# GUNFHELENAE <br> Export Control 

## ENERGY-MANEUVERABILITY (U)

## AIR PROVING GROUND CENTER EGLIN AFB FL

## MAR 1966

Distribution: Further dissemination only as directed by Air Proving Ground Center, Eglin AFB, FL, MAR 1966, or higher DoD authority. This document contains export-controlled technical data.

```
DECLASSIFIED UNDER AUTHORITY OF THE
INTERAGENCY SECURITY CLASSIFICATION APPEALS PANEL.
E.O. 13526, SEC'TION 5.3(b)(3)
ISCAP No. 2011-052,,Document 1 Date March 13.2013
```

10.MDR- 125

## -CONFIDENTIAL

## GUNFMVENHAL

## Export Control

## Redistribution Of DTIC-Supplied Information Notice

All information received from DTIC, not clearly marked "for public release" may be used only to bid on or to perform work under a U.S. Government contract or grant for purposes specifically authorized by the U.S. Government agency that is sponsoring access OR by U.S. Government employees in the performance of their duties.

Information not clearly marked "for public release" may not be distributed on the pub!ic/open Internet in any form, published for profit or offered for sale in any manner.

Non-compliance could result in termination of access.

## Reproduction Quality Notice

DTIC's Technical Reports collection spans documents from 1900 to the present. We employ 100 percent quality control at each stage of the scanning and reproduction process to ensure that our document reproduction is as true to the original as current scanning and reproduction technology allows. However, occasionally the original quality does not aliow a better copy.

If you are dissatisfied with the reproduction quality of any document that we provide, please free to contact our Directorate of User Services at (703) 767-9066/9068 or DSN 427-9066/9068 for refund or replaceinent.

## Do Not Return This Document To DTIC

## UNCLASSIFIED



The following notice applies to any unclassified (including originaliy classified and now declassified) technical reports released to "qualified U.S. contractors" under the provisions of DOD Directive 5230.25, Withholding of Unclassified Technical Data From Public Disclosure.

## NOTICE TO ACCOMPANY THE DISSEMINATION OF EXPORT-CONTROLLED TECHNICAL DATA

1. Export of information contained herein, which includes, in some circumstances, release to foreign nationals within the United States, without first obtaining approval or license from the Department of State for items controlled by the International Traffic in Arms Regulations (ITAR), or the Department of Commerce for items controlled by the Export Administration Regulations (EAR), may constitute a violation of law.
2. Under 22 U.S.C. 2778 the penalty for unlawful export of items or information controlled under the ITAR is up to ten years imprisonment, or a fine of $\$ 1,000,000$, or both. Under 50 U.S.C., Appendix 2410, the penalty for unlawful export of items or information controlled under the EAR is a fine of up to $\$ 1,000,000$, or five times the value of the exports, whichever is greater; or for an individual, imprisonment of up to 10 years, or a fine of up to $\$ 250,000$, or both.
3. In accordance with your certification that establishes you as a "qualified U.S. Contractor", unauthorized dissemination of this information is prohibited and may result in disqualification as a qualified U.S. contractor, and may be considered in determining your eligibility for future contracts with the Department of Defense.
4. The U.S. Government assumes no liability for direct patent infringement, or contributory patent infringement or misuse of technical data.
5. The U.S. Government does not warrant the adequacy, accuracy, currency, or completeness of the technical data.
6. The U.S. Government assumes no liability for loss, damage, or injury resulting from manufacture or use for any purpose of any product, articie, system, or material involving reliance upon any or all technical data furnished in response to the request for technical data.
7. If the technical data furnished by the Government will be used for commercial manufacturing or other profit potential, a license for such use may be necessary. Any payments made in support of the request for data do not include or involve any license rights.
8. A copy of this notice shall be provided with any partial or complete reproduction of these data that are provided to quailified U.S. contractors.

## DESTRUCTION NOTICE

For classified documents, follow the procedure in DoD 5220.22-M, National Industrial Security Program, Operating Manual, Chapter 5, Section 7, or DoD 5200.1-R, Information Security Program Regulation, Chapter 6, Section 7. For unclassified, limited documents, destroy by any method that will prevent disclosure of contents or reconstruction of the document.

## SECURITY

MARKING

# The classifiod of limited statys of this repert faglios to exek page, waless oiburwise mexad, <br> Sepasab pige piatost muSt be rartof accerdingly, 

THIS DOCLMENT CONTAINS INFORMATION AFFECTJNG THE NATIONAL DEFENSE OF THE UNITED STATES HITHIN THE MEANING OF THE ESPIONAGE LANS, TITLE I8, U.S.C., SECTJONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS COMTENTS IA ANY MANNER TO AN UNAUTIIORIZED PERSON IS PROHYBITED BY LAW.

Hotice: Then governent or othor dravings, spocifications or other data are used for any purpose other than in connection with a defi. nitely relsted government procurement operation, the U. S. Government thereby incurs no responsibilify, nor any obligation whatsoevsr; and the fact that the Government may have fermulated, furnished, or in any way supplied the said drawings, specifications, of other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporztion, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

## REPRODUCTION QUALITY NOTICE

This document is the best quality available. The copy furnished to DTIC contained pages that may have the following quality problems:

- Pages smalier or larger than normal.
- Pages with background color or light colored printing.
- Pages with small type or poor printing; and or
- Pages with continuous tone material or color photographs.

Due to various output media available these conditions may or may not cause poor legibility in the microfiche or hardcopy output you receive.

$\square$
If this block is checked, the copy furnished to DTIC contained pages with coler printing, that when reproduced in Black and White, may change detail of the original copy.
(U) This report on Project 0350T4 will be published In three volumes. Yolume I contains the theory, sample application, and the associated mathematical model. Thife effort conmenced on 15 dune 1565 and was coapleted on 15 January 2966. Volumes II and III will contain a few sample comparative analyses and the Energy-Naneuverability Diagrams of several aircraft fur clam and aiz-tomalr configurations. This report suparsedes APCCTDR-G4-35 (reference 8).
(V) The magnitude of this progran precludes the liating of all Indiviouals whose efforts have been invaluable to the progress of the work and the prepacation of this report. However, special recogaition oust be given to Mr. Carl Lavy, Mathematician, and Mrs, Anthony Eicla, Programer, of the Mathomatical Services Laboratory, and to Miss Betty Jo Salter, Illustrator, of the Graphics Section,

This technical report has been reutowed and is approved.

mLitef p. blouer, colonal, usha difactos, dif Forco hemant Lataratory


## UMCLASSIFIED ABSTRACJ

This report shuts how an aircraft's enerpy atate and energy rate capebilities are directly related to operational maneuverability and efftciency in tenns of energy-maneuverability theory. It denorstratea alao how energymaneuverability theory may be upplided to assist the tactician, commander, planiner, and designer in optimizing aireraft performance. Lozd factor veraus velocity (G-V) and altitude versus Mach number (H-H) diagrams are empicyed to obtain the interacting energy relationships furdanental to energymaneuverability theory. The G-V diagrams provide a measure of instantapeous maneuverability whle the HaM diagrams (the most valuable diagrams) show sustained meneuverability as a furction of energy rate, $g$, effictency, and range throughout an aircraft's performance envelope. the enargy dagrams, as the working tools of energy-maneuverability theory, may be used to determine operational maneuverbbility and efficiency of yarious amamentemginea airtrane combinations.

This docunent is subject to special expcrt controls and each transmittal to foreign govermants ar forcign nationals may be made only with prior approval of APG (PGIO), Egifn APS, Florida.

## COHTENTS

Section Page
I. Introduction ..... 1
II. IHETANTANDOUS NGMELVERABTLITY ..... 2
III. SUSTAIMED NANETNERAEILITY ..... 4
General ..... 4
Energy Rate ..... 6
0 ..... B
Efficiency ..... 10
Range ..... 13
IV. TACTICNL APPLICATIONS ..... 15
V. REQURFFENTS ..... 31
REFERERCES ..... 78
Ayprondix
 AA-2 MISSILES ..... 32II HRTHEATLCAL DERIVATTONS AMD MODELS FOR DEVDLDPING ENERGY.MOMEUVRAUILISY THDORY AND ASSOCLATED FLIGHT PATHS39
MII, A GOPARISON GE THE BRYOOMKELLEY ANO THE MDDIEIED RITOWSKItechniques74
ILUSTRGTIONS AND TABLES
Ftgure

1. F-4C G- Diagran at 30,000 Feet ..... 3
2. F-4C 5terdy-5tate Envelope ..... 5
3. F-40 LuE Energy Rate Diagran with Superimposed Rutowaki Minimbin Time Path ..... 7
4. F-liC l-G Energy Rate Diagran whoth Superimpoed Auleof- Thumb Path ..... 7
5. F-4C 3-6 Eneray Mate Dlagram ..... 9
6. F-í4 5-0 Energy Rote Pagram ..... 9

- 7. Constant (50\%) Fuel Em Efficiency Diagram ..... 12
8, F-tc Variable Fuel E-H Efficiency Diagram ..... 12

9. $\mathrm{F}-\mathrm{iC}$ Ratge Diagram ..... I4
10. F-4C G-Y Diagran at 30,000 Feet ..... 16
NASIC/ACAA
11. MIG-a3 C-Y Diagram at 30,000 Feet ..... 26ithis information nokongerneeds to be classified)

## Contents (Conthatd)



## SECTION I

INTRODUCTION
(0) Aireraft maneuverability can be deffned as the ability to change direction and/or magnitude of the velocity vector. Whtle this definitfon describes maneuverability accurately, it provides little feel for the fighter pilot ar enginees on how to acpuire best (optinum) maneduerability. Honever, fram experience, we krow that the best ray to maneuver for positing divantage' or to ieny this same advantage to an opponent depends on the cype of oramence used and the performance of the afrcraft. The type of ordnance enployed detentines the possible delivery conditions needed to effectively dellver thia ordnence, whether it be guided missiles, guns, or barby, Quentitatively, these delivery conditions can be depicted by Iaunch or firing envelopes, Uncc the intitial delivery conditions are know, the problem becomes one of maneuvering intu the effective launrh envelope. Such maneuverability in depencent upon the abillty of the pilot to control turn, altitude, airspead, and aeceleration.
(U) The purpose of the following discussion is to show how energymaneuverability is reiated to operationgl maneuverability ond how thia relarionalitp may be aplofted by the tactisian, comander, plannea, or dealgner" "M". in devclophng valid manewerirg end/or delivery tactice along with better aerial combat weppons syaters.

SECTICN II
LNSTATAMLOUS HANEVVERABILITY
(U) Tum can be described in terms of radius ( r ) and/or rate ( d ) at various airspeads ( $V$ ) and radial $g\left(W_{F}\right)$ by employing the relationships

$$
r=\frac{V^{2}}{g_{r}} \text { and } \omega=\frac{\mathrm{gN}_{r}}{V}
$$

in coxjunction with aerodynamic force syeten equations. From sucl equations, numprous charts depicting turn radius and rate can be developed to provide sume neasure of maneuverability. Nedless to say, the riunerous charts and associated contour's are difficult to digest. Cor simplification and clarity, load factor versus velocity ( $C-V$ ) diagrams are employed to depict turn in a mannar consistenm hith a pilot's background and his cockpit instrumentarion, (See Figure 1.)
(0) The intent of this diagran (Figure 1) is to ençle a pllot to detor-. mine maxinur tum in terms of a or load factor hy consulting the aerodyanaic Inint at the laft and the structural/stabilator limits at the top, bottom, and to the right. By an overlay comparisan of GW diagrams, a gilot an detemine if he, or a possible adversary, has a turn advantage, Any tum capability or advantage, extracted from such a diadram, provides only a relative messure of instantaneous maneuverability. The diagram fails to indicate the effect of pulling $g$ in terms of losing or gaining altitude and/or airapeed. As a result, no measure of sustained maneuverability can be agquired from a study of this diagram, To develop thí information, a look in a different direction is necessary.

## CONFIDENTAAL



Figure L. E..4C C-V Diagram at 30,000 Feet.

# CONFIDENTIAL 

## SECTION III

SUSTANED MNLIWERGILITY

## GEMERAL

(U) Nutitude, airspeed, and cbanges thereto are directly dependent upon the force system dating along, and normal to, the flight poch. By mathematically manipulating the expressions describing this Force system, altitade (h) ond astrpecd ( $V$ ) can be combined in tite expression for specific energy ( $E_{\mathrm{a}}$ ),

$$
E_{1}=h^{2}+\frac{y^{2}}{2 g}
$$

$+$
as sham in Appendix II.
(v) To h. Jneuver for a desired change or a combination of changes in direction, altitude, and airspeed, a pilot must disturl the foree gystem surrounding his aircraft. Therefore, from the above expression, we deduce that maneuverability is not only related to directional change (rum), but is also related to spectific energy in terms of altitude and airspeed. From this expression, we can also deduce that all maneuvering will be conducted between a maxinum energy level associated with a best altitude-dirspeed combination and a mininum anergy level associated with zero altitude and minimum airspeed, Tlese maximum and rinionlm enerry levelsanay be represented in an altitude versus Hach number ( $\mathrm{H}-\mathrm{M}$ ) diagram (Figure 2). The maximun energy'Ievel is located on Figure 2 at the print hhere the specffic energy ( $E_{0}$ ) contour is tangerit to the steady-state envelope, The minimun entrgy level is located on Figure 2 at sea level there the appropriate specficic energy contour intercepts the stady-state envelope. (The steady state enveloje is defined as the 2evel RIight operating boundary determined by anglear-attack limits, thrust available, Jrag, and stmeteral limits.)
(U) In an aif-to-air battle, offensive maneuvering advantage will belong to the pilot who can anter an engagement at a higher energy level and mointain more eunergy than his opponent wile locked tim a maneuver and counter-mbneuver duel. Manewering advantage will also belong to the pilot who enters an air-to-air battle at a louer energy level, but can gain more encrgy than his opponent duxing the course of the battle. From a perfomance standpointi, such an advantage is clear because the pilot wita the most encyby has a better opportunity to engage or lisengage at his own choosing. On the other hand, encrgy-loss manervers ean be employed defensively to nullify an attack or to gatn a temporary offonsive maneuvering position. Implicit in the entire


Figure 2. F-4C Steady-State Envelope.
discussion on energy staze and/or energy rate advantages is the fact that a pilot has enough internal energy (tuel) avalable to exploit these advantages.
 energy state as he is in maintaining energy while maneuvering with a wide assortment of stores on board. If he cannot maintain maneuvering energy, his choice of tactics/techniques becomes limited. In addition, if this same pilot is tapped by enemy air, his ability to evade or mullify the attack becemess questionable.
(נ) Observing the correlation of energy with maneuyerability, it follows that tactical maneuverability is relited to the amount of energy possessed and low wall that energy is managed. Erom a design standpoint, this means a fighter pilot must be given a vehicle wherein such factors as energy state, enerty rate, and the quantity of intermal energy available are projerly considered. For hest maneuverability, 'the fighter pilot must know when and how to move to a higher or lower energy level and how to hest conserve his internel energy when locked in an air-to-air or air-to-surface encounter.

## CONFIDENTIAL

## CHERCY RATE

(U) For an offensive maneuvering advantage, a figliter pllot must be at a higler encrgy level or be able to gatin energy more quickly titan his adversary before the mencuver and countermaneuver portion of the battle bagins. To gatn energy nore quickly-once GCI, radar, or visall contact is made-mecessitates a biest path for accumplishing this task.
(U) An appraximate method for finding a best flight path was discovered by E. S. Rutowsif (sec Re\#erence 1), Using his method, as ourlined in 'Appendix II, the best (Ratowski) path for gaining mancuvering energy may be reprem sentell on an altitude versus jach number (II-N) diagram containing energy rate (specific excess power) contours mithin tle steady-state envelope, as shown in Figure 3. Enerry gain is maximum ar the points where the specific encrgy (Ed). contours are tangent to the specific excess power ( $\mathrm{P}_{1}$ ) contouri, wilere

$$
\Gamma_{1}=\left(\frac{T_{1}-D}{W}\right) V,
$$

$T_{k}=$ thast available, $D=d r a g, V=$ veiocity, and $W=$ weight, A glance at Figure 3 shows a best (optimun) path for gaining energy most rapidly. Not nomally show is the best path when the starting point is located off the besic Rutouski path, A solution to this problem becones easy if the energy rate, off the Rutorski path, anside the steady-state envelope is assumed to be zero. Under this assumption, the pilot moves along the specific energy contory tonsistent ulth his energy levol until. interpept $1 s$ made with the Rutowsti juth. As shom In Figure 3, the best path consists' of two segments: the approprlate specific enerty contour and the basic Rutowski path. Using this procedure, pilots can deternine the best paths from any point in the envelope. However, these paths ara approxinate for tho reasons: (1) load factor is assumed constant (1 g) in developing the basic kutowski path and (2) energy rate is assumed to be zero in developing the best path from any point in the envelope.
(0) To provide a more exact solution, A. E. Bryson and H. J. Kelley (Reference $\dot{2}$ athd Appendix İ) have developed a direct method while $H$, $P$, lepamann (Reference 3) has developed an indirect methon for finding best paths. Fight paths, detemined by these methods, show that Rutowski is very nearly cortect. Rutowsi's mathod, when compared with the Brysan-Kelley and Heernamn methoda, reveals that a rule-of-thumb techrique can be usec by a pilot or engineer to find best energy paths. (See foperdx III,) The technique uscs the simple rule $\mathrm{Ab}_{\mathrm{s}}=\mathrm{kaH}$ for finding intercept curves and the subsonic-supersonic transition curve to the Rutowski path, (See Figure 4.) The value of $k=2 / 5$ when Mach number nust be decrgased and $k=-1$ nthen Mach number must be ineroesed to intpreept the basic Rutowski jath.

## CONFIDENTIAT



Figure 3. F-40 1-G Energy Rate Diagram with Superimposed Rutowski Hiuinum Tine lyath.


Tigure 4. F_lic lag Energy Rate Digyram with Superirmosed Rule-of-Thumb Path,

## CONFIDERTTIAL

(U) Ever thougla the path develaped by this procedure nay be satisfactury to the engineer, it still is not good enough to enable the pilot to fly the path, because of the constantly clanging altitude, , Sacin mumber, and pitela angle. ouseryation and analysis of the path just definod, however, suggast a way to avoid this predicamant. Conerally speaking, the sulsousic portion of the path can be represented by a constant Mach numuer climb, while the supersonic portiun may be approxinatpd by an average eonstant califurated airsped, To Intercopt che sulsonic or aupersonic segments, the pilot pulls op or pushes over, as indicated by the rule-of-thumb, until lie intercepts the basic juth. At intercept, the pilot ohould lead the Nach number or calibrated airspeed to provent a tenlporary loss of energy by pulling too much g, intercepte fron the subsonic segment to the suparsonic segment of the Rutawsk: path should be accomplished i at less than 2 g ; while intercepts so the subsonic portion of the Rutowsi path should be ascomplished at less than 3 or 4 g at lower altitudes and should dectiase correspondingly as altitude increases.
$\therefore$
(U) Although the HAN diagram is useful foz approximatiog bect energy rate flight paths, observation reveals that it can ulso be used for another purpose, The contours contained within tlie steady-state envelopo provide a measure of the ability to gain energy throughout the envelope. Since gaining energy is relatcd to maintalning maneuverability, the Zng Energy Rate diagram provides a measure of austained mancuverabilfty as a function of energy rate, By overlay techniques, the lag Energy Rate diagran can be used by the fighter pilot or tactician to detemine if he can gain energy nore quickly than sume adversary. Actual time values, depicting how rapidly the transfer takes place, can be provided by the previouslymentioned optimization programs. Such values will be provided in "Tactical Applications," Section IV of this report, then this infurmation is correlated with somm analysis yet to be presented, the pilot of tactician can then detemine the type of tactics or maneuvers to employ.

## G

(v) Energy Rate diagrams of more than 1 g can be helpful in detezmining the best tactics to employ in the maneuver and counter-paneuver portions of the fight. As shom in Figures 5 and 6 , these diagrams contain both posirive and negative energy fote ( $P_{1}$ ) contours within the steady-state envelope, as Euch, these diagrams portray the abillty to maintain energy wile pilling b; hence, they provide a measure of sustainell maneuverability as a funetion of E .
(t) Dnce again, by staple overlay or comparison techniques, regions of energy advantage and disedvantage can be easily detemined. If a fighter pilot can gain energy more quickly or lose it less rapidly tham some adversory in e moneuvering fight, he has offensive manewvering advantage, on the other land,

## CONFIDEANFIAL



Figure 5. F-40 3-G Energy Rate Diagram.


Figure 6. F-40 5-6 Enezgy Rate Diagram.
-CONFTDENTM:

## CONFIDENFIAL

if the enorgy values ore ceversed, the pilot, althourgh forced on the defensive, may employ energy loss .nmeuvers to his adyantage. In either case, the j-g and s-g Energy Rate diagrams graphicaly portray capabilities and limitations in the manauver and counter-maneuver portions of the finht. In the air-tossurface role, these diarams may be mployed to detemine aneuvering capabilities and limitations with a wde assortment of stores on boerd, Hith this information, pilots and tecticians can develop pre-attack and postatack ractics and manejuvers against a hostilie surface conplex.
(b) Even thougil the 3-g and 5-g diagramis berve as useful tools to deterarine advantages and disaduantages, they do not specify the ceact tactics or maneuvers needed. To decide wat maneuvery alould be amployed, a pilot must be uell versed in the theory, and proficient in the practice, of air-ta-afr and air-to-sur face tactics (see Roferences 4 and 5). With this background, a pilot can tranalate relative energy gain or loas relationships into valuable tactical आaneuvers.
(U) Recently, the Bryson-Kelley methori has been employad to develop best three_dimensional maneuvers (seu Reference 6). This method appars promising in findint specific optimum macuvers fur change of Cirection, zate of closure, and combinations thareof in minimum time or with minimum fuel antil wiegpone launch. Hovever, as presently developel, thio method falls to consider; (1) the best relative rebions within the flight envel.ope to maneuver and countermaneuver against a knom adversary; (2) plausible counternaneuvers by an idversary as he observes and/or antlelpates the optimum maneuvers: (5) a sequence of plausilile counter-maneuvers or maneuvsrs by an adversary 'for which a sequence of optimum maneuvers or counter-meneuvers mill be necessary; and (4) the poscibility that mose than one optimum manesver or counter-maneuver can be flown against a specific counter-maneuver or maneuver,
(B) Decause of these gerious deficiancles, the Bryson-Kelley method cannot be usad by itself to develop valid tactics for the airmonsir battle. How ever, there may be a possibillty of develoning near optimum tactics if the - qualitative knowiedge of the tactician concerning plausible maneuvers and counter-maneuvers is whed in conjunction with the qumtitative methods devel. oped by Rutowsi, Bryson-Kelley, and Heerman. The Deputy for Effectiveness Test, Ar Proving Ground Center, and the Air Foree Armament Laboratory, Rescarcis and Technology Division (ATD), Eglin Air Fore Base, Florida, are investigating the use of these methods for this purpose.

## EFFICIENCY

(J) Until now, the discussion has been concemed with energy stute (h, V) and/or energy rate ( $\mathrm{f}_{\mathrm{H}}$ ) in an offort to doscribe maneuverability and to gain ma-

## CONFIDENTIAT







$$
I_{\mathrm{KiL}}=\frac{\Gamma_{i}^{*}}{k_{f}} w_{p}
$$


 letailed development of this expression.) Two types of $E$ ( Effielency diagrams, incorforating thase eflicicncy cuntours within the steady-state envelope, may be constructed. In the firtit diagram (see Figure 7) ponstant fuel weight (500 interial) is assumed. In the second diagran (seefture 8) only the fuel remaiming at a given encrgy level is considered, after reducing the quantity of fued by tho minimum amount of fucl. needed to reach that enezgy level, The efficiency contouts depicted on thic diagram consider fuel avatable minus $5 \%$ interpal tuel aud 20 ainutes fuel for best laiter sped at 10,000 ft. Doth of these Len Criciuncy dipgrams ean be used to; (1) find the musit efficient (minimum fucl) paths by employing the sume rule-of-thuml techniques used with the Linctit Rate liagrams ind (a) Juturnine the amount of internal
 ciuncy of that cunyersian. Since the diagrams ean be employed in this foshim, they provide a measure of sustained mantuperability as a function of wiffermey. In addition, [-H Efficiuncy lingrams can le used extensively to deturnhate relative advantages and diavdvantages of competing trensport and Lombar dasigis. For these type aircraft, load factor and energy $r^{\text {ien }}$ are less impurtant measures of operational pertormance, The seconde efficiency山iagrom is more meaningful, siner variable fugl weight is considered through-
 is importatit in dutemindig regions of best effiesency, independent of etol liziturics. The rulative merit of these two warams will be disedssed in "Toetical Applicetions," Section IV of this report.
(v) By unioying comparative tuchniques, the tactician can fenerally sece whethur a fighter pilot or his adversary will conserve a greater percentage of (tely in mowirb fron unc ellersy level to illother. Higher numericol values indcate a greater uercentage of fuel remaining or a smaller percentage of fuel consumen, Thus, by correlating the E-N Eficiciency diagrams mith the Energy Rate diagrams, the tectician can dotermine to hbat Jegree a filot or his ad. versary can realistically maintain or employ ony energy state and/or energy rate duantages. To assist in this enteavor actual fuel percentage values can be provided (by the optinizatiun programs) to show hoy efficiently the

## CONFIDENATIAL



Figure 7. Constant (50\%) Fuel EnM Efficiency Diagran.


Figure 8. F-4C Variable Fuel Eminfticiency Diagram.
CONFHEEATHAL

## CONFIDENTIAL

energy transfer takes plece. Such infornation will be provided in iractieal Applications," Section IV of this report,

RGICE
(以) Thus far, maneuverability has been described direatly as a function of energy rate, 5 , end efficfency. However, to completely describe manewverability, we must consider indirect as well as direct influences, Range in directly influences maneuverability as it plays a vital part in deternining the area of maneuverability available over the earth's surface. Because of this relation:ohid, a combat pilot must have a good but annple measure of his available range at any altitude-airspeet conblnoion fithin the oteadystate envelope. To gain this information, he must consideri (i) the fulol copgumed and the distance traversed in reaching any altitude-alropeed conbination and (2) the renaining range available as a function of the fued zeraining at any altitude-alrspeed combination.
(U) By properiy considering this information, as outlined in Append.'x II, an H-K diagran depicting range can be developed (see Eigure 9). From thin diagram a pliot can deternine range at any altitude-airspead combination including the diatance traversed to reach that combination. The range contours depicted on this diagran are based on the seme fincl yogerve consideratious uaed in the variable fuel EM Efficiency diagran, The shaded area on the chart represente a tranalent region in the faight ervelope. In this region,
 and less than mininumiaftorburner thrust. For this reason, steadywate * : flight is not possible unless some device, e.g., gped 'brakes, is employed to increase drag.
(U) By using the Range diagram in conjunction wth the other energrmaneuverability diagrams, the tactioin can determize to that degree e pllot or his adveraary can reaistically gain advantage consistent witid distance from faiendly airfield of tanker expport. In dddition, plamers and debigners can evaluate the true operational performance of transport and buber type atrcraft by considering the Range diagram along witil the EH Efficiency diagrame.

## -CONFIDENTIAL



Ftgure 9. F-40 Range Diagram,

## SECRET

## SLCMION IV

TACTLCAL APPLICATIONS
(U) If tive anuray-nancuverabinity concept is incorporated in the present
 luatiou on low ine shathlameuver to gafn alvantade. The resulting information will then reveal to tur pilot how lae should best explait far maneuvering capa bilitios of his aircralt. Adlitionaliy, valid comparative analyses can be pertorach if similar diarrants are constructed fori potential encray fighters. The rulative regions of allyatage or disalvantage are found in terms of g, encrisy wats, ofliciency, and rante by periamance cumparisons throughout the flibith envolupe. Fron this compayisoin; the tactician of pilot can easily delemint willeh of two aircraft has the advantage in terns of ingtantoncous mancuycrability, shstained muncuverability, and range. Using this infornation, tive tactician oan deternine low to best maneuyer for adoatage.
(vi) For a sample comparison, we stall consider the $F$. $4 C$ versus the Soviet M10-Pl and determine maneuvering advantades and disadvantages. The conditions tor comparispn will be typical air-to-air confirurations, with 505 intenial
 depolymanic $g$ Jimit of the Mictel lies to the left of the same limit for the F-lic, At a glance, Figures lo and 21 indicate the NIG_2l has on enormoys-inatantanesus huneuverability anvantage over the F-4C. These diadrams also inse dicate that the MLGe2 lfas an advantage when comparing structural limits, The
NASICIACAA
DECLASSIFY
(This
information no
longer
needs to be
classilied):
: 1-: Enerify Rote diagrame (Figures 12 and 15) show that the" HIG-21 has the advantage within most of the subsonic purtion of the Elight envelupe and throughout all af the supersonic portion of the Gight envelofe. The only ragion of advantage for the $F-4 C$ lies $i$ a the subsonic nd transohic areas beion 15,000 rect. The magnitude of the maximun brow energy rate advantage ean be determined by consulting Table I.
class:med

## SECREF


NASIC/ACAA
DECLASSIFY
This information no longer
needs to be classified)
Figure 10. F-4C g-v Diagram at 30,000 Feet.


Figure 11. Mig-21 G-p Diagrem at 30,000 Feet.


Figure 13. HIG. 21 Haximum Power l-G Energy Rate Diagram.


In Figures 24 through 17, we note that the r-4C rctains most of its lounaltitude sulsonic/transonic sustained maneuvering advantage as 5 is incredsed from 1 to 5. In addition, these figutes reveal that the MiC-2 not only has a supersomic arl. vantage, but than is gaining regions of advantage subsonically since it can pull
 cates the HIG-2l can tun mote guickly than, or inside, the F-4C. This same conclusion was reachaed by studying the culdiadram. Logteally, this means the lefthad boundaries of the l-g, jog, and 5-g Encrgy Rate diagrams provide a ncasure of instantaneous maneuverability, winile entire diagrams provide a measure of sustained insmeuverahility as a function of energy rate and g .
(5) The military pohor l-g Energy Rate diagrams (Figures 18 and 19) show that the r_4C has the abilyty to gain energy more rapidly than the MIG-21 thruaghout most of the enve*ope. The manaritude of this advantage is depicted in Table IL.

TINLE II, FUTONSAI HINLMUN TIHE PATHS (HILTMARY POWER)

| NAS IC/ACAA DECLASSIFY (This information no longer needs to be classified | Type <br> Aireralt | $E_{1}=3,000$ ft to $L_{1}=45,000 \mathrm{ft}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Time } \\ & (\mathrm{sec}) \end{aligned}$ | $\begin{aligned} & \text { Fuel } \\ & \text { Weight Used } \\ & \text { (1b) } \end{aligned}$ | Fuel Used (d) |
|  | E-4C | 309 | 1342 | 12 |
|  | - $\mathbf{I I} G=21$ conf | 403 | 61.5 | $17^{\circ}$. |

(5) By consulting the military power 3-g and $5-\mathrm{g}$ Energy Rate diagrams (Figures 80 through 23), we olserve that the E-4C has a sustaimed maneuvering idvantage at the lower altitudes and higher Mac', numbers. These diagrams also reveal that the MIGmel regions of advantage spread to the loter portion of the envelope as $g$ is increased from 2 to 5 . In addition, thesc tiadrams (Figures 20 through 23) reveal that the M1G.2l can maneuver in regions unavailable to the F.4C. As mentioned previously, such a condition indicates that the MIO-Z1 can outturit the F.4C.
(5) Erom Figures 24 and 25, we observe that the E-4C has the subsonic advantage, while the HIG-2l has the supersonic adventage in tems of the maneuvering energy gained versus the percentage of intermal energy expended. On the otlier hand, the variable fuel E-H Efficiency diagrams (Figures a6 and 27) neveal that the $\mathrm{E}-4 \mathrm{C}$ increa'ses its subsomic advantage and acquires an advantage through most of the supersonic portion of the envelope. A matural quegtion arises at tilis point as to hby the F-4C appears to have a greater degree of edvantage in the variable fuel diagran tinan fin the


Figure 14. F-4C Kaximum Power 3-G Enargy Rate Diagram.


Figure 15. HIG-2l Raximum Power 3-G Energy Rate Diagram,

## 19 <br> SECRET



Figure 16, F-4C Maximum Pomer 5-G Energy Rate Diagram,


Eigure 17. HIG-21 Kaximum Power j-G Energy Rate Diagrem.

## 0 <br> SECRT

NASIC/ACA $\vec{A}$
Figure 18. F-4C Hilitary Power l-G Energy Rate Diagram,
DECLASSFY
(This information no
longer
needs to be ciassified)


Figura 19. MIG-2L Hilitary Power l-G Energy Rote Diagram

## SECRET




Figure 21. HTG-2l Hilitary Power 3-G Energy Rate Diagram.

## SECREF



Figure 22. $\quad$ - 4 C Cilitary Power 5-G Energy Rete Diagram.


Figuye 23: MIG-21 Military Power 5-G Energy Raie Diagram,


Figure 24. EuH Maximun' Puwer Constant (S0p) Euel Em Efficiency Dtagram.


Eigura 25, Mic-21 Haximum Porer Constant: (50\%) Buè E-M Efficiency 04ngram.

NASIC/ACAA
DECLASSIFY
(This information no longer needs to be class,fied)

## SECRET



Figure 26. Fwte Maximum Pomer Variable Euel E-H Efficiency Diagrant
NASIC/ACAA
DECLASSIFY
(This information no ronger needs to be classfied)


Flgure 27. HIC-2i Kaximun Power Veriable Fuel E-M Efficiency Diagram.

## SECREF

constant fuel diagran. The greater advantage attributed to the E-4C, in the variable fuel duacran resulus from the fact that the Pr4c has experided a smaller percentape of its.foel when it reaches the regions fherg the kigmer is more efficient. Certainly such an advantege hould not be realistic if the F-4c' entered the engagenent with a snaller percentage of fiel on board and/or. the engagement started at a high subsonic energy level. If such a condition existed, the efficiency advantage attributed to the MIO-2l in the supersonic region of the constant fuel diagtan would be realistic. This point of difference becomes important to the F-4C pilot when he encounters a MIG-2I in hostile territory. From the militnry power Entefficiency dragrans depicted
io Figures 28 throufis 37, we see tllat the $\mathrm{F}-4 \mathrm{C}$ has the anvantage in both cases, assuning, of course, that each aircraft has the same assumed or starting fuel percentages on board. Actual tine and percentage values of fuel expended aloni mininum fuel paths between energy levels for these two alreraft are displayed in Tables III and IV.


Fin The Range diagrans (Figures 32 and 33) roveal that the Falit has a substantial alvantade in the subsomic portion of the envelope and a lasser advantage in the supersonic portion of the envelope.



## SECRET



Figure 32. F-hC Range Diagram.


Figure 33. MIG-2l Range Diagram.

## SECRET

TSt By comparing the rule-of-thumb performance of the ALM-99/AB-2 and
 an enonous all-aspect, first-5hot advantage over the Mre-2l equipped with incernal gun(s) and two Al-2 missiles. This advantage prevails ageinst either armedvering or nomaneuverirg MIC-2l. However, in spite of this advantage, an F-4C pilot may find it difficult to employ ALM-7E di Ing a ra-attack or in an effort te nullify an attack, since the MiG-2l can easily outturn the F-4C as woll as maintain more energy hoile doing 60, By exploiting this dial adwantage, a skillful MIG-2l pilot may prevent a successful ALM-7E missile launch by simply maneuvering away from the front toward the rear hemisiphere of the Fillc. Far closewin maneuvering, the E-4C can mount a 20 millimeter centerline gun pod in addition to tie faur AIM-7 wissiles in an effort to get inside the missile minimum firing range restrictions. However, such a fix results in an zven greater margin of maneuvering superionity for the HIG-2l by reducing. the already inferior instantaneous and sustained maneuverability
 determined by consulting the energy diagraing in Volume III of tha report, to be publishod at a later date.
(6) Prom the foregoing analysis, it is clear that the MIG-2i enfoys an enormous instantancous meneuvaring advantage ard e substantial sugtained maneuvering advantage in terms of energy rate and g throughout the aupergonic portion of the flight envelope. Subsonically, at both masinum and adlitary power, the HIC-2l has a bustained manasvering advantage in the upper portions of the envelope that apread to the lower portions as fincreases. On the other hand, the F-4e has a sustained maneuvering advantage in terms of effielency throughout the entive subsonic portion of the envelope extending through most of the gupersonic envelope. Only In range and firstmahot capan biluty dees the F-4c enjoy o substantial advantage over the rico-2l.
(V) Waturally, for a complete analysis, adidtional infomation wist be developed. The taeticion needs enerty-mineuverability diagrans for various type conbat configurations. In addition, he needs comparative misaile ftring envelopes torather with radar and manelvering constraints thet may be inpoaed on the pilot or radar operator. If this information is provided, the tactician can deaign tastics by using energy-maneuverabillty methods, Assuring that Earaign Techalogy Divisions can provide reasonebly accurate data concering enemy perfomance, the tacticlan, for the first time, can develop effective tactics against any adversary. In addition, tactical comanders can use the energy-maneuverability comparative onalyses to gain meaningful perspective for decisions concerning the employment of friendly fighters against a knom eneny.

# CONFIDENATAA 

SECTION V
REQUTREMEHTS
(U) Presently, in order to meet misgion requiraments, Air force planners direct that mew designs meet eertain epecifications in temis of altitude, atrspeed, acceleration, $g$, and rarge. Contractorg, in on effort to satisfy the customer, produce designs to mept these specisications, However, no guerantee can ie rade that the design selected will be tha bent one alnce such speciflcations are paint data (derivatives) ara provide no indication of an aircraft's integrated perfomance and desian efficiency throughout the filght envelope.
(U) However, by applying energymaneuverability techniques, aloag with other infornation deemed necessary by the tactician, plamers would have the advantage of looking at complete perfomance (Including the previously mentioned point data) before making decisions conceming aircraft requirementa, As a result, true operational need mould be copsidered by both planners and destgners in detemining the best overail combination of amament, engine, and airframe in future designs,


ALH-7E MLSSILES
(G) MINLMLM RANGE. Minimum range ( $R_{\text {sio }}$ ) for nose quarter (NQ), obeain ( $A B$ ), and tail guarter (TV) attacks;

| Altitude ( Ft ) | Type of Attack |  |  |
| :---: | :---: | :---: | :---: |
|  | $\begin{gathered} N Q \\ (f t) \end{gathered}$ | $\begin{gathered} \mathrm{AB} \\ (\mathrm{ft}) \end{gathered}$ | $\begin{gathered} \tau Q \\ (\mathrm{ft}) \end{gathered}$ |
| SL | 8,500 | 7,600 | 5,100 |
| 10,000 | 9,250 | 3,100 | 5,400 |
| 20,000 | 10,100 | 8,600 | 5,300 |
| 30,000 | 11,400 | 9,200 | 6,300 |
| 40,000 | 12,800 | 9,800 | 6,800 |

For turns into the attack, add 1,000 ft to the above valuss. For turno away Erom the attack, suldract $1,000 \mathrm{ft}$.


Figure I-s, AH-TE Hissile Launch Envelope


Figure 1-2. AIM-9B/AA~2 Misaile Launch Envelope

## -CONFIDENTIAL

(C) HAXIHIM AAGGE. Against a Co-Speed Nonmanelvering Target. Haximum range ( $R_{n}$ ) and angle-off ( $4_{6}$ ) for a nose quarter, abegm, and tail quarter attack agaillst a cospeed nomaneuvering target:

Subsonic Target - Hach 0.9:

| Altitude <br> (ft) | $\mathrm{NQ} / \mathrm{k}$ <br> $(\mathrm{ft} / \mathrm{deg})$ | AB <br> $(\mathrm{ft})$ | $\mathrm{TQ} / \mathrm{k}$ <br> $(\mathrm{ft} / \mathrm{deg})$ |
| :---: | :---: | :---: | :---: |
| SL | $80,000 / 10$ | 15,000 | $9,000 / 30$ |
| 10,000 | $82,000 / 10$ | 22,000 | $13,000 / 30$ |
| 20,000 | $82,000 / 12$ | 30,000 | $19,000 / 30$ |
| 30,000 | $82,000 / 25$ | 40,000 | $26,000 / 50$ |
| 40,000 | $82,000 / 40$ | 52,000 | $35,000 / 30$ |

For a subsonic target Mach 0.5 as sea Ievel, $\mathrm{M} / 4=20,000 \pm / 10^{\circ}$, $\mathrm{AB}=24,000$ ft , and $\mathrm{T} / 4=18,000 \mathrm{ft} / 30^{\circ}$. For cach additional $10,000 \mathrm{ft}$, add $5,000 \mathrm{ft}$ to $\mathrm{NQ}, 8,000 \mathrm{ft}$ to AB , and 5,000 ft to TQ .

Supersonif Target:

| Altitude <br> $(\mathrm{ft})$ | $\mathrm{NQ} / \mathrm{f}$ <br> $(\mathrm{ft} / \mathrm{deg})$ | $A \mathrm{AD}$ <br> $(\mathrm{ft})$ | $\mathrm{TQ} / \mathrm{I}$ <br> $(\mathrm{ft} / \mathrm{deg})$ |
| :---: | :---: | :---: | :---: |
| SL | $82,000 / 10$ | - | - |
| 10,000 | $82,000 / 10$ | 12,000 | $7,000 / 30$ |
| 20,000 | $82,000 / 15$ | 20,000 | $24,000 / 30$ |
| 30,000 | $82,000 / 30$ | 30,000 | $20,000 / 30$ |
| 40,000 | $82,000 / 45$ | 40,000 | $28,000 / 30$ |

(8) hatanst Monmaneuvering Targets with Attacker Velocity Greater or Less thon Target Velonty, Haximum ranges for nose quater and tail quarter attacks against nopmaneuvering targets with attacker velocity greater or less than target velocity (delta Hach) follow:

Hose Quarter Atracks:

1. Add 3,00 feet to Ras for each 0.1 delta Hach below 10,000 feet when taxget velocity is greater than attacker velocity.
2. Add 1,500 feet to $R_{\text {ner }}$ for each 0.1 delta Mach above 10,000 feet men target velocity is greater than attacker velocity.
3. Add 1,000 Feet to $\mathrm{Rax}_{\mathrm{n}}$ for each 0.1 delta Hach what attacker velocity is graater than target velacity,


## CONFIDENYIAL

## Tail quarter Attacks:

1. Add 2,000 feet to $R_{\text {Lis }}$ for each 0.1 delta mach rate of closure.
2. Subtract 3,000 feet from $R_{\text {nex }}$ for each 0.1 delta mach separation.
(C) Against a Maneuvering Target,

Nose Quarter Attacks:

1. Reduce $R_{\text {nax }} 30,000$ feet when target maneuvers away from tle attack at 2 g .
2. Reduce Rax 5,000 feet/s for target maneuvers anay fram the attack with g greater than 2.
(6) Manesvers Akay frow. Attack, Tāil quarter Attacks:

| Altitude | Target $G$ |  |
| :---: | :---: | :---: |
| Below 20,000 ft | 3 | $2 / 3 \mathrm{R}_{\text {HR }}$ |
| Below 20,000 ft | 5 | 1/2 R $\mathrm{Rax}^{\text {a }}$ |
| Above 20,000 ft | 3 | $1 / 2 \mathrm{R}_{10}$ |
| Above $30,000 \mathrm{ft}$ | 5 | $1 / 3 \mathrm{R}_{\text {a }} \mathrm{k}$ |

The above values do not include backgzound clutter associated with a target at low altitude.
E.O. 13526, section 3.3(b)(4)

Withheld from public release under statutory authority of the Department of Defense FOIA 5 USC $8552(\mathrm{~b})(3)$ 10 USC \$ 130


## CONFIDENTIAL

E.O. 13526, section 3.3(b)(4)

> | Withheld from public release |
| :---: |
| under statutory authority |
| of the Department of Defense |
| FOIA 5 USC $\$ 552(b)(3)$ |
| 10 USC § 130 |



## CONFIDENTIAL

## E.O. 13526, section 3.3(b)(4)

## Withheld from public release

 under statutory authority of the Department of Defense FOIA 5 USC $\$ 552(\mathrm{~b})(3)$10 USC $\$ 130$

## APPEMDIS II

HATHEATICAL DERIWATIONS AND HODELS EOR DEVELOPDNG

(Appendix II is unclassified in its enturety)

In this appendix a discussion of the mathenatical methods employed to develop the Energy-Haneuverability Theory and associated flight patlis in the altitude-Hach number plane will be presented. For convenience, tha derivations will be described in tems of the following computer programs wheh have beet fonmulated to handle the computational aspects of the theory:

Part 1 - Bisic Energy-Haneuverability Computer Hodel
Part II - The Bryson-Kelley Steepest Ascent Optimization Program
Part III - Dymaic Profile Generator Prosram

## DERI VATIONS

IMSTANTANEDUS HANEIVERABIILTY, FOI any given aireraft, maximum load factor (nomal acceleration) may be computed as a function of altitude and airspeed;

```
            \(n_{L}=\frac{\mathrm{qSC}_{\mathrm{L}_{\text {垃 }}}}{H}\),
where \(\pi_{L}=\) ouximum nomai acceleration (dimensionless)
    \(q=\frac{2}{\mathrm{E}} \mathrm{p} \mathrm{v}^{2}\), dymanic pressure \(\left(\mathrm{lb} / \mathrm{ft} \mathrm{t}^{2}\right)\)
    p a atmospheric density (slugs/ft \({ }^{3}\) )
    \(V=\) true airspeed ( \(\mathrm{ft} / \mathrm{sec}\) )
    \(S=\) refercnce wing area \(\left\langle f^{2}\right\rangle\)
    \(\mathcal{C}_{\mathrm{L}_{\mathrm{rax}}}=\) maximun coefficient of lift (dimersionless)
    \(w=\) aircraft heiglt (lb)
```

Since calibrated airsped (CAS) is more meaningful to the pilot than true airspeed, the GV diagrams (see Figure 1, page 5) depict maximum nomal acce:eration versus cas.

SUSTATNED MANEVVLRABILITY. Energy Rate, The energy ( L ) possessad by an aircraft is the sum of its potentiai energy $\left(E_{p}\right)$ and its kinetic energy ( $E_{\alpha}$ ). Hathematically,

$$
\begin{aligned}
E & =E_{y}+E_{k} \\
& =w h+\frac{1}{2} m^{2} y^{z} \\
& =H\left(h+\frac{y^{2}}{2 g}\right)
\end{aligned}
$$

## where

$$
h=\text { altitude (ft) }
$$

$$
m=\text { aircraft mass (siugs) }
$$

$$
G=32.174 \mathrm{ft} / \mathrm{sec}^{2} \text {, the gravitational acceleration }
$$

The expression, $E=\varphi\left(\begin{array}{c}h \\ y^{2} \\ 2 g\end{array}\right)$ gaves us a mensure of the anergy stare of an aircraft at any altitude-airspeed combination, However, bince the main interest lies in comparing aireraft with different weights at the same altitudeairspeed eombinations, it is more meaningful to make the above expression indepentlent of aircraft weight. Dividing both sides of the above expression by w yields

$$
\frac{E}{w}=h+\frac{v^{2}}{2 g} .
$$

The term $E / w$ can be regarded as epecific energy $\left(\Sigma_{0}\right)$, with tha result that the energy state of an aircraft cen-now be expreesed as a function of altitude and airspeed:

$$
E_{i}=h+\frac{v^{2}}{2 g}
$$

The probilen of managing energy involves controliing the rate of transfer betwean energy levels. Differentiating the above expression resulta in

Where the dot ( $*$ ) Indieares the derivative with zespect to tifle, $\frac{d}{d t}$. To provide more insight into entergy rate, $\dot{E}_{n}$, we may empicy Figure II-l and write a force balance equation along the flight path.

0

$$
\mathrm{m} \psi=\mathrm{T}_{1}-\mathrm{D}-\mathrm{W} \sin \gamma,
$$

$$
T A-D=W \sin Y+\frac{H}{g} U,
$$

or

$$
\frac{T_{3} \rightarrow D}{H}=\sin Y+\frac{\dot{V}}{g} .
$$

Multurying both sides of this expression by $v$ yields

$$
\left(\frac{T}{W}\right) V=V \sin \gamma+\frac{W}{G}
$$



Figure II-1, Aircraft Force-Galance Dibgram.
since $b^{6}=\mathrm{y}$ sin $\gamma$ нe may write

$$
\left(\frac{T_{H}-\dot{B}}{\hat{H}}\right) v=\dot{h}+\frac{\tilde{W}}{g}
$$

The right side of the ubove expression is equgi to E. Recaling that mork is accomplished in transferring ficm one enccgy level to another, and that porer, by definition, is the time rate of duing mork, the left side of the pbove equation may be equated to specific expess power, $P_{y}$ :

$$
P_{n}=\vec{E}_{1}=\left(\frac{T_{4}-D}{W}\right) V
$$

In an attempt to counter an fmodtate threat, the energy-oriented fighter pinst hill strive to increase his maneuvering energy as quickly as possible. 7his anounts to maximizing the rate of transfer betheen energy levels, wich is egaivalent to marinizing the integral

$$
E_{1}=\int_{i_{1}}^{i_{2}} P_{1} d t_{t}
$$

Acoording to Rutowski (feference 1), this is accomplished wen

$$
\left[\frac{\partial P}{\partial Y}\right]_{E_{n}}=k=0
$$

0

$$
\left[\frac{\partial \Gamma_{1}}{\partial \hbar_{1}}\right]_{E_{4}=k}=0
$$

In the altitude-Nach number plane, these relationships are satisfied at those points where the $E$, contours are tangent to the $\operatorname{lng} P_{\text {, }}$ contours, Connecting thebe points regults in an spproximate minimun time path,

Energy-Haneuverability Lfficiency (E-HE). If the above-mentioned threst is not as dmanent, the pilot fill attempt to increase his maneuvering energy while conserving internal energy (fuel) for future maneuverability. This is achleved by raximizing the integral

$$
E_{4}=\int_{1}^{2} \frac{d E_{4}}{d H_{4}}
$$

$$
\begin{aligned}
& \text { Since } \mathrm{dt},=P_{\mathrm{s}} \mathrm{dt}, \\
& \text { and }
\end{aligned}
$$

we see that
and

$$
\frac{d E_{1}}{d w}=-\frac{P}{4}
$$

$$
E_{1}=-\int_{w_{1}}^{2} \frac{p_{2}}{L_{1}} d_{N_{1}}
$$

Mgan, by employlng Rutomsils technique, we obtain

$$
\left(\frac{\partial\left(P_{0} / \dot{H}_{0}\right)}{\partial V}\right)_{E_{1}=k}=0
$$

or

$$
\left(\frac{\alpha P_{0} / b_{t}}{\partial h_{i}}\right)_{C_{4}=k}=0
$$

These relationshipe are satisfled at those points in the altitude-Hach number Plane where the $E_{s}$ contours are tangent to the $l-g P_{i} / H_{f}$ contours. Connecting these points resulta in an approximate minimum fuel path.

The $P_{n} / \min _{4}$ contours suggest a measure of efficiency in view of the fact that they deplet the amount of gpectfic energy gained per pound of fuel ex+ pended. In order to acquire a more meandgifil measure of efficiency, these contours can be nodified to portray the amount of maneuvering energy gained for the internal energy (fuel) expended, this in defie by multiplying the $\mathrm{P}_{s} / 4$ contours by the welght of fuel avallabie, 4 , to abtain the resulting expresaton far Energy-Maneuverability Effichency:

$$
E-H E=\frac{F_{H}^{*}}{K_{4}} K_{4}
$$

where $P_{F}^{*}=$ the average $P_{s}$ over the fuel weightinterval $\psi_{0}-f_{C} \leq r_{f} \leq \psi_{t}$

i whare $H_{f}$. a inltial fuel weight (1b)
fe $=$ fuel consumed in fying form some reference energy level to any given altttude-Mach number point (lb)

$$
k_{t},=\text { tuel zeserve ( } 10 \text { ) }
$$

Raiden. Foc any altitude-airspred combination, avallable range for crisise condition may be expressed as

$$
R=\frac{w_{p}}{p_{0}^{*}} V+x,
$$

where $h_{0}^{*}=$ the average crutse fuel $\Omega_{0}, \psi_{s}$, over the fuel weight interwal

$$
H_{0}-f c \leq w \leq \leq 4,(\mathrm{lb} / \mathrm{sec})_{2}
$$

and $x=$ the borizontal distance traversed in flying from some reference energy level to any given altitude-airspeed combination.

COMPITATIONS
ENERGY RATC DIAGHANS. For any given aircraft, an Energy Rate diagrem may be constructed by dividing the altitule-Hach number plane into a reptangular brit), copiputiny energy rate ( $l_{1}$ ) values at all of the points of intersection of the grid lines, and then connecting points of equal $\mathrm{p}_{\text {, }}$, The contour deftred by $\rho_{1}=0$ represents the steddymstate boundary of the aircraft, An alrcraft carnot uperate outside this contour without losing energy, either in the form of altitude, airspeed, or some combination of both, The steady-state boundary is further restricted on the left by the buffet boundary (obtained by connecting polnts wiere $C_{L}=C_{L_{w a y}}$ ), and on the right by placard Limits (a combination of prossure [structural] limite and engine tangerature limits),

Considerable insight into the effects of pulling g within the alreraftrg Qight envelope can be gained by constructing Energy Rate diagrams of more than 1 g, These diagmas contain both positive and negative $P_{\text {, }}$ colteurs within the l-g stealy-state envelope, hs such, they provide a measure of sustained muteuvorbility as a result of fulling g whthin the envelope.
 In the same mannor as for the Energy Rate diagrams, except that now we compute and connect poidats of iequal Emile. Tho different types of c-ML diagrans are constructel), The first type is refarred to as the path independent (constant Fuel) L-ill diaram. Computations for all points in the anvelopo ara bascd on $50 \%$ fucl height. Sinec ruel weiglit is leeld constant, the expression

$$
E-M E=\frac{V_{i}^{*}}{b_{p}} \quad W_{q}
$$

reduces to

The diagram 15 called ${ }^{\text {poth-independent }}$ since the amount of fuel at the altitudeWach number points here canputations are made is Independent of the paths reguired to reach these points.

In the eepcond type of E-HE Ulagram, called the pathmependent (variatile fuel) EmE diagram, the anount of fuel required to reach any given altitudeWach number point is subtracted from the total fuel weight before EmE computations are made. The assumption is that the pllot ha's flown a ounmum fuel path from some reference energy level (we use $E_{1}$ (RFF $=3000$ feet) to the altitude. Hacin numer point uder consideration. A nore detailed diecussion of this ossumption will be given later $f$ n this appendtx and $\ddagger n$ Appendfx III. Addttonally, the amount of fuel upon which the path-dependent EME computations are based is reduced by a autable reserve (normully 5 of full tatemal plus 20 minutes loiter at $10,000 \mathrm{ft}$.

RANGE DIACRAHS, Again, the computational aspacts of this diagram are esontlally the 日ame as for the Energy hate and E-M Efficiency diagiame, To compute range, the program requires, os an additional input, a partital power aetting table, S.E., a table of cruisc fuel flow as a function of altitude, Wheh number, and irag (thrust required). The subsenic and supersonic porthons of the envelope are compated using partial military and partial afterburner power settings, respectively, A translent region is obselved between the subsonic and supersonic portions of the envelope. In this region, level unaccelerated fight is not possible as the thrust required is greater than military thrust avalzable, yet less than mininum afterburner thrust availabie,

For range, only a path-dependent (variable fuel) Range dlagram is constructed. For this diagram, the amount of fuel available at any given alcitudeMach number point is reduced by the anount of fuel required to fly a minimum fuel path from some reference erergy level (ogain, we use $\varepsilon_{\text {R }}=3000$ feet) to the point under consiberation, and by a suitable fuel resserve (c.g., spi of full intermal plus minutes Ioiter at 10,000 feat). The horizontal distance traversad in flying the abovementioned minimun fuel path, $x$, is constdered part of the avaliable range. A diacussion of the method used to conpute fucl consumed and horizontal distance traversed is given later in this appendix end In Appendix IIf,

EXTASLONS OF THE RUTOHSKI TECHNIQUE
Earller in this appendox, we saw that Rutowski calculated the lonation of the approximate minimum time and mininum fuel patha in the altitude-Mach numa bet plane. Rutowski did not, however, give any measure of the time required, fuel cunsumed, or horizontal distance traversed in flying along these approximate paths. His two basic assumptions wace that (1) the path be computed for 1 g and (2) the weight be held constant.

Rutowski's method has been extended to allow an approximation of the time required, fuel consumed, and horizontal distance traversed in flying along the mindaum time or minfmum fuel path. A byproduct of this extengion has been the renovel of his constant weight assumption,

Computations proced as follows, Progran inputs include initial, incre mental, 'and final values for $E_{g}$ and h:

$$
E_{1_{1}}\left(\Delta E_{4}\right) E_{L}
$$

$h_{I}\left(\Delta h^{\prime}\right) h_{L}$.

The specific energy equation,

$$
E_{1}=h+\frac{v^{2}}{2 g}
$$

Is rearranged to the form

$$
V=\left[2 g\left(\Gamma_{x}-h\right)\right]^{\frac{1}{2}}
$$

Then for $\Sigma_{4}=E_{41}, E_{s_{1}}+\Delta E_{11}, E_{a_{1}}+2 \Delta \Sigma_{1}, \ldots, E_{n_{L}}$, the following array Is eonstructed;

$$
\begin{aligned}
& \begin{array}{lllllllllll}
H_{1} & H_{2} & H_{1} & T_{L_{2}} & H_{2} & C_{L_{1}} & G_{D_{1}} & G_{1} & P_{1} & \left(P_{1} / H_{4}\right)
\end{array} \\
& \begin{array}{llllllllll}
h_{0} & H_{2} & H_{3} & T_{L_{0}} & i_{2} & C_{L_{0}} & C_{D_{2}} & D_{2} & P_{1} & \left(P_{3} / H_{1}\right)_{2}
\end{array}
\end{aligned}
$$

Actually the array does not an all the way from $h_{2}$ to $h_{L^{*}}$. The lower altitudes result in Mach numbers higher than the aircraft's capability, milen
 The ligher altitudes result in $\mathrm{C}_{\mathrm{L}}{ }^{\prime}{ }^{\prime}$ greater than $\mathrm{C}_{\mathrm{L}_{4} ;}{ }^{\text {is }}$. This fact elininates many of the lower lines of the array, Miditionally, when $h>E_{1}$, the quantity $\tilde{L} \mathrm{~g}\left(\mathrm{E}_{\mathrm{i}}^{2}-\mathrm{h}\right) \mathrm{J}^{\frac{1}{2}}$ becomes negative, eliminating lined of the array. Finally, other numerical tediniques, beyond the scope of this sppendix, bre employed to reduce the size of the above arrays, therelly decreasing the computer time required to constrict a path.

If a minimun time path is being conjuted, the progrom selects the altitudeNaels umber yoint for wifol $P$, is maximot in the erray and this point becomes a point. on the minimun tine path. Once the Ine containing the maxinum $p_{\text {a }}$ is selacted, it is used, along with a similar line ou the previous ariay, to apjroximata a time fucrmant, at, a fucl increment, $a^{4}$, and a horizontal distance increment, $\Delta x$, in the followiog mannet:

$$
\begin{aligned}
& \Delta H=\frac{\Delta I_{1}}{\left(l_{0} / G_{4}\right)} \\
& \Delta x=\left[\bar{v}^{2}-\left(\frac{\Delta l t}{\Delta t}\right)^{2}\right]^{\frac{1}{2}} \Delta t
\end{aligned}
$$

The methou outlined alove results in altitule-vach number points through hthich a minimum tive path nay be dram, The $\Delta t^{\prime} 5, \Delta H^{1} s$, and ax's are summed - over tle path to bive an approximation of the time required, fuel consuntert, and horizontal distance tisversed,

Cach time $E$, is ineramented, the weight used to compute the quantities in the ayray is first deeremented by the guantity an, computed in the previous array. This resultis in a variable weight path,

Computations for a minimum fual path proceed in aractly the same manncr, except that, for the minimun fuel poth, the line is chesen in the arrays where $P_{0} / 4$ instead al' $\Gamma_{\text {, }}$ is maximum.

To compute a path-bependent EANC or Range diagrany a Rutomski minimum fuel path must be compited first and the following talle butle;

where

$$
f c_{i}=\sum_{j=1}^{1} \Delta H_{2},
$$

and

$$
x_{i}=\sum_{j=1}^{ \pm} \Delta x_{j} .
$$

Then, when EwE or zange is computed for a given altitude-kach number point, ( $h, H)$, that ( $h, H$ ) determines an $E_{1}$, which, in turn, detemines an $f 0$ and an $x$, if we assume that the fuel consuned and hozizontal distance traversed in flying to any point on a constant specific energy line is the same. Thla is a rather bold assumption, however, and connot be accepted without further discusoion, Appendix III provides a detalled treatment of this assumption.

BART II. TILE ERYSON-KLLLEY STELPEST ASGENT OPTTALZATION IROCRAH

The second computer program enployed in these analyses is the Brysandelley Stecpest Ascent Optimization I'rogzan which provites dynamie Right profiles, in minindm time or with mininum expenditure of fuel. for transfer berween aro eneriy levels ( $\mathrm{H}_{1}, \mathrm{~K}_{1}$ ) and ( $\mathrm{H}_{3}, \mathrm{H}_{2}$ ).

By using the methods of E,S. Ratonski, tho somalled Rutowski puth, explained proviously under the Encrgy-fonewverability l'rogram, is obtained and is depicted in Figure 3, page 7 of this report, However, these metlods provide aothing mare tand vory good approximations to the solution of the minimam time or minimum fuel problens. They provide no insight into suelt parameters as load Factor or pitch angle along the path, In addition, the methods are predicated on l-g level flight parameters and do not consider the lorces acting perpendicular to the fight path, In essence, the Rutowsin mathod is a stotie method, in itself, but a very valuable tool leadiag to the more accurate dynamic optinum paths,

Even the more sophisticated Bynanic Profile Generptor Progran, diseussed in lart III of this Appendix, provides only approsimations to the desired solution to the minimum time or fuel ptoblem, Admittedly, the results of using the Rutcwski paths in conjunction with this program are mueh more realistic, as now both load factor and piteh angle are considered throughout the path. However, the techniques embodied in this program are still limited by the altitude-Mach number comuinations input into the program ats points describing the approximate pith, and yield aothing but a better appioximation to the desired optimurn path. The program is invaluable, though, as a generator of load fastor as a function of time for input into the Byson-Keney Steepest fiscent Progran as the nominal path.

This steepost-osent method of optimisation is an iterative scheme wich begins with acy nor-optimal path and proseeds to derive a slight improyement each iteration frem this nominal path. This slightly improyed path at pach itemation is used as the nen aminal path, and the process is repeated until. we are sufficiently close to the optimum for our purpose. In this process, each pew path is found by taking the trajectory which yields the largest gain in performance for a given size of perturbation in the control variables.

The yalue of a good first guess at the nominal patli is ipmediately evident. If this path is close to the optinum, the number of iterations neeessary to arrive at this profile will be small indeed when compared to those necessiry if the nomimal path is Ear from the optimum, The ability to input good hominal paths results in tremendous savings of valuable computer time,

The analysis of single stage trajectories by the steepest ascent method has been thoroughly treated in the litarature, One of the clearest treatnents available is that of blyson and Denlam In Reference 2. For convenience, a brier description of the gencral problem, as formulatad by them, is repeated here. Sone of the detailed derivations which are onitted here are presented in Referance 2.

After presentation of the general problem, the specific applications mide in formulating tive program at the hir Hroving Ground Genter are given in Jctail.

## GENLTAL PROBLEN

Detemine © ( $t$ ) in the time interval $t_{1} \leqslant t \leq t_{2}$. so as to maximize
(1) $t=\left[\begin{array}{l}\left.\left[t_{2}\right), t_{2}\right]\end{array}\right.$
subject to the constraints
(2) $\bar{T}=\bar{T}\left[\bar{x}\left(t_{a}\right), t_{2}\right]=0$

(4) the given initial souditions $\overline{\mathrm{x}}\left(\mathrm{t}_{1}\right)$
(5) $t_{z}$ detcmined by $u=b\left[\vec{x}\left(t_{z}\right), t_{2}\right]=0$.

The " over the symbols iluove indicates a mitrix quantityp, and a more detailed deseription of the above qunntities follows:



(9) $\Phi$ the pay-aff function, a kom function of $\bar{x}\left(t_{2}\right)$ and $t_{2}$,
(10) $F=\left[\begin{array}{c}e_{i} \\ f_{a} \\ 1 \\ \vdots \\ f_{a}\end{array}\right]$, an $\pi x 2$ matrix of known functlens of $\bar{x}(t)$, $\bar{d}(t)$, and $t$,
(11) $a=0$ is the stopping eondition that detemines final time $t_{2}$, and is a knom function of $\mathrm{X}\left(\mathrm{t}_{2}\right)$ and $\mathrm{t}_{\mathrm{E}}$.

The method proceeds as follows:

1. Choose a reasonable nominal control variable program, $\overline{\sigma^{*}}(\tau)$, and use It with the initial conditions (4) and the differertial equations (3) to calculate, by numarical methods, the 5 tute variable programs $\vec{x}^{*}(t)$ until $a=0$. In general, this nominal path will not satisfy tie teminal coneltions $\bar{f}=0$, of yield the maximur posaible value of $*$.
2. Conaidar snall perturbations $\delta \dot{o}(t)$ about the nominal control variable progran, $\tilde{\mathrm{a}}^{-1}(\mathrm{t})$, where
(12) $\delta \vec{a}(t)=\vec{a}(t)-\vec{a}^{\prime}(t)$.

As a result of these perturbations, the atate variable programa undergo perturbations $\bar{x}(t)$, where
(13) $5 \bar{x}(t)=\bar{x}(t)-\vec{x}{ }^{\prime}(t)$.

If the relations (12) and (13) are bubstituted into the differentiat equations, given by (3), the linear differential equations
(14) $\frac{d}{d E}(d \bar{x})=\bar{F}(t) \delta \bar{x}+\bar{\sigma}(t) \delta \bar{c}$
are obtained, aecurate to first order in the perturbations, whate

The symbol () fadicates that the enclosed partial deryatives are evaluated alang the noninei path.

Using the theory of adoint differential equations, the folloring expresaions may be written,



where the symbol "Indicates the transpose of the matrix and elements of the three $\bar{\lambda}$ matrices, appearing above, are obtained through the ntrnerteal integrathen of the differential equations adjoint to equations (3):
(c) $\frac{d \bar{\lambda}}{d t}=-\bar{F}^{\prime}(t) \bar{\lambda}(\tau)$
with the boundary conditions
where

$$
\text { (32) } \frac{\partial t}{\partial x}=\left[\begin{array}{ll}
\partial \phi \\
\partial x_{1}
\end{array}, \frac{\partial 1}{3 x_{2}}, \ldots, \frac{\partial \Phi}{\partial x_{4}}\right] \text {, }
$$

$i$

$$
\frac{d n}{d x} \cdot\left[\frac{\partial n}{d x_{1}}, \frac{\partial \infty}{\partial x_{0}}, \cdots, \frac{a n}{\partial x_{0}}\right]
$$

and

$$
\text { (23) } \|=\left(\frac{\partial s}{\partial t}+\frac{\partial \psi}{\partial \bar{x}} \bar{t}\right)_{t=t_{0}}^{*}, \quad t a\left(\frac{\partial \bar{t}}{\partial t}+\frac{\partial \bar{i}}{\partial \bar{x}}\right)_{t=t}^{*}
$$

$$
B=\left(\frac{\omega}{\partial t}+\frac{\partial \omega}{\partial \bar{x}} \bar{f}\right)_{t=t_{2}}^{*}
$$

Note that the $\vec{i}+\mathrm{a}$ are influence functions in that they tell hoo much a certain terminal condition is changed by a small change in some initial state variable. Note also that the adjoint equations ( E ) must be integrated backwards ene the boundary conditions (21) we given at the terminal point,

For eteepest ascent the $\delta$ o(t) program that maximizes the di in expression (17) must be founl, given values of dy and dom 0 in expreasions (18) and (19): respectively. This maximization must aiso be subject to a given value of the integral
(24)

$$
(d P)^{2}=\int_{11}^{a_{5}} \vec{\sigma}^{\prime}(t) \bar{H}(t) s \bar{\sigma}(t) d t
$$

The value of (dP) ${ }^{2}$ is chosen such that the perturbations will be small enough to insure that the neglect of second and higher order perturbations leading to equation (14) is reasonable. In adeltion, values of dipare gelected to bring the next solution closer to the desired termilial constraints, $\bar{T}=0$.

The $m \times n$ matrix $M(t)$ is symmetric and contains welghting functions as elements. They way be chosen arbitcarily to improve comperterce, In the usual ease (the APBC 7 rogram falls into this category), $\bar{W}(t)$ is taken equal to the identity matrix and (dP) becomes the integral of the square of the control variable perturbations, $\delta \overline{0}(\mathrm{t})$. . Observation reveala that all control variables should have the same dimenaiong for equation (24) to have any meaning. To meet this requirement the control variables aze nomaily required to be nondimensional.

A rather tnvolwed geries of mathematical manipulations (peesented in an orderly and cleax fashion in Reference 2, but coltted here for the sake of brevity) leads to the followig proper cholce of 6 (tit):

where
(26) $d \overline{\bar{p}}=\mathrm{d} \bar{\dagger}-\bar{\lambda}_{+2}^{\prime}\left(t_{2}\right) \delta \bar{x}\left(t_{2}\right)_{x}$

(28)

(29) $\bar{L}_{\phi}=\int_{1_{2}}^{\lambda_{2}} \alpha_{0} \bar{Q}^{-1} \overline{6} \lambda_{1} d t$,


and the ( $)^{-1}$ indicates the inverse matrix, and the + or - aiga before the radical in (25) is chosen if $\$$ is to be Increased or decreased, respectively.

If the selected ail is buch that $\overrightarrow{A B}$ is too large, then the rumerator in the
 $d P$ wust be ingobed, Since $d P$ is chosen to insure walid inneariation, the selected df nust also be 12mitted.

The predictad ehange in $f$ for the change in $-(\tau)$ given by (25) is

$$
\begin{align*}
& +\Gamma_{80}^{\prime}\left(t_{1}\right) \overline{0} \bar{x}\left(t_{1}\right) \text {. } \tag{32}
\end{align*}
$$

If $\overline{\mathrm{d}}=0$ (the teminal constrainte having been ${ }^{\text {satisfled }}$ ) and $\overline{\mathrm{tan}}\left(\mathrm{t}_{1}\right)=0$, then $d \bar{F}=\mathrm{C}$ and equation (32) becomes

which is a gradient in function space, since $d P$ is the length of the stap in the control variable program. As the opthum program is approathed and the feminal constralnts are met, $(d \bar{j}=6)$, this gradient must tend to zero, and expreasion (32) becones
3. A new control variable progras is now obtained as

$$
(35) \overline{\bar{c}}(t)=\bar{a}^{*}(t)+6 \overline{\bar{c}}(t) .
$$

Thif new ( $(t)$ is row used in the onighal nonlinear differentian equation given by (3), and the process is repeated unth the tominal constaints (e) are net and the pradient (35) beconen nearly zero,

THE EGLIN PROQRAM
Juring the apring of 1964，preparationa here begun for the phyaical test to walidate the Energy－klaneuveribility theory，as requested by the Tactical Air Command．The test was designed to show that the＂dip⿴囗十介odoodie＂menewvers asacclated with minimitu ame－ts－ciimb and ainimum fuel pathe did，in fast，rep－ resent the optimum flight proflleg for tranafer fom one energy level to anothcr．The above test has conducted under AFCC Project $0570 T 1$.

To adequately support this test，a program which could conpute these optimua paths was necessary．Artangenents had been oude to obtain the Bryson－Nelley Steepegt Ascent Computer Program fram the Flight Dynamics Liboratory at Hright－ Patterson AFB．The progran mas being iomulated and developed under Contract Ho．AF 33（657）－8Bag by Mchonsell Alecraft Corporation．then aduised that this progran pould not be ready in tire for the test，the decision was made to develop a program in－house at Eglin．Due to time 1 Inatationa，a somewhat stmple Aryson－Kelley Steepest Ascent Progrem was formulated in tao dimenoions and with one cantrol vardable．

The prodram was completed in August 1 g64 and used exteneively during the conduct of the test．Before giving the detaily of the Eglin foraulation，an explanation of sone key features of the program which are not prowided in the general formulation will be presented．

As mentioned before，the program has but one control variable，$n$ ，the nomnl accelaration in number of g＇g．Originally，the progran ong fomblated wh welocity and pitch angle serving as taminal constraints hand fa with altitude as the stopping condition $[$ ．However，using two terminal constraints led to trouble fith the matrix $\overline{\mathrm{I}} \phi \mathrm{j}$ ．This mateix was found to be nearly singu－ lar，and the exdstence of its inverge was，therefore，quite questionable．

Analyaia revealed that the constraint on plach angle was not vital and that two programs should be developed；one with velocity serving as the teminal con－ straint with altitude in the role of stopping condition，and the other with the roleg of velocity and altitude reversed．

In reducing the application of the Steepest Ascent Hethod to a routine computation，an autoratte scheme or cuntrol sybter for detemining the atep size，（dF）${ }^{2}$ ，must be deviged．In the Eglin progran，this control system Is as fullows：

1．Begin uitl a desired lmprovement in the quentity ta be optimized（tima for minimum the paths and zotal weight fur minimum fuel patha）．This debired improvemeut，dé，should be reflected in the next iteration if the teaninal con－
 tion of the given ds＊
2. If the teminal constraint las not been met, chenk to see if

 for bex ( $t$ ) given in (25).
5. The requested di is then nodified es each iteration cones closer to the optimum. This modification is controlled by the magnituds of the eradient given ly (33). As this marmirude grows smaller ant becones iess than predetemined values, the size of de is successively halved. The given values of the gradient, at thich the ds's are halved, are not readily obvious and appropriate values must be leamed through some experience with the progran,
4. Even with this sembautonatic control device, considerable time must be spent in deternining values of the gradient with the possilility that considerable enfluter time may still be consumed before a trae optimum path is reached, Several nther control systers are presentiy under investigation at Egifn AFB. It is hopeld thet a better automatic scheme will be fourd which will decrease both computer ruming time and the manpower required to eventually arrive at the optimum paths.

The fomulation of the Bryson-kelley technique, presently in use at Lglin AFB, is prosented here such that one may readily foilow it, having been nade acquainted with the genersi proilem previously.

1. Control Varfabie Hatrix $\bar{y}(t)$. ( $\quad(m=1\rangle$ Hatrix
$\bar{q}(t)=n(t)$,
where $a=1$ and $n=$ namal acceleration an nimber of gti (dipensionless).
2. State Varłalle Matrix $\bar{x}(t)$. ( $n \times 1$ ) Matrix
$\bar{x}(t)=\left[\begin{array}{l}x_{1} \\ x_{2} \\ x_{0} \\ x_{4}\end{array}\right]=\left[\begin{array}{l}h(t) \\ v(t) \\ y(t) \\ u(t)\end{array}\right]$, witere $n=4$,
ha altitude above HSL in fieet,
$Y=$ true airspeed in feet per second,
$Y=$ pitch angle (angle bctween velocity vector and reference horizontal platie) in radians,
$H=$ aircraft gross weight in pounds.
3. Teminal Constraint Hatrix $F=0 \quad 0 \quad(p \times 2)$ Nit:
a. $\bar{i}=1=h-h_{2}$ for Program 55, where $h$ is teraini: $\quad(j=1)$ constraint and $V$ is stoppling condfition.
b, $\mathcal{T}=\hat{y}=y=Y_{a}$ for Program 623, where $V$ is teminal $\quad(\rho=1)$ constiant and h is stopining condtt fon.
$h_{3}=$ desimé termsnal aitftude in feet,
$V_{2}=$ busirnd taminal velocity in feet per secend:
4. Pay-0ts Function 3
o. $\$=- \pm$ fer minimum time paths,
b. $t=\boldsymbol{H}$ z̈r minimuri fuel paths.
5. Time Derivative of State Varlable Matrix, f. ( $n \times 1$ ) Hatrix
$\bar{f}\left[\bar{x}, \pi, \bar{t} \bar{j}=\frac{d \bar{x}}{d t}\right.$.

a. $f_{2}=\dot{h}=V$ sill $Y$,
$f_{a}=\dot{\psi}=g\left[\frac{P_{0}-D}{w}-\sin Y\right]$,
$\tilde{r}_{3}=\dot{y}=\frac{g}{\eta}[\pi-\cos Y]$,

$$
f_{4}=\dot{v}=-\dot{4},
$$

where
$g=$ acceleration of gravity $=32,174 \mathrm{Et} / \mathrm{sec}^{2}$,
$T_{4}=$ thast available in pounds,
$D=$ drag in powise,
मि $u$ Edel flow in pounds per second.
6. Stopping Condition $0=0$.
3. $O=V-V_{a}, f o r$ Program 556 。
b. $\quad \mathrm{A}=\mathrm{h}-\mathrm{h}_{9}$, for Program 623.
7. $\vec{F}(t)=\frac{\partial \vec{f}(t)}{\partial \vec{X}(t)}$.
( $0 \times n$ ) Hatris
$E(t)=\left[f_{1 J}(t)\right]$ i, $j=1,2, \ldots, n$.
$E_{i j}=\frac{\partial f_{i}}{\partial x_{j}}=\frac{\partial \dot{x}_{1}}{\partial x_{j}}$
a, $f\left(I 2=\frac{\partial b}{\partial h}=\frac{\partial(y \sin y)}{\partial h}=0, \quad(\pi=4)\right.$
b. $f_{1 a}=\frac{\partial \hat{h}}{\partial y}=\sin \gamma$
e. $\quad f_{i 3}=\frac{\partial h}{\partial y}=V \cos y$,
d. $E_{L}=\frac{B h_{L}}{\partial w}=0$,
e, $\left.f_{a L}=\frac{\partial \dot{V}}{\partial h}=\frac{\partial\left\{\left\{\left\{\frac{T_{a}-D}{H}-\sin Y\right]\right.\right.}{\partial h}\right\} \frac{g}{\omega} \frac{\partial\left[T_{4}-D\right\}}{\partial h}$,
$f_{1} \quad f_{\partial z}=\frac{\partial V}{\partial V}=\frac{\underline{E}}{\Delta} \frac{\partial\left\{T_{a}-g\right\}}{\partial V}$,
E. $\quad f_{\text {ajg }}=\frac{\partial \overline{\partial y}}{\partial Y}-g \cos \varphi$
h. $f_{a t}=\frac{\partial X^{*}}{\partial H}=-\frac{E}{H}\left\{\frac{T_{N} \omega D}{W}+\frac{\partial D}{\partial H}\right\}$,
i) $\varepsilon_{s 1}=\frac{\partial \dot{Y}}{\partial h}=\frac{\partial\{\underline{Y}(n-\cos \gamma)\}}{\partial h}=D_{2}$
j. $\mathrm{f}_{3}=\frac{\mathrm{a} \psi}{\partial Y}=-\frac{g}{\gamma^{2}}\left(\pi-\frac{j}{\cos Y)}\right.$,
k. $f_{g 3}=\frac{\partial \dot{Y}}{\partial Y}=\frac{g}{Y} \sin Y_{Y}$

1. $f_{34}=\frac{\partial \dot{Y}}{\partial w}=0$,
m. $\mathrm{E}_{41}=\frac{\partial \dot{w}}{\partial h}=-\frac{\partial_{i}^{\prime}}{\partial h}$,

2. $f_{43}=\frac{\partial{ }^{2} \hat{d y}}{\partial y}=0$,
p. $f_{44}=\frac{\partial \hat{H}_{t}}{\partial_{t y}}=0$.
3. $\quad \bar{E}(t)=\frac{\partial \bar{f}(t)}{\partial \bar{E}(t)}$.
( $\mathrm{n} \times \mathrm{m}$ ) Hatrix
$\bar{Q}(t)=\frac{\partial \tilde{f}_{( }(t)}{\partial n}=\left[\frac{\partial f_{i}}{\partial n}\right], i=2, \ldots, 4,\left(\begin{array}{l}n=4 \\ n \\ m\end{array}\right]$
Let
$\bar{G}(t)=\left[g_{1}\right] \quad i=2, \ldots, 4$,
a. $g_{1}=\frac{\partial \dot{h}}{\partial_{n}} a \frac{\partial[Y \sin Y]}{\partial_{n}}=0$,



4. Lagrange Multipliers.
a. $\bar{\lambda}_{4}=\left[\begin{array}{l}\lambda_{\psi_{1}} \\ \lambda_{\psi_{2}} \\ \lambda_{4_{3}} \\ \lambda_{4_{4}}\end{array}\right]$,
$b_{1} \quad \dot{\lambda}_{1}=\left[\begin{array}{l}\lambda_{l_{1}} \\ \lambda_{l_{2}} \\ \lambda_{p_{3}} \\ \lambda_{b_{4}}\end{array}\right]$,
c. $\bar{A}_{0}=\left[\begin{array}{l}A_{n_{1}} \\ \lambda_{1} \\ \lambda_{1} \\ a_{3} \\ h_{n_{4}}\end{array}\right]$.
5. Adjoint Differential Equations for Lagrange Multipliers,
E. $\frac{d \bar{d}}{d t}=-\bar{F}^{\prime} \bar{A}_{1}$,

c. $\frac{d \vec{A}_{D}}{d t}=-\bar{F}^{\prime}{ }^{2}$

Now

$$
\bar{F}^{\prime}=\left[\begin{array}{llll}
0 & f_{21} & 0 & f_{42} \\
f_{2 a} & f_{3 a} & f_{3 a} & f_{43} \\
f_{13} & f_{a 3} & f_{33} & 0 \\
0 & f_{34} & 0 & 0
\end{array}\right]
$$

where the $f_{i j}$ are given in (7).
Performing the matrix multpifeations indicated in $7 a_{1} 7 b$, and 7 c , we have the following aet of differential equations for the individual eleqents of the Lagrange Kult iplier matricea:

$$
\mathrm{d}_{1} \quad X_{i_{1}}=-\sum_{1=1}^{4} X_{1}, f_{19}
$$

$$
j=2,2,3,4 .
$$

> (1) $\dot{i}_{1_{2}}=-\left(\lambda_{1_{2}} f_{31}+\lambda_{4_{4}} E_{42}\right)$,
> (2) $i_{f_{2}}=-\left(\lambda_{1_{2}} f_{17}+\lambda_{12} f_{82}+\lambda_{1_{3}} f_{33}+\lambda_{1_{4}} f_{43}\right)$,
> (3) $\dot{\lambda}_{\delta_{3}}=-\left(\lambda_{h_{2}} f_{25}+\lambda_{46} f_{23}+\lambda_{1,} f_{33}\right)$,
> (4) $\dot{\lambda}_{t_{4}}=\lambda_{1_{2}} f_{\text {a4* }}$
> e. $H_{1,}=-\sum_{i=1}^{4} \lambda_{i} f_{i j}$. $j=1,2,3,4$.

$$
\begin{aligned}
& \text { (3) } \lambda_{\phi_{s}}=-\left(\lambda_{1} f_{23}+\lambda_{\phi_{2}} f_{23}+\lambda_{H_{5}} f_{33}\right) \text {. } \\
& \text { (4) } \lambda_{14}=-\lambda_{f f} f_{04} \text {. } \\
& \text { f. } \quad \dot{i}_{a_{j}}=-\sum_{i=1}^{4} \lambda_{a_{1}} f_{1 j} \text {, } \\
& J=1,2,3,4, \\
& \text { (1) } \dot{i}_{Q_{4}}=-\left(M_{n_{2}} f_{a y}+\lambda_{n_{1}} f_{41}\right) \text {. }
\end{aligned}
$$

> (3) $K_{\Gamma_{6}}=\left(\lambda_{\Gamma_{2}} f_{l s}+\lambda_{n_{2}} f_{7 s}+\lambda_{\Gamma_{1}} f_{3 s}\right)$.
> (4) $\hat{R}_{8}=\lambda_{\Omega_{0}} f_{74}$.
11. 5oundary Conditiona for Legrange Multipliers.

(1) For maximizing a - t (miniman tiae fatis).

$$
\lambda_{k}\left(t_{a}\right)=\left[\begin{array}{c}
-\frac{\partial t}{\partial h} \\
-\frac{\partial t}{\partial v} \\
-\frac{\partial t}{\partial y} \\
-\frac{\partial t}{\partial v}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0 \\
0
\end{array}\right],
$$

or

$$
\lambda_{i_{1}}\left(t_{3}\right)=\lambda_{\phi_{2}}\left(t_{2}\right)=\lambda_{i_{2}}\left(t_{2}\right)=\lambda_{i_{4}}\left(t_{3}\right)=0 \text {. }
$$

(2) For maximizing $t=\mathrm{H}$ (mindmum fuel paths),

$$
A_{H}\left(t_{Z}\right)=\left[\begin{array}{l}
\frac{\partial H}{\partial h} \\
\frac{\partial H}{\partial Y} \\
\frac{\partial W}{\partial Y} \\
\frac{\partial W}{\partial H}
\end{array}\right] t=\left[\begin{array}{l}
0 \\
0
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]
$$

OI

$$
\begin{aligned}
\lambda_{H_{2}}\left(t_{1}\right) & =\lambda_{t_{2}}\left(t_{a}\right)=\lambda_{1_{3}}\left(t_{9}\right)=0, \text { and } \lambda_{t_{1}}\left(t_{a}\right)=1 . \\
\text { b. } \quad \bar{\lambda}_{1}\left(t_{2}\right) & =\left[\frac{\partial p_{1}}{\partial x}\right] t=t_{2}, \text { or } \lambda_{1}\left(t_{2}\right)=\left[\frac{\partial \phi_{1}}{\partial x_{i}}\right] t-t_{0}, \pm=1,2,5,4 .
\end{aligned}
$$

(1) For terminal constrajnt on h (Program 556),

or

$$
\lambda_{\phi_{4}}\left(t_{g}\right)=1, \text { and } \lambda_{\phi_{1}}\left(t_{a}\right)=\lambda_{\phi_{s}}\left(t_{7}\right)=\lambda_{\phi_{4}}\left(t_{2}\right)=0 \text {. }
$$

(c) For terminal constraint on V (Prograo 62)),

$$
\lambda_{i}\left(t_{a}\right)=\left[\begin{array}{l}
\frac{\partial v}{\partial L} \\
\frac{\partial V}{\partial v} \\
\frac{\partial v}{\partial y} \\
\frac{\partial V}{\partial w}
\end{array}\right]_{t=t_{D}}=\left[\begin{array}{l}
0 \\
1 \\
0 \\
0
\end{array}\right]
$$

or

$$
\begin{aligned}
& \lambda_{t_{2}}\left(t_{2}\right)=1 \text {, and } \lambda_{\phi_{1}}\left(t_{g}\right)=\lambda_{t_{3}}\left(t_{9}\right)=\lambda_{\phi_{1}}\left(t_{9}\right)=0 .
\end{aligned}
$$

(1) For Stopping Condition on Y (Progran 56),

$$
\bar{\lambda}_{n}\left(t_{a}\right)=\left[\begin{array}{c}
\frac{\partial V}{\partial h} \\
\frac{\partial v}{\partial Y} \\
\frac{\partial V}{\partial y} \\
\frac{\partial V}{\partial W}
\end{array}\right]_{t=t_{0}}=\left[\begin{array}{l}
0 \\
1 \\
0 \\
0
\end{array}\right],
$$

or

$$
\lambda_{\Omega_{0}}\left(t_{2}\right)=\eta_{2} \text { and } \lambda_{0_{2}}\left(t_{2}\right)=\lambda_{0_{0}}\left(t_{2}\right)=\lambda_{\Omega_{4}}\left(t_{2}\right)=0
$$

(2) For Stopping Gondition an h (Progran G23),

$$
\bar{M}_{\Delta}\left(t_{g}\right)=\left[\begin{array}{l}
\frac{\partial h}{\partial h} \\
\frac{\partial h}{\partial y} \\
\frac{\partial h}{\partial Y} \\
\frac{\partial h}{\partial u}
\end{array}\right]_{t=t_{g}}=\left[\begin{array}{l}
2 \\
0 \\
0 \\
0
\end{array}\right],
$$

or

$$
A_{\Omega_{1}}\left(t_{a}\right)=1, \text { and } \lambda_{\Omega_{2}}\left(t_{2}\right)=\lambda_{n_{8}}\left(t_{a}\right)=\lambda_{n_{4}}\left(t_{2}\right)=0
$$

12. The Hatrices $\bar{\lambda}_{\text {th }}$ and $\bar{\lambda}_{\text {th }}$,
B. $\quad \bar{x}_{W 0}=\bar{\lambda}_{i}=\frac{\dot{t}\left(t_{a}\right)}{\bar{\alpha}\left(t_{a}\right)} \bar{\lambda}_{n}$
$0:$
$\lambda_{(5)_{1}}=A_{3_{1}}-\frac{i\left(t_{3}\right)}{i\left(t_{3}\right)} A_{\Omega_{1},} 1=2,2,3,4$.
(1) For maximizing $=-\boldsymbol{t}$ (ninimunt tine paths),

$$
t\left(t_{a}\right)=-2 .
$$

(2) For maximizing $i=\mathrm{w}$ (minimum fuel paths),

$$
\dot{\Psi}\left(t_{a}\right)=-4\left(t_{a}\right)
$$

(3) For Stopping Condition on $v$ (Program 56).

$$
\delta\left(t_{a}\right)=V\left(t_{a}\right)=\left\{\left\{\frac{T_{4}-D}{H}-\sin \gamma\right\} t=t_{2} .\right.
$$

(4) For Stopping Condition on h (Program 6e3),

$$
\dot{x}\left(t_{a}\right)=h\left(t_{a}\right)=[v \sin \gamma]_{t=t_{a}}
$$

$$
\text { b. } \quad \vec{x}_{\alpha}=\bar{\lambda}_{t}-\frac{\phi\left(t_{a}\right)}{\bar{\alpha}_{( }\left(t_{3}\right)} \Sigma_{\Omega}
$$

or
(1) For terminal constraint on h (Program 556),

$$
\left\{\left(t_{2}\right)=h\left(t_{2}\right)=\{V \sin \gamma\}_{t=t_{2}}\right.
$$

(a) For terminal constraint on $V$ (Program 623),

$$
\text { . } \dot{\left(t_{a}\right)}=\dot{Y}\left(t_{A}\right)=g\left\{\frac{T_{L}-D}{H}-\sin \gamma\right\}_{t=t_{J}}
$$

15. The :latria Product $\left[\tilde{\lambda}_{\text {fin }}\right]^{\prime}$ G.
16. The thatrix Product $\left[\vec{h}_{0}\right]^{\prime} \mathrm{G}_{0}$

$$
\left[\bar{\lambda}_{\phi 2}\right]^{\prime} G=\sum_{i=1}^{1} \lambda_{n_{1}} g_{i}=\bar{G}^{\prime} \bar{\lambda}_{4 \Omega}=\lambda_{\alpha_{2}} E_{2}+\lambda_{m_{3} E_{5}}
$$



$$
\begin{aligned}
& \text { 和 } \vec{H}=1
\end{aligned}
$$

$$
\begin{aligned}
& =\int_{L_{1}}^{t_{2}}\left[\lambda_{1} n_{8} g_{2}+\lambda_{1 n_{0}} g_{3}\right]\left[\lambda_{m_{0}} g_{2}+\lambda_{n_{4} g_{3}}\right] d t_{1}
\end{aligned}
$$

$$
\begin{aligned}
& =\int_{i_{1}}^{s_{0}}\left[\lambda_{1 n_{0} g_{2}}+\lambda_{40_{0} g_{2}}\right]_{d t} .
\end{aligned}
$$



$$
\begin{aligned}
d \vec{\beta} & =d \vec{\eta}-\vec{\lambda} \vec{d}\left(t_{1}\right) b \bar{x}\left(t_{1}\right) \\
& =d \bar{\phi} a \in \bar{x}\left(t_{1}\right)=0 .
\end{aligned}
$$

a. d $=\Delta \mathrm{h}$ for Program $55 \%$,
b. $\quad \mathrm{d}=\mathrm{i}=\Delta \mathrm{Y}$ for Progran 623,
where $t_{0}$ and $\Delta V$ are the teminal condtion changes necessury to bring the next solution claser to the desired terminal constraints.
c, $\quad 4=h_{3}-h\left(t_{2}\right)$,
d. $\Delta V=V_{a}-V\left(t_{0}\right)$,
where $h\left(t_{3}\right)$ and $Y\left(t_{2}\right)$ ase the altitucie and tare alropeed at the final time point, $t=t_{3}$, of the previous salution.

Combining the regults from above,

$$
d \bar{\theta}^{\prime} \bar{I}=1 d \vec{i}=\frac{(d)^{2}}{I_{i}}
$$

17. The Hatrix Product $\overline{\mathrm{I}}$ /fot $\overline{\mathrm{I}}_{\mathrm{t}}$ 。


$$
I_{1}^{1_{\phi}} \bar{I}_{\phi \phi}^{-1} \bar{I}_{\phi \phi}=\frac{\left(I_{\phi \phi}\right)^{2}}{I_{\phi t}}
$$



19. The Expression $\langle\mathrm{dP})^{2}=d \beta^{\prime} \overline{\mathrm{I}_{t}^{\prime}}{ }^{2} \mathrm{~d} \overline{\mathrm{~B}}$.

Fror expression (32) is obtained the relationship

which can be reduced to the following exprescions, uaing the reaulta of previous paragraphs:

If the terninal constraint has been aatisfled, that is,

$$
\begin{aligned}
& \text { c. }|\Delta h| \leq 5_{1} \quad \text { for Frogram 566, or } \\
& \text { i. }|\Delta V| \leq E_{\mathrm{g}} \quad \text { for Program 623, }
\end{aligned}
$$

where $\mathrm{S}_{1}$ is sone predetemined tolerance within wheh the teminal constraint on altitude must fall, and so, the tolerance for a terminal constraint on velocity, then the expresefon (29,i.) is used in the expression for $\delta \mathbb{E}(t)$.

If the value of $|\Delta h|$ or $|\Delta V|$ is outside the predetemined toleranee, then



22. The Expression for $58(t)$.

Conbining the results of the preceding gages, the following expression for $\sigma$ (t) $(t)$ is obtained;

$$
\begin{aligned}
& \text { B. } \quad \delta \bar{\sigma}(t)= \pm\left[\varepsilon_{a}\left(\lambda_{\alpha_{2}}-\frac{\lambda_{\phi O_{2}} I_{\phi}}{I_{\phi i}}\right)\right] \\
& +\left[g_{0}\left(\lambda_{K_{0}}-\frac{\lambda_{\phi \phi_{b}} I_{\phi \phi}}{I_{\phi \phi}}\right)\right]\left[\left(\frac{I_{\phi d \phi}-I_{\phi \phi} d \phi}{I_{\phi t} I_{\phi \phi}-\left(I_{\phi \phi}\right)^{2}}\right)\right] \\
& +\frac{\left(\lambda_{1 \Omega_{3} g_{3}}+\lambda_{1 \Omega_{3} g_{3}}\right)}{I_{\psi}} d \psi
\end{aligned}
$$

 then
b. $\quad\left(\bar{a}(t)=\frac{\left(\lambda_{1} M_{2} E_{A}+\lambda_{1} D_{B} E_{j}\right)}{I_{\phi \dagger}} d \phi\right.$.

PART III. DYWHIN PROT: LE GENIRATOR

The Dymaic Prolile Generator uses the amodyamic and performance data for a given alrcraft to connect points in the altitudenach number plane with日月 approximate dynamic profile consistent with the capabintiles and Inditations of the bircraft. The progran provides a time history of the mormal acceleration, pitch angle, drag, feel consumed, and horizortal range traverged throughout the flight path.

The progron is deaigned to conpute normal acceleration in number of g's as a function of time, associated with the dyamic profile recessary to fly through the altitude-Hach number points defining a Kutowski paw for a given afreraft, This a schedule then serves as a good first guess for the namoal path in the Gryson-Kelley Steepest Ascent Program, leading to either a minimun tive or winimun fuel path for transfer between different energy levele.

Ao inttial aircraft gross weight, $H_{1}$, nomal acceleration, $\pi_{3}$, and pitch angle, $Y_{2}$ are assumed ot the fyret altitude-Mach numer point ( $\mathrm{h}_{3}, \mathrm{H}_{2}$ ) of the path. fine usual assumption is that the trausfer between enorgy levcis is fnitiated from level filghi, i,e., $\Pi_{1}=1,0 \mathrm{~g}$ and $Y_{1}=0$, Flgure II-2 depicts a typical path compnsed of $N(h, d)$ points.

To obtain the values for the quantities at the ith point $\left(h_{1}, H_{1}\right)$, we conploy an iterative predictormecriector process involving twe major steps: (1) predicting the value of scme of the quantitles acrogs the interval from ( $h_{f_{2}, 1}, h_{1-1}$ ) to ( $h_{1}, H_{1}$ ) besed on known valugs from the formet polnt and (ב) correcting these values during each of a number of iterations until a deafred level of convergence is atrained.

The tinc interval and fuel consumed are nstimated for the interval betwen the pointa, as diacussed previousiy in this appendix, and elther of two methods employe to dotermine the nomal accelaration required by the aireraft to Ry between the pointa.

The flabt method involves ablving bimultareously the equations of motion along add perpendicular to the path to prowide on e re son for the coefficient of ifit, $\mathrm{G}_{\mathrm{L}}$ :

$$
\bar{\Gamma}_{L}=\overline{A C}_{L}+\sqrt{\frac{\overline{T_{L}}-\overline{\bar{u}}(a i n \bar{Y}+\dot{V} / \mathrm{g})}{\overline{\mathrm{S}} \overline{\mathrm{~K}}}-\frac{\overline{\bar{C}_{D_{t}}}}{\overline{\mathrm{~K}}}}
$$



Figure Il-2. Typical Rutaiaki Fath Used by the Dymamic Profile Generatar.

## where $C_{D}$, is the roroulft drag coefficient,

$X$ is the induced drag parameter,
$\Delta C_{L}$ is the value of $C_{L}$ for which $C_{D}$ is minimion
$T_{4}$ is the thrust available in pounds,
$\%$ is the aircraft gross weight in pomde,
$y$ is the aircraft pitch angle,
i is the acceleration along the flight path in $\mathrm{ft} / \mathrm{gec}^{2}$,
F It the acceleration due to gravity,
q It the dynamic pressure in $\mathrm{lb} / \mathrm{f}^{2}$,
$s$ is the aircraft reference wing area $5 n \mathrm{ft}^{2}$.
The bar notation ( $\overline{\mathrm{C}}_{\mathrm{L}}$ or $\overline{\mathrm{T}}_{\mathrm{A}}$ ) indicate日 the value of this quantity at the widpotat of the interval,

The pitch angle $\bar{y}$ ) at thlu moldoint is approxmated by the following eqression:

$$
\operatorname{Gin} \bar{Y}=\frac{h_{i}-h_{I-x}}{\overline{Y_{\Delta t}}},
$$

## SONFIDENTIAL

shere $\Delta t$ Is the time increment_required to fly between $\left(h_{i-1}, M_{1-1}\right)$ and ( $h_{1}, M_{1}$ ). This value for sin $\bar{Y}$ along with the other required information are used to determine $\bar{C}_{L}$ which provides the value of $n$, the normal acceleration in number of $\xi^{\prime} s$ across the interval.

An alternate method enploys an estimate of sin $Y_{1}$ by the expression:

$$
\sin Y_{s}=\frac{2 \Delta h}{\bar{Y} \Delta t}
$$

to obtain the angular rate $\}$ :

$$
\dot{Y}=\frac{Y_{1}-Y_{i-1}}{\Delta t}
$$

This expression is then used to compute n:

$$
n=\frac{\overline{\mathrm{V}} \dot{\mathrm{Y}}}{\mathrm{~g}}+\cos \overline{\mathrm{Y}} .
$$

A more detailed breakdom of these methods is readily avallable from the airthors upon request.

# -CONFIDENFIAL 



APPEDTE III

(v) In computing the path-dependent (variable fuel) E-re and Range diam grams; a aethod is needed for computing the fuel consumed and horirental dian tance traversed in flying from bome reference point ( $h_{0}, H_{0}$ ) to any point ( $h, H$ ) inside the steady-state envelope of an atreratt, This fimplies eome path coruecting tile points ( $h, h_{0}$ ) and ( $h, 4$ ) in the eltitude-Hach number plane. of courae, any number of "flyable' paths can be drawn wich connect ( $h_{0}, M_{0}$ ) and ( $h$, M). The foregoing EH Efficfency and range corgiderations augeest a simpll fication by connectirg ( $h_{0}, H_{0}$ ) and ( $h, H$ ) with a rifimu fuel path.
(L) Use of a bophisticated technigue, such as the one credted to Bryson and Kelley, becones prohibitive because of the amomt of corputer time involved, For this reason, an approximate techrique becomes expedingly desirable.
(U) Heermann (Reference 3) observed that eurves of conatant minimum time are approxinate curves of conetant specific enersy, $E_{3,}$ in the sititude-Mach number plane $\mathrm{n}_{\text {, More rent }}$ recenvestigations by Heenamandicate that the asme is true for curves of constant minimim fuel.
(V) Heenann's resulta, coupled with experience gained with the Rutowski mathod, extended in the minner describet in Appendix II, suggest that fuel consumed in traverbing a Rutowal mintmun fuel path to a giyen energy level is alrost independent of the altitude Hach number combiration on that energy level. Additionally, for ratge confutations, the horizontal distance traversed in climbing to a glven altitude-flach number point is small in comparison with the range reaining ond, hence, a gocerhat leas aecurate approximation of horizontal distance is acceptable.
(U) Investigationt huve revealed that the Rutowsh approximations are extrenely good oncs. That is, in part, due to the fact that fuel and distance errors tend to compensate for each other.
 atranarized and ofll be diacused hee to support these remarks.
(6) The firat exomple attosts to the accuracy of the Putowsid approximb-
 $H=1.854$ at 44,900 feet $\left(E_{1}=95,000\right.$ feet $)$, with an Initial weight of 40,392 pounde. The Brysormelley parh indtated 4,017 pounds of fuel cmanmed and 60.4 natical wiles traversed, corpared with 3,993 pounds of fyel nonsumed and 64.8 noutical alles traversed vis the Rutomoki progrer for the exwe cose, A
difference of $2^{i s}$ pourds ( 0.68 ) and 3.6 miles ( 5.3 ) exata betrern the Brysorm Kelley and the Rutowni patho. The Brybon-Xelley path requifed 92 notnuter of computer time versus 1.4 minutes for the futomki puth.
(U) The following cases fllustrate the effecta on fuel coribloned and ditu tance traversed wien the taminal ( $h, H$ ) ILes above or below the Rutoweki path at $E_{0}=95,000$ feet. Since the gubsonic portions of the patha for the $F-40$ were identical, orly the superaonic portione nerc considered. The ame state ment appLies to the MiG-21 pathb. Figure 1I-1, page 46, indestea the general ghape of the patha in the altitudemach number plane.
(6) Whree Bryeon-Kalley paths each were run for the F-HC and the mile-21 frow $H=1.0$ at 39,000 feet and $M=1.0$ at 44,700 feet, setpectively, to $\varepsilon_{\text {, }}=95,000$ feet. Tesminal conditions are given in Table IILlu,


NASC/ACAA |
DECLASSIFY
(This information no longer needs to be classified)

| Case | Armaft | Altitude (ft) | Macr ${ }^{\text {No. }}$ |
| :---: | :---: | :---: | :---: |
| 1 | F-40 | 36,900 | 1.997 |
| 2 | F-46 | 44,900 | 1.854 |
| 3 | C-4c | 52,900 | 1.700 |
| 4 | KICOI | 39,800 | 1.946 |
| 5 | HIC.21 | 42,800 | 1.592 |
| 6 | KIC-2] | 50,000 | 1.741 |
| - |  |  |  |

(t) Cases 1, 2, and 5 ternm"ted 8,000 feet below, on, and 8,000 feet above the Rutorixi path at $\mathrm{I}_{4}=95,000$ feet, reppectively. Caser 4,5 , and 6 terminvted 3,000 feet below, on, and 8,000 feet fiove the futowki path at $\mathcal{E}_{\mathrm{A}}=95,000$ feet, respectively. Plaeard constrainta prevented the termind tion of care 4 below 39,000 feet.
(S) Table 1II-2 presents a comprifan of thr Bryaon-kelley and Rntowsi pathe. Thit table depicts fuel conamed for the Brymon-Xelley path, $\mathrm{fc}_{\mathrm{BK}}$, and for the Rutouski path, fcR; hordzontri datance over tha gromd for the Bryoon-kelley puth, $x_{B X}$, apd for the liutowixi path, $x_{\mathrm{g}}$. Alno show are the weight differences, $\delta v$, and percentage weight differenced, $\$ \Delta y$, betwen the Bryanmalley ond Putowsil paths, well an the liatance differences, ax, and purcentage diatance differmees, \$bx. All welahta are in pound, and nil distances are fn matical miles.


Figure III-1. Rutoubki Path Compariaon Schupe.

TAGLE III-2, COHPARISOH OF ERYSOH-KELEEY AHD RUTOWSKI PRTHS

(V) Observation revesils that the fuel conaumed via the Ratowoti method Ia consiatently less than the ruel consumed via the Brysor-Kelley mathod. The mane consiatency holda, hovever, for the homzontal diotance traversed. The errors introduced by uoing this approximation technique tond to comenante for ench other.
(U) For the entire range comptation, the percentage errore fhon above become inaignificant afnce the climb input in only a part of the tote? input. however, if wore exact conputations for the Range and [-Y Efficiency diagrams are necesaary, the Bryon-helley (or the flemann) pathe can be emloyed in stead of the Rutrmaki paths,

## REFERTNCES

1. Rutowal, E. S., Energy Appronch to the Gensral Alrcraft Performance Problem, Journal of Aeronalitical Science, Volume $\overline{21}, \overline{H o}, \overline{3}$, Harch 1954,
2. Bryson, A. E, and Denhan, W, F, A Steepect Ascent Methad for Solving. Optimun Protreming Pxoblens, Raytheon Comany ER-2393, April 1963.
3. Heeman, $H$. and Kretainger, P., The Minfinum Tiree Problen, Jounal of

4. Boyd, John R., Aerial Atuack Study, Nellis AYB 50-10-6e, Revised 11 August 1964,
5. Tactical Als Comand Manual 3al (teat), Counter Air Interdiction and Cloap Support, Masch $296 \%$.
6. Demitson, J, M., Analytisal Approsoh to T-4 Maneuvering in Alr-to-Atr Conbat, HeDonself Aircraft Corporation E-165, 26 Harch 1965.
7. AIM TD/alh TE Haneuvering Tatcit and Hintman Range Definition Study, Patheor Eampary, Missfle Sybtars Divieiun, BR 3361, 26 4prit 1965.
B. Boyd, Join Re and Chrisile, Thanas Pe, Enorgy-Honeuverability Theory, APCC-7DR-64-35, Hay 1964,
8. Boyd, John R and Christie, Thomas P., Energy-Hanteuverablity Theory and Applicationg, Paper for wath Annul Air Fonce Sca ence and Enginepring 5ymposium, 9 october 1965.

1

## CONFIDENTIAL Export Control

[This page is intentionally left blank.]

# CONFIDENTIAL <br> Export Control 

[ This page is intentionally left blank.]

## CONFIDENTIAL

## Export Control



