RANK ORDER CORRELATION

Spearman's rank order correlation is used in nonparametric inferences to determine if two random variables are independently distributed. Therefore, no assumptions are made about the underlying distributions. When Spearman's correlation coefficient is significantly different from zero, it can be interpreted as an association between two variables, just like the ordinary Pearson product-moment correlation. However, the Spearman's rank correlation coefficients do not appear explicitly in the formula for the total-cost standard deviation, and therefore their impact upon the spread of the total-cost distribution is not generally understood.

The Spearman Rank Order correlation is computed from the ranking of the elements in the ordered pairs, as opposed to the actual values. If some of the ranks are identical, all of the ties are assigned the average of the ranks that they would have had if their values had been slightly different. In this situation, some of the ranks could be partial integers. In all cases the sum of all assigned ranks will be the same as the sum of the integers from 1 to n where n represents the number of elements, namely $n(n + 1)/2$.

To calculate the Spearman Rank correlation coefficient, let R_i be the rank of x_i among the other x 's, let S_i be the rank of y_i among the other y 's, and let ties being assigned the appropriate average as described above. Then the rank-order correlation coefficient is defined to be the linear correlation coefficient of the **ranks**, and is given below:

Equation 22

$$r_s = \frac{\sum_i (R_i - \bar{R})(S_i - \bar{S})}{\sqrt{\sum_i (R_i - \bar{R})^2} * \sqrt{\sum_i (S_i - \bar{S})^2}}$$

If there are no ties in the ranking, then the equation can be reduced to:

Equation 23

$$r_s = 1 - \frac{6 * \sum d^2}{n * (n^2 - 1)}$$

Where:

d = difference between the paired ranks

n = number of paired ranks

A.11 DETERMINISTIC

Deterministic refers to events that have no random or probabilistic aspects but proceed in a fixed predictable fashion. A deterministic model consists of an exact relationship; for example, if the labor rate (LabRate\$) is known and the man-hours (Mhrs) are known, then the cost is known by calculating LabRate\$*Mhrs. Unlike a stochastic model, a deterministic equation has no random error. However, there may be error in the variables, meaning LabRate\$ and Mhrs may not be known exactly. In this case, however, there is still no error associated with the equation.

A.12 STOCHASTIC

Stochastic refers to patterns or processes resulting from random factors. A stochastic model consists of random functions to simulate a deterministic system. Unlike a deterministic model, the equations or their parameters are not known with certainty but only with some amount of probability. For example, the labor rate and the man-hours are only known within some degree of probability and therefore the cost calculated by LabRate\$*Mhrs is only known with probability.

A.13 OBLIGATION VS EXPENDITURE

An obligation reserves funds pending completion of a contract. An expenditure is a disbursement of obligated funds. Estimates and budgets represent obligations NOT expenditures.

A.14 SCENARIO BASED METHOD LOGNORMAL EQUATION DERIVATION

The Scenario Based Method can be statistically augmented with a few assumptions. This section presents an approach assuming an underlying lognormal distribution. Start with the assumption that the probability distribution of $Cost_{pgm}$ is lognormally distributed and the point (x_{PE}, α_{PE}) falls along this lognormal. There are two steps involved in computing the mean and standard deviation of $Cost_{pgm}$. The first is to compute the mean and standard deviation of $\ln(Cost_{pgm})$. The second is to translate these values into the mean and standard deviation of $Cost_{pgm}$, so the units are in dollars instead of “log-dollars.”

Step 1: Derive the Log Space Mean and Standard Deviation

Formulas for the mean and standard deviation of $\ln(Cost_{pgm})$ are

Equation 24

$$\mu_{\ln \text{Cost}_{pgm}} = \ln x_{PE} - z_{PE} \sqrt{\ln(1 + D^2)}$$

Equation 25

$$\sigma_{\ln \text{Cost}_{pgm}} = \sqrt{\ln(1 + D^2)}$$

where D is the CV, x_{PE} is the program's point estimate cost, z_{PE} is the value such that $P(Z \leq z_{PE}) = \alpha_{PE}$ and Z is the standard normal random variable; that is, $Z \sim N(0,1)$. The value for z_{PE} .

Step 2: Derive the Unit Space Mean and Standard Deviation

Once $\mu_{\ln \text{Cost}_{pgm}}$ and $\sigma_{\ln \text{Cost}_{pgm}}$ are computed, they need to be translated into "dollar-units."

Equation 26

$$\mu_{\text{Cost}_{pgm}} = e^{\mu_{\ln \text{Cost}_{pgm}} + \frac{1}{2}\sigma_{\ln \text{Cost}_{pgm}}^2}$$

Equation 27

$$\sigma_{\text{Cost}_{pgm}} = \sqrt{e^{2\mu_{\ln \text{Cost}_{pgm}} + \sigma_{\ln \text{Cost}_{pgm}}^2} (e^{\sigma_{\ln \text{Cost}_{pgm}}^2} - 1)}$$

Once $\mu_{\text{Cost}_{pgm}}$ and $\sigma_{\text{Cost}_{pgm}}$ are computed, the entire distribution function of the lognormal can be specified, along with the probability that Cost_{pgm} may take a particular outcome.

SBM Computational Example: Suppose the distribution function for Cost_{pgm} is lognormal. Suppose the point estimate cost of the program is 100 (\$M) and this cost was assessed to fall at the 25th percentile. Suppose the type and phase of the program is such that 30 percent variability (CV) in cost around the mean has been historically seen. Suppose the protect scenario was defined and determined to cost 145 (\$M). Given this,

Compute $\mu_{\text{Cost}_{pgm}}$ and $\sigma_{\text{Cost}_{pgm}}$.

From **Equation 24** and **Equation 25**, it follows that

$$\mu_{\ln \text{Cost}_{pgm}} = \ln x_{PE} - z_{PE} \sqrt{\ln(1 + D^2)} = \ln(100) - (-0.6745) \sqrt{\ln(1 + (0.3)^2)} = 4.80317$$

$$\sigma_{\ln \text{Cost}_{pgm}} = \sqrt{\ln(1 + D^2)} = \sqrt{\ln(1 + (0.3)^2)} = 0.29356$$

From **Equation 26** and **Equation 27** translate the above mean and standard deviation into dollar units; that is,

$$\mu_{\text{Cost}_{pgm}} = e^{\mu_{\ln \text{Cost}_{pgm}} + \frac{1}{2}\sigma_{\ln \text{Cost}_{pgm}}^2} = e^{4.80317 + \frac{1}{2}(0.29356)^2} \approx 127.3 \text{ ($M)}$$

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$$\begin{aligned}\sigma_{Cost_{Pgm}} &= \sqrt{e^{2\mu_{\ln Cost_{Pgm}} + \sigma_{\ln Cost_{Pgm}}^2} (e^{\sigma_{\ln Cost_{Pgm}}^2} - 1)} \\ &= \sqrt{e^{2(4.80317) + (0.29356)^2} (e^{(0.29356)^2} - 1)} \approx 38.2 \text{ (\$M)}\end{aligned}$$

A.15 FACTOR METHODS

A.15.1 IC CAIG FACTOR APPROACH

Tables Table A-2 and **Table A-3** present the Risk Scoring Matrices used in a methodology employed by the Intelligence Community Cost Analysis Improvement Group (IC CAIG) and the Missile Defense Agency (MDA) in the past (**References 18, 30, 18, and 23**). In this particular application the scores run from 0 to 10. **Tables Table A-4 – Table A-7** present some additional tables used for a specific MDA system. These are not part of the core method but are shown for reference only. **Figure A-9** shows how the risk scores are mapped into distribution parameters. Note these are triangular shaped distributions and the bounds are absolute lows and highs.

Table A-2 Hardware Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Technology Advancement	Completed (State of the Art)	Minimum Advancement Required	Modest Advancement Required	Significant Advancement Required	New Technology
2 Engineering Development	Completed (Fully Tested)	Prototype	HW/SW Development	Detailed Design	Concept Defined
3 Reliability	Historically High for Same Item	Historically High on Similar Items	Known Modest Problems	Known Serious Problems	Unknown
4 Producibility	Production & Yield Shown on Same Item	Production & Yield Shown on Similar Items	Production & Yield Feasible	Production Feasible & Yield Problems	No Known Production Experience
5 Alternate Item	Exists or Availability on Other Items Not Important	Exists or Availability of Other Items Somewhat Important	Potential Alternative Under Development	Potential Alternative in Design	Alternative Does Not Exist & is Required
6 Schedule	Easily Achievable	Achievable	Somewhat Challenging	Challenging	Very Challenging

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Table A-3 Software Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Technology Approach	Proven Conventional Analytic Approach; Standard Methods	Undemonstrated Conventional Approach, Standard Methods	Emerging Approaches, New Applications	Unconventional Approach, Concept Under Development	Unconventional Approach, Unproven
2 Design Engineering	Design Completed & Validated	Specifications Defined & Validated	Specifications Defined	Requirements Defined	Requirements Partially Defined
3 Coding	Fully Integrated Code Available & Validated	Fully Integrated Code Available	Modules Integrated	Modules Exist but are Not Integrated	Wholly New Design; No Modules Exist
4 Integrated Software	Thousands of Instructions	Tens of Thousands of Instructions	Hundreds of Thousands of Instructions	Millions of Instructions	Tens of Millions of Instructions
5 Testing	Tested with System	Tested by Simulation	Structured Walk-Throughs Conducted	Modules Tested (Not as a System)	Untested Modules
6 Alternatives	Alternatives Exist; Alternative Design is Not Important	Alternatives Exist; Design is Somewhat Important	Potential Alternatives are Under Development	Potential Alternatives are Under Consideration	Alternative Does Not Exist but is Required
7 Schedule & Management	Relaxed Schedule, Serial Activities, High Review Cycle Frequency; Early First Review	Modest Schedule, Few Concurrent Activities; Reasonable Review Cycle	Modest Schedule, Many Concurrent Activities; Occasional Reviews Scheduled Late First Review	Fast Track but on Schedule; Numerous Concurrent Activities	Fast Track with Missed Milestones; Review Only at Demonstrations; No Periodic Reviews

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Table A-4 IA&T Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Technology (Highest Level in System)	Off the Shelf Old Technology	Off the Shelf State of the Art Technology	Modest Advancement Required	Significant Advancement Required	New Technology Development
2 Engineering Development (Hardware)	System Complete Fully Tested	System Incomplete & Untested	Hardware Development	Detailed Design Completed	Preliminary Design Completed
3 Engineering Development (Software)	Software Complete Fully Tested	Beta Version Tested	Software Development	HW/SW Interfaces Defined	Preliminary Architecture Defined
4 Interfaces Complexity	Standards Based; Few Simple Interfaces	Standards Based; Many Simple Interfaces	Limited Standards; Many Interfaces	Limited Standards; More Complex Interfaces	No Standards; Many Complex Interfaces
5 Subsystem Integration	All Subsystems Integrated and Tested	Subsystems Integrated; Not Tested	OTS/MOTS Subsystems & Interfaces Defined	New Development Subsystems & Interfaces Defined	Subsystem Requirements Defined
6 Major Component Production	Production and Yield Demonstrated on Same System	Production and Yield Demonstrated on Similar System	Production Plan Established; Yield Feasible	Production Feasible; Yield Potential Unknown	No Known Production Experience
7 Schedule (Hardware)	Achievable; No Critical Paths; Adequate Resources	Achievable; Few Critical Paths; Adequate Resources	Challenging; Few Critical Paths; Limited Resources	Challenging; Many Critical Paths; Limited Resources	Very Challenging; Many Critical Paths; Resources Shortfall
8 Schedule (Software)	Not Time Critical	Critical Path; Below Average SLOC per Day	Critical Path; Average SLOC per Day	Critical Path; Above Average SLOC per Day; Resources Available	Very Challenging; Many Critical Paths; Resources Shortfall
9 Reliability	High Reliability Demonstrated; Predicted High	High Reliability on Similar Systems; Predicted High	Known Modest Problems; Predicted High	Known Serious Problems; Predicted High	Unknown/Serious Problems; Predicted Low

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Table A-5 System Engineering and Integration Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Technology Advancement	No New Tech or COTS	Minimum Advancement	Modest Advancement	Significant Advance	New Technology
2 Engineering Development	Completed or COTS	Prototype	HW/SW Development	Detailed Design	Concept Defined
3 Coordination Required	None, Single Source	Minimum Std I/F	Modest MIS Connection	Significant, Many Sources	New Team Multiple Source
4 Analytical Toolset	Fully Automated COTS	Automated Minimum Customization	Custom to Integrate	Custom Development	Manual Analysis
5 Interface Control	Fully Standard Interfaces	Specifications Frozen	"Plug & Play" Interfaces	Interface SW to Develop	Interface to Enhance Performance
6 Schedule	Easy	Achievable	Some Challenge	Challenging Risky Path	Difficult Critical Path

Table A-6 System Test and Evaluation Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Test Hardware Tech Instrumentation Tech	Existing TE Suite	Assemble Proven Tech	Special Instrumentation	Special Instr & Calibration	New Eqpt & Instruments
2 Simulation Technology	All Test No Simulation	Validated Sim Used Before	Validated New Application	Expand Sim & Validate	New Simulation
3 Software Development	No Software Required	Data Reduction or Existing	Data Collection Real-Time S/W	Test Driver Integration	New Test Driver Real-Time S/W
4 Completeness	Comprehensive Coverage	Key Parameters Comprehensive	Mathematically Validated	Modern Test Theory Applied	New Test Methodology
5 Test Environment	Full Realism Real Players	Parametric Environment	Hardware & HWIL Simulation	HWIL/SWIL Environment	Sim Players or Environment
6 Schedule	Easy No Uncontrolled Factors	Achievable Accounts for Uncontr Factors	Some Challenge Uncontr Factors Not Accounted	Challenging Concurrency of Components	Difficult Severe Concurrency

Table A-7 System Common Risk Scoring Matrix

Risk Categories	Risk Scores (0=Low, 5=Medium, 10=High)				
	0	1-2	3-5	6-8	9-10
1 Technology Advancement	No New Tech or COTS	Minimum Advancement	Modest Advancement	Significant Advance	New Technology
2 Engineering Development	Completed or COTS	Prototype	HW/SW Development	Detailed Design	Concept Defined
3 Material Handling	Routine Done Before	No Hazards	HAZMAT Experienced	HAZMAT Change Proc	New HAZMAT Handling
4 Information Systems	Existing/COTS or None	Integrate COTS Components	Large Network or Diverse HW	New Network Topology	Design & Dev New Component
5 Consumables Management	Automated Experienced	Automated Similar	Manual or New Automation	Expand System Experience	New Area
6 Schedule	Easy	Achievable	Some Challenge	Challenging Risky Path	Difficult Critical Path

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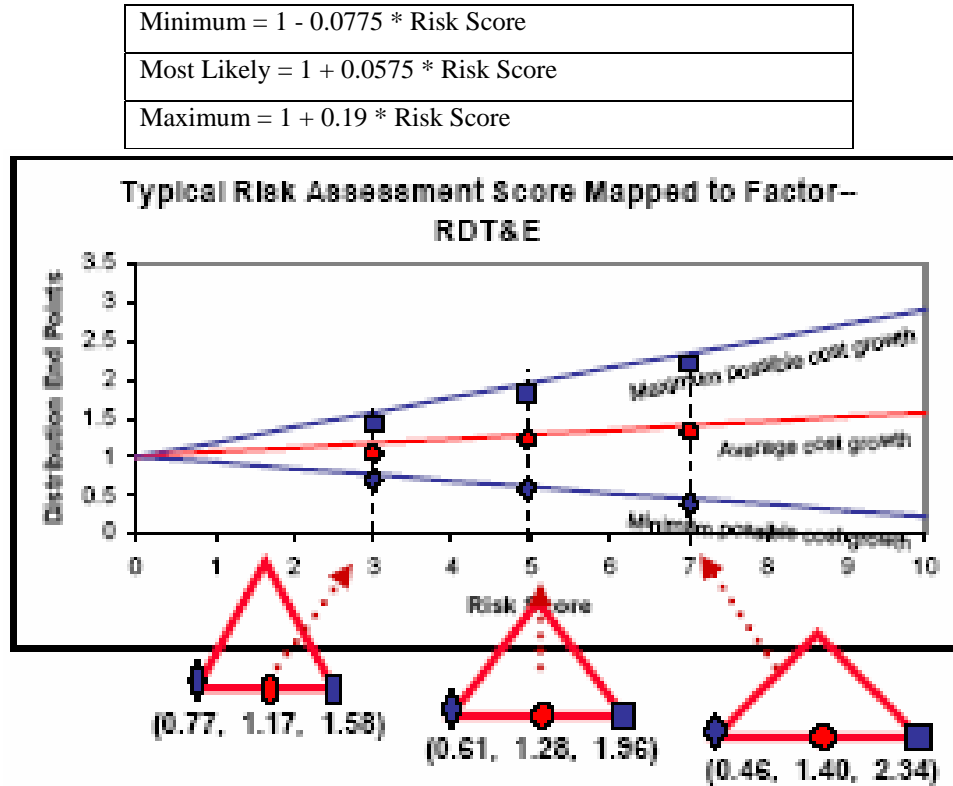


Figure A-9 Risk Scores to Distribution Bounds

A.15.2 Cost Growth Factor

The Cost Growth Potential Factor (GCPF) method assesses cost elements scores of Low, Medium, or High in Technical, Schedule, and Budget categories. Hardware elements and Software elements have separate risk assessment scoring matrices as shown in **Table A-8** and **Table A-9**. In this application, the risk scores are simply Low, Medium, and High. The overall scores are not weighted or graduated further. The overall assessment score is mapped into a CGPF as depicted in **Table A-10**. The CGPFs are not interpolated further – the point is that the insight into potential cost growth is limited to the three-point scale and attempts to fine tune the gradation further implies more knowledge than is the case. The score is multiplied by the work remaining in each cost element. **Table A-11** presents CGPFs for space programs with confidence interval bounds. At the present time, bounds for the non-Space commodities have not been computed.

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Table A-8 Hardware CGPF Matrix

Hardware and IA&T Schedule/Technical Risk Categories and Scores			
Categories of Cost Growth Potential	Assessment Scores		
	Low	Medium	High
Technical	<ul style="list-style-type: none"> Technology is completed, state of the art or requires a minimum advancement 	<ul style="list-style-type: none"> Technology advancement requires modest advancement. 	<ul style="list-style-type: none"> New or significant technology advancement required
	<ul style="list-style-type: none"> Engineering development is completed and fully tested or a prototype exists 	<ul style="list-style-type: none"> Engineering requires HW/SW development. 	<ul style="list-style-type: none"> Engineering development has concept defined or detailed design
	<ul style="list-style-type: none"> The reliability for the same or similar items has been historically high 	<ul style="list-style-type: none"> The reliability has known modest problems. 	<ul style="list-style-type: none"> Reliability has known serious problems or is unknown
	<ul style="list-style-type: none"> Production and yield for the same or similar items has been shown 	<ul style="list-style-type: none"> Production and yield is feasible. 	<ul style="list-style-type: none"> Production is feasible but serious yield problems are known or there is no known production experience
	<ul style="list-style-type: none"> Alternate items exist or availability of alternate items is only somewhat important 	<ul style="list-style-type: none"> Potential alternatives are under development. 	<ul style="list-style-type: none"> Potential alternative in design or no alternative exists but is required
	IA&T Elements	<ul style="list-style-type: none"> Interface complexity based on standards with simple interfaces only 	<ul style="list-style-type: none"> Interface complexity based on limited standards with average interfaces
Schedule	Easily achievable	Moderately challenging	Challenging
Budget	Easily achievable	Moderately challenging	Challenging

Table A-9 Software CGPF Matrix

Software Schedule/Technical Risk Categories and Scores			
Categories of Cost	Assessment Scores		
	Low	Medium	High
Technical	<ul style="list-style-type: none"> Proven conventional analytic approach or undemonstrated conventional technology approach with standard methods 	<ul style="list-style-type: none"> Emerging technology approach with new applications 	<ul style="list-style-type: none"> Unconventional technology approach with developmental or unproven concept
	<ul style="list-style-type: none"> Engineering design with defined <i>and validated</i> specifications 	<ul style="list-style-type: none"> Engineering design with defined <i>specifications</i> 	<ul style="list-style-type: none"> Engineering design with defined <i>requirements</i>
	<ul style="list-style-type: none"> Fully integrated code available 	<ul style="list-style-type: none"> Coding modules integrated 	<ul style="list-style-type: none"> Coding modules exist but not integrated
	<ul style="list-style-type: none"> Tens of thousands (or less) lines of code for integrated SW 	<ul style="list-style-type: none"> Hundreds of thousands lines of code for integrated SW 	<ul style="list-style-type: none"> Millions lines of code for integrated SW
	<ul style="list-style-type: none"> Tested by simulation or with system 	<ul style="list-style-type: none"> Structured walk through 	<ul style="list-style-type: none"> Modules tested individually, not as a system
	<ul style="list-style-type: none"> Alternate exists design exists or availability of alternate design is only somewhat important 	<ul style="list-style-type: none"> Potential alternatives are under development 	<ul style="list-style-type: none"> Potential alternative under consideration or no alternative exists but is required
Schedule	Relaxed schedule with few concurrent activities and a reasonable review cycle	Modest schedule with many concurrent activities and a occasional reviews; Late first review	Aggressive schedule with numerous concurrent activities and no periodic review
Budget	Easily achievable	Moderately challenging	Challenging

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Table A-10 Cost Growth Potential Factors

	CGPF Range			
CGPF Category	Space	Aircraft	Missile	C4ISR
Low	1.27	TBD	TBD	TBD
Medium	1.66	TBD	TBD	TBD
High	2.47	TBD	TBD	TBD

Table A-11 Cost Growth Potential Factors for Space Programs (95% Confidence Interval)

	CGPF Range			
CGPF Category	-CI	Most Likely	+ CI	% +/-
Low	1.01	1.27	1.53	20%
Medium	1.22	1.66	2.10	26%
High	1.44	2.47	3.50	42%

A.15.3 Risk Assessment Process**A.15.3.1 Risk Assessment Process Overview**

One of the major considerations in cost estimating is how to assess and quantify Technical, Schedule and Configuration (TSC) Risks. Certainly, there are many complex methods and formulas that do so, but these risks are ultimately subjective and judgmental in nature, no matter how they are developed and applied. The intent of this instruction is to provide a means and rationale for estimating TSC risk using a common-sense, non-statistical approach that generates results that correlate well with more mathematically rigorous methods. The TSC risk is evaluated and applied separately from cost risk and further skews the cost risk to simulate TSC unknowns for events occurring outside of the tailored CERs.

A baseline technical description for a program is developed and usually documented in a Cost Analysis Requirements Document (CARD) which contains the WBS lists for the entire program. Based on the WBS lists, each item in the list is assessed in terms of the Probability of Failure and the Consequence of Failure. Both the Probability and Consequence of Failures are judged in terms of schedule, technical, and configuration changes. This process is done for both the development and production phases of the program.

Figure A-10 shows the overall risk assessment process for an example space and control segment program.

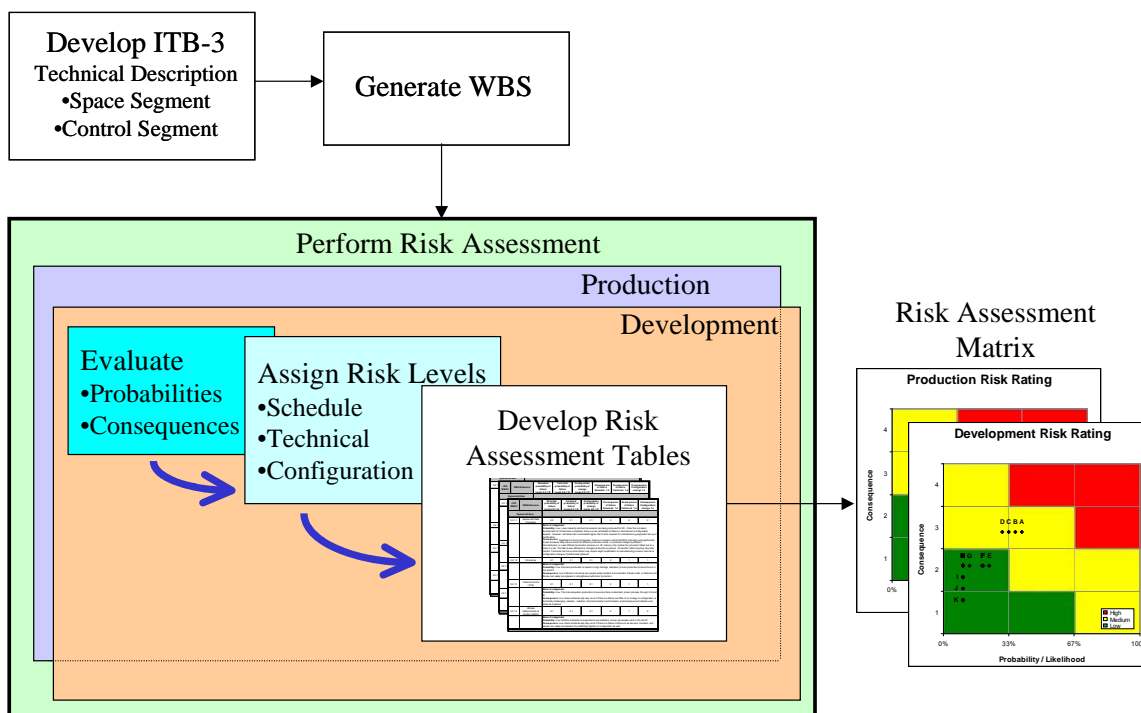


Figure A-10 Sample Risk Assessment Process

A.15.3.2 Risk Category Definitions

Technical Uncertainty -- the variation in the effort and the type and quantity of materials required to meet the technical requirements of a WBS item. There are two main components to technical uncertainty: content uncertainty and execution uncertainty. Content uncertainty refers to our understanding of the effort to be performed. What are the exact performance requirements? What is the level of maturity of any enabling technology? How complex will the final design be? What are the producibility issues associated with the item? How much of the design can be borrowed from similar items? Execution uncertainty refers to our understanding of the capabilities of the organization performing the effort. How well qualified are the people available to perform the effort? Is the capacity and quality of the available tooling adequate? Is the management structure of the performing organization suitable for the effort?

Schedule Uncertainty -- The variation in the key dates associated with a WBS item. The elemental units of a program's schedule are called activities. A WBS item is normally comprised of several schedule activities. The duration of the individual schedule activities is driven by three factors: the technical difficulty of the work to be performed, the qualifications of the people performing the work, and the availability of an adequate number of people to do the work. In other words, schedule uncertainty is driven by technical uncertainty. Schedule activities influence each other through precedence relationships (e.g., Activity C cannot begin until Activities A and B are complete). Schedule precedence relationships often cut across WBS items. Because of these interrelationships, schedule slips in one WBS item can impact the duration of activities in other WBS items, and can actually increase the labor hours in the impacted items. Programs that have a high degree of technical interrelationship between activities as well

as a high degree of concurrence have inherently high schedule uncertainty. In other words, the topology of the program schedule has a strong impact on schedule uncertainty.

Configuration Uncertainty-- the variation in the fundamental technical characteristics of a WBS item. From the cost estimator's perspective, configuration uncertainty falls into two categories: uncertainty in input parameters that are captured by the cost model and uncertainty in parameters or basic configuration features that cannot be addressed without modifying the structure of the cost model. An example of the first type of uncertainty event would be design changes that resulted in reducing the weight of an antenna by 12 lb. An example of the second kind of uncertainty event would be a requirements change that required a switch from a mechanically-steered antenna to a phased array antenna.

A.15.3.3 Risk Assessment Rating Guideline

Using the WBS list, the risk assessment and ratings are performed in collaboration with the technical personnel who are familiar with the development and the production phases of the subject program. Risk assessment tables for the development and production phases are generated. For each WBS item, the following ranking guideline is used:

- Probability of Failure Range (Technical, Schedule, and Configuration):
 - 0.0 to 0.33 - Unlikely to minimal chance of failure
 - 0.34 to 0.67 - Average chance of failure
 - 0.68 to 1.0 - Above average to near certainty of failure
- Consequence of Failure Range: (Technical, Schedule, and Configuration):
 - 1. No significant performance degradation, no schedule slip, and no major technical characteristics increase
 - 2. Minor performance degradation, minor schedule slip but recoverable, minor technical characteristics growth
 - 3. Significant performance degradation, significant schedule slip but partially recoverable, significant technical characteristics growth
 - 4. Major performance shortfall, major schedule slip and unlikely to recover, and major SWaP growth

Step 1: Develop List of Risk Issues

Develop a list of specific risk issues that have the potential to cause TSC problems for the program and that will ultimately affect the cost of the program. Prioritize this list from most significant to least significant. This list should then be used as an overall framework for the evaluation in the following steps.

Step 2: Identify Probability of Failure

Input: List of risk issues and your primary products (box level and/or card level), interfaces, or tasks (e.g. OCS transition, SV integration, etc). Typically we use the WBS as the starting level for risk assessment.

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Process: Evaluation against given criteria, brainstorming, current and future state of technology, project maturity and prototyping. Consideration of the project unique uncertainties surrounding the potential technical, configuration, and schedule impacts throughout the life of the project. *Cost uncertainties should not be addressed in this exercise.*

Use Table A-12-

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Table A-13 as a brainstorming tool to flesh out potential issues and risks. The purpose of the tables is to make sure you have considered problems that may arise from a variety of sources. The tables' entries are not in themselves risks. As you rate an item in the various categories, record in a comments column what issue or problem is driving, or is driven by, the rating in the "probable" or "frequent" columns.

Generally, an issue/problem causes high rating in a number of different places. When you are done, look through the comments and group the issues into distinct items. These are the identified risks. They should be stated as specific problems you foresee. They should have enough of a potential for happening or degree of impact to cause a concern. Statements need to be specific enough that you can associate a general time frame for its occurrence, assess likelihood and impact of its occurrence in step 2.

These tables do not necessarily capture all risks. If there are any specific risks that you are concerned about, record them in the risk list.

Output: Average probability by WBS of the failure to achieve planned results.

Step 3: Identify Consequence of Failure

Input: List of risks and WBS by WBS probability of failure.

Process: Evaluate & quantify likelihood of occurrence and magnitude of impact on technical, configuration, and schedule project goals.

Likelihood: Use the probability ratings the tables give you as a starting point.

Impact: Assume that the risk has occurred. Consider what would be the damage, and what would be needed to recover from it. Look at the "*Consequences of Occurrence*" table for help in identifying a level of impact from the definitions given there.

Document your estimates of likelihood and impact and rationale in the Risk Matrix files. Use your estimates of impact and the "*Consequences of Occurrence*" table to determine the level of impact (level 1, 2, 3, and 4). Use the "Risk Rating Matrix" to assign a qualitative risk level: High, Moderate, or Low.

Output: Average consequence impacts by WBS.

Step 4: Determine Risk Rating

By plotting the average probability of failure vs. the average consequence of failure on Table A-16, the overall TSC risk rating (low, medium, or high) by WBS can be assigned.

Step 5: Apply Risk Rating to Point Estimate

The cost estimating team then uses the overall TSC risk rating in combination with other cost estimating uncertainty parameters to derive the overall cost estimating confidence level.

Table A-12 Probability of Reduced Technical Performance

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Performance	Improbable 0.0 < P ≤0.33	Probable 0.33 < P ≤0.67	Frequent 0.67 < P ≤1.0
Requirements			
Complexity	Simple or easily allocated	Moderate, allocable	Significant or difficult to allocate
Size	Small or easily divisible	Medium, can be divided	Large, not divisible
Stability	Little or no baseline change	Some changes in baseline expected	Changing rapidly or no baseline
Support	Support concept defined	Roles & missions issues unresolved	No support concept or major unresolved issues
Reliability/ Maintainability	Allocable to HW and SW components	Requirements can be defined	Addressable only at total system level
Derating criteria	Comprehensive and in-place	Limited, but in-place	None in place
Use of Std parts/Pgm parts list	In place & complete	In place. Missing some classes	Not in place
Constraints			
Resources	Mature, growth capacity within design, flexible	Available, some growth capacity	No growth capacity, new development, inflexible
Personnel	Available, in place, stable, experienced	Available but not in place, some experience	Not available, little or no experience, high turn over
Standards	Appropriate for application	Some tailoring, not all reviewed for applicability	No tailoring, none applied to the contract
GFE/GFP	Meets requirements, available	May meet requirements, uncertain availability	Incompatible with system req'ts, unavailable
Environment	Little or no impact on design	Some impact on design	Major impact on design
Perf envelopes	Operations well within boundaries	A few operations at boundaries	Continuous operations at boundaries
Technology			
Software	Mature, approved higher order language	Approved or non-approved higher order language	Significant use of assembly language
Hardware	Mature, available	Some development or available	Totally new development
Tools	Documented, validated, in-place	Available, validated, some development	Invalidated, major development, proprietary
Data rights	Fully compatible with support and follow on	Minor incompatibilities with support and follow on	Incompatible with support & follow on
Experience	Greater than 4 years	Less than 4 years	Little or none

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Dev-approach			
Prototype	Used, documented for use sufficiently	Some documentation and use	No use or no documentation
Documentation	Correct and available	Some deficiencies, available	Non-existent
Environment	In place, validated, experienced with use	Minor modifications, tools available	Major development effort
Management	Existing products and process controls	Product and process controls need enhancement	Weak or nonexistent
Integration			
Requirements	Completely defined and funded. No special design issues	Partially defined or not funded. Some unique design problems, solutions identified	Not defined, critical compatibility or design problem--no solution identified
External interfaces	Simple & well defined. No unique development required	Minor development needed or some interfaces need more definition	Major development required and/or major interfaces not defined
Internal interfaces	Defined; loose coupling between system element	More definition needed or some tight coupling	Major interface ill defined or stringent coupling needed
HW/SW interfaces	Clearly defined & formally documented	Partially defined or some documentation incomplete	Not defined or clearly documented

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Table A-13 Probability of Schedule Growth Occurrence

Schedule Drivers	Improbable $0.0 < P \leq 0.33$	Probable $0.33 < P \leq 0.67$	Frequent $0.67 < P \leq 1.0$
Requirements			
Complexity	Compatible with existing technology	Some dependency on new technology	Incompatible with existing technology
Stability	Little or no change projected	Controllable change projected	Rapid or uncontrolled change
Definition	Known, baselined	Baselined, some unknowns	Unknown, no baseline
Need Dates			
Threat	Verified projections	Some unstable aspects	Rapidly changing
Economic	Stable commitments	Some uncertain commitments	Unstable, fluctuating commitments
Political	Little projected sensitivity	Some limited sensitivity	Extreme sensitivity
GFE/GFP	Available, certified	Certification or delivery questions	Unavailable and/or uncertified
Tools	In place, available	Some deliveries in question	Uncertain delivery dates
Technology			
Availability	In-place	Some aspects still in development	Totally still in development
Maturity	Application verified	Some applications verified	No evidence of application
Experience	Extensive application	Some application	Little or none
Resources			
Personnel	Good discipline mix in place	Some disciplines not available	Questionable mix and/or availability
Facilities	Existent, little or no modification	Existent, some modification	Nonexistent, extensive changes
Financial	Sufficient budget allocated	Some questionable allocations	Budget allocation in doubt

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Testing			
Program schedule	Sufficient to allow retest/redesign of a reasonable number of failures	Minimal reserve for retest/redesign	Based on “all successes” in design validation and testing
Test schedule	based on realistic test rate; adequate margin for retest	Test rates slightly optimistic; minimal retest margin	Test rate optimistic; no margins for retest
Test Assets	Sufficient procured, including spares & test schedule overlap	Marginal number procured	Insufficient number procured
Test Integration	All tests part of integrated test program	Some integration of testing	Test program compilation of independent tests
Concurrency			
Use of	All system development completed before production started	Long lead items ordered before qualification completed (on those items)	Major production initiated before testing completed (on those items)

Table A-14 Consequences of Occurrence

Impact Level	Tech Performance Impact (includes producibility & supportability)	Cost Impact	Schedule Impact
Level 1	No/minimal reduction in element technical performance No reduction in system performance	Minor cost growth absorbable within budget	Minor schedule variance No milestone impacts
Level 2	Minimum or slight reduction in element or system technical performance. All requirements still met	Cost growth exceeds authorized budget Sufficient management reserves available	Some schedule slips that are recoverable at program level No major program delivery impacted
Level 3	Decrease in system technical performance. Eliminates all margin. Mission success questionable	Cost growth exceeds authorized budget. Management reserve is inadequate to cover	Significant schedule slip, partially recoverable at system level Upcoming major program delivery impacted
Level 4	Significant shortfall in system technical performance. System requirement not achieved. Mission success unattainable	Cost growth greatly exceeds authorized budget Large funding increase necessary	Major impact to system schedule. Schedule growth is unacceptable Subsequent scheduled launch impacted

A.15.3.4 Sample Risk Assessment Table

Figure A-11 shows the description of the risk assessment matrix table using an example.

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WBS Element		Probability of Failure (Sched, Tech, Config)			Consequence of Failure (Sched, Tech, Config)		
JFOVBS#	WBS Elements	Schedule probability of failure (scale 0.0-1.0)	Technical probability of failure (scale 0.0-1.0)	Configuration probability of change (scale 0.0-1.0)	Consequence of failure-Schedule 1-4	Consequence of failure-Technical 1-4	Consequence of failure-Configuration 1-4
Spacecraft Bus							
321.1	Spacecraft Data Processor	0.5	0.3	0.2	3	3	3
		<p>Basis of Judgment</p> <p>Probability: Low to moderate risk. RAD750 based processor is planned for a flight in early 2004. 12 SV programs (non-GPS) are lined up to use the processor, and BAE is continuing the development for higher performance processor. Its commercial version has been on the market for some time. For 2012 launch date, the required tech freeze date is 2007. 2004 is well within this freeze date, but due to the fact that it has never flown, poses a medium risk in terms of schedule, but a lower risk for technical and configuration changes since designs already exist.</p> <p>Consequence: Significant. Failure on RAD750 performance will significantly impact schedule, technical and configuration changes. May have to resort to a fallback processor that is ratchard but less capable. Parallel processor may enable increased performance, thus additional processor will moderately impact the configuration. Fallback may require additional shielding of the processor units.</p>					
321.2	Structure	0.1	0.1	0.1	1	1	1

Ranges 0.1 to 0.9

Ranges 1 to 4

Figure A-11 Sample Risk Assessment Table Description

For each WBS number and element name, the risk assessment ratings are assigned appropriately for both Probability of Failure and Consequence of Failure according to their impacts on schedule, technical, and configuration changes

Spacecraft Data Processor Example - Development (Refer to Figure A-11)

For this component, the risk assessment ratings are as follows:

Probabilities of Failure:

Schedule = 0.5

Technical = 0.3

Configuration = 0.2

Overall Probability of Failure = 0.33.

Basis of Judgment:

Low to moderate risk. RAD750-based processor is planned for a flight in early 2004. Twelve SV programs (non-GPS) are lined up to use the processor, and BAE is continuing the development for higher performance processor. Its commercial version has been on the market for some time. For 2012-launch date, the required technical freeze date is 2007. The 2004 flight is well within this freeze date, but due to the fact that it has never flown, poses a medium risk in terms of schedule, but a lower risk for technical and configuration changes since designs already exist.

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Consequences of Failure in:

Schedule = 3

Technical = 3

Configuration = 3

Overall Consequence of Failure = 3.

Basis of Judgment:

Significant. Failure on RAD750 performance will significantly impact schedule, technical, and configuration changes. May have to resort to a fall-back processor that is radiation-hard but less capable. Parallel processing may enable increased performance, thus additional processor will moderately impact the configuration. Fall-back may require additional shielding of the processor units.

This point is in the yellow area (Medium Risk) of the Risk Rating Matrix Figure 3. The derivation of each point in the sample matrix in Table 4 is based on the average values for the Probabilities of Failure and the Consequences of Failure.

Table A-15 SS WBS Plotted Points -- Development

SS WBS Elements	DEVELOPMENT		
Spacecraft Bus	<u>Risk Rating</u>	<u>PROB</u>	<u>CONS</u>
Spacecraft Data Processor	Medium	0.33	3.0
Structures & Mechanisms	Low	0.10	1.0
Thermal Control (TCS)	Low	0.10	1.7
Attitude Det. & Control Subsy (ADACS)			
Attitude Sensors	Low	0.10	1.3
Mechanical RCS	Low	0.10	1.7
Electrical Power System (EPS)	Low	0.13	2.0
Power Generation			
Power Storage			
Power Cond. & Dist.			
Telemetry, Tracking & Control (TT&C)	High	0.70	3.0
Propulsion	Medium	0.50	2.0

Table A-16 Risk Rating Matrix

Consequence Level	Probability/Likelihood of Occurrence, P		
	Improbable ($0.0 < P \leq 0.33$)	Probable ($0.33 < P \leq 0.67$)	Frequent ($0.67 < P \leq 1.0$)
Level 4	Medium	High	High
Level 3	Medium	Medium	High
Level 2	Low	Medium	Medium
Level 1	Low	Low	Medium

A.15.3.5 How Risk Rating is Used to Estimate Cost Risk

The resulting quantification of the risk boundary distributions are summarized as:

Low = 0.9 to 1.1 (90% to 110%)

Medium = 0.9 to 1.4 (90% to 140%)

High = 0.9 to 2.0 (90% to 200%)

To complete the process, the application of these TSC ranges should be modeled as triangular distributions in conjunction with the cost risk utilizing Monte Carlo type simulations to determine the overall deviation from point estimate at various confidence levels.

A.15.4 A Shortened Risk Assessment Process

From the program's risk analysis matrix, assign each cost element a high medium, or low grade based on the potential impact due to critical risk items. This process is best accomplished in coordination with the Program Manager and engineers. In the simulation model, define a uniform distribution to be multiplied times each cost element in addition to the element's cost risk distributions. Define the bounds for each uniform distribution as follows:

High 1.0 – 2.0

Medium 1.0 – 1.33

Low 1.0 – 1.1

Note these distributions are in addition to cost uncertainty distributions.

A.16 NEEDS-BASED ALLOCATION

An alternative to the standard deviation based allocation method described in **Section 5.2** is one that favors allocation to those elements with the largest right-hand tail to fill and also takes into account the correlation between elements. In other words, it tries to allocate risk dollars to the

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elements where they are “needed.” The steps in this “needs-based” approach from reference 47 are:

1. Compute the dollar need of each WBS element, $Need_j$
2. Compute correlation between risk dollar requirements of elements i and j , $Corr_{ij}$
3. Compute Total Need Base

$$\text{Need Base} = \sum_{i=1}^n \sum_{j=1}^n Corr_{ij} Need_i Need_j$$

4. Need “Portion” for Project-Element k is

$$- \sum_{i=1}^n Corr_{ik} Need_i Need_k$$

5. Risk Dollars Allocated to Project-Element k

$$- = \left(\sum_{i=1}^n Corr_{ik} Need_i Need_k \div \text{Base} \right) \times \text{Risk Dollars}$$

$$- = \text{A percentage of total risk dollars}$$

Figure A-12 presents an application of this technique to the missile example.

WBS Element	Low	Point Estimate	High	Need	Need Portion	Allocation	60 Percentile Estimate
Sys Dev & Demo Phase							205680
Air Vehicle							139001
Design & Development	17484	25000	47806	6174	518972561	294.4	25294
Prototypes	3222	9749	35257	7429	748307276	424.5	10173
Software	15408	80452	583086	164797	40687815319	23081.9	103534
Sys Engineering/Program Management	12516	21000	41671	5145	467051143	265.0	21265
System Test and Evaluation	6362	23040	231140	71313	16434679101	9323.3	32363
Training	1858	5760	40119	11428	2740295944	1554.5	7315
Data	696	2304	16905	4871	1165846634	661.4	2965
Support Equipment	831	2304	12492	3294	821961105	466.3	2770

Figure A-12 Needs Based Allocation

APPENDIX B DETAILED GUIDANCE

The purpose of this appendix is to expand the information in the body of the handbook with additional detail that would have otherwise made it too cumbersome. The paragraphs in this appendix correspond to those of the handbook body for quick reference.

B.1 INTRODUCTION

B.1.1 PURPOSE

It is only natural for most cost analysts, Program Managers, and other decision makers to develop or expect a single number when seeking or presenting a cost estimate. The use of a single number rather than a range of numbers is understandable from the point of view of those responsible for preparing and managing a budget. Programs are funded and executed using discrete dollars, not ranges of dollars. However, unless each component of the estimate is known with certainty, such a **point estimate** represents only one of several possible outcomes.

A decision-maker who must decide on a single value to submit as a budget input for example, should make that decision in the context of the point estimate with respect to all other outcomes. **Cost risk** is a measure of the chance that, due to unfavorable events, a given point estimate will be exceeded. That is the central point of cost uncertainty analysis -- quantifying the possible outcomes and their likelihood so that an informed choice can be made.

B.1.2 THE DIFFERENCE BETWEEN UNCERTAINTY AND RISK

Uncertainty analysis involves assessing both positive and negative “uncertain” or random events that can lead to impacts on a program. These events might be:

- Technical in nature, such as failing to achieve a performance objective or utilization of emerging technologies that leads to favorable outcomes,
- Programmatic in nature, such as schedule slips due to contractor labor difficulties or over optimism
- Budget/cost in nature, such as overruns due to overly optimistic estimates, budget cuts, uncertainty due to the estimating methods themselves, etc

Cost uncertainty analysis is the process of assessing the cost implications (positive or negative) associated with each of the identified uncertainties. Cost risk analysis involves assessing the overall cost uncertainty in the program and quantifying the likely range of negative impacts. The goal of this analysis is to enable decision makers to go forward with budget estimates that include an assessment of the risk (probability) of overruns (unfavorable events).

B.1.3 THE REQUIREMENT FOR COST RISK ANALYSIS

Budget documentation must be developed in such a way that organizational obligations can be succinctly defined. Obligations entail commitments and these commitments must be made in discrete dollars, not ranges of dollars. The estimates upon which the budgets were based may have been completed long before the reality of program execution is realized. By the time programs are executed, any number of management, technical or schedule changes may have been made and the final cost invariably is much different from the original point estimate (see paragraph **B.1.4**).

Decision makers require insight into the potential for cost growth (or savings) of the estimate for each alternative considered by the project. The least expensive alternative based upon the arithmetic sum of the alternative's elements may not be the least expensive when cost growth potential is taken into account. Cost risk analysis is used to estimate cost growth (or savings) potential.

And finally, risk analysis is necessary because it is mandated in **Reference 11**, paragraph C.2.2.7, which states

Areas of cost estimating uncertainty will be identified and quantified. Uncertainty will be quantified by the use of probability distributions or ranges of cost. The presentation of this analysis should address cost uncertainty attributable to estimating errors; e.g., uncertainty inherent with estimating costs based on assumed values of independent variables outside database ranges, and uncertainty attributed to other factors, such as performance and weight characteristics, new technology, manufacturing initiatives, inventory objectives, schedules, and financial condition of the contractor. The probability distributions, and assumptions used in preparing all range estimates, should be documented and provided to the CAIG.

B.1.4 POINT ESTIMATE

B.1.4.1 Defining the Technical Baseline Point Estimate

Unless each child of the cost estimate is known with certainty, the point estimate represents only one of several possible outcomes. A decision-maker who must decide on the "official" budget should make that decision in the context of the point estimate with respect to all other outcomes. That is the central point of risk analysis -- quantifying the possible outcomes and their likelihood so that an informed choice can be made. Recognizing that there is more than one outcome raises two interesting questions. One, what causes there to be more than one outcome and, two, if there is more than one outcome, which one does the aggregated point estimate represent?

The answer to the first question is simply that estimating techniques are not sufficiently precise to capture all the vagaries associated with producing an estimate. To the extent historical data is used, uncertainty creeps in because no two programs or projects are identical: each is unique unto itself. The technology employed, the schedule, the contractor(s), and the budget climate all contribute to the unique character of each program and its data points. Moreover, even if the past is perfectly known, the future is not. Considering the educated guesses that an analyst must make in developing an estimate, it is no surprise that a cost estimate is just that: an estimate.

The second question can be even more perplexing. If the point estimate is composed of several subsidiary estimates, what is its likelihood? There are at least three possible results. In the first case, if each cost element is estimated at its most likely cost and the uncertainty is symmetric and centered on each cost (that is, for each element, the mean, median, and mode are the same.), it would be reasonable to expect that the total point estimate represents a cost where there is a 50-percent chance of overrun and a 50-percent chance of underrun. But suppose the uncertainty surrounding each of the cost elements varies and is not always symmetric. The sum of those cost elements no longer produces a point estimate with a 50/50 overrun/underrun chance, because it is not known what is being summed. In the second case, it could be argued that the most likely (mode) value for each element is being summed, but this does not necessarily result in the most likely (mode) total. The third case is that the sum is a combination of means, medians, and

modes. For cases 2 and 3 (which are the most common in cost estimates), the resulting sum is not at the 50/50 overrun/underrun position nor is the overrun/underrun position of that sum known or readily deduced. Therefore, if the likelihood of the point estimate is unknown, the likelihood of the other outcomes is also unknown; a decision-maker using such a value decides "in the blind." Risk analysis addresses this question.

B.1.4.2 Role of Sensitivity Analysis

See main body.

B.1.5 Acceptable Methods For Calculating Cost Estimate Uncertainty

B.1.5.1 Overview of Cost risk Analysis Methods

See main body.

B.1.5.2 Simulation Based Cost Risk Analysis

The Monte Carlo simulation method encompasses any technique of statistical sampling employed to approximate solutions to quantitative problems (for details see **A.9**).

B.1.5.3 Scenario-Based Method (SBM)

See main body.

B.1.6 MODELING COST ESTIMATE UNCERTAINTY ANALYTICALLY

There are a variety of analytical methods available to compute the point estimate uncertainty. One of the most well known is the Formal Risk analysis (FRISK) method first introduced in **Reference 10**. It is an approach that allows the user to fit a lognormal or normal distribution to the point estimate total cost. Also known as the "method of moments," FRISK is a popular method by which the analyst can obtain a reasonable estimate of the uncertainty for simple to complex cost models. For example, FRISK has been embedded into the NASA/Air Force Cost Model (NAFCOM). FRISK relies on several simplifying assumptions. The authors recommend using the normal distribution to approximate the total cost distribution when the WBS is large and elements are thought to be uncorrelated. The basis for this recommendation is the central limit theorem: the fit to the normal distribution is improved as the number of elements increases. When the number of cost elements is small, the lognormal is recommended as a better approximation than the normal. In general, if the number of elements is 20 or less, use lognormal. If greater and not very correlated, use normal.

FRISK can be an efficient method to apply to simple cost estimates (such as summing of ten to twenty throughput costs). Many analysts will use FRISK if time is short and the modeling requirement is straight forward. More sophisticated cost models involving many correlated CERs and correlated cost drivers can become very complicated to solve. For most analysts, in these situations, one of the three acceptable methods described in **B.1.5** remains the better choice.

B.2 INPUTS-BASED SIMULATION METHOD

B.2.1 OVERVIEW

B.2.1.1 Uncertainty to be Captured

See main body.

B.2.1.2 Uncertainty That May not be Captured

See main body.

B.2.1.3 Uncertainty That Should not be Captured

See main body.

B.2.2 ESTIMATING METHODOLOGIES

B.2.2.1 Overview

See main body.

B.2.2.2 Uncertainty for Parametric CERs

B.2.2.2.1 Overview

See main body.

B.2.2.2.2 Use Lognormal to Describe Parametric CER Uncertainty

There are many methods to determine which distribution shape to use. In some cases, analysts have been known to capture the residuals (difference between the actual data and results predicted by the CER) and pass them through a “best fit” process to find the distribution that most accurately describes the spread in the residuals. While it can be a useful approach, it is fundamentally flawed in so far as there are rarely a sufficient number of data points to make a clear assessment. This process can be tested by generating data using a known spreadsheet distribution function (for instance “normal”). Applying a best fit process to approximately twenty data points generated from a normal distribution function has returned uniform, beta or Weibull as the “best fit.” These tools appear to need hundreds, perhaps thousands of data points to reliably estimate the “real” distribution. Cost analysts generally have less than 15 data points to work with. Consequently, using “best fit” should be treated with caution.

Left with choosing a default between the six named distributions, beta and Weibull were eliminated as they were considered too complex to be used as a default. Normal was eliminated because in cost analysis, rarely is there an equal chance of an underrun as there is an overrun (normal distributions are symmetrical). Uniform was eliminated because an equal likelihood across the entire range seemed too pessimistic. Triangular was eliminated to avoid having to deal with situations where the lower bound may be a negative number. Also, triangular distributions require an assumption about “skew” in order to derive upper and lower bounds.

Lognormal distributions have a defined lower bound that is never less than zero. They have an upper bound of infinity, thus providing at least some probability of a large cost overrun. The skew of a lognormal is pre-defined. The only drawback to the lognormal is that some analysts may find the mathematics associated with it somewhat confusing. All three tools (CB, @Risk, and ACE) use different conventions to describe a lognormal. However, with care, the identical lognormal distribution can be successfully replicated in all three.

This handbook recommends the lognormal distribution as the “default” selection to describe parametric CER.

B.2.2.3 Detailed Approach For Modeling Parametric CER Uncertainty

B.2.2.3.1 Overview

The probability distribution of the estimating error in any regression method is an assumption of the statistical model, not a statistical fact. While the regression method will yield an objective assessment of the error distribution spread, there is no objective basis to select one distribution shape from another. However, there are conventions that have been adopted by the community that are a reasonable place to start. The most well known of these is the assumption that OLS generated CERs have normally distributed errors. Again, this is not a statistical fact, but an assumption made by statisticians in order to calculate statistics to characterize the CER fit. When the number of data points is less than 30, it is preferable to use the student-t distribution. **Table B-1** summarizes commonly accepted regression method characteristics. The information in **Table B-1** represents how industry experts “typically” interpret and model a parametric CER. There remains considerable debate on many of these assumptions. Never-the-less, **Table B-1** is currently the best information available. For details on each of these regression methods, see **A.3 Regression Methods**.

Table B-1 Common Regression Method Characteristics

Regression Method	Linear OLS	Log-Linear OLS	MUPE	ZMPE
Functional Forms CER Produces	$a + b \cdot \text{Var}$ or $b \cdot \text{Var}$	$a \cdot \text{Var}^b$ or Var^b	Any	Any
Correction to estimate CER Mean	Mean	Median	Mean	Mode
	None	PING factor	None	Unknown
Error Distribution Shape	Student-t or Normal	Lognormal	Student-t or Normal	Symmetrical Triangle

B.2.2.3.2 Calculating the Prediction Interval for Linear OLS CERs

If the CER has the form: $a + b \cdot \text{Var1} + c \cdot \text{Var2} + \text{etc}$ and it was derived using OLS, then it is acceptable to assume the CER produces the **mean** and the uncertainty distribution shape is the student t or normal. To estimate the bounds of the distribution, calculate the prediction interval based upon the point estimate value for the input variables.

Table B-2 Statistic Package Prediction Interval For A Linear OLS CER

UC1 = 30.15 + 1.049 * WarheadWt		
WarheadWt	12.00	
Confidence Level (%)	80.00%	
	Result	PE Multiplier
Lower Bound	35.09	82.11%
Estimate	42.73	Mean
Upper Bound	50.38	117.89%

Many statistical packages will calculate a lower and upper bound for the OLS generated CER based upon a specified value for the independent variable(s). As illustrated in **Table B-2**,

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entering 12 lbs as the value for the point estimate and choosing 80%, the statistical package calculates the upper and lower bound. In this case, these bounds are associated with the 10 and 90 percent probability levels. If needed, or as a check, **Table B-3** identifies the data required and the Excel formulas used to manually calculate the bounds.

Table B-3 Manual Calculation of Prediction Interval For A Linear OLS CER

	UC1	Warhead Wt	
System #1	31.00	6.00	
System #2	46.00	8.00	
System #3	36.00	9.00	
System #4	48.00	13.00	
System #5	40.00	14.00	
System #6	50.00	17.00	
System #7	55.00	20.00	
System #8	54.00	27.00	
System #9	58.00	30.00	
System #10	67.00	31.00	

Model Form:	Unweighted Linear model
Number of Observations Used:	10
Equation in Unit Space:	$UC1 = 30.15 + 1.049 * \text{WarheadWt}$

Element	Range Name	Value	Formula
Confidence Level (%)	ConfLvl	80%	Arbitrary, but 80% is consistent with calculating the 10/90 bounds.
Degrees of Freedom	DegOfFreedom	8	Number of observations minus number of coefficients estimated.
Student t	StudentT	1.397	$TINV(1-\text{ConfLvl}, \text{DegOfFreedom})$
Std Error (SE)	StdErr	5.126	See Appendix
# of Observations	NumObs	10	
TBE Warhead Wgt	TBEWgt	12.00	User Input
Ave Warhead Wgt	AveWgt	17.50	$AVERAGE(\text{WarheadWtObservations})$
Warhead Wgt Stdev	Sx	8.73	$STDEVP(\text{WarheadWtObservations})$
Delta to Bound	Delta	7.64	$StdErr * StudentT * \sqrt{1 + 1/NumObs + ((TBEWgt - AveWgt)/Sx)^2 / NumObs}$
Lower Bound		35.09	TBE-Delta
Point Estimate	TBE	42.73	$UC1 = 30.15 + 1.049 * \text{WarheadWt}$
Upper Bound		50.38	TBE+Delta

If the statistical package and/or the data in **Table B-3** are unavailable, the analyst may estimate the bounds based upon the CER result and the Standard Error for the CER. This simple procedure is illustrated in **Table B-4**. For details on “SEEAdjust” see 2.2.2.3 and A.8.3. NORMINV is an Excel function that returns the inverse of the normal cumulative distribution for the specified mean and standard deviation.

Table B-4 Estimating a Prediction Interval For A Linear OLS CER

Element	Range Name	Value	Formula
Standard Error	SEE	5.126	From Statistical Package
Adjust for Sample Size & Relevance	SEEAdjust	1.267	10 datapoints, source data different than project
Adjusted Standard Error	SEEUnitSpace	6.495	SEE * SEEAdjust
Mean	Mean	42.733	
Lower Bound (10% level)		34.41	NORMINV(0.10,Mean,SEEUnitSpace)
Upper Bound (90% level)		51.06	NORMINV(0.90,Mean,SEEUnitSpace)
Lower Bound as % of PE		80.52%	
Upper Bound as % of PE		119.48%	

All simulation packages will allow a normal distribution to be defined with the mean and some other probability level. Since the normal is symmetrical, only one of the bounds plus the mean (the CER result) is required.

If information about the CER is so sparse that even the standard error is unavailable then the analyst must resort to a **subjective** assessment of the CER uncertainty bounds. Default subjective uncertainties (see paragraph 2.3.3) are considerably larger than the examples in this section.

B.2.2.3.3 Calculating the Prediction Interval for Log-Linear OLS CERs

A very common mistake is to assume OLS log-linear CERs produce the “mean” of the uncertainty distribution. This is not true. The back-transformed unit-space CER produces a value that is closer to the median. There is no requirement to calculate the mean. The lognormal distribution based upon the median and the CER statistics will be identical to one calculated based upon the mean of the same distribution. The simulation result will be identical.

The mathematics involved in adjusting the result to reflect the mean, while not complicated, does add a further layer of unnecessary calculations. For completeness, the details of how to calculate this adjustment are contained in A.3.2.3.

If the CER has the form: $a \cdot \text{Var}1^b \cdot \text{Var}2^c \cdot \text{etc}$ and was derived using OLS in log space (see A.3.2 for details), then the CER produces the **median** and the uncertainty distribution shape is **lognormal**. To estimate the bounds of the lognormal distribution, calculate the prediction interval based upon the Technical Baseline point estimate for the input variables.

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Table B-5 Prediction Interval For Log-Linear OLS CER

UC1 = 1.618 * MotorWt ^ 0.6848		
MotorWt	200.00	
Confidence Level	80.00%	
	Result	PE Multiplier
Lower Bound	49.52	81.30%
Estimate	60.91	Median
Upper Bound	74.92	123.01%

Many statistical packages will calculate a lower and upper bound for the OLS generated CER based upon a specified value for the independent variable(s). As illustrated in **Table B-5**, entering 200 lbs as the value for the point estimate and choosing 80%, the statistical package calculates the upper and lower bound. In this case, these bounds are associated with the 10 and 90 percent probability levels.

Table B-6 Manual Calculation of Prediction Interval For A Log-Linear OLS CER

Element	Range Name	Value	Formula
Confidence Level (%)	ConfLvl	80%	Arbitrary, but 80% is consistent with calculating the 10/90 bounds.
Degrees of Freedom	DegOfFreedom	8	Number of observations minus number of coefficients estimated.
Student t	StudentT	1.397	TINV(1-ConfLvl,DegOfFreedom)
Std Error (SE)	StdErr	0.1413	See Appendix A
# of Observations	NumObs	10	
TBE Motor Wgt	TBEwgt	5.30	User Input LN(200)
Ave Motor Wgt	AveWgt	5.25	AVERAGE(MotorWtObservations)
Motor Wgt Stdev	Sx	0.43	STDEVP(MotorWtObservations)
Delta to Bound	Delta	0.21	StdErr*StudentT*SQRT(1+1/NumObs+((TBEwgt-AveWgt)/Sx)^2/NumObs)
Lower Bound	LowerBound	3.90	TBE-Delta
Point Estimate	TBELogSpace	4.11	UC1 = 1.618 * MotorWt ^ 0.6848
Upper Bound	UpperBound	4.32	TBE+Delta

Lower Bound		49.52	EXP(LowerBound)
Point Estimate	TBE	60.91	EXP(TBELogSpace)
Upper Bound		74.93	EXP(UpperBound)

If the number of observations (and degrees of freedom) are not known, the upper and lower bound can be estimated using the normal distribution similar to the example shown in **Table B-4**.

If even the standard error is unavailable then the analyst must resort to a **subjective** assessment of the CER uncertainty bounds (however, lognormal should be used). Default subjective uncertainties (see paragraph **2.3.3**) are considerably larger than the example in this section.

B.2.2.3.4 Calculating the Prediction Interval for MUPE CERs

Regardless of the CER form, if it was derived using the MUPE method, it can be assumed that the CER produces the **mean** and the uncertainty distribution is **normal**. To estimate the bounds of the normal distribution for a linear MUPE CER, calculate the prediction interval based upon the Technical Baseline point estimate for the input variables. Details on a closed form solution are provided in **A.3.3.2**.

If the bounds cannot be derived analytically, then the analyst must resort to a **subjective** assessment of the CER uncertainty bounds (however, normal should be used). Default subjective uncertainties (see paragraph **2.3.3**) are generally considerably larger than calculated values.

B.2.2.3.5 Calculating the Prediction Interval for ZMPE CERs

To date, there is no objective distribution assumption for CERs derived using the ZMPE regression method. There is no objective interpretation of the CER result, although a common assumption is that it is the mode. While any distribution shape can be used, many have chosen the triangular as most appropriate. Unfortunately there is no objective way to establish the bounds on these CERs either, although progress is being made in this area (see **Reference 46**). Thus, the analyst must make “subjective” selections.

B.2.3 UNCERTAINTY FOR NON-PARAMETRIC CERS AND PARAMETRIC CER INPUTS

B.2.3.1 Overview

See main body.

B.2.3.2 “Standard” Distribution Shapes to Model Subjective Uncertainty

See main body.

B.2.3.3 Distribution Boundaries For Subjective Distributions

When establishing bounds for a specific distribution, it is very important to establish the meaning or interpretation of these bounds. For many years, a common assumption has been to interpret subjective bounds as the 10% and 90% values, meaning that there is a 10% chance the actual low could be lower than the expert opinion and a 10% chance the actual high could be higher. In contrast to this, at least one study has concluded that this assumption is too optimistic (**Reference 6**). In that study, the conclusion was that experts rarely identify 60% of the possible range and never did better than 70%.

B.2.4 ELICITATION OF SUBJECTIVE BOUNDS FROM SUBJECT MATTER EXPERTS

See main body.

B.2.5 DEFAULT SUBJECTIVE DISTRIBUTION BOUNDS

B.2.5.1 Characterizing Low/Medium/High/Extreme High Bounds

The characterization of low/medium/high was loosely derived from noting that the CVs of regressed CERs tend to fall in the 0.15 to 0.35 range (good to not so good fits) for most commodities. For space systems, however, CVs of 0.45 and above are not uncommon. The defaults are based upon the following assumptions:

- For Lognormal, the SEE in log-space is 0.15 for low, .25 for medium, .35 for high and .45 for extreme high dispersion
- For Weibull, the probability level of the point estimate 0.15 for low, .25 for medium, .35 for high

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- For all other distributions, the standard deviation divided by the mean (i.e. the coefficient of variation – CV) is .15 for low, .25 for medium, .35 for high. .45 for extreme high and center skew
- The CV based upon the point estimate is calculated as the standard deviation divided by the point estimate. By adding this convention, uncertainty of the estimate will scale with the point estimate.
- Skew is defined as (Mode – Low) / (High – Low). Left skew is 0.25, symmetrical is 0.50, right skew is 0.75.

Table B-7 Default Bounds For Subjective Distributions

Distribution	Point Estimate Interpretation	Point Estimate and Probability	Mean	CV based on mean	CV Based on PE	15%	85%
Lognormal Low	Median	1.0 (50%)	1.011	0.151	0.153	0.856	1.168
Lognormal Med	Median	1.0 (50%)	1.032	0.254	0.262	0.772	1.296
Lognormal High	Median	1.0 (50%)	1.063	0.361	0.384	0.696	1.437
Lognormal EHigh**	Median	1.0 (50%)	1.107	0.474	0.525	0.627	1.594
Normal Low	Mean	1.0 (50%)	1.000	0.150	0.150	0.845	1.155
Normal Med	Mean	1.0 (50%)	1.000	0.250	0.250	0.741	1.259
Normal High	Mean	1.0 (50%)	1.002	0.346	0.347	0.640	1.363
Normal EHigh	Mean	1.0 (50%)	1.015	0.426	0.432	0.555	1.470
Weibull Low	Mode	1.0 (25%)	1.158	0.179	0.208	0.956	1.370
Weibull Med	Mode	1.0 (20%)	1.393	0.332	0.463	0.956	1.855
Weibull High	Mode	1.0 (15%)	2.104	0.572	1.204	1.000	3.277
Triangle Low Left	Mode	1.0 (75%)	0.878	0.178	0.156	0.695	1.041
Triangle Low	Mode	1.0 (50%)	1.000	0.150	0.150	0.834	1.166
Triangle Low Right	Mode	1.0 (25%)	1.122	0.139	0.156	0.959	1.305
Triangle Med Left	Mode	1.0 (75%)	0.796	0.327	0.260	0.492	1.069
Triangle Med	Mode	1.0 (50%)	1.000	0.250	0.250	0.723	1.277
Triangle Med Right	Mode	1.0 (25%)	1.204	0.216	0.260	0.931	1.508
Triangle High Left*	Mode	1.0 (75%)	0.745	0.448	0.334	0.347	1.103
Triangle High	Mode	1.0 (50%)	1.000	0.350	0.350	0.612	1.388
Triangle High Right	Mode	1.0 (25%)	1.286	0.283	0.364	0.903	1.711
Triangle EHigh Left*	Mode	1.0 (75%)	0.745	0.496	0.370	0.300	1.150
Triangle EHigh	Mode	1.0 (50%)	1.004	0.444	0.446	0.509	1.500
Triangle EHigh Right	Mode	1.0 (25%)	1.367	0.343	0.468	0.876	1.914
Uniform Low Left	Mode	1.0 (75%)	0.870	0.172	0.150	0.688	1.052
Uniform Low	Mode	1.0 (50%)	1.000	0.150	0.150	0.818	1.182
Uniform Low Right	Mode	1.0 (25%)	1.130	0.133	0.150	0.948	1.312
Uniform Med Left	Mode	1.0 (75%)	0.784	0.319	0.250	0.480	1.087
Uniform Med	Mode	1.0 (50%)	1.000	0.250	0.250	0.697	1.303
Uniform Med Right	Mode	1.0 (25%)	1.217	0.206	0.250	0.913	1.520
Uniform High Left	Mode	1.0 (75%)	0.697	0.502	0.350	0.273	1.121
Uniform High	Mode	1.0 (50%)	1.000	0.350	0.350	0.576	1.424
Uniform High Right	Mode	1.0 (25%)	1.303	0.269	0.350	0.879	1.728
Uniform EHigh Left	Mode	1.0 (75%)	0.695	0.577	0.401	0.2085	1.181
Uniform EHigh	Mode	1.0 (50%)	1.000	0.450	0.450	0.454	1.546
Uniform EHigh Right	Mode	1.0 (25%)	1.390	0.324	0.450	0.844	1.935
Beta Low Left	Mode	1.0 (61%)	0.939	0.160	0.150	0.775	1.099
Beta Low	Mode	1.0 (50%)	1.000	0.150	0.150	0.838	1.162
Beta Low Right	Mode	1.0 (39%)	1.061	0.142	0.150	0.901	1.225
Beta Med Left	Mode	1.0 (63%)	0.883	0.283	0.250	0.605	1.152
Beta Med	Mode	1.0 (50%)	1.000	0.250	0.250	0.726	1.274
Beta Med Right	Mode	1.0 (37%)	1.117	0.224	0.250	0.848	1.396
Beta High Left	Mode	1.0 (66%)	0.808	0.419	0.339	0.412	1.186
Beta High	Mode	1.0 (50%)	1.000	0.350	0.350	0.605	1.395
Beta High Right	Mode	1.0 (33%)	1.202	0.291	0.350	0.816	1.606

** EHigh = Extreme High

* To match these parameters, tools must be set to truncate the distribution at zero.

B.2.5.2 Calculating Alternative Specifications for Distributions

There is sufficient information in Table 2-5 (Table B-7 provides additional information) to model the distributions in any of the available tools. However, analysts (and tools) have preferred ways for defining a distribution. In this section, several of the more common translations are provided.

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WBS Element	Unit Space Input For CB or @Risk			Log Space		Unit Space	
	Mean	Sdev	Median	Mean	SEE	85% Value	85% Bound
CER Result	62.0154	11.8710	60.91	4.1094	0.1897	74.14	121.73%
As function of "1"	1.0182	0.1949	1.00	0.0000	0.1897	1.22	121.73%

Unit Space Mean	$EXP(\text{MeanLogSpace} + \text{SElogSpace}^2/2)$
Unit Space Stdev	$((EXP(\text{SElogSpace}^2) - 1) * \text{MeanUnitSpace}^2)^{0.5}$
Unit Space Median	CER result
Log Space Mean	$LN(\text{MedianUnitSpace})$
Log Space SE	From regression report
85% of Median	$LOGINV(0.85, \text{MeanLogSpace}, \text{SElogSpace}) / \text{MedianUnitSpace}$

Figure B-1 How to Calculate Lognormal Parameters Given Unit Space Median & Log Space SEE

WBS Element	Unit Space Input For CB or @Risk			Log Space		Unit Space	
	Mean	Sdev	Median	Mean	SEE	85% Value	85% Bound
CER Result	62.0154	11.8710	60.91	4.1094	0.1897	74.14	121.73%
As function of "1"	1.0182	0.1949	1.00	0.0000	0.1897	1.22	121.73%

Unit Space Mean	From regression report
Unit Space Stdev	From regression report
Unit Space Median	$Exp(\text{MeanLogSpace})$
Log Space Mean	$LN(\text{MeanUnitSpace}) - \text{SElogSpace}^2/2$
Log Space SE	$SQRT(LN(\text{StdevUnitSpace}^2 / (\text{MeanUnitSpace}^2 + 1)))$
85% of Median	$LOGINV(0.85, \text{MeanLogSpace}, \text{SElogSpace}) / \text{MedianUnitSpace}$

Figure B-2 How to Calculate Lognormal Parameters Given Unit Space Mean & Sdev

WBS Element	Unit Space Input For CB or @Risk			Log Space		Unit Space	
	Mean	Sdev	Median	Mean	SEE	85% Value	85% Bound
CER Result	62.0154	11.8710	60.91	4.1094	0.1897	74.14	121.73%
As function of "1"	1.0182	0.1949	1.00	0.0000	0.1897	1.22	121.73%

Unit Space Mean	$\text{MedianUnitSpace} * EXP((\text{SElogSpace}^2)/2)$
Unit Space Stdev	From regression report
Unit Space Median	From regression report
Log Space Mean	$LN(\text{MedianUnitSpace})$
Log Space SE	$SQRT(LN((1 + SQRT(1 + 4 * (\text{SdevUnitSpace} / \text{MedianUnitSpace})^2)) / 2))$
85% of Median	$LOGINV(0.85, \text{MeanLogSpace}, \text{SElogSpace}) / \text{MedianUnitSpace}$

Figure B-3 How to Calculate Lognormal Parameters Given Unit Space Median & Sdev

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		Guidance and Control	Sys Test & Eval	Source or Formula
Low Bound	Low	595	225	Expert Opinion
Mode	Mode	700	250	Expert Opinion
High Bound	High	980	300	Expert Opinion
Uncertainty Captured	UncertCap	70%	70%	Expert Opinion
Skew	Skew	0.27	0.33	$(\text{Mode}-\text{Low})/(\text{High}-\text{Low})$
Uncertainty Not Captured	UncertNotCap	30%	30%	$1 - \text{UncertCap}$
Lower Interpretation	LowInterp	8%	10%	$\text{Skew} * \text{UncertNotCap}$
High Interpretation	HighInterp	78%	80%	$\text{UncertCap} + \text{LowInterp}$
High Tail Area	HighTail	22%	20%	$1 - \text{HighInterp}$
Absolute High	AbsHigh	1268.8	349.5	$\text{High} + (\text{High}-\text{Mode}) * \text{SQRT}(\text{UncertNotCap}) / (1 - \text{SQRT}(\text{HighTail}))$
Absolute Low	AbsLow	486.7	200.2	$(\text{Mode}-\text{Skew} * \text{AbsHigh}) / (1 - \text{Skew})$

Figure B-4 How to Calculate Triangular Absolute Bounds Given Expert Opinion

		Integ, Assembly, Test & Check Hrs/Unit	Source or Formula
Low Bound	Low	120	Expert Opinion
Mode	Mode	120	Expert Opinion
High Bound	High	192	Expert Opinion
Uncertainty Captured	UncertCap	70%	Expert Opinion
Skew	Skew	0.00	$(\text{Mode}-\text{Low})/(\text{High}-\text{Low})$
Uncertainty Not Captured	UncertNotCap	30%	$1 - \text{UncertCap}$
Lower Interpretation	LowInterp	0%	$\text{Skew} * \text{UncertNotCap}$
High Interpretation	HighInterp	70%	$\text{UncertCap} + \text{LowInterp}$
High Tail Area	HighTail	30%	$1 - \text{HighInterp}$
Absolute High	AbsHigh	222.9	$\text{High} + (\text{High}-\text{Mode}) * \text{UncertNotCap} / (1 - (\text{UncertNotCap}))$
Absolute Low	AbsLow	120.0	$(\text{Mode}-\text{Skew} * \text{AbsHigh}) / (1 - \text{Skew})$

Figure B-5 How to Calculate Uniform Absolute Bounds Given Expert Opinion

B.2.6 GUIDANCE ON OBJECTIVE OR SUBJECTIVE DISTRIBUTION BOUNDARY LIMITS

There is significant debate on the guidance to “truncate at zero”. Many believe you should not truncate at zero because it artificially changes the mean and other statistics associated with the distribution that the analyst has determined is the “best fit” for the data.” Simply choosing one that does not permit a negative tail (like lognormal) and preserve the mean and standard deviation is viewed as unacceptable by some if the analysis has shown the “best fit” to be something else (that does permit the negative tail). The essence of the argument is that we should not arbitrarily change mathematically proven results merely because “we don’t like” the idea of negative values influence the simulation.

A pertinent question in this debate is “how big an impact does it have?” The answer, of course, has to be “it depends”. In the case of the CRUH missile example, changing the model to allow negative tails caused the percentile results to be between 3% lower (at the 5 percentile level) to 0.5% lower at the 95% level. In this situation, the combination of the mean shifting to the right overcame the net reduction in variance to produce a more conservative S curve when negative tails are truncated. This would have to be studied further to determine if this is a “typical” result.

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Those that recommend the truncate at zero convention generally cite one or more of the following reasons:

1. Allowing cost and technical parameters to have a negative tail implies that negative numbers are a plausible occurrence which is generally not the case.
2. Granted, mathematical purity is retained by allowing simulation draws to result in negative costs, but allowing non-plausible draws is undesirable from the standpoint of logic and explanatory worth.
3. If negative values are not generally encountered in the real world, and there is not a single data point in the data set that is negative, the simulation should not be forced to encounter cases where it does happen.
4. Cost models where the negative number is an input to a CER where the negative value is to be raised to a power less than 1 will fail to run. (for instance, Excel and ACE will report an error if a simulation iteration requires a calculation that includes the square root of a negative number). This problem was encountered during experiments applying negative tail distributions on inputs in the CRUH example model.
5. Since negative values are undesirable, the key to avoiding them is to select distributions with tails that do not stretch into the negative. Choosing shapes such as the lognormal where the mean and standard deviation are preserved and the lower limit of the distribution does not fall below zero, brings clarity and realism to the entire cost prediction discussion.
6. Most of the time, the CER or independent variable statistics are based upon less than 30 observations. Therefore it is difficult to argue sticking hard and fast to a particular shape or the mean and standard deviation is a "better choice" than selecting one that avoids negative draws. Maintaining that the original choice for a distribution shape should be preserved (with a negative tail) because it best fits the data could be misplaced loyalty to the "math" that led to the selection.

The debate essentially asks the analyst to choose between two "wrong" approaches. The fact that the analyst will be forced to remove negative tails when their presence precludes the ability for the simulation to run suggests introducing inconsistent assumptions (i.e. selective truncating). This handbook encourages analysts to avoid negative tails.

B.2.7 DISCRETE UNCERTAINTIES

See main body.

B.2.8 UNCERTAINTY VIA THIRD PARTY TOOLS

See main body.

B.2.9 SPECIFIC TECHNICAL/SCHEDULE UNCERTAINTY CONSIDERATIONS

It has been recognized for many years that a significant impact to cost can arise due to schedule and technology push assumptions in a program. While this is germane primarily to development efforts, there are a few occasions in production efforts where these considerations are important

(e.g., technology associated with manufacturing process). While some schedule slips are expected and can be absorbed in the program without significant cost impact, other slips may cause considerable cost increases. Schedule slips on some activities have only minimal impact on the cost of future activities while some slips can lead to increased costs on many future activities in the program.

However, before any adjustments to the cost risk assessment are made to account for schedule or technology risk consideration, it is necessary to compare the program being estimated to the source data used to generate the CER. Most, if not all development programs experience cost impacts due to schedule and technology “issues.” Most CERs, estimating methods, and even analogy and expert opinion estimating processes have been colored by and influenced by past, real programs. As such, most estimating approaches include “nominal” amounts of cost impact due to these factors. In fact, it could be argued that the magnitude of error term in normal OLS is partially due to these factors. So, it is necessary for each analyst to realistically assess the degree to which his or her schedule and technology risk considerations are unusual relative to past experience. This is generally going to be a subjective assessment.

It is also necessary to consider if the technology and schedule cost risk impacts are independent, or, as is the much more likely case, highly interrelated. Problems with technology invariably lead to redesign efforts, additional test cycles, and numerous other program changes. These in turn lead to schedule changes. In any subjective process, to account for the cost impact due to schedule and technology considerations, it is critically important that the underlying justification or reason for the potential impact is clearly understood to minimize the likelihood of double counting.

The challenge in deriving adjustments that reflect independent and separate assessments of schedule and technical difficulties is that there are no normative, quantitative measures of schedule or technical difficulty, either separately or in combination. It is difficult to separate the impact to cost/schedule overruns, due to purely technical or schedule problems, because technical difficulties manifest themselves in schedule extensions and that will have cost consequences.

To be sure, multipliers such as these would only be recommended for use in cases where better information is unavailable upon which to base the choice of a multiplier to capture the effects of extraordinary schedule or technical uncertainty.

B.3 COMPLETE THE SIMULATION

See main body.

B.4 HOW TO “BUY” CERTAINTY

Section 4.0 defines one method for calculating “risk dollars.” If the simulation approach is used, it is possible to define risk dollars in another way as illustrated in **Figure B-6**. Under this method, the TBE results are not an integral part of presentation.

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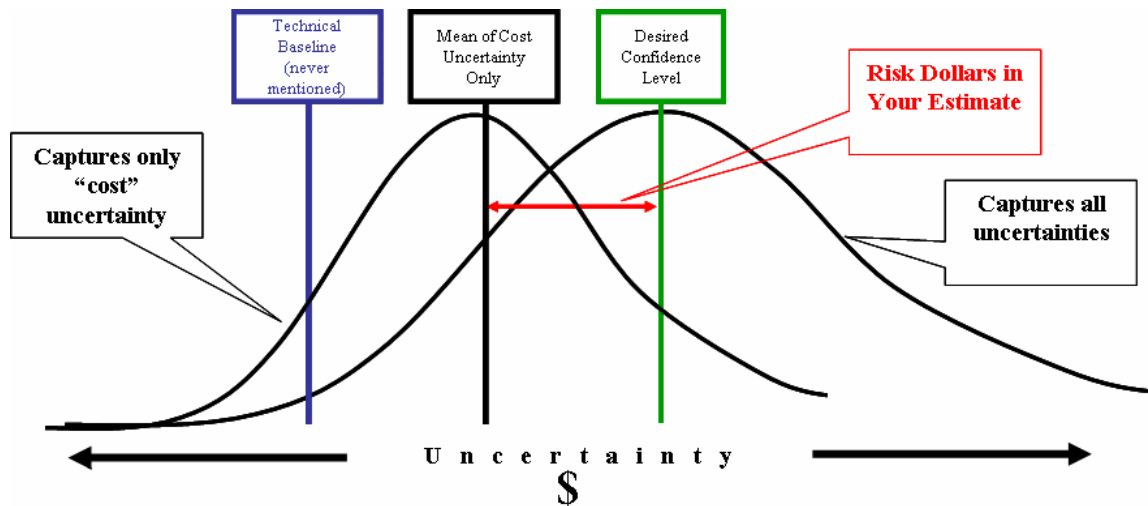


Figure B-6 Defining “Risk Dollars” Based on the Mean of “Cost” Uncertainty

The steps include:

1. Determine the TBE dollars only as a basis for defining cost uncertainties.
2. Define uncertainties and correlation for all WBS methodologies.
3. Run the simulation thereby determining a mean. This is the reference point for calculating risk dollars.
4. Complete defining the remaining uncertainty in the model and re-run the simulation to produce the total uncertainty curve and create the cdf.
5. Select a confidence level.
6. Determine the dollars at the cdf x-axis value at the point of the confidence level.
7. Compute risk dollars as the difference between the desired confidence level and the cost uncertainty mean.

This method has the advantage of associating risk dollars with the all the uncertainty in the model over and above the cost uncertainty. It also has the advantage of establishing a reference point that already captures cost uncertainty. It has several disadvantages, however, that include:

- The need to run two simulations rather than one for each case under consideration
- The added complexity of adding logic to a model in order to “remove” and then “include” specific uncertainties
- Clearly defining what is in and what is out of the “cost uncertainty” in a consistent way across cases within a project and projects within a portfolio

B.5 HOW TO ALLOCATE AND TIME PHASE RISK DOLLARS

See main body.

B.6 HOW TO PRESENT THE “RISK STORY”

See main body.

B.7 ALTERNATIVES TO THE INPUTS-BASED SIMULATION METHOD**B.7.1 OVERVIEW**

See main body.

B.7.2 OUTPUTS-BASED SIMULATION METHOD

See main body.

B.7.3 SCENARIO BASED METHOD

See main body.

B.7.3.1 Overview

See main body.

B.7.3.2 Approach & Assumptions

See main body.

B.7.3.3 Estimating The Point Estimate Probability

For the statistical SBM, the probability

Equation 28

$$P(\text{Cost}_{pgm} \leq x_{PE}) = \alpha_{PE}$$

where Cost_{pgm} is the true, but unknown, total cost of the program and x_{PE} is the program's point estimate cost (PE) is needed. Here, the probability α_{PE} is a judgmental or subjective probability. It is assessed by the engineering and analysis team. In practice, α_{PE} often falls in the interval $0.10 \leq \alpha_{PE} \leq 0.50$.

B.7.3.4 Estimating the Coefficient of Variation (CV)*

The coefficient of variation (CV) is a statistical measure defined as the ratio of distribution's standard deviation σ to its mean μ . It is the recommended way to characterize the variability of the distribution (at one standard deviation around its mean) in a unitless and consistent manner. The general form of the CV is given by

Equation 29

$$CV = \frac{\sigma}{\mu}$$

To date, there are no commodity specific statistics available. Centers are encouraged to set their own standards for selecting the confidence level of the PE and CV for specific commodities. It is not until the last step of this process that these measures come into play. As will be shown in the forthcoming examples, the distribution function of the program's total cost can be derived

* The coefficient of variation is also known as the coefficient of dispersion.

from just the three values. Specifically, with just the point estimate cost (PE), α_{PE} , and CV, the underlying distribution function of the program's total cost can be determined. With this, other possible program costs, such as the protect scenario cost, can be mapped onto the function. From this, the confidence level of the protect scenario and its implied cost reserve can be seen.

B.7.3.5 Statistical SBM With An Assumed Underlying Lognormal

Although there is a host of distribution forms that may be suitable to model the distribution at the top most parent level, the lognormal has been selected as the default because:

- it will not fall below zero
- it is similar in shape to distributions observed from simulations
- it provides for the possibility of a large overrun
- it is simpler to understand and use (compared to the more robust Weibull)
- it is possible to derive the full specification from a few basic assumptions

B.7.3.6 Determine Probability of the Protect Scenario

To determine the confidence level of the protect scenario find $\alpha_{x_{PS}}$ such that

Equation 30

$$P(\text{Cost}_{pgm} \leq x_{PS} = 145) = \alpha_{x_{PS}}$$

Finding $\alpha_{x_{PS}}$ is equivalent to solving

Equation 31

$$\mu_{\ln \text{Cost}_{pgm}} + z_{x_{PS}} (\sigma_{\ln \text{Cost}_{pgm}}) = \ln x_{PS}$$

for $z_{x_{PS}}$. From the above, the following expression may be written

Equation 32

$$z_{x_{PS}} = \frac{\ln x_{PS} - \mu_{\ln \text{Cost}_{pgm}}}{\sigma_{\ln \text{Cost}_{pgm}}}$$

Since $x_{PS} = 145$, $\mu_{\ln \text{Cost}_{pgm}} = 4.80317$, and $\sigma_{\ln \text{Cost}_{pgm}} = 0.29356$ it follows that

Equation 33

$$z_{x_{PS}} = \frac{\ln x_{PS} - \mu_{\ln \text{Cost}_{pgm}}}{\sigma_{\ln \text{Cost}_{pgm}}} = \frac{\ln 145 - 4.80317}{0.29356} = 0.59123$$

From the standard normal look-up table it can be found that $P(Z \leq z_{x_{PS}} = 0.59123) \approx 0.723$

Therefore, the protect scenario cost of 145 (\$M) falls at approximately the 72nd percentile of the distribution with a cost reserve (CR) of 45 (\$M).

B.7.3.7 SBM Worked Example

The SBM method will be applied to the Missile example. In this example the Missile System PE is \$696 M\$ BY2006. The PE mean and standard distribution will be derived using the PE probability 30% and CV 28%. **Figure B-7** illustrates how to solve this problem using Excel.

	Range Name	Value	Equation
Point Estimate	PE	\$696	Protect Scenario Case
Point Estimate Probability	PEcl	0.30	
Coefficient of Variation	CV	0.28	
Log Space PE Mean	LnPEmean	6.68942	$\text{LN}(\text{PE}) - \text{NORMSINV}(\text{PEcl}) * \text{SQRT}(\text{LN}(1 + \text{CV}^2))$
Log Space PE Standard Dev	LnPEsdev	0.27473	$\text{SQRT}(\text{LN}(1 + \text{CV}^2))$
PE Mean	PEmean	\$834.77	$\text{EXP}(\text{LnPEmean} + 0.5 * \text{LnPEsdev}^2)$
PE Standard Deviation	PEstdev	\$233.74	$\text{SQRT}(\text{EXP}(2 * \text{LnPEsdev}^2) * (\text{EXP}(\text{LnPEsdev}^2) - 1))$

Figure B-7 SBM Implementation in Excel

A plot that describes the program cost distribution is shown in **Figure B-8**. This is a plot of a lognormal distribution with mean \$834.77 \$M and standard deviation \$233.74 \$M. The Excel function to obtain the SBM data for this plot is $\text{LOGNORMDIST}(\text{Cost}, \text{LnPEmean}, \text{LnPEsdev})$.

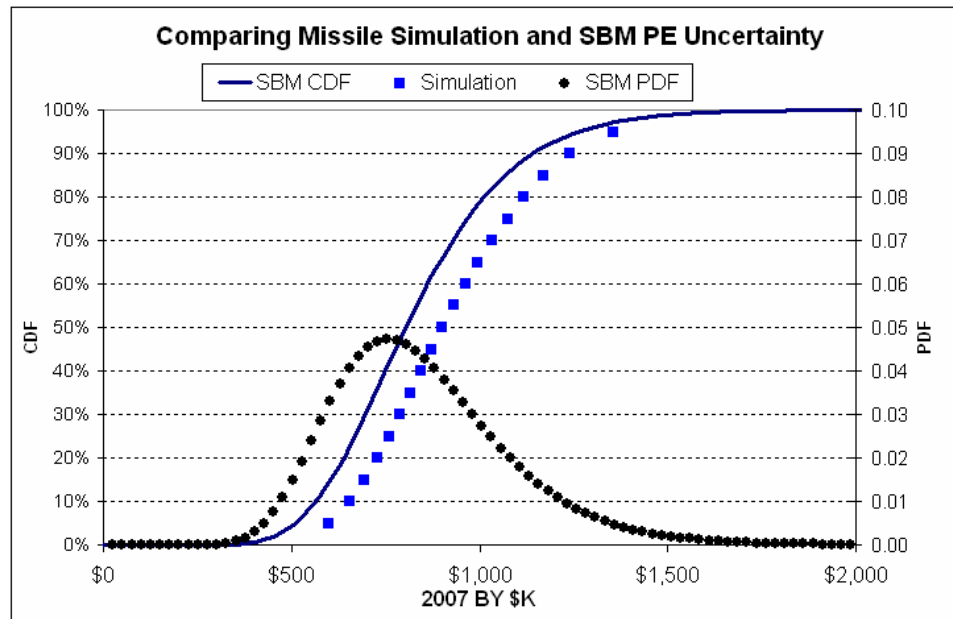


Figure B-8 Plot SBM Implementation in Excel ($\alpha_{PE} = 0.30$, $CV = 0.28$)

The SBM approach does not approximate the full Monte-Carlo solution very well in this example. But the results are very sensitive to the choice of α_{PE} and CV. If α_{PE} is assumed to be 0.20 and CV is 0.30, then the results would be as shown in **Figure B-9**.

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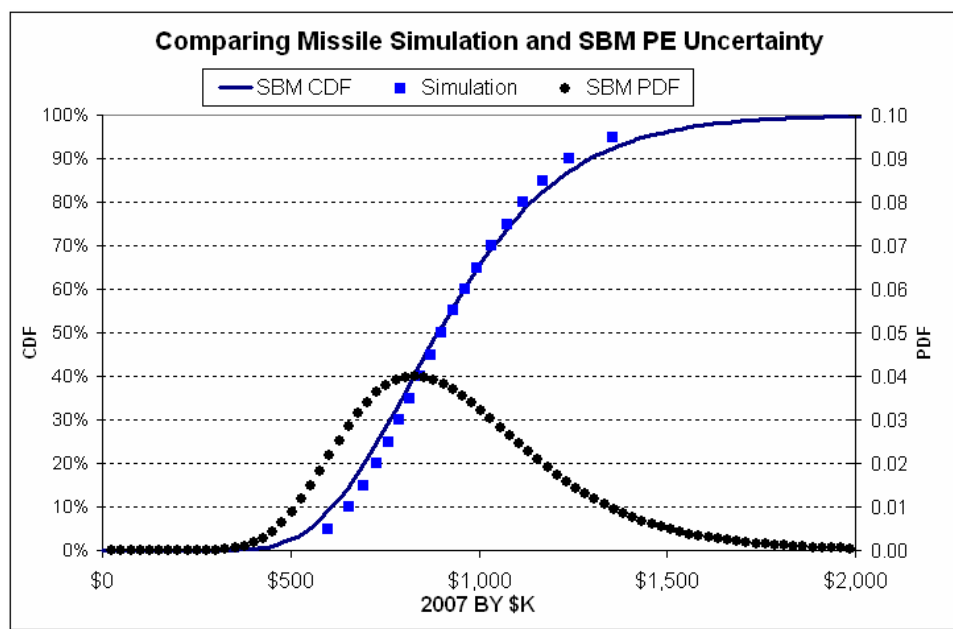


Figure B-9 Plot SBM Implementation in Excel ($\alpha_{PE}=0.20$, $CV=0.30$)

B.8 A DISCUSSION ON HOW TO SELECT A FUNDING LEVEL

See main body.

APPENDIX C MISSILE MODEL IN @RISK, ACE AND CRYSTAL BALL

C.1 EXAMPLE WORKED USING @RISK

The section presents an example case using @RISK. This section is NOT intended to be a @RISK tutorial but is meant only to illustrate the use of guidance contained in the body of this handbook. **Figure C-1** presents the final product. The row and column headings are shown in the figure to be referenced in the text. The remainder of this section illustrates implementation of the concepts described in the body of the handbook as applied to a hypothetical missile system estimate.

AFCAA Cost Risk Analysis Handbook

WBS Description	Range Name	Estimate Method	Point Estimate	CER Uncertainty	Distribution Form	Low	High	Low Interpretation	High Interpretation																																																																																																
* DETAILED ESTIMATE																																																																																																									
Missile System			\$921,910																																																																																																						
SDD Phase			\$218,207																																																																																																						
Air Vehicle	AV_PMP		\$142,378																																																																																																						
Design & Development		[Throughput] 25000	\$30,106	1.20624117	Triangular	0.931	1.508	0.15	0.85																																																																																																
Prototypes		[Factor] ProdT1 * Step	\$14,809																																																																																																						
Software		[Third Party Tool] ThirdPartyToolsSWManMonths * SWLaborRates	\$97,463																																																																																																						
Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LaborRate * SDD_MonthsDur	\$25,195																																																																																																						
System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP	\$36,596																																																																																																						
Training		[Factor] SDD_Trng_Fac * AV_PMP	\$7,993																																																																																																						
Data		[Factor] SDD_Data_Fac * AV_PMP	\$3,197																																																																																																						
Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP	\$2,848																																																																																																						
Production Phase			\$703,703																																																																																																						
Air Vehicle	AV_Prod		\$411,327																																																																																																						
Payload		[Parametric CER: OLS Linear] 30.15 * 1.049 * WarheadWt	\$14,599	1.000	Normal	0.8210	1.1790	0.10	0.90																																																																																																
Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt * 0.6848	\$20,528	1.013	Lognormal	0.8130	1.2501	0.10	0.90																																																																																																
Airframe		[Parametric CER: MUPE Triad] 256.2 * 0.05682 * AirFrameWt ^ 1.374	\$115,732	1.018	Lognormal	1.018	0.18967	mean	std dev																																																																																																
Guidance and Control		[Throughput] 700	\$251,347	1.18610625	Triangular	0.850	1.400	0.08	0.78																																																																																																
Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRate * ProdQty	\$9,126																																																																																																						
Engineering Changes		[Factor] ECO_Fac * AV_ProdS	\$23,309																																																																																																						
Sys Engineering/Program Management		[Factor] SEPM_Fac * AV_ProdS	\$144,536																																																																																																						
System Test and Evaluation		[Throughput] 250 times 4 years of production	\$1,046																																																																																																						
Training		[Factor] Trng_Fac * AV_ProdS	\$49,538	1.0460109	Triangular	0.900	1.200	0.15	0.85																																																																																																
Data		[Factor] Data_Fac * AV_ProdS	\$9,237																																																																																																						
Peculiar Support Equipment		[Factor] PSE_Fac * AV_ProdS	\$9,237																																																																																																						
Common Support Equipment		[Discrete] CSES	\$55																																																																																																						
Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_ProdS	\$55,423																																																																																																						
* INPUT VARIABLES																																																																																																									
* DEVELOPMENT VARIABLES																																																																																																									
SDD Duration (Months)	SDD_MonthsDur		60	64.424	Triangular	54	72	0.10	0.80																																																																																																
Prototype Quantity	DevQty		90	5																																																																																																					
Development Learning Slope	DevSlp		90	90																																																																																																					
Step Increase over Production Cost	Step		1.8	2.216	Triangular	1.500	3.000	0.15	0.85																																																																																																
Software Manmonths From Third Party Tool	ThirdPartyToolsSWManMonths		6400	7187.530	Lognormal	0.8	1.50	PE factor @80%																																																																																																	
Software Labor Rate (\$/hr)	SWLaborRate		12	13.56	Uniform	0.9480	1.3120	0.00	1.00																																																																																																
SEPM Headcount	SDD_SEPM_LOE		25	34.787	Triangular	22,000	27,000	0.18	0.88																																																																																																
SEPM Labor Rate (\$/mo)	SDD_SEPM_LaborRate		14000	15820	Uniform	0.9480	1.3120	0.00	1.00																																																																																																
Sys Test Eval Factor	SDD_STE_Fac		0.2	0.2570	Triangular	0.903	1.710	0.15	0.85																																																																																																
Training Factor	SDD_Trng_Fac		0.05	0.0561	Triangular	0.959	1.505	0.15	0.85																																																																																																
Data Factor	SDD_Data_Fac		0.02	0.0225	Triangular	0.959	1.505	0.15	0.85																																																																																																
Support Equipment Factor	SDD_SptEquip_Fac		0.02	0.02	None																																																																																																				
* PRODUCTION VARIABLES																																																																																																									
Production Quantity	ProdQty		600	600																																																																																																					
Production Learning Slope	ProdSlp		90	90																																																																																																					
Warhead Weight (lbs)	WarHeadWt		12	17.216	Triangular	11	20	0.03	0.73																																																																																																
Motor Weight (lbs)	MotorWt		200	229.488	Triangular	190	250	0.05	0.75																																																																																																
Airframe Weight (lbs)	AirFrameWt		330	337.835	LogNormal	0.8	1.20	PE factor @80%																																																																																																	
Integration, Assembly, Test and Checkout	IACO_HsPerUnit		120	130	Uniform	100	160	0.00	1.00																																																																																																
Manufacturing Labor Rate	MfgLaborRate		90	90	None																																																																																																				
Engineering Changes Factor	ECO_Fac		0.05	0.057	Triangular	0.0200	0.1000	0.00	1.00																																																																																																
SEPM Factor	SEPM_Fac		0.28	0.374	Triangular	0.0285	0.6235	0.10	0.90																																																																																																
Training Factor	Trng_Fac		0.11	0.1204	Triangular	0.931	1.508	0.15	0.85																																																																																																
Data Factor	Data_Fac		0.02	0.0225	Triangular	0.959	1.505	0.15	0.85																																																																																																
PSE Factor	PSE_Fac		0.02	0.0225	Triangular	0.959	1.505	0.15	0.85																																																																																																
Initial Spares Factor	InitSpares_Fac		0.12	0.1347	Triangular	0.959	1.505	0.15	0.85																																																																																																
* Estimating T1 for Production at Vehicle Level																																																																																																									
* Only valid if all production learning slopes are the same																																																																																																									
Prod T1	ProdT1			AV_ProdS.FYTOT / LC_Area.FYTOT				1539.903587																																																																																																	
Converting Development slope to an index				Since slope = 2b * 100, then b = ln(slope/100) / ln(2)				-0.15200309																																																																																																	
Production Quantity Adjusted for Learning	LC_Area			1				267.1122089																																																																																																	
Converting Production Slope to an index				Since slope = 2b * 100, then b = ln(slope/100) / ln(2)				-0.15200309																																																																																																	
								4.339192708																																																																																																	
* Schedule/Technical Penalty Factors																																																																																																									
Payload	PenaltyPayload			1.3	Triangular	0.9	1.5	0	100																																																																																																
Propulsion	PenaltyProp			1.3	Triangular	0.9	1.5	0	100																																																																																																
Airframe	PenaltyAirFrame			1.0	Triangular	0.9	1.1	0	100																																																																																																
Guidance and Control	PenaltyGuidance			1.3	Triangular	0.9	1.5	0	100																																																																																																
Integration, Assembly, Test and Checkout	PenaltyIATC			1.2	Triangular	0.9	2	0	100																																																																																																
<table border="1"> <thead> <tr> <th></th> <th>Point Estimate</th> <th>Discrete Probability</th> <th>Mean Value</th> <th>X1</th> <th>X2</th> <th>P1</th> <th>P2</th> </tr> </thead> <tbody> <tr> <td>Total CSE</td> <td></td> <td>347</td> <td>55</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>CSE Item #1</td> <td></td> <td>21</td> <td>60%</td> <td>21</td> <td>0</td> <td>60%</td> <td>40%</td> </tr> <tr> <td>CSE Item #2</td> <td></td> <td>34</td> <td>60%</td> <td>34</td> <td>0</td> <td>60%</td> <td>40%</td> </tr> <tr> <td>CSE Item #3</td> <td></td> <td>54</td> <td>10%</td> <td>54</td> <td>0</td> <td>10%</td> <td>90%</td> </tr> <tr> <td>CSE Item #4</td> <td></td> <td>13</td> <td>30%</td> <td>13</td> <td>0</td> <td>30%</td> <td>70%</td> </tr> <tr> <td>CSE Item #5</td> <td></td> <td>45</td> <td>10%</td> <td>45</td> <td>0</td> <td>10%</td> <td>90%</td> </tr> <tr> <td>CSE Item #6</td> <td></td> <td>23</td> <td>50%</td> <td>23</td> <td>0</td> <td>50%</td> <td>50%</td> </tr> <tr> <td>CSE Item #7</td> <td></td> <td>42</td> <td>50%</td> <td>42</td> <td>0</td> <td>50%</td> <td>50%</td> </tr> <tr> <td>CSE Item #8</td> <td></td> <td>49</td> <td>10%</td> <td>49</td> <td>0</td> <td>10%</td> <td>90%</td> </tr> <tr> <td>CSE Item #9</td> <td></td> <td>39</td> <td>40%</td> <td>39</td> <td>0</td> <td>40%</td> <td>60%</td> </tr> <tr> <td>CSE Item #10</td> <td></td> <td>27</td> <td>50%</td> <td>27</td> <td>0</td> <td>50%</td> <td>50%</td> </tr> </tbody> </table>											Point Estimate	Discrete Probability	Mean Value	X1	X2	P1	P2	Total CSE		347	55					CSE Item #1		21	60%	21	0	60%	40%	CSE Item #2		34	60%	34	0	60%	40%	CSE Item #3		54	10%	54	0	10%	90%	CSE Item #4		13	30%	13	0	30%	70%	CSE Item #5		45	10%	45	0	10%	90%	CSE Item #6		23	50%	23	0	50%	50%	CSE Item #7		42	50%	42	0	50%	50%	CSE Item #8		49	10%	49	0	10%	90%	CSE Item #9		39	40%	39	0	40%	60%	CSE Item #10		27	50%	27	0	50%	50%
	Point Estimate	Discrete Probability	Mean Value	X1	X2	P1	P2																																																																																																		
Total CSE		347	55																																																																																																						
CSE Item #1		21	60%	21	0	60%	40%																																																																																																		
CSE Item #2		34	60%	34	0	60%	40%																																																																																																		
CSE Item #3		54	10%	54	0	10%	90%																																																																																																		
CSE Item #4		13	30%	13	0	30%	70%																																																																																																		
CSE Item #5		45	10%	45	0	10%	90%																																																																																																		
CSE Item #6		23	50%	23	0	50%	50%																																																																																																		
CSE Item #7		42	50%	42	0	50%	50%																																																																																																		
CSE Item #8		49	10%	49	0	10%	90%																																																																																																		
CSE Item #9		39	40%	39	0	40%	60%																																																																																																		
CSE Item #10		27	50%	27	0	50%	50%																																																																																																		

Figure C-1 Completed Missile Example in @RISK

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The completed point estimate as described in **Section 1.4** is depicted in Figure C-2.

WBS Description	Range Name	Estimate Method	Point Estimate
Missile System			\$696,110
SDD Phase			\$164,898
Air Vehicle	AV_PMP		\$111,549
Design & Development		[Throughput] 25000	\$25,000
Prototypes		[Factor] ProdT1 * Step	\$9,749
Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$	\$76,800
Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LabRate * SDD_MonthsDur	\$21,000
System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP	\$22,310
Training		[Factor] SDD_Trng_Fac * AV_PMP	\$5,577
Data		[Factor] SDD_Data_Fac * AV_PMP	\$2,231
Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP	\$2,231
Production Phase			\$531,212
Air Vehicle	AV_Prod		\$333,396
Payload		[Parametric CER: OLS Linear] 30.15 + 1.049 * WarheadWt	\$11,416
Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt ^ 0.6848	\$16,271
Airframe		[Parametric CER: MUPE Triad] 256.2 + 0.05682 * AirFrameWt ^ 1.374	\$112,250
Guidance and Control		[Throughput] 700	\$186,979
Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRate\$ * ProdQty	\$6,480
Engineering Changes		[Factor] ECO_Fac * AV_Prod\$	\$16,670
Sys Engineering/Program Management		[Factor] SEPM_Fac * AV_Prod\$	\$93,351
System Test and Evaluation		[Throughput] 250 times 4 years of production	\$1,000
Training		[Factor] Trng_Fac * AV_Prod\$	\$33,340
Data		[Factor] Data_Fac * AV_Prod\$	\$6,668
Peculiar Support Equipment		[Factor] PSE_Fac * AV_Prod\$	\$6,668
Common Support Equipment		[Discrete] CSES	\$113
Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_Prod\$	\$40,007

Figure C-2 Complete the Point Estimate

Figure C-3, **Figure C-4,** and

Figure C-5 depict the application of objective uncertainty to three types of parametric CERs as

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described in Section 2.2.2. These uncertainty ranges are applied to rows 25, 26, and 27 in the example.

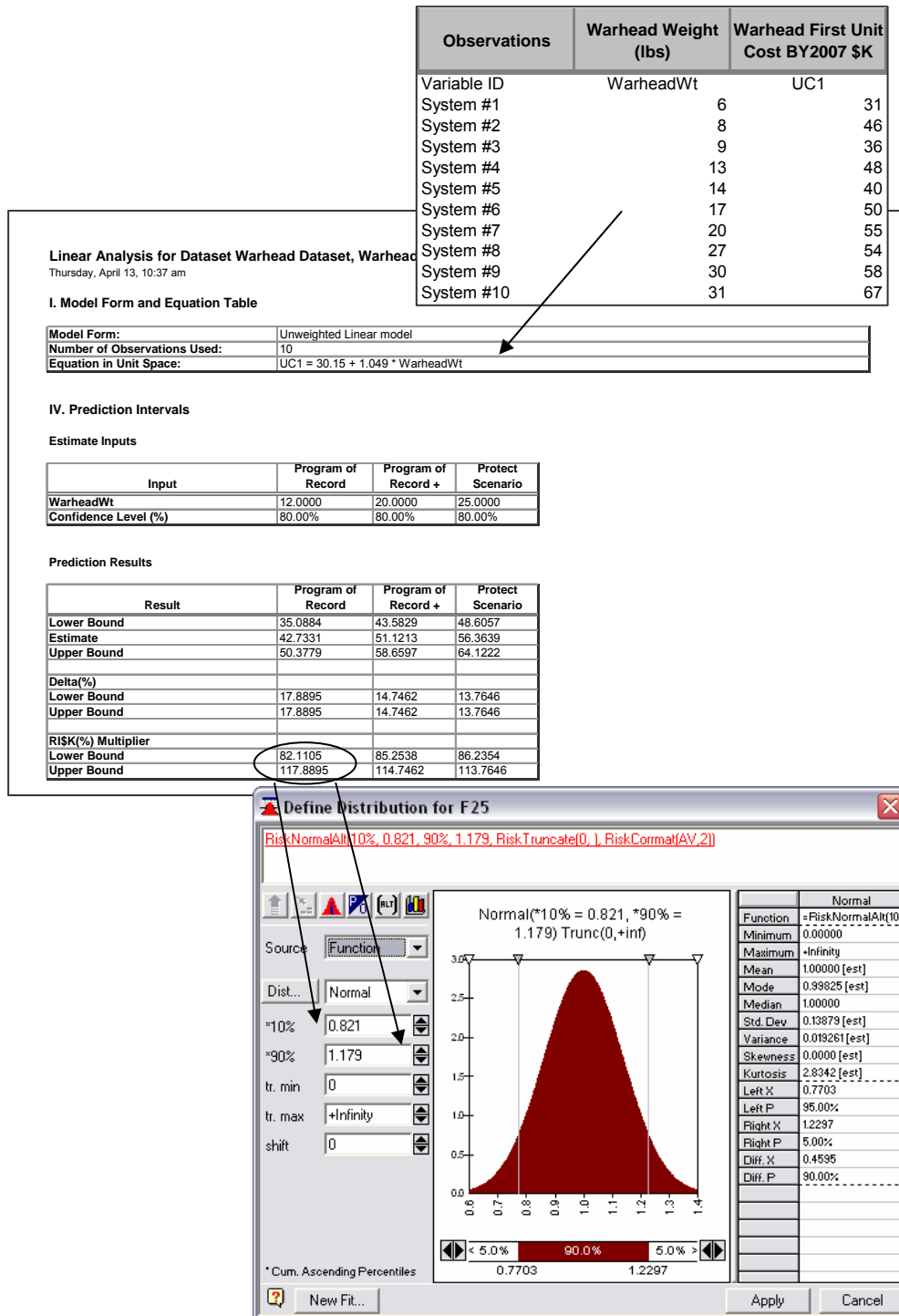


Figure C-3 Linear CER

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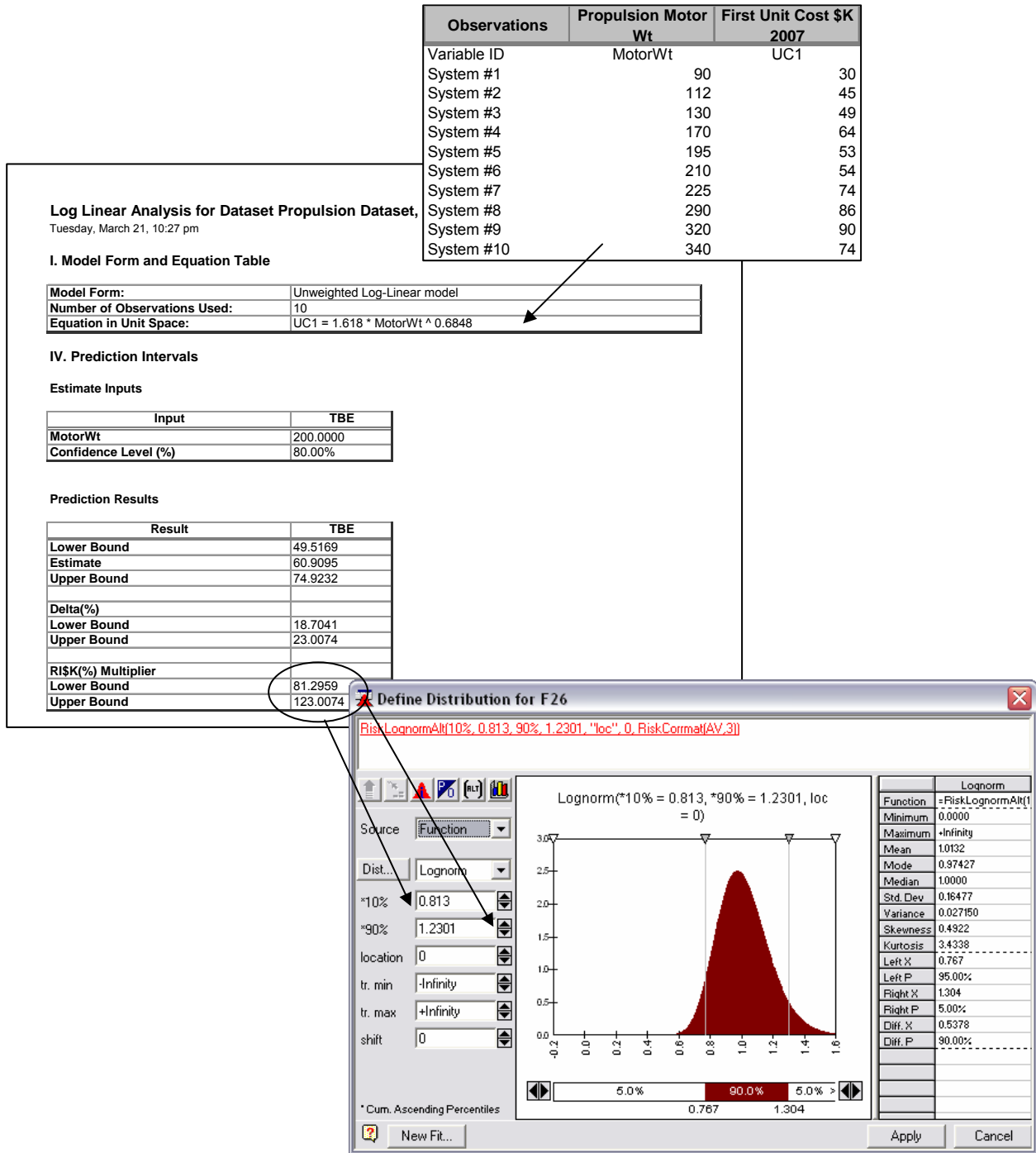


Figure C-4 Log CER

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Observations	Airframe Wt (lbs)	First Unit Cost \$K2007
Variable ID	AirFrame Wt	AirUC1
System #1	911	858.6
System #2	888	894.1
System #3	1080	1139.4
System #4	912	1001.4
System #5	1416	1426.6
System #6	533	457.8
System #7	515	577.6
System #8	868	840.9
System #9	1060	892.7
System #10	802	1059.0

Non Linear Analysis for Dataset Airframe Dataset, Case 5

Thursday, July 27, 1:46 pm

I. Equation Form & Error Term

Model Form:	Weighted Non-Linear Model
Non-Linear Equation:	AirUC1 = 256.2 + 0.05682 * AirFrameWt ^ 1.374
Error Term:	MUPE (Minimum-Unbiased-Percentage Error)
Minimization Method:	Modified Marquardt

II. Fit Measures

Coefficient Statistics Summary

Variable/Term	Coefficient Estimate	Approximate Std Error	Approximate Lower 95% Confidence	Approximate Upper 95% Confidence
b	0.0568	0.0977		
c	1.3736	0.2515		

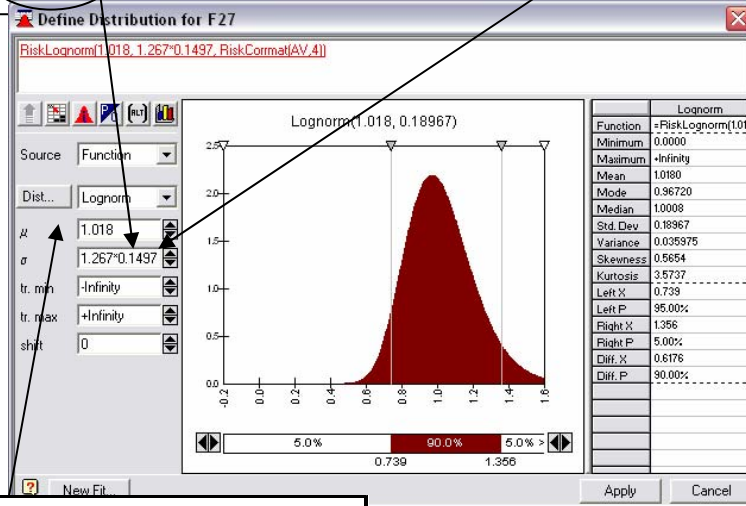
Least Squares Minimization Summary Statistics

Source	DF	Sum of Squares (SS)
Residual (Error)	8	385238.8779
Total (Corrected)	9	2080481.3927

Goodness-of-Fit Statistics

Std. Error (SE)	Approx. R-Squared	Approx. R-Squared (Adj)
0.1497	81.48%	79.17%

SE Multiplier	Number of Data Points in Sample								
	5	10	15	20	25	30			
0.00	1.095	1.049	1.033	1.025	1.020	1.017	Very Similar		
0.25	1.101	1.052	1.035	1.026	1.021	1.018			
0.50	1.118	1.061	1.041	1.031	1.025	1.021			
0.75	1.146	1.075	1.051	1.038	1.031	1.026	Similar		
1.00	1.183	1.095	1.065	1.049	1.039	1.033			
1.25	1.230	1.121	1.082	1.062	1.050	1.042			
1.50	1.285	1.151	1.103	1.078	1.063	1.053	Dissimilar		
1.75	1.346	1.186	1.127	1.097	1.078	1.066			
2.00	1.414	1.225	1.155	1.118	1.095	1.080			
2.25	1.487	1.267	1.185	1.142	1.115	1.096	Different		
2.50	1.565	1.313	1.218	1.167	1.136	1.114			
2.75	1.647	1.362	1.253	1.195	1.159	1.134			
					291	1.225	1.183	1.155	Very Different



$EXP(0+(1.267*0.1497)^2/2)$

Figure C-5 Triad MUPE CER

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Figure C-6 depicts the application of subjective bounds to an expert opinion estimate as described in **Section 2.3**. This technique applies to Rows 45, 48, and 58 in the example. The use of **Figure 2-5**'s template is illustrated for row SEPM Headcount.

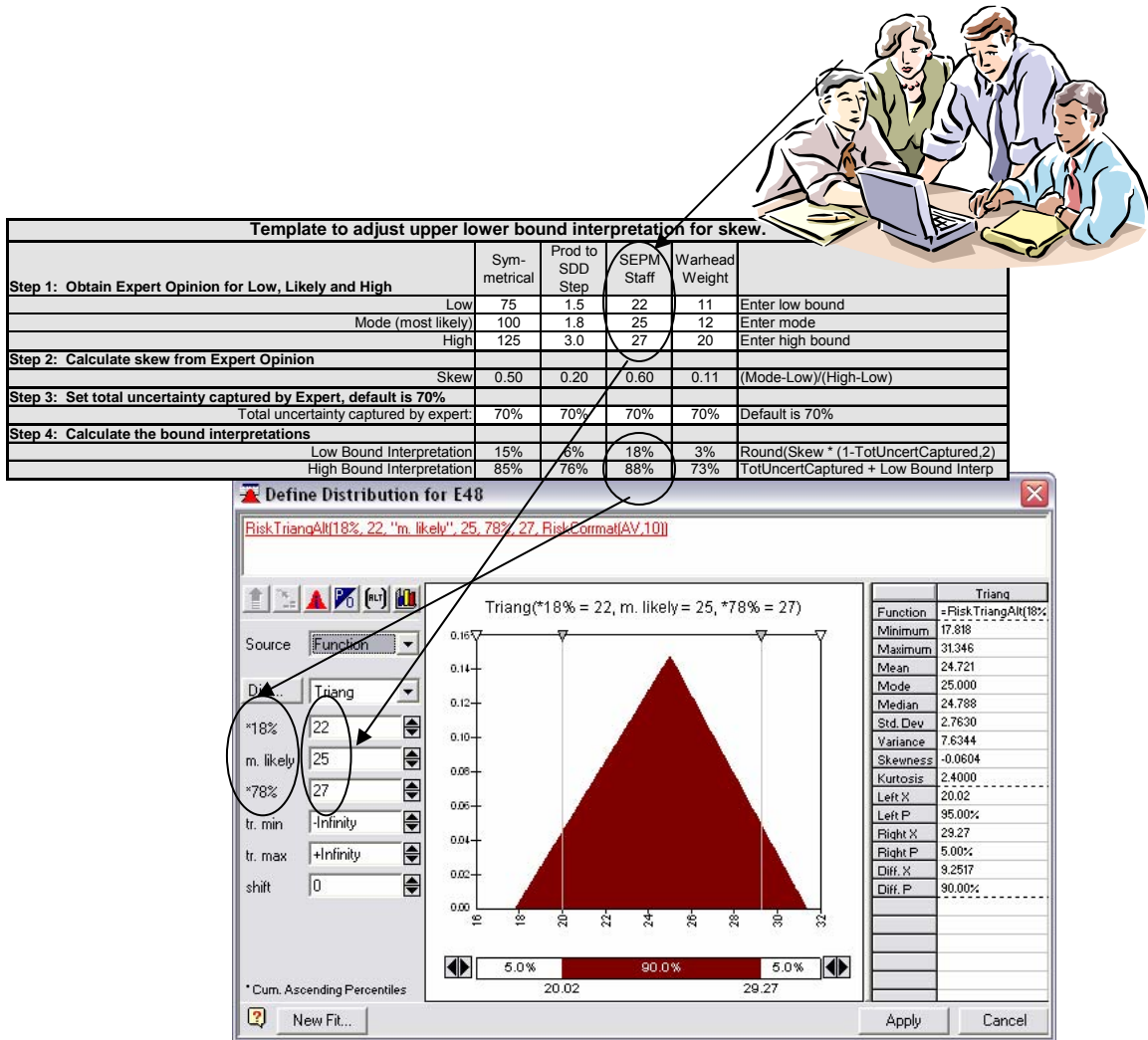


Figure C-6 Subjective Uncertainty

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Rows 25-27 also illustrate the application of combined uncertainty as discussed in **Section 2.3**. Cost estimating uncertainty is applied to the CERs in those rows and applied to their cost drivers in rows 58-60. This is depicted in **Figure C-7**.

WBS Elements			Form	Adj SE	Low	Low Intrp	High	High Intrp	Comment
Payload	\$11,416	$30.15 + 1.049 * \text{WarheadWt}$	Normal		82.11%	10	117.90%	90	Regression Result
Propulsion	\$16,271	$1.618 * \text{MotorWt} ^ 0.6848$	LogNormal		81.30%	10	123.01%	90	Regression Result
Airframe	\$112,250	$256.2 + 0.05682 * \text{AirFrameWt} ^ 1.374$	LogNormal	0.1897					Regression Result
Guidance and Control	\$186,979	700	Triangular		85%	8	140%	78	Expert Opinion
Warhead Weight (lbs)	12.0	WarheadWt	Triangular		11	3	20	73	
Motor Weight (lbs)	200.0	MotorWt	Triangular		190	5	250	75	
Airframe Weight (lbs)	330.0	AirFrameWt	LogNormal				120%	80	

Figure C-7 Combining Uncertainties

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Default subjective bounds as discussed in **Section 2.5** are applied to several elements in the example. **Figure C-8** depicts this for the Design & Development cost throughput on row 14. In this case the estimate is deemed to be triangular distribution right skewed with medium uncertainty. A segment of **Table 2-5** is shown. This technique is applied to cost factors on rows 50-52 and 65-68 and to labor rates on rows 47 and 49.

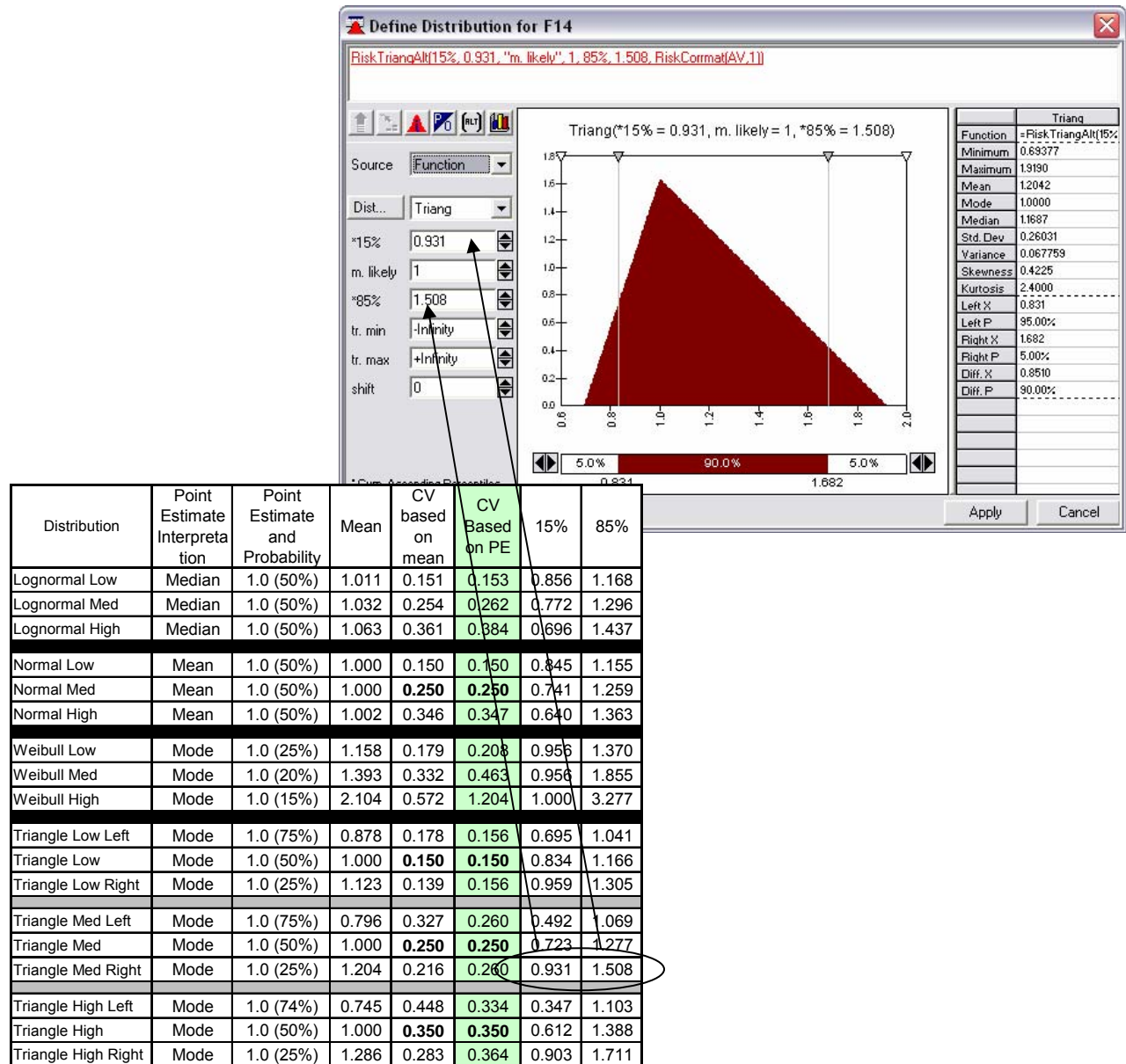


Figure C-8 Default Subjective Bounds

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Figure C-9 illustrates the treatment of an input from a feeder model on row 46 as described in **Section 2.8**. In this example the mean value was obtained from the feeder model. The 80% value was determined to be 150% of the mean from use of the feeder model's own internal risk modeling capability.

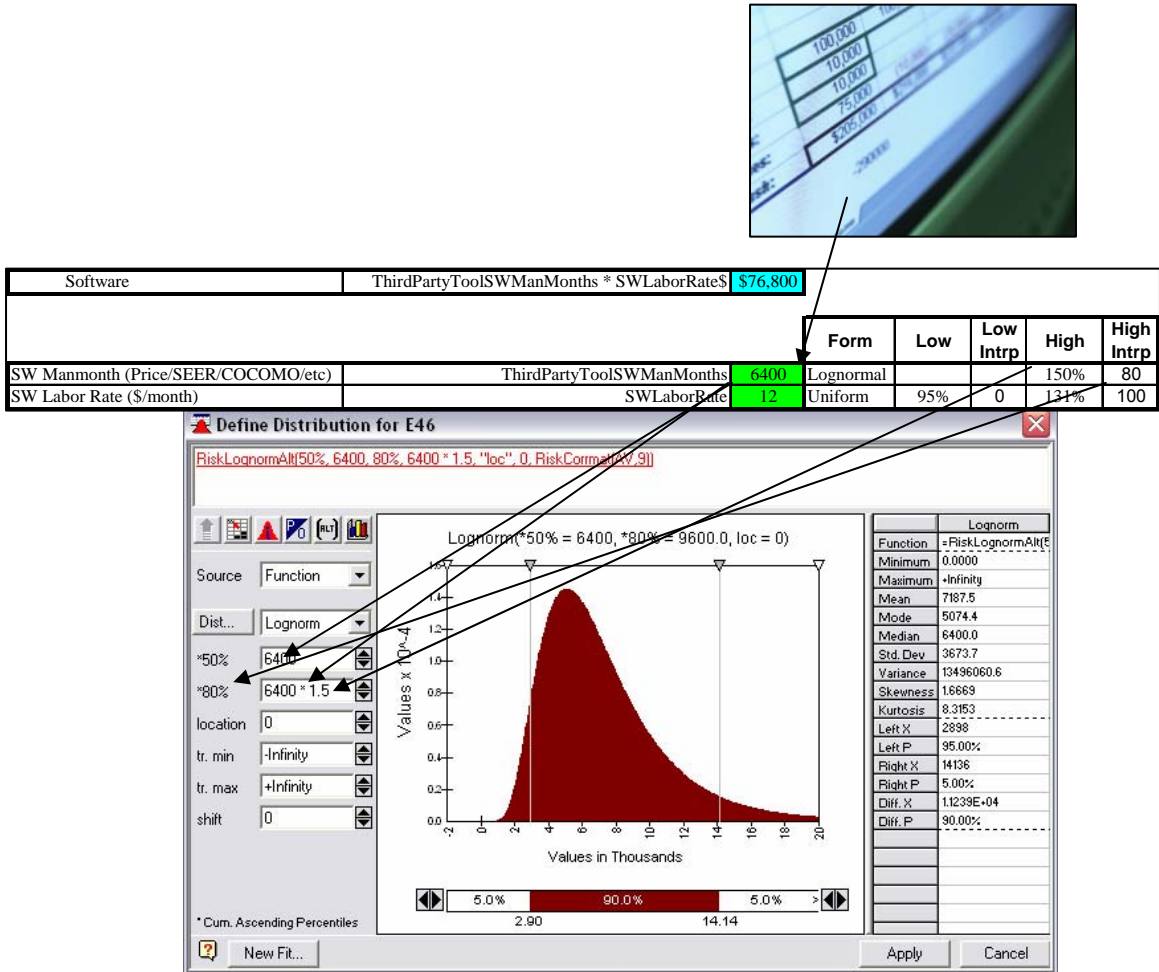


Figure C-9 Feeder Model

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Application of specific technical/schedule uncertainty considerations as described in **Section 2.9** is illustrated in **Figure C-10**. This technique was applied to rows 25-29 in the example model.

WBS Elements

Payload	\$11,416	$(30.15 + 1.049 * \text{WarheadWt}) * \text{PenaltyPayload}$
Propulsion	\$16,271	$1.618 * \text{MotorWt}^{0.6848} * \text{PenaltyProp}$
Airframe	\$112,250	$(256.2 + 0.05682 * \text{AirFrameWt}^{1.374}) * \text{PenaltyAirFrame}$
Guidance and Control	\$186,979	$700 * \text{PenaltyGuidance}$
Integration, Assembly, Test and Checkout	\$6,480	$\text{IACO_HsPerUnit} * \text{MfgLaborRate\$} * \text{ProdQty} * \text{PenaltyIATC}$

Schedule/Technical Penalty Factors			Absolute Bounds	
			Low	High
Payload	1.0	Triangular	0.9	1.5
Propulsion	1.0	Triangular	0.9	1.5
Airframe	1.0	Triangular	0.9	1.1
Guidance and Control	1.0	Triangular	0.9	1.5
Integration, Assembly, Test and Checkout	1.0	Triangular	0.9	2.0

Figure C-10 Penalty Method

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Figure C-11 presents an application of discrete risk as described in **Section 2.7**. In this example ten items of CSE are will either be needed at the certain cost shown in column D or will not be needed as a cost of zero. Column E contains the probability the item will be needed. The figure also shows use of the @RISK Discrete distribution with the probability of an item being needed linked to cell E91. Column F rolls up to the CSE value utilized in row 36 of the estimate.

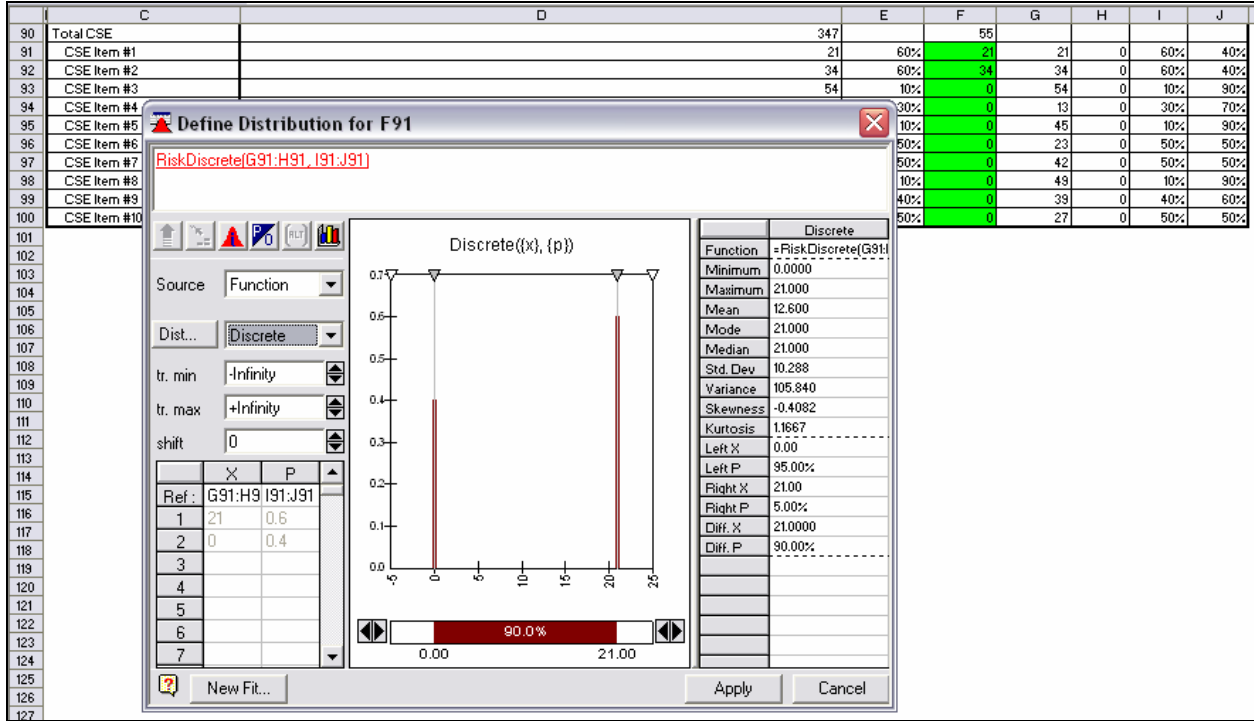


Figure C-11 Discrete Distribution

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Figure C-12 depicts the measurement of correlation due to functional relationships in the SDD phase of the missile example as described in **Section 3.2.2**.

	Missile System	SDD Phase	Air Vehicle	Design & Development	Prototypes	Software	Sys Eng/ PM	Sys Test and Eval	Training	Data
@Risk Resultant Pearson Product Moment Correlation										
Missile System	1.00	0.72	0.68	0.41	0.68	0.60	0.47	0.69	0.70	0.70
SDD Phase		1.00	0.99	0.39	0.46	0.96	0.44	0.92	0.95	0.95
Air Vehicle			1.00	0.37	0.43	0.98	0.37	0.86	0.95	0.95
Design & Dev				1.00	0.31	0.24	0.28	0.38	0.39	0.39
Prototypes					1.00	0.31	0.36	0.45	0.46	0.46
Software						1.00	0.32	0.83	0.92	0.92
Sys Eng/P M							1.00	0.39	0.40	0.40
Sys Test and Eval								1.00	0.84	0.84
Training									1.00	0.91
Data										1.00

@RISK Output Data Report										
Output Data										
	Missile System	SDD Phase	Air Vehicle	Design & Development	Prototypes	Software	Sys Eng/ PM	Sys Test and Eval	Training	Data
Output Simulation Iteration / Cell	1	1	1	1	1	1	1	1	1	1
	\$E\$11	\$E\$12	\$E\$13	\$E\$14	\$E\$15	\$E\$16	\$E\$17	\$E\$18	\$E\$19	\$E\$20
1	\$635,032	\$210,311	\$146,886	\$33,105	\$10,218	\$103,563	\$23,484	\$24,555	\$8,645	\$3,802
2	\$1,178,031	\$215,199	\$133,405	\$30,371	\$17,869	\$85,166	\$28,878	\$38,337	\$8,152	\$3,758
3	\$695,423	\$113,337	\$68,708	\$23,799	\$5,404	\$39,505	\$18,814	\$19,490	\$3,544	\$1,406
4	\$886,406	\$327,644	\$227,875	\$26,806	\$16,670	\$184,399	\$16,758	\$62,524	\$11,413	\$4,515
5	\$932,319	\$188,491	\$128,584	\$30,499	\$10,019	\$88,065	\$19,423	\$27,684	\$7,826	\$2,403
6	\$861,790	\$180,502	\$119,379	\$30,837	\$20,392	\$68,150	\$27,000	\$22,417	\$6,993	\$2,325
7	\$1,234,580	\$315,612	\$183,699	\$44,775	\$28,928	\$109,996	\$31,683	\$79,150	\$12,603	\$4,803
8	\$769,173	\$183,203	\$121,182	\$24,860	\$14,788	\$81,535	\$22,183	\$28,231	\$6,739	\$2,444
9	\$735,429	\$152,790	\$107,770	\$24,047	\$8,218	\$75,504	\$18,319	\$17,341	\$5,161	\$2,045
10	\$779,498	\$148,790	\$102,647	\$36,350	\$6,433	\$59,864	\$16,160	\$21,439	\$4,623	\$1,867
11	\$1,156,703	\$205,139	\$127,170	\$37,298	\$24,099	\$65,773	\$26,150	\$37,842	\$8,208	\$3,225

Figure C-12 Measured Correlation

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Figure C-13 depicts the specification of additional correlation as described in **Section 3.2.3**. In this case all the elements shown were deemed to be weakly correlated and assigned values of 0.25. In the example case four sets of correlation matrices are specified. **Figure C-47**, **Figure C-48**, and **Figure C-49** present the other three correlation matrices.

The screenshot shows the @RISK - Model - [Correlation] window. The main area displays a correlation matrix for the 'AV (20x20)' model. The matrix is a lower triangular matrix where the diagonal elements are 1.000 and all other elements are 0.250. The parameters listed in the rows are:

AV (20x20)	AFCAA Sample Missile EstimatelF14 [Throughput] 25000 / CER	AFCAA Sample Missile EstimatelF25 [Parametric CER: OLS Linear]	AFCAA Sample Missile EstimatelF26 [Parametric CER: OLS Loglinear]	AFCAA Sample Missile EstimatelF27 [Parametric CER: MUPE Triad]	AFCAA Sample Missile EstimatelF28 [Throughput] 700 / CER	AFCAA Sample Missile EstimatelF32 [Throughput] 250 times 4 years	AFCAA Sample Missile EstimatelE42 SDD_MthsDur	AFCAA Sample Missile EstimatelE45 Step	AFCAA Sample Missile EstimatelE46 ThirdPartyToolsSWMonths	AFCAA Sample Missile EstimatelE48 SDD_SEPM_LDE	AFCAA Sample Missile EstimatelF50 SDD_STE_Fac / CER	AFCAA Sample Missile EstimatelF51 SDD_Tmg_Fac / CER	AFCAA Sample Missile EstimatelF52 SDD_Data_Fac / CER	AFCAA Sample Missile EstimatelF54
AFCAA Sample Missile EstimatelF14 [Throughput] 25000 / CER	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF25 [Parametric CER: OLS Linear]	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF26 [Parametric CER: OLS Loglinear]	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF27 [Parametric CER: MUPE Triad]	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF28 [Throughput] 700 / CER	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF32 [Throughput] 250 times 4 years	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelE42 SDD_MthsDur	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelE45 Step	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelE46 ThirdPartyToolsSWMonths	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelE48 SDD_SEPM_LDE	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF50 SDD_STE_Fac / CER	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250	0.250
AFCAA Sample Missile EstimatelF51 SDD_Tmg_Fac / CER	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250	0.250
AFCAA Sample Missile EstimatelF52 SDD_Data_Fac / CER	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000	0.250
AFCAA Sample Missile EstimatelF54	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1.000

Figure C-13 Enter Correlation

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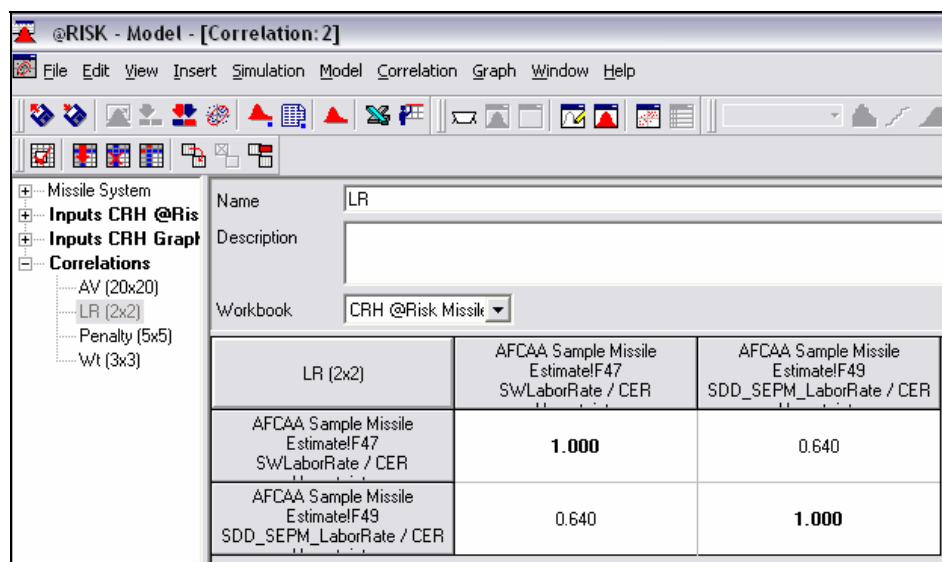


Figure C-14 Labor Rate Correlation Matrix

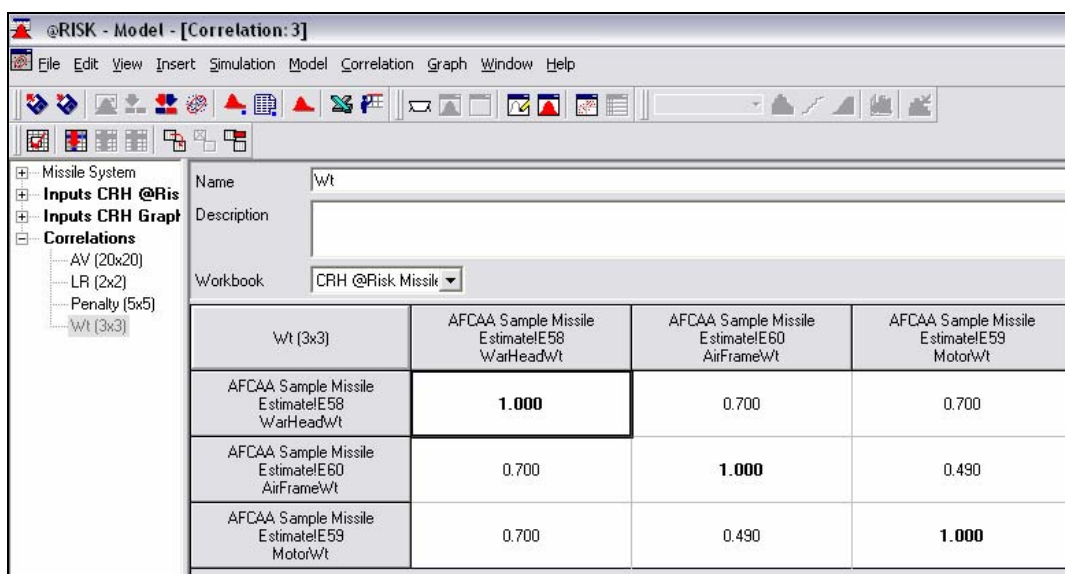


Figure C-15 Weight Correlation Matrix

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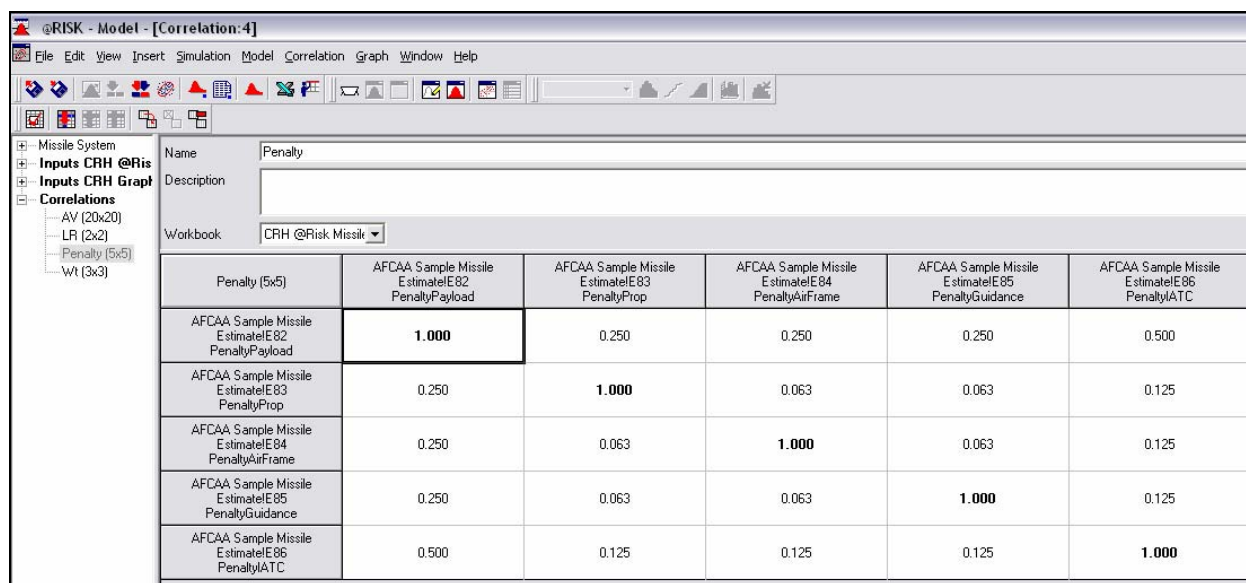


Figure C-16 Schedule/Technical Penalty Correlation Matrix

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Figure C-17 illustrates the calculation of risk dollars as described in Section 4.1. The SDD phase and the Production phase risk dollars are calculated using by subtracting the selected confidence level (60%) values from the point estimate.

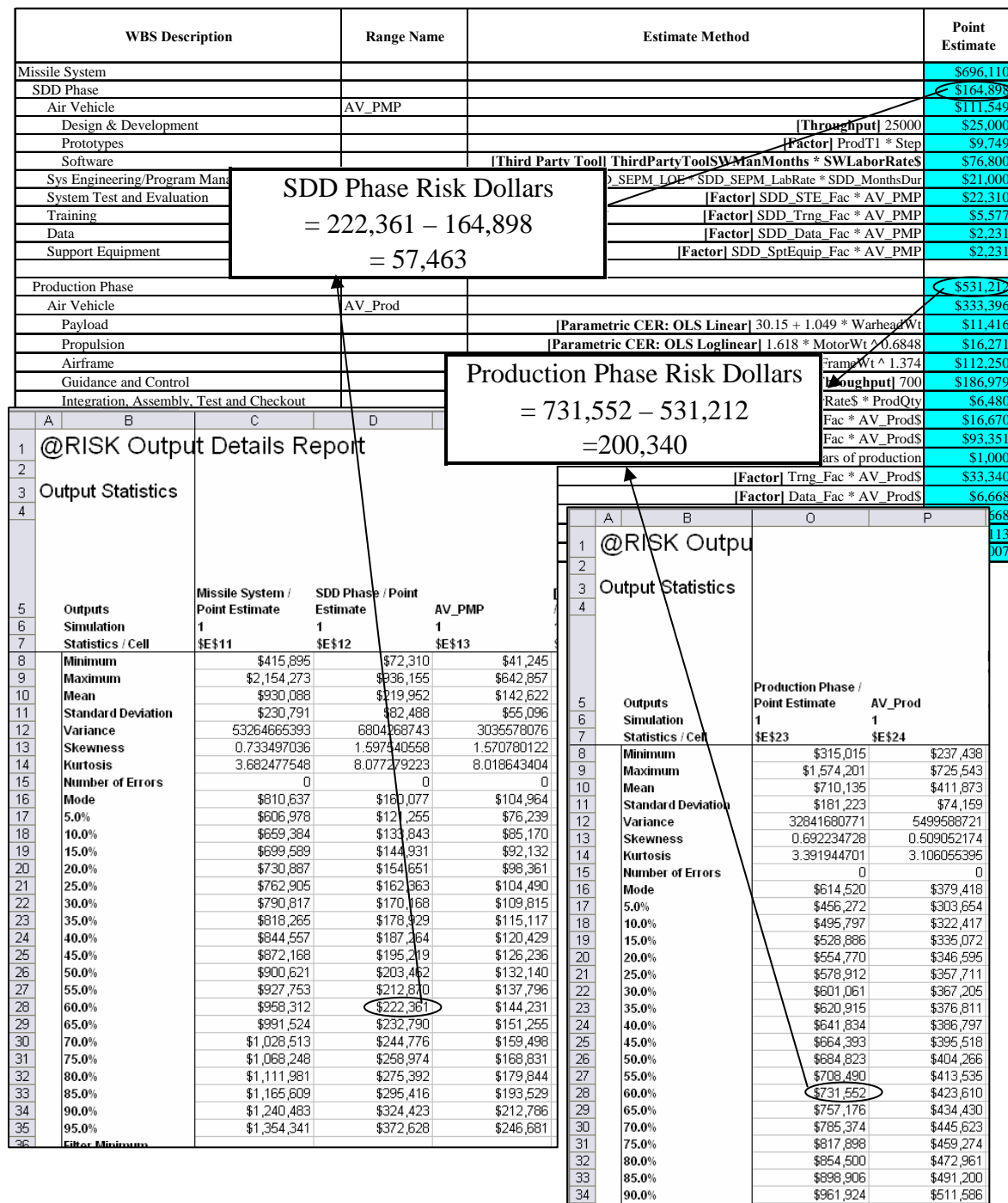


Figure C-17 Calculate Risk Dollars

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Figure C-18 presents the allocation of the risk dollars to the child elements as described in **Section 5.2**.

	Point Estimate	Std Dev	60% Confidence Level From Risk Model	Sum of Children	Difference Between Parent and Sum of Children	Sum of Children's Std Dev	Prorated Difference to Each Child	Difference Between Parent (Incl its Proration) and Sum of Children	Prorated Difference to Each Child	Sum of 60% From Risk Model plus Proration	Difference from Statistical Result
Missile System	\$696,110	\$231,798	\$963,145								
Sys Dev & Demo Phase	\$164,898	\$81,542	\$221,766	\$221,595	\$171	\$87,163				\$221,766	0.0%
Air Vehicle	\$111,549	\$54,857	\$144,041	\$144,322		\$63,262	\$108	(\$173)		\$144,149	0.1%
Design & Development	\$25,000	\$6,509	\$31,196						(\$18)	\$31,178	-0.1%
Prototypes	\$9,749	\$6,044	\$15,674						(\$17)	\$15,657	-0.1%
Software	\$76,800	\$50,709	\$97,452						(\$139)	\$97,313	-0.1%
Sys Engineering/Program Management	\$21,000	\$4,958	\$25,732				\$10			\$25,742	0.0%
System Test and Evaluation	\$22,310	\$21,091	\$37,562				\$41			\$37,603	0.1%
Training	\$5,577	\$3,680	\$8,112				\$7			\$8,119	0.1%
Data	\$2,231	\$1,480	\$3,267				\$3			\$3,270	0.1%
Support Equipment	\$2,231	\$1,097	\$2,881				\$2			\$2,883	0.1%
			0								
Production Phase	\$531,212	\$181,997	\$734,632	\$739,591	(\$4,959)	\$213,354				\$734,632	0.0%
Air Vehicle	\$333,396	\$74,435	\$424,253	\$426,297		\$98,189	(\$1,730)	(\$3,774)		\$422,523	-0.4%
Propulsion	\$11,416	\$3,006	\$15,065						(\$116)	\$14,949	-0.8%
Payload	\$16,271	\$4,499	\$21,116						(\$173)	\$20,943	-0.8%
Airframe	\$112,250	\$26,776	\$119,916						(\$1,029)	\$118,887	-0.9%
Guidance and Control	\$186,979	\$61,745	\$260,818						(\$2,373)	\$258,445	-0.9%
Integration, Assembly, Test and Chec	\$6,480	\$2,163	\$9,382						(\$83)	\$9,299	-0.9%
Engineering Changes	\$16,670	\$9,092	\$24,799				(\$211)			\$24,588	-0.9%
Sys Engineering/Program Management	\$93,351	\$94,298	\$160,801				(\$2,192)			\$158,609	-1.4%
System Test and Evaluation	\$1,000	\$135	\$1,074				(\$3)			\$1,071	-0.3%
Training	\$33,340	\$16,003	\$51,664				(\$372)			\$51,292	-0.7%
Data	\$6,668	\$2,400	\$9,613				(\$56)			\$9,557	-0.6%
Peculiar Support Equipment	\$6,668	\$2,424	\$9,611				(\$56)			\$9,555	-0.6%
Common Support Equipment	\$113	\$47	\$124				(\$1)			\$123	-0.9%
Initial Spares and Repair Parts	\$40,007	\$14,520	\$57,652				(\$337)			\$57,315	-0.6%

Figure C-18 Allocate Risk Dollars

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C.2 EXAMPLE WORKED USING ACE

The section presents an example case using ACE 7.0. This section is NOT intended to be an ACE tutorial but is meant only to illustrate the use of guidance contained in the body of this handbook. **Figure C-19** presents one screen of the final product. The remainder of this section illustrates implementation of the concepts described in the body of the handbook as applied to a hypothetical missile system estimate.

WBS/ACES Description	Unique ID	Equation / Throughput	Program of Record	Phasing Method	Fiscal Year	Units	Start Date	Finish Date	Approp
* DETAILED ESTIMATE									
Missile System			\$ 696,344 (15%) *						
Sys Dev and Demo	SDD\$		\$ 164,898 (26%) *						3600
Air Vehicle	AV_PMP\$		\$ 111,549 (32%) *						3600
Design & Development		[Throughput] 25000	\$ 25,000 (25%) *	BE	2007	\$K	SDD_Start	SDD_End	3600
Prototypes		[Factor] ProdTI * Step	\$ 9,749 (20%) *	R					3600
Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$	\$ 76,800 (41%) *	BE			SDD_Start	SDD_End	3600
Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LaborRate\$ * SDD_MonthsDur * FYTrapz(SDD_Start, SDD_End, 0, 1)	\$ 21,000 (24%) *	F					3600
System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP\$	\$ 22,310 (23%) *	F					3600
Training		[Factor] SDD_Tng_Fac * AV_PMP\$	\$ 5,577 (25%) *	F					3600
Data		[Factor] SDD_Data_Fac * AV_PMP\$	\$ 2,231 (25%) *	F					3600
Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP\$	\$ 2,231 (32%) *	F					3600
Production Phase	Prod\$		\$ 531,446 (16%) *						3020
Air Vehicle	AV_Prod\$		\$ 333,396 (15%) *						3020
Payload		[Parametric CER: OLS Linear] (30.15 + 1.049 * WarheadWt)	\$ 11,416 (14%) *	R	2007	\$K			3020
Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt * 0.6948	\$ 16,271 (17%) *	R	2007	\$K			3020
Airframe		[Parametric CER: MUPE Triad] (256.2 + 0.05682 * [Expert Opinion] 700 * PenaltyGuidance	\$ 186,979 (15%) *	R	2007	\$K			3020
Guidance and Control		[Build-up] IACO_HiPerUnit * MtgLaborRate\$ * ProdQty *	\$ 6,480 (9%) *	F	2007	\$			3020
Integration, Assembly, Test and Checkout		[Factor] ECO_Fac * AV_Prod\$	\$ 16,670 (24%) *	F					3020
Engineering Changes		[Factor] SEPM_Fac * AV_Prod\$	\$ 93,351 (32%) *	F					3020
Sys Engineering/Program Management		[Throughput Cost per FY Production] If(ProdQty > 0, 250, 0)	\$ 1,000 (40%) *	F	2007	\$K			3020
System Test and Evaluation									
Training		[Factor] Tng_Fac * AV_Prod\$	\$ 33,340 (13%) *	F					3020
Data		[Factor] Data_Fac * AV_Prod\$	\$ 6,668 (13%) *	F					3020
Peculiar Support Equipment		[Factor] PSE_Fac * AV_Prod\$	\$ 6,668 (12%) *	F					3020
Common Support Equipment		[Discrete] CSE\$ * FYCRel@AV_Prod\$	\$ 347 (99+%)*	F					3020
Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_Prod\$	\$ 40,007 (12%) *	F					3020
* INPUT VARIABLES	*IN_VAR								
Start of System Development and Demonstration (SDD_Start	01Oct2007	01OCT2007 *	C					
End of SDD	SDD_End	DateAdd(SDD_Start, 0, SDD_MonthsDur, SDD_DaysDur) - 1	30SEP2012 (33%) *	C					
Prototype Quantity	DevQty	FYSlide(DateF(SDD_Start) + 1)	5 *	F					
Production Quantity	ProdQty	FYSlide(DateF(SDD_End))	600 *	F					
* DEVELOPMENT VARIABLES									
SDD Duration (Months)	SDD_MonthsDur	60	60 (33%) *	C					
Development Learning Slope	DevSlp	90	90 *	C					
Step Increase over Production Cost	Step	1.8	1.80 (31%) *	C					
Software Manmonth From Price/SEER/COCOMO/	ThirdPartyToolSWManMonths	6400	6,400.00 (30%) *	C					
Software Labor Rate (\$/month)	SWLaborRate\$	12,455.30 BY2005 \$ (14%) *		C	2007	\$K			3600
SEPM Headcount	SDD_SEPM_LOE	25	25 (60%) *	C					
SEPM Labor Rate (\$/mo)	SDD_SEPM_LaborRate\$	14000	\$ 14 (14%) *	C	2007	\$			3600
Sys Test Eval Factor	SDD_STE_Fac	.20	0.200 (25%) *	C					
Training Factor	SDD_Tng_Fac	.05	0.050 (25%) *	C					

Figure C-19 Completed Missile Example in ACE

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The completed point estimate as described in **Section 1.4** is depicted in **Figure C-20**.

	WBS/CES Description	Unique ID	Equation / Throughput	Program of Record
25	* DETAILED ESTIMATE			
26	Missile System			\$ 696,344 *
27	Sys Dev and Demo	SDD\$		\$ 164,898 *
28	Air Vehicle	AV_PMP\$		\$ 111,549 *
29	Design & Development		[Throughput] 25000	\$ 25,000 *
30	Prototypes		[Factor] ProdT1 * Step	\$ 9,749 *
31	Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$	\$ 76,800 *
32	Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LaborRate\$ * SDD_MonthsDur * FYTrapz(SDD_Start, SDD_End, 0, 1)	\$ 21,000 *
33	System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP\$	\$ 22,310 *
34	Training		[Factor] SDD_Tmg_Fac * AV_PMP\$	\$ 5,577 *
35	Data		[Factor] SDD_Data_Fac * AV_PMP\$	\$ 2,231 *
36	Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP\$	\$ 2,231 *
37				
38	Production Phase	Prod\$		\$ 531,446 *
39	Air Vehicle	AV_Prod\$		\$ 333,396 *
40	Payload		[Parametric CER: OLS Linear] (30.15 + 1.049 * WarheadWt) *	\$ 11,416 *
41	Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt ^ 0.6848 *	\$ 16,271 *
42	Airframe		[Parametric CER: MUPE Triad] (256.2 + 0.05682 * AirFrameWt ^	\$ 112,250 *
43	Guidance and Control		[Expert Opinion] 700 * PenaltyGuidance	\$ 186,979 *
44	Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRate\$ * ProdQty *	\$ 6,480 *
45	Engineering Changes		[Factor] ECO_Fac * AV_Prod\$	\$ 16,670 *
46	Sys Engineering/Program Management		[Factor] SEPM_Fac * AV_Prod\$	\$ 93,351 *
47	System Test and Evaluation		[Throughput Cost per FY Production] If(ProdQty > 0, 250, 0)	\$ 1,000 *
48	Training		[Factor] Tmg_Fac * AV_Prod\$	\$ 33,340 *
49	Data		[Factor] Data_Fac * AV_Prod\$	\$ 6,668 *
50	Peculiar Support Equipment		[Factor] PSE_Fac * AV_Prod\$	\$ 6,668 *
51	Common Support Equipment		[Discrete] CSE\$ * FYCRel(@AV_Prod\$)	\$ 347 *
52	Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_Prod\$	\$ 40,007 *

Figure C-20 Complete the Point Estimate

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Figure C-21, Figure C-22, and Figure C-23 depict the application of objective uncertainty to three types of parametric CERs as described in Section 2.2.2. These uncertainty ranges are applied to rows 40, 41, and 42 in the example.

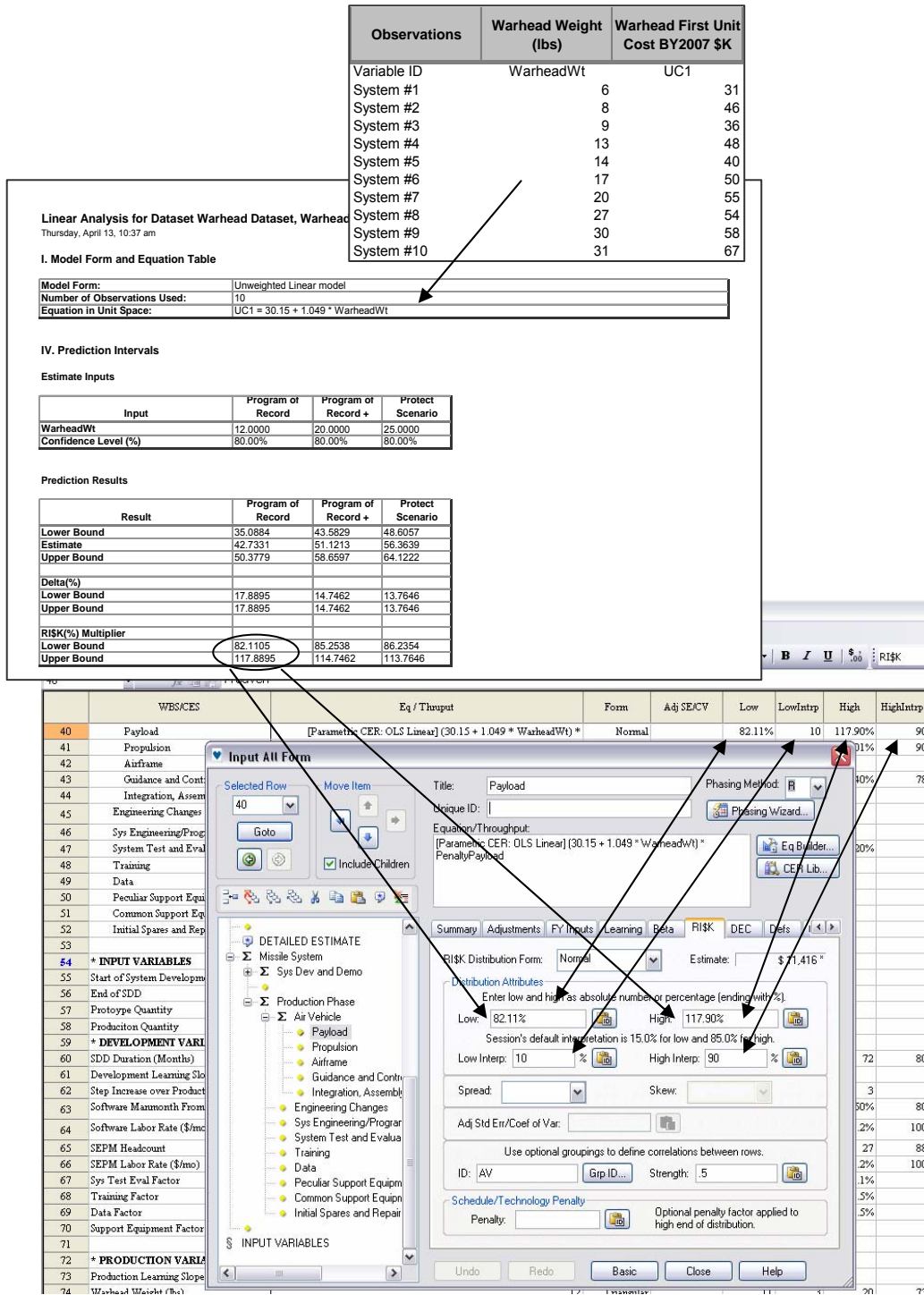


Figure C-21 Linear CER

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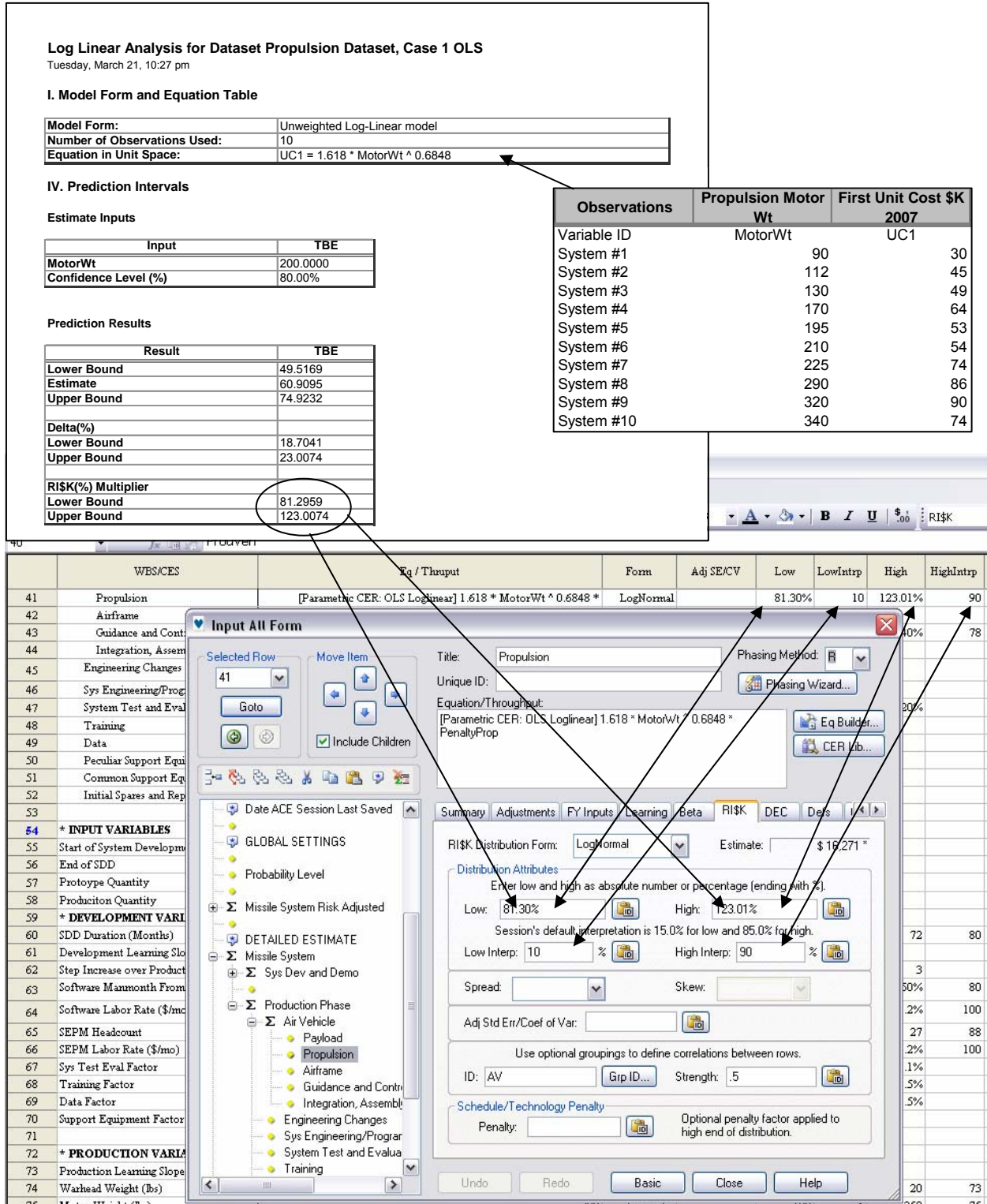


Figure C-22 Log CER

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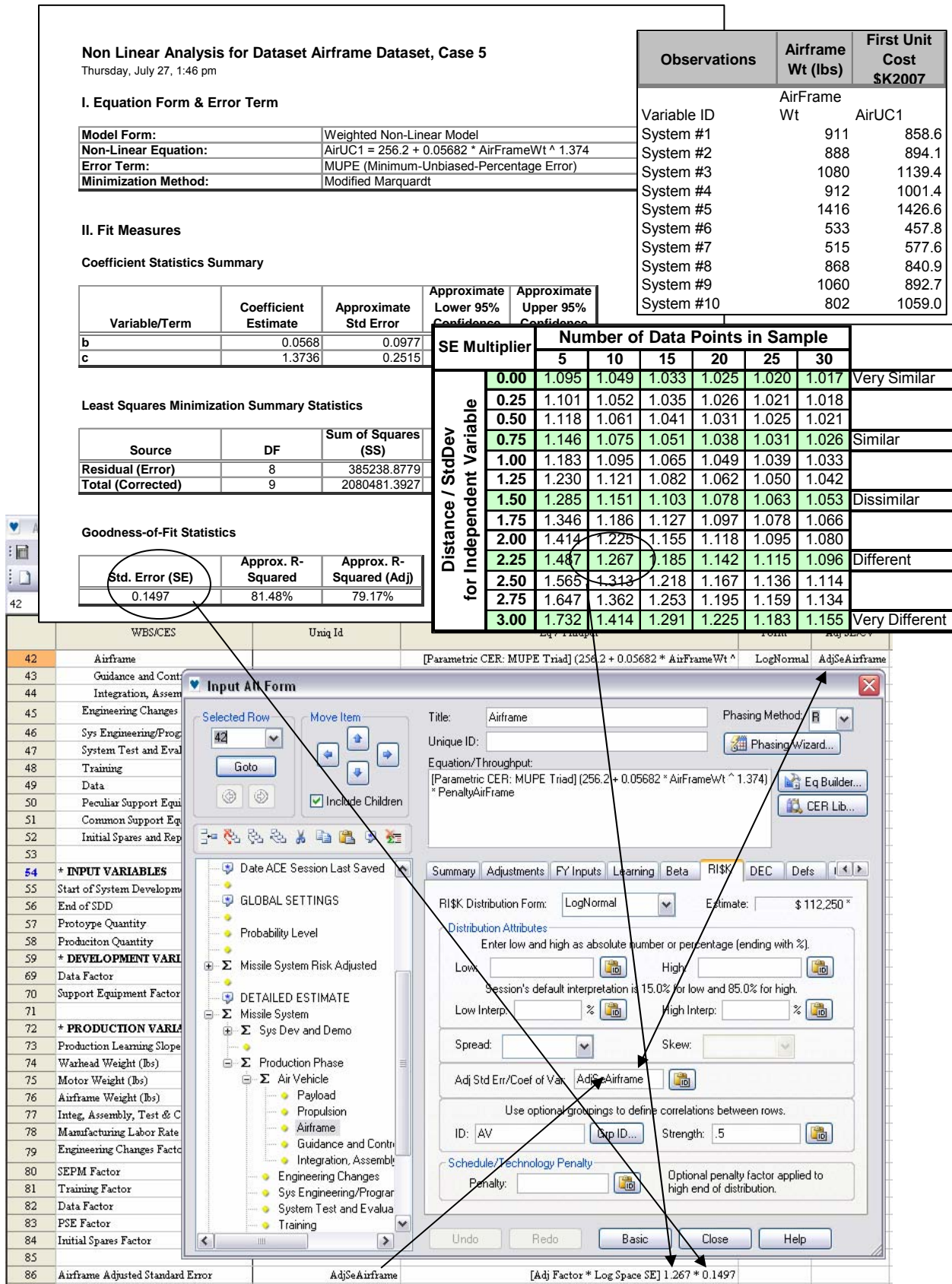


Figure C-23 Triad MUPE CER

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Figure C-24 depicts the application of subjective bounds to an expert opinion estimate as described in Section 2.3. This technique applies to Rows 62, 65, and 74 in the example.

Template to adjust upper lower bound interpretation for skew.

	Symmetrical	Prod to SDD Step	SEPM Staff	Warhead Weight	
Step 1: Obtain Expert Opinion for Low, Likely and High					
Low	75	1.5	22	11	Enter low bound
Mode (most likely)	100	1.8	25	12	Enter mode
High	125	3.0	27	20	Enter high bound
Step 2: Calculate skew from Expert Opinion					
Skew	0.50	0.20	0.60	0.11	(Mode-Low)/(High-Low)
Step 3: Set total uncertainty captured by Expert, default is 70%					
Total uncertainty captured by expert:	70%	70%	70%	70%	Default is 70%
Step 4: Calculate the bound interpretations					
Low Bound Interpretation	15%	6%	18%	3%	Round(Skew * (1-TotUncertCaptured_2)
High Bound Interpretation	85%	76%	88%	73%	TotUncertCaptured + Low Bound Interp

Input All Form

Selected Row: 65

Title: SEPM Headcount

Unique ID: SDD_SEPM_LOE

Equation/Thruput: 25

RI\$K Distribution Form: Triangular

Estimate: 25 *

Distribution Attributes

Enter low and high as absolute number or percentage (ending with %).

Low: 22 High: 27

Session's default interpretation is 15.0% for low and 85.0% for high.

Low Interp: 18% High Interp: 88%

Spread: Skew:

Adj Std Err/Coef of Var:

Use optional groupings to define correlations between rows.

ID: AV Grp.ID: Strength: .5

Schedule/Technology Penalty

Penalty: Optional penalty factor applied to high end of distribution.

Figure C-24 Subjective Uncertainty

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Rows 40-42 also illustrate the application of combined uncertainty as discussed in **Section 2.3**. Cost estimating uncertainty is applied to the CERs in rows 40-42 and applied to their cost drivers in rows 74-76. This is depicted in **Figure C-25**.

ACE 7.0 - [CRH Missile Model Draft 3 1Aug06.aceit - RISK (BY20075K)]									
File Edit View Documentation Calc Cases Reports Tools Window Help									
100% Times New Roman 8 B I U \$00 RISK									
89									
WBS/CES	Uniq Id	Eq / Thruput	Form	Adj SECV	Low	LowIntrp	High	HighIntrp	
40	Payload	[Parametric CER: OLS Linear] $(30.15 + 1.049 * \text{WarheadWt}) *$	Normal		82.11%	10	117.90%	90	
41	Propulsion	[Parametric CER: OLS Loglinear] $1.618 * \text{MotorWt} * 0.6848 *$	LogNormal		81.30%	10	123.01%	90	
42	Airframe	[Parametric CER: MUPE Triad] $(256.2 + 0.05682 * \text{AirFrameWt} *$	LogNormal	AdjSeAirframe					
43	Guidance and Control	[Expert Opinion] $700 * \text{PenaltyGuidance}$	Triangular		85%	8	140%	78	
44	Integration, Assembly, Test and C	[Build-up] $\text{IACO}_B \text{PerUnit} * \text{MfgLbssRate} * \text{ProdQty} *$							
45	Engineering Changes	[Factor] $\text{ECO_Fac} * \text{AV_Prod} \$$							
46	Sys Engineering/Program Manageme	[Factor] $\text{SEPM_Fac} * \text{AV_Prod} \$$							
47	System Test and Evaluation	[Throughput Cost per FY Production] $\text{If}(\text{ProdQty} > 0, 250, 0)$	Triangular		90%		120%		
48	Training	[Factor] $\text{Trng_Fac} * \text{AV_Prod} \$$							
49	Data	[Factor] $\text{Data_Fac} * \text{AV_Prod} \$$							
50	Peculiar Support Equipment	[Factor] $\text{PSE_Fac} * \text{AV_Prod} \$$							
51	Common Support Equipment	[Discrete] $\text{CSE} * \text{FYCRel}(\text{AV_Prod} \$)$							
52	Initial Spares and Repair Parts	[Factor] $\text{InitSpares_Fac} * \text{AV_Prod} \$$							
53									
54	* INPUT VARIABLES	*IN_VAR							
72	* PRODUCTION VARIABLES								
73	Production Learning Slope	ProdSlp	90						
74	Warhead Weight (Lbs)	WarheadWt	12	Triangular		11	3	20	73
75	Motor Weight (Lbs)	MotorWt	200	Triangular		190	5	250	75
76	Airframe Weight (Lbs)	AirFrameWt	330	LogNormal				120%	80

Figure C-25 Combining Uncertainties

AFCAA Cost Risk Analysis Handbook

Default subjective bounds as discussed in **Section 2.5** are applied to several elements in the example. **Figure C-26** depicts this for the Design & Development cost throughput on row 29. In this case the estimate is deemed to be triangular distribution right skewed with medium uncertainty. This technique is applied to cost factors on rows 67-69 and to labor rates on rows 64 and 66.

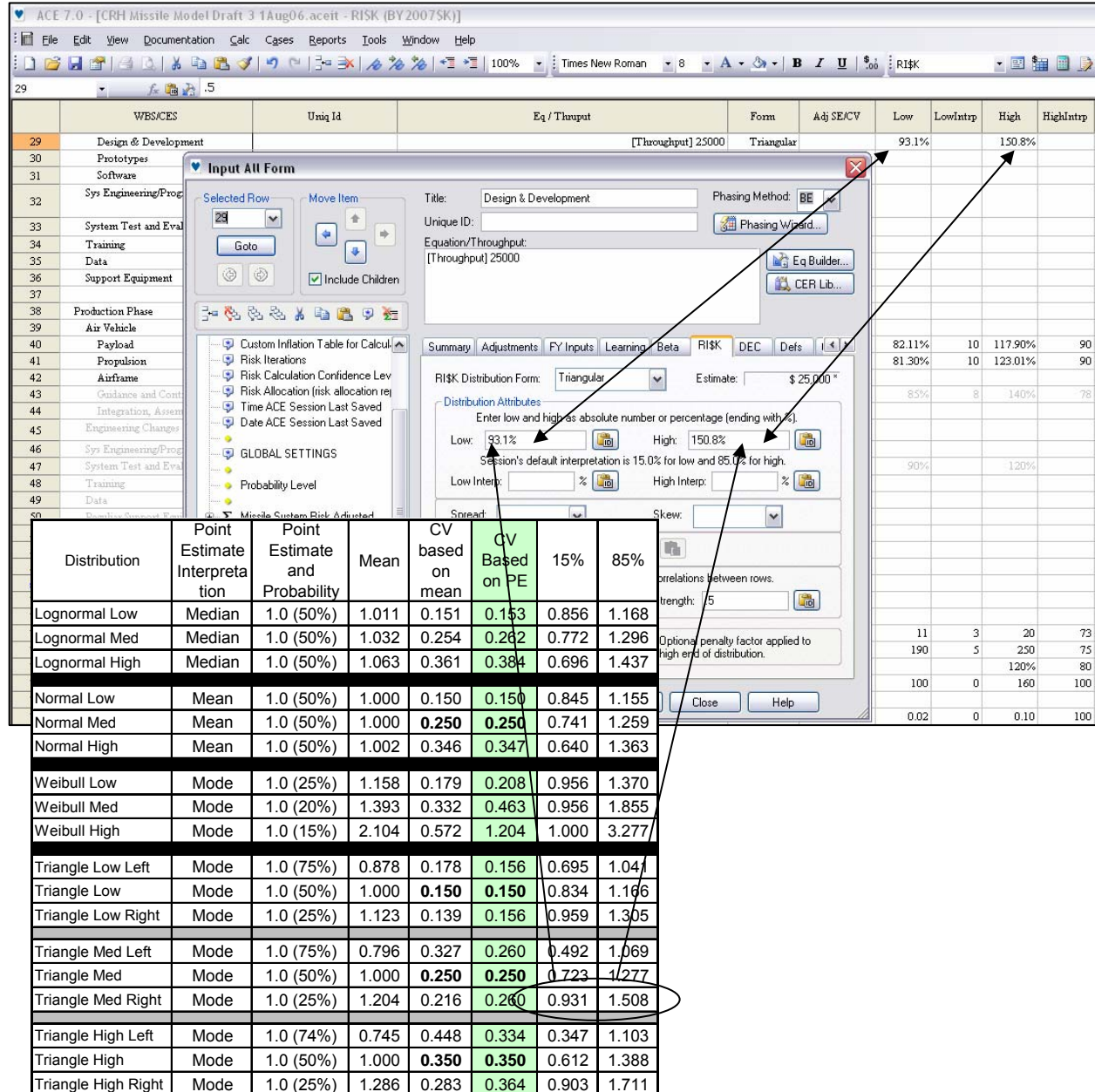



Figure C-26 Default Subjective Bounds

AFCAA Cost Risk Analysis Handbook

Figure C-27 illustrates the treatment of an input from a feeder model on row 63 as described in **Section 2.8**. In this example the mean value was obtained from the feeder model. The 80% value was determined to be 150% of the mean from use of the feeder model's own internal risk modeling capability.



	WBS/CES	Eq / Throughput	Form	Adj SE/ CV	Low	LowIntrp	High	HighIntrp
31	Software	[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$						
53								
54	* INPUT VARIABLES							
59	* DEVELOPMENT VARIABLES							
63	Software Manmonth From Price/SEER/COCOMO/etc	6400	LogNormal				150%	80
64	Software Labor Rate (\$/month)	12	Uniform		94.8%	0	131.2%	100

Figure C-27 Feeder Model

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Application of specific technical/schedule uncertainty considerations as described in **Section 2.9** is illustrated in **Figure C-28**. This technique was applied to rows 40-44 in the example model using penalty distributions in rows 96-100.

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103								
	WBSACES	Uniq Id	Eq / Thruput	Form	Low	LowIntrp	High	HighIntrp
38	Production Phase	Prod\$						
39	Air Vehicle	AV_Prod\$						
40	Payload		[Parametric CER: OLS Linear] (30.15 + 1.049 * WarheadWt) * PenaltyPayload	Normal	82.11%	10	117.90%	90
41	Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt ^ 0.6848 * PenaltyProp	LogNormal	81.30%	10	123.01%	90
42	Airframe		[Parametric CER: MUPE Triad] (256.2 + 0.05682 * AirFrameWt ^ 1.374) * PenaltyAirFrame	LogNormal				
43	Guidance and Control		[Expert Opinion] 700 * PenaltyGuidance	Triangular	85%	8	140%	78
44	Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRate\$ * ProdQty * PenaltyIATC					
94								
95	* Schedule/Technical Penalty Factor							
96	Payload	PenaltyPayload		1	Triangular	.9	0	1.5
97	Propulsion	PenaltyProp		1	Triangular	.9	0	1.5
98	Airframe	PenaltyAirFrame		1	Triangular	.9	0	1.1
99	Guidance and Control	PenaltyGuidance		1	Triangular	.9	0	1.5
100	Integration, Assembly, Test and Checkout	PenaltyIATC		1	Triangular	.9	0	2.0

Figure C-28 Penalty Method

AFCAA Cost Risk Analysis Handbook

Figure C-29 presents an application of discrete risk as described in **Section 2.7**. In this example ten items of CSE are will either be needed at the certain cost shown in rows 123-133 or will not be needed as a cost of zero. Rows 111-121 contains the probability the item will be needed in the DEC column labeled DiscRiskProb. Rows 111-121 also contain the function to multiply the cost times the distribution in the form of a UDF. UDF details are in rows 104,105, and 135-145. Row 111 rolls up to the CSE value utilized in row 51 of the estimate.

101	WBS/CES	Uniq Id	Eq / Throughput	DiscRiskProb	RowIndex	Form	Adj SE/CV	Low	LowIntp	High	HighIntp
103	* Discrete Uncertainties										
104	UDF Discrete Risk Cost Calculator	DiscreteRiskCost\$(@Risk)	I{D\$Sumif@Risk}>100-DiscRiskProb,D\$Sumif@CSECost,0)								
105	UDF Sumif	D\$Sumif@Target(WBS)	Sumif(RowIndex,RowIndex,@Target(WBS)								
106											
107	Mean For Discrete Risks		RiskMean(@CSE\$)								
108	Confidence Level of Mean		RiskConf@CSE\$, RiskMean(@CSE\$))								
109	Discrete Risk \$ at Selected Probability Level		RiskCost(@CSE\$, ProbLvl)								
110											
111	Total For Discrete Uncertainties	CSE\$									
112	CSE Item #1		DiscreteRiskCost\$(@CSERisk)	60	1						
113	CSE Item #2		DiscreteRiskCost\$(@CSERisk)	60	2						
114	CSE Item #3		DiscreteRiskCost\$(@CSERisk)	10	3						
115	CSE Item #4		DiscreteRiskCost\$(@CSERisk)	30	4						
116	CSE Item #5		DiscreteRiskCost\$(@CSERisk)	10	5						
117	CSE Item #6		DiscreteRiskCost\$(@CSERisk)	50	6						
118	CSE Item #7		DiscreteRiskCost\$(@CSERisk)	50	7						
119	CSE Item #8		DiscreteRiskCost\$(@CSERisk)	10	8						
120	CSE Item #9		DiscreteRiskCost\$(@CSERisk)	40	9						
121	CSE Item #10		DiscreteRiskCost\$(@CSERisk)	50	10						
122											
123	Consequence of Discrete Uncertainties	CSECost									
124	Cost CSE Item #1			21	1						
125	Cost CSE Item #2			34	2						
126	Cost CSE Item #3			54	3						
127	Cost CSE Item #4			13	4						
128	Cost CSE Item #5			45	5						
129	Cost CSE Item #6			23	6						
130	Cost CSE Item #7			42	7						
131	Cost CSE Item #8			49	8						
132	Cost CSE Item #9			39	9						
133	Cost CSE Item #10			27	10						
134											
135	Probabilities of Discrete Uncertainties	CSERisk									
136	Need Probability New CSE Item #1			100	1	Uniform		0	0	100	100
137	Need Probability New CSE Item #2			100	2	Uniform		0	0	100	100
138	Need Probability New CSE Item #3			100	3	Uniform		0	0	100	100
139	Need Probability New CSE Item #4			100	4	Uniform		0	0	100	100
140	Need Probability New CSE Item #5			100	5	Uniform		0	0	100	100
141	Need Probability New CSE Item #6			100	6	Uniform		0	0	100	100
142	Need Probability New CSE Item #7			100	7	Uniform		0	0	100	100
143	Need Probability New CSE Item #8			100	8	Uniform		0	0	100	100
144	Need Probability New CSE Item #9			100	9	Uniform		0	0	100	100
145	Need Probability New CSE Item #10			100	10	Uniform		0	0	100	100
146											
147											

Figure C-29 Discrete Distribution

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Figure C-30 depicts the measurement of correlation due to functional relationships in the SDD phase of the missile example as described in **Section 3.2.2**.

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	WBS/CES	Row 26: Missile System	Row 27: Sys Dev and Demo	Row 28: Air Vehicle	Row 29: Design & Development	Row 30: Prototypes	Row 31: Software	Row 32: Sys Engineering/Pro gram Management	Row 33: System Test and Evaluation	Row 34: Training	Row 35: Data	Row 36: Support Equipment
26	Missile System	1.00	0.72	0.68	0.42	0.69	0.60	0.46	0.70	0.71	0.71	0.68
27	Sys Dev and Demo		1.00	0.99	0.40	0.45	0.96	0.41	0.92	0.95	0.95	0.99
28	Air Vehicle			1.00	0.37	0.42	0.98	0.34	0.85	0.95	0.95	1.00
29	Design & Development				1.00	0.31	0.24	0.27	0.39	0.39	0.39	0.37
30	Prototypes					1.00	0.29	0.35	0.45	0.45	0.45	0.42
31	Software						1.00	0.29	0.82	0.92	0.92	0.98
32	Sys Engineering/Program Ma							1.00	0.37	0.38	0.38	0.34
33	System Test and Evaluation								1.00	0.84	0.84	0.85
34	Training									1.00	0.92	0.95
35	Data										1.00	0.95
36	Support Equipment											1.00

Figure C-30 Measured Correlation

AFCAA Cost Risk Analysis Handbook

Figure C-31 depicts the specification of additional correlation as described in **Section 3.2.3**. Columns Group and Strength are used to specify correlation. The content of the Group column groups all rows containing the same unique identifier (e.g. “AV”). The content of the Strength column contains the strength of the correlation (e.g. 0.5 representing correlation of 0.25).

WBS/CES	Uniq Id	Eq / Thrupt	Form	Low	LowIntrp	High	HighIntrp	Group	Strength
25									
26									
27	SDD\$								
28	AV_PMP\$								
29		[Throughput] 25000	Triangular	93.1%		150.8%		AV	.5
30		[Factor] ProdT1 * Step							
31		[Third Party Tool] ThirdPartyTool\$WManMonths * SWLaborRate\$							
32		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LaborRate\$ *							
33		[Factor] SDD_STE_Fac * AV_PMP\$							
34		[Factor] SDD_Trmg_Fac * AV_PMP\$							
35		[Factor] SDD_Data_Fac * AV_PMP\$							
36		[Factor] SDD_SptEquip_Fac * AV_PMP\$							
37									
38	Prod\$								
39	AV_Prod\$								
40		[Parametric CER: OLS Linear] (30.15 + 1.049 * WzheadWt) *	Normal	82.11%	10	117.90%	90	AV	.5
41		[Parametric CER: OLS Loglinear] 1.618 * MotorWt * 0.6848 *	LogNormal	81.30%	10	123.01%	90	AV	.5
42		[Parametric CER: MUPE Triad] (256.2 + 0.05682 * AirFrameWt ^	LogNormal					AV	.5
43		[Expert Opinion] 700 * PenaltyGuidance	Triangular	85%	8	140%	78	AV	.5
44		[Build-up] IACO_HsParUnit * MfgLaborRate\$ * ProdQty *							
45		[Factor] ECO_Fac * AV_Prod\$							
46		[Factor] SEPM_Fac * AV_Prod\$							
47		[Throughput Cost per FY Production] IfProdQty > 0, 250, 0)	Triangular	90%		120%		AV	.5
48		[Factor] Trmg_Fac * AV_Prod\$							
49		[Factor] Data_Fac * AV_Prod\$							
50		[Factor] PSE_Fac * AV_Prod\$							
51		[Discrete] CSE\$ * FYCRek(@AV_Prod\$)							
52		[Factor] IntSpares_Fac * AV_Prod\$							
53									
54	* INPUT VARIABLES	*IN_VAR							
55		SDD_Start				01Oct2007			
56		SDD_End				DateAdd(SDD_Start, 0, SDD_MonthsDur, SDD_DaysDur) - 1			
57		DevQty				FYISide(DateYr(SDD_Start) + 1)			
58		ProdQty				FYISide(DateYr(SDD_End))			
59	* DEVELOPMENT VARIABLES								
63		ThirdPartyTool\$WManMonths				6400			
64		SWLaborRate\$	LogNormal				150%	80	AV
65		SDD_SEPM_LOE	Uniform	94.8%	0	131.2%	100	LR	.8
66		SDD_SEPM_LaborRate\$	Triangular	22	18	27	88	AV	.5
67		SDD_SEPM_LaborRate\$	Uniform	94.8%	0	131.2%	100	LR	.8
68		SDD_STE_Fac	Triangular	90.3%		171.1%		AV	.5
69		SDD_Trmg_Fac	Triangular	95.9%		130.5%		AV	.5
70		SDD_Data_Fac	Triangular	95.9%		130.5%		AV	.5
71		SDD_SptEquip_Fac	None						

Figure C-31 Enter Correlation

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Figure C-32 illustrates the calculation of risk dollars as described in **Section 4.1**. The SDD phase and the Production phase risk dollars are calculated using by use of the RiskCost function on rows 20 and 23.

ACE 7.0 - [CRH Missile Model Draft 3 1Aug06.aceit - Methodology (BY2007SK)]					
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47	WBS/CES Description	Unique ID	Equation / Throughput	Program of Record	Phasing Method
15	Probability Level	ProbLvl	60	60 *	
16					
17	Missile System Risk Adjusted			\$ 956,398 *	
18	SDD Phase Risk Adjusted			\$ 221,766 *	
19	SDD Phase TBE		SDD\$	\$ 164,898 *	F
20	SDD Phase Risk Dollars		RiskCost(@SDD\$, ProbLvl, 1)	\$ 56,868 *	BE
21	Production Phase			\$ 734,632 *	
22	Production Phase TBE		Prod\$	\$ 531,446 *	F
23	Production Phase Risk Adjusted		RiskCost(@Prod\$, ProbLvl, 1)	\$ 203,186 *	BE
24					
25	* DETAILED ESTIMATE				
26	Missile System			\$ 696,344 (15%) *	
27	Sys Dev and Demo	SDD\$		\$ 164,898 (26%) *	
28	Air Vehicle	AV_PMP\$		\$ 111,549 (32%) *	
29	Design & Development		[Throughput] 25000	\$ 25,000 (25%) *	BE
30	Prototypes		[Factor] ProdT1 * Step	\$ 9,749 (20%) *	R
31	Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$	\$ 76,800 (41%) *	BE
32	Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LaborRate\$ * SDD_MonthsDur * FYTrapz[SDD_Start, SDD_End, 0, 1]	\$ 21,000 (24%) *	F
33	System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP\$	\$ 22,310 (23%) *	F
34	Training		[Factor] SDD_Trng_Fac * AV_PMP\$	\$ 5,577 (25%) *	F
35	Data		[Factor] SDD_Data_Fac * AV_PMP\$	\$ 2,231 (25%) *	F
36	Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP\$	\$ 2,231 (32%) *	F
37					
38	Production Phase	Prod\$		\$ 531,446 (16%) *	

Figure C-32 Calculate Risk Dollars

AFCAA Cost Risk Analysis Handbook

Figure C-33 presents the allocation of the risk dollars to the child elements as described in **Section 5.2**. This is accomplished using the Reports | Allocation menu.

ACE 7.0 - [CRH Missile Model Draft 3 1Aug06.aceit - Budgetary Report in BYS (FY2007 SK, Budgetary, Case: Program of Record, 60% CL allocated at Level 2)]										
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Cost Element	BUDGET BY1/CY2006	BUDGET BY2 2007	POM FY 2008	POM FY 2009	POM FY 2010	POM FY 2011	POM FY 2012	POM FY 2013	To Complete	TOTAL
26 Sys Dev and Demo			\$ 45,637	\$ 74,852	\$ 66,591	\$ 25,198	\$ 9,488			\$ 221,766 (60%)
27 Air Vehicle			\$ 29,581	\$ 51,149	\$ 45,598	\$ 14,656	\$ 3,165			\$ 144,150 (60%)
28 Design & Development			\$ 7,178	\$ 11,536	\$ 8,141	\$ 3,556	\$ 768			\$ 31,178 (60%)
29 Prototypes				\$ 3,609	\$ 12,049					\$ 15,658 (60%)
30 Software			\$ 22,403	\$ 36,005	\$ 25,408	\$ 11,100	\$ 2,397			\$ 97,313 (60%)
31 Sys Engineering/Program Management			\$ 5,157	\$ 5,143	\$ 5,143	\$ 5,143	\$ 5,157			\$ 25,741 (60%)
32 System Test and Evaluation			\$ 7,900	\$ 13,454	\$ 11,489	\$ 3,914	\$ 845			\$ 37,604 (60%)
33 Training			\$ 1,706	\$ 2,905	\$ 2,481	\$ 845	\$ 183			\$ 8,119 (60%)
34 Data			\$ 687	\$ 1,170	\$ 999	\$ 340	\$ 73			\$ 3,269 (60%)
35 Support Equipment			\$ 606	\$ 1,032	\$ 881	\$ 300	\$ 65			\$ 2,883 (60%)
36										
37 Production Phase							\$ 40,058	\$ 32,870	\$ 661,703	\$ 734,632 (60%)
38 Air Vehicle							\$ 22,909	\$ 18,773	\$ 380,840	\$ 422,522 (59%)
39 Payload							\$ 818	\$ 668	\$ 13,464	\$ 14,949 (59%)
40 Propulsion							\$ 1,145	\$ 936	\$ 18,862	\$ 20,943 (59%)
41 Airframe							\$ 6,502	\$ 5,312	\$ 107,073	\$ 118,887 (59%)
42 Guidance and Control							\$ 14,134	\$ 11,548	\$ 232,764	\$ 258,445 (59%)
43 Integration, Assembly, Test and Checkout							\$ 310	\$ 310	\$ 8,679	\$ 9,299 (59%)
44 Engineering Changes							\$ 1,334	\$ 1,093	\$ 22,160	\$ 24,588 (59%)
45 Sys Engineering/Program Management							\$ 8,608	\$ 7,052	\$ 142,948	\$ 158,608 (59%)
46 System Test and Evaluation							\$ 268	\$ 268	\$ 536	\$ 1,071 (59%)
47 Training							\$ 2,784	\$ 2,281	\$ 46,228	\$ 51,292 (59%)
48 Data							\$ 519	\$ 425	\$ 8,613	\$ 9,557 (59%)
49 Peculiar Support Equipment							\$ 519	\$ 425	\$ 8,611	\$ 9,555 (59%)
50 Common Support Equipment							\$ 7	\$ 5	\$ 111	\$ 123 (58%)
51 Initial Spares and Repair Parts							\$ 3,111	\$ 2,548	\$ 51,656	\$ 57,315 (59%)

Figure C-33 Allocate Risk Dollars

AFCAA Cost Risk Analysis Handbook

C.3 EXAMPLE WORKED USING CRYSTAL BALL

The section presents an example case using Crystal Ball. This section is NOT intended to be a Crystal Ball tutorial but is meant only to illustrate the use of guidance contained in the body of this handbook. **Figure C-34** presents the final product. The row and column headings are shown in the figure to be referenced in the text. The remainder of this section illustrates implementation of the concepts described in the body of the handbook as applied to a hypothetical missile system estimate.

	B	C	D	E	F	G	H	I	J	K
	WBS Description	Range Name	Estimate Method	Point Estimate	CER Uncertainty	Distribution Form	Low	High	1st Param Interpretation	2nd Param Interpretation
7										
8										
9										
10	* DETAILED ESTIMATE									
11	Missile System			\$696,110						
12	SDD Phase			\$164,898						
13	Air Vehicle	AV_PMP		\$111,549						
14	Design & Development		[Throughput] 25000	\$25,000		Triangular	0.951	1.508	0.15	0.85
15	Prototypes		[Factor] ProdT1 * Step	\$9,749						
16	Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRateS	\$76,800						
17	Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LabRate * SDD_MonthsDur	\$21,000						
18	System Test and Evaluation		[Factor] SDD_STE_Fac * AV_ProdS	\$22,210						
19	Training		[Factor] SDD_Trmg_Fac * AV_PMP	\$5,577						
20	Data		[Factor] SDD_Data_Fac * AV_PMP	\$2,231						
21	Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP	\$2,231						
22										
23	Production Phase			\$531,212						
24	Air Vehicle	AV_Prod		\$333,396						
25	Payload		[Parametric CER: OLS Linear] 30.15 * WarheadWt	\$11,416		Normal	0.8211	1.1790	0.10	0.90
26	Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt ^ 0.6848	\$16,271	1.000	Lognormal	0.8130	1.2301	0.10	0.90
27	Airframe		[Parametric CER: MUPE Triad] 256.2 * AirFrameWt ^ 1.374	\$112,250	1.000	Lognormal	1.018	0.18967	mean	std dev
28	Guidance and Control		[Throughput] 700	\$186,979		Triangular	0.850	1.400	0.08	0.78
29	Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRateS * ProdQty	\$6,480						
30	Engineering Changes		[Factor] ECO_Fac * AV_ProdS	\$16,670						
31	Sys Engineering/Program Management		[Factor] SEPM_Fac * AV_ProdS	\$93,251						
32	System Test and Evaluation		[Throughput] 250 times 4 years of production	\$1,000		Triangular	0.900	1.200	0.15	0.85
33	Training		[Factor] Trmg_Fac * AV_ProdS	\$33,340						
34	Data		[Factor] Data_Fac * AV_ProdS	\$6,668						
35	Peculiar Support Equipment		[Factor] PSE_Fac * AV_ProdS	\$6,668						
36	Common Support Equipment		[Discrete] CSES	\$113						
37	Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_ProdS	\$40,607						
38										
39	* INPUT VARIABLES									
40										
41	* DEVELOPMENT VARIABLES									
42	SDD Duration (Months)	SDD_MonthsDur		60	0.5	Triangular	54	72	0.10	0.80
43	Prototype Quantity	DevQty		90						
44	Development Learning Slope	DevSlp		90						
45	Step Increase over Production Cost	Step		1.8	1.500	Triangular	1.5	3	0.15	0.85
46	Software Manmonth From Price/SEER/COCOMO/etc	ThirdPartyToolSWManMonths		6400	6400	Lognormal	150%		PE factor @80%	
47	Software Labor Rate (\$/month)	SWLaborRate		12	12	Uniform	0.9480	1.3120	0.00	1.00
48	SEPM Headcount	SDD_SEPM_LOE		25	25	Triangular	22	27	0.18	0.88
49	SEPM Labor Rate (\$/mo)	SDD_SEPM_LaborRate		14000	14000	Uniform	0.9480	1.3120	0.00	1.00
50	Sys Test Eval Factor	SDD_STE_Fac		0.2	0.200	Triangular	0.903	1.710	0.15	0.85
51	Training Factor	SDD_Trmg_Fac		0.05	0.050	Triangular	0.959	1.305	0.15	0.85
52	Data Factor	SDD_Data_Fac		0.02	0.020	Triangular	0.959	1.305	0.15	0.85
53	Support Equipment Factor	SDD_SptEquip_Fac		0.02	0.02	None				
54										
55	* PRODUCTION VARIABLES									
56	Production Quantity	ProdQty		600						
57	Production Learning Slope	ProdSlp		90						
58	Warhead Weight (lbs)	WarHeadWt		12	12	Triangular	11	20	0.03	0.73
59	Motor Weight (lbs)	MotorWt		200	200	Triangular	190	250	0.05	0.75
60	Airframe Weight (lbs)	AirFrameWt		330	330	LogNormal	120%		PE factor @80%	
61	Integration, Assembly, Test and Checkout Hours per Unit	IACO_HsPerUnit		120	120	Uniform	100	160	0.00	1.00
62	Manufacturing Labor Rate	MfgLaborRate		90	90	None				
63	Engineering Changes Factor	ECO_Fac		0.05	0.05	Triangular	0.0200	0.1000	0.00	1.00
64	SEPM Factor	SEPM_Fac		0.25	0.250	Triangular	0.0283	0.6233	0.10	0.90
65	Training Factor	Trmg_Fac		0.1	0.100	Triangular	0.9511	1.508	0.15	0.85
66	Data Factor	Data_Fac		0.02	0.020	Triangular	0.959	1.305	0.15	0.85
67	PSE Factor	PSE_Fac		0.02	0.020	Triangular	0.959	1.305	0.15	0.85
68	Initial Spares Factor	InitSpares_Fac		0.12	0.120	Triangular	0.959	1.305	0.15	0.85
69										
70	* Estimating T1 for Production at Vehicle Level									
71	* Only valid if all production learning slopes are the same									
72	Prod T1	ProdT1		AV_ProdS / I.C_Area	1248.148					
73										
74	Converting Development slope to an index			Since slope = 2b * 100, then b = ln(slope/100) / ln(2)	-0.152					
75	Production Quantity Adjusted for Learning	I.C_Area		1	267.112					
76										
77	Converting Production Slope to an index			Since slope = 2b * 100, then b = ln(slope/100) / ln(2)	-0.152					
78					4.339					
79										
80										
81	* Schedule/Technical Penalty Factors									
82	Payload	PenaltyPayload		1.0		Triangular	0.9	1.5	0	100
83	Propulsion	PenaltyProp		1.0		Triangular	0.9	1.5	0	100
84	Airframe	PenaltyAirFrame		1.0		Triangular	0.9	1.1	0	100
85	Guidance and Control	PenaltyGuidance		1.0		Triangular	0.9	1.5	0	100
86	Integration, Assembly, Test and Checkout	PenaltyIATC		1.0		Triangular	0.9	2	0	100
87										
88										
89										
90		Total CSE		Point Estimate	Discrete Probability	Expected Value				
91		CSE Item #1		347		113.3				
92		CSE Item #2		21	6%	12.6				
93		CSE Item #3		34	6%	20.4				
94		CSE Item #4		54	10%	5.4				
95		CSE Item #5		13	3%	3.9				
96		CSE Item #6		45	10%	4.5				
97		CSE Item #7		23	5%	11.5				
98		CSE Item #8		42	5%	21				
99		CSE Item #9		49	10%	4.9				
100		CSE Item #10		30	6%	15.6				

Figure C-34 Completed Missile Example in Crystal Ball

AFCAA Cost Risk Analysis Handbook

The completed point estimate as described in **Section 1.4** is depicted in **Figure C-35**.

WBS Description	Range Name	Estimate Method	Point Estimate
Missile System			\$696,110
SDD Phase			\$164,898
Air Vehicle	AV_PMP		\$111,549
Design & Development		[Throughput] 25000	\$25,000
Prototypes		[Factor] ProdT1 * Step	\$9,749
Software		[Third Party Tool] ThirdPartyToolSWManMonths * SWLaborRate\$	\$76,800
Sys Engineering/Program Management		[Build-up] SDD_SEPM_LOE * SDD_SEPM_LabRate * SDD_MonthsDur	\$21,000
System Test and Evaluation		[Factor] SDD_STE_Fac * AV_PMP	\$22,310
Training		[Factor] SDD_Trng_Fac * AV_PMP	\$5,577
Data		[Factor] SDD_Data_Fac * AV_PMP	\$2,231
Support Equipment		[Factor] SDD_SptEquip_Fac * AV_PMP	\$2,231
Production Phase			\$531,212
Air Vehicle	AV_Prod		\$333,396
Payload		[Parametric CER: OLS Linear] 30.15 + 1.049 * WarheadWt	\$11,416
Propulsion		[Parametric CER: OLS Loglinear] 1.618 * MotorWt ^ 0.6848	\$16,271
Airframe		[Parametric CER: MUPE Triad] 256.2 + 0.05682 * AirFrameWt ^ 1.374	\$112,250
Guidance and Control		[Throughput] 700	\$186,979
Integration, Assembly, Test and Checkout		[Build-up] IACO_HsPerUnit * MfgLaborRate\$ * ProdQty	\$6,480
Engineering Changes		[Factor] ECO_Fac * AV_Prod\$	\$16,670
Sys Engineering/Program Management		[Factor] SEPM_Fac * AV_Prod\$	\$93,351
System Test and Evaluation		[Throughput] 250 times 4 years of production	\$1,000
Training		[Factor] Trng_Fac * AV_Prod\$	\$33,340
Data		[Factor] Data_Fac * AV_Prod\$	\$6,668
Peculiar Support Equipment		[Factor] PSE_Fac * AV_Prod\$	\$6,668
Common Support Equipment		[Discrete] CSES	\$113
Initial Spares and Repair Parts		[Factor] InitSpares_Fac * AV_Prod\$	\$40,007

Figure C-35 Complete the Point Estimate

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Figure C-36, Figure C-37, and depict the application of objective uncertainty to three types of parametric CERs as described in Section 2.2.2. These uncertainty ranges are applied to rows 25, 26, and 27 in the example.

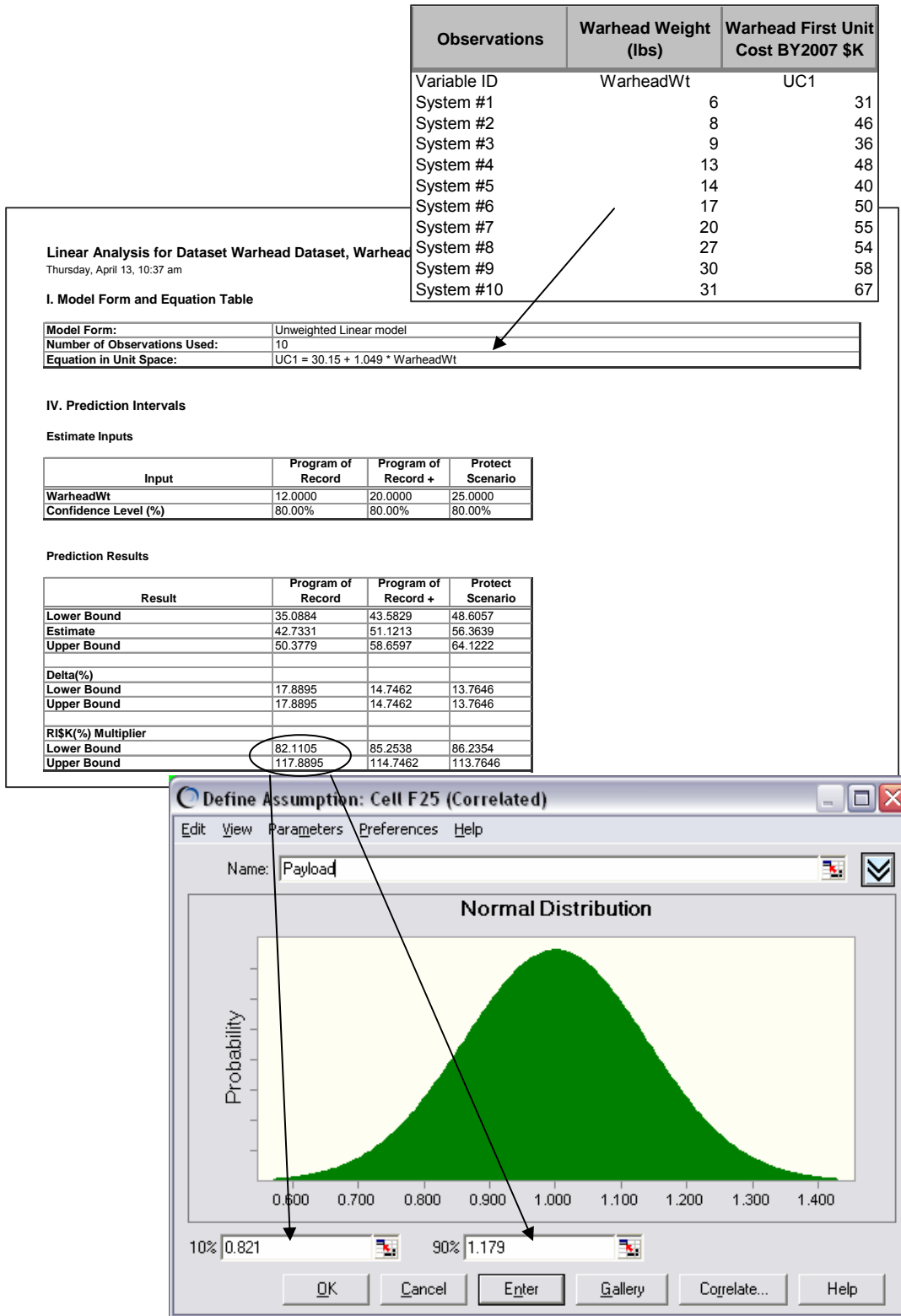


Figure C-36 Linear CER

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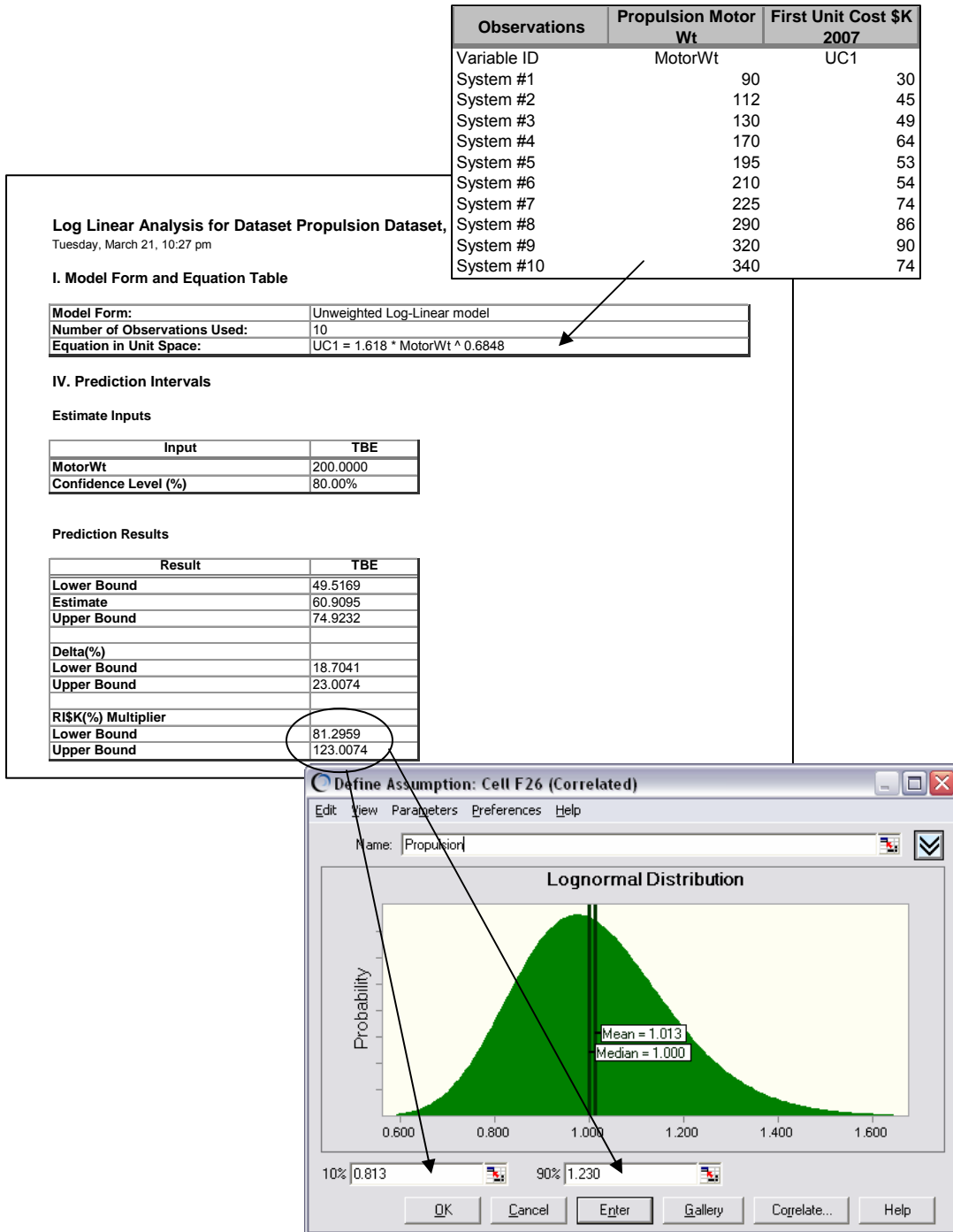


Figure C-37 Log CER

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Observations	Airframe Wt (lbs)	First Unit Cost \$K2007
Variable ID	AirFrame Wt	AirUC1
System #1	911	858.6
System #2	888	894.1
System #3	1080	1139.4
System #4	912	1001.4
System #5	1416	1426.6
System #6	533	457.8
System #7	515	577.6
System #8	868	840.9
System #9	1060	892.7
System #10	802	1059.0

Non Linear Analysis for Dataset Airframe Dataset, Case 5

Thursday, July 27, 1:46 pm

I. Equation Form & Error Term

Model Form:	Weighted Non-Linear Model
Non-Linear Equation:	AirUC1 = 256.2 + 0.05682 * AirFrameWt ^ 1.374
Error Term:	MUPE (Minimum-Unbiased-Percentage Error)
Minimization Method:	Modified Marquardt

II. Fit Measures

Coefficient Statistics Summary

Variable/Term	Coefficient Estimate	Approximate Std Error	Approximate Lower 95% Confidence	Approximate Upper 95% Confidence
b	0.0568	0.0977		
c	1.3736	0.2515		

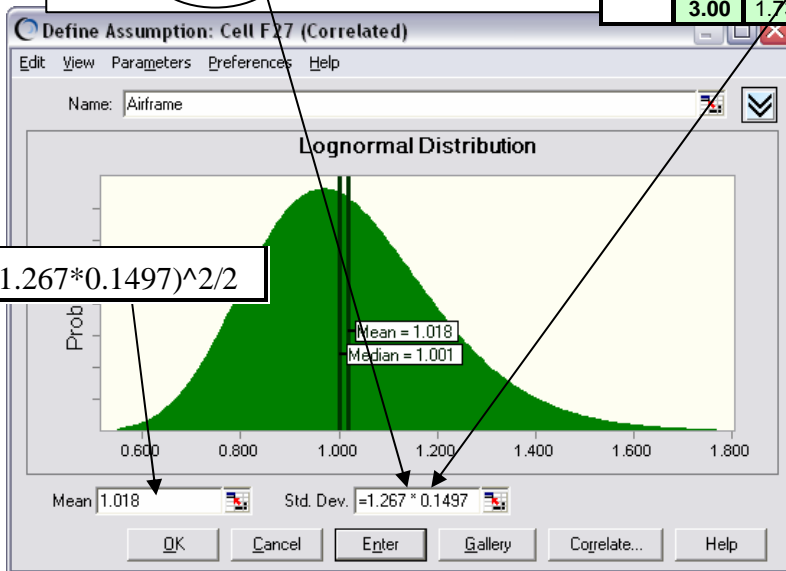
Least Squares Minimization Summary Statistics

Source	DF	Sum of Squares (SS)
Residual (Error)	8	385238.8779
Total (Corrected)	9	2080481.3927

Goodness-of-Fit Statistics

Std. Error (SE)	Approx. R-Squared	Approx. R-Squared (Adj)
0.1497	81.48%	79.17%

SE Multiplier	Number of Data Points in Sample						
	5	10	15	20	25	30	
0.00	1.095	1.049	1.033	1.025	1.020	1.017	Very Similar
0.25	1.101	1.052	1.035	1.026	1.021	1.018	
0.50	1.118	1.061	1.041	1.031	1.025	1.021	
0.75	1.146	1.075	1.051	1.038	1.031	1.026	Similar
1.00	1.183	1.095	1.065	1.049	1.039	1.033	
1.25	1.230	1.121	1.082	1.062	1.050	1.042	
1.50	1.285	1.151	1.103	1.078	1.063	1.053	Dissimilar
1.75	1.346	1.186	1.127	1.097	1.078	1.066	
2.00	1.414	1.225	1.155	1.118	1.095	1.080	
2.25	1.487	1.267	1.185	1.142	1.115	1.096	Different
2.50	1.565	1.313	1.218	1.167	1.136	1.114	
2.75	1.647	1.362	1.253	1.195	1.159	1.134	
3.00	1.732	1.414	1.291	1.225	1.183	1.155	Very Different



$EXP(0+(1.267*0.1497)^2/2)$

Figure C-38 Triad MUPE CER

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Figure C-39 depicts the application of subjective bounds to an expert opinion estimate as described in **Section 2.3**. This technique applies to Rows 45, 48, and 58 in the example. The use of **Figure 2-5**'s template is illustrated for row SEPM Headcount.

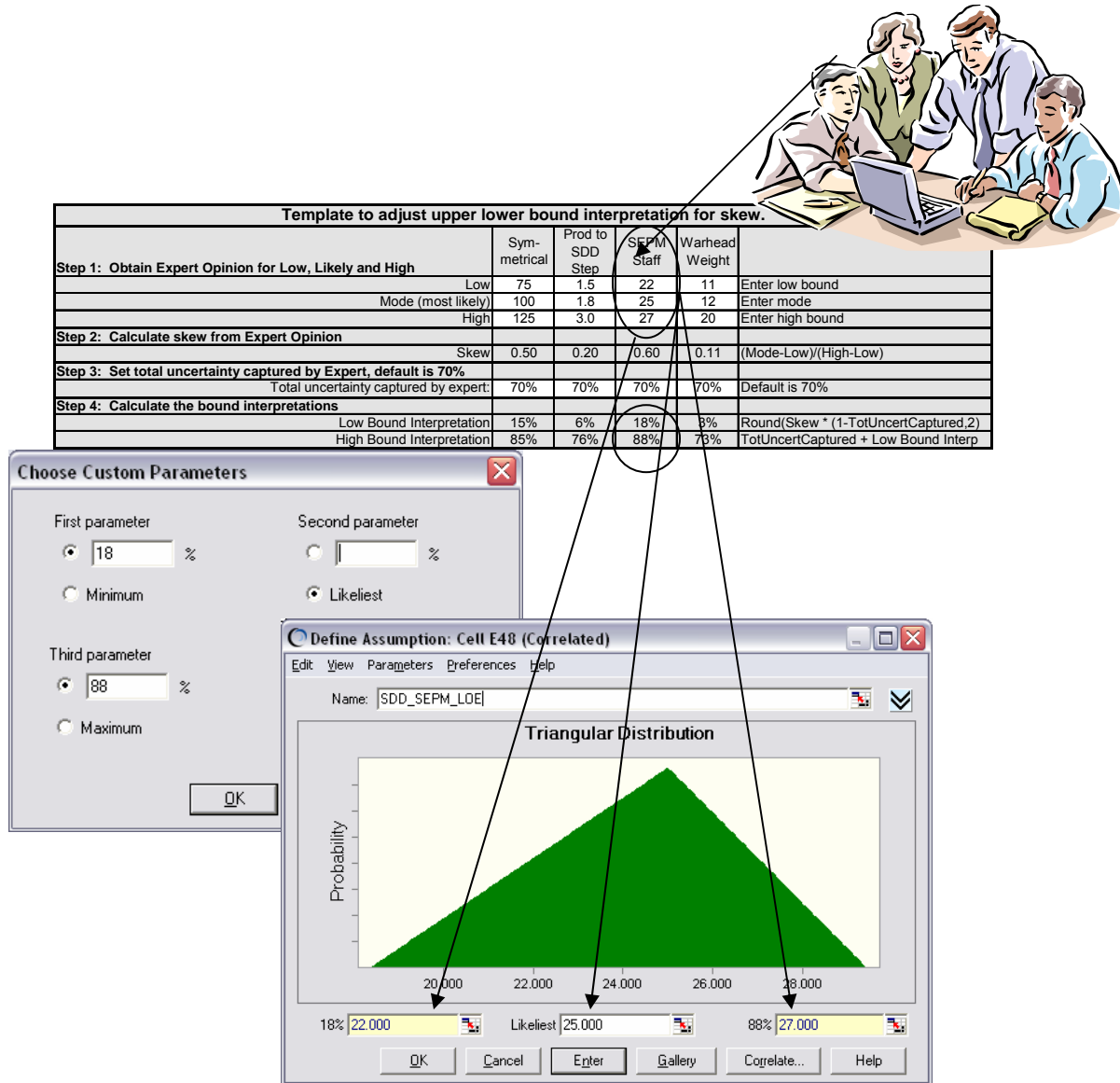


Figure C-39 Subjective Uncertainty

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Rows 25-27 also illustrate the application of combined uncertainty as discussed in **Section 2.3**. Cost estimating uncertainty is applied to the CERs in those rows and applied to their cost drivers in rows 57-59. This is depicted in **Figure C-40**.

WBS Elements			Form	Adj SE	Low	Low Intrp	High	High Intrp	Comment
Payload	\$11,416	$30.15 + 1.049 * \text{WarheadWt}$	Normal		82.11%	10	117.90%	90	Regression Result
Propulsion	\$16,271	$1.618 * \text{MotorWt} ^ 0.6848$	LogNormal		81.30%	10	123.01%	90	Regression Result
Airframe	\$112,250	$256.2 + 0.05682 * \text{AirFrameWt} ^ 1.374$	LogNormal	0.1897					Regression Result
Guidance and Control	\$186,979	700	Triangular		85%	8	140%	78	Expert Opinion
Warhead Weight (lbs)	12.0	WarheadWt	Triangular		11	3	20	73	
Motor Weight (lbs)	200.0	MotorWt	Triangular		190	5	250	75	
Airframe Weight (lbs)	330.0	AirFrameWt	LogNormal				120%	80	

Figure C-40 Combining Uncertainties

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Default subjective bounds as discussed in **Section 2.5** are applied to several elements in the example. **Figure C-41** depicts this for the Design & Development cost throughput on row14. In this case the estimate is deemed to be triangular distribution right skewed with medium uncertainty. A segment of **Table 2-5** is shown. This technique is applied to cost factors on rows 50-52 and 65-68 and to labor rates on rows 47 and 49.

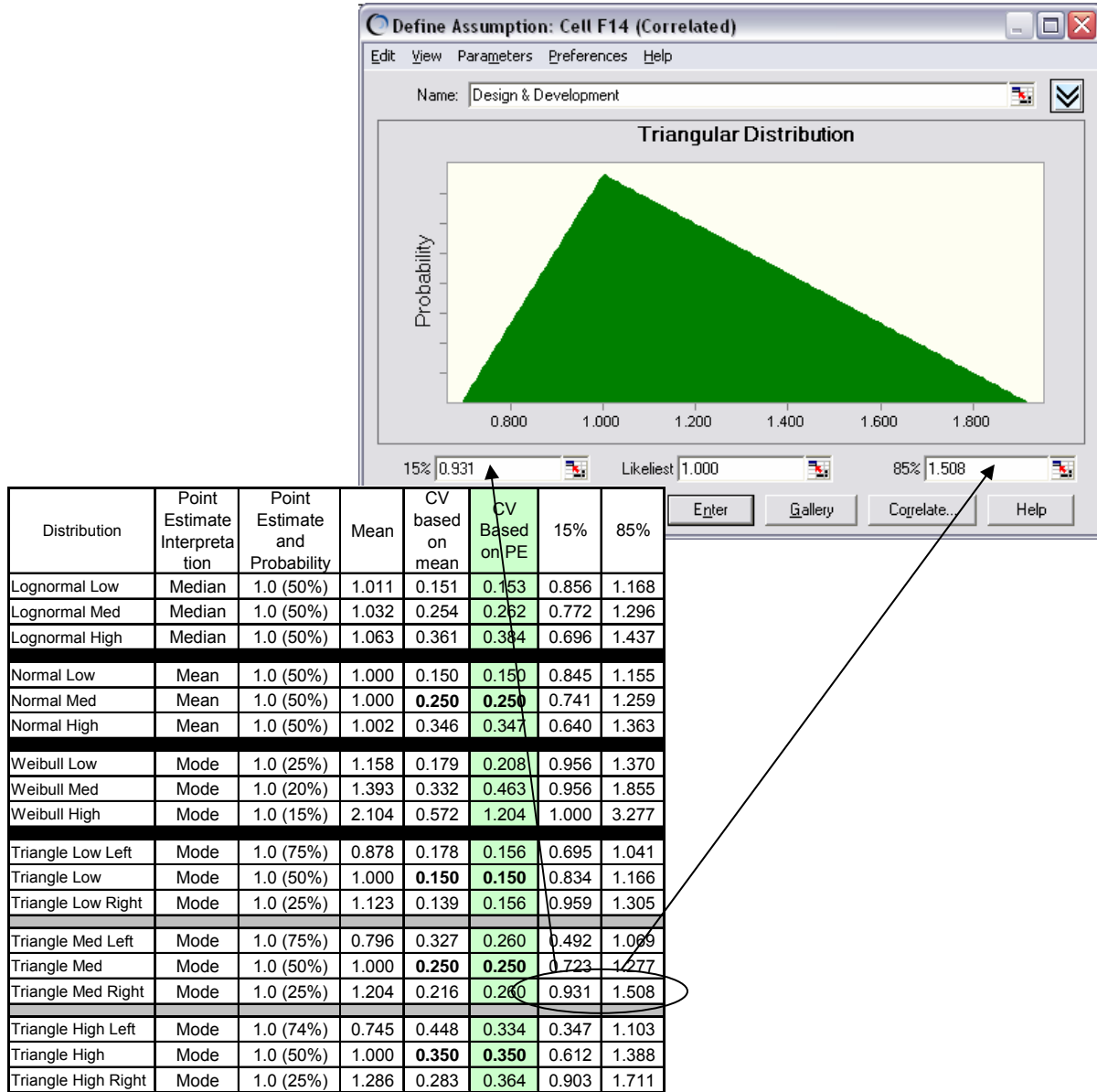


Figure C-41 Default Subjective Bounds

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Figure C-42 illustrates the treatment of an input from a feeder model on row 46 as described in **Section 2.8**. In this example the mean value was obtained from the feeder model. The 80% value was determined to be 150% of the mean from use of the feeder model's own internal risk modeling capability.

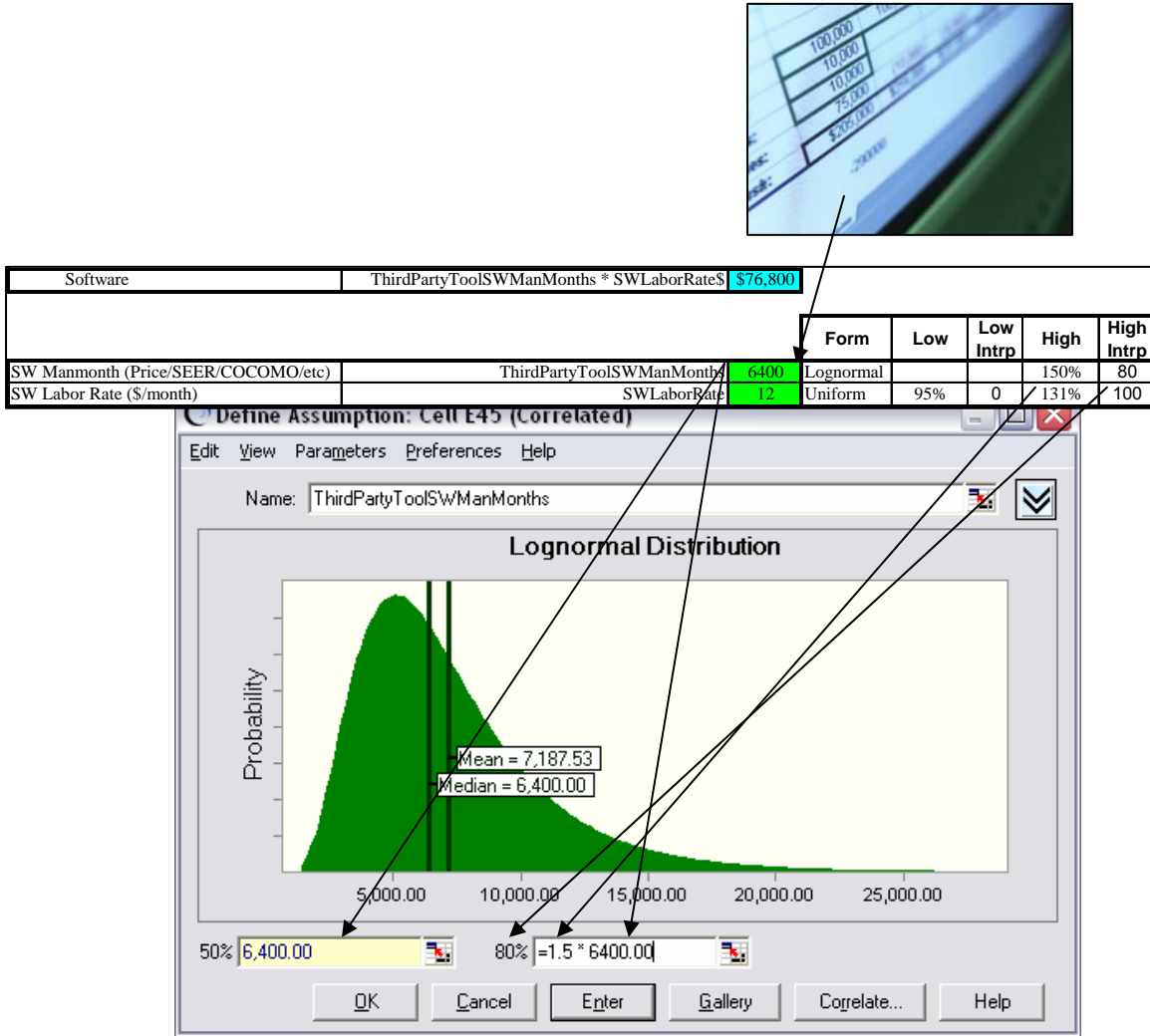


Figure C-42 Feeder Model

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Application of specific technical/schedule uncertainty considerations as described in **Section 2.9** is illustrated in **Figure C-43**. This technique was applied to rows 25-29 in the example model.

WBS Elements

Payload	\$11,416	$(30.15 + 1.049 * \text{WarheadWt}) * \text{PenaltyPayload}$
Propulsion	\$16,271	$1.618 * \text{MotorWt}^{0.6848} * \text{PenaltyProp}$
Airframe	\$112,250	$(256.2 + 0.05682 * \text{AirFrameWt}^{1.374}) * \text{PenaltyAirFrame}$
Guidance and Control	\$186,979	$700 * \text{PenaltyGuidance}$
Integration, Assembly, Test and Checkout	\$6,480	$\text{IACO_HsPerUnit} * \text{MfgLaborRate\$} * \text{ProdQty} * \text{PenaltyIATC}$

Schedule/Technical Penalty Factors			Absolute Bounds	
			Low	High
Payload	1.0	Triangular	0.9	1.5
Propulsion	1.0	Triangular	0.9	1.5
Airframe	1.0	Triangular	0.9	1.1
Guidance and Control	1.0	Triangular	0.9	1.5
Integration, Assembly, Test and Checkout	1.0	Triangular	0.9	2.0

Figure C-43 Penalty Method

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Figure C-44 presents an application of discrete risk as described in **Section 2.7**. In this example ten items of CSE are will either be needed at the certain cost shown in column D or will not be needed as a cost of zero. Column E contains the probability the item will be needed. The figure also shows use of the Crystal Ball Yes-No distribution with the probability of yes linked to cell E91. Column F contains formulas to multiply the result of this distribution times the estimated cost. Column F rolls up to the CSE value utilized in row 36 of the estimate.

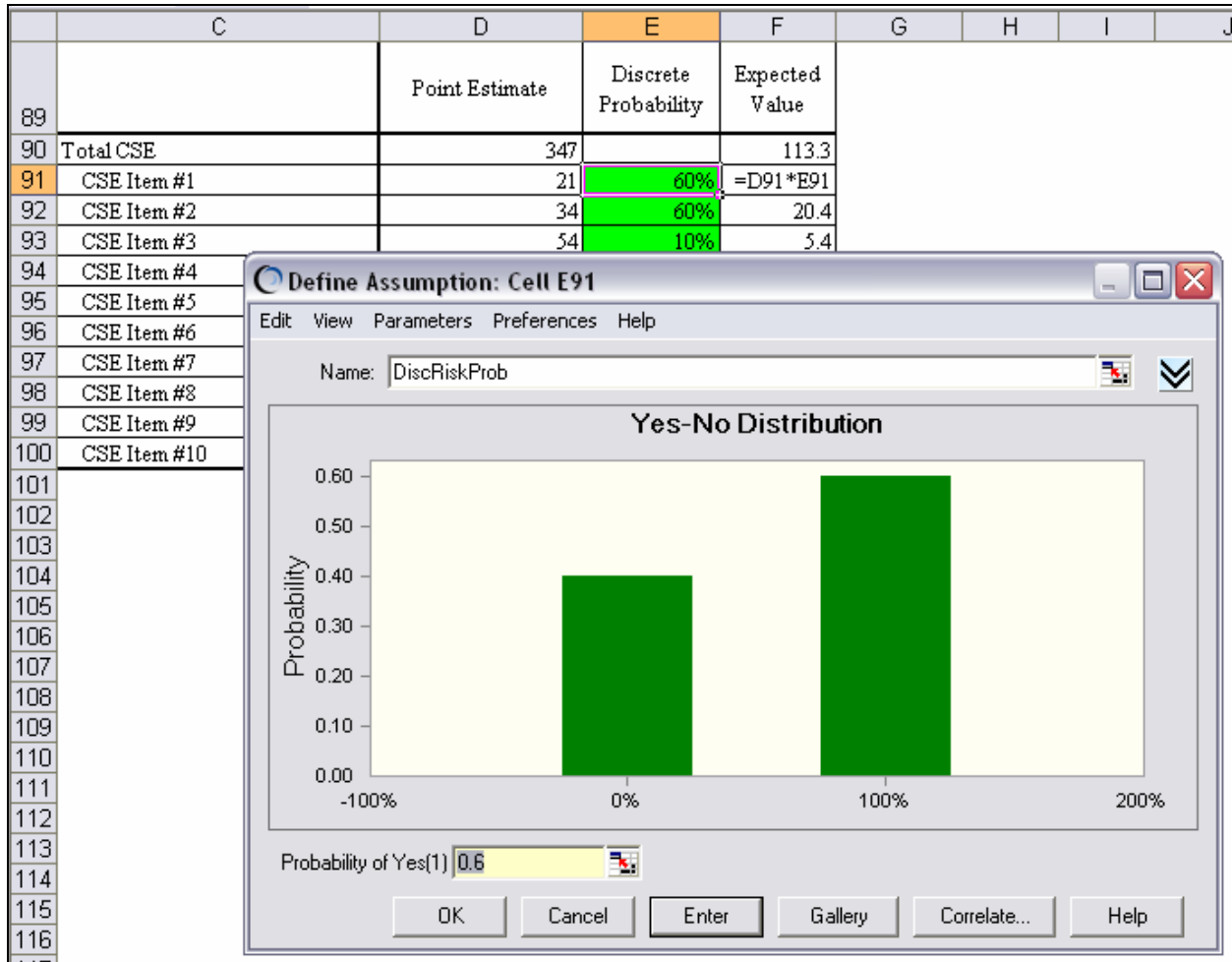


Figure C-44 Discrete Distribution

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Figure C-45 depicts the measurement of correlation due to functional relationships in the SDD phase of the missile example as described in **Section 3.2.2**.

E5		fx =CORREL(\$D46:\$D10045,E46:E10045)										
	B	C	D	E	F	G	H	I	J	K	L	M
1												
3			Missile System	SDD Phase	Air Vehicle	Design & Development	Prototypes	Software	Sys Eng/ PM	Sys Test and Eval	Training	Data
4			CB Resultant Pearson Product Moment Correlation									
5		Missile System	1.00	0.72	0.69	0.40	0.69	0.61	0.47	0.70	0.71	0.71
6		SDD Phase		1.00	0.99	0.38	0.46	0.96	0.44	0.92	0.95	0.95
7		Air Vehicle			1.00	0.36	0.43	0.98	0.37	0.86	0.95	0.95
8		Design & Dev				1.00	0.30	0.22	0.28	0.36	0.38	0.38
9		Prototypes					1.00	0.31	0.37	0.45	0.47	0.47
10		Software						1.00	0.32	0.83	0.92	0.92
11		Sys Eng/P M							1.00	0.40	0.41	0.40
12		Sys Test and Eval								1.00	0.84	0.85
13		Training									1.00	0.92
14		Data										1.00
15												
45		Trial values	Missile System	SDD Phase	Air Vehicle	Design & Development	Prototypes	Software	Sys Engineering/Program Management	System Test and Evaluation	Training	Data
46	1		689,394	148,175	91,070	28,180	8,369	54,521	23,500	23,277	6,915	1,591
47	2		1,174,842	234,314	137,494	24,271	13,369	99,853	31,237	52,221	7,581	3,032
48	3		759,341	173,267	112,051	29,281	8,082	74,688	24,179	26,108	6,136	2,552
49	4		1,241,785	329,314	204,515	35,917	21,965	146,633	30,196	75,417	10,484	4,612
50	5		1,234,821	225,320	136,249	43,848	12,903	79,498	32,834	44,154	6,399	2,899
51	6		672,883	194,449	133,569	23,793	7,403	102,373	25,203	24,086	6,185	2,735
52	7		1,051,703	250,325	173,252	26,210	21,592	125,450	25,101	35,836	9,399	3,331
53	8		657,203	187,666	129,138	31,273	8,788	89,076	26,408	20,873	5,916	2,748
54	9		679,379	164,680	104,576	32,323	9,470	62,783	27,634	22,072	6,299	2,007

Figure C-45 Measured Correlation

AFCAA Cost Risk Analysis Handbook

Figure C-46 depicts the specification of additional correlation as described in Section 3.2.3. In this case all the elements shown were deemed to be weakly correlated and assigned values of 0.25. In the example case four sets of correlation matrices are specified. Figure C-47, Figure C-48, and Figure C-49 present the other three correlation matrices.

The screenshot shows the Crystal Ball Tool interface. A 'Run' menu is open, highlighting 'Tools' and 'Correlation Matrix...'. A dialog box titled 'Select assumptions (step 1 of 2)' is open, showing a 'Correlation Matrix' window. The dialog box contains a list of 'Available Assumptions' and a 'Selected Assumptions' list. Below the dialog box, a spreadsheet displays a correlation matrix for various assumptions. The diagonal elements are 1.000, and all other elements are 0.250.

	Design & Development (AFCAA Sample Missile Estimate)	Payload (AFCAA Sample Missile Estimate)	Propulsion (AFCAA Sample Missile Estimate)	Airframe (AFCAA Sample Missile Estimate)	Guidance and Control (AFCAA Sample Missile Estimate)	System Test and Evaluation (AFCAA Sample Missile Estimate)	SDD_MthsDur (AFCAA Sample Missile Estimate)	Step (AFCAA Sample Missile Estimate)	ThirdPartyToolSWManMonths (AFCAA Sample Missile Estimate)	SDD_SEPM_LOE (AFCAA Sample Missile Estimate)	Sys Test Eval Factor (AFCAA Sample Missile Estimate)	Training Factor (AFCAA Sample Missile Estimate)	Data Factor (AFCAA Sample Missile Estimate)	ECO Factor (AFCAA Sample Missile Estimate)	IACO_HsPerUnit (AFCAA Sample Missile Estimate)	SEPM_Fac (AFCAA Sample Missile Estimate)	Training Factor (AFCAA Sample Missile Estimate)	Data Factor (AFCAA Sample Missile Estimate)	PSE Factor (AFCAA Sample Missile Estimate)	Initial Spares Factor (AFCAA Sample Missile Estimate)	
Design & Development (AFCAA Sample Missile Estimate)	1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Payload (AFCAA Sample Missile Estimate)		1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Propulsion (AFCAA Sample Missile Estimate)			1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Airframe (AFCAA Sample Missile Estimate)				1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Guidance and Control (AFCAA Sample Missile Estimate)					1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
System Test and Evaluation (AFCAA Sample Missile Estimate)						1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
SDD_MthsDur (AFCAA Sample Missile Estimate)							1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Step (AFCAA Sample Missile Estimate)								1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
ThirdPartyToolSWManMonths (AFCAA Sample Missile Estimate)									1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
SDD_SEPM_LOE (AFCAA Sample Missile Estimate)										1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Sys Test Eval Factor (AFCAA Sample Missile Estimate)											1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Training Factor (AFCAA Sample Missile Estimate)												1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
Data Factor (AFCAA Sample Missile Estimate)													1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
ECO Factor (AFCAA Sample Missile Estimate)														1.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250
IACO_HsPerUnit (AFCAA Sample Missile Estimate)															1.000	0.250	0.250	0.250	0.250	0.250	0.250
SEPM_Fac (AFCAA Sample Missile Estimate)																1.000	0.250	0.250	0.250	0.250	0.250
Training Factor (AFCAA Sample Missile Estimate)																	1.000	0.250	0.250	0.250	0.250
Data Factor (AFCAA Sample Missile Estimate)																		1.000	0.250	0.250	0.250
PSE Factor (AFCAA Sample Missile Estimate)																			1.000	0.250	0.250
Initial Spares Factor (AFCAA Sample Missile Estimate)																				1.000	0.250

Figure C-46 Enter Correlation

AFCAA Cost Risk Analysis Handbook

	Software Labor Rate (\$/month) (AFCAA Sample Missile Estimate)	SEPM Labor Rate (\$/mo) (AFCAA Sample Missile Estimate)
Software Labor Rate (\$/month) (AFCAA Sample Missile Estimate)	1.000	0.640
SEPM Labor Rate (\$/mo) (AFCAA Sample Missile Estimate)		1.000

Figure C-47 Labor Rate Correlation Matrix

	WarHeadWt (AFCAA Sample Missile Estimate)	MotorWt (AFCAA Sample Missile Estimate)	AirFrameWt (AFCAA Sample Missile Estimate)
WarHeadWt (AFCAA Sample Missile Estimate)	1.000	0.700	0.700
MotorWt (AFCAA Sample Missile Estimate)		1.000	0.490
AirFrameWt (AFCAA Sample Missile Estimate)			1.000

Figure C-48 Weight Correlation Matrix

	PenaltyPayload (AFCAA Sample Missile Estimate)	PenaltyProp (AFCAA Sample Missile Estimate)	PenaltyAirFrame (AFCAA Sample Missile Estimate)	PenaltyGuidance (AFCAA Sample Missile Estimate)	PenaltyIATC (AFCAA Sample Missile Estimate)
PenaltyPayload (AFCAA Sample Missile Estimate)	1.000	0.250	0.250	0.250	0.500
PenaltyProp (AFCAA Sample Missile Estimate)		1.000	0.0625	0.0625	0.125
PenaltyAirFrame (AFCAA Sample Missile Estimate)			1.000	0.0625	0.125
PenaltyGuidance (AFCAA Sample Missile Estimate)				1.000	0.125
PenaltyIATC (AFCAA Sample Missile Estimate)					1.000

Figure C-49 Schedule/Technical Penalty Correlation Matrix

AFCAA Cost Risk Analysis Handbook

Figure C-50 illustrates the calculation of risk dollars as described in **Section 4.1**. The SDD phase and the Production phase risk dollars are calculated using by subtracting the selected confidence level (60%) values from the point estimate.

WBS Description	Point Estimate
Missile System	\$696,110
SDD Phase	\$164,898
Air Vehicle	\$111,549
Design & Development	\$25,000
Prototypes	\$9,749
Software	\$76,800
Sys Engineering/Program Management	\$21,000
System Test and Evaluation	\$22,310
Training	\$5,577
Data	\$2,231
Support Equipment	\$2,231
Production Phase	\$531,212
Air Vehicle	\$333,396
Payload	\$11,416
Propulsion	\$16,271
Airframe	\$112,250
Guidance and Control	\$186,979
Integration, Assembly, Test and Checkout	\$6,480
Engineering Changes	\$16,670
Sys Engineering/Program Management	\$93,351
System Test and Evaluation	\$1,000
Training	\$33,340
Data	\$6,668
Peculiar Support Equipment	\$6,668
Common Support Equipment	\$113
Initial Spares and Repair Parts	\$40,007

CRYSTAL BALL STATISTICS							
Forecast Name	10%	20%	30%	40%	50%	60%	70%
Missile System	655,192	729,664	786,570	842,820	900,616	959,446	1,025,579
SDD Phase	133,277	152,565	169,119	185,197	202,405	221,470	244,162
Air Vehicle	84,792	98,614	109,790	120,551	131,549	144,286	160,084
Design & Development	27,325	29,216	31,196	33,443	35,882	38,486	41,250
Prototypes	12,582	14,066	15,759	17,667	19,700	21,869	24,174
Software	76,482	86,320	97,931	111,893	128,055	145,551	164,807
Sys Engineering/Program Management	23,056	24,341	25,705	27,195	28,760	30,351	31,969
System Test and Evaluation	28,535	32,871	37,531	43,706	50,255	57,146	64,375
Training	6,028	6,577	7,214	8,004	8,919	9,861	10,831
Data	2,638	2,931	3,260	3,678	4,136	4,625	5,144
Support Equipment	1,696	1,960	2,188	2,411	2,631	2,886	3,202
Production Phase	496,775	552,160	596,276	642,025	687,678	735,289	789,817
Air Vehicle	320,089	346,631	366,022	384,997	403,906	425,107	446,933
Payload	13,544	14,289	15,064	15,975	16,916	17,887	18,887
Propulsion	18,719	20,048	21,138	22,436	23,800	25,131	26,529
Airframe	106,808	112,918	119,655	127,794	136,300	145,146	154,332
Guidance and Control	227,156	242,482	260,135	280,332	302,000	325,146	349,775
Integration, Assembly, Test and Checkout	8,313	8,819	9,399	10,067	10,813	11,636	12,546
Engineering Changes	20,299	22,364	24,634	27,429	30,299	33,254	36,299
Sys Engineering/Program Management	110,993	133,850	158,943	190,241	227,790	270,241	318,241
System Test and Evaluation	1,000	1,036	1,074	1,118	1,162	1,206	1,250
Training	31,654	36,025	39,844	43,613	47,339	51,548	56,805
Data	6,437	7,164	7,790	8,381	8,997	9,650	10,396
Peculiar Support Equipment	6,459	7,193	7,799	8,415	9,002	9,620	10,405
Common Support Equipment	55	73	86	99	112	124	137
Initial Spares and Repair Parts	38,710	43,120	46,799	50,263	53,795	57,733	62,019

SDD Phase Risk Dollars
 $= 221,470 - 164,898$
 $= 56,572$

Production Phase Risk Dollars
 $= 735,289 - 531,212$
 $= 204,077$

Figure C-50 Calculate Risk Dollars

Figure C-51 presents the allocation of the risk dollars to the child elements as described in **Section 5.2**.

	Point Estimate	Std Dev	60% Confidence Level From Risk Model	Sum of Children	Difference Between Parent and Sum of Children	Sum of Children's Std Dev	Prorated Difference to Each Child	Difference Between Parent (Incl its Additional Proration) and Sum of Children	Prorated Difference to Each Child	Sum of 60% From Risk Model plus Proration	Difference from Statistical Result
Missile System	\$696,110	\$231,798	\$963,145								
Sys Dev & Demo Phase	\$164,898	\$81,542	\$221,766	\$221,595	\$171	\$87,163				\$221,766	0.0%
Air Vehicle	\$111,549	\$54,857	\$144,041	\$144,322		\$63,262	\$108	(\$173)		\$144,149	0.1%
Design & Development	\$25,000	\$6,509	\$31,196						(\$18)	\$31,178	-0.1%
Prototypes	\$9,749	\$6,044	\$15,674						(\$17)	\$15,657	-0.1%
Software	\$76,800	\$50,709	\$97,452						(\$139)	\$97,313	-0.1%
Sys Engineering/Program Management	\$21,000	\$4,958	\$25,732			\$10				\$25,742	0.0%
System Test and Evaluation	\$22,310	\$21,091	\$37,562			\$41				\$37,603	0.1%
Training	\$5,577	\$3,680	\$8,112			\$7				\$8,119	0.1%
Data	\$2,231	\$1,480	\$3,267			\$3				\$3,270	0.1%
Support Equipment	\$2,231	\$1,097	\$2,881			\$2				\$2,883	0.1%
			0								
Production Phase	\$531,212	\$181,997	\$734,632	\$739,591	(\$4,959)	\$213,354		(\$3,774)		\$734,632	0.0%
Air Vehicle	\$333,396	\$74,435	\$424,253	\$426,297		\$98,189	(\$1,730)			\$422,523	-0.4%
Propulsion	\$11,416	\$3,006	\$15,065						(\$116)	\$14,949	-0.8%
Payload	\$16,271	\$4,499	\$21,116						(\$173)	\$20,943	-0.8%
Airframe	\$112,250	\$26,776	\$119,916						(\$1,029)	\$118,887	-0.9%
Guidance and Control	\$186,979	\$61,745	\$260,818						(\$2,373)	\$258,445	-0.9%
Integration, Assembly, Test and Chec	\$6,480	\$2,163	\$9,382						(\$83)	\$9,299	-0.9%
Engineering Changes	\$16,670	\$9,092	\$24,799				(\$211)			\$24,588	-0.9%
Sys Engineering/Program Management	\$93,351	\$94,298	\$160,801				(\$2,192)			\$158,609	-1.4%
System Test and Evaluation	\$1,000	\$135	\$1,074				(\$3)			\$1,071	-0.3%
Training	\$33,340	\$16,003	\$51,664				(\$372)			\$51,292	-0.7%
Data	\$6,668	\$2,400	\$9,613				(\$56)			\$9,557	-0.6%
Peculiar Support Equipment	\$6,668	\$2,424	\$9,611				(\$56)			\$9,555	-0.6%
Common Support Equipment	\$113	\$47	\$124				(\$1)			\$123	-0.9%
Initial Spares and Repair Parts	\$40,007	\$14,520	\$57,652				(\$337)			\$57,315	-0.6%

Figure C-51 Allocate Risk Dollars

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Figure C-52 illustrates an outputs-based simulation applied to the same missile point estimate (Column D) as discussed in **Section 7.2**. Column E contains the CB distributions that are described in columns F-J. Column C contains formula multiplying columns D and E. Column C also rolls-up the simulated total. The distributions are also correlated as shown in **Figure C-53**.

	B	C	D	E	F	G	H	I	J
7	WBS Description	Estimate (Result)	Point Estimate (Outputs)	Output Uncertainty	Distribution Form	Low	High	1st Param Interpretation	2nd Param Interpretation
11	Missile System	\$696,110							
12	SDD Phase	\$164,898							
13	Air Vehicle	\$111,549							
14	Design & Development	\$25,000	\$25,000	1	Lognormal	0.772	1.296	0.15	0.85
15	Prototypes	\$9,749	\$9,749	1	Lognormal	0.772	1.296	0.15	0.85
16	Software	\$76,800	\$76,800	1	Lognormal	0.696	1.437	0.15	0.85
17	Sys Engineering/Program Management	\$21,000	\$21,000	1	Lognormal	0.772	1.296	0.15	0.85
18	System Test and Evaluation	\$22,310	\$22,310	1	Lognormal	0.696	1.437	0.15	0.85
19	Training	\$5,577	\$5,577	1	Lognormal	0.856	1.168	0.15	0.85
20	Data	\$2,231	\$2,231	1	Lognormal	0.856	1.168	0.15	0.85
21	Support Equipment	\$2,231	\$2,231	1	Lognormal	0.856	1.168	0.15	0.85
22									
23	Production Phase	\$531,212							
24	Air Vehicle	\$333,396							
25	Payload	\$11,416	\$11,416	1	Lognormal	0.856	1.168	0.15	0.85
26	Propulsion	\$16,271	\$16,271	1	Lognormal	0.856	1.168	0.15	0.85
27	Airframe	\$112,250	\$112,250	1	Lognormal	0.772	1.296	0.15	0.85
28	Guidance and Control	\$186,979	\$186,979	1	Lognormal	0.772	1.296	0.15	0.85
29	Integration, Assembly, Test and Checkout	\$6,480	\$6,480	1	Lognormal	0.856	1.168	0.15	0.85
30	Engineering Changes	\$16,670	\$16,670	1	Lognormal	0.772	1.296	0.15	0.85
31	Sys Engineering/Program Management	\$93,351	\$93,351	1	Lognormal	0.696	1.437	0.15	0.85
32	System Test and Evaluation	\$1,000	\$1,000	1	Lognormal	0.856	1.168	0.15	0.85
33	Training	\$33,340	\$33,340	1	Lognormal	0.856	1.168	0.15	0.85
34	Data	\$6,668	\$6,668	1	Lognormal	0.856	1.168	0.15	0.85
35	Peculiar Support Equipment	\$6,668	\$6,668	1	Lognormal	0.856	1.168	0.15	0.85
36	Common Support Equipment	\$113	\$113	1	Lognormal	0.856	1.168	0.15	0.85
37	Initial Spares and Repair Parts	\$40,007	\$40,007	1	Lognormal	0.772	1.296	0.15	0.85

Figure C-52 Outputs-based Simulation Missile Example

	Initial Spares and Repair Parts (AFCAA Sample Missile Estimate)	Common Support Equipment (AFCAA Sample Missile Estimate)	Peculiar Support Equipment (AFCAA Sample Missile Estimate)	Data (AFCAA Sample Missile Estimate)	Training (AFCAA Sample Missile Estimate)	System Test and Evaluation (AFCAA Sample Missile Estimate)	Engineering Changes (AFCAA Sample Missile Estimate)	Integration, Assembly, Test and Checkout (AFCAA Sample Missile Estimate)	Guidance and Control (AFCAA Sample Missile Estimate)	Airframe (AFCAA Sample Missile Estimate)	Propulsion (AFCAA Sample Missile Estimate)	Payload (AFCAA Sample Missile Estimate)	Support Equipment (AFCAA Sample Missile Estimate)	Data (AFCAA Sample Missile Estimate)	Training (AFCAA Sample Missile Estimate)	System Test and Evaluation (AFCAA Sample Missile Estimate)	Sys Engineering/Program Management (AFCAA Sample Missile Estimate)	Software (AFCAA Sample Missile Estimate)	Prototypes (AFCAA Sample Missile Estimate)	Design & Development (AFCAA Sample Missile Estimate)	
Design & Development (AFCAA Sample Missile Estimate)																					1.000
Prototypes (AFCAA Sample Missile Estimate)																					0.500
Software (AFCAA Sample Missile Estimate)																					0.500
Sys Engineering/Program Management (AFCAA Sample Missile Estimate)																					0.500
System Test and Evaluation (AFCAA Sample Missile Estimate)																					0.500
Training (AFCAA Sample Missile Estimate)																					0.500
Data (AFCAA Sample Missile Estimate)																					0.500
Support Equipment (AFCAA Sample Missile Estimate)																					0.500
Payload (AFCAA Sample Missile Estimate)																					0.500
Propulsion (AFCAA Sample Missile Estimate)																					0.500
Airframe (AFCAA Sample Missile Estimate)																					0.500
Guidance and Control (AFCAA Sample Missile Estimate)																					0.500
Integration, Assembly, Test and Checkout (AFCAA Sample Missile Estimate)																					0.500
Engineering Changes (AFCAA Sample Missile Estimate)																					0.500
Sys Engineering/Program Management (AFCAA Sample Missile Estimate)																					0.500
System Test and Evaluation (AFCAA Sample Missile Estimate)																					0.500
Training (AFCAA Sample Missile Estimate)																					0.500
Data (AFCAA Sample Missile Estimate)																					0.500
Peculiar Support Equipment (AFCAA Sample Missile Estimate)																					0.500
Common Support Equipment (AFCAA Sample Missile Estimate)																					0.500
Initial Spares and Repair Parts (AFCAA Sample Missile Estimate)																					0.500

Figure C-53 Outputs-based Simulation Correlation Matrix

APPENDIX D ACRONYM LIST

ACAT	Acquisition Category
ACE	Automated Cost Estimator
ACEIT	Automated Cost Estimating Integrated Tools
BY	Base Year
CARD	Cost Analysis Requirements Description
CB	Crystal Ball
CDF	Cumulative Distribution Function
CER	Cost Estimating Relationship
CGPF	Cost Growth Potential Factor
COCOMO	Constructive Cost Model
CR	Cost Reserve
CRUH	Cost Risk and Uncertainty Handbook
CV	Coefficient of Variation
df	Degrees of freedom
ECO	Engineering Change Order
FY	Fiscal Year
GERM	Generalized Error Regression Model
IC CAIG	Intelligence Community Cost Analysis Improvement Group
IRLS	Iteratively Reweighted Least Squares
LOLS	Ordinary Least Squares, Log Space
MDA	Missile Defense Agency
MPE	Minimum Percentage Error
MSE	Mean Squared Error
MUPE	Minimum Unbiased Percentage Error
OLS	Ordinary Least Squares
PDF	Probability Density Function
PE	Point Estimate
PF	Ping Factor
PI	Prediction Interval
PS	Protect Scenario
QAIV	Quantity as an Independent Variable

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SBM	Scenario-Based Method
SDD	System Development and Demonstration
SE	Standard Error
SEE	Standard Error of the Estimate
SEPM	Systems Engineering and Program Management
SME	Subject Matter Expert
SW	Software
TBE	Technical Baseline Point Estimate
TY	Then Year
WBS	Work Breakdown Structure
ZMPE	Zero Bias Minimum Percent Error

ATTACHMENT 1

A Scenario-Based Method for Cost Risk Analysis

Paul R. Garvey

A Scenario-Based Method for Cost Risk Analysis

Paul R. Garvey
The MITRE Corporation
MP 05B0000023, September 2005

Abstract

This paper presents an approach for performing an analysis of a program's cost risk. The approach is referred to as the scenario-based method (SBM). This method provides program managers and decision-makers an assessment of the amount of cost reserve needed to protect a program from cost overruns due to risk. The approach can be applied without the use of advanced statistical concepts, or Monte Carlo simulations, yet is flexible in that confidence measures for various possible program costs can be derived.

1.0 Introduction

This paper* introduces an analytical, non-Monte Carlo simulation, approach for quantifying a program's cost risks and deriving recommended levels of cost reserve. The approach is called the Scenario-Based Method (SBM). This method emphasizes the development of written scenarios as the basis for deriving and defending a program's cost and cost reserve recommendations.

The method presented in the paper grew from a question posed by a government agency. The question was *Can a valid cost risk analysis (that is traceable and defensible) be conducted with minimal (to no) reliance on Monte Carlo simulation or other statistical methods?* The question was motivated by the agency's unsatisfactory experiences in developing and implementing Monte Carlo simulations to derive "risk-adjusted" costs of future systems.

This paper presents a method that addresses the question posed by the agency. The method reflects a "minimum acceptable" approach whereby a technically valid measure of cost risk can be derived without Monte Carlo simulations or advanced statistical methods. A "statistically-light" analytical augmentation to

* *This paper was written for the United States Air Force Cost Analysis Agency.*

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this method is also presented that enables one to assess probabilities that a program's cost will (or will not) be exceeded.

2.0 Some Basic Terms and Definitions

Throughout this paper certain technical terms and distinctions between them are used. This section presents these terms and explains the subtleties between their meanings. First, we'll briefly discuss the concept of a subjective probability. This will be followed by a discussion of risk versus uncertainty and the differences between them.

Subjective Probability Assessments [1]: Probability theory is a well-established formalism for quantifying uncertainty. Its application to real-world systems engineering and cost analysis problems often involves the use of subjective probabilities. Subjective probabilities are those assigned to events on the basis of personal judgment. They are measures of a person's degree-of-belief an event will occur.

Subjective probabilities are associated with one-time, non-repeatable events, those whose probabilities cannot be objectively determined from a sample space of outcomes developed by repeated trials, or experimentation. Subjective probabilities must be consistent with the axioms of probability [1]. For instance, if an engineer assigns a probability of 0.70 to the event "*the number of gates for the new processor chip will not exceed 12000*" then it must follow the chip will exceed 12000 gates with probability 0.30. Subjective probabilities are conditional on the state of the person's knowledge, which changes with time.

To be credible, subjective probabilities should only be assigned to events by subject matter experts, persons with significant experience with events similar to the one under consideration. Instead of assigning a single subjective probability to an event, subject experts often find it easier to describe a function that depicts a distribution of probabilities. Such a distribution is sometimes called a

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subjective probability distribution. Subjective probability distributions are governed by the same mathematical properties of probability distributions associated with discrete or continuous random variables.

Subjective probability distributions are most common in cost uncertainty analysis, particularly on the input-side of the process. Because of their nature, subjective probability distributions can be thought of as “belief functions.” They describe a subject expert’s belief in the distribution of probabilities for an event under consideration. Probability theory provides the mathematical formalism with which we operate (add, subtract, multiply, and divide) on these belief functions.

Risk versus Uncertainty [1]: There is an important distinction between the terms risk and uncertainty. Risk is the chance of loss or injury. In a situation that includes favorable and unfavorable events, risk is the *probability an unfavorable event occurs*. Uncertainty is the *indefiniteness* about the outcome of a situation. We analyze uncertainty for the purpose of measuring risk.

In systems engineering the analysis might focus on measuring the risk of: failing to achieve performance objectives, overrunning the budgeted cost, or delivering the system too late to meet user needs. Conducting the analysis involves varying degrees of subjectivity. This includes defining the events of concern, as well as specifying their subjective probabilities.

Given this, it is fair to ask whether it’s meaningful to apply rigorous procedures to such analyses. In a speech before the 1955 Operations Research Society of America meeting, Charles Hitch addressed this question. He stated [2]:

Systems analyses provide a framework which permits the judgment of experts in many fields to be combined to yield results that transcend any individual judgment. The systems analyst [cost analyst] may have to be content with better rather than optimal solutions; or with devising and costing sensible methods of hedging; or merely with

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discovering critical sensitivities. We tend to be worse, in an absolute sense, in applying analysis or scientific method to broad context problems; but unaided intuition in such problems is also much worse in the absolute sense. Let's not deprive ourselves of any useful tools, however short of perfection they may fail.

Given the above, it is worth a brief review of what we mean by cost uncertainty analysis and cost risk analysis. *Cost uncertainty analysis is a process of quantifying the cost impacts of uncertainties associated with a system's technical definition and cost estimation methodologies. Cost risk analysis is a process of quantifying the cost impacts of risks associated with a system's technical definition and cost estimation methodologies. Cost risk is a measure of the chance that, due to unfavorable events, the planned or budgeted cost of a project will be exceeded.*

Why conduct the analysis? There are many answers to this question; one answer is to produce a defensible assessment of the level of cost to budget such that this cost has an acceptable probability of not being exceeded.

3.0 The Scenario-Based Method (SBM): A Non-statistical Implementation

Given the "what" and "why" of cost risk analysis, *a minimum acceptable method* is one that operates on specified scenarios that, if they occurred, would result in costs higher than the level planned or budgeted. These scenarios do not have to represent worst cases; rather, they should reflect a set of conditions a program manager or decision-maker would want to have budget to guard against, should any or all of them occur. For purposes of this discussion, we'll call this minimum acceptable method the "Scenario-Based Method" (SBM) for cost risk analysis.

The Scenario-Based Method derives from what could be called "sensitivity analysis", but with one difference. Instead of arbitrarily varying one or more variables to measure the sensitivity (or change) in cost, the Scenario-Based Method involves specifying *a well-defined set of technical and programmatic conditions that collectively affect a number of cost-related variables and associated work breakdown structure (WBS) elements in a way that increase cost beyond what was*

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planned. Defining these conditions and integrating them into a coherent risk “story” for the program is what is meant by the term “*scenario*”.

The process of defining scenarios is a good practice. It builds the supportive rational and provides a traceable and defensible analytical basis behind a “derived” measure of cost risk; this is often lacking in traditional simulation approaches. Visibility, traceability, defensibility, and the cost impacts of specifically identified risks is a principal strength of the Scenario-Based Method.

Figure 1 illustrates the process flow behind the non-statistical SBM.

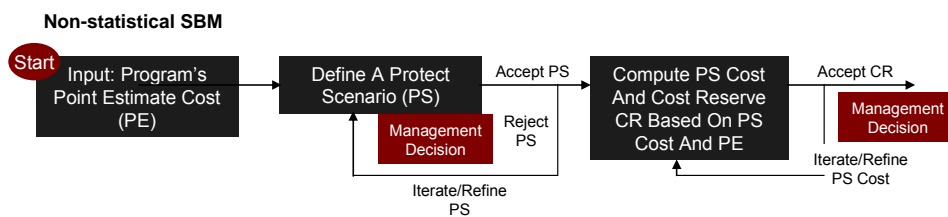


Figure 1. A Non-statistical Scenario-Based Method

The first step (see Start) is input to the process. It is the program’s point estimate cost (PE). For purposes of this paper, the point estimate cost is defined as the cost that does not include an allowance for cost reserve. It is the sum of the cost element costs summed across the program’s work breakdown structure *without adjustments for uncertainty*. Often, the point estimate is developed from the program’s cost analysis requirements description (CARD).

Next, is the effort to define a protect scenario (PS). The key to a “good PS” is one that identifies, not an extreme worst case, but a scenario that captures the impacts of the major known risks to the program – those events the program manager or decision-maker must monitor and guard the costs of the program against. Thus, the PS is not arbitrary. It should reflect the above, as well as

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provide a possible program cost that, in the opinion of the engineering and analysis team, has *an acceptable chance of not being exceeded*.

In practice, it is envisioned that management will converge on a protect scenario after a series of discussions, refinements, and iterations from the initially defined scenario. This part of the process, if executed, is to ensure all parties reach a consensus understanding of the risks the program faces and how they are best represented by the protect scenario.

Once the protect scenario has been defined and agreed to its cost is then determined. The next step is computing the amount of cost reserve dollars (CR) needed to protect the program's cost against identified risk. This step of the process defines cost reserve as the difference between the PS cost and the point estimate cost, PE. Shown in figure 1, there may be additional refinements to the cost estimated for the protect scenario, based on management reviews and considerations. This too may be an iterative process until the reasonableness of the magnitude of this figure is accepted by the management team.

A Valid Cost Risk Analysis

This approach, though simple in appearance, is a valid cost risk analysis; *why?* The process of defining scenarios is a valuable exercise in identifying technical and cost estimation risks inherent to the program. Without the need to define scenarios, cost risk analyses can be superficial with its basis not well-defined or carefully thought through. Scenario definition encourages a discourse on program risks that otherwise might not be held. It allows risks to become fully visible, traceable, and "costable" to program managers and decision-makers.

Defining, iterating, and converging on a protect scenario is valuable for understanding the "elasticity" in program costs and identifying those sets of risks (e.g., weight growth, software size increases, schedule slippages, etc.) the program must guard its costs against. Defining scenarios, in general, builds the

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supportive rationale and provides a traceable and defensible analytical basis behind a “derived” measure of cost risk; this is often lacking in traditional simulation approaches. Visibility, traceability, defensibility, and the cost impacts of specifically identified risks is a principal strength of the Scenario-Based Method.

The non-statistical SBM described above does come with limits. Mentioned earlier, cost risk, by definition, is a measure of the chance that, due to unfavorable events, the planned or budgeted cost of a program will be exceeded. A non-statistical SBM does not produce confidence measures. The chance that the cost of the protect scenario, or the cost of any defined scenario, will not be exceeded is not explicitly determined. The question is *Can the design of the SBM be modified to produce confidence measures while maintaining its simplicity and analytical features?* The answer is yes. A way to do this is described in the section that follows.

4.0 The Scenario-Based Method (SBM): A Statistical Implementation

This section presents a statistical, non-Monte Carlo simulation, implementation of the SBM. It is an optional augmentation to the methodology discussed above. It can be implemented with lookup tables, a few algebraic equations, and some appropriate technical assumptions and guidance.

There are many reasons to implement a statistical SBM. These include (1) a way to develop confidence measures; specifically, confidence measures on the dollars to plan so the program’s cost has an acceptable chance of not being exceeded (2) a means where management can examine changes in confidence measures, as a function of how much reserve to “buy” to ensure program success from a cost control perspective and (3) a way to assess where costs of other scenarios of interest different than the protect scenario fall on the probability distribution of the program’s total cost.

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Approach & Assumptions

Figure 2 illustrates the basic approach involved in implementing a statistical SBM. Observe that parts of the approach include the same steps required in the non-statistical SBM. So, the statistical SBM is really an augmentation to the non-statistical SBM. The following explains the approach, discusses key technical assumptions, and highlights selected steps with computational examples.

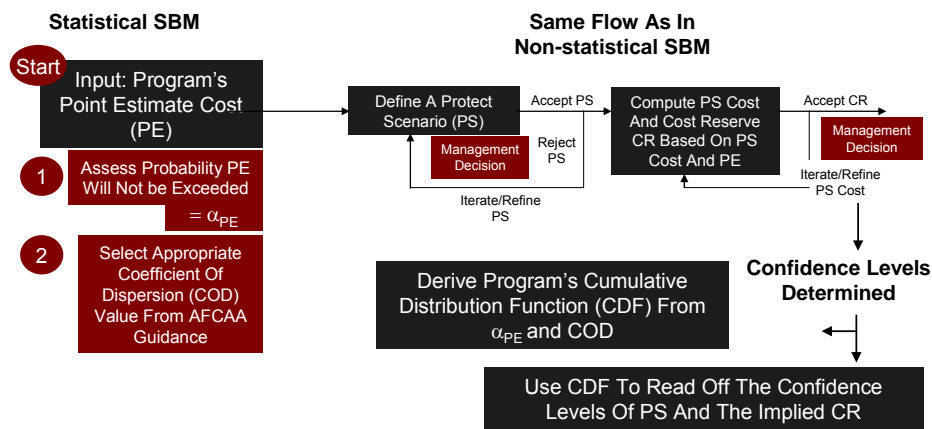


Figure 2. A Statistical Scenario-Based Method

Mentioned above, the statistical SBM follows a set of steps similar to the non-statistical SBM. In figure 2, the top three activities are essentially the same as described in the non-statistical SBM, with the following exception. Two statistical inputs are needed. They are the probability the point estimate cost (PE) will not be exceeded α_{PE} and the coefficient of dispersion (COD). We'll next discuss these a little further.

Point Estimate Probability

For the statistical SBM, we need the probability

$$P(\text{Cost}_{\text{Pgm}} \leq x_{\text{PE}}) = \alpha_{\text{PE}} \quad (4-1)$$

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where $Cost_{pgm}$ is the true, but unknown, total cost of the program and x_{PE} is the program's point estimate cost (PE). Here, the probability α_{PE} is a judgmental or subjective probability. It is assessed by the engineering and analysis team. In practice, α_{PE} often falls in the interval $0.10 \leq \alpha_{PE} \leq 0.50$.

*Coefficient of Dispersion (COD)**

What is the coefficient of dispersion? The coefficient of dispersion (COD) is a statistical measure defined as the ratio of distribution's standard deviation σ to its mean μ . It is one way to look at the variability of the distribution at one standard deviation around its mean. The general form of the COD is given by equation 4-2.

$$D = \frac{\sigma}{\mu} \quad (4-2)$$

Figure 3 illustrates this statistical measure.

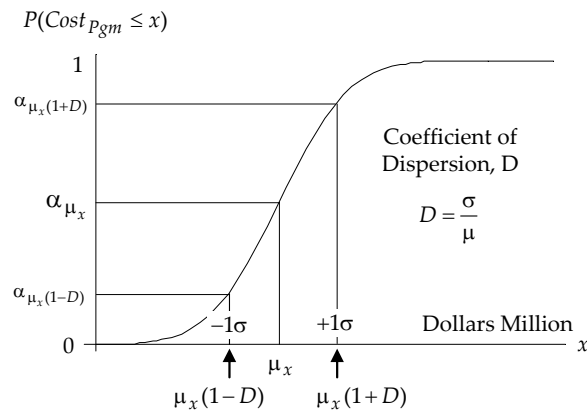


Figure 3. Coefficient of Dispersion

* The coefficient of dispersion is also known as the coefficient of variation.

Here, the COD statistic is a judgmental value but one guided by Air Force Cost Analysis Agency (AFCAA) and industry experiences with programs in various stages or phases of the acquisition process. As will be discussed later in this paper, a sensitivity analysis should be conducted on both statistical inputs, namely α_{PE} and COD, to assess where changes in assumed values affect cost risk and needed levels of reserve funds.

The next two steps along the top of the process flow, in figure 2, follow the procedures described in the non-statistical SBM. Notice these two steps do not use the statistical measures α_{PE} and COD. It is not until you reach the last step of this process that these measures come into play.

As will be shown in the forthcoming examples, the distribution function of the program's total cost can be derived from just the three values identified on the far-left side of the process flow in figure 2. Specifically, with just the point estimate cost PE, α_{PE} , and COD the underlying distribution function of the program's total cost can be determined. With this, other possible program costs, such as the protect scenario cost, can be mapped onto the function. From this, the confidence level of the protect scenario and its implied cost reserve can be seen.

This completes an overview description of the statistical SBM process. The following presents two computational examples that illustrate how the statistical SBM works.

4.1 Formulas: Statistical SBM With An Assumed Underlying Normal

Here, we assume the underlying probability distribution of $Cost_{Pgm}$ is normally distributed and the point (x_{PE}, α_{PE}) falls along this normal. If we're given just the point estimate PE, α_{PE} , COD, then the mean and standard deviation of $Cost_{Pgm}$ are given by the following equations.

$$\mu_{Cost_{Pgm}} = x_{PE} - z_{PE} \frac{Dx_{PE}}{1 + Dz_{PE}} \quad (4-3)$$

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$$\sigma_{Cost_{Pgm}} = \frac{Dx_{PE}}{1 + Dz_{PE}} \quad (4-4)$$

where D is the coefficient of dispersion (COD), x_{PE} is the program's point estimate cost, z_{PE} is the value such that $P(Z \leq z_{PE}) = \alpha_{PE}$ and Z is the standard normal random variable; that is, $Z \sim N(0,1)$. The value for z_{PE} derives from the look-up table in Appendix A.

Once $\mu_{Cost_{Pgm}}$ and $\sigma_{Cost_{Pgm}}$ are computed, the entire distribution function of the normal can be specified, along with the probability that $Cost_{Pgm}$ may take any particular outcome, such as the protect scenario cost. The following illustrates how these equations work.

Computational Example 4-1: Assumed Normal

Suppose the distribution function for $Cost_{Pgm}$ is normal. Suppose the point estimate cost of the program is 100 (\$M) and this cost was assessed to fall at the 25th percentile. Suppose the type and phase of the program is such that 30 percent variability in cost around the mean has been historically seen. Suppose the protect scenario was defined and determined to cost 145 (\$M). Given this,

- Compute $\mu_{Cost_{Pgm}}$ and $\sigma_{Cost_{Pgm}}$.
- Plot the distribution function of $Cost_{Pgm}$.
- Determine the confidence level of the protect scenario cost and its associated cost reserve.

Solution

- From the information given and from equations 4-3 and 4-4 we have

$$\mu_{Cost_{Pgm}} = x_{PE} - z_{PE} \frac{Dx_{PE}}{1 + Dz_{PE}} = 100 - z_{PE} \frac{(0.3)(100)}{1 + (0.3)z_{PE}}$$

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$$\sigma_{Cost_{pgm}} = \frac{Dx_{PE}}{1 + Dz_{PE}} = \frac{(0.3)(100)}{1 + (0.3)z_{PE}}$$

We need z_{PE} to complete these computations. From the information given, we know $P(Z \leq z_{PE}) = 0.25$. Since Z is assumed to be a standard normal random variable, we can look-up the values for z_{PE} from table A-1 (refer to Appendix A). In this case, it follows that

$$P(Z \leq z_{PE} = -0.6745) = 0.25$$

therefore, with $z_{PE} = -0.6745$ we have

$$\mu_{Cost_{pgm}} = x_{PE} - z_{PE} \frac{Dx_{PE}}{1 + Dz_{PE}} = 100 - z_{PE} \frac{(0.3)(100)}{1 + (0.3)z_{PE}} = 125.4 \text{ (\$M)}$$

$$\sigma_{Cost_{pgm}} = \frac{Dx_{PE}}{1 + Dz_{PE}} = \frac{(0.3)(100)}{1 + (0.3)z_{PE}} = 37.6 \text{ (\$M)}$$

b) A plot of the distribution function of $Cost_{pgm}$ is shown in figure 4. This is a plot of a normal distribution with mean 125.4 (\$M) and standard deviation 37.6 (\$M).

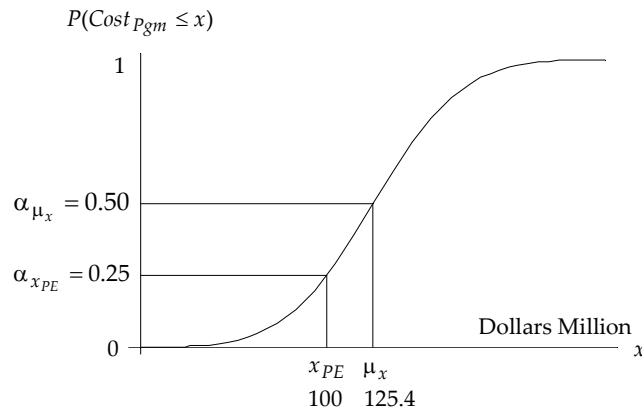


Figure 4. A Plot of the Normal Distribution: Mean 125.4, Sigma 37.6

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c) To determine the confidence level of the protect scenario we need to find $\alpha_{x_{PS}}$ such that

$$P(\text{Cost}_{pgm} \leq x_{PS} = 145) = \alpha_{x_{PS}}$$

Finding $\alpha_{x_{PS}}$ is equivalent to solving

$$\mu_{\text{Cost}_{pgm}} + z_{x_{PS}} (\sigma_{\text{Cost}_{pgm}}) = x_{PS}$$

for $z_{x_{PS}}$. From the above, we can write the expression

$$z_{x_{PS}} = \frac{x_{PS} - \mu_{\text{Cost}_{pgm}}}{\sigma_{\text{Cost}_{pgm}}} = \frac{x_{PS}}{\sigma_{\text{Cost}_{pgm}}} - \frac{1}{D} \quad (4-5)$$

Since $x_{PS} = 145$, $\mu_{\text{Cost}_{pgm}} = 125.4$, and $\sigma_{\text{Cost}_{pgm}} = 37.6$ it follows that

$$z_{x_{PS}} = \frac{x_{PS} - \mu_{\text{Cost}_{pgm}}}{\sigma_{\text{Cost}_{pgm}}} = \frac{x_{PS}}{\sigma_{\text{Cost}_{pgm}}} - \frac{1}{D} = \frac{145}{37.6} - \frac{1}{(0.3)} = 0.523$$

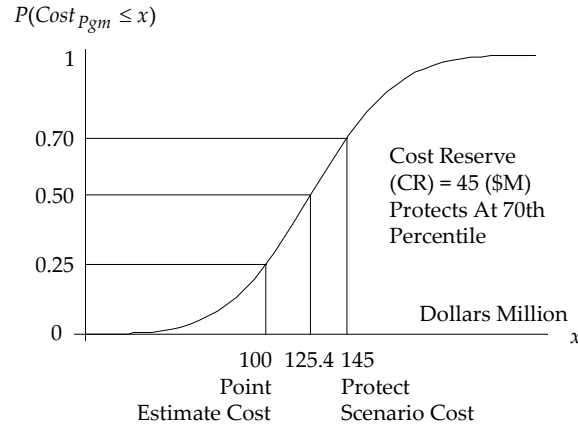
From the look-up table in Appendix A we see that

$$P(Z \leq z_{x_{PS}} = 0.523) \approx 0.70$$

Therefore, the protect scenario cost of 145 (\$M) falls at approximately the 70th percentile of the distribution with a cost reserve (CR) of 45 (\$M). Figure 5 shows these results graphically. This concludes example 4-1. ♣

The following provides formulas for the mean and standard deviation of Cost_{pgm} if the underlying distribution of possible program costs is represented by a lognormal. The lognormal is similar to the normal in that the $\ln(\text{Cost}_{pgm})$ is normally distributed instead of Cost_{pgm} being normally distributed. The lognormal is different than the normal distribution because it is skewed towards the positive end of the range, instead of being symmetric about the mean.

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Figure 5. Example 4-1 Illustrated: Assumed *Normal* Distribution

Numerous studies [1] have empirically shown the normal or lognormal to be excellent approximations to the overall distribution function of a program's total cost, even in the presence of correlations among cost element costs. The decision to use one over the other is really a matter of analyst judgment. In practice, it is simple enough to execute an analysis using both distributions to examine if there are significant differences between them. Then, use judgment to select the distribution that best reflects the cost and risk conditions of the program.

4.2 Formulas: Statistical SBM With An Assumed Underlying LogNormal

Here, we assume the underlying probability distribution of Cost_{pgm} is lognormally distributed and the point (x_{PE}, α_{PE}) falls along this lognormal. There are two steps involved in computing the mean and standard deviation of Cost_{pgm} . The first is to compute the mean and standard deviation of $\ln(\text{Cost}_{pgm})$. The second is to translate these values into the mean and standard deviation of Cost_{pgm} , so the units are in dollars instead of "log-dollars".

Step 1: Formulas for the Mean and Standard Deviation of $\ln(\text{Cost}_{pgm})$

$$\mu_{\ln \text{Cost}_{pgm}} = \ln x_{PE} - z_{PE} \sqrt{\ln(1 + D^2)} \quad (4-6)$$

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$$\sigma_{\ln \text{Cost}_{p_{gm}}} = \sqrt{\ln(1 + D^2)} \quad (4-7)$$

where D is the coefficient of dispersion (COD), x_{PE} is the program's point estimate cost, z_{PE} is the value such that $P(Z \leq z_{PE}) = \alpha_{PE}$ and Z is the standard normal random variable; that is, $Z \sim N(0,1)$. The value for z_{PE} derives from the look-up table in Appendix A.

Step 2: Once $\mu_{\ln \text{Cost}_{p_{gm}}}$ and $\sigma_{\ln \text{Cost}_{p_{gm}}}$ are computed, they need to be translated into "dollar-units". Equation 4-8 and equation 4-9 provide this translation [1].

$$\mu_{\text{Cost}_{p_{gm}}} = e^{\mu_{\ln \text{Cost}_{p_{gm}}} + \frac{1}{2}\sigma_{\ln \text{Cost}_{p_{gm}}}^2} \quad (4-8)$$

$$\sigma_{\text{Cost}_{p_{gm}}} = \sqrt{e^{2\mu_{\ln \text{Cost}_{p_{gm}}} + \sigma_{\ln \text{Cost}_{p_{gm}}}^2} (e^{\sigma_{\ln \text{Cost}_{p_{gm}}}^2} - 1)} \quad (4-9)$$

Once $\mu_{\text{Cost}_{p_{gm}}}$ and $\sigma_{\text{Cost}_{p_{gm}}}$ are computed, the entire distribution function of the lognormal can be specified, along with the probability that $\text{Cost}_{p_{gm}}$ may take a particular outcome. The following illustrates how the last four equations work.

Computational Example 4-2: Assumed LogNormal

Suppose the distribution function for $\text{Cost}_{p_{gm}}$ is lognormal. Suppose the point estimate cost of the program is 100 (\$M) and this cost was assessed to fall at the 25th percentile. Suppose the type and phase of the program is such that 30 percent variability in cost around the mean has been historically seen. Suppose the protect scenario was defined and determined to cost 145 (\$M). Given this,

- a) Compute $\mu_{\text{Cost}_{p_{gm}}}$ and $\sigma_{\text{Cost}_{p_{gm}}}$.
- b) Plot the distribution function of $\text{Cost}_{p_{gm}}$.
- c) Determine the confidence level of the protect scenario cost and its associated cost reserve.

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Solution

a) From equations 4-6 and 4-7, and example 4-1, it follows that

$$\mu_{\ln \text{Cost}_{Pgm}} = \ln x_{PE} - z_{PE} \sqrt{\ln(1+D^2)} = \ln(100) - (-0.6745) \sqrt{\ln(1+(0.3)^2)} = 4.80317$$

$$\sigma_{\ln \text{Cost}_{Pgm}} = \sqrt{\ln(1+D^2)} = \sqrt{\ln(1+(0.3)^2)} = 0.29356$$

From equations 4-8 and 4-9 we translate the above mean and standard deviation into dollar units; that is,

$$\mu_{\text{Cost}_{Pgm}} = e^{\mu_{\ln \text{Cost}_{Pgm}} + \frac{1}{2}\sigma_{\ln \text{Cost}_{Pgm}}^2} = e^{4.80317 + \frac{1}{2}(0.29356)^2} \approx 127.3 \text{ (\$M)}$$

$$\begin{aligned} \sigma_{\text{Cost}_{Pgm}} &= \sqrt{e^{2\mu_{\ln \text{Cost}_{Pgm}} + \sigma_{\ln \text{Cost}_{Pgm}}^2} (e^{\sigma_{\ln \text{Cost}_{Pgm}}^2} - 1)} \\ &= \sqrt{e^{2(4.80317) + (0.29356)^2} (e^{(0.29356)^2} - 1)} \approx 38.2 \text{ (\$M)} \end{aligned}$$

b) A plot of the distribution function of Cost_{Pgm} is shown in figure 6. This is a plot of a lognormal distribution with mean 127.3 and standard deviation 38.2

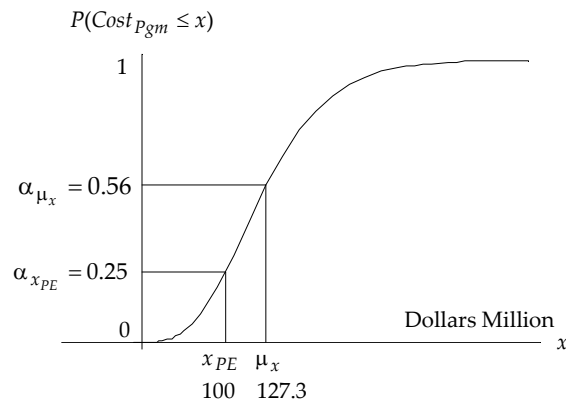


Figure 6. A Plot of the LogNormal Distribution: Mean 127.3, Sigma 38.2

c) To determine the confidence level of the protect scenario we need to find $\alpha_{x_{PS}}$ such that

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$$P(\text{Cost}_{pgm} \leq x_{PS} = 145) = \alpha_{x_{PS}}$$

Finding $\alpha_{x_{PS}}$ is equivalent to solving

$$\mu_{\ln \text{Cost}_{pgm}} + z_{x_{PS}} (\sigma_{\ln \text{Cost}_{pgm}}) = \ln x_{PS}$$

for $z_{x_{PS}}$. From the above, we can write the expression

$$z_{x_{PS}} = \frac{\ln x_{PS} - \mu_{\ln \text{Cost}_{pgm}}}{\sigma_{\ln \text{Cost}_{pgm}}}$$

Since $x_{PS} = 145$, $\mu_{\ln \text{Cost}_{pgm}} = 4.80317$, and $\sigma_{\ln \text{Cost}_{pgm}} = 0.29356$ it follows that

$$z_{x_{PS}} = \frac{\ln x_{PS} - \mu_{\ln \text{Cost}_{pgm}}}{\sigma_{\ln \text{Cost}_{pgm}}} = \frac{\ln 145 - 4.80317}{0.29356} = 0.59123$$

From the look-up table in Appendix A we see that

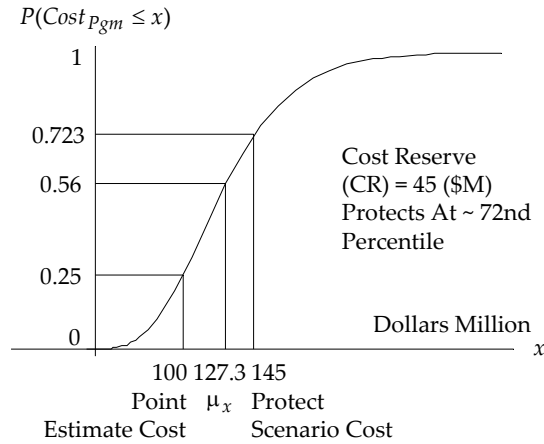
$$P(Z \leq z_{x_{PS}} = 0.59123) \approx 0.723$$

Therefore, the protect scenario cost of 145 (\$M) falls at approximately the 72nd percentile of the distribution with a cost reserve (CR) of 45 (\$M). Figure 7 shows these results graphically. This concludes example 4-2. ♣

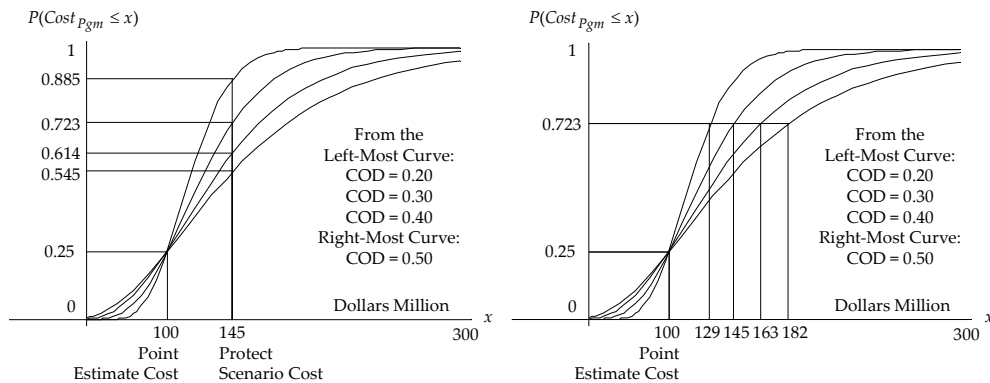
4.3 A Sensitivity Analysis

There are many ways to design and perform a sensitivity analysis on the SBM, particularly the statistical SBM. In this mode, one might vary the statistical inputs, namely α_{PE} and/or the COD. From experience, we know α_{PE} will often fall in the interval $0.10 \leq \alpha_{PE} \leq 0.50$. For this paper, we set $\alpha_{PE} = 0.25$ and the COD equal to 0.30 to illustrate the statistical aspects of the SBM. In practice, these measures will vary for each program – not only as a function of the program's type (e.g., space, C4ISR) but its maturity and phase along the acquisition timeline.

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Figure 7. Example 4-2 Illustrated: Assumed *LogNormal* Distribution

The following shows a sensitivity analysis on the statistical SBM with varying levels of the coefficient of dispersion, COD. This is done in the context of example 4-2. Figure 8 illustrates how either the confidence level can vary as a function of the COD or how the dollar level can vary as a function of the COD. Here, the left-most family of lognormal distributions, in figure 8, shows for a protect scenario cost of 145 (\$M) the confidence level can range from 0.545 to 0.885 depending in the magnitude of the COD.

Figure 8. A Sensitivity Analysis on the Coefficient of Dispersion:
Families of *LogNormal* Distributions

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The right-most family of lognormal distributions, in figure 8, shows for a confidence level of just over 70 percent the dollars can range from 129 (\$M) to 182 (\$M), depending on the magnitude of the COD.

The above analysis is intended to demonstrate the sensitivity of the analysis results to wide variations in the coefficient of dispersion. In practice, a program would not experience such wide swings in COD values. However, it is good practice to vary the COD by some amount around the “point” value to see what possible variations in confidence levels or dollars results*.

As a good practice point a sensitivity analysis should always be conducted, especially when implementing the statistical SBM. The analysis can signal where additional refinements to scenarios, and the underlying analytical assumptions, may be needed. This is what good analysis is all about!!

5.0 Summary

This paper presented an approach for performing an analysis of a program’s cost risk. The approach is referred to as the scenario-based method (SBM). It provides program managers and decision-makers a scenario-based assessment of the amount of cost reserve needed to protect a program from cost overruns due to risk. The approach can be applied without the use of advanced statistical concepts, or Monte Carlo simulations, yet is flexible in that confidence measures for various possible program costs can be derived.

Features of this approach include the following:

** This analysis was based on the assumption that a program’s cost uncertainty could be represented by a lognormal distribution. It is important to note the lognormal is bounded by zero; hence, cost will always be non-negative. In a sensitivity analysis, such as the one presented here, it is possible the coefficient of dispersion could be so large as to drive program costs into negative values if an underlying normal is assumed, since the normal distribution is an infinite distribution at both tails. As the SBM is tested and implementation experiences with the approach are collected, it may be decided the lognormal distribution assumption is the “better” of the two, in most cases.*

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- Provides an analytic argument for deriving the amount of cost reserve needed to guard against well-defined “scenarios”;
- Brings the discussion of “scenarios” and their credibility to the decision-makers; this is a more meaningful topic to focus on, instead of statistical abstractions the classical analysis can sometimes create;
- Does not require the use of statistical methods to develop a valid measure of cost risk reserve; this is the non-statistical SBM;
- Percentiles (confidence measures) can be designed into the approach with a minimum set of statistical assumptions;
- Percentiles (as well as the mean, median (50th%), variance, etc.) can be calculated algebraically and thus can be executed in near-real time within a simple spreadsheet environment; Monte Carlo simulation is not needed;
- Does not require analysts develop probability distribution functions for all the uncertain variables in a WBS, which can be time-consuming and hard to justify;
- Correlation is indirectly captured in the analysis by the magnitude of the coefficient of dispersion applied to the analysis;
- The approach fully supports traceability and focuses attention on key risk events that have the potential to drive cost higher than expected.

In summary, the Scenario Based Method encourages and emphasizes a careful and deliberative approach to cost risk analysis. It requires the development of scenarios that represent the program’s “risk story” rather than debating what percentile to select. Time is best spent building the case arguments for how a confluence of risk events might drive the program to a particular percentile. This is where the debate and the analysis should center. This is how a program manager and decision-maker can rationalize the need for cost reserve levels that may initially exceed expectations. It is also a vehicle for identifying where risk mitigation actions should be implemented to reduce cost risk and the chances of program costs becoming out of control.

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References

- [1] Garvey, Paul R., *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, 2000, Marcel Dekker, Inc., Taylor & Francis Group, Chapman-Hall/CRC Press, New York, N.Y., 10016.
- [2] Hitch, C. J. 1955. *An Appreciation of Systems Analysis*, P-699. Santa Monica, California: The RAND Corporation.

About the Author...

Paul R. Garvey is Chief Scientist, and a Director, for the Center for Acquisition and Systems Analysis at The MITRE Corporation. Mr. Garvey is internationally recognized and widely published in the application of decision analytic methods to problems in systems engineering risk management. His articles in this area have appeared in numerous peer-reviewed journals, technical books, and recently in John Wiley & Son's Encyclopedia of Electrical and Electronics Engineering.

Mr. Garvey authored the textbook "Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective", published in 2000 by Marcel Dekker, Inc., New York, NY. He is currently authoring a second textbook "Analytical Methods for Risk Management: A Systems Engineering Perspective" to be published by Taylor & Francis Group, Dekker/Chapman-Hall/CRC-Press, New York, NY.

Mr. Garvey completed his undergraduate and graduate degrees in mathematics and applied mathematics at Boston College and Northeastern University, where for ten years he was a member of the part-time faculty in the Department of Mathematics.

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Appendix A

Cumulative Distribution Function of the Standard Normal Random Variable

The tables below are values of the cumulative distribution function of the standard normal random variable “Z”. Here, $Z \sim N(0, 1)$. The columns with three-digits represent values for “z”. The columns with the eight-digits are equal to the probability given by the integral below.

$$P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

Since $Z \sim N(0, 1)$ the following is true; $P(Z \leq -z) = P(Z > z) = 1 - P(Z \leq z)$.

0.00	0.5000000	0.21	0.5831661	0.42	0.6627572	0.63	0.7356528
0.01	0.5039894	0.22	0.5870644	0.43	0.6664021	0.64	0.7389138
0.02	0.5079784	0.23	0.5909541	0.44	0.6700314	0.65	0.7421540
0.03	0.5119665	0.24	0.5948348	0.45	0.6736448	0.66	0.7453732
0.04	0.5159535	0.25	0.5987063	0.46	0.6772419	0.67	0.7485712
0.05	0.5199389	0.26	0.6025681	0.47	0.6808225	0.68	0.7517478
0.06	0.5239223	0.27	0.6064198	0.48	0.6843863	0.69	0.7549030
0.07	0.5279032	0.28	0.6102612	0.49	0.6879331	0.70	0.7580364
0.08	0.5318814	0.29	0.6140918	0.50	0.6914625	0.71	0.7611480
0.09	0.5358565	0.30	0.6179114	0.51	0.6949743	0.72	0.7642376
0.10	0.5398279	0.31	0.6217195	0.52	0.6984682	0.73	0.7673050
0.11	0.5437954	0.32	0.6255158	0.53	0.7019441	0.74	0.7703501
0.12	0.5477585	0.33	0.6293000	0.54	0.7054015	0.75	0.7733727
0.13	0.5517168	0.34	0.6330717	0.55	0.7088403	0.76	0.7763728
0.14	0.5556700	0.35	0.6368306	0.56	0.7122603	0.77	0.7793501
0.15	0.5596177	0.36	0.6405764	0.57	0.7156612	0.78	0.7823046
0.16	0.5635595	0.37	0.6443087	0.58	0.7190427	0.79	0.7852362
0.17	0.5674949	0.38	0.6480272	0.59	0.7224047	0.80	0.7881447
0.18	0.5714237	0.39	0.6517317	0.60	0.7257469	0.81	0.7910300
0.19	0.5753454	0.40	0.6554217	0.61	0.7290692	0.82	0.7938920
0.20	0.5792597	0.41	0.6590970	0.62	0.7323712	0.83	0.7967307

Table A-1. Table of Standard Normal Values (continued on next page)

Example Computations

1. $P(Z \leq z = -0.525) = P(Z > z = 0.525) = 1 - P(Z \leq z = 0.525) = 1 - 0.70 = 0.30$
2. $P(Z \leq z = -0.675) = P(Z > z = 0.675) = 1 - P(Z \leq z = 0.675) = 1 - 0.75 = 0.25$
3. $P(Z \leq z = 0.525) = 0.70$

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0.84	0.7995459	1.05	0.8531409	1.26	0.8961653	1.47	0.9292191
0.85	0.8023375	1.06	0.8554277	1.27	0.8979576	1.48	0.9305633
0.86	0.8051055	1.07	0.8576903	1.28	0.8997274	1.49	0.9318879
0.87	0.8078498	1.08	0.8599289	1.29	0.9014746	1.50	0.9331928
0.88	0.8105704	1.09	0.8621434	1.30	0.9031995	1.51	0.9344783
0.89	0.8132671	1.10	0.8643339	1.31	0.9049020	1.52	0.9357445
0.90	0.8159399	1.11	0.8665004	1.32	0.9065824	1.53	0.9369916
0.91	0.8185888	1.12	0.8686431	1.33	0.9082408	1.54	0.9382198
0.92	0.8212136	1.13	0.8707618	1.34	0.9098773	1.55	0.9394292
0.93	0.8238145	1.14	0.8728568	1.35	0.9114919	1.56	0.9406200
0.94	0.8263912	1.15	0.8749280	1.36	0.9130850	1.57	0.9417924
0.95	0.8289439	1.16	0.8769755	1.37	0.9146565	1.58	0.9429466
0.96	0.8314724	1.17	0.8789995	1.38	0.9162066	1.59	0.9440826
0.97	0.8339768	1.18	0.8809998	1.39	0.9177355	1.60	0.9452007
0.98	0.8364569	1.19	0.8829767	1.40	0.9192433	1.61	0.9463011
0.99	0.8389129	1.20	0.8849303	1.41	0.9207301	1.62	0.9473839
1.00	0.8413447	1.21	0.8868605	1.42	0.9221961	1.63	0.9484493
1.01	0.8437523	1.22	0.8887675	1.43	0.9236414	1.64	0.9494974
1.02	0.8461358	1.23	0.8906514	1.44	0.9250663	1.65	0.9505285
1.03	0.8484950	1.24	0.8925122	1.45	0.9264707	1.66	0.9515428
1.04	0.8508300	1.25	0.8943502	1.46	0.9278549	1.67	0.9525403

1.68	0.9535214	1.89	0.9706211	2.10	0.9821356	2.31	0.9895559
1.69	0.9544861	1.90	0.9712835	2.11	0.9825709	2.32	0.9898296
1.70	0.9554346	1.91	0.9719335	2.12	0.9829970	2.33	0.9900969
1.71	0.9563671	1.92	0.9725711	2.13	0.9834143	2.40	0.9918025
1.72	0.9572838	1.93	0.9731967	2.14	0.9838227	2.50	0.9937903
1.73	0.9581849	1.94	0.9738102	2.15	0.9842224	2.60	0.9953388
1.74	0.9590705	1.95	0.9744120	2.16	0.9846137	2.70	0.9965330
1.75	0.9599409	1.96	0.9750022	2.17	0.9849966	2.80	0.9974448
1.76	0.9607961	1.97	0.9755809	2.18	0.9853713	2.90	0.9981341
1.77	0.9616365	1.98	0.9761483	2.19	0.9857379	3.00	0.9986500
1.78	0.9624621	1.99	0.9767046	2.20	0.9860966	3.10	0.9990323
1.79	0.9632731	2.00	0.9772499	2.21	0.9864475	3.20	0.9993128
1.80	0.9640697	2.01	0.9777845	2.22	0.9867907	3.30	0.9995165
1.81	0.9648522	2.02	0.9783084	2.23	0.9871263	3.40	0.9996630
1.82	0.9656206	2.03	0.9788218	2.24	0.9874546	3.50	0.9997673
1.83	0.9663751	2.04	0.9793249	2.25	0.9877756	3.60	0.9998409
1.84	0.9671159	2.05	0.9798179	2.26	0.9880894	3.70	0.9998922
1.85	0.9678433	2.06	0.9803008	2.27	0.9883962	3.80	0.9999276
1.86	0.9685573	2.07	0.9807739	2.28	0.9886962	3.90	0.9999519
1.87	0.9692582	2.08	0.9812373	2.29	0.9889894	4.00	0.9999683
1.88	0.9699460	2.09	0.9816912	2.30	0.9892759	5.00	0.9999997

Table A-1. Table of Standard Normal Values (concluded)