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FORCE LIMITED VIBRATION TESTING

NASA TECHNICAL HANDBOOK

FOREWORD

This handbook is approved for use by NASA Headquarters and all Centers and is intended to provide a common framework for consistent practices across NASA programs.

The primary goal of vibration tests of aerospace hardware is to identify problems that, if not remedied, would result in flight failures. This goal can only be met by implementing a realistic (flight-like) test with a specified positive margin. In most cases, the goal is not well served by traditional acceleration-controlled vibration tests that indeed screen out flight failures, but in addition may cause failures that would not occur in flight. The penalty of overtesting is manifested in design and performance compromises, as well as in the high costs and schedule overruns associated with recovering from artificial test failures.

It has been known for 30 years that the major cause of overtesting in aerospace vibration tests is associated with the infinite mechanical impedance of the shaker and the standard practice of controlling the input acceleration to the frequency envelope of the flight data. This approach results in artificially high shaker forces and responses at the resonance frequencies of the test item. To alleviate this problem it has become common practice to limit the acceleration responses in the test to those predicted for flight, but this approach is very dependent on the analysis that the test is supposed to validate. Another difficulty with response limiting is that it requires placing accelerometers on the test item at many critical locations, some of which are often inaccessible.

The advent of piezoelectric triaxial force gages has made possible an alternative, improved vibration-testing approach based on measuring and limiting the reaction force between the shaker and test item. Piezoelectric force gages are robust, relatively easy to install between the test item and shaker, and require the same signal conditioning as piezoelectric accelerometers commonly used in vibration testing. Also vibration test controllers now provide the capability to limit the measured forces and thereby notch the input acceleration in real time. To take advantage of this new capability to measure and control shaker force, a rationale for predicting the flight-limit forces has been developed and applied to many flight projects during the past five years. Force limited vibration tests are conducted routinely at the Jet Propulsion Laboratory (JPL) and also at several other NASA Centers, Government laboratories, and many aerospace contractors.

This handbook describes an approach that may be used to facilitate and maximize the benefits of applying this relatively new technology throughout NASA in a consistent manner. A NASA monograph, NASA-RP-1403, which provides more detailed information on the same subject, is also available for reference.

Requests for information, corrections, or additions to this handbook should be directed to the Mechanical Systems Engineering and Research Division, Section 352, Jet Propulsion Laboratory, 4800 Oak Grove Dr., Pasadena, CA 91109. Requests for general information on standards should be sent to NASA Technical Standards Program Office, ED41, MSFC, AL, 35812 (telephone 256-544-2448). This and other NASA standards may be viewed and downloaded, free-of-charge, from our NASA Standards Homepage: <http://standards.nasa.gov>.

W. Brian Keegan
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FORCE LIMITED VIBRATION TESTING

1. SCOPE

1.1 Purpose. This handbook establishes a methodology for conducting force limited vibration tests for all NASA flight projects. The purpose is to provide an approach which may be consistently followed by those desiring to use force limiting, without having to conduct an extensive literature search or research and development effort before conducting the test. A monograph on Force Limited Vibration Testing is available for reference and is recommended for those needing more detailed technical information (NASA-RP-1403).

1.2 Applicability. This handbook recommends engineering practices for NASA programs and projects. It may be cited in contracts and program documents as a technical requirement or as a reference for guidance. Determining the suitability of this handbook and its provisions and providing for correct implementation is the responsibility of program/project management and the performing organization. Individual provisions of this handbook may be tailored (i.e., modified or deleted) by contract or program specifications to meet specific program/project needs and constraints.

For the purpose of this handbook, a force limited vibration test is any vibration test in which the force between the test item and shaker is measured and controlled. The recommended means of measuring the force is with piezoelectric force gages, but other means, e.g., shaker armature current or strain gages, may be useful in special situations. Similarly, the control of the force is preferably accomplished in real time, but iterative, off-line control may be employed as a stepping stone. If the force is not measured, the test is not considered a force limited vibration test, and this handbook does not apply. This distinction is important because in the past some have found it convenient to simulate a force limited test and then to use the analytical results to notch the acceleration input in the test. The simulation approach is not recommended because measurement of the force is considered to be the essential element of the force limiting approach.

This handbook is applicable to all force limited vibration tests of NASA flight hardware including aircraft and launch vehicle equipment, spacecraft and space experiment hardware.

Since the purpose of force limiting is to mitigate the effect of test item resonances in the vibration test, the technique is most useful for structure-like equipment and for fragile equipment such as optics and complex instruments. References 2, 5, 7, and 10 provide examples of the use of force limiting in vibration tests of spaceflight hardware.

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2. REFERENCE DOCUMENTS

2.1 Government documents.

| | | |
|----------------|---|---|
| NASA-HDBK 7005 | - | Handbook for Dynamic Environmental Criteria, Final Draft #2. May 2000 |
| NASA-RP-1403 | - | Force Limited Vibration Testing Monograph, May 1997 |
| NASA-STD-7001 | - | Payload Vibroacoustic Test Criteria, June 21, 1996 |
| NASA-STD-7002 | - | Payload Test Requirements, July 10, 1996 |

2.2 References:

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3. DEFINITIONS

| | |
|--|--|
| Accelerance | Complex frequency response function which is ratio of acceleration to force. |
| Acceleration of Center of Gravity (CG) | Acceleration of instantaneous centroid of distributed masses (equal to external force divided by total mass, according to Newton's 2nd Law). |
| Apparent Mass | Complex frequency response function which is ratio of force to acceleration. |
| Control System | The hardware and software that provide means for the test operator to translate vibration specifications into the drive signal for the shaker. |
| Design Verification Test | Test to see if as-built test item can survive design loads. |
| Dual Control | Control of both force and acceleration. |
| Dynamic Absorber | Single degree-of-freedom system (SDFS) tuned to excitation frequency to provide reaction force which reduces motion at attachment point. |
| Effective Mass | Masses in model consisting of SDFS's connected in parallel to a common base, so as to represent the apparent mass of a base-driven continuous system. The sum of the effective modal masses equals the total mass. |
| Extremal Control | A shaker controller algorithm based on control of the maximum (extreme) of a number of inputs in each frequency control band. |
| Flight Limits | Definition of accelerations or forces that are believed to be equal to the maximum flight environment, often P(95/50). |
| Force Limiting | Reduction of the reaction forces in a vibration test to specified values, usually to the interface forces predicted for flight, plus a desired margin. |
| Impedance | Complex frequency response function which is ratio of force to velocity quantities. (Sometimes used to refer to ratio of force to any motion quantity.) |
| Level | Test input or response in decibels (dB), $\text{dB} = 20 \log \text{amplitude} = 10 \log \text{power}$. |
| Margin | Factor to be multiplied times, or decibels to be added to, the flight limits to obtain the test specification. |

| | |
|--|---|
| Notching | Reduction of acceleration input spectrum in narrow frequency bands, usually where test item has resonances. |
| Payload | Vibration test item. |
| Quality Factor | The amplification (Q) of a SDFS at resonance, equal to the reciprocal of twice the critical damping ratio. |
| Quasi-Static Acceleration | Combination of static and low frequency loads into an equivalent load specified for design purposes as the CG acceleration. |
| Residual Mass | Sum of the effective masses of all modes with resonance frequencies greater than the excitation frequency. |
| Response Limiting | Reduction of input acceleration to maintain measured response at or below specified value. |
| Shaker | The machine that provides vibratory motion to the test item, usually electrodynamic in aerospace testing (can also be hydraulic or rotary). |
| Single Degree-of-Freedom System (SDFS) | Vibration model with one mass attached to a base. |
| Source | Test item support structure that provides in-flight vibration excitation. |
| Tap Test | Measurement of apparent mass or acceleration by tapping on structure with small rubber or plastic-tipped hammer that incorporates force transducer. |
| Test Fixture | Adapter hardware that allows test item to be mounted to shaker. |
| Three Axis Load Cell | Force gage which measures the three perpendicular components of force simultaneously. |
| Two Degree-Of-Freedom System (TDFS) | Vibration model with two masses attached to a base. |

4. GENERAL REQUIREMENTS

4.1 Criteria for Force Limiting. The purpose of force limiting is to reduce the response of the test item at its resonances on the shaker in order to replicate the response at the combined system resonances in the flight-mounting configuration. References 1, 3, and 6 provide background information and rationale on the need for force limiting in vibration tests. Force limiting is most useful for structure-like test items that exhibit distinct, lightly damped resonances on the shaker. Examples are: complete spacecraft, cantilevered structures like telescopes and antennas, lightly damped assemblies such as cold stages, fragile optical components, and equipment with pronounced fundamental modes such as a rigid structure with flexible feet. The amount of relief available from force limiting is greatest when the structural impedance (effective mass) of the test item is equal to, or greater than, that of the mounting structure. However, it is recommended that notches deeper than 14 dB be implemented only with appropriate peer review. Force limiting is most beneficial when the penalties of an artificial test failure are high. Sometimes this is after an initial test failure.

4.2 Instrumentation.

4.2.1 Piezoelectric Force Gages. The use of piezoelectric force gages for force limiting is highly recommended over other types of force measurement means, such as strain gages, armature current, etc. The advent of piezoelectric, quartz force gages has made the measurement of force in vibration tests almost as convenient and accurate as the measurement of acceleration. The high degree of linearity, dynamic range, rigidity, and stability of quartz make it an excellent transducer material for both accelerometers and force gages. Similar signal processing, charge amplifiers and voltage amplifiers, may be used for piezoelectric force gages and accelerometers. However, there are several important differences between these two types of measurement. Force gages must be inserted between (in series with) the test item and shaker and therefore require special fasteners, whereas accelerometers are placed upon (in parallel with) the test item or shaker. The total force into the test item from several gages placed at each shaker attachment may be obtained by simply using a junction to add the charges before they are converted to voltage, whereas the output of several accelerometers is typically averaged after the charge is converted to voltage. Finally, piezoelectric force gages tend to put out more charge than piezoelectric accelerometers because the force gage crystals experience higher loading forces, so sometimes it is necessary to use a charge attenuator before the charge amplifier.

4.2.2 Force Gage Preload. Piezoelectric force gages must be preloaded so that the transducer always operates in compression. This preload is achieved using a threaded bolt or stud, which passes through the inside diameter of the transducer. The static compression force in the transducer is balanced by the static tension in the bolt. Having a high preload and a smooth transducer and mating surfaces minimizes several common types of measurement errors, e.g., bending moments being falsely sensed as tension/compression. However, using flight hardware and fasteners, it is usually impossible to achieve the force gage manufacturer's recommended preload. In addition, sometimes it is necessary to tradeoff transducer preload and dynamic load carrying capability, particularly for moments. The three requirements for selecting the preload are: (1) that the preload is sufficient to prevent unloading due to the dynamic forces and moments, e.g., heel-to-toe rocking; (2) that the maximum stress on the transducers does not exceed that associated with the maximum load set specified by the manufacturer; and (3) that the preload is sufficient to carry the shear loads via friction, without slip.

When the bolt preload is critical, which is often the case for large test items like spacecraft, the force transducers may be used to measure the bolt preload while the bolts are torqued. (The actual preload resulting from a specified value of torque may vary by a factor of two or more, depending on friction and lubrication.) Piezoelectric force transducers are dynamic, not static sensors. However, when charge amplifiers with high input resistance and a long time constant option are utilized, the preload force values for each gage may be held steady for hours, which is ample time for the bolt torquing sequence.

4.2.3 Force Gage Calibration. The force gage manufacturer provides a nominal calibration for each transducer, but the sensitivity of installed units must generally be determined insitu, because the preload bolt carries a portion of the dynamic load. (Some transducer manufacturers offer standard preload hardware, but for aerospace applications the preload configuration is usually tailored to the installation; often a flight-like bolt is used.) The transducer and the bolt act like two springs in parallel, and the fluctuating load carried by each is proportional to their stiffness. Therefore the sensitivity of the transducer is reduced by the ratio of the transducer

stiffness to the sum of the bolt plus the transducer stiffness. (The flexibility of any structural elements, mounting feet, etc., in these two load paths must also be included.) The transducer stiffness is available from the manufacturer and the stiffness of the bolt and structural elements can be estimated from strength-of-materials or finite-element-model (FEM) calculations.

Insitu calibration of force gages may be accomplished either statically or dynamically. The recommended method of calibrating the transducers for a force limited vibration test is to conduct a preliminary low-level sine-sweep or random run. The low frequency asymptote of the apparent mass is compared with the known total mass of the test item. (The relevant apparent mass is the ratio of total force to the input acceleration in the shake direction.) If it is not possible to start the sweep at a frequency sufficiently below the first resonance of the test item, it may be necessary to use the single degree-of-freedom system (SDFS) transmissibility to determine a correction (amplification) factor for the low frequency asymptote. Typically, the force measured before correcting for the preload bolt will be approximately 80 to 90 percent of the test item total weight in the axial direction and 90 to 95 percent of the total weight in the lateral directions.

Alternately, using the charge amplifier configuration discussed in Section 4.2.2 for measuring preload, the transducer installation may be calibrated statically using weights, springs, or hydraulic loads. It is recommended that a static calibration be performed by first loading the transducers, zeroing out the charge, and then removing the load, in order to minimize any transient overshoot associated with the load application.

4.2.4 Force Gage Combinations. It is recommended that the total force in the shaker excitation direction be measured in a force limited vibration test. The total force from a number of gages in parallel is readily obtained using a junction box that sums the charges, and therefore the forces, before conditioning the signal with a charge amplifier. An alternative is to specify limits for the force at individual attachment positions, but this is not recommended. Since vibration tests are normally conducted sequentially in three perpendicular axes, it is convenient to employ triaxial force transducers. Sometimes it is necessary to limit the cross-axis force and the moments in addition to the in-axis force; this is particularly the case in tests of large eccentric test items such as spacecraft. For these applications, the six force resultant forces and moments for a single node may be measured with a combination, commonly four, of triaxial force transducers and a special voltage summing amplifier, which is generally available from the force gage manufacturers.

4.2.5 Accelerometers. Accelerometers on the fixture are also required in force limited vibration tests in order to control the acceleration input to the acceleration specification at frequencies other than at the test item resonances. In addition, it is often convenient to use a limited number of accelerometers to measure the response at critical positions on the test item. These response accelerometers may be used only for monitoring or if justified by appropriate rationale, for response limiting in addition to the force limiting.

4.3 Fixtures. The preferred method of configuring the force gages is to sandwich one gage between the test item and conventional test fixture at each attachment position and use fasteners that are longer than the conventional ones to accommodate the thickness of the gages. In this configuration, there is no fixture weight above the transducers and the gage force is identical to the force into the test item. Sometimes the preferred approach is impractical; e.g., if there are too many attachment points or the attachments involve shear pins in addition to bolts. In these cases it may be necessary to use one or more adapter plates to interface the

transducers to the test item. The requirement is that the total weight of the adapter plates above the force gages does not exceed 10 percent of the weight of the test item. This limitation is necessary because the force gages read the sum of the force required to accelerate the interface plate and that delivered to the test item. If the fixture weight exceeds the 10 percent criterion, force limiting will only be useful for the first one or two modes in each axis. Use of a circuit to subtract the interface plate force in real time is not recommended because of the phase errors that result when the interface plate is not rigid. The use of armature current to measure shaker force is also not recommended, because the weight of the armature and fixtures typically are much greater than 10 percent of that of the test item and also because of phase errors associated with electromechanical resonances.

4.4 Force Specifications. Force limits are analogous and complementary to the acceleration specifications used in conventional vibration testing. Just as the acceleration specification is the frequency spectrum envelope of the inflight acceleration at the interface between the test item and flight mounting structure, the force limit is the envelope of the in-flight force at the interface. In force limited vibration tests, both the acceleration and force specifications are needed, and the force specification is generally based on and proportional to the acceleration specification. Therefore, force limiting does not compensate for errors in the development of the acceleration specification, e.g., too much conservatism or the lack thereof. These errors will carry over into the force specification. Since in-flight vibratory force data are scarce, force limits are often derived from coupled system analyses and impedance information obtained from measurements or finite element models (FEM). Fortunately, considerable data on the interface forces between systems and components are becoming available from system acoustic and vibration tests of development test models. Semi-empirical methods of predicting force limits are currently being developed on the basis of the limited flight and system test data.

4.4.1 Analytical Force Limits. Analytical models and methods of obtaining impedance information to use in these models are discussed in Section 5.0. Here, the general requirements for analytical force limits are discussed. It is required that analytical models used to predict force limits take into account the resonant behavior of both the source (mounting structure) and the payload (test item) and that the models incorporate impedance information, from test data or finite element models (FEM's), on both the source and the payload. The models discussed in 5.0 are two degree-of-freedom system (TDFS) models, in which the coupled source and payload are each described by a single resonant mode. In more complex models, the source and payload may have many modes. In the early stages of a program, before hardware exists, strength of materials or FEM's are often used to determine the modal parameters of the source and payload. Later in the program, before the vibration tests of flight hardware, it is recommended that the modal parameters be updated with impedance data measured in tap tests on the mounting structure and in the shaker tests of the test item. The coupled source and payload models are exercised with some representative excitation of the source, and the envelope (or peak values) of the interface acceleration and interface force frequency response functions (FRF) are calculated, preferably in one-third octave bands. The ratio of the interface force envelope to the acceleration envelope is calculated from the model. Finally, the force limit specification is calculated by multiplying the conventional acceleration specification by this ratio. (It is essential that the ratio of the envelopes or peaks, not of the actual FRF's, be calculated.) The simple and complex TDFS methods discussed in Section 5.1 and illustrated in Figures 1-3 are examples of this approach.

4.4.2 Semi-Empirical Force Limits. The alternative semi-empirical approach to deriving force limits is based on the extrapolation of interface force data for similar mounting structure and test items. A general form for a semi-empirical force limit for sine or transient tests is given in Reference 4.

$$F_s = C M_o A_s \quad (1a)$$

where F_s is the amplitude of the force limit, C is a constant which depends on the configuration, M_o is the total mass of the payload (test item), and A_s is the amplitude of the acceleration specification. The form of Equation (1a) appropriate for random vibration tests is:

$$S_{FF} = C^2 M_o^2 S_{AA} \quad (1b)$$

where S_{FF} is the force spectral density and S_{AA} is the acceleration spectral density. (Comparing Equation (1b) with Figure 2, which is discussed in detail in Section 5.1.1, it may be apparent that C^2 is equivalent to the ordinate in Figure 2, and that the constant C replaces the quality factor Q of the isolated payload system.)

Reference 2 presents interface force data measured between three equipment items and JPL's Cassini spacecraft during acoustic tests of the development test model spacecraft. Reference 10 presents interface force data measured between a 65 pound instrument (CRIS) and the Advanced Composition Explorer (ACE) spacecraft during the actual launch of the spacecraft. The force data measured on the CASSINI and ACE spacecraft are enveloped by Equation (1b) with C approximately equal to the square-root of two at frequencies up to and including the fundamental resonance of the test item and with C decreasing as one over frequency at frequencies above the fundamental resonance. In order to use the semi-empirical method to predict force limits for a new hardware configuration, it is required to show similarity between the new configuration and the reference data configuration. (Figure 2 may also be used to scale values of the constant C based on the ratio of the payload to source masses.)

4.4.3 Quasi-Static Design Verification. The design of aerospace components is often based on a prediction of the maximum acceleration of the center of gravity (CG) of the component expected in flight, i.e., the quasi-static limit load. However, the CG of a flexible body is a virtual (not a real) point and its acceleration is very difficult to measure with an accelerometer in a vibration test, particularly at frequencies above the fundamental resonance. However, Equation (1a) with C equal to unity and A_s equal to the CG acceleration is Newton's 2nd law. Thus, limiting the measured external force to the product of the total mass of the test item times the quasi-static load limit is the recommended method of limiting the CG acceleration in vibration tests. (Sometimes the limit load is multiplied by a factor of 1.2 or 1.25 for a design verification test.) This approach is valid at all frequencies. In addition, force gages may be used to control the applied forces or moments in a vibration test to the values derived from the coupled-loads analysis.

4.5 Control System. Most relatively new vibration test controllers have the two capabilities needed to automatically implement force limiting. (Force limiting is viewed as a form of response limiting by most of the controller manufacturers.) To implement force limiting, the controller must first be capable of "extremal control," sometimes called maximum or peak control by different vendors. In extremal control, the largest of a set of signals is limited to a single reference spectrum. Most controllers used in aerospace testing laboratories support extremal control. The second capability, which is highly desirable but not absolutely essential, is that the controller should accommodate different reference spectra for limiting individual

response (and force) measurement channels. Controllers that support response limiting are now available from most vendors, and in addition, upgrade packages are available to retrofit some of the older controllers for this capability.

If the controller has extremal-control capability, but not response-limiting capability, there are still two possibilities for automatic force limiting. First, the gain of the charge amplifier used to condition the force signal may be adjusted to achieve the desired amount of force limiting. The problem with this approach is that the force limit is then independent of frequency, and the limiting and notching is generally restricted to the fundamental mode, since the force associated with the higher order modes decreases with increasing frequency. A second more elaborate approach, used in the early days of force limiting, is to use a graphic equalizer type of filter to shape the frequency spectrum of the force signal.

If the controller does not have either the extremal-control or response-limiting capability, notching of the acceleration specification to limit the force to the force specification must be done manually based on the force measured in low-level runs.

4.6 Test Planning Considerations. Several considerations need to be addressed in the test planning if force limiting is to be employed. First, the size, number, and availability of the force transducers need to be identified as well as any special fixture requirements to accommodate the transducers. Next, the approach for deriving and updating the force specification needs to be decided. Finally the control strategy, which in special cases may include cross-axis force, moment, individual force, and response limiting in addition to the in-axis force, must be decided and written into the test plan. In some instances, the control strategy will be limited by the control system capabilities. In all cases, it is recommended that the control strategy be kept as simple as possible, in order to expedite the test and to minimize the possibility of mistakes. If required, the rationale for force limiting should be discussed and concurrence obtained from the appropriate organizations, which may include the customer, Government overseer, quality control, etc.

5. DETAILED IMPLEMENTATION

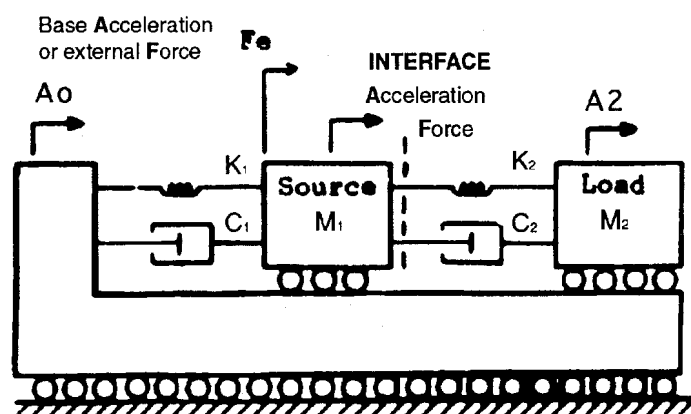
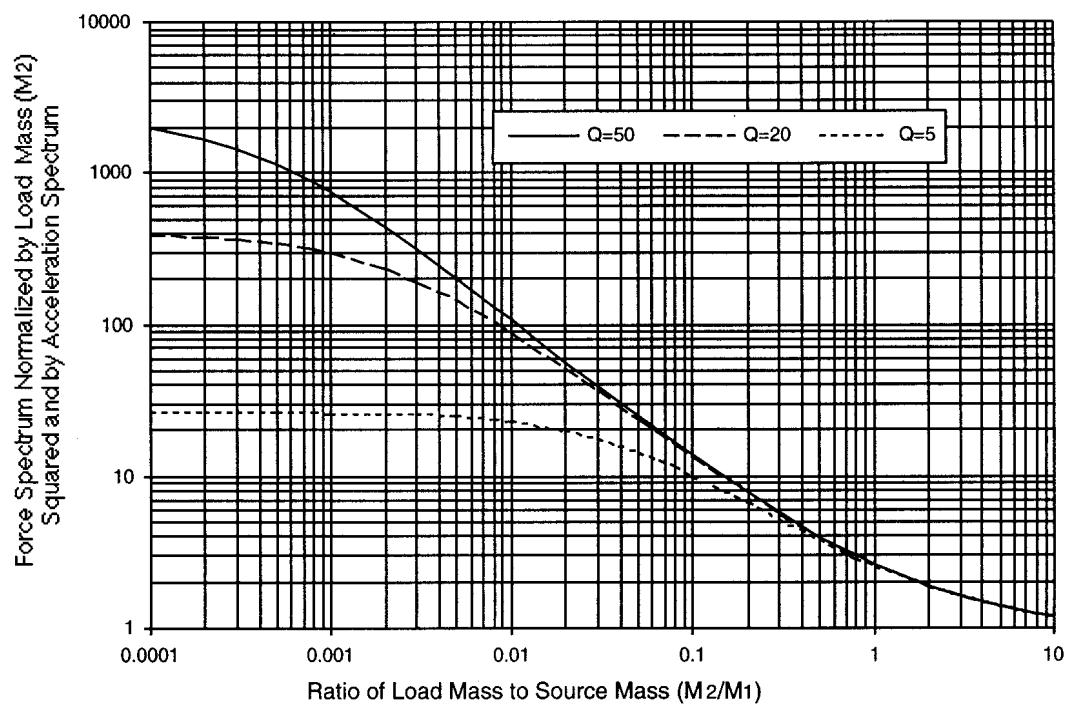
5.1 Derivation of Force Limits. As the force limiting technology matures, there may eventually be as many methods of deriving force limits as there are of deriving acceleration specifications. Several acceptable methods are described herein.

Force spectra have typically been developed in one-third octave bands (see example in Section 6.2), but other bandwidths, e.g., octave or one-tenth octave bands, may also be used. Force limiting is usually restricted to the frequency regime encompassing approximately the first three modes in each axis; which might be approximately 100 Hz for a large spacecraft, 500 Hz for an instrument, or 2000 Hz for a small component. It is important to take into account that the test item resonances on the shaker occur at considerably higher frequencies than in flight. Therefore, care must be taken not to roll off the force specification at a frequency lower than the fundamental resonance on the shaker and not to roll off the specification too steeply; i.e., it is recommended that the roll-off of the force spectrum be limited to approximately 9 dB/octave.

5.1.1 Simple TDFS. The simple TDFS method of deriving force limits is described in Reference 8. The basic model is shown in Figure 1. The model represents one vibration mode of the source (system 1) coupled with one vibration mode of the payload (system 2). Figure 2 shows the ratio of the interface force spectral density S_{FF} to the input acceleration spectral density S_{AA} , normalized by the payload mass M_2 squared, as a function of the mass ratio M_2/M_1 , calculated from the simple TDFS. When this mass ratio is very small, there is no force limiting effect; the force spectral density asymptote is the payload mass M_2 squared times the input acceleration spectral density times the quality factor Q_2 squared. The ratio of this asymptotic value of the force to the force limit at larger values of M_2/M_1 , is the expected amount of notching, sometimes called the knockdown factor. The force limit is insensitive to damping at values of M_2/M_1 greater than 0.4, but the peak value of the unnotched force spectrum, and therefore the notch depth resulting from force limiting, will be proportional to the actual quality factor Q_2 squared. To use Figure 2, the source and payload masses must be determined from FEM analyses or measurements as a function of frequency. It is recommended that one-third octave frequency bands be utilized. In the simple TDFS method, it is recommended for conservatism that these masses be taken as the residual masses rather than the modal masses. Appendix A gives the equations for replicating the curves in Figure 2.

5.1.2 Complex TDFS. The complex TDFS method of deriving force limits is also described in Reference 8. The complex TDFS model is shown in Figure 3; it requires both the modal (m) and the residual (M) masses of the source and payload. Table 1 tabulates the normalized ratio of interface force spectral density to input acceleration spectral density for a complex TDFS with $Q=20$, which is a good nominal value for most practical applications. It is recommended that both the simple and complex TDFS models be used and that the larger of the two calculations be used in each one-third-octave frequency band. It will generally be found that the simple TDFS gives the larger result off the payload resonances and the complex TDFS gives the larger result at the payload resonances. Notice that the normalized force in Table 1 is equal to unity for $m_2/M_2 = 0$, i.e. no payload modal mass, and should be interpolated for $0 < m_2/M_2 < 0.125$.

Table 1 does not include results for ratios of the source modal to residual mass (m_1/M_1) less than 0.25, because it is felt that these cases represent local source modes, which may not be relevant to the interface environment. However, values for $m_1/M_1 < 0.25$ are tabulated in the monograph "Force Limited Vibration Testing," listed in Section 2.1.

FIGURE 1. Simple Two-Degree-of-Freedom System (TDFS) of Coupled OscillatorsFIGURE 2. Normalized Force Specification from Simple TDFS

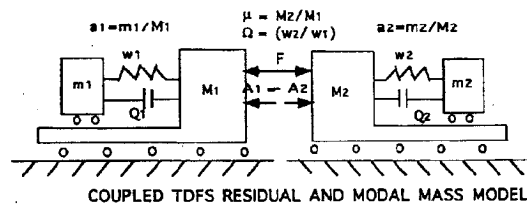
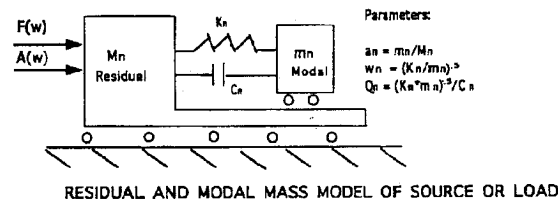
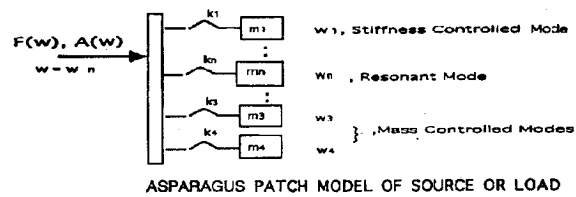
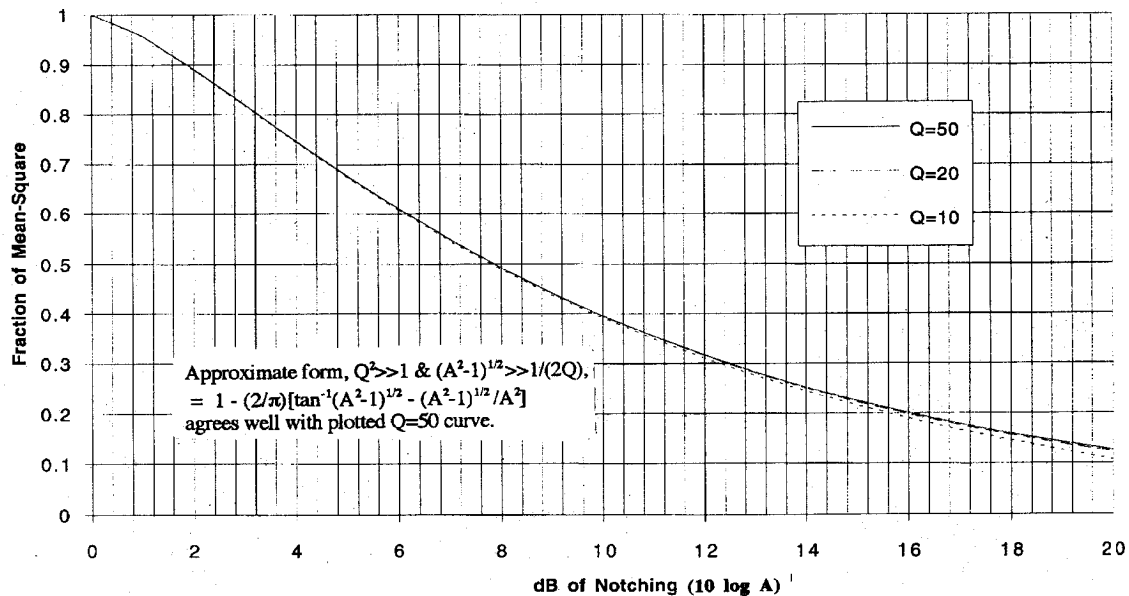
FIGURE 3. Complex TDFS with Residual and Modal MassesFIGURE 4. Reduction of SDFS Mean-Square Response By Notching

Table I. Normalized Force Spectrum for Complex TDFS with Q=20

| Ratio of modal to residual mass $m1/M1, m2/M2$ | Residual mass ratio, $M2/M1$ | | | | | | | | |
|--|------------------------------|-------|------|------|------|------|------|------|------|
| | 0.001 | 0.003 | 0.01 | 0.03 | 0.1 | 0.3 | 1 | 3 | 10 |
| 8.0, 8.0 | 932 | 933 | 936 | 948 | 1001 | 1180 | 1240 | 1234 | 1238 |
| 8.0, 4.0 | 233 | 233 | 233 | 235 | 239 | 256 | 294 | 265 | 250 |
| 8.0, 2.0 | 58 | 58 | 58 | 58 | 59 | 60 | 68 | 73 | 68 |
| 8.0, 1.0 | 15 | 15 | 15 | 15 | 15 | 15 | 17 | 23 | 22 |
| 8.0, 0.5 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 7 | 6 |
| 8.0, 0.25 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 5 |
| 8.0, 0.125 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 3 |
| 8.0, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4.0, 8.0 | 871 | 867 | 858 | 849 | 904 | 1042 | 1067 | 1110 | 1229 |
| 4.0, 4.0 | 218 | 218 | 217 | 216 | 220 | 250 | 254 | 250 | 252 |
| 4.0, 2.0 | 55 | 55 | 55 | 55 | 56 | 61 | 72 | 68 | 67 |
| 4.0, 1.0 | 14 | 14 | 14 | 14 | 14 | 16 | 21 | 23 | 22 |
| 4.0, 0.5 | 3 | 3 | 4 | 4 | 4 | 4 | 6 | 10 | 10 |
| 4.0, 0.25 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 5 | 5 |
| 4.0, 0.125 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3 |
| 4.0, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.0, 8.0 | 1586 | 1478 | 1260 | 1061 | 990 | 946 | 982 | 1099 | 1201 |
| 2.0, 4.0 | 406 | 391 | 355 | 305 | 272 | 259 | 238 | 236 | 254 |
| 2.0, 2.0 | 103 | 101 | 97 | 88 | 79 | 82 | 70 | 65 | 62 |
| 2.0, 1.0 | 26 | 26 | 26 | 25 | 24 | 25 | 25 | 23 | 22 |
| 2.0, 0.5 | 7 | 7 | 7 | 7 | 7 | 9 | 10 | 10 | 10 |
| 2.0, 0.25 | 2 | 2 | 2 | 2 | 2 | 3 | 5 | 5 | 6 |
| 2.0, 0.125 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 4 |
| 2.0, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1.0, 8.0 | 11041 | 5731 | 2714 | 1486 | 967 | 901 | 984 | 1095 | 1181 |
| 1.0, 4.0 | 3869 | 2206 | 1105 | 567 | 332 | 247 | 233 | 238 | 248 |
| 1.0, 2.0 | 1228 | 826 | 432 | 226 | 125 | 83 | 71 | 66 | 64 |
| 1.0, 1.0 | 359 | 283 | 166 | 100 | 50 | 34 | 26 | 23 | 23 |
| 1.0, 0.5 | 100 | 89 | 63 | 42 | 24 | 15 | 12 | 11 | 11 |
| 1.0, 0.25 | 28 | 27 | 23 | 17 | 11 | 8 | 6 | 6 | 6 |
| 1.0, 0.125 | 8 | 8 | 8 | 7 | 5 | 5 | 4 | 4 | 4 |
| 1.0, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.5, 8.0 | 13889 | 7720 | 3501 | 1726 | 1023 | 880 | 974 | 1093 | 1171 |
| 0.5, 4.0 | 4516 | 2895 | 1417 | 695 | 357 | 247 | 225 | 240 | 244 |
| 0.5, 2.0 | 1346 | 1003 | 561 | 283 | 136 | 89 | 70 | 64 | 65 |
| 0.5, 1.0 | 377 | 319 | 211 | 117 | 59 | 39 | 27 | 24 | 22 |
| 0.5, 0.5 | 102 | 95 | 74 | 48 | 27 | 17 | 12 | 11 | 10 |
| 0.5, 0.25 | 28 | 27 | 25 | 19 | 13 | 8 | 7 | 6 | 6 |
| 0.5, 0.125 | 8 | 8 | 8 | 8 | 6 | 5 | 4 | 4 | 4 |
| 0.5, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.25, 8.0 | 17378 | 9978 | 4092 | 1944 | 1017 | 833 | 936 | 1092 | 1166 |
| 0.25, 4.0 | 5194 | 3725 | 1805 | 812 | 380 | 249 | 225 | 241 | 242 |
| 0.25, 2.0 | 1455 | 1205 | 741 | 359 | 173 | 93 | 71 | 66 | 65 |
| 0.25, 1.0 | 391 | 354 | 269 | 160 | 74 | 43 | 28 | 23 | 22 |
| 0.25, 0.5 | 103 | 99 | 86 | 63 | 38 | 22 | 14 | 12 | 11 |
| 0.25, 0.25 | 28 | 28 | 27 | 23 | 16 | 10 | 8 | 7 | 7 |
| 0.25, 0.125 | 8 | 8 | 8 | 8 | 7 | 5 | 5 | 4 | 4 |
| 0.25, 0.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

5.1.3 Multiple Degree-of-Freedom Systems. In general, a multiple degree-of-freedom model of the source and payload may be utilized as in References 11 and 12. The model parameters are determined from modal mass and resonance frequency information for the source and payload. The ratio of the interface force envelope to the interface acceleration envelope should be evaluated as with simpler models, and the force limit should be determined by multiplying this ratio by the acceleration specification obtained as in conventional vibration tests.

5.1.4 Alternative Methods. Just as there are now many ways of developing acceleration specifications, in the future there will be many ways of deriving force limits. In time, a data base of flight and system test force data and validated semi-empirical methods will be available, but for the present, force limits must often be derived from analytical models with structural impedance data. Although the methods recommended in this handbook are preferred, other methods may be acceptable if they are rational and result in a desired margin over flight. One alternative method is to use the blocked force, which is the force that the source will deliver to an infinite impedance payload. Unfortunately for light loads, the blocked force is too large to result in much limiting as shown in Reference 9. Another method, which is recommended for low frequency testing, is to base the force limit on the CG acceleration used for quasi-static design, as discussed in Section 4.4.3.

5.2 Apparent and Effective Mass. The frequency response function (FRF), which is the ratio of the reaction force to applied acceleration at the base of a structure, is called "apparent mass." The apparent mass is a complex impedance-like quantity which reflects the mass, stiffness, and damping characteristics of the structure. The modal models recommended herein require only the "effective" masses, which are real quantities and therefore are much simpler.

5.2.1 Effective Mass Concept. The concept of effective mass was introduced in Reference 13. Consider the drive point apparent mass of the model consisting of the set of SDFS's connected in parallel to a rigid, massless base as shown in Figure 3, which is from Reference 8. The modal contribution to this drive-point apparent mass, divided by the SDFS frequency response factor, is called the effective mass of that mode. The sum of the effective modal masses for each excitation axis is equal to the total mass of the distributed system. The sum of the effective masses of the modes with resonance frequencies above the excitation frequency is called the effective residual mass. Appendix B provides a more general definition of effective mass and a procedure for using NASTRAN to calculate the effective masses.

5.2.2 Shaker Measurement of Payload Effective Mass. The payload effective residual mass should be measured and used to update the calculated force limits before conducting a force limited vibration test of flight hardware, because the force limits in both the simple and complex TDFS models are proportional to the payload effective residual mass. Fortunately, the payload effective residual mass can be readily measured with a low level sine sweep, or random, test run when the payload is mounted with force gages on the shaker. First, the magnitude of the drive point apparent mass, the ratio of total reaction force in the excitation direction to the input acceleration, is measured. Then this apparent mass function is smoothed (a moving average in frequency) to eliminate the resonance peaks. The resulting smooth curve, which must be a decreasing function of frequency by Foster's theorem, is taken as the effective residual mass. In addition, the effective modal mass for each distinguishable mode may be evaluated by equating the corresponding peak in the apparent mass curve to the sum of the residual mass and the product of the effective modal mass times the quality factor Q , which may be determined from half-power bandwidth.

5.2.3 Tap Test Measurement of Source Effective Mass. The source effective residual mass is determined in a similar manner by smoothing the FRF's of the magnitude of the drive point apparent mass of the source. The source FRF's may be measured with a modal hammer incorporating a force gage. The measurements involve tapping at representative positions where the payload attaches and computing the FRF of the ratio of the force to the acceleration, which is measured with an accelerometer mounted temporarily on the source structure near the hammer impact point. (The payload must not be attached to the source structure during these measurements.) Some judgment is involved in combining the apparent masses measured at multiple attachment points to obtain a single-node model of the effective mass. At low frequencies, each point yields the total mass, unless rotations are introduced. At high frequencies, the apparent masses from multiple points should be added, usually by adding the sum of the squares. Also when calculating or measuring the apparent mass of a mounting structure, it is important to decide how much of the adjacent structure it is necessary to consider. It is necessary to include only enough of the mounting structure so that the source effective modal and residual masses are accurately represented in the frequency range of the payload resonances.

6. NOTES

6.1 Reduction of Mean-Square Response Due to Notching. It is often important to know how much the mean-square response, or force, will be reduced when a resonance is limited by notching the input acceleration. Limiting a response to the spectral density peak value, divided by the factor A squared, results in a notch of depth A squared in the input spectral density at the resonance frequency. The reduction in the mean-square response resulting from notching is considerably less than that associated with reducing the input spectral density at all frequencies. (In the latter case the response is reduced proportionally, e.g. a flat 6 dB reduction in the input spectrum yields a 6 dB reduction in the response spectrum and a factor of four reduction in the mean-square response.) The reduction in mean-square response of an SDFS resulting from notching the input $\text{dB} = 20 \log A$ at the response resonance frequency is shown in Figure 4, which is from Reference 9. Note from Figure 4 that a notch of approximately 14 dB is required to reduce the mean-square response by a factor of four! (A factor of four reduction in the mean-square corresponds to a factor of two reduction in the r.m.s. value and therefore in the peak value of the response.)

6.2 Force Specification Example. Appendix C is a spread sheet calculation of the force specification for an instrument (CRIS) mounted on a honeycomb panel of a spacecraft (ACE) using three methods: the simple TDFS, the complex TDFS, and the semi-empirical. Note that the acceleration PSD values in the table are rounded off to two significant figures, but the exact acceleration PSD values are used to calculate the force specifications in the table.

6.3 Definition of Symbols

| | | |
|-----------------|---|---|
| A | = | interface acceleration |
| A_b | = | base acceleration |
| A_o | = | free acceleration of source |
| A_s | = | acceleration specification |
| c | = | dashpot constant |
| C | = | constant in semi-empirical method |
| F | = | interface force |
| F_s | = | force specification or limit |
| k | = | spring stiffness |
| \mathbf{k} | = | physical stiffness matrix |
| M_o | = | total mass |
| M | = | residual mass |
| m | = | modal mass |
| \underline{M} | = | apparent mass, F/A |
| \mathbf{m} | = | physical mass matrix |
| \mathbf{M} | = | modal mass matrix |
| Q | = | quality factor (amplification at resonance of SDFS) |
| S_{AA} | = | acceleration spectral density |
| S_{FF} | = | force spectral density |
| u | = | absolute displacements |
| U | = | generalized modal displacement |
| ϕ | = | mode shape |
| ω | = | radian frequency |
| ω_o | = | natural frequency of uncoupled oscillator |

Subscripts

| | | |
|---|---|--------------------------------------|
| 1 | = | source oscillator |
| 2 | = | payload oscillator |
| F | = | unrestrained (free) |
| P | = | prescribed motion and rigid body set |
| N | = | modal set |
| n | = | single mode |
| p | = | reaction force direction |
| q | = | prescribed acceleration direction |

Appendix A

Equations for Calculating the Simple TDFS Force Limits

The force limit is calculated for the TDFS in Figure 1 with different masses for the source and the payload oscillators. For this TDFS, the maximum response of the payload and therefore the maximum interface force occur when the uncoupled resonance frequency of the payload equals that of the source. For this case, the characteristic equation is that of a classical dynamic absorber, from Reference 8:

$$(\omega/\omega_0)^2 = 1 + (m_2/m_1)/2 \pm [(m_2/m_1) + (m_2/m_1)^2/4]^{0.5} \quad (A1)$$

where ω_0 is the natural frequency of one of the uncoupled oscillators, m_1 is the mass of the source oscillator, and m_2 is the mass of the load oscillator in Figure 1. The ratio of the interface force S_{FF} to acceleration S_{AA} spectral densities, divided by the magnitude squared of the payload dynamic mass m_2 , is:

$$S_{FF}/(S_{AA} m_2^2) = [1 + (\omega/\omega_0)^2/Q_2^2] / \{[1 - (\omega/\omega_0)^2]^2 + (\omega/\omega_0)^2/Q_2^2\} \quad (A2)$$

where Q_2 is the quality factor, one over twice the critical damping ratio, of the payload.

The force spectral density, normalized by the payload mass squared and by the acceleration spectral density, at the two-coupled system resonances is obtained by combining Equations (A1) and (A2). For this TDFS, the normalized force is just slightly larger at the lower resonance frequency of Equation (A1). The maximum normalized force spectral density, obtained by evaluating Equation (A2) at the lower resonance frequency, is plotted against the ratio of payload to source mass for three values of Q_2 in Figure 2.

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Appendix B

Calculation of Effective Mass

Applying the rationale of Reference 13 and subdividing the displacement vector into unrestrained absolute displacements u_F and prescribed absolute displacements u_P , the equilibrium equation is:

$$\begin{bmatrix} m_{FF} & m_{FP} \\ m_{PF} & m_{PP} \end{bmatrix} \begin{Bmatrix} d^2 u_F / dt^2 \\ d^2 u_P / dt^2 \end{Bmatrix} + \begin{bmatrix} k_{FF} & k_{FP} \\ k_{PF} & k_{PP} \end{bmatrix} \begin{Bmatrix} u_F \\ u_P \end{Bmatrix} = \begin{Bmatrix} f_F \\ f_P \end{Bmatrix} \quad (B1)$$

$$\text{Let: } \{u\} = \phi U = \begin{bmatrix} \phi_N & \phi_P \\ 0 & I_{PP} \end{bmatrix} \begin{Bmatrix} U_N \\ U_P \end{Bmatrix} \quad (B2)$$

Where ϕ_N are normal modes and ϕ_P are rigid body modes associated with a kinematic set of unit prescribed motions, and U_N is the generalized modal relative displacement and U_P is the generalized prescribed absolute displacement. Substituting and pre-multiplying by ϕ_N^T yields:

$$\begin{bmatrix} M_{NN} & M_{NP} \\ M_{NP}^T & M_{PP} \end{bmatrix} \begin{Bmatrix} d^2 U_N / dt^2 \\ d^2 U_P / dt^2 \end{Bmatrix} + \begin{bmatrix} \omega_N^2 M_{NN} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_N \\ U_P \end{Bmatrix} = \begin{Bmatrix} F_N \\ F_P \end{Bmatrix} \quad (B3)$$

$$\text{where: } M_{NN} = \phi_N^T m_{FF} \phi_N \quad (B4)$$

$$M_{NP} = \phi_N^T m_{FF} \phi_P + \phi_N^T m_{FP} I_{PP} \quad (B5)$$

$$M_{PP} = I_{PP} m_{PP} I_{PP} + I_{PP} m_{PF} \phi_P + \phi_P^T m_{FP} I_{PP} + \phi_P^T m_{FF} \phi_P \quad (B6)$$

$$F_P = I_{PP} f_P + \phi_P^T f_F \quad (B7)$$

$$F_N = \phi_P^T f_F \quad (B8)$$

$$\text{For: } d^2 U_P / dt^2 = U_P = F_N = 0, \quad d^2 U_n / dt^2 = -\omega_n^2 U_n, \quad \text{and } U_n = 1:$$

$$M_{nP}^T = -F_P / \omega_n^2 \quad (B9)$$

where n indicates a single mode. (Note that M_{nP}^T is in mass units.) M_{nP}/M_{nn} is sometimes called the **elastic-rigid coupling** or the **modal participation factor** for the n th mode. If the model is restrained at a single point, the reaction (F_P) in (B8) is the SPCFORCE at that point in a NASTRAN modal analysis.

The initial value of M_{PP} is the rigid body mass matrix. If a Gaussian decomposition of the total modal mass in (B3) is performed, it subtracts the contribution of each normal mode, called the **effective mass**:

$$M_{nP}^T M_{nn}^{-1} M_{nP}, \quad (B10)$$

from M_{PP}^n , which is the **residual mass** after excluding the mass associated with the already processed n modes.

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Consider the ratio of the reaction force in direction p , to the prescribed acceleration in direction q . The effective mass is the contribution of the n th mode to this ratio, divided by the SDFS frequency response factor. The sum of the common-direction effective masses for all modes is equal to the total mass, or moment of inertia for that direction. The effective masses are independent of the modal normalization.

Appendix C

Force Specification Example

