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EXPERIMENTAL MODAL ANALYSIS AND DYNAMIC COMPONENT SYNTHESIS

VOL I - Summary of Technical Work

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This report is one of six reports that represent the final technical report on the work involved with United States Air Force Contract F33615-83-C-3218, Experimental Modal Analysis and Dynamic Component Synthesis. The reports that are part of the documented work include the following:

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- Vol. I Summary of Technical Work,
- Vol. II Measurement Techniques for Experimental Modal Analysis,
- Vol. III Modal Parameter Estimation,
- Vol. IV System Modeling Techniques,
- Vol. V Universal File Formats,
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SUMMARY

Volume I introduces the work contained in Volume II through Volume VI of this Technical Report. This includes a state-of-the-art review in several areas connected with experimental modal analysis and dynamic component synthesis. It comprehends frequency response measurement techniques, experimental modal analysis methods, modal parameter estimation, modal modeling, sensitivity analysis, and component mode synthesis. All discussion and development of this material is documented using a consistent set of nomenclature. Several new modal parameter estimation algorithms and a new superelement component dynamic synthesis method were developed as part of this effort.

With respect to the material contained in Volume II, this report reviews the area of measurement techniques applicable to experimental modal analysis. Primarily, this is concerned with the accurate measurement of frequency response functions on linear, time invariant, observable structural systems. When attempting to experimentally determine the dynamic properties (natural frequency, damping, and mode shapes) of a structure, one of the most important aspects is to collect and process data that represent the structure as accurately as possible. These data can then be used as input to a number of parameter estimation algorithms and could also be used in modal modeling algorithms. Volume II of this Technical Report describes in detail the procedures used to collect the data. Many of the potential errors are discussed as well as techniques to eliminate or reduce the effects of these errors on the quality of the results. If the steps described in this Technical Report are followed, data can be collected, as input to modal parameter estimation algorithms, that will yield accurate dynamic properties of the test structure. With care and attention to theoretical limitations, these dynamic properties can be used to construct a modal model.

Regarding the material contained in Volume III, this report documents the area of modal parameter estimation in terms of a review of efforts - over the past twenty-five years - in developing several new multiple reference methods, and in attempting to provide a common basis and understanding for all of the modal parameter estimation procedures developed to date. The summary of modal parameter estimation includes a substantial literature examination and the presentation of earlier methods, such as the Least Squares Complex Exponential, as special cases of general techniques, such as the Polyreference Time Domain method. Several new modal parameter estimation methods are developed and presented using consistent theory and nomenclature. The methods that are described in this manner include: Polyreference Time Domain, Polyreference Frequency Domain, Multiple Reference Ibrahim Time Domain, Multiple Reference Orthogonal Polynomial, and Multi MAC. These techniques, in terms of general characteristics, are also compared to others such as the Least Squares Complex Exponential, Ibrahim Time Domain, Eigensystem Realization Algorithm, and Direct Parameter Estimation methods. These methods are all similar in that they involve the decomposition of impulse response functions (time domain), frequency response functions (frequency domain), or forced response patterns (spatial domain) into characteristic functions in the appropriate domain. These characteristic functions are the single degree of freedom information in the respective domain.

Concerning the material contained in Volume IV, this report lines out the theoretical basis for the current methods used to predict the system dynamics of a modified structure or of combined structures based upon a previously determined, modal or impedance, model of the structure(s). The methods reviewed were: Modal modeling technique, local eigenvalue modification, coupling of structures using eigenvalue modification, complex mode eigenvalue modification, sensitivity analysis, impedance modeling technique, building block approach, dynamic stiffness method, and the frequency response method. The effects of measurement errors, modal parameter estimation error, and truncated modes in the application of modal modeling technique are evaluated. Some of the experimental modal model validation methods are also presented. Several methods to normalize the measured complex modes were reviewed including both time domain and frequency domain techniques. A new component mode synthesis method (Superelement Component Dynamic Synthesis) developed by the University of Dayton Research Institute is presented.

PREFACE

This volume is one of six Technical Reports that represent the final report on the work involved with United States Air Force Contract F33615-83-C-3218, Experimental Modal Analysis and Dynamic Component Synthesis. The reports that are part of the documented work include the following:

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For a complete understanding of the research conducted under this contract, all of the Technical Reports should be referenced.

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1. OVERVIEW

1.1 Introduction

Experimental modal analysis is utilized in a variety of applications in the development of aircraft and aerospace systems. Initially, only a qualitative comparison of the experimentally derived parameters with the analytically derived parameters was considered sufficient for verification of the analytical model. Currently, it is desirable to be able to correct, refine, or even define the analytical model from the experimental data. Experimentally derived structural dynamic models are frequently desired for the calculation of the vibrational responses or the dynamic stability resulting from known input forces to the structure or the determination of the input forces or loads, once the operational displacements, velocities, or accelerations are available. Other applications of experimental modal analysis involve the prediction of modal parameters of the complete structure when only the modal parameters of the individual structural components are known from test. This may include structural and component modifications to obtain desired dynamic properties of the total structure and may involve active vibration control systems.

Experimentally derived information concerning structural and generalized mass and stiffness for each degree of freedom is also required for the purpose of verification of the analytical model. These requirements dictate a compatibility of the experimental modal analysis theory, test configuration, and modal parameter estimation algorithms with the analytical modeling approach. This concept necessitates coordination between the test and the analysis. However, the overriding consideration, in terms of utilizing experimental results in the evaluation of the analytical model is the accuracy of the experimental modal analysis approach. This requires that the experimental modal analysis approach be of primary concern and that the analytical modeling approach must conform to the experimental procedures that result in the best possible accuracy.

This document (Volume I) serves to overview the technical material presented in Volume II through Volume VI of this Technical Report. Section 2 though Section 6 of this Volume provide an overview of Volume II through Volume VI, respectively. Complete technical details are provided only in the individual Volumes.

1.2 Program Objective

The objective of this effort is a refinement of the experimental modal analysis approach with the particular constraint of applicability to structural modeling approaches including direct dynamic modeling, model verification, model perturbation, and component synthesis. This refinement of experimental modal analysis can be based upon experimental parameters with minimized errors and predictable error bounds. This broad objective has been attempted in relation to four major tasks and several subtasks as defined originally by the U.S. Air Force.

- Modal Parameter Identification
  - State-of-the-Art Review
  - Experimental Procedure
  - Modal Parameter Estimation

- Generalized Parameter Estimation
- Structural Parameter Estimation
- Implicit Force Methods

- Experimental-Analytical Coordination
- Dynamic Component Synthesis
  - Modal Synthesis
  - Modal Sensitivity Analysis
  - Structural Modification
  - Modal Truncation
- Software Development

Two specific goals included an updated state-of-the-art bibliography review in each of the technical areas included in this effort and a development of a consistent nomenclature that would be used to present and review technical material in all the areas covered by this effort. The results of both of these goals are summarized at the end of this document.

1.3 Program Team

The University of Cincinnati, in order to perform effectively and respond to all technical requirements identified under this research effort, decided to approach this effort on a team basis with another research facility. The program team consists of the University of Cincinnati Structural Dynamics Research Laboratory (UC-SDRL) and the University of Dayton Research Institute (UDRI).

1.4 Program Team Experience

The University of Cincinnati Structural Dynamics Research Laboratory (UC-SDRL) has been involved in numerous investigations involving experimental modal analysis. The previous studies include involvement as a subcontractor to The Boeing Company on the investigation entitled *Improved Ground Vibration Test Method* and involvement as contractor on the investigation entitled *Simultaneous Multiple Random Input Study*. The particular interest in almost every study has been the sensitivity of each portion of the experimental modal analysis approach to refinement and the evaluation of the sources of error that contribute to invalid estimates of modal parameters. Particular examples of this approach has led to the development of the modal assurance criterion, global least squares modal parameter estimation, and the use of multiple inputs in the estimation of frequency response functions.

1.5 Program Considerations

This research program encompassed investigations of experimental modal analysis and structural modeling approaches which are optimized in terms of data base organization, error minimization, and accuracy assessment concerned with estimated structural and dynamic parameters. The research program involved software development as a necessary element of this evaluation. The approaches that have been investigated will be applicable to full scale aircraft and aerospace structures, dynamically scaled models, structural sections removed from the total structure with different boundary conditions, and the modification of the dynamic properties of substructures or components of the total structure.

The experimental modal analysis approaches investigated will begin to permit the identification of the sources of error. The sources of error will include all forms of measurement and data processing error as well as deviations from the theory involved as a basis for the experimental and analytical methods. One obvious example of this is the identification of nonlinearities which cause significant inaccuracy due to deviation from the linear models used in almost every experimental or analytical approach. Particular experimental procedures were reviewed and a new procedure has been developed to detect the presence of nonlinearities, identify the general characteristic of the nonlinearities, and to minimize the effects of the nonlinearities in the presence of a linear model.

The amount of time permitted for the experimental test is frequently a limiting factor of the accuracy of the modal parameters that are estimated with any experimental modal analysis approach. While this time constraint is often a function of particular experimental test instrumentation and analysis equipment, methods that allow for shortened or minimal experimental test requirements have been considered favorably.

Test instrumentation and analysis equipment, while not of primary concern in this study, has been considered in terms of general criteria that affect the quality of the resulting experimental data. Some of the criteria that will be considered are as follows: computer word size, analog-to-digital conversion word size, autoranging transducer amplification, transducer calibration, actual versus effective dynamic range, system noise, error generation, error reduction, error accumulation, parallel or multiplexed signal acquisition, input-output flexibility, data processing, analysis and storage capabilities, software generation, and etc.

The research has been primarily based upon the *frequency response function* approach to experimental modal analysis. This approach will include the estimation of frequency response functions from single or multiple inputs. Other experimental modal analysis approaches that are not based upon frequency response functions will be investigated, particularly damped complex exponential approaches that can conveniently be modified to handle impulse response function data.

Applicable software required for evaluation of experimental modal analysis methods, error determination, accuracy evaluation, and dynamic modeling method will be developed in the appropriate computer system during the research phases of the proposed work. Software that contributes to the goals of this research proposal will be made available in the HP-5451-C Fourier System, either under Basic Control System (BCS) or Real Time Executive IV (RTE-IV) environments. Only software that must be available during the actual acquisition of the data will be developed in the BCS environment. Analysis oriented software will be made available under the RTE-IV environment compatible with the RTE Modal Program currently in use by UC-SDRL and by Eglin AFB. While most structural modeling software can also be made available in the RTE-IV environment, including modal modification and sensitivity software, the structural modeling software such as component mode synthesis using modal or impedance models may only be operational on larger computer systems. If this situation occurs, magnetic tape formats for required data bases will be developed and supplied.

2. MEASUREMENT TECHNIQUES - EXPERIMENTAL MODAL ANALYSIS

2.1 Introduction

The most fundamental phase of any experimental analysis is to acquire data that are relevant to defining, understanding, and solving the problem. When attempting to define a structure dynamically (usually in terms of impedance functions or in terms of natural frequencies, damping ratios, and modal vectors), this normally involves measuring a force input to the structure and the system response to that input as either displacement, velocity or acceleration (all of which are related through differentiation and/or integration).

These data are sometimes observed, measured and analyzed in the time domain using equipment as simple as a volt meter and an oscilloscope and forcing functions that are well defined such as single frequency sine waves. The natural frequencies are estimated by observing peaks in the response amplitude. Damping can be estimated by a log decrement equation and mode vectors are estimated by measuring the response at various points of interest on the structure. Phase resonance testing, or forced normal mode testing, used extensively in the aircraft industry, is a refined version of this approach.

Advances in hardware and software allowed for the computation of the fast Fourier transform (FFT), the single input, single output frequency response function and the ability to use these measured and stored frequency response functions as inputs to parameter estimation algorithms which could "automatically" estimate natural frequency, damping, and mode shapes and even display "animated" mode shapes on display terminals. The digital computer, mass storage medium, and the FFT allowed band limited random noise to be used as the forcing function so that the structure could be tested faster and the data analyzed or re-analyzed at a later time. But these new techniques also caused many potential errors, particularly signal processing errors.

In recent years, more advances in the speed, size and cost of mini-computers and other test related hardware have made multi-input, multi-output frequency response function testing a desirable testing technique.

Volume II of this Technical Report is concerned with the measurement techniques that are widely used in experimental modal analysis. Although some history is presented, a more complete history can be found by reviewing the literature identified in the Bibliography that is offered as part of this report. Also, some present research in the areas of frequency response function estimation, multiple input considerations, and non-linear vibration considerations is cited.

2.2 Modal Test Objectives

The objectives of a modal test are to make measurements that, as accurately as possible represent the true force input and system response so that accurate frequency response functions are computed. These frequency response functions are the input to parameter estimation algorithms. If the data used as input to these algorithms are not accurate, the parameters estimated by the algorithms are also not accurate.

2.3 Terminology

Throughout this report, the nomenclature will follow, as close as possible, the nomenclature found at the end of this report. Any exceptions will be noted at the time they are introduced.

One potential point of confusion is the concept of system degree of freedom versus a measurement degree of freedom.

A system degree of freedom is the more classical definition of the number of independent coordinates needed to describe the position of the structure at any time with respect to an absolute coordinate frame. Therefore, every potential physical point has six (three linear and three rotational) degrees of freedom. Therefore, the structure has an infinite number of system degrees of freedom. While the theoretical number of system degrees of freedom is infinite, the number of system degrees of freedom can be considered to be finite since a limited frequency range will be considered. This number of system degrees of freedom in the frequency range of interest is referred to in the following sections as N.

A measurement degree of freedom is a physical measurement location (both in terms of structure coordinates as well as measurement direction) where data will be collected. Therefore, for a typical modal test, the number of measurement degrees of freedom will not necessarily be related to the number of system degrees of freedom. It is apparent that the number of measurement degrees of freedom must be at least as large as the number of system degrees of freedom. In general, since three translational motions are measured at every physical measurement location and since these physical locations are distributed somewhat uniformly over the system being tested, the number of measurement degrees of freedom will be much larger than the number of system degrees of freedom expected in the frequency range of interest. This, though, does not guarantee that all modal information in the frequency range of interest will be found.

The number of measurement degrees of freedom (the number of physical measurement locations multiplied times the number of transducer orientations at each physical measurement location) is referred to in the following sections as m. Note that the number of measurement degrees of freedom can be used to describe input or output characteristics.

2.4 Modal Testing

The basic goal of any modal test is to determine the damped natural frequency, damping, and in most cases mode shapes, of a test structure. These are known as the *modal properties* or *dynamic properties* of a system and are unique to the system and the boundary conditions under which it was tested. In some cases it is also necessary to compute generalized or modal mass and modal stiffness. Therefore, by measuring these dynamic properties, the system is defined The results from the modal test are historically used for one of several purposes. Some of these are:

- Troubleshooting
- Finite element model verification
- Finite element model correction
- Experimental modal modeling
- Experimental impedance modeling

One of the most fundamental aspects of a modal test is to decide what the purpose of the modal test is to be before the test. Too often, the purpose of the test is not stated or is stated too broadly so that the modal test will be compromised from the start.

In all cases, the modal tests start with acquiring data (usually input and output) from the structure. Because of the one to one relationship between the time domain and the frequency domain, the data, which is always measured in the time domain, may be converted to the frequency domain.

In the time domain, free decay data or impulse response functions, $h(t)$, are used in the estimation the dynamic properties.

In the frequency domain, frequency response functions, $H(\omega)$, are estimated. The frequency response function is then the input to a parameter estimation algorithm used to estimate the dynamic properties.

There are also modal parameter estimation methods that do not require that intermediate functions be computed; these methods utilize long time records. Due to practical limitations concerning archival and retrieval of data in this format, these methods are not addressed in this report.

2.4.1 Test Structure Set-up

The first decision that must be made before any data is collected is the test configuration. Since the modal parameters that are estimated are for the test structure in the configuration in which it is tested, the test structure should be in a configuration that, as close as possible, represents the desired data. This means that the boundary conditions are an important consideration when setting up the test. If the structure is in a free-free configuration, then the modal parameters estimated are for the free-free case. This is especially important when attempting to verify a finite element model. If the structure is tested in a configuration that is different from the configuration that was modeled, there is no chance of correlation. Since the modal parameters that are estimated are for that configuration, a structure may need to be tested more than once to completely define the structure in its various operating configurations.

Also in this initial phase of the test, the points to be tested are identified, marked, and measured in physical coordinates. In most cases, the physical points and associated directions where acceleration (displacement) is to be measured are selected to give physical significance to the animation. But it is important that any critical points that need to be measured are also identified.

Another factor that may need to be considered at this time is the ability to access the measurement degrees of freedom that need to be tested. This may require some ingenuity so that the test configuration is changed as little as possible during the data collection.

2.4.2 Hardware Set-up

In all cases, it is only possible to *estimate* the dynamic properties of the system. This is directly a result of only being able to *estimate* "true" inputs and responses of the system. It is therefore imperative that the "best" possible data is collected.

For single input, single output frequency response function testing, a force input to the system must be measured as well as the system response to that input.

One of the first decisions to be made is the frequency range (f_{min} to f_{max}) for the test. The frequency range must, of course, include all important modes that are to be identified. But, because of the constraints of the parameter estimation algorithms, the number of modes (modal density) should be kept to a minimum. This may mean that more than one test, using different frequency ranges, needs to be conducted. Many times, the test frequency range cannot be determined until initial measurements have been made. An important consideration is that, when using modal modeling techniques, it is important to identify modes that are higher or lower in frequency than the test frequency. This will yield more accurate modal models.

Next, the type of excitation and the form of the forcing function must be selected. Sometimes, the structure may determine the type of excitation. Other times, the use of the data may determine the excitation. If the purpose of the test is troubleshooting, an impact test may be the best form of excitation. If a modal model is to be built, more precise input must be used.

If an impact test is to be conducted, the size of hammer and hardness of the impact surface must be selected. This will determine the frequency range of the usable frequency response.

If a shaker is to be used to excite the structure, a forcing signal needs to be selected. This could include sine, pure random, periodic random, or burst random as well as others that may more closely match operating conditions. Section 4 of Volume II of this Technical Report presents many common excitation signals and their strong and weak points.

The force input location(s) must be selected to excite all the important modes in the frequency range to be tested. For multi-input testing, there are other constraints that must be satisfied. Section 6 of Volume II of this technical report has a complete review of these constraints.

In a typical test, load cells are used to measure the force input and accelerometers to measure acceleration values which can be related to displacement.

The transducers generally have their own power supply and signal conditioning hardware. Accelerometers need to be selected such that they have sufficient sensitivity but also low mass to make measurements of acceleration that accurately define the acceleration of the structure at that point.

These signals, force and acceleration, are then passed through low-pass anti-aliasing filters, analog-to-digital converters, and into the analysis computer.

The computer calculates fast Fourier transforms and all necessary auto and cross spectra needed to compute a single frequency response function. This is normally stored to disc and another output selected.

In the case of multiple inputs, many of the potential errors arise from the additional hardware needed to collect the data required to compute frequency response functions. The potential "bookkeeping" to be certain that the correct auto and cross spectra are being used in the computations can in itself be bothersome. A two input, 6 response test necessitates the calculation of 8 auto spectra and 13 cross spectra, each with a real and imaginary part, in addition to the 12 frequency response functions that are estimated in one acquisition session. There are also 2 load cells, 6 accelerometers, 16 cables, 8 transducer power supplies, 2 exciter systems, 2 signal generators, 8 anti- aliasing filters, and 8 ADC channels that all have the possibility of failure during the test. It is therefore important to have a technique to check various components used in the test at selected intervals. Most of these errors can be eliminated by good measurement practice.

2.4.3 Initial Measure nents

Once the structure is defined and an input point(s) selected, it is necessary to take initial measurements to be certain that the input point excites the structure reasonably well over the analysis range and to be certain that all hardware is operating properly.

Usually, the "driving point" is measured first. This is because the form of the driving point measurement is well known and defined and also because this should be a "clean" measurement. In a driving point frequency response function, all peaks in the imaginary part should be of the same sign (positive or negative), each resonance should be followed by an anti-resonance, and all circles in the Argand plot lie on the same half of the plane. Many potential problems can be averted based on this one measurement.

Once the driving point measurement is satisfactory, measurements at remote points are made. This will ensure that the structure is satisfactorily excited at all points for that force level. In a typical test, the level of excitation is not changed over the duration of the test. In fact, if the structure is highly non-linear, this would make the analysis overly complicated.

2.4.4 Non-linear Check

Another important step in a successful modal test is to check for linearity. The basic theory of modal analysis requires a linear structure. Seldom is the structure under test linear over all but a limited force range. Linearity is easily checked by exciting the structure at various force levels. If a shift in natural frequency occurs for different force levels, the structure exhibits some form of non-linear stiffness. If the amplitude of the frequency response function changes, the structure exhibits non-linear damping. Section 7 of Volume II of this technical report is an extensive review of non-linear considerations and of non-linear detection methods.

2.4.5 Modal Test

Once the initial set up is complete, the actual testing phase is simply a process of collecting, processing, and storing the relevant information. This data will then be used in parameter estimation algorithms and potentially modal modeling algorithms. For an in-depth review of these areas, other technical reports, found in the preface, should be consulted.

2.5 Modal Data Acquisition

Acquisition of data that will be used in the formulation of frequency response functions or in a modal model involves many important technical concerns. One primary concern is the digital signal processing or the converting of analog signals into a corresponding sequence of digital values that accurately describe the time varying characteristics of the inputs to and responses from a system. Once the data is available in digital form, the most common approach is to transform the data from the time domain to the frequency domain by use of a discrete Fourier transform algorithm. Since this algorithm involves discrete data over a limited time period, there are large potential problems with this approach that must be well understood.

2.5.1 Digital Signal Processing

The process of representing an analog signal as a series of digital values is a basic requirement of modern digital signal processing analyzers. In practice, the goal of the analog to digital conversion (ADC) process is to obtain the conversion while maintaining sufficient accuracy in terms of frequency, magnitude, and phase. When dealing strictly with analog devices, this concern was satisfied by the performance characteristics of each individual analog device. With the advent of digital signal processing, the performance characteristics of the analog device is only the first criteria of consideration. The characteristics of the analog to digital conversion now become of prime importance.

This process of analog to digital conversion involves two separate concepts, each of which are related to the dynamic performance of a digital signal processing analyzer. *Sampling* is the part of the process related to the timing between individual digital pieces of the time history. *Quantization* is the part of the process related to describing an analog amplitude as a digital value. Primarily, sampling considerations alone affect the frequency accuracy while both sampling and quantization considerations affect magnitude and phase accuracy.

2.5.2 Transducer Considerations

The transducer considerations are often the most overlooked aspect of the experimental modal analysis process. Considerations involving the actual type and specifications of the transducers, mounting of the transducers, and calibration of the transducers will often be some of the largest sources of error.

Transducer specifications are concerned with the magnitude and frequency limitations that the transducer is designed to meet. This involves the measured calibration at the time that the transducer was manufactured, the frequency range over which this calibration is valid, and the magnitude and phase distortion of the transducer, compared to the calibration constant over the range of interest. The specifications of any transducer signal conditioning must be included in this evaluation.

Transducer mounting involves evaluation of the mounting system to ascertain whether the mounting system has compromised any of the transducers specifications. This normally involves the possibility of relative motion between the structure under test and the transducer. Very often, the mounting systems which are convenient to use and allow ease of alignment with orthogonal reference axes are subject to mounting resonances which result in substantial relative motion between the transducer and the structure under test in the frequency range of interest. Therefore, the mounting system which should be used depends heavily upon the frequency range of interest and upon the test conditions. Test conditions are factors such as temperature, roving or fixed transducers, and surface irregularity. A brief review of many common transducer mounting methods is shown in Table 1.

Transducer calibration refers to the actual engineering unit per volt output of the transducer and signal conditioning system. Calibration of the complete measurement system is needed to verify that the performance of the transducer and signal conditioning system is proper. Obviously, if the measured calibration differs widely from the manufacturers specifications, the use of that particular transducer and signal conditioning path should be questioned. Also, certain applications, such as impact testing, involve slight changes in the transducer system (such as adding mass to the tip of an instrumented hammer) that affect the associated calibration of the transducer.

Ideally, on-site calibration should be performed both before and after every test to verify that the transducer and signal conditioning system is operating as expected. The calibration can be performed using the same signal processing and data analysis equipment that will be used in the data acquisition. There are a number of calibration methods which can be used to calibrate the transducer and signal conditioning. Some of these methods yield a calibration curve, with magnitude and phase, as a function of frequency while other methods simply estimate a calibration constant. Most of the current calibration methods are reviewed in Table 2. Note that some of the methods are more suited for field calibration while other methods are more suited for permanent installations in calibration laboratories [4-9].

2.5.3 Error Reduction Methods

There are several factors that contribute to the quality of actual measured frequency response function estimates. Some of the most common sources of error are due to measurement mistakes. With a proper measurement approach, most of this type of error such as overloading the input, extraneous signal pick-up via ground loops or strong electric or magnetic fields nearby, etc., can be avoided. Violation of test assumptions are often the source of another inaccuracy and can be viewed as a measurement mistake. For example, frequency response and coherence functions have been defined as parameters of a linear system. Nonlinearities will generally shift energy from one frequency to many new frequencies, in a way which may be difficult to recognize. The result will be a distortion in the estimates of the system parameters, which may not be apparent unless the excitation is changed. One way to reduce the effect of nonlinearities is to randomize these contributions by choosing a randomly different input signal for each of the n measurements. Subsequent averaging will reduce these contributions in the same manner that random noise is reduced. Another example involves control of the system input. One of the most obvious requirements is to excite the system with energy at all frequencies for which measurements are expected. It is important to be sure that the input signal spectrum does not have "holes" where little energy exist. Otherwise, coherence will be very low, and the variance on the frequency response function will be large.

Assuming that the system is linear, the excitation is proper, and obvious measurement mistakes are avoided, some amount of noise will be present in the measurement process. Noise is a general designation describing the difference between the true value and the estimated value. A more exact designation is to view this as the total error comprised of two terms, variance and bias. Each of these classifications are merely a convenient grouping of many individual errors which cause a specific kind of inaccuracy in the function estimate. The variance portion of the error essentially is Gaussian distributed and can be reduced by any form of synchronization in the measurement or analysis process. The bias or distortion portion of the error causes the expected value of the estimated function to be different from the true value. Normally, bias errors are removed if possible but, if the

Method	Frequency range (Hz)	Main advantage	Main disadvantage
Hand-held	20–1000	"Quick look"	Poor measuring quality for long sample periods
Putty	0–200	Alignment, ease of mounting	Low-frequency range, creep problem during measurement
Wax	0–2000	Ease of application	Temperature limitations, frequency range limited by wax thickness, alignment
Hot glue	0–2000	Quick setting time, good alignment	Temperature-sensitive transducers
Magnet	0–2000	Quick setup	Requires magnetic material, alignment, bounce a problem during impact, surface preparation important
Adhesive film	0–2000	Quick setup	Alignment, flat surface
Epoxy-cement	0–5000	Mounts on irregular surface, alignment	Long curing time
Stud mount	0–10,000	Accurate alignment if carefully machined	Difficult setup
		Approximate freq ranges, depends on transducer mass, and contact conditions	

TABLE 1. Transducer Mounting Methods

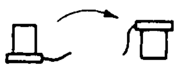
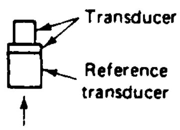
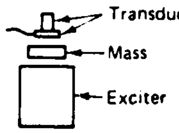
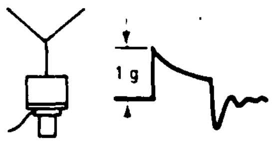
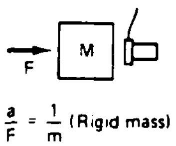
Method			Remarks
Inversion test		Constant	Can only be used with transducer that has stable dc output; calibration against local earth's gravity
Comparison method		Frequency response	Calibration against reference transducer
Reciprocity method		Constant and/or frequency response	Calibration against mass-loaded shaker
Drop method		Constant	Calibration against local earth and gravity; used for ac-coupled transducers
Ratio method		Frequency-response ratio	Calibration against known a/F for a rigid mass

TABLE 2. Calibration Methods

form and the source of a specific bias error is known, many techniques may be used to reduce the magnitude of the specific bias error.

Four different approaches can be used to reduce the error involved in frequency response function measurements in current fast Fourier transform (FFT) analyzers. The use of averaging can significantly reduce errors of both variance and bias and is probably the most general technique in the reduction of errors in frequency response function measurement. Selective excitation is often used to verify nonlinearities or randomize characteristics. In this way, bias errors due to system sources can be reduced or controlled. The increase of frequency resolution through the zoom fast Fourier transform can improve the frequency response function estimate primarily by reduction of the leakage bias error due to the use of a longer time sample. The zoom fast Fourier transform by itself is a linear process and does not involve any specific error reduction characteristics compared to a baseband fast Fourier transform(FFT). Finally, the use of weighting functions(windows) is widespread and much has been written about their value [1-3,10,11]. Primarily, weighting functions compensate for the bias error(leakage) caused by the analysis procedure.

2.6 Excitation Techniques

When exciting a structure to determine its modal properties, it is important to remember that the form of the excitation will have an effect on the validity of the estimates of the modal properties. If the frequency response estimates contain errors, then the estimates of the modal properties will also contain errors. There are many signals that can be used to excite structures for modal testing. Some have many advantages over others. The accuracy of the estimates of the frequency response functions and the time to acquire the data are only some of the differences between the signals.

2.6.1 Excitation Constraints

While there is no well developed theory involving the excitation of structures for the purpose of estimating the frequency response functions, there are a number of constraints that must be considered in order to yield an estimate of the frequency response function that is unbiased [17-23].

The first constraint that is important to the estimation of the frequency response function is concerned with digital signal processing. Since most modern data acquisition equipment is based upon digital data acquisition and Discrete Fourier Transforms, unique requirements are placed on the excitation signal characteristics. This digital approach to processing the input and response signals, with respect to the frequency domain, assumes that starting at a minimum frequency and ending at a maximum frequency the analysis is going to proceed only at integer multiples of the frequency resolution, therefore matching the limits of the Discrete Fourier Transform. Therefore, this constraint first indicates that any excitation signal should only contain frequency information between the minimum and maximum frequency. It also implies that, ideally, either the frequency content should be discrete and located only at integer multiples of the frequency resolution or that the excitation should be a totally observed transient.

Both of these methods match the Discrete Fourier Transform equally well, but there are advantages and disadvantages to both. If the data contains information only at multiples of the frequency resolution, it is impossible to use a zoom Fourier Transform to achieve a smaller frequency resolution on the same excitation function. If a new excitation function is created that contains information only at integer multiples in the zoom band, it is possible to zoom. If the data are a transient, the signal-to-noise ratio may become a problem.

The second constraint that is important to the estimation of the frequency response function is concerned with the requirements of the modal parameter estimation algorithms. A fundamental assumption in modal analysis is that the structure under evaluation is a linear system or at least behaves linearly for some force level. While this is never absolutely true, parameter estimation algorithms are written as though this assumption is valid. With the increasing complexity of the modal parameter estimation algorithms, violation of this characteristic within the frequency response function data base renders these algorithms impotent. Therefore, the modal parameter estimation constraint requires that the excitation signal yield the best linear estimate of the frequency response function even in the presense of small nonlinear characteristics or a significant nonlinear characteristic being evaluated around an operating point.

An additional restriction that is important when using multiple inputs is the requirement that the inputs be uncorrelated. This can be achieved by using deterministic signals, such as sinusoids, with different magnitude, phases, and frequencies for each input during each average involved with the estimation of the frequency response function. Normally, uncorrelated inputs are achieved by using a different random excitation signal for each input. Assuming that a significant number of averages is involved, the use of uncorrelated random signals, is a simple solution to the requirement that the excitation signals be uncorrelated.

2.6.2 Excitation Signals

Inputs which can be used to excite a system in order to determine frequency response functions belong to one of two classifications. The first classification is that of a random signal. Signals of this form can only be defined by their statistical properties over some time period. Any subset of the total time period is unique and no explicit mathematical relationship can be formulated to describe the signal. Random signals can be further classified as stationary or non-stationary. Stationary random signals are a special case where the statistical properties of the random signals do not vary with respect to translations with time. Finally, stationary random signals can be classified as ergodic or non-ergodic. A stationary random signal is ergodic when a time average on any particular subset of the signal is the same for any arbitrary subset of the random signal. All random signals which are commonly used as input signals fall into the category of ergodic, stationary random signals.

The second classification of inputs which can be used to excite a system in order to determine frequency response functions is that of a deterministic signal. Signals of this form can be represented in an explicit mathematical relationship. Deterministic signals are further divided into periodic and non-periodic classifications. The most common inputs in the periodic deterministic signal designation are sinusoidal in nature while the most common inputs in the non-periodic deterministic designation are transient in form.

The choice of input to be used to excite a system in order to determine frequency response functions depends upon the characteristics of the system, upon the characteristics of the parameter estimation, and upon the expected utilization of the data. The characterization of the system is primarily concerned with the linearity of the system. As long as the system is linear, all input forms should give the same expected value. Naturally, though, all real systems have some degree of nonlinearity. Deterministic input signals result in frequency response functions that are dependent upon the signal level and type. A set of frequency response functions for different signal levels can be used to document the nonlinear characteristics of the system. Random input signals, in the presence of nonlinearities, result in a frequency response function that represents the best linear representation of the nonlinear characteristics for a given level of random signal input. For small nonlinearities, use of a random input will not differ greatly from the use of a deterministic input.

The characterization of the parameter estimation is primarily concerned with the type of mathematical model being used to represent the frequency response function. Generally, the model is a linear summation based upon the modal parameters of the system. Unless the mathematical representation of all nonlinearities is known, the parameter estimation process cannot properly weight the frequency response function data to include nonlinear effects. For this reason, random input signals are prevalently used to obtain the best linear estimate of the frequency response function when a parameter estimation process using a linear model is to be utilized.

The expected utilization of the data is concerned with the degree of detailed information required by any post-processing task. For experimental modal analysis, this can range from implicit modal vectors, needed for trouble-shooting, to explicit modal vectors used in an orthogonality check. As more detail is required, input signals, both random and deterministic, will need to match the system characteristics and parameter estimation characteristics more closely. In all possible uses of frequency response function data, the conflicting requirements of the need for accuracy, equipment availability, testing time, and testing cost will normally reduce the possible choices of input signal.

With respect to the reduction of the variance and bias errors of the frequency response function, random or deterministic signals can be utilized most effectively if the signals are periodic with respect to the sample period or totally observable with respect to the sample period. If either of these criteria are satisfied, regardless of signal type, the predominant bias error, leakage, will be eliminated. If these criteria are not satisfied, the leakage error may become significant. In either case, the variance error will be a function of the signal-to-noise ratio and the amount of averaging.

Many signals are appropriate for use in experimental modal analysis. Some of the most commonly used signals are described in Volume II of this Technical Report. For those excitation signals that require the use of a shaker, Figure 1 shows a typical test configuration; Figure 2 shows a typical test configuration when an impact form of excitation is to be used. The advantages and disadvantages of each excitation signal are summarized in Table 3.

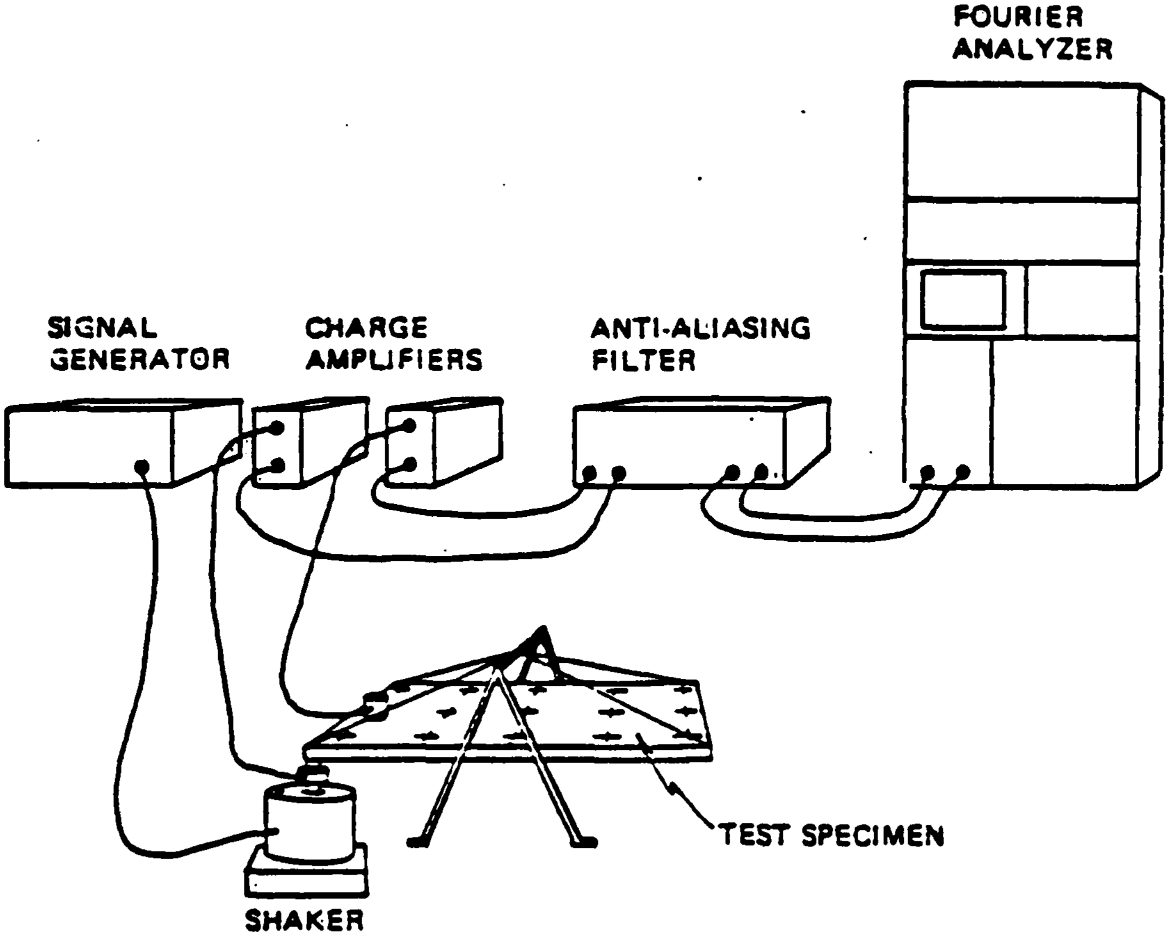


Figure 1. Typical Test Configuration: Shaker

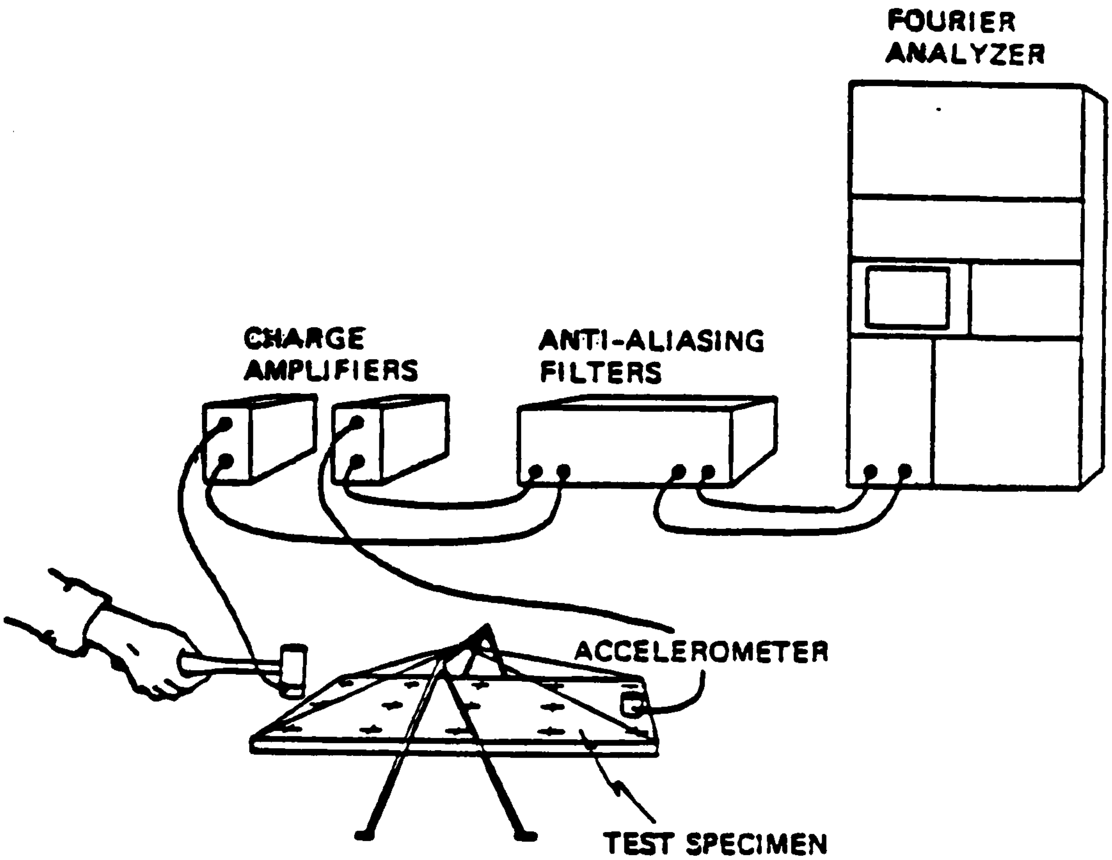


Figure 2. Typical Test Configuration: Impact Hammer

	Type of Excitation							
	Steady State Sine	Pure Random	Psuedo Random	Random	Fast Sine	Impact	Burst Sine	Burst Random
Minimize Leakage	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Signal-to-Noise Ratio	Very High	Fair	Fair	Fair	High	Low	High	Fair
RMS-to-Peak Ratio	High	Fair	Fair	Fair	High	Low	High•	Fair
Test Measurement Time	Very Long	Good	Very Short	Fair	Fair	Very Good	Very Good	Very Good
Controlled Frequency Content	Yes	Yes •	Yes •	Yes •	Yes •	No	Yes •	Yes •
Controlled Amplitude Content	Yes	No	Yes •	No	Yes •	No	Yes •	No
Removes Distortion	No	Yes	No	Yes	No	No	No	Yes
Characterize Nonlinearity	Yes	No	No	No	Yes	No	Yes	No

• Requires special hardware

TABLE 3. Summary of Excitation Signals

2.7 Frequency Response Function Estimation

The theoretical foundation for the estimation of modal parameters has been well documented. Historically, modal testing was first done using the phase resonance, or forced normal mode testing method. Using this method, the structure was forced into a normal mode by a number of single frequency force inputs. The frequency, damping, and modal vector could then be estimated.

With advances in computer technology (both hardware and software), and especially the development of the Fast Fourier Transform, it became practical to estimate frequency response functions for random data. The theoretical foundation for the computation of frequency response functions for any number of inputs has be well documented [1-3,25-29]. A single input, single output frequency response function was estimated for all test points. This greatly reduced test time. But, in order to insure that no modes had been missed, more than one input location should be used.

Starting in about 1979, the estimation of frequency response functions for multiple inputs has been investigated [27,30-36]. The multiple input approach has proven to have advantages over the single input approach. When large numbers of responses are measured simultaneously, the estimated frequency response functions are consistent with each other.

2.7.1 Theory

Consider the case of N_i inputs and N_o outputs measured during a modal test on a dynamic system as shown in Figure 3. Equation 1 is the governing equation.

$$X(\omega) = H(\omega) * F(\omega) \tag{1}$$

For simplicity, the ω will be dropped from the equations. Since the actual measured values for input and output may contain noise, the measured values are:

$$F = \hat{F} - \upsilon$$

and

$$X = \hat{X} - \eta$$

Therefore, a more general model for the computation of frequency response functions for N_i inputs and N_o outputs could be at response location p:

$$\hat{X}_p - \eta_p = \sum_{q=1}^{N_i} H_{pq} * (\hat{F}_q - \upsilon_q) \tag{2}$$

Where:

F $= \hat{F} - \upsilon$ Actual input
X $= \hat{X} - \eta$ Actual output
$\hat{X}_p$ = Spectrum of the p-th output, measured
$\hat{F}_q$ = Spectrum of the q-th input, measured
H_{pq} = Frequency response function of output p with respect to input q
υ_q = Spectrum of the noise part of the input
η_p = Spectrum of the noise part of the output

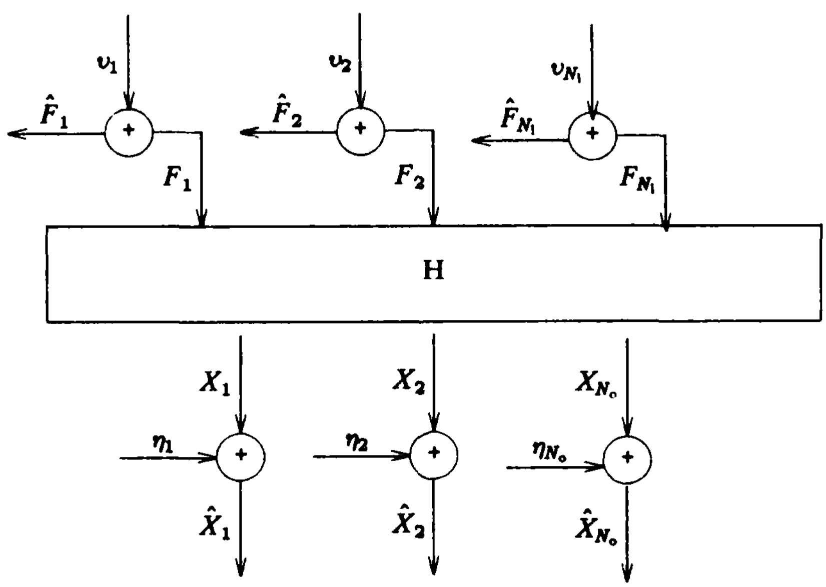


Figure 3. Multiple Input System Model

If $N_i = N_o = 1$, Equation 2 reduces to the classic single input, single output case. With N_i not equal 1, the equation is for the multiple input case.

For the multiple input case, the concept of coherence must be expanded to include ordinary, partial, and multiple coherence functions [30,35]. Each of the coherence functions is useful in determining the validity of the model used to describe the system under test or, as discussed in Section 6 of Volume II of this Technical Report, to evaluate how well the inputs conform to the theory.

Ordinary coherence is defined as the correlation coefficient describing the possible causal relationship between any two signals. Ordinary coherence can be calculated between any two forces or between any force and any response In this calculation, the contribution of all other signals is ignored Therefore, the interpretation of the ordinary coherence functions must be made with great care. The use and interpretation of the ordinary coherence function between forces will be discussed in Section 6 of Volume II of this Technical Report. The ordinary coherence function between an input and an output is of little use in determining the validity of the model This is because the output in the multiple input case is due to a number of inputs so that the ordinary coherence will not have the same useful interpretation as in the single input case.

Partial coherence is defined as the ordinary coherence between any two conditioned signals. The signals are conditioned by removing, in a systematic manner, the contribution(s) of other signals. The order of conditioning has an effect on the degree of correlation. A partial coherence function can be calculated between conditioned inputs, a conditioned output and a conditioned input, or with multiple outputs, between conditioned outputs. Typically, the input and output are conditioned by removing the potential contributions to the output and input from other input(s). The removal of the effects of the other input(s) is formulated on a linear least squares basis. There will be a partial coherence function for every input/output combination for all permutations of conditioning. The usefulness of partial coherence with respect to frequency response function estimation is to determine the degree of correlation between inputs. The use and interpretation of the partial coherence will be discussed in Section 6.

Multiple coherence is defined as the correlation coefficient describing the possible causal relationship between an output and all known inputs. There will be one multiple coherence function for every output. Multiple coherence is used similarly to the ordinary coherence in the single input case. The multiple coherence function should be close to unity throughout the entire frequency range of the estimated frequency response function. A low value of multiple coherence at resonance indicates possible measurement error, unknown inputs, unmeasured inputs, or signal processing errors such as leakage. However, a low value of multiple coherence is not expected at an antiresonance since there should be sufficient signal-to-noise ratio at these frequencies (antiresonance is not a global property of the system).

2.7.2 Mathematical Models

Depending on the where the noise is assumed to enter the measurement process, there are at least three different mathematical models that can be used to estimate the frequency response functions. It is important to remember that the system determines its own frequency response function for a given input/output pair and the boundary conditions for the test. In the limit, if all noise were removed, any estimation technique must give the same result.

2.7.2.1 H_1 Technique

Assuming that there are no measurement errors on the input forces, let the measurement errors on the response signal be represented by $\{\eta\}$. The H_1 least squares technique aims at finding the solution $[H]$ of Equation 3 that minimizes the Euclidean length of $\{\eta\}$, the "squared error". This solution is also called the least squares estimate. Writing Equation 2, using all measured values *(the ^ has been dropped for simplicity)* in a form more readily recognized yields [31-34,36]:

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} = \{X\}_{N_o \times 1} - \{\eta\}_{N_o \times 1} \tag{3}$$

The subscripts refer to the size of the matrix. It is well known that the solution $[H]$ can be found as the solution of the set of "normal equations" formed by post multiplying by $\{F\}^H$ [37-39].

$$[H]\{F\}\{F\}^H = \{X\}\{F\}^H - \{\eta\}\{F\}^H \tag{4}$$

$||\eta||_2$ is minimum

Where:

H : complex conjugate transpose (Hermitian)
$||..||_2$: Euclidean norm

Equation 4 can be reduced to Equation 5 by assuming that the noise on the outputs are uncorrelated with the inputs and that with sufficient averages, the normalized noise spectra are close to zero.

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} \{F\}^H_{1 \times N_i} = \{X\}_{N_o \times 1} \{F\}^H_{1 \times N_i} \tag{5}$$

The elements of the coefficient matrix and right hand matrix in Equation 5 are readily identified, when expanded, with the auto and cross power spectra of input forces and response signals [30-35].

When the matrix multiplications of Equation 5 are expanded to form Equations 6 or 7, the form is more readily recognized as a frequency response function estimation.

$$[H][GFF]=[GXF] \tag{6}$$

or

$$[H]=[GXF][GFF]^{-1} \tag{7}$$

Where:

[H] = Frequency response function matrix

$$= \begin{bmatrix} H_{11} & H_{12} & . & . & H_{1N_i} \\ H_{21} & . & & & . \\ . & . & & & . \\ . & . & & & . \\ H_{N_o1} & . & . & . & H_{N_oN_i} \end{bmatrix}$$

$[GXF]$= Input/output cross spectra matrix

$$= \{X\}\{F\}^H$$

$$= \begin{Bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_{N_o} \end{Bmatrix} [F_1^* \; F_2^* \; . \; . F_{N_i}^*]$$

$*$: complex conjugate

$[GFF]$= Input cross spectra matrix

$$= \{F\}\{F\}^H$$

$$= \begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_{N_i} \end{Bmatrix} [F_1^* \; F_2^* \; . \; . \; F_{N_i}^*]$$

$$= \begin{bmatrix} GFF_{11} & . \; . \; . & GFF_{1N_i} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ GFF_{N_i 1} & . \; . \; . & GFF_{N_i N_i} \end{bmatrix}$$

$GFF_{ik} = GFF_{ki}^*$ (Hermitian matrix)

The ordinary coherence function can be formulated in terms of the elements of the matrices defined previously. The ordinary coherence function between the p-th output and the q-th input can be computed from Equation 8.

$$COH_{pq} = \frac{|GXF_{pq}|^2}{GFF_{qq}\, GXX_{pp}} \tag{8}$$

Where:

GXX_{pp} = Auto power spectrum of the output

The magnitude of the error vector that corresponds to the least squares solution is a measure of how well the response signal is predicted by the input forces. When compared with the magnitude of the response signal, a normalized measure, known as the multiple coherence function, can be defined by Equation 9 [34,35].

$$MCOH_p = \sum_{s=1}^{N_i} \sum_{t=1}^{N_i} \frac{H_{ps}\, GFF_{st}\, H_{pt}^{*}}{GXX_{pp}} \tag{9}$$

Where:

H_{ps} = Frequency Response Function for output p and input s
H_{pt} = Frequency Response Function for output p and input t

For more than 2 inputs, Equation 6 can be expanded, as an example, for six inputs to yield Equation 10. Note that Equation 10 has been put in transposed form in which the frequency response functions appear as a column instead of a row. Equation 10 is recognized as a set of simultaneous equations with the frequency response functions as the unknowns.

$$\begin{bmatrix} GFF_{11} & \cdot\ \cdot & GFF_{61} \\ GFF_{12} & & \cdot \\ \cdot & & \cdot \\ GFF_{16} & \cdot\ \cdot & GFF_{66} \end{bmatrix} \begin{bmatrix} H_{p1} \\ H_{p2} \\ \cdot \\ H_{p6} \end{bmatrix} = \begin{bmatrix} GXF_{p1} \\ GXF_{p2} \\ \cdot \\ GXF_{p6} \end{bmatrix} \tag{10}$$

Equation 10 could be solved for the frequency response functions by inversion of the [GFF] matrix but the computational time and possible dynamic range errors may make the inversion technique undesirable [34,40]. Computational techniques for solution of the equation are discussed in Volume II of this Technical Report.

As before, ordinary coherence functions can be defined between any two forces or any force with the response giving a total of 21 possible ordinary coherence functions. In a systematic way, 4 partial coherence functions between forces can also be defined and one multiple coherence function can be defined by Equation 9. The partial coherence functions are defined and discussed in Section 6 of Volume II of this Technical Report.

2.7.2.2 H_2 Technique

If all measurement errors are assumed to be confined to the inputs, let the errors associated with the inputs be represented by $\{v\}$. The H_2 least squares technique aims at finding the solution $[H]$ of Equation 11 that minimizes the length of $\{v\}$. The basic model for the H_2 technique is shown in Figure 4 [41,42].

$$[H]_{N_o \times N_i} \{ \{F\}_{N_i \times 1} + \{\upsilon\}_{N_i \times 1} \} = \{X\}_{N_o \times 1} \tag{11}$$

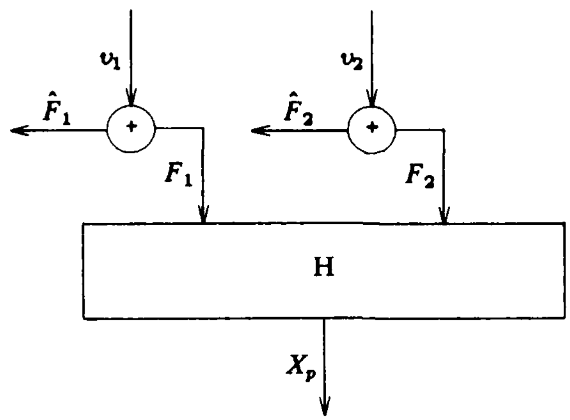


Figure 4. System Model for H_2 Technique

To find the solution of Equation 11 in a similar manner as in the H_1 case, postmultiply by $\{X\}^H$.

$$[H]\{ \{F\} + \{\upsilon\} \} \{X\}^H = \{X\}\{X\}^H \tag{12}$$

If the noise on the input $\{\upsilon\}$ is assumed not correlated with the output and if sufficient averages are taken so that the noise matrix approachs zero, Equation 12 can be written as:

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} \{X\}^H_{N_o \times 1} = \{X\}_{N_o \times 1} \{X\}^H_{1 \times N_o} \tag{13}$$

The elements of the matrices are now identified as the cross power spectra between inputs and outputs and the output auto power spectra.

To investigate the potential uses of the H_2 technique for multiple inputs, it will be helpful to expand the equations for two cases. One is when the number of inputs and the number of outputs are equal and another when the number of outputs is greater than the number of inputs.

For the case of two inputs and two responses, $N_o = N_i = 2$, Equation 13 can be solved for the frequency response functions by inverting the input/output cross spectra matrix at every frequency in the analysis range and solving the set of simultaneous equations. For the case of two inputs and three responses, $N_o = 3$ and $N_i = 2$, Equation 13 suggests that a 3×3 matrix must be multiplied by a 2×3 matrix. For the equation to be valid, a generalized inverse must be used. Therefore, unique frequency response functions cannot be estimated from this set of data. Therefore, an added constraint on the H_2 technique is that the number of outputs must equal the number of inputs [42,43]. For the single input, single output case, this constraint is not a disadvantage. But, for the multiple input technique, this constraint makes the H_2 technique impractical for many testing situations (for example a 2 triaxial response test with 2 inputs). Also, the major advantage of the H_2 technique is to reduce the effects of noise on the input. This can also be accomplished by selective excitation that is investigated in Section 4 of Volume II of this Technical Report or by formulating the H_v frequency response estimate which is better suited for multiple inputs. Therefore, the H_2 technique was not heavily investigated for the multiple input case.

2.7.2.3 H_v Technique

Assume now that measurement errors are present on both the input and the response signals, represented by $\{\upsilon\}$ for the noise on the input and $\{\eta\}$ for the noise on the output. The H_v least squares technique aims at finding the solution $[H]$ in Equation 2 that minimizes the sum of the Euclidean lengths of $\{\eta\}$ and $\{\upsilon\}$, or the "total squared error" [42,44-46]. This solution is referred to as the Total Least Squared estimate. It is proved in the literature that it can be identified with the elements of the matrix $[GFFX]$ defined by Equation 14. Again the elements of this matrix are readily identified with the auto and cross power spectra of input forces and response signals.

$$[H]\{\{F\}-\{\upsilon\}\}=\{X\}-\{\eta\} \tag{13}$$

$$[GFFX]=[\{F\}\{X\}]^H\ [\{F\}\{X\}] \tag{14}$$

$$[GFFX]=\begin{bmatrix}[GFF] & [GFX]\\ [GFX]^H & GXX\end{bmatrix} \tag{14}$$

The matrix $[GFFX]$ is Hermitian; its eigenvalue decomposition is therefore defined by Equation 15. The Total Least Squared estimate for $[H]$ is then defined by Equation 16.

$$[GFFX]=[V]\lceil \Lambda \rfloor [V]^H \tag{15}$$

Where:

$$\lceil \Lambda \rfloor = diag(\lambda_1, \lambda_2,\dots \lambda_m)$$

$$[V]^h[V]=I$$

$$\{H\}=\begin{Bmatrix}-V_{1\ p+1}\,/\,V_{p+1\ p+1}\\ \vdots\\ -V_{p\ p+1}\,/\,V_{p+1\ p+1}\end{Bmatrix} \tag{16}$$

Notice that the Total Least Squares solution does not exist if V_{p+1p+1} equals 0. This however can only happen if the submatrix $[GFF]$ of $[GFFX]$ is singular [44]: that is, if the input forces are correlated. Verifying that the input forces are not correlated is therefore sufficient to warrant the existence of the Total Least Squares solution.

Corresponding to the Total Least Squares estimate, there will be errors on both input forces and response signals. The magnitude of the errors on the response signal can be expressed by Equation 17. If this error is substituted into Equation 18, one calculates a measure of how well the response signal is predicted by the input forces, considering now however also errors on the input forces.

$$G\eta\eta = \lambda_{p+1} V^{*}_{p+1\ p+1} V_{p+1\ p+1} \tag{17}$$

$$MCOH = 1 - \frac{G\eta\eta}{GXX} \tag{18}$$

2.7.2.4 H_s Technique

In a similar fashion, a "scaled" frequency response function has been proposed by Wicks and Vold [47].

Starting with Equation 2 for a single input (the equations can be readily expanded to the multiple input case):

$$\hat{X} - \eta = H^{*}(\hat{F} - \upsilon) \tag{19}$$

Expanding for the single input case and collecting error terms yields:

$$\eta\eta^{*} + (HH^{*})(\upsilon\upsilon^{*}) = (H\hat{F} - \hat{X})(H\hat{F} - \hat{X})^{*} \tag{20}$$

If the error terms of Equation 22 are equal in magnitude, a least squares minimization can be applied to Equation 22. To insure that the magnitudes are equal, either the input or the output can be scaled. Assuming that the input is scaled by S, Equation 21 can be written as:

$$\hat{X} - \eta = HS^{*}(\hat{F} - \upsilon)$$

If the scaling constant is carried throughout the development, an equation can be written for a "scaled" frequency response function.

$$H_S = \frac{(\hat{X}\hat{X}^{*} - S^2\hat{F}\hat{F}^{*}) + \sqrt{(S^2\hat{F}\hat{F}^{*} - \hat{X}\hat{X}^{*})^2 + 4S^2\hat{X}^{*}\hat{F}\hat{F}^{*}\hat{X}}}{2S\hat{X}^{*}\hat{F}} \tag{21}$$

2.7.3 Comparison of H_1, H_2, and H_v

The assumption that measurement errors are confined totally to the input forces or totally to the response signals is sometimes unrealistic. But, it is important to understand why H_1, H_2, and H_v yield different estimates of the same input/output frequency response function for a given system. It is important also to remember that a linear system has only one theoretical frequency response for any given input/output pair. Table 4 compares the different assumptions and solution techniques.

TABLE 4. Comparison of H_1, H_2, and H_v

Technique	Solution	Assumed location of noise	
	Method	Force Inputs	Response
H_1	LS	no noise	noise
H_2	LS	noise	no noise
H_v	TLS	noise	noise

It is also important to realize that if the noise on the inputs $\{\eta\}$ and the noise on the responses $\{\upsilon\}$ are eliminated, H_1 equals H_2 and they are approximately equal to H_v. Therefore, it is important to spend time to acquire data that is noise free and that fits the assumptions of the Discrete Fourier Transform rather than accept the errors and try to minimize their effect by the solution technique.

From the standpoint of frequency response function estimation, the H_1 technique, at resonances, underestimates the height of the peak amplitude and therefore overestimates the damping. In the Argand plane, the circles look "flat". The H_2 technique, at resonances, overestimates the amplitude and therefore underestimates damping. The circles look oblong in the H_2 technique. The H_v technique gives, at resonance, an estimate of the frequency response function that is between the H_1 and H_2 estimates. At antiresonances, the reverse is true, H_1 gives the lowest estimate and H_2 gives the highest estimate with H_v in the middle. Away from resonance, all three give the same estimate. It is important to remember that in all three cases, the value computed is only an estimate of the theoretical frequency response function. If other measurement errors or violation of system assumptions are present, all three estimators will give erroneous results. It is therefore important to spend as much time as possible to reduce known errors before data acquisition begins.

2.8 Multiple Input Considerations

From the theory of multiple input frequency response function estimation, the equations for the computation of the frequency response functions all require that the input cross spectra matrix $[GFF]$ not be singular [1-3,25-30]. Unfortunately, there are a number of situations where the input cross spectra matrix $[GFF]$ may be singular at specific frequencies or frequency intervals. When this happens, the equations for the frequency response functions cannot be used to solve for unique frequency response functions at those frequencies or in those frequency intervals even though the equations are still valid.

One potential reason for the input cross spectra matrix $[GFF]$ to be singular is when one or more of the input force auto power spectrum is zero at some frequency or some frequency interval. If an input has a zero in the auto power spectrum, the associated cross spectrums calculated with that force will also have zeros at the same frequency or frequency interval. The primary reason for this to occur would be because of an impedance mismatch between the exciter system and the system under test. Unfortunately, this situation occurs at system poles that have a low value of damping where a good estimate of the frequency response is desired. Therefore, it is imperative to check the input cross spectra matrix for zeros. For the two input case where the determinant is calculated, a good check is to be sure that the determinant does not have zeros in it.

Another way that the input cross spectra matrix may be singular is if two or more of the inputs are totally coherent at some frequency or over some frequency interval. A good method to check for coherent forces is to compute the ordinary and conditioned partial coherence functions among the inputs [25-30]. A technique is also presented in Section 6.2.2 of Volume II of this Technical Report that computes the principal auto power spectra of the input forces [48]. This technique uses an eigenvalue decomposition to determine the dimensionality of the input cross spectra matrix $[GFF]$. If two of the inputs are fully coherent, then there are no unique frequency response functions associated with those inputs at those frequencies even though the Equation 7 is still valid. This is because the frequency response is now estimated using a singular matrix that will yield infinite solutions that are combinations of each other. Although the signals used as inputs to the exciter system are uncorrelated random signals, the response of the structure at resonance, combined with the inability to completely isolate the exciter systems from this response will result in the ordinary or conditioned partial coherence functions to have values other than zero, particularly, at the system poles. As long as the coherence functions are not unity at any frequency, the equations will give a correct estimate of the frequency response function. It is therefore necessary to have a method to evaluate the inputs to assure that there are neither holes in the auto power spectrum nor perfectly coherent inputs.

2.8.1 Optimum Number Of Inputs

When considering the estimation of frequency response functions in the presence of multiple inputs, more time must be spent to determine the number of inputs, the input directions, and the input locations.

An advantage of the multiple input technology is that, for most structures, all important modes can be excited in one test cycle. For example, in a typical test of an aircraft structure, if existing single input technology is used, at least two complete tests must be conducted in order to get sufficient energy into both the vertical and lateral fuselage modes. If two symmetric, correlated inputs with zero or 180 degree phase difference are used, even though the number of degrees of freedom that the parameter estimation algorithm must deal with is reduced, at least two complete tests must also be conducted to define all the modes of the structure. With uncorrelated random multiple inputs, since there is no constraint on the input directions, one input could be vertical and the other horizontal. In this way, both the vertical and lateral modes will be excited in the same test cycle. By exciting at symmetric locations, the frequency response estimates can be added or subtracted to enhance in phase and out of phase modes. Since the original frequency response estimates are not destroyed, effectively, four pieces of useful information have been estimated for the structure under test in one test cycle.

But, as the number of inputs is increased, so too is the potential for problems with the excitation forces. One such problem is that, due to the structural response, the inputs may be correlated by one or more exciters driving the other exciters. This happens most often if the exciters are placed at locations that have a high amplitude of motion particularly at resonance. Also, depending on the size of the structure, there is a diminishing return on more inputs. The advantage of two inputs to one input has been apparent in almost every test case. For more than two inputs, particularly on smaller structures, the added inputs mean that more averages must be taken to compute "clean" frequency response functions. In practice, fighter aircraft have been tested with as many as six inputs with no adverse effects. For automobiles, three inputs appears to be a practical limit.

2.9 Non-linear Considerations

The vibration of structures is a very natural phenomenon and although much work has been dedicated to the analysis and understanding of it, there exist an infinite number of vibrational problems which cannot be predicted. Because of this, experimental testing is needed to describe the vibrational characteristics of many structures. In this research, a linear system will be defined as a system in which the responses are linearly proportional to the input forces. A nonlinear system will be defined as having responses that are not linearly proportional to the input forces. In the analysis of linear systems, the responses can be predicted and an explicit mathematical model can be generated to represent the physical characteristics of the system. In the analysis of nonlinear systems, the responses can not adequately predict the measured dynamic characteristics.

It is accepted that most real structures exhibit nonlinear characteristics. Practical experience suggest that in many cases, this nonlinear term is negligible and a linear system can be assumed. However, as structures become more complicated and more accurate results are required, the nonlinear component is no longer negligible.

In modal analysis, frequency response functions are calculated based on a linear model of the structure. Thus, it is important to first accurately determine the contribution of the nonlinear components to the system. As described in Figure 5, the contribution of nonlinear components varies from system to system.

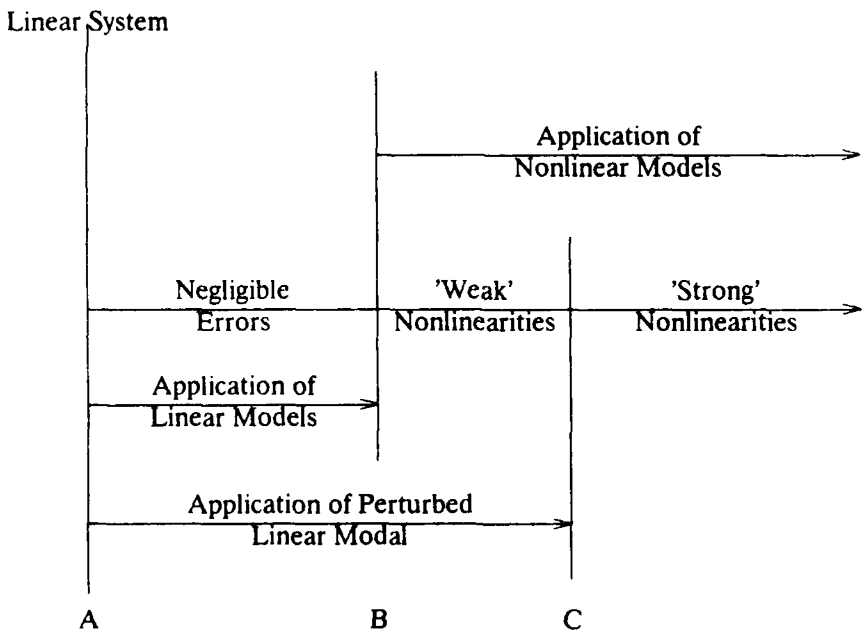


Figure 5. Evaluation of Linear and Nonlinear Systems

At position "A", there exists a totally linear system having no contamination of nonlinear components. Between positions "A" and "B" the system is composed of linear and nonlinear terms; however, the nonlinear component in this case is small enough to be negligible. Between positions "B" and "C", the nonlinear term can no longer be negligible - the system should not be assumed to be linear. Finally, past position "C" the system is considered to highly nonlinear [49].

2.9.1 Objectives

One of the primary objectives of this research is to investigate previous studies dealing with nonlinearities as related to the modal analysis field. A review of current studies which deal with nonlinearities was made in terms of a literature search. Because of the difficulty involved in creating a physical structure with a known nonlinearity, most of the past research has only dealt with basic mathematical nonlinear structures having only one or two degrees of freedom.

Because of the increasing utilization of the multiple input estimation technique, it is the ultimate goal of this research to investigate a method for *detecting* nonlinearities in a system when using the multiple input estimation technique combined with random excitation signals. What is eventually desired is a detection method which could be programmed into the modal acquisition software and accessed to evaluate the degree of nonlinearity within a test structure.

2.9.2 Modal Analysis and Nonlinearities

In the field of experimental modal analysis there are three basic assumptions that are made about a structure. First, the structure is assumed to be time invariant. This means that the modal parameters that are to be determined will be constants of the structure. In general, a structure will have components whose mass, stiffness, or damping depend on factors which are not measured or included in the model. For example, in some structures, the components could be temperature dependent. In this case, temperature could be described as a time varying parameter; therefore, each of the temperature dependent components would be considered to be time varying. Thus, for a time varying structure, the same measurements made at different times would be inconsistent.

The second basic assumption is that the structure is observable. By observable, it is meant that the input/output measurements that are made contain enough information to generate an adequate behavioral model of the structure. For example, if a structure has several degrees of freedom of motion that are not measured, then the structure is considered to be not observable. Such a case would be that of a partially filled tank of liquid when sloshing of the fluid occurs [50].

Finally, the third basic assumption is that the structure is either linear or can be approximated as linear over a certain frequency range. This essentially means that the response of the structure due to the simultaneous application of two or more excitation forces is a linear combination of the responses from each of the input forces acting separately. This relationship is shown in Figure 6. If a particular input signal, $a(t)$, causes an output signal, $A(t)$, and a second input signal, $b(t)$, causes a different output signal, $B(t)$; then, if both input signals, $a(t)$ and $b(t)$, are applied to a linear system, the output signal will be the summation of the individual signals, $A(t)+B(t)$ [51].

2.9.3 Basic Nonlinear Systems

A linear system with several degrees of freedom can be modeled completely by a frequency response function which can be defined as the Fourier transform of the output signal divided by the Fourier transform of the input signal. In mechanical systems, the input signal is a type of force while the

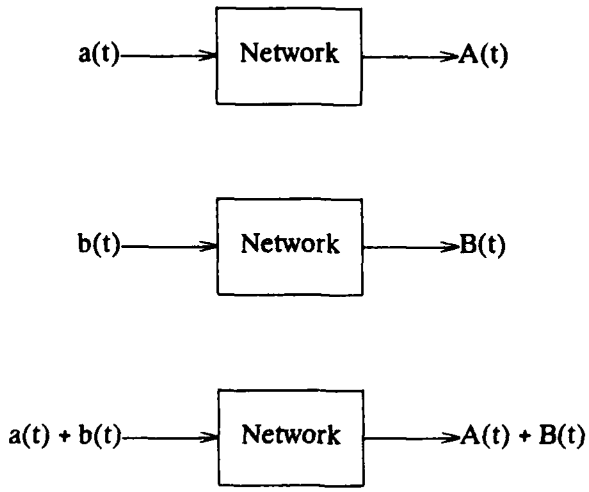


Figure 6. Linear Network

output signal is a quantity such as displacement, velocity, or acceleration. The frequency response function of these different output quantities are referred to as receptance, mobility, or compliance respectively, see Table 5.

TABLE 5. Frequency Response Measurements

Receptance	$\frac{\text{Acceleration}}{\text{Force}}$
Effective Mass	$\frac{\text{Force}}{\text{Acceleration}}$
Mobility	$\frac{\text{Velocity}}{\text{Force}}$
Impedance	$\frac{\text{Force}}{\text{Velocity}}$
Dynamic Compliance	$\frac{\text{Displacement}}{\text{Force}}$
Dynamic Stiffness	$\frac{\text{Force}}{\text{Displacement}}$

As was previously stated, a linear structure can be characterized by its frequency response function and, as long as the structure does not physically change, this function will remain constant. This is not the case for a nonlinear structure; the system is no longer characterized by a single response function. In this case, the structure is very dependent to the time varying variables in the inputs of the system.

Although there exist very linear systems, most real structures have nonlinear components. Practical experience shows that the degree of nonlinearity of a structure varies according to the characteristics of the system. That is, welded structures will usually exhibit a linear response; where a riveted or spot welded structure exhibits a very nonlinear response [52].

However, this linear characteristic property is not found in many systems. In a simple coil spring, a nonlinear behavior will occur when the spring is overly compressed or extended. In either case, the elastic spring will exhibit a nonlinear characteristic such that the spring force increases more rapidly than the spring deformation; this is referred to as a hardening spring. On the other hand, certain systems such as the simple pendulum exhibit a softening characteristic.

Nonlinear behavior in structures can be related to such characteristics as backlash, nonlinear stiffness, nonlinear damping, nonlinear material properties, or friction. These nonlinearities can be classified as "limited" or "nonlimited" nonlinearities. In the "limited" case, the nonlinearity is limited within a particular force level range. Nonlinearities due to backlash could be classified as a "limited" nonlinearity. "Nonlimited" nonlinearities refer to those nonlinearities which are independent of force level. Nonlinear damping is an example of a "nonlimited" nonlinearity [54].

2.9.4 Excitation Techniques

When a structure is to be tested to determine the modal parameters, one of the most important considerations is the excitation method to be used. For a linear structure, the frequency response function is independent of the amplitude and type of the excitation signal. This is not the case for a nonlinear structure; the selection is crucial since the method of excitation can either minimize or enhance the nonlinear behavior of the structure. The different excitation signals can be divided into two classifications; the deterministic excitation signals and the random excitation signals.

The deterministic signal is one which can be described by an explicit mathematical relationship. These signals are then divided into two other classifications, periodic or non-periodic. A signal is periodic if it repeats itself at equal time intervals. Frequency response functions that result from deterministic signals are dependent upon the signal level and type. Therefore, these signals are very useful in detecting nonlinearities in structures. Table 6 gives a summary of the different excitation signals and their classification.

The use of a sine wave, which is a deterministic signal, to excite a structure is very common [55]. The main advantage of a swept sine test is that the input force can be precisely controlled. It is this characteristic that makes this method particularly useful when trying to identify nonlinear systems. If a particular system is nonlinear, by varying the input force levels, one can compare several frequency response functions and identify inconsistencies. A major disadvantage with this method is that it gives a very poor linear approximation of a nonlinear system. This causes a serious problem if the data is to be used to estimate modal parameters. Therefore, this method is adversely affected by nonlinearities.

Another deterministic excitation signal is an impact signal. The impact testing technique is very useful for trouble-shooting and preliminary modal surveys. However, this technique should not be utilized with nonlinear structures because of the difficulties in controlling the impact force and insufficient energy to properly excite the structure [56].

TABLE 6. Excitation Signal Summary

Deterministic Excitation	Random Excitation
Slow Sinusoidal Sweep Fast Sinusoidal Sweep Periodic Chirp Impulse (Impact) Step Relaxation	True (Pure) Random Pseudo Random Periodic Random Burst Random (Random Transient)

The random signal is one which can only be described by its statistical properties over a given time period; no explicit mathematical relationship exists. For structures with small nonlinearities, the frequency response functions from a random signal will not differ greatly from that of a deterministic signal. However, as the nonlinearity in the structure increases, the random excitation gives a better linear approximation of the system, since the nonlinearities tend to be averaged out. The random excitation signal has been increasingly useful since it enables the structure to be investigated over a wide frequency range, unlike the sine wave [9].

2.9.5 Detection of Non-linearities

It is apparent that in physical mechanical systems, there exists a linear and nonlinear response due to some force input. In many cases, this nonlinear behavior can be neglected. However, in other cases, the nonlinear response cannot be ignored. It is and has been essential to perform some type of linearity check in order to make this evaluation. As the use of dynamic models based on experimental data becomes more extensive, the detection and eventually the characterization of nonlinearities becomes even more important.

The work reviewed in Volume II of this Technical Report provides several nonlinearity detection methods which are currently being implemented. Each of these different techniques has certain limitations which effect the accuracy of the detection method. As an alternative detection method, this work researched the possibility of detecting nonlinearities by utilizing higher order terms in conjunction with the multiple input/output estimation theory. By using higher order terms of a measured input force, the nonlinear behavior of a system can be detected. This alternative technique was researched as a fast and valid method to give an indication of the linearity of a system when implementing a random type of excitation signal.

This research demonstrated that for a theoretical single degree-of-freedom system, as the amount of nonlinearity increases, the nonlinear detection functions become more predominant. This preliminary study indicates that this nonlinear detection technique is sensitive to different types of nonlinearities if the correct number of higher order nonlinear terms are utilized. Further research is needed to determine the actual number of higher order terms necessary to accurately model a particular nonlinear system. To further validate this detection technique, the frequency response functions which are estimated utilizing the multiple input theory should be compared to the frequency response functions estimated by a single input/output algorithm. Although initial investigations demonstrated that the utilized forces were uncorrelated in the frequency domain, additional research is needed to study the relationship between these forces. As in most cases, this detection method is only valid for the measured response and input force points; it is still necessary to check several critical points of the structure for linearity and then assume the entire system behaves accordingly. The higher order detection technique does however eliminate having to perform a linearity check at different force levels for comparison purposes. The technique will give an indication of the amount of nonlinearity present in the structure at a particular force level. Further research is needed to evaluate an acceptable level at which linearity within the system can be assumed.

2.10 Summary - Measurement Techniques for Experimental Modal Analysis

In the material contained in Volume II of this Technical Report, the area of measurement techniques applicable to experimental modal analysis is discussed in some detail. This review is primarily concerned with the accurate measurement of frequency response functions on linear, time invariant, observable structural systems. Much effort was spent on the understanding of the many different algorithms used to estimate frequency response functions. It is most important to understand that if the noise in the measurement problem is reduced to zero (variance and bias errors) all of the frequency response functions reduce to the same form. When attempting to experimentally determine the dynamic properties (natural frequency, damping, and mode shapes) of a structure, one of the most important aspects is to collect and process data that represent the structure as accurately as possible. These data can then be used as input to an number of parameter estimation algorithms and could also be used in modal modeling algorithms. Volume II of this Technical Report describes in detail the procedure used to collect these data. Many of the potential errors are discussed as well as techniques to eliminate or reduce the effects of these errors on the quality of the results. If the procedures described in this Technical Report are followed, data can be collected, as input to modal parameter estimation algorithms, that will yield accurate dynamic properties of the test structure. With care and attention to theoretical limitations, these dynamic properties can be used to construct a modal model.

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3. MODAL PARAMETER ESTIMATION

3.1 Introduction

Modal parameter estimation is the determination of frequency, damping, and modal coefficients from the measured data which may be in: (1) relatively raw form in terms of force and response data in the time or frequency domain, or (2) in a processed form such as frequency response or impulse response functions. Most modal parameter estimation is based upon the measured data being the frequency response function; or the equivalent impulse response function, typically found by inverse Fourier transforming the frequency response function. Regardless of the form of the measured data, the modal parameter estimation techniques have traditionally been divided into two categories: (1) single degree-of-freedom (SDOF) approximations, and (2) multiple degree-of-freedom (MDOF) approximations. Since the single degree-of-freedom equations are simply special cases of the multiple degree-of-freedom equations, all theoretical discussion is made only in terms of the multiple degree-of-freedom case.

The current effort in the modal parameter estimation area is concerned with a unified theory that explains any previously conceived modal parameter estimation method as a subset of a general theory. This unified theory concept would eliminate the confusing nomenclature that currently exists and simplify the understanding of the strengths and weaknesses of each method. The modal parameter estimation methods that have been developed over the past several years involve multiple measurement, multiple reference concepts that can be viewed as an interaction between the temporal domains (time, frequency, etc.) and the spatial domains (physical coordinates, modal coordinates, etc.) in order to achieve the "best" estimate of the modal parameters. Volume III of this Technical Report presents this background and provides a complete development, using a consistent set of nomenclature, of most multiple reference modal parameter estimation algorithms in use at the present time.

3.2 Historical Overview

While engineers have tried to estimate the vibration characteristics of structures since the turn of the century, the actual history of experimental modal parameter estimation is normally linked to the work by Kennedy and Pancu[1] in 1947. Until this time, the instrumentation that had been available was not sufficiently refined to allow for detailed study of experimental modes of vibration. As the instrumentation and analysis equipment has improved over the last forty years, major improvements in modal parameter estimation techniques have followed. Specifically, the development of accurate force and response transducers, the development of test equipment based upon digital computers and the development of the fast Fourier transform (FFT) have been the key advances that have initiated bursts of development in the area of modal parameter estimation.

During this time period, efforts in modal parameter estimation have involved two concepts. The first concept involved techniques oriented toward the forced normal mode approach to modal parameter estimation. This approach to the estimation of modal parameters involves exciting the system into a single mode of vibration by using a specific sinusoidal forcing vector. Since the success of this method is determined by the evaluation of the phase characteristics with respect to the characteristics occurring at resonance, this approach can be broadly classified as the phase resonance method. In order to refine the phase resonance method, particularly the force appropriation aspect of the method, efforts began to estimate the modal information on a mode by mode basis, using measured impedance, or frequency response functions. Much of the early work on this concept centered on using the phase information as a means of identifying the effects of separate modes of vibration in the measurement. For this reason, this concept has become known as the phase separation method. Most methods that are in use today can be classified as phase separation methods since no effort is made to excite only one mode of vibration at a time.

With respect to the phase resonance methods, the basic theory was first documented by Lewis and Wrisley[2] in 1950. This theory was refined and presented in a more complete manner by F. de Veubeke[3] in 1956. Significant advances in the approach to force appropriation were documented by Trail-Nash[4] and Asher[5] in 1958. Significant improvements and refinements in the phase resonance methods have taken place in the last thirty years, particularly in the automation of the force appropriation and the use of digital computers, but the basic theoretical concept has not changed since 1950.

With respect to the phase separation methods, much effort has occurred over the last forty years and continues to the present day. Kennedy and Pancu[1] in 1947 documented that the presence of two modes of vibration could be detected by observing the rate of change of the phase in the area of resonance. Since this method was developed based upon a plot of the real part of the impedance function versus the imaginary part of the impedance function, this method is referred to as a circle-fit method based upon these characteristics in the Argand plane. Broadbent[6] applied this concept to flight flutter data in 1958. Since the data acquisition process was largely analog until 1970, most of the work until that time was oriented towards trying to fit a single degree of freedom model to portions of the analog data. The significant contributors during this time period began with Stahle [7] in 1958, and continued with Bishop and Gladwell [8] and Pendered and Bishop [9-11] in 1963, and Mahalingham [12] in 1967. Once data began to be collected and stored in a digital fashion, the phase separation methods migrated to multiple degrees of freedom approaches. The initial work involving multiple degree of freedom models was documented by Klosterman [13] in 1971 Richardson and Potter [14] in 1974 and Van Loon [15] in 1974. While the work during this period evolved the basic polynomial and partial fraction models that are the basis of modern experimental modal parameter estimation methods, the algorithms were basically unstable, iterative approaches to the solution for the unknown parameters. Also, these methods used only one measurement at a time in the estimation of the modal parameters. In 1978, Brown [16] documented work on the Least Squares Complex Exponential method that was a two stage approach to the estimation of modal parameters using all of the available data. In the first stage, the frequency and damping values are estimated; in the second stage, the modal coefficients are estimated. Ibrahim [17], also in 1977, documented the initial version of the Ibrahim Time Domain Method, which formulated the solution for the modal parameters into an eigenvalue-eigenvector solution approach. These last approaches represent conceptual approaches that have been extended today into similar methods involving multiple references. The significant advances in the multiple reference, or polyreference, methods used and being developed at the present time were first documented by Vold [18] in 1982 with the Polyreference Time Domain method. Since that time, several other polyreference methods have been developed. Detailed documentation of the multiple reference methods is contained in later sections.

In summary, over the last forty years, many experimental modal parameter estimation methods have been developed that can be classified as either phase resonance or phase separation methods. Often, it seems that these methods are very different and unique. In reality, the methods all are derived from the same equation and are concerned with the decomposition of a composite function into its constituent parts. This decomposition may occur in the time domain in terms of damped complex exponentials, in the frequency domain in terms of single degree-of-freedom functions, or in the modal domain in terms of modal vectors. This decomposition may occur during the test, as in the phase resonance methods, or occur during analysis, as in the phase separation methods. The various modal parameter estimation methods are enumerated in the following list:

- Forced Normal Mode Method [2-5,19,20]
- Quadrature Amplitude [7,8,11]
- Kennedy-Pancu Circle Fit [1,13,21-23]
- Single Degree-of-Freedom Polynomial [14,21,22,24,25]

- Nonlinear Frequency Domain [13,14,21,22]
- Complex Exponential [26,27]
- Least-Squares Complex Exponential (LSCE) [16]
- Ibrahim Time Domain (ITD) [17,28-32]
- Eigensystem Realization Algorithm [33-35]
- Orthogonal Polynomial [36,37]
- Global Orthogonal Polynomial [38]
- Polyreference Time Domain [18,39,40]
- Polyreference Frequency Domain [40-43]
- Direct Parameter Identification: Time Domain [40]
- Autoregressive Moving Average (ARMA) [44-48]
- Direct Parameter Identification: Frequency Domain [40,49]

3.3 Multiple-Reference Terminology

3.3.1 Mathematical Models

The most general model that can be used is one in which the elements of the mass, damping, and stiffness matrices are estimated, based upon measured forces and responses. Thus, the model that is used is based upon a matrix differential equation transformed into the domain of interest.

Time domain:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \tag{24}$$

Frequency domain:

$$-\omega^2[M]\{X(\omega)\} + j\omega[C]\{X(\omega)\} + [K]\{X(\omega)\} = \{F(\omega)\} \tag{25}$$

Laplace domain:

$$s^2[M]\{X(s)\} + s[C]\{X(s)\} + [K]\{X(s)\} = \{F(s)\} \tag{26}$$

If Eq. (24), (25), or (26) is used as the model for parameter estimation, the elements of the unknown matrices must first be estimated from the known force and response data measured in the time, frequency or Laplace domain. Once the matrices have been estimated, the modal parameters can be found by the solution of the classic eigenvalue-eigenvector problem [38,42,49]. Due to truncation of the data in terms of frequency content, limited numbers of degrees-of-freedom, and measurement errors, the matrices found by Eq. (24), (25), or (26) are, in general, not directly comparable to matrices determined from a finite element approach. Instead, the matrices that are estimated simply yield valid input-output relationships and valid modal parameters. This is because there is an infinite number of sets of mass, damping, and stiffness matrices that yield the same modal parameters over a reduced frequency range limited to the dynamic range of the measurements. For this reason, Eqs. (24), (25), and (26) are often pre-multipled by the inverse of the mass matrix so that the elements of the two matrices, the dynamic damping matrix [D] and dynamic stiffness matrix [E] are estimated:

Time domain:

$$[I]\{\ddot{x}(t)\} + [D]\{\dot{x}(t)\} + [E]\{x(t)\} = \{F'(t)\} \tag{27}$$

Frequency domain:

$$-\omega^2[I]\{X(\omega)\} + j\omega[D]\{X(\omega)\} + [E]\{X(\omega)\} = \{F'(\omega)\} \tag{28}$$

Laplace domain:

$$s^2[I]\{X(s)\} + s[D]\{X(s)\} + [E]\{X(s)\} = \{F'(s)\} \tag{29}$$

Existing modal parameter estimation methods used in commercial modal analysis systems most often employ a model based upon measured impulse response (time domain) or frequency response (frequency domain) functions. While the exact model used as the basis for modal parameter estimation varies, almost all models used in conjunction with frequency response function data can be described by a general model in the time domain, frequency domain, or Laplace domain. The general model in the time domain is a damped complex exponential model (often the impulse response function) while the general model in the frequency domain is the frequency response function. The general model in the Laplace domain is the transfer function. For general viscous damping, the mathematical models for each domain for a multiple degree degree-of-freedom mechanical system can be stated as:

Time Domain:

$$h_{pq}(t) = \sum_{r=1}^{N} A_{pqr}\, e^{\lambda_r t} + A^*_{pqr}\, e^{\lambda^*_r t} \tag{30}$$

Frequency Domain:

$$H_{pq}(\omega) = \sum_{r=1}^{N} \frac{A_{pqr}}{j\omega-\lambda_r} + \frac{A^*_{pqr}}{j\omega-\lambda^*_r} \tag{31}$$

Laplace Domain:

$$H_{pq}(s) = \sum_{r=1}^{N} \frac{A_{pqr}}{s-\lambda_r} + \frac{A^*_{pqr}}{s-\lambda^*_r} \tag{32}$$

where:

s	=	Laplace variable
s	=	$\sigma + j\omega$
σ	=	angular damping variable (rad/sec)
ω	=	angular frequency variable (rad/sec)
p	=	measured degree-of-freedom (response)
q	=	measured degree-of-freedom (input)
r	=	modal vector number
N	=	number of modal frequencies
A_{pqr}	=	residue
A_{pqr}	=	$Q_r \psi_{pr} \psi_{qr}$
A_{pqr}	=	$\psi_{pr} L_{qr}$
Q_r	=	complex modal scaling coefficient for mode r

ψ_{pr} = modal coefficient for measured degree-of-freedom p and mode r

L_{qr} = modal participation factor for reference degree-of-freedom q and mode r

λ_r = system pole

λ_r = $\sigma_r + j\omega_r$

The models described in Eqs. (30) through (32) have many other equivalent forms based upon expansion of the terms under the summation. Also, the models take on slightly different forms under assumptions concerning specific physical damping mechanisms (hysteretic, etc.) [13,14,40]. Other forms of these models are also used where certain assumptions or mathematical relationships are utilized. For example, an equivalent model can be found when the common denominator of Eq. (31) is formed yielding a polynomial numerator and polynomial denominator of maximum order "2N" [13,14,22]. The denominator polynomial is then a function of the system poles. Often, an assumption is made concerning the modal vectors being normal (real) rather than complex. This reduces the number of unknowns that must be estimated from "2N" to "N".

3.3.2 Sampled Data

The mathematical models described in the previous section are all developed based upon the concept that the temporal variable (time or frequency) is continuous. In reality the temporal variable must be thought of as sampled in each domain. This restriction requires special consideration when applying the models developed in Eqs. (24) through (32). Differential equations must now be thought of as finite difference equations; continuous integral transforms are replaced by discrete transforms such as the Fast Fourier Transform (FFT) and the Z Transform. The concepts affecting the numerical processing of sampled data with respect to the continuous models represented in Eqs. (24) through (32) are exactly the same as the concepts that are the basis of the area of digital signal analysis with respect to the measurement of the data. The limitation of the frequency information creates special processing problems that are related to Shannon's Sampling Theorem; the limitations of the dynamic range of the measured data and of the computer precision yield special numerical problems with respect to the solution algorithm.

In general, the numerical considerations often determine which mathematical model will be most effective in the estimation of modal parameters. Time domain models tend to provide the best results when a large frequency range or large numbers of modes exist in the data. Frequency domain models tend to provide the best results when the frequency range of interest is limited and when the number of modes is small. While these are general considerations, the actual numerical implementation determines the ability of the algorithm to estimate modal parameters accurately and efficiently.

3.3.3 Consistent Data

Modal parameter estimation methods all assume that the system that is being investigated is linear and time invariant. While this is often nearly true, these assumptions are never exactly true. *Consistent data* refers to the situation where the data is acquired so as to best satisfy these two assumptions. Problems associated with nonlinearity can be minimized by maintaining a prescribed force level and/or using excitation methods that give the best linear approximation to the nonlinear characteristic (random excitation). Problems associated with the time invariance constraint can be minimized by acquiring all of the data simultaneously using multiple excitations [50-54]. This reduces mass loading and boundary condition variations that can be caused by moving a transducer around the structure or by changing the location of the excitation.

3.3.4 Residuals

With respect to spatial geometry, continuous systems have an infinite number of degrees-of-freedom but, in general, only a finite number of modes can be used to describe the dynamic behavior of a system. The theoretical number of degrees-of-freedom can be reduced by using a finite frequency range (f_a, f_b). Therefore, for example, the frequency response function can be broken up into three partial sums, each covering the modal contribution corresponding to modes located in the frequency ranges $(0, f_a)$, (f_a, f_b), and (f_b, ∞) as shown in Figure 7.

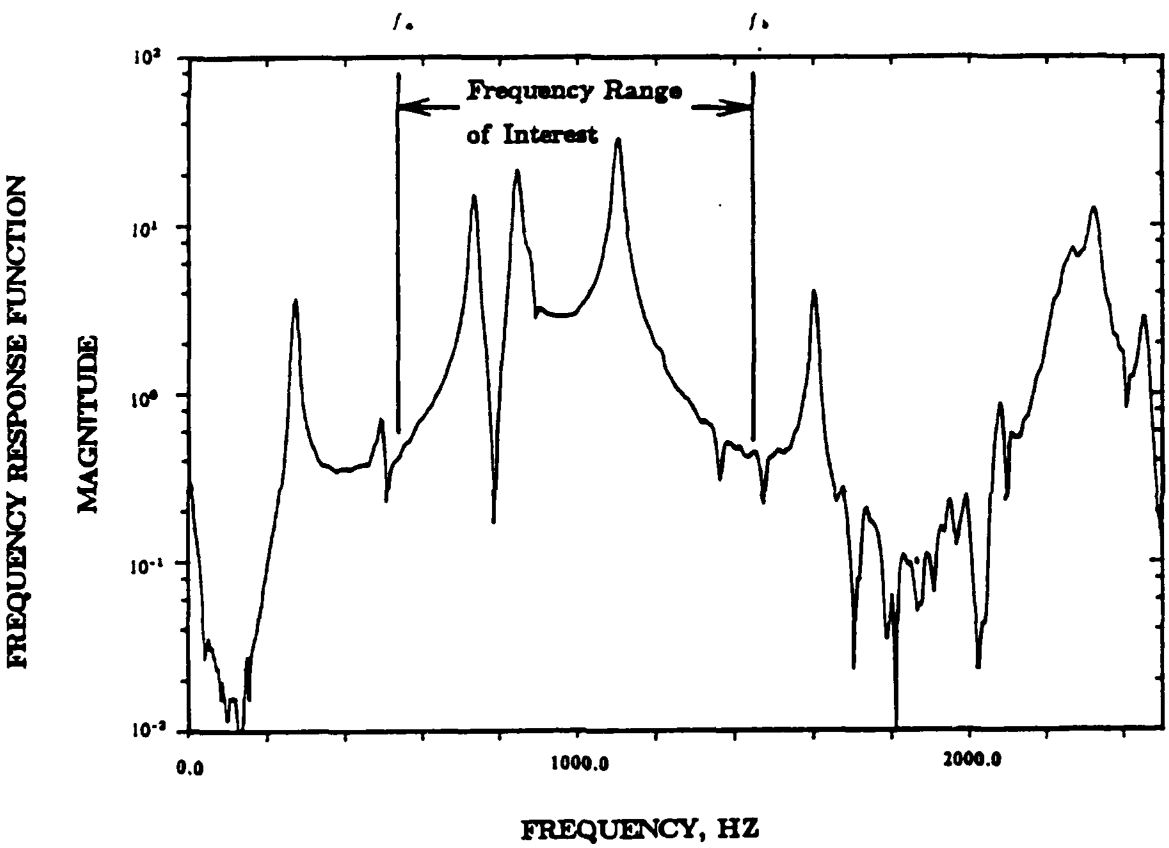

Figure 7. Frequency Range of Interest

In the frequency range of interest, the modal parameters can be estimated to be consistent with Eq. (33). In the lower and higher frequency ranges, residual terms can be included to handle modes in these ranges. In this case, the general frequency response function model can be stated:

$$H_{pq}(\omega) = R_{Ipq}(\omega) + \sum_{r=1}^{N} \frac{A_{pqr}}{j\omega-\lambda_r} + \frac{A^*_{pqr}}{j\omega-\lambda^*_r} + R_{Fpq} \tag{33}$$

where:

$R_{Ipq}(\omega)$ = residual effect of lower frequency modes
R_{Fpq} = residual effect of higher frequency modes (constant with ω)

In many cases the lower residual is called the *inertia restraint,* or *residual inertia,* and the upper residual is called the *residual flexibility* [13]. In this common formulation of residuals, both terms are real-valued quantities. The lower residual is a term reflecting the inertia or mass of the lower modes and is an inverse function of the frequency squared. The upper residual is a term reflecting the flexibility of the upper modes and is constant with frequency. Therefore, the form of the residual is based upon a physical concept of how the combined system poles below and above the frequency range of interest affect the data in the range of interest. As the system poles below and above the range of interest are located in the proximity of the boundaries of the frequency range of interest, these effects are not the simple real-valued quantities noted in Eq. (33). In these cases, residual or computational modes may be included in the model to partially account for these effects. When this is done, the modal parameters that are associated with these computational poles have no physical significance because the poles are not structural modes of the system, but may be required in order to compensate for strong dynamic influences from outside the frequency range of interest. Using the same argument, the lower and upper residuals can take on any mathematical form that is convenient as long as the lack of physical significance is understood. Power functions of frequency (zero, first, and second order) are commonly used within such a limitation. In general, the use of residuals is confined to frequency response function models. This is primarily due to the difficulty of formulating a reasonable mathematical model and solution procedure in the time domain for the general case that includes residuals.

3.3.5 Global Modal Parameters

Theoretically, modal parameters are considered to be unique based upon the assumption that the system is linear and time invariant. Therefore, the modal frequencies can be determined from any measurement and the modal vectors can be determined from any reference condition. If multiple measurements or reference conditions are utilized, the possibility of several, slightly different, answers for each modal parameter exists. The concept of *global modal parameters,* as it applies to modal parameter estimation, means that there is only one answer for each modal parameter and that the modal parameter estimation solution procedure enforces this constraint. Every frequency response or impulse response function measurement theoretically contains the information that is represented by the characteristic equation (modal frequencies and damping). If individual measurements are treated in the solution procedure independent of one another, there is no guarantee that a single set of modal frequencies and damping are generated. In a like manner, if more than one reference is measured in the data set, redundant estimates of the modal vectors can be estimated unless the solution procedure utilizes all references in the estimation process simultaneously. Most modal parameter estimation algorithms estimate the modal frequencies and damping in a global sense but few estimate the modal vectors in a global sense.

3.3.6 Modal Participation Factors

A *modal participation factor* is a complex-valued scale factor that is the ratio of the modal coefficient at one reference degree-of-freedom to the modal coefficient at another reference degree-of-freedom. A more general view of the modal participation factor is that it represents the relationship between the residue and the eigenvector coefficient as in the following equations:

$$A_{pqr} = Q_r \psi_{pr} \psi_{qr} \tag{34}$$

$$L_{qr} = Q_r \psi_{qr} \tag{35}$$

$$A_{pqr} = \psi_{pr} L_{qr} \tag{36}$$

where:

p	=	measured degree-of-freedom (response)
q	=	measured degree-of-freedom (reference)
r	=	modal vector number
A_{pqr}	=	residue
Q_r	=	complex modal scaling coefficient for mode r
ψ_{pr}	=	modal coefficient for measured degree-of-freedom p and mode r
ψ_{qr}	=	modal coefficient for reference degree-of-freedom q and mode r
L_{qr}	=	modal participation factor for reference degree-of-freedom q and mode r

From a mathematical standpoint, the modal participation matrix is equal to the left eigenvectors of the transfer matrix [H] as shown in Eq. (37):

$$\left[H\right] = \left[\Psi\right] \lceil \Lambda \rfloor \left[L\right] \tag{37}$$

where:

$\left[H\right]$	=	transfer function matrix
$\left[\Psi\right]$	=	complex modal vector matrix
$\lceil \Lambda \rfloor$	=	diagonal matrix with poles

Note that for Eq. (35) the modal participation factor represents the product of a modal scaling coefficient and another term from the right eigenvector for reference degree-of-freedom q. This will always be true for reciprocal systems since the left and right eigenvectors for a given mode are equal. For non-reciprocal systems, the modal participation factor is the appropriate term from the left eigenvector. Note also that the modal participation factor, since it is related to the eigenvector, has no absolute value but is relative to the magnitudes of the other elements in the eigenvector.

Modal participation factors reflect the interaction of the spatial domain with the temporal domain (time or frequency). Modal participation factors can be computed any time that multiple reference data are measured and such factors are used in multiple reference modal parameter estimation algorithms. Modal participation factors relate how well each modal vector is excited from each of the reference locations. This information is often used in a weighted least squares error solution procedure to estimate the modal vectors in the presence of multiple references. Theoretically, these modal participation factors should be in proportion to the modal coefficients of the reference degrees of freedom for each modal vector. Modal participation factors in a solution procedure enforce the constraint concerned with Maxwell's reciprocity between the reference degrees of freedom. Most multiple reference, modal parameter estimation methods estimate modal participation factors as part of the first stage estimation of global modal frequencies and damping.

3.3.7 Order of the Model

The *order* of the model equals the number of unknowns that must be estimated in the model. In the modal parameter estimation case, this refers to the frequency, damping, and complex modal coefficient for each mode of vibration at every measurement degree-of-freedom plus any residual terms that must be estimated. Therefore, the order of the model is directly dependent on the number of modal frequencies, "N", that are to be estimated. For example, for a system with "N" modes of vibration, assuming that no residuals were required, "4N" unknowns must be estimated. For cases involving measured data, the order of the model is extremely important. Estimates of modal parameters are affected by the order of the model. A problem arises from the inability to be certain that the correct order of the model has been chosen during the initial estimation phase. If the number of modes of vibration is more or less than "N", modes of vibration will be found that do not exist physically or modes of vibration will be missed that actually do occur. In addition, the values of frequency, damping, and complex modal coefficient for the actual modes of vibration will be affected.

The number of modes of vibration is normally chosen between one and an upper limit, dependent on the memory limitations of the computational hardware. The true number of system poles is a function of the frequency range of the measurements used to estimate the modal parameters. By observing the number of peaks in the frequency response function, the minimum number of system poles can be estimated. This estimate is normally low, based upon poles occurring at nearly the same frequency (pseudo-repeated roots), limits on dynamic range, and poorly excited modes. For these reasons, the estimate of the correct order of the model is often in error. When the order of the model is other than optimum, the estimate of the modal parameters will be in error.

Many of the parameter estimation techniques that are used assume that only one mode exists in a limited range of interest and all of the other modes appear as residual terms. For this case Eq. (33) can be rewritten as:

$$H_{pq}(\omega) = R_{I_{pq}}(\omega) + \frac{A_{pq}}{j\omega - \lambda_1} + \frac{A^*_{pq}}{j\omega - \lambda^*_1} + R_{F_{pq}} \tag{38}$$

3.3.8 Solution Procedure

Equations (30) through (32) are nonlinear in terms of the unknown modal parameters. This can be noted from the unknowns in the numerator and denominator of Eq. (31) and the unknowns as the argument of the transcendental functions of Eq. (30). The nonlinear aspect of the model must be treated in one of two ways: (1) by the use of an iterative solution procedure to solve the nonlinear estimation problem, allowing all modal parameters to vary according to a constraint relationship until an error criterion reaches an acceptably low value, or (2) by separating the nonlinear estimation problem into two linear estimation problems. For the case of structural dynamics, the common technique is to estimate "2N" frequencies and damping values in a first stage and then to estimate the "4N" modal coefficients plus any residuals in a second stage.

In the iterative technique in the solution of the nonlinear estimation approach, a set of starting values must be chosen to initiate the sequence. The number and value of these starting values affect the final result. Poor initial estimates can lead to problems of convergence, as a result of which, close operator supervision usually is required for a successful use of this technique.

An alternative method is to reformulate the nonlinear problem into a number of linear stages so that each stage is stable. The actual data that are used in the estimate of the modal parameters also affect the results. Based on the choice of the order of the model, "N", there are "4N" modal parameters to be estimated. If residuals, in one form or another, are also included, the number of modal parameters to be estimated will be slightly higher. The common method of solving for these unknown modal parameters is to find an equation involving known information for every unknown to be found. In this case, the measured frequency response function or impulse response function provides the known information and Eq. (30) or Eq. (31) can be repeated for different frequencies or time values in order to obtain a sufficient number of equations. These equations, for the linear case, can then be solved simultaneously for the unknown modal parameters. As an illustration of this relationship, consider a common modal parameter situation in which the number of modes in the frequency range of interest is between 1 and 30. Assuming the highest ordered model means that slightly more than 120 modal parameters must be estimated. From a single frequency response measurement, 1024 known values of the function will be available (512 complex values at successive values of frequency). Many more equations, based on the known values of frequency response, can be formed than are needed to find the unknown modal parameters. An obvious solution is to choose enough equations to solve for the modal parameters. The problem arises in determining what part of the known information is to be involved in the solution. As different portions of the known data (data near a resonance compared to an anti-resonance, for example) are used in the solution, the estimates of modal parameters vary. As the quality of the data becomes marginal, this variance can be quite large. When the modal parameters that are estimated appear to be non-physical, this is often the reason.

To solve this problem, all or a large portion of the data can be used if a pseudo-inverse type of solution procedure is used. One procedure that is used is to formulate the problem so as to minimize the squared error between the data and the estimated model. This least-squares error method to the solution is the most commonly used technique in the area of modal parameter estimation. If there are many more known pieces of information than unknowns that must be estimated, many more equations can be formed than are needed to solve for the unknowns. The least-squares error method to the solution allows for all of these redundant data to be used to estimate the modal parameters in a computationally efficient manner. The least-squares error method usually can be derived directly from the linear equations using a normal equations approach. In general, this procedure does not significantly increase the memory or computational requirements of the computational hardware. Any solution procedure that can be used is only estimating a "best" solution based upon the choice of the model, the order of the model, and the known, measured data used in the model.

3.4 Characteristic Polynomial

The impetus of this section is to show that for discrete data, a difference characteristic equation can be formulated in order to solve for the poles of the system. Further, it will be shown that the difference equation can be formulated directly from the impulse response function data. By solving for the polynomial coefficients and the roots of the polynomial equation, the modal parameters, frequency and damping, are determined. The characteristic polynomial will be formulated for the continuous case, as a differential function, and then extended to the discrete case, as a difference function.

3.4.1 Differential Theory

The homogeneous differential equation for a single degree of freedom system is:

$$m\,\ddot{x}(t) + c\,\dot{x}(t) + k\,x(t) = 0 \tag{39}$$

In order to solve the differential equation assume a solution of the of the form $x(t) = X\,e^{st}$, where X

is a scalar value. Substituting the appropriate derivatives of the assumed solution into Eq.(39):

$$(m s^2 + c s + k) X e^{st} = 0 \tag{40}$$

Thus, the differential equation is transformed into an algebraic polynomial equation, called the characteristic equation.

$$m s^2 + c s + k = 0 \tag{41}$$

The complex valued roots of the characteristic equation will yield the characteristic solutions, λ_1 and λ_2. The real part is the damping and the imaginary part is the eigenfrequency, or damped natural frequency. Thus, the solution to the governing differential equation is:

$$x(t) = \sum_{r=1}^{2} X_r \, e^{\lambda_r t} \tag{42}$$

The scalar magnitudes, X_1 and X_2, are determined from the system initial conditions. Note that *any* exponential function will satisfy the differential equation. One such function of particular interest, is the impulse response function.

$$h(t) = \sum_{r=1}^{2} A_r \, e^{\lambda_r t} \tag{43}$$

A system with N degrees-of-freedom can be described by a set of N, coupled, second order differential equations. The characteristic equation for this system is represented by the following polynomial:

$$a_{2N} s^{2N} + a_{2N-1} s^{2N-1} + a_{2N-2} s^{2N-2} + \ldots + a_1 s + a_0 = 0 \tag{44}$$

Solving this polynomial equation will yield $2N$ complex valued roots, or, characteristic solutions λ_r. Then the solutions to the differential equations will be complex exponentials of the form:

$$x_{pq}(t) = \sum_{r=1}^{2N} X_r \, e^{\lambda_r t} \tag{45}$$

where:

- p = response location degree-of-freedom
- q = reference location degree-of-freedom

Thus, impulse response functions,

$$h_{pq}(t) = \sum_{r=1}^{2N} A_{pqr} \, e^{\lambda_r t} \tag{46}$$

will also satisfy the differential equations. Consider a few impulse response functions for various reference and response points.

$$h_{11}(t) = \sum_{r=1}^{2N} A_{11r} \, e^{\lambda_r t} \tag{47}$$

$$h_{12}(t) = \sum_{r=1}^{2N} A_{12r} \, e^{\lambda_r t} \tag{48}$$

$$h_{13}(t) = \sum_{r=1}^{2N} A_{13r}\, e^{\lambda_r t} \tag{49}$$

$$h_{pq}(t) = \sum_{r=1}^{2N} A_{pqr}\, e^{\lambda_r t} \tag{50}$$

The common characteristic in each of the above equations is that every impulse response function is a linear superposition of identical damped complex exponentials, $e^{\lambda_r t}$ for $r = 1 \rightarrow 2N$. That is, the roots of the characteristic polynomial are common to all reference and response locations. Thus, the characteristic solutions are global system parameters, since they are independent of reference or, response location. The important result is that since *each* $e^{\lambda_r t}$ is a characteristic solution to the homogeneous linear differential equation,

$$a_{2N} D^{2N}\left(A_{pqr}\, e^{\lambda_r t}\right) + a_{2N-1} D^{2N-1}\left(A_{pqr}\, e^{\lambda_r t}\right) + a_{2N-2} D^{2N-2}\left(A_{pqr}\, e^{\lambda_r t}\right) + \ldots + a_1 D\left(A_{pqr}\, e^{\lambda_r t}\right) + a_0 = 0$$

where,

- $D^n[f(t)] = \dfrac{d^n[f(t)]}{dt^n}$ (differential operator)

that a linear superposition of characteristic solutions will also be a solution. That is, $h_{pq}(t)$ will also satisfy the differential equation. Actually, a set of N second order linear differential equations must be satisfied, but, a differential equation of order $2N$ can be found that will have the same roots as the set of N second order equations.

Substituting a few impulse response functions for various reference and response points, a number of differential equations are obtained.

$$a_{2N} D^{2N}\left(h_{11}(t)\right) + a_{2N-1} D^{2N-1}\left(h_{11}(t)\right) + a_{2N-2} D^{2N-2}\left(h_{11}(t)\right) + \ldots + a_1 D\left(h_{11}(t)\right) + a_0 = 0$$

$$a_{2N} D^{2N}\left(h_{12}(t)\right) + a_{2N-1} D^{2N-1}\left(h_{12}(t)\right) + a_{2N-2} D^{2N-2}\left(h_{12}(t)\right) + \ldots + a_1 D\left(h_{12}(t)\right) + a_0 = 0$$

$$a_{2N} D^{2N}\left(h_{13}(t)\right) + a_{2N-1} D^{2N-1}\left(h_{13}(t)\right) + a_{2N-2} D^{2N-2}\left(h_{13}(t)\right) + \ldots + a_1 D\left(h_{13}(t)\right) + a_0 = 0$$

$$a_{2N} D^{2N}\left(h_{pq}(t)\right) + a_{2N-1} D^{2N-1}\left(h_{pq}(t)\right) + a_{2N-2} D^{2N-2}\left(h_{pq}(t)\right) + \ldots + a_1 D\left(h_{pq}(t)\right) + a_0 = 0$$

Note that the coefficients a_0 to a_{2N} do not vary with reference or response location and thus, can be estimated from a combination of various number of reference and response points.

3.4.2 Difference Theory

From an experimental standpoint, the data are sampled, which means instead of continuous knowledge of the system, the values obtained are for distinct discrete temporal points. The impulse response functions are generally obtained by inverse Fourier transforming the frequency response functions. Thus, from the discrete impulse response functions the pole information, frequency and damping, is determined. The model for the discrete impulse response function is:

$$h_{pq}(t_k) = \sum_{r=1}^{2N} A_{pqr}\, e^{\lambda_r t_k} = \sum_{r=1}^{2N} A_{pqr}\, z_r^k \tag{51}$$

where,

- $t_k = k\,\Delta t$
- Δt is the sample interval
- $k = 1 \rightarrow blocksize$
- $z_r = e^{\lambda_r \Delta t}$.

It should be noted, for discrete data, that the sample interval, Δt, limits the frequency for which valid information can be determined, whereas, in the analysis of continuous data, there are no frequency constraints. In other words, theoretically, characteristics can be determined to infinite frequency for continuous functions, but, the process of digitally sampling continuous data causes a maximum frequency for which characteristics can be determined. The frequencies above this maximum will alias back into the sampled bandwidth, and thus bias the results. For this reason, low-pass filters are used to exclude information above the maximum frequency.

Recall the characteristic equation for an N degree-of-freedom system:

$$a_0\, s^0 + a_1\, s^1 + a_2\, s^2 + \ldots + a_{2N-1}\, s^{2N-1} + a_{2N}\, s^{2N} = 0 \tag{52}$$

will have $2N$ characteristic solutions, λ_r, for $r = 1 \rightarrow 2N$. The characteristic polynomial is not unique in that, many polynomials can be constructed that will yield the same characteristic solutions, even though the coefficients will be different. For this reason, another polynomial can be formulated that will have characteristic solutions that are related to the characteristic solutions of Eq.(52). The polynomial has the form:

$$a'_0\, z^0 + a'_1\, z^1 + a'_2\, z^2 + \ldots + a'_{2N-1}\, z^{2N-1} + a'_{2N}\, z^{2N} = 0 \ . \tag{53}$$

The relationship between z and s is $z = e^{s \Delta t}$. Analogous to Eq.(52), there are also $2N$ characteristic solutions of Eq.(53), z_r for $r = 1 \rightarrow 2N$. The roots of the two equations are related by $z_r = e^{\lambda_r \Delta t}$ where, z_r are precisely the values of z for which the characteristic equation, Eq.(53), is zero. Note that z_r is simply the sampled form of the continuous exponential solution in the differential case. Thus, by knowing the system characteristics, z_r, the desired parameters, λ_r, can be determined. If the coefficients are known, Eq.(53) could be solved, but, from an experimental aspect, both the coefficients and the system characteristics are unknown. Thus, in order to determine the system characteristics, the a' coefficients must be determined first. This is accomplished by substituting a characteristic solution of the system, z_r, into Eq.(53).

$$a'_0\, A_{pqr}\, z_r^0 + a'_1\, A_{pqr}\, z_r^1 + a'_2\, A_{pqr}\, z_r^2 + \ldots + a'_{2N-1}\, A_{pqr}\, z_r^{2N-1} + a'_{2N}\, A_{pqr}\, z_r^{2N} = 0 \tag{54}$$

Substituting $z_r = e^{\lambda_r \Delta t}$ into the above equation,

$$a'_0\, A_{pqr}\, (e^{\lambda_r \Delta t})^0 + a'_1\, A_{pqr}\, (e^{\lambda_r \Delta t})^1 + a'_2\, A_{pqr}\, (e^{\lambda_r \Delta t})^2 + \ldots + a'_{2N}\, A_{pqr}\, (e^{\lambda_r \Delta t})^{2N} = 0 \tag{55}$$

or,

$$a'_0\, A_{pqr}\, e^0 + a'_1\, A_{pqr}\, e^{\lambda_r \Delta t} + a'_2\, A_{pqr}\, e^{\lambda_r 2 \Delta t} + \ldots + a'_{2N}\, A_{pqr}\, e^{\lambda_r 2N \Delta t} = 0 \ . \tag{56}$$

The important result is that since *each* $e^{\lambda_r \Delta t}$ is a characteristic solution to the homogeneous linear difference equation, that a linear superposition of characteristic solutions will also be a solution of Eq.(56), which means that, in general, Eq.(51) can be substituted into Eq.(56). Once again, a set of N second order linear difference equations must be satisfied, but, a difference equation of order $2N$ can be found that will have the same roots as the set of N second order equations.

Consider a number of equations for various reference and response locations:

$$a'_0\, h_{11}(t_0) + a'_1\, h_{11}(t_1) + a'_2\, h_{11}(t_2) + \ldots + a'_{2N\text{-}1}\, h_{11}(t_{2N\text{-}1}) + a'_{2N}\, h_{11}(t_{2N}) = 0 \tag{57}$$

$$a'_0\, h_{12}(t_0) + a'_1\, h_{12}(t_1) + a'_2\, h_{12}(t_2) + \ldots + a'_{2N\text{-}1}\, h_{12}(t_{2N\text{-}1}) + a'_{2N}\, h_{12}(t_{2N}) = 0 \tag{58}$$

$$a'_0\, h_{13}(t_0) + a'_1\, h_{13}(t_1) + a'_2\, h_{13}(t_2) + \ldots + a'_{2N\text{-}1}\, h_{13}(t_{2N\text{-}1}) + a'_{2N}\, h_{13}(t_{2N}) = 0 \tag{59}$$

$$a'_0\, h_{pq}(t_0) + a'_1\, h_{pq}(t_1) + a'_2\, h_{pq}(t_2) + \ldots + a'_{2N\text{-}1}\, h_{pq}(t_{2N\text{-}1}) + a'_{2N}\, h_{pq}(t_{2N}) = 0 \tag{60}$$

Note that the coefficients a'_0 to a'_{2N} do not vary with reference or response location and thus, can be estimated from a combination of various number of reference and response points. Once the a' coefficients are estimated from a set of equations similar to the ones above, the poles, z_r, and hence λ_r, can be estimated from the $2N$ solutions of the characteristic equation,

$$a'_0\, z^0 + a'_1\, z^1 + a'_2\, z^2 + \ldots + a'_{2N\text{-}1}\, z^{2N\text{-}1} + a'_{2N}\, z^{2N} = 0 \tag{61}$$

where:

- $z_r = e^{\lambda_r \Delta t}$.

In summary, a series of $2N$ linear difference equations with constant coefficients are formed from the sampled impulse response function data in order to solve for the common constant coefficients. These coefficients are then used in the characteristic equation to solve for the system characteristics, z_r, which contain the desired parameters, λ_r.

Note that the characteristic polynomial for the continuous, or discrete case, is of order $2N$, that is, twice the number of modes. This results in a time domain differential, or difference equation of order $2N$. For this reason, from a numerical analysis concept, for large numbers of modes, N, or large differences in modal frequency (λ_1 compared to λ_N), time domain methods are numerically better conditioned.

3.5 Characteristic Space Concepts

A new way of conceptualizing the area of parameter identification was developed during the course of the work under this contract. One of the objectives of the contract was to summarize existing modal parameter estimation methods and develop new ones. In the process of performing this task, it became obvious that most of the current algorithms could be described conceptually in terms of a three-dimensional complex space of the system's characteristics. Modal parameter estimation is the process of deconvolving measurements defined by this space into the system's characteristics.

The frequency and/or unit impulse response function matrix which describes a system, can be expressed in terms of the convolution of three fundamental characteristic functions; two complex spatial, and one complex temporal. The spatial characteristics are a function of geometry and the temporal corresponds to either time or frequency. Mathematically the frequency response matrix and the impulse response matrix can be expressed as follows:

$$[H(\omega_k)] = [\Psi] \lceil \Lambda_k \rfloor [L] \qquad [h(t_k)] = [\Psi] \lceil e^{\lambda t_k} \rfloor [L] \tag{62}$$

where:

- $[H(\omega_k)]_{N_o \times N_i}$ = frequency response matrix (element $H_{pq}(\omega_k)$)
- $[h(t_k)]_{N_o \times N_i}$ = unit impulse response matrix (element $h_{pq}(t_k)$)

- $[\Psi]_{N_o \times 2N}$ = modal vector matrix (function of spatial variable p, element ψ_{pr})
- $[L]_{2N \times N_i}$ = modal participation factor (function of spatial variable q, element L_{rq})
- $[\Lambda_k]_{2N \times 2N}$ = diagonal matrix of characteristic roots (element $\frac{1}{j\,\omega_k - \lambda_r}$)
- $[e^{\lambda t_k}]_{2N \times 2N}$ = diagonal matrix of characteristic roots (element $e^{\lambda_r t_k}$)
- ω_k = frequency temporal variable ($k = 1 \rightarrow$ *blocksize* $/2$, may be unequally spaced)
- t_k = time temporal variable ($t_k = k\,\Delta t$)
- p = response degree-of-freedom spatial variable
- q = reference degree-of-freedom spatial variable
- r = temporal degree-of-freedom variable
- N = number of modes (system poles, indexed by r)
- N_o = number of responses (indexed by p)
- N_i = number of references (indexed by q)

The frequency response function matrix consists of a three-dimensional complex space, which for a real system is a continuous function of the three characteristic variables (p,q,ω). However, in terms of measurements the functions consist of sampled data where, p,q and ω_k are sampled characteristic variables. In other words, the frequency response function is measured at discrete input, or reference points (q), output response points (p), and discrete frequency (ω_k), or time points (t_k).

A summary of the characteristic vectors are:

- The response characteristic functions consist of a set of vectors which are proportional to the eigenvectors of the system. The eigenvectors are indexed by r and the elements of the vectors are indexed by p.

- The reference characteristic functions consist of a set of vectors which are proportional to the modal participation factors, which are in turn proportional to the system eigenvectors at the reference degrees-of-freedom. The modal participation vectors are indexed by r and the elements of the vectors are indexed by q.

- The temporal characteristic functions consist of vectors which are equivalent to sampled single degree-of-freedom frequency response functions, or unit impulse response functions. The index on the vector is r and the index on the sampled element of each vector is ω_k, or t_k.

The variable r is the index on the characteristic. For a given r there is a discrete characteristic space. The summation, or superposition with respect to r defines; the measured, or sampled frequency response, or impulse response matrix, or, in other words, the three-dimensional complex space.

This concept is difficult to visualize, since the matrix is represented by three-dimensional complex characteristic space. The easiest method is to describe the variation along lines parallel to axes of the space. Lines parallel to the temporal axis correspond to individual frequency response functions, or unit impulse response functions. These frequency response functions consist of a summation of the temporal characteristics, weighted by the two spatial characteristics, which define the other two axis of the characteristic space.

Lines parallel to the response axis correspond to forced modes of vibration. These forced modes consist of a summation of the system eigenvectors weighted by the input characteristic and the temporal characteristic.

Likewise, lines parallel to the input, or reference axis consist of a summation of the system eigenvectors weighted by the response characteristic and the temporal characteristic. The variation along these lines are referred to in the literature as the modal participation factors.

Modal parameter estimation is the process of deconvolving this sampled space into the basic characteristic functions which describe the space. In practice, there are many more measured, or sampled points in the space than there are elements in the three characteristic vectors, therefore, the parameter estimation process is over determined. As a result, one of the important steps in the process has been the reduction of the data to match the number of unknowns in the parameter identification process. This data reduction has historically been done by using a pseudo inverse, or a principal component method, with least squares being the most common pseudo inverse method.

The early single degree-of-freedom (SDOF) and multiple degree-of-freedom (MDOF) modal parameter estimation methods used subsets of the sampled data and extracted one of the characteristic functions at a time, normally the temporal characteristic. For example, the very early methods like the complex exponential were used to fit individual frequency response, or unit impulse response functions for the temporal characteristics (eigenvalue) and the residues (convolution of the response and input characteristics). For these cases, each frequency response measurement gave a different estimation of the system eigenvalues, or temporal characteristics. Since the measurements were taken one function at a time some of this variation was due to inconsistencies in the data base and the rest of the variation due to noise and distortion errors.

Later methods started to use either, least square, or principal component methods to condense the data over a number of sampled frequency response functions, into small subsets parallel to the temporal axis (for example the Least Square Complex Exponential and/or the Polyreference Time Domain methods). These methods then give global estimates of the eigenvalues, or temporal characteristic functions. The Least Squares Complex Exponential parameter estimation algorithm reduced the information to a single function parallel to the temporal axis and as a result, only estimated the temporal characteristic in a global sense. The Polyreference Time Domain algorithm estimates several functions parallel to the temporal axis at the input, or reference points. As a result this method also gives global estimates of the input characteristic functions, or modal participation factors.

The more recent methods use larger subsets of the sampled data and utilize simultaneous data from all three axis resulting in global estimates of all three characteristics. In order to use these global methods, it is important that a consistent data base be measured.

3.6 Summary - Modal Parameter Estimation

One of the conclusions reached in a previous Air Force Contract (F33615-77-C3059) was that the area of modal parameter estimation will, in the future, advance rapidly due to technology transfer from other fields involved in parameter estimation. This certainly has occurred as indicated by the drastic increase in the number of parameter estimation algorithms which have been described in the literature in the last five years. This effort has been international in scope, with many of the newer techniques being variations of each other. Volume III of this Technical Report reviews multiple reference modal parameter estimation in detail.

These methods range from single reference single degree-of-freedom (SDOF) methods to sophisticated multi-reference, multi-response, multiple degree-of-freedom (MDOF) methods. The algorithm of choice depends upon a number of conditions:

- Modal Application

  - Trouble Shooting--For many of the problem solving, or trouble shooting applications, the simplier SDOF, or single reference MDOF methods are used, since simple test procedures and a quick look are desirable.

  - Model Verification--There has been increased emphasis on finite element verification. These applications require a higher level test and parameter identification procedures.

  - Model Generation, or Correction--There is also increased emphasis on; the generation of modal models based upon experimental data, and/or the correction of existing models. These applications require the highest level of test and parameter identification procedures.

- Equipment Considerations

  The parameter identification methods reviewed in this report depend heavily upon the testing methods (single input, or multiple input) and testing equipment. These new algorithms place a severe requirement upon the testing methods to obtain consistent data bases, particularly for the more advanced multi-input multi-output methods.

- Wideband vs Narrowband

  Wideband verses narrowband refers to the frequency bandwidth of the frequency response measurements. In general, for very broad frequency range measurements, time domain algorithms work well, while frequency domain algorithms seem to perform well for the narrow, or zoom bands. Recently, there has been increased emphasis in sine testing. Sine testing, not in the classical sense, but in terms of multi-input multi-output test and parameter estimation methods. This emphasis will provide the impetus to refine the frequency domain algorithms to efficiently use the increased spatial information that multi-input multi- output sine testing yields.

- Modal Density

  The choice of the parameter estimation method depends heavily upon the modal density. For cases with low modal density, single input SDOF or MDOF methods work well. For the high modal density cases the multi-input methods, especially ones which use spatial information, are the methods of choice. It should be noted that the advanced methods require consistent data and place additional constraints on the testing methods.

A summary of the characteristics of the modal parameter identification methods is shown in Table 7. All of the methods which were discussed in detail in Volume II of this Technical Report are briefly summarized in this Table.

It should be again noted that all of the methods covered in this report can be described in terms of a characteristic space, where a particular parameter identification algorithm uses as input, measured values in this characteristic space, to deconvolve the systems characteristics. The more advanced methods use information from all three axes of the characteristic space simultaneously. From the measurement standpoint, it is increasingly more important that the measured data be consistent.

TABLE 7. Summary of Modal Parameter Estimation Methods

Modal Parameter Estimation Characteristics							
	Time, Frequency, or Spatial Domain	Single or Multiple Degrees-of-Freedom	Global Modal Frequencies and Damping Factors	Repeated Modal Frequencies and Damping Factors	Global Modal Vectors	Global Modal Participation Factors	Residuals
Quadrature Amplitude	Frequency	SDOF	No	No	No	No	No
Kennedy-Pancu Circle Fit	Frequency	SDOF	No	No	No	No	Yes
SDOF Polynomial	Frequency	SDOF	Yes/No	No	No	No	No
Non-Linear Frequency Domain	Frequency	MDOF	No	No	No	No	Yes
Complex Exponential	Time	MDOF	No	No	No	No	No
Least Squares Complex Exponential (LSCE)	Time	MDOF	Yes	No	No	No	No
Ibrahim Time Domain (ITD)	Time	MDOF	Yes	No	Yes	No	No
Multi-reference Ibrahim Time Domain (MITD)	Time	MDOF	Yes	Yes	Yes/No	No/Yes	No
Eigensystem Realization Algorithm (ERA)	Time	MDOF	Yes	Yes	Yes	Yes	No
Orthogonal Polynomial	Frequency	MDOF	Yes	No	No	No	Yes
Multi-reference Orthogonal Polynomial	Frequency	MDOF	Yes	Yes	Yes	Yes	Yes
Polyreference Time Domain	Time	MDOF	Yes	Yes	No	Yes	No
Polyreference Frequency Domain	Frequency	MDOF	Yes	Yes	Yes	Yes	Yes
Time Domain Direct Parameter Identification	Time	MDOF	Yes	Yes	Yes	Yes	No
Frequency Domain Direct Parameter Identification	Frequency	MDOF	Yes	Yes	Yes	Yes	Yes
Multi-MAC	Spatial	SDOF	No	Yes	Yes	No	No
Multi-MAC / CMIF / Enhanced FRF	Spatial	MDOF	Yes	Yes	Yes	Yes	No

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4. SYSTEM MODELING

4.1 Introduction

The goal of Volume IV of this Technical report is to document the review of the current methods used to predict the system dynamics of an altered structure or of combined structures based upon a previously defined, modal or impedance, model of the structure(s). Of particular interest is the performance of such modeling methods with respect to experimentally based models.

Volume IV investigates several system modeling techniques to determine their capabilities and limitations from a theoretical and practical viewpoint. Several experimental techniques, practical aspects of the analytical and approximate techniques, test results, modeling results, and analysis of the results are presented to compare and evaluate the various modeling methods. This study presents all of the techniques in a consistent manner from the same origin, using consistent nomenclature, to clearly highlight the similarities and differences inherent in their development which form the basis of the strengths and weaknesses of each technique. To gain practical insights, all of the techniques presented in Sections 2 through 4 of Volume IV of this Technical Report are compared with experimental results. Section 5 of Volume IV presents the new superelement method of dynamic component synthesis as developed by the University of Dayton Research Institute (UDRI).

4.2 System Modeling

System modeling is a computer based technique that is used to represent the dynamic characteristics of a structure. This representation takes the form of either experimental data, modal data, or analytical data. Once the dynamic characteristics of a structure are used to form a model or system model, several uses of the model are possible. First, the effects of design changes or hardware changes to the original structure could be studied. Second, the structure could be coupled with another structure to determine the overall resultant dynamic behavior. Finally, the model can be used analytically to apply forces and determine the forced response characteristics of the structure.

The main objective of system modeling is to use a mathematical representation of the dynamic characteristics of a structure in a computer environment to effectively develop a design or trouble shoot a particular problem of a design. Several techniques of various origin have evolved with the advancement of computer technology. Depending on the situation, each is very effective if properly utilized. Design development is generally considered an extensive long range process that results in an optimally designed structure given the constraints of the project. Trouble shooting involves the evaluation of failures or design flaws which must be corrected quickly.

The most obvious way to classify system models is into analytical and experimental methods. The primary approach to analytical modeling is commonly known as finite element Analysis. Finite element analysis is analytical in nature because only knowledge of the physical properties of the structure is used to build a dynamic representation. This is done by subdividing the structure into discrete elements and assembling the linear second order differential equations by estimating the mass, stiffness, and damping matrices from the physical coordinates, material properties, and geometric properties.

Finite element analysis is extremely useful because no physical test object is necessary to compute resonant frequencies and mode shapes, forced response, or hardware modifications. Therefore, this method comes in very handy in the development cycle, where it can be used to correct major flaws in the dynamic characteristics of a structure before a prototype is built. Since finite element analysis is an approximate analytical technique, experimental modal analysis data is obtained as soon as possible so the analytical model can be validated. Detailed finite element analysis method is not covered in this report.

Experimental modeling techniques are further subdivided into two groups. They are modal models and impedance models. Modal modeling is an experimentally based technique that uses the results of an experimental modal analysis to create a dynamic model based on the estimated poles of the system. The result is a computationally efficient model that is uncoupled due to transformation of the physical coordinates into the modal coordinates which renders the system into a number of lumped mass, single degree of freedom components. This concept is fundamental to modal analysis. This model is then used to investigate hardware changes, couple structures, or compute the forced response. This method is developed fully in Section 2 of Volume IV of this Technical Report.

Impedance modeling uses measured impedance functions or frequency response functions to represent the dynamic characteristics of a structure in the frequency domain. This method uses the experimentally measured functions to compute the effects of hardware changes or to couple together several components. To use this modeling technique, measurements at the constraint or connection points, driving point, and cross measurement between the two are needed to compute a modified frequency response function. Impedance modeling is fully developed for the compliance method and stiffness method in Section 4 of Volume IV of this Technical Report.

Both of the experimental modeling methods are quick and easy to use in their basic implementations. Therefore, they are extremely useful in trouble shooting situations but have limited application in the design cycle.

The final classification of system models is experimental/analytical models or mixture methods. Two techniques are considered mixture methods. The first is sensitivity analysis. Sensitivity analysis is an approximate technique that uses the first term or first two terms of a power series expansion to determine the rate of change of eigenvalues or eigenvectors with respect to physical changes (mass, stiffness, and damping). Therefore, this method is used for trend analysis, selection of hardware modification location, and design optimization. This technique is considered a mixture method because it computes sensitivity values for modal parameters which result from either experimental modal analysis or finite element analysis.

The other experimental/analytical method is component mode synthesis. Component mode synthesis and the building block approach are techniques that use experimental or analytical modal representations of the components of a large or complex structure to predict the resultant dynamic characteristics of the entire structure. Furthermore, this technique has evolved to the point where components are combined in either physical or modal coordinates. Therefore, this technique is truly a mixture method where experimental and analytical data are used to optimize a design. This method is very useful in the design cycle of industries that produce large structures, such as the automotive and aerospace industries. A new component mode synthesis method (not a classical mode synthesis method such as SYSTAN) is discussed in detail in Section 5 of Volume IV and the building block approach is discussed in Section 4 of Volume IV along with the development of the impedance modeling technique.

In summary, system modeling is a diverse field that involves many aspects of structural dynamics. One of the primary goals of Volume IV of this Technical Report is to present this material in a concise and consistent manner to reduce unnecessary confusion and better relate the various factions involved. Furthermore, each technique is applied to a structure to gain further insight into the practical aspects of system modeling.

4.3 Boundary Conditions

In the application of the system modeling techniques, there are three test configurations used to obtain the frequency response functions or the derived modal data base. These three test configurations involve the boundary condition and can be summarized as free-free, constrained, and actual operating boundary conditions. In terms of analytical modeling technique, such as finite element analysis, various boundary conditions can be easily simulated in the mathematical model to predict the dynamic characteristics of a system. Therefore, this gives the analysts the luxury to evaluate the model using desired boundary condition. In terms of experimental modeling technique, in a laboratory environment, usually it is very difficult and too costly to implement the test fixture to simulate the actual operating condition of a complete system, or the constrained boundary condition of a component. Therefore, most of the modal tests are performed under an environment simulating free-free boundary conditions.

Besides the boundary conditions mentioned above, modal tests can be performed on the mass-loaded structures to predict the shifted dynamic characteristics of the original structure. The advantages of adding lumped masses at the connecting or attachment points of a component under testing are: (1) modal coefficients associated with those connecting degrees of freedom can be more accurately excited and described under the mass loading effect, (2) rotational degrees of freedoms at the connecting degrees of freedom can be computed using rigid body computer programs if sufficient number of accelerometers are mounted on the additive masses, (3) analytically, added masses can be removed from the mass-loaded testing configuration, and the enhanced modal parameters of the original structure can be obtained, (4) if the dynamic characteristics of the original structures are available through analytical or experimental method, then more accurate generalized masses can be obtained through these two sets of data. Examples of applying the mass additive technique can be found in References [1] and [2].

4.4 Modal Modeling

This section reviews the system modeling technique known as modal modeling. Modal modeling is also known as the Snyder Technique, Local Eigenvalue Modification, Structural Modifications, Dual Modal Space Structural Modification Method, and Structural Dynamic Modification [3-16]. The common thread of the research mentioned is that all utilize a model in generalized or modal coordinates from experimental data upon which to investigate structural changes. Structural changes are transformed into modal coordinates and added to the structure and the result is resolved to yield the modified modal parameters.

The modal modeling technique was initially published by Kron [3] in 1962 and extended by Weissenberger [4], Simpson and Taborrok [5], and Hallquist, Pomazal and Snyder [6]. Early researchers in this area restricted themselves to a local modification eigensolution technique. Several software packages have been developed employing this technique since the implementation and widespread use of digital Fourier analysis. Notably, Structural Measurement Systems, Inc. first released Structural Dynamics Modification [7] using the local modification procedure. The local modification procedure allows only simple mass, stiffness and damping changes between two general points.

Recent research has progressed in several areas. First, Hallquist and Snyder [8], Luk and Mitchell [9] and SMS [7] have used the local modification technique for coupling two or more structures together. Hallquist [10], O'Callahan [11] and De Landseer [12] have expanded the method to include complex modes. O'Callahan [13] and SMS [14] have found methods to approximate more realistic modifications using local modification for trusses and beams. Mitchell and Elliot [15] and O'Callahan and Chou [16] have developed different methods that use full six degree of freedom representations that depend on the approximation or measurement of the rotational degrees of freedom to make beam or plate modifications. Complete technical details of modal modeling methods can be found in Section 2 of Volume IV of this Technical Report.

4.4.1 Limitations of Modal Modeling

Since the structural dynamic modification method uses the modal parameters of the unmodified structure to predict the effects of the modification , it is apparent that the accuracy of the results is dependent on the validity of the data base supplied. For this reason, it is important that all errors associated with the data acquisition and processing of the unmodified structure are minimized.

Among the sources of error that must be addressed are nonlinearity, standard FFT errors (aliasing and leakage), scaling errors, as well as truncation errors of the modal model itself. Only the most serious of these errors will be addressed at this time.

One of the assumptions in the experimental modal analysis is linearity of the system. Therefore, all methods discussed here are based on this assumption. In reality, inaccurate results will arise when linear system coupling algorithm is used to predict the dynamic characteristics of structure(s) coupled together by nonlinear joints.

Leakage is a measurement error that arises from the processing of signals that are not periodic in the time window of the signal analyzer. Because of this truncation in the time domain, the Fourier coefficients of the sampled signal do not lie on the Δf of the analyzer. This causes energy at a specific frequency to spread out into adjacent frequency bands, and results in an amplitude distortion at the actual frequency. It is this amplitude error that causes scaling errors in the modal mass and stiffness estimates associated with each mode. This in turn affects the modification process by scaling the predicted modes of the modified structure by an amount that is proportional to the amplitude error of the original data.

This leakage problem can be reduced by: (1) using periodic excitation, or transient excitation signals such as burst random, or, (2) using the H_v frequency response function procedure, or, (3) using cyclic averaging technique in the measurement stage, or, (4) taking data with smaller Δf, or, (5) using windows on the time domain measurement, such as, impact with exponential window applied to the response signals. If exponential window is used, it must be accounted for in the parameter estimati on because it adds artificial damping to the structure. Care must be taken not to overcompensate for this damping allowing the poles of the system to become negatively damped.

Modification errors often arise from using a modal data base that is not properly scaled relative to the system of units used by the modeling software. This type of error will occur if the modal parameters are estimated using frequency response functions which were measured using improper transducer calibrations. This improper scaling once again results in improper estimates of modal mass and stiffness used by the modeling software.

Up to this point, it has been shown that the accuracy of the results is dependent on the amount of error in the data base. Another concern that needs to be addressed is the validity of the modal model. Because the effects of a structural modification are calculated in modal space, if an insufficient number of modes are included in the original data base, there will be a limit to the number of modal vectors that can be predicted. This phenomena is known as modal truncation and should be considered in choosing a frequency range for thc analysis. It may be desirable in some cases to extend the frequency range of data acquisition above the actual frequency range of interest for the data base to include a few extra modes. This extended frequency range will improve the calculation of out of band residuals, and may help for the case where these out of band modes are shifted into the frequency range of interest by the modification. Care must be taken in extending the frequency range, to prevent an excessive loss of frequency resolution.

Another concern in the development of the modal model is the number of degrees of freedom to be included in the analysis. By definition, the number of degrees of freedom must be equal to or greater than the number of modes in the frequency range of interest. Realistically, the number of DOF should be much larger than the number of modes of interest to accurately define the individual modal vectors. Since modal coefficients exist only at points where data has been taken, it is possible to miss nodal lines of the structure if too few points are included in the analysis. This generally becomes a more significant problem for higher order mode shapes.

Once the original data base has been established, the modal parameters can be estimated using any of several existing parameter estimation algorithms. All of these algorithms attempt to yield a best estimate of the actual parameters. Because there will always be some degree of experimental error in the data, the resulting estimates of modal parameters will be subject to error. In order to minimize this error, it is advantageous to use some sort of least squares implementation to yield a best estimate of modal parameters.

The use of SDOF versus MDOF parameter estimation algorithms is determined by the modal density of the structure being analyzed. If a SDOF method is used for a structure with closely coupled modes, poor estimates of modal mass and stiffness are obtained, and the modification routine will yield poor results.

The various errors mentioned in the previous paragraphs are commonly committed, and easily overlooked when performing a modal test. This is not intended to be an exhaustive list of errors affecting modal modeling, but an indication of the types of things that must be kept in mind when establishing a valid modal data base. Without good estimates of the original structures modal parameters, there can be no serious attempt at accurately predicting the characteristics of the modified structure.

4.4.2 Validation of Experimental Modal Models

As mentioned in the previous section, the accuracy of the modal modeling or structural modification results is dependent on the validity of the modal model supplied. There will always be some degree of experimental and modal parameter estimation errors in the data base. Therefore, it is important for the users to qualitatively, and if possible, quantitatively, examine the validity and errors of the modal model before it is used to predict the system modeling or modification results. Although perfect results should not be desired in the application of modal modeling technique, it is important to realize that any modal model obtained experimentally is far from being perfect. Therefore, it is suggested that the experimental modal model be validated, or optimized , before the model is input to any modal modeling algorithm. Some of the validation methods are briefly summarized in the following sections.

4.4.2.1 Frequency Response Function Synthesis

Synthesizing a frequency response function (not used in the estimation of modal parameters) using extracted modal parameters and compared with a measured frequency response function at the synthesized measurement degree of freedom is, in general, a common practice during the modal parameter estimation process. If a good match exists between these two sets of frequency response functions, then it is a good indication that the extracted modal parameters are agreeable with the measurement data. But this doe s not guarantee that there is no error exist in the modal data base.

4.4.2.2 Modal Assurance Criterion

Modal Assurance Criteria (MAC) [29] is commonly used to check the consistency of the extracted modal parameters, when more than one estimate of each mode is available.

4.4.2.3 Detection of Mode Overcomplexity

This method qualifies each mode by a number called the Mode Overcomplexity Value (M.O.V.) and the global Modal Model by the Mode Overcomplexity Ratio (M.O.R.) [30]. The basic idea of the Mode Overcomplexity test is that, for good modal models with complex modes, the frequency sensitivity for an added mass change should be negative. If it happens that the sensitivity is positive, it is caused by either an incorrect scale factor (modal mass) or by the fact that the phase angle of the complex modes compared to the normal mode phase angle exceeds a certain limit; in other words, it is due to an overcomplexity of the mode shape.

The MOV is defined as the ratio of the number of positive frequency sensitivities over the number of all the frequency sensitivities for a particular mode. To give more weight to points with a high modal displacement compared to points with a small modal displacement, a weighted sum is introduced to give a more general evaluation of the modal model. The value of MOV is between 1 and 0, the bigger the value, the modal model is more overcomplex.

The MOR is defined as the ratio of $\sum_{i=1}^{m} MOV_i$ over ($1 - \sum_{i=1}^{m} MOV_i$) which gives a one figure assessment of the modal model with respect to its overcomplexity. The MOR ranges from zero to infinity. A low MOR value indicates good modal data, while a large MOR indicates a scale factor problem or a overcomplexity problem.

4.4.2.4 Mass Additive/Removal Technique

This technique employs a mass additive or removal procedure to verify or validate experimental modal model in the application of modal modeling technique. Modal model can be obtained from either the original structure, or, mass-loaded structure [2]. If a modal model is obtained from the original structural configuration, then, comparisons can be made between the analytically predicted dynamic characteristics of the mass-loaded structure - from the modal modeling algorithm - and the test results (such as modified resonant frequencies) obtained from the physically modified mass-loaded structure. If there is no good agreement between these two sets of results for the mass-loaded structure, then this is an indication that global or local scaling errors, or overcomplexity of some measured complex modes, exist in the experimental modal model. If high quality data are desired in predicting the system dynamics of the altered structure or combined structure(s), then the previously determined experimental modal model needs to be validated, if possible, or, a new set of data needs to be recollected before any modeling application is attempted.

For the second case, i.e., if a modal model is obtained from the physically mass-loaded structure, then comparisons can be made between the analytically predicted and experimentally measured dynamic characteristics of the mass-removed structure. Similarly, if there is no good agreement between these two sets of data, this indicates some errors exist in the original modal model. In Reference [2], using approximated real modes from the measured complex modes, a modal scaling procedure can be used to correct the global scaling errors in the experimental modal model.

The number of masses and the size/weight of each additive mass that can be added to the structure is dependent on the total mass and size of the structure(s). In general, the following rules can be used as guidelines in considering the number and size(s) of the additive mass(es):

- The added mass(es) can be considered rigid in the frequency range of interest.
- With small amount of mass(es) added or removed to or from the structure, the mode shapes can be considered unchanged before and after the modification.
- Sensitivity of the change of system dynamics is dependent on the location(s) of the added mass(es). In other words, if there is only one mass added to a large structure, then some of the

modes may not be sensitive enough to alter their frequencies due to the fact that the added single mass is near the nodal points of such modes.

- Rotational degrees freedom, if permitted, can be extracted from the rigid body motion of the lumped mass(es). This information is very useful if the mass mounting point(s) is(are) the connection or coupling point(s) of the structure(s).

4.4.2.5 Improvement of Norms of Modal Vectors

Zhang and Lallement [31] proposed a method to improve the norms of the measured modal vectors and then calculate the generalized modal masses of the original structure. This method will correct modal scaling errors in the modal data base. This method requires a set of modal data from the initial structure and a set of data from the mass loaded (perturbated) structure.

4.4.3 Modal Modeling Summary

In summary, modal modeling has been discussed from its inception by Kron through present day research inolving beam modifications in the modal domain. Modal modeling is a technique that is very quick, because the generalized coordinates have a reduced number of degrees of freedom. Therefore, many modifications can be investigated in a short time. Earlier, this technique was presented mainly as a trouble shooting technique. In fact, researchers [16] have found this method to be three to six times faster than analytical approaches. This ratio increases with the size of the problem. The speed of this technique and its interactive implementation make it well-suited for on-site problem solving and initial design cycle work.

Many limitations are apparent in the development. First, if experimental data are used, the frequency response functions must be carefully calibrated. This technique is extremely sensitive to experimental errors in general. Data must be carefully acquired to avoid bias errors such as leakage and aliasing. Errors made in the estimation of the frequency response functions translate into errors in the modal model and modal matrix $[\Psi]$. Modal parameter estimation is extremely critical in modal modeling. Parameter estimation is a two-stage process that estimates eigenvalues which are used to compute the modal model and the modal vectors which make up the transformation matrix.

Recall that a convenient form of the model is for unity modal mass or unity scaling. Examination of the modal stiffness and damping matrix reveals that the estimate of the damping ratio ζ is involved in both matrices. Unfortunately, damping is a difficult parameter to estimate. This is one of the major limitations of the accuracy of an experimental modal model. Fortunately, if great care is taken in the measurements, the magnitudes of this error are not great enough to cause more variation than found in normal experimental error.

Another source of error is truncation. Errors occur in two forms: geometry and modal truncation. Geometry truncation is a problem that occurs when not enough physical coordinates are defined to adequately describe the dynamics of the structure. Higher order mode shape patterns are not properly defined unless enough points are defined along the shape to describe it. A good rule of thumb is to apply Shannon's sampling theorem to the highest order mode expected. Geometry truncation also occurs when all pertinent translational and rotational degrees of freedom are not measured. If a structure exhibits motion in all translational degrees of freedom and only one is measured, the associated error is defined as geometry truncation. In general, the number of data points should be much greater than the number of modes of interest to avoid geometry truncation.

Modal truncation refers to the number of modes included in the data set. Since the modified mode shapes are a linear combination of the original mode shapes, the rank of the original modal matrix limits the possible dynamic changes that can be calculated. The lower limit of the number of modes required for even simple structures is six [32], to have sufficient rank to accurately predict the results for the first few modes. A good rule of thumb is to include several modes beyond the frequency range of interest to insure the validity of the results within the frequency range of interest. Another serious modal truncation error occurs when the rigid body modes of a free-free structure are not included when that structure is tied to ground.

Based on the preceding discussion it is apparent that the use of modal modeling programs with experimental data requires carefully acquired data and good parameter estimation results. These problems can be overcome by carefully designing the modal test and using the proper parameter estimation algorithms for the given data [33]. This technique works equally well with analytical data and has been implemented in this manner by Structural Measurement Systems [34].

The issue of complex versus real modes has been debated greatly in recent years. To be completely accurate the complex form of the modal modeling technique should be used when nonproportional or heavy damping exists in a structure. Using a real normal mode in this case will cause erroneous results [11]. One compromise is to use a normalized set of real modes derived from the measured complex modes described in Section 2.7 of Volume IV of this Technical Report.

The use of beam modifications greatly increases the capability of modal modeling. Simple scalar modifications and lumped masses are limiting and unrealistic. Beam modifications require rotational information at the modification points. This information is not readily available but can be obtained with some effort experimentally or analytically. Once rotational information is readily available from experimental sources, modal modification will become a more powerful trouble shooting tool.

4.5 Sensitivity Analysis

Sensitivity analysis is an approximate technique that determines the rate of change of eigenvalues and eigenvectors using a Taylor expansion of the derivatives. This technique was developed by Fox and Kappor [35] and Garg [36] initially in the late Sixties and early Seventies. Van Belle and VanHonacker [20,37] further developed its use with mechanical structures and implemented it for use directly on modal parameters. This technique is approximate because only one term (differential) or two terms (difference) of the series expansion are used to approximate the derivative.

Sensitivity analysis is useful in two ways. First, if a certain type of modification of a structure is required, Sensitivity analysis determines the best location to make effective structural changes. Sensitivity analysis also is used to predict the amount of change by linearly interpolating the amount of change from the sensitivity to achieve the desired dynamic behavior. This last method is very time consuming, especially when using difference sensitivities to maintain accuracy.

4.5.1 Limitations of Sensitivity Analysis

The detailed development of the sensitivity analysis equations is provided in Section 3 of Volume IV of this Technical Report. Different expressions are developed for the sensitivity of modal parameters to changes in mass, stiffness and damping. For the calculation of the sensitivity of an eigenvalue, λ_k, only the corresponding eigenvector is necessary. Calculation of eigenvalue derivations do not require complete information on the dynamics of the structure [38].

Finite difference or difference sensitivities use a second term in the approximation to account for the amount of the physical parameter change. Nevertheless, an improved estimation is obtained only when the change in the physical parameter is small. If the magnitude of change is increased beyond a certain value, the result will be even worse. The difference sensitivity equations involve a term, $\frac{1}{(\lambda_k - \lambda_m)}$, that is involved with the second order derivative of an eigenvalue. If there are two close modes, this term will become large so that the second order derivative contribution dominates the approximation. Further more, if two adjacent modes are very close to each other, the term will diverge so the result will often be unacceptable. Hence, care must be taken when a set of modal data shows repeated eigenvalues. The current sensitivity analysis method does not attempt to account for repeated eigenvalues. Since, for repeated eigenvalues no unique definition of the modal vectors associated with the repeated roots exists, sensitivity analysis cannot be used for modal data containing repeated eigenvalues unless a normalization for the repeated eigenvalue case can be defined.

From this discussion, it is seen that the expressions used to compute differential or difference sensitivity from modal parameters are in the form of the transfer function matrix. Because only one or two terms are used from the Taylor expansion, the technique is an approximate one. Since only modal parameters are necessary, this technique is equally applicable to experimental or analytical data. Currently it is implemented with experimental data [38].

VanHonacker [38] has shown this method to be accurate for only small incremental changes. The differential method is far less accurate than the difference method. For small changes of mass, stiffness, or damping the differential technique will accurately predict the eigenvalue shift. For more significant parameter changes, the difference technique is recommended. Therefore, sensitivity analysis has only limited application in the prediction of the effects of structural modifications.

Sensitivity is extremely useful as a preprocessor to Modal Modeling or Finite Element Analysis techniques. The sensitivities of a structure can be computed rapidly from the modal parameters to determine the optimal location at which to investigate a modification. Furthermore, the sensitivity value is useful in determining how much of a modification is required. Therefore, Sensitivity Analysis is a valuable tool in the optimization of a design.

This technique has several limitations. First, the results are only as good as the modal parameters used in the calculations. Therefore, all of the experimental errors and parameter estimation limitations which hinder other modeling methods apply to sensitivity techniques as well. Notably, a limited number of modes are available from zero to the maximum frequency measured. Although not currently implemented with analytical data, any inaccuracies in an analytical model would similarly deteriorate the calculations when used with modal data. In the experimental case, geometry truncation errors are significant due to the exclusion of rotational degrees of freedom.

As a preprocessor to other modeling techniques, sensitivity has advantages. The computations are fast and stable, especially when compared with a complete eigensolution. It is intuitive in nature because it provides rates of change that allow the selection of the best type and location of modification as well as a comparison of different modifications. This provides a large amount of information that offers much insight into the dynamic behavior of a structure.

4.6 Impedance Modeling

The general impedance method was first introduced by Klosterman and Lemon [39] in 1969. Due to the state of measurement equipment at that time the method was not pursued further. As the ability of Fourier analyzers to accurately measure frequency response functions improved in the late Seventies, the interest in General Impedance Techniques was renewed. Two techniques are developed in this chapter using experimental data. Both methods use measured frequency response functions or synthesized frequency response functions.

The general impedance technique is formulated in two ways: frequency response and dynamic stiffness. The dynamic stiffness approach was initially developed by Klosterman [40] and implemented by Structural Dynamics Research Corporation as SABBA (Structural Analysis using the Building Block Approach) [41]. This technique is primarily used to couple together structures to predict the total dynamic characteristics using the concept of superposition. Thus, the phrase building block approach was applied to this technique.

The frequency response method was published and implemented by Crowley and Klosterman [42] at the Structural Dynamics Research Corporation in 1984 and referred to as SMURF (Structural Modifications Using Response Functions). It is primarily a trouble shooting technique. It operates on frequency response functions; therefore, no modal model is necessary to investigate structural modifications. Problems may be solved by acquiring several frequency response functions and investigating the effects of modifications. The modified frequency response function is computed as a function of frequency by simple block operation using the original frequency response function and a frequency representation of the structural modification.

4.6.1 Limitations of Impedance Modeling

The general impedance technique is a method that employs the use of frequency response data to investigate coupling of structures and structural modifications. The dynamic stiffness approach is the more powerful of the two. The advantage of this technique is that a large array of investigations may be conducted. Also, the method is not as cumbersome as the component modal synthesis technique because it has a reduced number of degrees of freedom.

Due to the dynamic stiffness formulation, modal models of experimental or analytical origin may be combined or modified using an array of mass, stiffness, damping, beam, or matrix elements represented in impedance form. This brings more analytical capability directly to an engineer in a mini-computer environment as implemented currently [41] when compared with modal modeling techniques.

Due to numerical problems, Klosterman recommends use of synthesized frequency response functions to build the dynamic stiffness matrix. This introduces errors made in modal parameter estimation, but reduces numerical problems associated with noisy frequency response functions measurements because the parameter estimation process fits a smooth curve through the measured data. Modeling of this type requires carefully acquired, and properly calibrated, data to obtain the best modal model possible.

One of the major problems with the dynamic stiffness approach is the determination of the $[H]^{-1}$ matrices. The inverse of the matrix must exist. This problem forces the use of modal data because the inversion process is numerically unstable [43]. The number of modes must be much greater than the number of constraint points to insure the existence of the inverse.

The most serious limitation of the dynamic stiffness approach is the computational speed and stability. When implemented initially, only individual frequency response functions were computed for the resultant structure. This implementation is efficient, but the computations are somewhat unstable unless synthesized frequency response functions are used. This led to the application of a determinant search algorithm to compute the resultant eigenvalues and eigenvectors. This algorithm is not computationally efficient because the equations are solved frequency by frequency. This fact has led to more widespread use of component modal synthesis techniques that are described in Section 5 of Volume IV of this Technical Report.

The frequency response method is implemented for single constraint situations to avoid the matrix inversion problem. This makes it useful for trouble shooting situations. Since measured frequency response functions are used in the calculations, no modal model is necessary. Therefore, if the necessary frequency response functions exist, modifications may be made directly to obtain the modified frequency response. This makes the frequency response technique the fastest trouble shooting technique, but only modified frequency response functions are computed not modal data. This technique is slower than modal modeling when modal data is desired.

Data is required for driving points and cross measurements at the constraints, response point, and driving point to compute the modified frequency response. Therefore, the impact testing or multiple reference techniques are most convenient for the frequency response method. Synthesized frequency response functions can be used in this technique when the desired frequency responses are not available, but the advantage of avoiding modal parameter estimation is lost. Modal modeling is a better alternative at this point because the entire set of modified modal parameters is computed. Only individual modified frequency response functions result from the frequency response method.

The frequency response technique is sensitive to measurement errors such as leakage, aliasing, and random noise. Wang [44] has found the errors largest at anti-resonance. This is due to the fact that the signal to noise ratio is poor at anti-resonance. Therefore, as the magnitude of modification or constraint increases, the more the accuracy of the calculations will deteriorate because frequencies will shift closer to or past anti-resonances.

4.7 Component Dynamic Synthesis

Component dynamic synthesis is an analytical procedure for modeling dynamical behavior of complex structures in terms of the properties of its components or substructure. The procedure involves explicitly every component in the structure the advantages of which are many-fold: analysis and design of different components of the structure can proceed independently, component properties can be obtained from tests and/or analysis, and the size of the built-up structure analysis problem can be reduced to manageable proportions.

Component synthesis with static condensation [45,46] has long been used for improving efficiency of static analysis. In this method, known as substructuring technique, unique or functionally distinct components of a structural system are analyzed separately, condensed, and then combined to form a reduced model. This reduced model, having fewer degrees of freedom, is generally more economical to analyze than the original structural model. The static condensation is an exact reduction procedure.

In dynamic analysis, exact reduction of an individual component is dependent upon the natural frequencies of the total structural system which are yet unknown at the component level. Frequency independent or iterative reduction methods must therefore be used, which introduce approximations. The various reduction methods are collectively known as component dynamic synthesis or modal synthesis (CMS).

The objectives of this section are to review the state of the art in component dynamic synthesis and to develop and implement an improved dynamics synthesis procedure.

4.7.1 Dynamic Synthesis Methods

In order to review the existing dynamic synthesis procedures, it is necessary to define certain frequently-used terms. A component or substructure is one which is connected to one or more adjacent components by redundant interfaces. Discrete points on the connection interface are called boundary points and the remainder are called interior points. The following classes of modes are commonly used as basis components in component dynamic model definition. Details of these mode sets is given later in this report.

1. Normal Modes: These are free vibration eigenmodes of an elastic structure that result in a diagonal generalized mass and stiffness matrix. The normal modes are qualified as free or fixed interface modes, depending on whether the connection interfaces are held free or fixed. Loaded interface normal modes simulate intermediate fixity of the interfaces.

2. Constraint Modes: These are static deflection shapes resulting from unit displacements imposed on one connection degree of freedom and zero displacements on all the remaining degrees of freedom.

3. Attachment Modes: The attachment modes are static deflection shapes defined by imposing a unit force on one connection degree of freedom while the remaining connection degrees-of-freedom (DOFs) are force free. If the structure is unrestrained, this mode set will consist of inertia relief modes. Attachment modes are also static modes.

4. Rigid-body Modes: These are displacement shapes corresponding to rigid body degrees of freedom. They may be considered a subset of normal modes corresponding to null eigenvalues or else defined directly by geometrical consideration.

5. Admissible Shape Functions: These are any general distributed coordinates or space functions, linear combinations of which simultaneously approximate the displacement of all points of an elastic structure. The only requirements are that the admissible functions satisfy geometry boundary conditions of the component over which they are defined, and satisfy certain differentiability conditions. These are analogous to finite element shape functions.

Static condensation or Guyan Reduction [46] is the simplest of all component dynamic synthesis techniques. The approach is a direct extension of static condensation. The transformation matrix of static constraint modes which is used to reduce the order of the stiffness matrix in static analysis is also used to reduce the order of the structure mass matrix. The kinetic energy of the interior nodes is represented by only static mode shapes. Drawbacks of this approach are obvious. The static modes are not the best Ritz modes for component dynamic representation.

The concept of Component Modal Synthesis (CMS) was first proposed by Hurty [47]. Component members were represented by admissible functions (low- order polynomials) to develop a reduced order model. This procedure is essentially the application of the Rayleigh-Ritz procedure at the component level. Hurty extended the method to include discrete finite element models[48]. This method proposed that the connect DOF of a component were fixed or had a zero displacement. Hurty then partitioned the modes of the structure into rigid body modes, constraint modes, and normal modes. The constraint and rigid body modes were found by applying unit static load to each of the connection points individually to obtain static deformation shapes of the structure. These modes were added to the constrained normal modes to form a truncated mode set used in the synthesis of the entire structure. A simplification of Hurty's fixed interface method was presented by Craig and Bampton [49]. Substructure component modes were divided into only two groups: constraint modes and normal modes. This resulted in a procedure which is conceptually simpler, easier to implement in analysis software. Bamford [50] further increased the accuracy of the method by adding attachment modes which improve the convergence of the method. The attachment modes are the displaced configurations of a component when a unit force is applied to one boundary degree of freedom while all other boundary DOF remain free of loads.

Goldman [51] introduced the free interface method, employing only rigid body modes and free-free normal modes in substructure dynamic representation. This technique eliminates the computation of static constraint modes, but their advantage is negated by the poor accuracy of the method. Hou [52] presented a variation of Goldman's free-interface method in which no distinction is made between rigid body modes and free-free normal modes. Hou's approach also includes an error analysis procedure to evaluate convergence.

Gladwell [53] developed "branch mode analysis" by combining free interface and fixed interface analysis to reduce the order of the stiffness and mass matrices for individual substructures. The reduction procedure depends upon the topological arrangement of the substructures in the model. Thus, reduction of any one substructure requires knowledge of the arrangement of all substructures in the model.

Bajan, et al. [54] developed an iterative form of the fixed interface method. He showed that significant improvements in synthesis accuracy can be achieved by repeating the reduction, based on updated estimates of system frequencies and mode shapes.

Benfield and Hruda [55] introduced inertia and stiffness loading of component interfaces to account for adjacent substructures. The use of loaded interface modes is shown to have superior convergence characteristics.

Motivated by the need to use experimental test data, MacNeal [56] introduced the use of hybrid modes and inertia relief modes for component mode synthesis. Hybrid modes are substructure normal modes computed with a combination of fixed and free boundary conditions. Inertia relief attachment modes are attachment modes for components with rigid body freedoms. MacNeal also included residual inertia and flexibility to approximate the static contribution of the truncated higher order modes of a component. Rubin [57] extended the residual flexibility approach for free interface method by introducing higher order corrections to account for the truncated modes. Klosterman [58] more fully developed the combined experimental and analytical method introduced by MacNeal. Hintz discussed the implications of truncating various mode sets and developed guidelines for retaining accuracy with a reduced size model[59].

Many authors have compared the techniques discussed. No method clearly appears to be superior to the other. The constrained interface method of Craig and Bampton and Hurty is expected to be the most accurate when the connect degrees of freedom have little motion. The free interface method with the use of residuals as proposed by Rubin appears to be more accurate than the constrained approach.

Recent research has centered on comparisons of the various methods. Baker [60], for example, compares the constrained and free-free approach using experimental techniques and also investigates using mass additive techniques and measured rotational DOF [45]. This investigation was motivated by a need to find the best method for rigidly connected flexible structures. In this connection, the constrained method produced the best result. Klosterman [58] has shown the free-free method to be accurate for relatively stiff structures connected with flexible elements. This supports Rubin's conclusion [57] that the free-free method is at least as accurate when residual effects are accounted for. These conclusions are intuitive because the type of boundary condition imposed in the analysis that best represents the boundary of the assembled structure provides the best accuracy in the modal synthesis.

Meirovitch and Hale [61] have developed a generalized synthesis procedure by broadening the definition of the admissible functions proposed by Hurty [47]. This technique is applicable to both continuous and discrete structural models. The geometric compatibility conditions at connection interfaces are approximately enforced by the method of weighted residuals.

The method due to Klosterman [58] has been implemented in an interactive computer code SYSTAN [62] and that due to Herting [63] is available in NASTRAN. The latter is the most general of the modal synthesis techniques. It allows retention of an arbitrary set of component normal modes, inertia relief modes, and all geometric coordinates at connection boundaries. Both the fixed-interface method of Craig and Bampton, and the MacNeal's residual flexibility method, are special cases of the general method. Other analyses presented in the literature based on modal synthesis techniques are not incorporated into general structural analysis codes. In general, there is a lack of sophistication in available software.

4.7.2 Damping Synthesis Methods

The methods of dynamic synthesis are particularly useful and sometimes the only alternative available in damping prediction for built-up structures. Most frequently, damping has been synthesized in the manner analogous to stiffness and mass synthesis with the assumption of proportional damping. Hasselman and Kaplan [64] used complex modes of components with nonproportional damping. Obtaining damping matrices in the presence of general energy dissipating mechanisms in a complex structure is one of the complicating factors, however. In such cases an average damping behavior can be obtained from tests in the form of cyclic energy dissipated versus peak stored energy correlation or damping law. Kana et al. [65] synthesized system damping based on substructure stored energy at the system modal frequency. Soni [66] developed a substructure damping synthesis method applicable to cases where substructure damping varies greatly and irregularly from mode to mode. The procedure has been validated in experimental studies [67]. Jezequel [68] employed fixed interface component modes together with mass loaded interface modes replacing the static constraint mode in his damping synthesis method. Mass loading results in an improved representation of interface flexibility and dissipation; however, the use of constrained interface modes make it difficult to implement it in experimental testing.

The subject of component dynamic synthesis has received increasing attention in recent years. Reference [69] presents several detailed reviews, applications, and case histories, with particular emphasis on experimental characterization of component dynamics.

4.7.3 A Comparison of the Synthesis Methods

While differing in their detailed treatment, all the synthesis methods have the following objectives: (1) to efficiently predict the dynamics of a structure within required accuracy for a minimum number of DOFs; (2) to analyze the components totally independent of other components so the design process is uncoupled, and (3) to use component properties derived from tests and/or analyses. The various methods discussed in the preceding paragraphs only partially satisfy the three basic requirements. Common to all modal synthesis methods discussed in the preceding is the complexity of the matrix manipulations involved in setting up the coupling and assembly procedure to obtain the final reduced equation system.

The major limitation on the use of the existing modal synthesis methods is their lack of compatibility with practical experimental procedures. Although various types of component dynamic representations have been proposed, only those requiring normal modal properties are practical. Test derived modal representation is, in general, incomplete; the component normal modes obtained assuming any type of support conditions at the interfaces are, in general, different from those occurring when the components are acting within the compound of the total system. Since only a limited set of modal data is obtainable, the interface flexibility is not adequately modeled. Depending upon the synthesis method used, additional information is therefore usually required to approximate the effect of interface condition or modal truncation.

Fixed interface mode synthesis methods employ static deflection shapes. In an experimental setup, constraining interface degrees of freedom proves impractical, particularly when large dimensions or a large number of connection points are involved. Also, damping data associated with static modes is unavailable. For these reasons the free interface based modal synthesis methods are best suited for achieving test compatibility. These methods also lend themselves to accuracy improvements via the artifice of interface loading or by augmenting the normal modal data with residual flexibility and inertia effects of truncated modes.

4.7.4 Superelement Method

The objective of this work is to investigate and develop component dynamic synthesis procedures and associated computer software which (a) combines component dynamic characteristics obtained from modal tests or analyses or both; (b) accounts for the effects of differences in interface boundary constraints of the component structures in the modal test and in the comparison of total structure; and c) reduces inaccuracies due to modal truncation.

In view of the above objectives, and the assessment of existing synthesis methods, reviewed in the preceding section, the free interface modal synthesis methods are studied further in this work. For completeness, the fixed interface and the discrete element representations are also considered. For certain components the use of constrained interface conditions may be unavoidable. Structural components such as panels, stringers, simple masses, vibration control devices, etc. are conveniently input to the synthesis procedure via discrete elements.

A principal feature of the work developed here is the component dynamic model reduction procedure that leads to an exact and numerically stable synthesis. In order to affect component coupling, neither the specification of external coupling springs nor an user-specified selection of independent coordinate is required. Existing synthesis procedures suffer from these drawbacks. Component dynamic models considered include free-free normal modes with or without interface loading, up to second order stiffness and inertia connections accounting for the effect of modal truncation, fixed interface modes, and also the physical coordinate components. The modal reduction procedure involves interior boundary coordinate transformations which explicitly retain connection interface displacement coordinates in the reduced component dynamic representations. Interior coordinates may include physical, modal, or any admissible coordinates. Components in this reduced form are termed "superelement" because they are a generalization of the conventional finite elements of structural mechanics. The problem of component dynamic synthesis is then reduced to the assembly of the superelements. The direct stiffness approach and all subsequent processing operations of the finite element method are then applicable.

In order to develop an improved component dynamics synthesis procedure, there are two key issues to focus on: the modeling of component dynamics and the coupling of component coordinates. As seen in the review, no one method of component dynamic modeling is shown to be superior to any other. The methods of synthesis developed in the literature use one or other type of component representation. Aerospace structures involve a wide variety of components and any single component dynamic modeling method may not be uniformly suitable to all the components. With this in mind, a generalized synthesis method was developed which permits different types of component models and an associated coupling procedure. This material is reviewed in detail in Section 5.2 of Volume IV of this technical Report.

4.7.5 Summary - Superelement Method

A set of consistent Ritz transformations was derived that lead to an exact, efficient and unified procedure for coupling component dynamic models. A broad class of test and/or analysis derived component dynamic models were considered in this work. These dynamic models are compatible with the state of the art experimental modal testing as well as analytical procedures and permit improvement of synthesis accuracy through the inclusion of flexibility, inertia and damping corrections of truncated high frequency modes in the component dynamic models. The synthesis procedure developed in this work may be considered as a generalization of the Craig-Bampton method to include free interface normal modes, residual flexibility attachment modes, loaded interface normal modes, and any general type of component modes or admissible shapes that adequately represent the dynamics of a component. Several existing methods such as MacNeal's method, and Rubin's method are shown to be the particular cases of the generalization presented in this work. As a result of this generalization, the different components of a built-up system may be characterized using any type of dynamic model which is most convenient regardless of the manner in which the other components are characterized. Furthermore the component coupling is exact and computationally efficient; no artificial coupling element or the user specification of independent coordinates is required. The developed procedure is implemented in a stand alone computer program COMSYN which is documented in Appendix C of Volume IV of this Technical Report.

The component dynamics reduction method developed in this work transforms the component dynamic coordinates to the superelement coordinates containing the physical coordinates of the connection interfaces as well as any desired noninterface points. As a result of this reduction procedure the component subsystems take the form of a finite element. It is therefore possible to obtain system synthesis even with nonlinear components. The addition of the necessary data handling and solution algorithms to treat nonlinear components will greatly enhance the capabilities of COMSYN. It is recommended as a further work.

4.8 Summary - System Modeling

Experimental modal analysis developed in the past decade can provide a valid data base used in the application of system modeling techniques. The success of applying system modeling techniques in improving the engineering quality of the industrial products through a design cycle, is dependent on the quality of the experimental data, and the accuracy of the system modeling algorithm used to predict the altered system dynamics of a structure or combined structure(s).

Generally speaking, all system modeling techniques, which include modal or impedance modeling method, sensitivity analysis, and the component mode synthesis method, can predict satisfactory results if a complete and perfect experimental model can be obtained from testing and used as data base for system modeling predictions. In reality, there exist many uncertainties and difficulties in obtaining a complete modal or impedance model representing a physical structure. Difficulties in simulating actual boundary conditions in the testing laboratory, lack of rotational degrees of freedom measurement, incomplete modal model due to limited testing frequency range, nonlinearities existing in the structure under test, scaling errors, and mode overcomplexity, could seriously affect the quality and completeness of the experimentally-derived modal or impedance model. These deficiencies in obtaining a reliable and complete experimental model make the system modeling technique a much less powerful tool in the application of engineering design. In other words, currently, the weakness of applying system modeling techniques comes from those limitations and uncertainties to obtain a desired modal or impedance model of physical structure(s).

At the present time, many efforts have been dedicated by researchers to overcome those deficiencies in obtaining a desired experimental modal or impedance model, such as the development of rotational transducers. Further research and practices are still needed to develop a well-defined engineering procedure and criterion to make the use of the system modeling techniques a more powerful and reliable tool in engineering practices.

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5. UNIVERSAL FILE STRUCTURE

5.1 Introduction

One of the significant problems of experimental and analytical structural analysis involves combining, comparing, and correlating data that exists in different formats, in different software and in different hardware. This problem is not a technological problem so much as it is a logistical problem. In order to address this problem, a standardized data base structure needs to be identified and supported by all of the organizations operating in the structural dynamics area. While this goal cannot be accomplished, ultimately, until an official standard exists, it is possible to alleviate the problem by identifying the basis for a data base structure and providing this information to the organizations that would eventually be involved in the development of an official standard. The objective of Volume V of this Technical Report is to document a data base standard that can provide a means for data exchange.

The requirements for a data base standard that can be applicable to different software and hardware environments must be very general so that any level of user can support the data base standard. For this reason, an eighty character per record, ASCII format is the only basis for the data base structure that can be supported in the required environments. It is important to note that this data base format is not intended to be used as an internal format within software or as the basis of a hardware format. This sort of format is only useful as a mechanism for input and output to media that are compatible with the different environments that may need to be utilized.

5.2 Format Development

In order to develop the data base structure, the types of formats or capabilities that were needed were first identified. The basic requirements included a file structure that could define the geometry of the nodal degrees of freedom, measurement data at the nodal degrees of freedom, and modal parameters associated with the nodal degrees of freedom. In addition to these basic requirements, information concerned with the source of the file information and the units of the data is needed to qualify the information in the files that belong to a specific data base.

Once the basic requirements were identified, existing data base structures were evaluated to determine whether a current format would be sufficient or could be modified to meet the basic requirements. In this regard, consideration was given to the basic requirement that the format be ASCII, to whether the data base already included the required formats, to whether the data base is being utilized at the present time, etc. Several possibilities existed with respect to an internal data base developed at the University, to data bases utilized by finite element programs, and to data bases utilized by experimentally based programs. For example, the University of Cincinnati Structural Dynamics Laboratory (UC-SDRL) had developed an ASCII format data base in order to compare finite element and experimental test data. This format was limited to nodal geometry and modal parameters and would have to be expanded in order to service all of the needs that exist in the analytical and experimental structural dynamics area.

As a result of this review and deliberation, the Universal File[1,2] concept utilized by Structural Dynamics Research Corporation (SDRC) was adopted as the basis for the data base structure. In general, this Universal File concept addressed the needs of both the analytical and experimental aspects of the structural dynamics area. Also, there is considerable experience and history of the use of this Universal File structure in both the analytical and experimental programs that SDRC has developed. The structure of the Universal File is documented very well and has already been adopted by other organizations as the basis for internal data base structures. Additionally, SDRC supported the concept of a wider application of the Universal File concept and has added Universal

File structures to address potential needs that previously had not been identified. For example, the Units File (File Type 156) has been added to facilitate the different units that occur when data originates from different hardware and software vendors.

5.3 Universal File Concept

A Universal File is a physical file, card deck, magnetic tape, paper tape, etc. containing symbolic data in physical records with a maximum record length of 80 characters.

On the physical file, data is contained in logical data sets with the following characteristics:

a. The first record of the data set contains "-1" right justified in columns 1 through 6. Columns 7 through 80 of the physical record are blanks.

b. The second record of the data set contains the data type number, numeric range 1 through 32767, right justified in columns 1 through 6. Columns 7 through 80 of this physical record are blanks.

c. The last record of the data set contains "-1" right justified in columns 1 through 6. Columns 7 through 80 of the physical record are blanks.

d. The specification of data on the remaining records of the data set are totally dependent on the data set type.

For example:

```
    -1
   xxx
    .
    .
 (data pertaining to the data set type)
    .
    .
    -1
```

Although the data organization is built around 80 character records, the capacity for data record blocking has been provided. Its principle use would be for magnetic tapes where the overhead associated with 80 character records is excessive. As such, a preferred physical/logical record blocking of 80 logical records per physical record is recommended. This improves system capacity and response dramatically.

5.4 Future Considerations

If further data base structures become necessary, several options can be pursued. First of all, the Universal Files documented in later sections of this report are a subset of the Universal Files supported by SDRC. Other Universal File formats may already exist which satisfy future requirements.[1,2] If another Universal File format does not already exist to service the intended needs, a new format can be developed as long as the Universal File format number is unique.

Another future consideration is the development of other similar formats. A similar concept to Universal Files is being developed in Europe, called Neutral Files and Meta Files,[3-6] to serve the same purpose. If future standards are developed and adopted, conversion programs to convert from the Universal File format to the new formats should be straight forward.

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6. SOFTWARE DOCUMENTATION

6.1 Introduction

Volume VI of this Technical Report describes briefly the history and current state of development of the Real Time Executive (RTE) Modal Program at the University of Cincinnati Structural Dynamics Research Laboratory (UC-SDRL). The RTE Modal Program serves as the kernal for much of the software development under this research effort. The purpose of Volume VI is to provide a reference for the operation of the RTE Modal Program and to provide a reference for future program development.

The RTE Modal Program has been developed as a replacement of an earlier program (User Program 9) that was written for the HP-5451-B Fourier system. The original concept of an RTE based program began in 1978 but was not realized in a working form until early in 1981. Based on the operating system of the HP-5451-B, Basic Control System (BCS), continued expansion of that software is prohibitive due to the inflexible programming environment and the memory limitations. To address these problems, the RTE Modal Program utilizes the overhead functions of the File Management Program under (RTE), an operating system available on Hewlett Packard computers, to provide flexibility that does not have to be built into the modal software The emphasis of the modal software development in the RTE environment is toward supportability rather than efficiency. For future development reasons and based upon the research nature of the Structural Dynamics Research Laboratory, the ability of graduate students to extend and enhance the current software is always the primary consideration. In this way, the modal software can eventually support any type of data acquisition system as well as interface through file structures to related software such as finite element analysis packages.

Future development of this software will be based upon a graphics workstation concept, utilizing a Unix operating system. This project has already been initiated in order to allow the developments in modal analysis software to be more readily available in several hardware configurations.

6.2 Software Compatibility

The UC-SDRL and the UDRI believe that the success of this effort in providing an efficient and user-oriented analytical tool is highly dependent upon the program development philosophy which was adopted during this effort. The more important guidelines that were followed during this effort will include the following:

- Programming Language

  All program development is compatible with the current version of ANSI standard FORTRAN (1977), when possible. Exceptions to this would be assembly level software required by particular hardware or software operating systems.

- Structured Programming

  All software developed during the research effort has been written in "structured" FORTRAN and therefore will be arranged in short modular subroutines for faster compilation, less memory requirements, and easier modification.

- Internal Documentation

The programs are internally documented containing a "block" of comments at the top of each module and "step comments" within each module. The block comments provide a concise statement of the function of the subroutine, algorithm used, input and output arguments, names and meaning of the important variables, subprograms called, and any peculiar features of the subroutines. Included is an identification of system dependent code with an explanation of the purpose of the system dependent operation.

- External Documentation

External documentation in the form of a User's Manual is provided. This consists of an appendix in Volume IV and all of Volume VI of this Technical Report. All units, modules, programs, systems and interactions between them will be complete. Information sufficient for a user of the program to prepare data, run the program, and assess the results are included in the manual. Also included will be statement of the program function, names and functions of the principal modules, call sequence of modules, list of modules called by each module and name and purpose of major variables.

- Compatibility with Modal Analyzers (Data Acquisition)

All programs are compatible with the data base generated by HP-5451-B and HP-5451-C Fourier Systems. Through the use of Universal Files, almost any Fourier analyzer can be made to be compatible. The component synthesis program, in particular, is also compatible with the format generated by the NASTRAN finite element program. The format for specification of component data to the synthesis program is described in an appendix to Volume IV of this Technical Report. for ease of developing interface with any other modal analysis software. All magnetic tape formats that are developed will be based upon an 80 ASCII character record (card image) format. While this format produces a somewhat longer data tape, the ability of most computer systems to read such a format with standard I/O subroutines is a stronger consideration.

6.3 Data Acquisition Hardware Environment

The HP-5451-C Fourier System was originally the primary target for the initial version of the RTE Modal Program. This system provides a BCS programming environment for the estimation of frequency response functions and the storage of the frequency response functions to disc media compatible with the RTE environment. Current software is compatible with HP-1000 systems with either 21-MX-E or 21-MX-F processors or HP A Series computers such as the A-700 or A-900. In this mode of operation, data acquisition will be provided by a HP-5451-B/C, a HP-5420-A, a HP-5423-A an S/K-LMS FMON, or a Genrad 2515 Fourier System. Data will be available on disc media via the FMTXX structure defined by the HP-5451 Fourier Systems. Compatibility of data from these as well as other Fourier systems is always available through the Universal File Structure supported by SDRC and UC-SDRL. Documentation on this file structure may be found in Volume VI of this Technical Report.

6.4 Modal Analysis Hardware Environment

The RTE Modal Program is designed to be executed on an HP-5451-C Fourier System with multiple HP-7900 Discs, an HP-7906 Disc or an HP-7925 Disc. The minimum memory configuration is 128K words but portions of the RTE Modal Program will run more efficiently if more memory is available (256K words or larger). At the present time, the Extended Memory Area (EMA) and the Vector Instruction Set (VIS) are not utilized in any of the primary programs. These capabilities are utilized in some of the advanced parameter estimation and modal animation programs. Due to the increasing memory requirements and computational load of many of the parameter estimation algorithms currently under evaluation, these options will be utilized even more in the future.

6.4.1 Memory Requirements

The RTE Modal Program involves the operation of multiple programs through a series of monitors. Programs may be suspended as other programs are executed or multiple programs may be executed simultaneously. For this reason, the optimum memory size currently would require five partitions of 28K words available to the RTE Modal Program at one time. This allows all dormant, suspended programs as well as active programs to be memory resident and reduces the program swapping time. If this much memory is not available, dormant programs will be swapped to disc to allow active programs to be executed. Therefore, in this situation, more work track area will be required on the system discs to swap dormant programs.

6.4.2 Disc Requirements

The RTE Modal Program is designed to run most efficiently on a multiple HP-7900 Disc system, a HP-7906 Disc, or a HP-7925 Disc, all of which are supported as BCS environment options on the HP-5451-C Fourier System. The RTE Modal Program will run on a HP-5451-C Fourier System with only one HP-7900 Disc but file storage is minimal.

6.4.3 Graphics Display Requirements

Originally, the HP-5460-A Display Unit was the primary graphics vector display that was supported as part of the RTE Modal Program for data evaluation and modal vector animation. Additionally, several other graphics vector display devices are currently supported. The HP-1351 Vector Graphics Generator is supported as an optional display for the HP-1000 systems that do not normally include a high speed vector display. Both the HP-5460 and the HP-1351 displays are controlled from RTE using the Universal Interface Driver (DVM72) supported by Hewlett-Packard as part of the RTE operating system. Both displays are interfaced via the Data Control Interface Card (HP-05460-60025). The HP-1351 Vector Graphics Generator requires the 16 Bit Parallel Interface (Option 002) to operate in this format. Operation of the HP-1351 Graphics Vector Generator also requires the maximum amount of memory available for the unit.

In addition to these two displays, support of the HP-134x displays has recently been added. Support for the HP-1345 involves a 16 bit parallel interface with the use of the Universal Interface Driver and support for the HP-1347 involves an IEEE-488 interface with the use of the appropriate HP-IB driver.

6.4.4 Plotter Requirements

All HP plotters interfaced via the HP-IB, the HP-7210 Digital Plotter, and all Tektronix 40xx Terminals will operate with the current software. Logical units have been defined within the RTE Modal Program to include up to five plotter logical units to allow for future plot flexibility. The tentative plan is to eventually include the HP-264X Graphics Terminal.

6.5 Modal Analysis Software - Operating System Environment

The RTE Modal Program currently runs in any revision of RTE later than Revision 2140 of RTE-4-B. RTE software is not part of the standard HP-5451-C Fourier System. Therefore, any group or facility that would wish to run the RTE Modal Program in this environment must purchase this software from Hewlett-Packard. This software can be generated on either a session or non-session basis. The non-session structure is for a limited number of users with no accounting feature. The session structure is for multiple users and uses an account structure to restrict access to portions of the system. In the session type of environment, the RTE Modal Program runs in a multi-user situation, allowing multiple copies of a program to run and managing resources such as modal animation devices and data logical units based upon the workstation that is in use.

6.5.1 RTE-4-B (Non-session)

RTE-4-B (Non-Session) is an RTE environment that is currently supported by Hewlett-Packard. This is compatible with the FSDS systems that are supported with the HP-5451-C systems but includes a newer revision operating system and the loader program.

6.5.2 RTE-4-B (Session)

RTE-4-B (Session) is an RTE environment for multiple users that is currently supported by Hewlett-Packard. While this operating system is not the same as RTE-4-B (Non-Session), the RTE Modal Program will currently run in this environment.

6.5.3 RTE-6-VM

RTE-6-VM is the virtual memory RTE environment which is available as of Revision 2201. While this is not a true virtual memory operating environment, this system is expected to reduce the overhead of working with large arrays. It is expected that conversion to the RTE-6-VM will require changes that will not be downward compatible but, due to the attractive characteristics of the operating system, the eventual target environment will most likely be RTE-6-VM.

6.5.4 RTE-A

RTE-A is the virtual memory RTE environment available for the A Series Hewlett Packard computers. This operating system is very similar to the RTE-6-VM operating system.

6.5.5 Operating System Requirements

Within the structure of the RTE Operating System, certain system capabilities must be available. First of all, the RTE Modal Program makes use of a minimum of 432 blocks of 128 words as a temporary area for the storage of arrays during program execution. This working space is located on disc and serves as the database for the RTE Modal Program. Therefore, if sufficient disc space is not available, the program will terminate execution at the initialization stage. Additionally, if memory is at a minimum, more disc space will be required by the RTE Operating System to swap dormant programs to the disc in order to run active programs. If sufficient disc space is not available, a currently active program will not be able to schedule a son program without suspending the RTE Modal Program while waiting for disc space to become available. Unfortunately, it is unlikely that any activity, except for the removal of a dormant program from the program stack with the 'OF,NAMR,1', will ever release disc space so that the suspended program can continue. Therefore, in minimum memory configurations, more disc space must be made available so the RTE Modal Program cannot be suspended. The current version of the software requires a minimum of 25 work tracks for operation in a 96K word RTE Operating System.

The only other system capability that is used by the RTE Modal Program is the System Available Memory (SAM). This buffer in the system must be at least 3000 words in length for class I/O data transfers used by the RTE Modal Program.

6.6 Modal Analysis Software Overview

The RTE Modal Program development is structured to emphasize simplicity rather than efficiency. For this reason, approximately 90% of the software code is in Fortran, ANSI 1966 or ANSI 1977. Many operations could proceed faster or more efficiently if written in Assembly language but, as the software and hardware changes in the future, the overhead required to recode these operations is not efficient in the long term sense and would not be efficient with regards to the long term goals of the research program at the University of Cincinnati.

Much of the function of the RTE Modal Program is designed to facilitate access to other related programs and their data sets as well as to provide other programs access to the data sets created from the RTE Modal Program. In this way, the RTE Modal Program can use or provide information from/to a finite element program or alternate experimental data analysis techniques.

The structure of the monitor and commands within the RTE Modal Program is intended to facilitate a tutorial approach to the use of the program. Each monitor has a help feature where the available commands can be determined as well as a short description covering the use of each command. The individual commands often involve multiple optional parameters which provide the experienced user with the ability to streamline the use of the command and answer a minimum number of questions.

6.6.1 Monitor Structure

The RTE Modal Program is structured as a nested set of monitors where each monitor exits to the next higher monitor until the File Manager (FMGR) monitor is reached. At the current time, no capability of sequencing commands either within or among the monitors in an automatic way is provided. In the future, this type of programming is an obvious extension to the current capability.

Each monitor contains a user help feature that gives the user access to an on-line user manual. This help feature can be accessed in each monitor to determine what commands are available and specifically how to exercise the command.

6.6.2 RTE File Structure

The RTE Modal Program generates and uses two types of FMGR files in order to facilitate data storage and retrieval as well as to provide data sets to other programs. The two file types are designated as Project Files and Modal Files. The use of Project Files is intended to provide data storage and retrieval for the RTE Modal Program while the use of Modal Files is to create a file format that is documented (Appendix D of Volume VI of this Technical Report) to be used to transfer modal data files between the RTE Modal Program and other programs. Modal Files are also convenient for storing only a small portion of the total modal data set. Component definition information, coordinates, display sequence, frequency and damping information or a subset of the modal vectors may individually stored in a modal file. Refer to the File Store Command for details.

6.6.2.1 Project Files

Project Files are binary files consisting of 128 word records. Within the FMGR concept, this is designated as a Type 1 File. The Project File is a block image of the data storage area managed by the RTE Modal Program. Note that a block is defined as 128 words of storage either in memory or on disc. Effectively, this data area contains the current state of all important variables and data arrays so that the operation of the program can be restarted in a given state very easily.

6.6.2.2 Modal Files

Modal Files are binary files consisting of 16 word records. Within the FMGR concept, this is designated as a Type 2 File. The Modal File is a structured copy of a specific part of the modal data set that exists at the time the file is created. Within the RTE Modal Program, five Modal Files have been defined currently which can be stored in this manner.

6.6.2.3 Universal Files

Data can be written to or read from other system types and other programs by means of universal files. Universal files are ASCII files with defined formats for storing data, including modal parameters, structure geometry, display sequences, frequency response functions and general measurements. For a complete description of available universal file formats see Appendix I of Volume VI of this Technical Report.

This concept thus allows communication between any programs supporting universal files such as data acquisition, parameter estimation, modal modification and finite element programs.

These universal file formats were originally developed at Structural Dynamics Research Corporation.

6.6.3 Data Acquisition

Data acquisition was originally expected to take place on a HP-5451-B/C Fourier System. The resulting frequency response function data is placed on a data disc according to a table contained within the subroutine FMTXX. This table, DIFS, is used by the BCS operating environment to determine where any record of any of nine file types is located on the data disc. This same subroutine, FMTXX, is loaded with the RTE Modal Program so that the same DIFS table is available to the RTE Modal Program. This table can be altered at any time thru use of the Measurement Format Command to accomodate users with multiple FMTXX structures.

Data acquisition is also now supported on several other devices. First of all, any device that supports the Universal File structure can serve as a source of modal data using File Type 58. This Universal File Structure is documented in Appendix I. In addition to this possible form of support, data acquired from the HP-5423-A, data acquired and coded from SMS modal software, and data acquired from the S/K-LMS Fourier System (FMON) is supported by way of the . surement Format Command and the Measurement. Header Command. Data acquisition can take place on a HP-5420-A or a HP-5423-A if the data can be moved to the data disc in a format compatible with the HP-5451 Fourier System. User programs exist for the HP-5451-C Fourier system to do this in a BCS operating environment. The programs for the HP-5423-A are User Program 80 and 81 while the programs for the HP-5420-A are User Programs 82 and 83. The standard versions of these programs do not provide any modal information in the header of the resulting HP-5451-C Fourier System data record. This information must be added using the Data Setup Command. The versions of the User Programs 80 and 81 in use at the University of Cincinnati for the HP-5423-A automatically insert the 63 header words from the HP-5423-A in words 14 through 76, inclusive, of the 128 word header of the HP-5451-C Fourier System data record. In this way, modal data taken on a HP-5423-A can immediately be processed by choosing the proper format using the Measurement Source Command.

6.6.4 Graphics Displays

Within the RTE Modal Program, all data and display animations occur on one of several graphics vector displays. Graphics vector displays are used due to the higher quality of the vector displays compared to raster scan displays. Currently, several graphics displays (HP-5460, HP-1345, HP-1347, HP-1351) are supported. Any number of graphics vector displays in any combination may be present in the system at any time in order to support multiple display requirements as well as multiple users.

The user is often required to interact with the RTE Modal Program by providing information based upon the data currently displayed on the graphics vector display unit. This interaction normally occurs via control of the cursor, mode, and scaling functions of the graphics vector display unit.

6.7 Frequency - Damping Estimation

The task of determining damped natural frequencies can be performed using one of the following methods:

- Manual (spectral line)
- Cursor (spectral line)
- Least Squares Complex Exponential (frequency and damping)
- Polyreference Time Domain (frequency and damping)
- Polyreference Frequency domain (frequency,damping and modal vectors)
- Orthogonal Polynomial (frequency and damping)
- Multi-Mac (frequency and modal vectors)
- Modified Ibrahim Time Domain (frequency and damping)

The first two methods, manual and cursor, are single degree-of-freedom (SDOF) approximation methods. With these methods, only one frequency response function can be used at a time. Therefore, it is wise to scan at least one frequency response from all major structure components so that no important modes are inadvertently missed. Operation of the cursor automatically stores the spectral line and frequency with the designated mode.

The remaining methods; Least Squares Complex Exponential (LSCE), Polyreference Time Domain (PTD), Polyreference Frequency Domain (PFD), Orthogonal Polynomial (OP), Multi-Mac (MM), and Modified Ibrahim Time Domain (MITD), are all multiple degree-of-freedom methods. In addition, the last five methods are multi-reference methods. However, they can also be used on single reference data.

The Least Squares Complex Exponential and the Polyreference Time Domain algorithm are basically the same methods. The last one is an extension of the first one to multiple references. They are both linear least squares time domain methods based upon complex exponentials. In the process of determining the frequency and damping, any and/or all of the measurements can be involved. An additional feature of the Polyreference Time Domain, as compared with the Least Squares Complex Exponential, is that the poles in the frequency range of interest can be determined based on different numbers of degrees-of-freedom (DOF), which can be sometimes advantageous.

The Polyreference Frequency Domain, Orthogonal Polynomial, and Multi-Mac methods are frequency domain methods. They have the advantage that any arbitrary frequency window can be selected out of the measured frequency range. They can also handle frequency response function data with variable frequency spacing. The disadvantage of these methods is that they become numerically unstable for wide frequency ranges and for high numbers of modes. The Polyreference Frequency Domain algorithm estimates the damping and damped natural frequency as well as the associated modal vectors in a single process. So this technique is a one-stage technique, while for all other methods, with the exception of Multi-Mac, the modal vectors are obtained in a second stage. Multi-Mac is the only method of these three methods that does not calculate the damping. Similar to the Least Squares Complex Exponential and Polyreference Time Domain, in the Polyreference Frequency Domain all measurements, or a subset of the measurements, can be included in the estimation of frequency and damping.

The Modified Ibrahim Time Domain algorithm is similar to the Polyreference Time Domain technique. Specifically, both are time domain techniques based upon complex exponentials, but the Modified Ibrahim Time Domain has the advantage of computing fewer computational poles. However, due to the fact that more memory is needed to calculate the frequency and damping values, the algorithm may not be able to simultaneously process all measurements. Therefore, data sets containing many measurements may have to be reduced to a subset, in order to use this method.

For all of the algorithms, the location of the poles in the frequency range of interest is very important. In general, poor damping values are estimated for poles too close to the edges of the frequency range. An exception to the previous constraint is the Orthogonal Polynomial algorithm.

A difficult task in modal parameter estimation is the determination of the order of the model, or the number of degrees of freedom of the system, such that, the estimating algorithm will find all structural poles. Three features are implemented to help in the process of deciding this value; an error chart, a stabilization diagram, and a rank estimate chart. These features will provide approximate values for the order, or degree of freedom of the system, but, in general, some judgement is still necessary to determine the "best" number for acceptable frequency/damping estimates.

The time domain algorithms tend to produce more computational poles than the frequency domain algorithms. On the other hand, frequency domain methods like Multi-Mac and Polyreference Frequency Domain, which force the modal vectors to be orthogonal, tend to have difficulties estimating the correct pole values for closely coupled poles, or for very local modes.

6.7.1 Error and Rank Chart

Most of the advanced algorithms use an error chart and/or a rank estimate chart, to aid the user when a decision has to be made about the order of the model. An error chart basically explains what the error will be in predicting the next point in an impulse response function, based on the information of the previous points. The number of previous points used is, in this case, related to (2 or 4 times) the estimated order, or degree-of-freedom of the model. The error chart may be interpreted in the following way. In general, the error chart will have an area where the error rolls off drastically with increasing degree-of-freedom. This area can be approximated by a straight line with a slope equal to the roll off. In addition, there will be a second part in the error chart where the error will stabilize. This range can be approximated by another straight line. The two lines will intersect each other at the approximate order of the model. For the frequency domain methods this is approximately the number of degrees-of-freedom that has to be entered in order to get a good estimate of the poles in the frequency range of interest. For the time domain methods, this value will generate, in general, a reasonable estimate for the frequency values of the poles in the frequency range of interest. However, quite often a poor estimate of the damping value of the poles will be obtained for this degree-of-freedom. But, by entering this number of degree-of-freedom an idea is obtained about the number of effective poles in the frequency range of interest. This can be helpful later on, to distinguish the real poles from the computational poles when a higher degree-of-freedom is entered in the algorithm. For the time domain methods, the best pole estimates are obtained when the number of degrees-of-freedom chosen is equal to 1.5 to 2 times the estimated order of the model.

Some algorithms provide a rank estimate chart. This chart comes from a singular-value decomposition of a matrix, which is related, or equivalent, to the system matrix. The rank of this matrix is once again equal to the order of the model. The rank estimate chart is interpreted in much the same way as the error chart (see previous paragraph).

6.7.2 Measurement Selection Option

A subset of the data set can be selected in the frequency/damping estimation phase. At times it may be desirable to exclude some measurements from the data set in the frequency/damping estimation process. For example, the estimation of a mode local to a specific direction, component, or set of points would be enhanced if only the direction, component, or points active in that mode are included in the estimation process. If all measurements are included, the local mode may be dominated by another structural mode and the algorithm might be unable to detect the local mode, or estimate it accurately. In the case of multiple references, a single reference may be excluded from the estimation process and instead used to synthesize frequency response functions in order to verify the modal model. For these and many other reasons, the measurement selection option is implemented. The measurement selection consists of the following options:

- Measurement Direction
- Components
- Point Numbers
- References

If a subset of the measurements is desired, one of the four options can be invoked. With the first three options, parameters can be input individually (N1), or sequentially (N1,N2) for all frequency/damping methods. The selection of references to be used is somewhat different for the multiple reference algorithms, but similar to the first three options for single degree-of-freedom and the Least-Squares Time Domain methods. In all cases, only the parameters entered for the option chosen are used to form the subset and the other options remain unchanged, unless they too are invoked. In other words, if the point number option is selected, only the point numbers entered would be used to form the subset (all other point numbers are excluded), but all directions, components and references remain active. To exit an option, zero is entered. "Continue" is selected after selecting the desired subset.

By using the measurement selection option, a subset of the measurements defined in the measurement directory can be selected for the estimation of frequency and damping values. This subset remains active only for the Frequency/Damping Estimation Monitor and all measurements in the measurement directory remain active for the estimation of modal coefficients, except for the Polyreference Frequency Domain method. For this method, the modal vectors will be determined ONLY for the same subset, since all modal parameters are determined in a single solution process.

6.8 Modal Vector Estimation

The task of estimating modal coefficients can be performed by one of the following methods:

- Complex Magnitude
- Real Part of Frequency Response Function
- Imaginary Part of Frequency Response Function
- Real Circle Fit
- Complex Circle Fit
- Least-Squares Frequency Domain
- Polyreference Time Domain
- Polyreference Frequency Domain

The first five methods, complex magnitude, real part, imaginary part, real circle fit and complex circle fit, are single degree-of-freedom methods. The Least-Squares frequency domain method is a multiple degree-of-freedom method, but similar to the first five methods, does not estimate global modal vectors. The two polyreference methods are multiple degree-of-freedom, multiple reference methods and estimate global modal vectors.

At the present time, the RTE Modal Program is capable of estimating complex modal coefficients using a floating point word for the real part and a floating point word for the imaginary part. The modal vectors are actually stored, regardless of the method used to estimate the modal coefficients, as the diameter of the complex circle that can be used to describe the single degree of freedom and with the units of the data from which the modal vectors were estimated. Within the RTE Modal Program, if the modal vectors are rescaled, the actual values of the modal vectors are never altered; a complex scale factor is altered from unity to account for any scaling required. All values that are output from the RTE Modal Program include this complex scale factor in a transparent manner.

The ability to animate the modal vectors is possible in any of four formats. The possibilities allow the user to view the modal vectors in complex or one of three real formats. Options are available in the real formats to view the complex magnitude, real component, or imaginary component so that all data types (D/F,V/F,A/F,D/D,V/V,A/A) can be used to determine modal vectors. This also gives the user the possibility to view the out-of-phase portion of the modal vector to determine whether a complex modal vector is a function of reasonable structure characteristics or a function of poor excitation energy distribution.

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NOMENCLATURE

Matrix Notation

{..}	braces enclose column vector expressions
$\{..\}^T$	row vector expressions
[..]	brackets enclose matrix expressions
$[..]^H$	complex conjugate transpose, or Hermitian transpose, of a matrix
$[..]^T$	transpose of a matrix
$[..]^{-1}$	inverse of a matrix
$[..]^+$	generalized inverse (pseudoinverse)
$[..]_{q \times p}$	size of a matrix: q rows, p columns
$\lceil .. \rfloor$	diagonal matrix

Operator Notation

A^*	complex conjugate
F	Fourier transform
F^{-1}	inverse Fourier transform
H	Hilbert transform
H^{-1}	inverse Hilbert transform
ln	natural logrithm
L	Laplace transform
L^{-1}	inverse Laplace transform
Re + jIm	complex number: real part "Re", imaginary part "Im"
$\dot{x}$	first derivative with respect to time of dependent variable x
$\ddot{x}$	second derivative with respect to time of dependent variable x
$\overline{y}$	mean value of y
$\hat{y}$	estimated value of y
$\sum_{i=1}^{n} A_i B_i$	summation of $A_i B_i$ from i = 1 to n
$\frac{\partial}{\partial t}$	partial derivative with respect to independent variable "t"
det[..]	determinant of a matrix
$\|\|..\|\|_2$	Euclidian norm

Roman Alphabet

A_{pqr}	residue for response location p, reference location q, of mode r
C	damping
COH	ordinary coherence function†
COH_{ik}	ordinary coherence function between any signal i and any signal k†
COH^n	conditioned partial coherence†
e	base e (2.71828...)
F	input force

F_q	spectrum of q^{th} reference†
GFF	auto power spectrum of reference†
GFF_{qq}	auto power spectrum of reference q†
GFF_{ik}	cross power spectrum of reference i and reference k†
$[GFFX]$	reference power spectrum matrix augmented with the response/reference cross power spectrum vector for use in Gauss elimination
GXF	cross power spectrum of response/reference†
GXX	auto power spectrum of response†
GXX_{pp}	auto power spectrum of response p†
$h(t)$	impulse response function†
$h_{pq}(t)$	impulse response function for response location p, reference location q †
$H(s)$	transfer function†
$H(\omega)$	frequency response function, when no ambiguity exist, H is used instead of $H(\omega)$†
$H_{pq}(\omega)$	frequency response function for response location p, reference location q, when no ambiguity exist, H_{pq} is used instead of $H_{pq}(\omega)$†
$H_1(\omega)$	frequency response function estimate with noise assumed on the response, when no ambiguity exist, H_1 is used instead of $H_1(\omega)$†
$H_2(\omega)$	frequency response function estimate with noise assumed on the reference, when no ambiguity exist, H_2 is used instead of $H_2(\omega)$†
$H_S(\omega)$	scaled frequency response function estimate, when no ambiguity exist, H_S is used instead of $H_S(\omega)$†
$H_v(\omega)$	frequency response function estimate with noise assumed on both reference and response, when no ambiguity exist, H_v is used instead of $H_v(\omega)$†
$[I]$	identity matrix
j	$\sqrt{-1}$
K	stiffness
L	modal participation factor
M	mass
M_r	modal mass for mode r
$MCOH$	multiple coherence function†
N	number of modes
N_i	number of references (inputs)
N_o	number of responses (outputs)
p	output, or response point (subscript)
q	input, or reference point (subscript)
r	mode number (subscript)
R_I	residual inertia
R_F	residual flexibility
s	Laplace domain variable
t	independent variable of time (sec)
t_k	discrete value of time (sec) $t_k = k\,\Delta t$
T	sample period
x	displacement in physical coordinates
X	response
X_p	spectrum of p^{th} response†
z	Z domain variable

Greek Alphabet

$\delta(t)$	Dirac impulse function
Δf	discrete interval of frequency (Hertz or cycles/sec)

Δt	discrete interval of sample time (sec)
ϵ	small number
η	noise on the output
λ_r	r^{th} complex eigenvalue, or system pole $\lambda_r = \sigma_r + j\omega_r$
$\lceil \Lambda \rfloor$	diagonal matrix of poles in Laplace domain
ν	noise on the input
ω	variable of frequency (rad/sec)
ω_r	imaginary part of the system pole, or damped natural frequency, for mode r (rad/sec) $\omega_r = \Omega_r \sqrt{1 - \zeta_r^2}$
Ω_r	undamped natural frequency (rad/sec) $\Omega_r = \sqrt{\sigma_r^2 + \omega_r^2}$
ϕ_{pr}	scaled p^{th} response of normal modal vector for mode r
$\{\phi\}_r$	scaled normal modal vector for mode r
$[\Phi]$	scaled normal modal vector matrix
$\{\psi\}$	scaled eigenvector
ψ_{pr}	scaled p^{th} response of a complex modal vector for mode r
$\{\psi\}_r$	scaled complex modal vector for mode r
$[\Psi]$	scaled complex modal vector matrix
σ	variable of damping (rad/sec)
σ_r	real part of the system pole, or damping factor, for mode r
ζ	damping ratio
ζ_r	damping ratio for mode r
†	vector implied by definition of function

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