NASA SPACE VEHICLE DESIGN CRITERIA (STRUCTURES)

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NATURAL VIBRATION MODAL ANALYSIS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FOREWORD

NASA experience has indicated a need for uniform criteria for the design of space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment

Structures

Guidance and Control

Chemical Propulsion.

Individual components of this work will be issued as separate monographs as soon as they are completed. A list of all previously issued monographs in this series can be found on the last page of this document.

These monographs are to be regarded as guides to design and not as NASA requirements, except as may be specified in formal project specifications. It is expected, however, that the criteria sections of these documents, revised as experience may indicate to be desirable, eventually will become uniform design requirements for NASA space vehicles.

This monograph was prepared under the cognizance of Langley Research Center. The Task Manager was G. W. Jones, Jr. The author was J. S. Archer of TRW Systems. A number of other individuals assisted in development and review. In particular, the significant contributions of the following are acknowledged: V. L. Alley, Jr., and J. S. Mixson of NASA Langley Research Center; R. Chen, D. L. Keeton, and C. H. Perisho of McDonnell Douglas Corporation; W. H. Gayman of Jet Propulsion Laboratory, California Institute of Technology; T. J. Harvey and R. J. Herzberg of Lockheed Missiles & Space Company; F. C. Hung of North American Rockwell Corporation; W. C. Hurty of the University of California, Los Angeles; G. Morosow of Martin Marietta Corporation; C. D. Pengelley of General Dynamics Corporation; C. P. Rubin of Hughes Aircraft Company; and M. J. Turner of The Boeing Company.

Comments concerning the technical content of these monographs will be welcomed by the National Aeronautics and Space Administration, Office of Advanced Research and Technology (Code RVA), Washington, D. C. 20546.

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NATURAL VIBRATION MODAL ANALYSIS

1. INTRODUCTION

Natural vibration modal data describe the linear dynamic characteristics of the space vehicle structure. A natural mode of vibration occurs when each point in the structure executes harmonic motion about a point of static equilibrium, every point passing through its equilibrium position at the same instant and reaching its maximum displacement at the same instant. The nature of the displacement (deflection, rotation, and slope) is generally known as the natural mode shape, and the frequency of the harmonic motion is generally known as the natural mode frequency. In addition to the natural (normal) vibration-mode shapes and frequencies, these data include the modal (generalized, integrated) mass, the internal loads associated with the natural vibration modes and frequencies, and the structural damping associated with the modes.

Knowledge of these data is basic to an understanding of the dynamic transient accelerations, velocities, and displacements of the vehicle under any kind of excitation. This knowledge is required for stability analyses made in control-system design. It is essential in dynamic-response analyses where the structural modes are used in the determination of loads on the structure. It is also essential in planning and implementing a meaningful program of inflight-loads determination. For spin-stabilized space vehicles, knowledge of modal data is essential in designing to preclude critical whirl conditions.

Provision of accurate data has made it possible to correct conditions that caused several major vehicle failures and near-failures. Some of these are listed below:

- Control-system coupling with a launch-vehicle structure in the launch mode required engine shutdown and system redesign to prevent failure associated with excitation of the cantilevered mode of the launch stand.

- Launch-vehicle roll-control jets were overpowered by a runaway roll instability which was associated with a rotary sloshing vibration mode.

- Torsion oscillation during staging of a major launch vehicle required close attention to payload torsional vibration to minimize torsion loads and accelerations on the spacecraft structure.

- The minimum-frequency specification for a new spacecraft was not satisfied due to inadequate modal vibration analysis; the result was a costly modification of design.

- A spacecraft in orbit experienced continuing low-frequency vibration oscillations that interfered with onboard experiments due to coupling with the control system and orbital environment effects. This problem has eluded resolution because of inadequate knowledge of the system's vibration characteristics.

- Several launch vehicles have experienced POGO-type longitudinal oscillation phenomena which result from unstable coupling of propulsion feed systems with longitudinal structure-vibration modes.

- Failure of primary structure in space vehicles has occurred from fatigue caused by repeated vibration testing. With adequate knowledge of the primary structure resonances, the maximum structural responses can be limited and the fatigue damage caused by testing can thus be reduced.

- Catastrophic failure of a launch vehicle occurred because the spin rate was coincident to the fundamental flexural natural frequency, resulting in spin resonance.

- Premature exhaustion of control fuel has resulted from structural-mode feedback to the autopilot system, causing periodic pulsing of controls.

This monograph is concerned with the determination and evaluation of natural vibration modal data for space vehicle structure. The monograph presents analytical and experimental methods for obtaining these data and judging their accuracy, and provides means of demonstrating the validity of the data.

Three groups of physical parameters have a dominant effect on the character of the natural vibration modal data:

1. The magnitude and distribution of the masses and inertias in the structure.
2. The load-deflection properties of the structure.
3. The boundary conditions of the structure.

Each of these parameters may vary considerably during the operational life of the vehicle. The definition of the boundary conditions is complicated by the "contractual interface" problem associated with space-vehicle procurement practices. The problem is how to determine valid modal data of complete space vehicles when separate stages of the vehicle are under development by different industrial contractors, and contractor direction is administered by different organizational segments of NASA.

The basic approach to determining natural vibration modal data of vehicle structure is to rely on theoretical data verified by experimental methods. An important part of the analysis is the derivation of the conceptual (analytical or mathematical) model of the real structure and formulation of the equations of motion representing this structure. The methods by which the equations are solved are well established. Significant load-displacement data and boundary conditions of the analytical model are verified by static tests on engineering-development structure, and by modal tests on prototype structure. Modal testing permits evaluation of the validity of modes derived from analysis, and in some cases extends the analytical results. Also, modal testing remains the only known way to establish modal damping factors.

This monograph is related to other planned monographs in this series which treat the inputs to and responses of vehicle structure as it encounters various natural and induced environments. These related monographs require the use of analyses which call for determination of natural vibration modal data as one of the steps in dealing with their specific problem areas. For example, planned monographs on structural vibration, shock, and dynamic instabilities such as POGO require a determination of modal data as a step in the response analyses that determine the severity of loads on space vehicle structure caused by these environments.

2. STATE OF THE ART

A vast amount of material has been published on methods of calculating theoretical natural-vibration modes for dynamic systems of varying degrees of complexity (e.g., refs. 1 to 5). The literature also provides modal data for common mechanical systems (refs. 6 and 7). However, exact solutions for modal-data problems in complex structures are beyond the state of the art. In practice, simplifications are introduced to allow solution of the problem with available methods.

Space-vehicle modal analysis is generally confirmed by test. Sometimes the natural frequencies and perhaps the first or second mode shape of a spacecraft can be conveniently established utilizing the test setup for the qualification sine-sweep testing. The experimental determination of more precise modal data for launch vehicles or highly flexible spacecraft, however, requires a specially designed test program and equipment. Techniques for the experimental evaluation of modal parameters are well documented. References 8 to 11 are examples of the literature available on both ground-test and flight-test techniques. The application of multiple shakers for determining modal parameters was formulated by Beckley in 1946 and was developed for use on aircraft by Lewis and Wrisley (ref. 12) in 1950. More recent application of multiple-shaker testing to space vehicle structures is given in references 13 and 14.

2.1 Analytical Determination of Modal Data

Methods of analytical determination of modal parameters are most readily examined in terms of the various phases of the analysis. The major steps in the analysis are the following:

- Modeling of structure.
- Formulation of equations of motion.
- Solution for modal data.

The derivation of the mathematical model of the structure and the associated choice of coordinate system are linked with the formulation of the equations of motion. In all practical cases, at least parts of the structure have distributed masses, and so the number of degrees of freedom required to represent exactly its dynamic motion is infinite. The reduction of the real structure to systems of finite degrees of freedom represents one of the basic simplifications in the modeling process. This can be done by such techniques as lumping of the masses (springs), by application of finite-element techniques, or by a Ritz (Galerkin) approach. The application of these techniques results finally in the equations of motion which reduce to a matrix eigenvalue problem by removal of the time factor.

The methods by which these equations are solved are independent of the derivation of the mathematical model and the formulation of the equations of motion.

2.1.1 Modeling of Structure

The mathematical model is the prime factor in obtaining satisfactory modal definition for the structure. If the model is of poor quality, mathematical rigor in the solution of the equations of motion will not improve results. The following basic factors are given careful consideration in the synthesis of the mathematical model:

- Stiffness distribution.
- Mass distribution.
- Boundary conditions.

Neglect of any of these considerations may result in a model that is not dynamically similar to the actual structure.

2.1.1.1 Stiffness Distribution

The definition of the stiffness distribution is the most difficult task in the synthesis of a mathematical structural dynamic model. For structures having continuously distributed properties (beams, plates, simple shells, and similar elements), a model exhibiting those same properties may be utilized (ref. 2). Models of this type are used with displacement functions to obtain modal parameters.

Most structures are complex and contain discontinuities in stiffness. For these structures, mathematical models are used which are composed of independently modeled structural components (finite elements) joined at coordinate points through common displacement and force components. These are defined as finite-element models. Finite-element models are available for beams (ref. 15), flat-plate elements (refs. 16 to 21), curved-plate elements (refs. 22 to 24), and sandwich-plate elements (ref. 21). These elements are used in conjunction with displacement and force coordinates which define the geometry of the elements. The finite-element model generally results in the definition of a stiffness or flexibility matrix for the discretized system (ref. 2).

The finite-element technique is the most generally applicable of the available analytical methods. It is readily adapted to digital-computer solution and takes maximum advantage of matrix notation in mathematical manipulations to obtain solutions. This technique is being rapidly improved and has been adopted in all major computer programs for general structural analysis. The basic problem of computer storage limitations is a dominant consideration in development and utilization of such programs. Modal data can be obtained for almost any configuration, using the finite-element technique.

Additional factors that affect the stiffness parameters of the structure include pressure in shells (refs. 23, 25, and 26), axial loads that approach the critical level (refs. 15 and 23), and temperature effects on material properties. The effect of typical airframe-type joints on modal-parameter computation is discussed in reference 9.

In the analysis of systems with many component structures and often inherently large differences in stiffness parameters, the modal parameters of component structures are used to synthesize the overall structural characteristics (ref. 27). Some advantages of using this technique are that it solves lower-order eigenvalue problems, generates the stiffness and mass parameters for smaller substructures, minimizes numerical difficulties due to the ill-conditioning of the stiffness matrix, resolves contractual interface problems, and conserves computer storage.

2.1.1.2 Mass Distribution

Mass distribution depends on the physical system under consideration and the method of analysis. For a system with a uniform or piecewise uniform mass distribution (i.e., beams, rods, plates, and panels), commonly handled by an analysis based on continuous-system equations, the mass distribution is readily defined by the actual structural distribution.

Several methods are used to define the mass distribution. These include the lumped-mass method, the consistent-mass method, and a number of approaches that use various velocity-interpolation functions to define a mass matrix (ref. 1).

The lumped-mass method distributes the element masses in concentrations located at the coordinate points in a physically reasonable manner which maintains the center of mass of the structure (refs. 1 and 28 to 30). This method of distribution is well suited for the analysis of structures with preponderantly concentrated masses. Local rotary masses are frequently used with this method to represent the effect of significant transverse-mass distributions. The disadvantage of the method is the relatively large number of coordinate points required for accurate analysis of systems with preponderantly distributed masses.

The consistent-mass technique represents the mass in a manner consistent with the actual distribution of mass in the structure (refs. 15, 16, 22, 23, 31, and 32). Although only recently codified, this technique is being widely adopted and incorporated in modern analytical computer programs. The consistent-mass-distribution technique has been shown to yield more accurate results than the concentrated-mass technique for systems where the mass is largely distributed in the structure (refs. 28 and 31). Its disadvantage is the nondiagonal mass matrix, which tends to increase the complexity of the analysis.

The treatment of the nonstructural mass of liquids in a fuel tank requires special consideration, since this item may be more significant than any other in contributing to uncertainties in the calculation of the lower modes for liquid-propelled boosters. The mechanical coupling of the fluid mass with the structure is generally accomplished through an equivalent pendulum analogy that simulates the free-surface lateral sloshing effect (ref. 33). Longitudinal mechanical coupling is accomplished through an equivalent spring for the tank-end bulkhead which supports the fluid (ref. 25). More complex longitudinal coupling may be accomplished with finite-element models that couple the changes in the cross-sectional area of the tank and deflection of the bottom bulkhead with motion of the fluid center of gravity (ref. 23).

2.1.1.3 Boundary Conditions

Because natural modes of a structure are sensitive to boundary conditions (ref. 34), the same boundary conditions are imposed on the model as on the actual structure, insofar as feasible. Frequently, static tests must be performed to determine the influence coefficients defining the boundary conditions at an interstage connection with the supporting stage structure. Experimental influence-coefficient data are readily obtained by static test when necessary. In some cases of large full-scale structures, these data are approximated by use of replica models.

2.1.2 Formulation of Equations of Motion

The methods of formulating the equations of motion (refs. 1 and 2) can be classified under the following categories:

- Integral equation methods.
- Differential equation methods.
- Energy methods.

Each method may include either the theoretically exact or approximate approach and can be used to handle both the distributed and discrete structural models. In all but a few special cases, however, the mathematically exact solution to the equations of motion cannot be found and the analyst must resort to numerical techniques.

2.1.2.1 Integral Equation Methods

Integral equation methods of formulating the equations of motion make use of an influence-coefficient function that defines the displacement of any point of a supported structure in terms of the applied load. The displacement under the inertial loads is obtained through integral equations which become the equations of motion. The advantage of this method is that it includes the boundary conditions in the basic equation of motion. The derivation of the influence function, however, is achievable for simple structures only. Approximate formulations can be made by techniques such as the Galerkin method described in references 1 and 2.

2.1.2.2 Differential Equation Methods

The natural modes and frequencies of a structure can be determined through differential equations which relate the structure's distortions to the inertial forces on the structure. This is a classical approach, and modal data obtained by this approach are available in the literature for a large variety of simple configurations. However, it is difficult to apply this method to a complex structure.

Approximate methods of formulating these differential equations for the nonuniform beam were proposed and developed by Myklestad (ref. 35), Thomson (ref. 36), and Holzer (refs. 1 and 37). These methods utilize a series of connected uniform beams that approximate the stiffness distribution of the nonuniform beam. Discrete masses may be located at the intersection of the uniform beam elements, but not necessarily at every intersection. A frequency-dependent relationship between the boundary conditions at the two ends of the beam results from the basic analysis. This relationship identically satisfies the boundary conditions only for the natural frequencies of the system. The addition of secondary relationships of shear deformation and rotary inertia results in a set of differential equations known as the Timoshenko beam theory. A practical method of treating the Timoshenko beam equations applicable to space-vehicle vibrations is given in reference 38.

The use of equations of motion of component members to define the total structural dynamic characteristics has been generalized in the transfer-matrix method. The formulation of transfer matrices and their application to a wide variety of structural problems is detailed in reference 39. Transfer-matrix techniques extend the usefulness of the approximate differential equation method to such structures as frames and built-up shells. The transfer-matrix technique can be applied to any type of linear structure for which the elemental transfer matrices can be derived. The effects of shear deformation and rotary inertia are easily included.

2.1.2.3 Energy Methods

Energy methods for formulation of the equations of motion are based on energy principles of mechanics, such as conservation of energy, virtual work, Lagrange's equation, and Hamilton's principle (ref. 40). In this approach, displacement functions that approximate the mode shape are used to represent the structure behavior. While the chosen functions are not theoretically restricted to those satisfying the geometric boundary conditions for the mode shapes, the accuracy of the solution depends strongly on how well the geometric boundary conditions are satisfied. The advantage of energy methods lies in their versatility; they can be applied to any structural configuration and can approximate the structural behavior to any desired degree of precision.

Rayleigh's energy method (ref. 41) provides a technique for determining the first mode of simple structures when a reasonable estimate of the mode shape can be made. Ritz (refs. 1 and 42) extended the Rayleigh method to allow for the calculation of higher modes. This method uses a set of functions, combined to provide a closer approximation of the natural mode shape. If the functions are properly chosen, accurate natural-frequency approximations are obtained, as well as definition of higher-order mode data. A disadvantage is that the accuracy of the modal data obtained by the Rayleigh-Ritz procedure (ref. 1) depends on the validity of the assumed mode shapes. However, modern techniques,by which the system modes are synthesized from easily derived shapes for the elements or components of the system through imposition of continuity conditions,render this problem easily tractable by means of almost automatic procedures. These procedures are involved in the displacement method of analysis using finite elements, or in the method of component-mode synthesis. Thus, the Rayleigh-Ritz method can make use of such synthetic assumed modes to overcome its disadvantage.

2.1.3 Solution for Modal Data

The equations of motion can be solved to obtain the modal data by several methods. Hand solution is usually limited to small-order systems because of the overwhelming amount of numerical labor involved. The solution of large-order systems is generally readily achieved on electronic computers.

Exact solutions are available for linear differential equations with constant coefficients, such as equations used to characterize systems with both uniform mass and stiffness properties. Where these properties are not uniformly distributed, exact solutions are not always possible and approximate solutions are obtained. The solutions to the basic differential equation of motion are detailed in the literature for various boundary conditions (refs. 3, 7, and 43). Both mode shapes and natural frequencies are readily available for the single-span uniform beam.

The two methods generally used to obtain the natural vibration mode shapes and frequencies are expansion of the characteristic determinant and matrix iteration. Reference 44 describes a step-by-step procedure for expansion and solution of the characteristic determinant. A detailed description of the matrix-iteration procedure can be found in references 1, 45, and 46. Several other techniques, such as the Jacobi and Householder methods, are described in references 47 and 48. The Jacobi method readily provides data for any mode of the system, whereas the Householder method is most efficient for providing data for only the lower-frequency modes. The modal mass and the normalized internal loads consistent with the natural vibration modes and frequencies are readily obtained, once the mode shapes and frequencies are determined (ref. 2). Modal damping is an exception in that it must be determined primarily by previous experience and/or by experiment (refs. 1, 3, 12, and 49). Knowledge of the distribution of damping throughout a structure is not generally deterministic. However, the assumption that the damping distribution is proportional to either the mass matrix or the stiffness matrix (or both) is frequently made for the expediency of uncoupling the equations of motion for linearly damped systems (ref. 2). When such a distribution is made, it is defined as proportional damping.

A numerical problem encountered in the solution of natural mode problems by digital computer is the loss of significant digits, which is related to the size and numerical ill-conditioning of the matrix equations (ref. 50). A major source of ill-conditioning is extreme variation in stiffness or flexibility of adjacent individual components in the structural matrix. For example, in finite elements the axial and "inplane" stiffnesses are usually many orders of magnitude greater than transverse stiffnesses. Suitable coordinate choices can generally be applied to isolate the effects of these widely separated stiffnesses and to improve matrix conditioning. Part of the problem relating to the limiting case of infinite axial stiffness with consequent kinematic redundancies among the modal coordinates is discussed in reference 2.

2.1.4 Accuracy of Analytical Modal Data

It should be noted that the accuracy requirements for modal data depend upon the problem to be solved. For example, data that may be adequate for control-systems analysis may be inadequate for detailed loads analysis. Furthermore, no single type of analytical model representation can adequately describe all configurations of a space vehicle. The common-beam analogy that may be fully adequate to represent a given booster and spacecraft configuration for analyses during launch is probably inadequate for representing the complex structure of the spacecraft alone during other phases of flight.

The accuracy of modal data obtainable by analysis decreases as the structure of the vehicle becomes more complex, and also decreases with increase in the number of the modes for which data are desired. For example, an accuracy of 1% on the fundamental frequency of a simple space vehicle is probably readily attainable, whereas 5% on the fundamental frequency of a complex vehicle having multiple branched beams with redundant interconnections might be unreasonable. Similarly, 10% on the frequency of the fourth or fifth mode of the simple vehicle would be well within the state of the art, but 10% on the frequency of the fourth or fifth mode of the complex structure would probably be unfeasible.

2.2 Experimental Determination of Modal Data

The only positive method of evaluating analytically determined modal parameters is by test. The parameters of interest are:

- Modal frequencies.
- Modal displacements.
- Modal mass.
- Modal damping ratios.

Modal parameters can be determined by experiment in several ways, the most convenient of which is to excite the structure with one or more shakers located near the predicted antinodes. The function of the shaker in modal testing is to provide, as nearly as possible, a force distribution which opposes the distributed damping forces in the vibrating structural system.

The determination of modal data with several shakers is well documented. Reference 12 describes the use of 24 shakers in tests of aircraft structures; reference 51, the use of 10 shakers to determine the first four lateral bending modes of a full-scale Minuteman missile; and reference 52 describes the use of two shakers to determine the first three lateral bending modes of a third-stage Minuteman motor. Bisplinghoff (ref. 2) discusses the use of shakers that are shifted during each modal search to provide optimum development of a mode.

A good understanding of the structure and of the test techniques is required to locate shakers over the structure and "tune in" modes by adjusting the frequency and force amplitude. The equipment required for modal testing varies with the size and complexity of the space vehicle, and with the boundary conditions to be imposed. Experimentally obtained modes are generally required for the fixed-base condition and for the free-free condition.

As noted in references 12, 34, 51, and 52, careful attention is given to the simulation of boundary conditions. Valid results are obtained from tests only when the test boundary conditions simulate the actual service conditions. As an example, the free-free modes of a structure can be obtained experimentally with the structure mounted in a soft suspension. The damping inherent in the suspension and the suspension effect on frequency and mode shape must be taken into account in computing the modal data from the test results.

Mode shapes, frequencies, and damping are obtained directly from experimental data. Modal mass is derived from experimentally obtained modal frequency and displacement data, utilizing known mass distribution throughout the structure.

The most common method for obtaining modal damping consists of interrupting the vibration forces simultaneously and measuring the resulting logarithmic decrement in the various response signals (ref. 1). Other methods are based on measurements of resonant response bandwidth (ref. 1). Stahle (ref. 49) developed a method whereby response signals are separated into real and imaginary components with reference to the input; damping is then determined as a function of the resonant peaks from the real or inphase response plots. Lewis and Wrisley (ref. 12) devised a method whereby modal damping is determined from measurements of the input force at resonance.

Even though efficient test equipment and test techniques are used, the identification of modes may still be a problem. Where the modes are widely separated, simple amplitude and phase studies of response time histories will identify each mode. The natural frequency at which the responses were recorded is first determined from decay-response plots obtained during interruptions of the exciting forces. An absence of beats between the modal and forcing frequencies indicates coincidence of these frequencies (ref. 52). Where modes are close together in the frequency regime, modal identification is difficult. Heavy reliance is usually placed on the ability to move the shakers to locations which will provide optimum development of the mode. Stahle (ref. 49) showed that the imaginary or quadrature responses (with respect to the input function) peak more rapidly than total response. This provides a more direct identification of modal frequency and response magnitude than can be obtained from a total response plot. The natural frequency could also be checked against the real or inphase component plot. Several manufacturers have developed electronic devices which convert response signals into inphase and quadrature components (relative to an input signal).

To avoid phase errors from multiple instrumentation channels, a signal conditioner, recording channel, and two transducers are sometimes used in "probing" the structure to establish mode shape. One transducer is fixed at a point of large modal amplitude to give a phase reference for determination of the algebraic sign to be assigned to the other transducer, which is used to measure the response at a number of locations along the structure.

After the natural frequency of the mode has been established, successive runs are made at resonance with the transducer attached to the specimen at a different location for each run. Any differences in phase from location to location are then attributable to causes other than the instrumentation system. The mode-amplitude survey data are made at sufficient points to provide displacement data at locations of all significant concentrated and distributed masses in the structure. These comprehensive modal-displacement data are required for calculation of modal masses for the generalized mass matrix and for modal checks.

A final step in the modal test program is evaluation of the consistency of the experimental data by examination of the orthogonality of the modal data*. The relative orthogonality of the modal data is generally determined as each successive mode is obtained. This is accomplished by utilizing the experimentally determined mode shapes and the distributed mass of the system to compute the generalized mass matrix. Each of the individual mass-matrix coefficients is obtained from an integrated or summed double product of two experimental mode shapes and the known mass of the system.

Ideally, the nondiagonal elements in the mass matrix should be zero, but this is seldom the case. Procedures such as the one discussed in reference 53 are utilized to make small adjustments in mode shape.

The determination of experimental frequencies, mode shapes, modal mass, and modal damping is generally difficult to accomplish in a reliable and repeatable manner. In many ways, vibration testing is more of an art than a science. The dependence of the desired data on the amplitude of the excitation; the existence in the structure of such nonlinear effects as hysteresis, dead zones, joints, and friction; and the difficulties associated with accurate measurement of the desired phenomena all contribute to a healthy suspicion of the accuracy of experimental data unless they have been obtained under carefully controlled conditions.

3. CRITERIA

3.1 General

Natural vibration modal data used in the design of a space vehicle shall be determined in sufficient quantity and with sufficient accuracy to support adequately any aspect of vehicle design or operation for which the modal data are necessary.

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*The concept of orthogonality for the mode shapes of a beam (neglecting rotary inertia) is mathematically defined (ref. 1) as

$$\int_0^{\ell} W_i(y)W_j(y)m(y)dy = 0 \qquad \text{for } i \neq j$$

where $W_i(y)$ and $W_j(y)$ are any two mode shapes of a beam of length ℓ with deflection $W(y)$ and mass-per-unit length $m(y)$ provided as functions of position y. It is said that the functions $W_i(y)$ and $W_j(y)$ are orthogonal to each other with respect to the weighting function $m(y)$.

3.2 Guides for Compliance

3.2.1 Data Required

A determination shall be made of the type, amount, and required accuracy of the natural vibration modal data needed to support the various aspects of vehicle design or operation.

3.2.2 Analysis

Modal analyses shall be performed to provide the required data. In that part of each modal analysis where the mathematical model of the structure is synthesized, the stiffness distribution, mass distribution, and boundary conditions for the structure shall be represented so as to ensure a model which is dynamically similar to the actual structure. The complexity of the mathematical model shall depend upon the modal data needed for the particular problems to be investigated. The equations of motion (which incorporate the mathematical model) shall be formulated by methods suited for the particular problems considered. The modal data shall be obtained by solving the free vibration equations of motion. It shall be demonstrated that the computed modal data satisfy the previously determined requirements of type, amount, and accuracy.

3.2.3 Tests

The major or significant analytical load-displacement characteristics of the vehicle structure, where not otherwise established, shall be verified by static tests.

If modal data obtained by analysis cannot otherwise be demonstrated to be adequate, the analytical data shall be replaced or confirmed by the results of ground or inflight dynamic tests on a realistic structure, on a dynamically scaled replica model, or on both.

4. RECOMMENDED PRACTICES

Natural vibration modal data should be determined as early as possible in the vehicle-design procedure.

The following tasks should be accomplished to obtain natural vibration modal data:

- Determine the type, amount, and required accuracy of modal data needed to support the desired analyses.

- Construct mathematical model (or models) which represents the vehicle structure.
- Establish proper boundary conditions for the various operations phases, such as prelaunch transportation and testing, launch and exit, stage separation, docking, free space flight, entry, and touchdown; then formulate and solve the equations of motion to obtain the natural vibration modal data.
- Conduct tests on vehicle structures to verify the model used in the analysis and to confirm the adequacy of the analytical modal data.

4.1 Data Required

All environmental sources of disturbance must be considered in determining the type of modal data required. Natural vibration modal data for the vehicle structure should be determined as required for all phases of the vehicle operation. These data include at least the following:

- Longitudinally and laterally supported launch-vehicle stages for prelaunch handling, transportation, and captive firings.
- Lateral and longitudinal launch-stand/space-vehicle modes for ground-wind loads and for launch ignition.
- Longitudinal, lateral, and torsional unsupported space-vehicle modes for configurations at the times of discrete events during the launch-and-exit phase; e.g., liftoff, wind-shear and gust encounter, engine shutdown, stage separation, and engine ignition.
- Normal modes of the cantilevered spacecraft in its launch configuration for use in synthesizing the requisite mathematical models of the complete space vehicle.
- Normal modes for all phases of free flight, including orbital maneuvers, appendage deployment, docking, entry, and touchdown, as applicable.

Modal-data accuracy requirements should be allowed to vary in accordance with the particular problem to be solved. No single type of analytical model representation should be used to describe all configurations of a space vehicle. Since (as explained in Sec. 2) the accuracy of modal data obtainable varies widely with complexity of the structure and the order of the mode being analyzed, the numbers recommended in the next two paragraphs should be regarded as guides only and in no case should be taken as requirements without a study to determine their suitability to the individual problem.

In general, modal frequencies for stability analyses of control systems should be accurate to 10% on modes higher than the fundamental. The fundamental-mode frequency should preferably be accurate to 5%. For flexible, high-fineness-ratio launch vehicles, a minimum of the three lower-frequency bending modes, plus at least the lowest-frequency slosh mode, is recommended for control-system stability analyses. For relatively stiff, low-aspect-ratio vehicles, such as the Apollo Service and Command Module stage, only the lowest-frequency slosh mode may be required. On the other hand, a singularly flexible (local flexure) configuration, such as the docked LEM/Apollo Service and Command Module, would probably require only the fundamental lateral bending mode, the lowest-frequency slosh mode, and the fundamental torsional mode for evaluation of the control system's stability. In all cases, a sufficient number of the lower-frequency modes should be furnished to provide a frequency range of response which encompasses the frequency characteristics of the control system and of the exciting forces.

Modal data for analyses of the vehicle load should be accurate to 5% on the fundamental frequency and 10% on other frequencies of interest. Lateral-load analyses of launch vehicles generally require a minimum of six of the lowest-frequency lateral bending modes. Similarly, a minimum of six of the lowest-frequency longitudinal modes is recommended for longitudinal-load analyses. When longitudinal and lateral modes are coupled, additional modes should be used in the analysis. For detailed investigation of loads in a particular stage or in the payload, the low-frequency modes in which the component of interest has a relatively large response should also be used. For the analyses of responses resulting from localized load sources, such as an injection rocket, the use of the lower-frequency modes with large-amplitude displacements at the source and in the direction of the exciting force is recommended. In all cases, a sufficient number of the lowest-frequency modes should be furnished to provide a frequency range of response that encompasses the frequency range of the exciting forces.

4.2 Analysis

4.2.1 Modeling of Structure

The mathematical model should represent the linear characteristics of the structure. It should account for all stress-strain effects that influence the structural distortions, including beam shear, torsion, and axial extension, as well as plate shear and twist, unless their effect on the modal data has been proven negligible (refs. 38 and 40).

The effects of initial internal forces which modify the load-displacement characteristics should be included in the mathematical model. These initial internal forces may result from dead loads, quasi-static accelerations during boost, built-in preloads, and other static or quasi-static loads applied to the structure.

In evaluating the effective stiffness of the model, the effect of local structure, such as joints between interstage adapters and vehicle stages, trusses on which payload or engines are mounted, or play in joints such as engine gimbal blocks when the engine is not under thrust, should be carefully scrutinized. Depending upon the characteristics of the joint, the combination of axial and bending loads could lead to variations in stiffness during different periods of operation, both in flight and on the ground. The variation of joint stiffness under these conditions is difficult to determine by analysis and should be ascertained by test.

The mathematical model should represent the actual distribution of mass throughout the structure. The use of distributed-mass models, such as the consistent-mass-matrix technique (ref. 31), is recommended. If a lumped-mass technique is used, it should be demonstrated by a parameter-variation study that a sufficient number of discrete mass points are used to represent adequately the modal characteristics (refs. 54 and 55). For accurate modal-data determination of a lumped-mass unsupported one-dimensional system, the number of discrete masses should be about 10 times the order of the highest mode to be determined. For more complex structure, such as multidimensional frames, the relationship between the number of discrete points and the accuracy of the computed frequency is not established, and reliance must be placed on the experience of the analyst.

Mechanical models of tanks containing fluids should simulate at least the first-mode lateral sloshing effect. A mass-spring model based on the pendulum analogy is recommended to simulate this phenomenon (ref. 33). Longitudinal mechanical coupling of the fluid with the structure must also be provided for in the longitudinal-mode analysis (ref. 25). Care must be exercised to assure that only the effective portion of the fluid mass is represented in the model (i.e., a smooth cylindrical tank rotating about its geometric axis of revolution does not cause rotation of any contained fluid in a linear model).

Finite-element techniques are recommended for modeling complex structures mathematically. In this approach, the elastic model should be defined in terms of assumed displacement functions in component parts leading to direct construction of a structural-stiffness matrix, or else in terms of assumed force distributions in component parts leading to direct construction of a structural-flexibility matrix. The relative proportion of adjacent individual components in the structural model must be chosen with care to minimize extreme variations in stiffness or flexibility which result in loss of accuracy in the structure-matrix coefficients (ref. 50). Unless the equivalent of IBM 360 double-precision arithmetic is used, one should not allow the ratio of numerical values between diagonal elements in the elastic matrix to exceed 1:1000.

4.2.2 Formulation of Equations of Motion

The approach used for formulating the equations of motion should take advantage of simplicities inherent in configurations which have symmetry, uniform geometry, or uniformly distributed properties. The use of continuous models is recommended for configurations with uniform geometry and uniformly distributed properties (such as entry cones, fairings, and engine nozzles). The formulation may follow any of several classical methods (ref. 1): solution of integral equations with the associated influence functions, solutions of differential equations with associated boundary conditions, or an energy approach using assumed modes, such as the Rayleigh-Ritz method.

The use of matrix notation and matrix manipulation is recommended. The differential equations of motion then take the matrix form of

$$[M] \ \{\ddot{q}_i\} + [K] \ \{q_i\} = \{Q_j\} \tag{1}$$

where $[M]$ and $[K]$ are square matrices of the mass and stiffness coefficients, respectively, and $\{q_i\}$ and $\{Q_j\}$ are column matrices of the coordinate displacements and applied dynamic forces, respectively (ref. 2). The data should be frequently updated to reflect changes in structural parameters.

4.2.3 Solution for Modal Data

The modal data should be obtained from the solution of the following equation of motion or its equivalent:

$$-\omega_i^2 \ [M] \ \{q_i\} + [K] \ \{q_i\} = \{0\} \tag{2}$$

The basic modal data consist of the sets of natural frequencies, ω_i, and the corresponding mode shapes, $\{q_i\}$, which satisfy equation (2). Generally, only the lower-frequency modes and mode shapes are required.

The modal internal forces and stresses and the generalized mass are subsequently obtained on the basis of the mode shapes, $\{q_i\}$, and the mass and stiffness matrices, $[M]$ and $[K]$, of the structure.

It is recommended that the modal data be solved by analytical techniques which compute only the required low-frequency data, such as matrix-iteration techniques or the Householder method. For large-order multidegree-of-freedom systems, the analysis should be performed with IBM 360 double-precision arithmetic or its equivalent. The modal data provided for each mode from the analysis should consist of frequency; mode shape, including displacements and rotations; internal forces; internal stress; and generalized mass. Damping data should also be provided for each mode from experience and verified by experiment. The sloshing-mode damping should be obtained as recommended in reference 33 and the planned monograph on slosh suppression systems.

For complex structural systems in which the separate components are physically identifiable, the method of component modes is recommended for solution of the modal data. This is particularly desirable when the appropriate modal properties of the separate components are known or can be readily determined.

4.3 Tests

4.3.1. Basic Recommendations

Static tests should be conducted to verify (where not adequately established) the analytically derived major or significant load-displacement characteristics and boundary conditions of the analytical model of the vehicle structure. Furthermore, if modal data obtained by analysis cannot otherwise be demonstrated to be adequate, the analytical data should be replaced or confirmed by the results of modal tests. The design of the test program depends on the confidence placed in the analytical results. Extensive testing is recommended where a radically new configuration without prior experience is involved. On the other hand, simple changes of payload on a standard launch vehicle may require only an analytical determination of the new natural vibration modal data. In general, changes in mass can be adequately handled by changes in the mathematical model without additional tests, whereas significant changes in stiffness usually require test verification.

If the actual design incorporates seriously nonlinear features, such as looseness in joints and backlash in gears, then its behavior cannot be adequately predicted by a linear analysis. In fact, linear characteristics such as normal modes might not even exist at all, in which case tests on such a structure could never reveal such nonexistent properties. If such features exist in the design, either by intent for a good reason or by virtue of uncontrollable factors, then the analyst, designer, and experimentalist, together, must be careful in applying linear analytic approximations to nonlinear real-life test results.

4.3.2 Static Testing

Tests of static-load displacements and boundary-condition influence coefficients should be performed on full-scale engineering models that have primary-structure static characteristics identical to those of the prototype and flight structure. If full-scale tests are not feasible, data from replica models, such as the 1/5-scale Saturn replica model (ref. 56), may be sufficient. The load conditions under which the displacement data are measured should simulate the quasi-static conditions expected for the time of flight for which the modal data will apply.

Load-displacement data should be obtained to determine, at a minimum, the elastic characteristics for the primary load-carrying element in the structure,with loads applied at the location of the primary masses or at major attachment points. For a simple spacecraft structure, the load-displacement characteristics may be determined for only a single major load point, as at the cantilevered end of the major structural element, at the location of the major equipment platform cantilevered from a central cylinder, or at the support points for an injection rocket attachment. For a launch vehicle or a large spacecraft structure, load-displacement measurements should be determined also for interstage structure between attach points, at major transverse bulkheads, for engine-support trusses, and for payload-support points.

The load-displacement characteristics and boundary-condition data obtained should be compared with the mathematical model. These data should correlate within about 20% for distortion under a given load. If necessary, the mathematical model should be changed to agree with the static-load-displacement test data. These load-displacement tests (or influence-coefficient measurements) should not be confused with structural qualification tests.

4.3.3 Dynamic Testing

To provide a basis for evaluating the quality of analytically derived modal data, dynamic tests should preferably be performed on a full-scale engineering model, prototype, or on flight-test structures which have dynamic characteristics identical to those of the flight structure. Scaled replica models may also be used where full-scale tests are not feasible or to supplement full-scale tests (ref. 56). Low-level qualification-type sine-sweep tests are recommended on spacecraft for verifying analytical modal frequencies and fundamental mode shapes, and for determining modal damping.

If analytical data are not available, or if more precise experimental data on large spacecraft and launch vehicles are desired than are available from sine-sweep tests, then modal-vibration survey tests should be performed to obtain the required modal data. A meaningful modal-survey test can involve large expenditures of time and money, and thus should be justified and carefully planned.

The equipment generally required for dynamic modal testing consists of a suspension or *support system,* a *shaker system, instrumentation,* and *data-acquisition equipment.*

For determination of fixed-base modes, a large rigid *support system* for the vehicle should be used. Particular attention should be given to the duplication of flight-vehicle joint flexibility at the interface. For measurement of free-free modes, a suspension system should be used which gives "rigid-body" suspension frequencies that are well below the elastic frequencies of the vehicle. Acceptable suspension techniques include flexible coil springs, air springs, bungee cord, and, for vehicles with unusually low natural frequencies, nearly buckled columns (refs. 14 and 57 to 59).

The *shaker system* requirements should be determined by the type of suspension and nature of the modal tests. It is recommended that the methods treated in reference 12 be used as a guide for establishing the power and force requirements for modal testing. The exciters are frequently used without flexure connections between the armature and housing, necessitating precise alignment of the housings. For this reason, shaker supports should be carefully designed, possibly with vernier adjustments for vertical and lateral positioning.

Instrumentation is required to measure the forces being applied to the structure and the displacement or acceleration responses. The electrical current supplied to each exciter is proportional to force, so the current measurement may be used to measure the force, if convenient. As an alternative, simple load cells may be used. Small accelerometers should be mounted at each shaker location and a "roving" accelerometer should be used to obtain the mode shapes. The roving accelerometer should be temporarily attached to the structure at the test point with double-backed tape or a light plastic vacuum cup. Precalibrated fixed accelerometers should be used for inaccessible locations and for internal measurements.

Data-acquisition equipment can range from complex "mode-lock" servomechanisms and complete instrumentation displays to a single dual-beam oscilloscope and a direct-writing oscillograph. If the latter system is used, a switching device should be employed with the oscilloscope to permit successive comparisons of force and acceleration measurements at each operating shaker. Display of shaker force on one axis of a single-beam scope and the singly integrated acceleration signal on the other axis gives a Lissajous pattern that is a fully collapsed ellipse when the shaker force is in phase with local velocity. The direct-writing oscillograph permits comparisons of all responses and measurements of the logarithmic decrement at each location for evaluation of structural damping.

Modal tests should include (1) *frequency surveys* to establish approximate resonant frequencies; (2) *isolation of each mode* by adjusting shaker locations and forces; (3) *damping characteristics* of each mode; (4) *mode-shape* measurement of each mode; and (5) evaluation of the *orthogonality* condition.

Frequency surveys should be performed with a single shaker at two or three locations. The responses of the fixed accelerometers and the magnitude of the input force should be recorded on an oscillograph for detailed evaluation of resonant frequencies.

For *isolation of each mode,* external forces should be applied so that shaker force is in phase with local velocity as shown by the Lissajous pattern. The procedure should be started by exciting the structure with a single shaker at the approximate resonant frequency. The frequency is adjusted until shaker force and local velocity at the point of excitation are in phase.

Responses at other locations on the vehicle are then compared on the oscilloscope with the response of the reference accelerometer to establish phase angles. If a control system like that described in reference 12 is not available, the second exciter should be located at the point having the largest phase-angle error (from 0° or 180°). Amplitude and sense of the second force are adjusted to optimum, again using the phase relationship between response locations as a criterion. The roving accelerometer is of considerable help in determining regions of large phase-angle error. This method becomes increasingly complex with mode number because the higher modes are generally more difficult to excite properly.

In addition to phase angle, the *damping characteristics* of a mode should be determined to establish "purity." A well-defined mode will decay cleanly at the modal frequency without beating or shifting to another frequency. Decay records of all the fixed accelerometers should be recorded for evaluation of modal damping.

Once the mode has been established, the *mode shape* should be determined at each significant mass point with the roving accelerometer. Responses should be measured with respect to an arbitrarily selected station, generally the location of a fixed accelerometer. A good "bookkeeping" system should be used for recording data to avoid the necessity of repeating a complicated shaker setup.

One additional step in the modal test is evaluation of the consistency of the experimental data by examination of the modal data *orthogonality*. It is recommended that the relative orthogonality of the modal data be determined as each successive mode is obtained. The generalized mass matrix obtained from an integrated double product of the experimental mode shape and the known mass of the system should have off-diagonal elements no larger than about 10% of the major diagonal elements. If this limit is exceeded, a procedure such as that given in reference 53 should be utilized to make small adjustments in mode shape.

For large and complex systems, where tests of the complete system are not feasible because of size, boundary conditions, or other factors, it is recommended that dynamic tests of separate components be combined with suitable static tests to provide the required data. Inflight maneuvers to excite the fundamental modes on instrumented test vehicles are recommended where ground tests of the complete system are not feasible.

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SP-8001	(Structures)	Buffeting During Launch and Exit, May 1964
SP-8002	(Structures)	Flight-Loads Measurements During Launch and Exit, December 1964
SP-8003	(Structures)	Flutter, Buzz, and Divergence, July 1964
SP-8004	(Structures)	Panel Flutter, May 1965
SP-8005	(Environment)	Solar Electromagnetic Radiation, June 1965
SP-8006	(Structures)	Local Steady Aerodynamic Loads During Launch and Exit, May 1965
SP-8007	(Structures)	Buckling of Thin-Walled Circular Cylinders, September 1965
SP-8008	(Structures)	Prelaunch Ground Wind Loads, November 1965
SP-8009	(Structures)	Propellant Slosh Loads, August 1968
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