MIL-STD-471A
NOTICE 1
10 January 1975

MILITARY STANDARD

MAINTAINABILITY VERIFICAION/DEMONSTRATION/EVALUATION

TO ALL HOLDERS OF MIL-STD-471A

1. The following are new and revised pages of this standard. The revised pages supersede the original. pages:

NEW PAGE	DATE	SUPERSEDED PAGE	DATE
iii	27 March 1973 (Reprinted without change)		
iv	10 January 1975	iv	27 March 1973
19	10 January 1975	19	27 March 1973
20	10 January 1975	20	27 March 1973
21	10 January 1975	21	27 March 1973
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25	27 March 1973 (Reprinted without change)		
26	10 January 1975	26	27 March 1973
26a	10 January 1975	New Page	
27	10 January 1975	27	27 March 1973
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55-77	10 January 1975	New Pages	

2. RETAIN THIS NOTICE PAGE AND INSERT BEFORE THE TABLE OF CONTENTS.

FSC - MISC

3. Holders of MIL-STD-471A will verify that page changes and additions indicated above have been entered. The notice page will be retained as a check sheet. This insurance, together with appended pages, is a separate publication. Each notice is to be retained by stocking points until the Military Standard is completely revised or cancelled.

Custodians:
Army - EL
Navy - As
Air Force - 17

Preparing Activity:
Air Force - 17

Project MISC-0953

Review Activities:
Army - MI, SC, TE
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APPENDIX A

MAINTENANCE TASK SAMPLING FOR USE WITH FAILURE SIMULATION

A.10 Scope.

A.10.1 Purpose. This appendix outlines a procedure for the selection of a sample of corrective maintenance tasks for maintainability demonstration when the tasks result from failure simulation.

A.10.2 Application. The procedure described herein is applicable only when failure simulation is to be used to generate maintenance tasks. The procedure is applicable to the equipment level and it is assumed that system level maintainability requirements have been allocated to the equipment level for demonstration. The mean estimates for equipment may be employed to determine achievement of system maintainability requirements. If sampling of preventive maintenance tasks or servicing is permitted, a procedure and tables similar to that illustrated in this appendix for corrective maintenance must be developed for each type of task (i.e., preventive maintenance, servicing).

A.10.3 Sample Stratification. The major objectives of stratification in this standard are to: (a) allow for the selection of maintenance tasks in such a manner that the selection simulates the failure frequency of the test unit in actual operation, (units with low MTBF's will be selected more frequently than units with higher MTBF's), (b) insure that a proportionately representative sample of task types/times are selected. Proportional stratified sampling may be used for selection of maintenance tasks to be demonstrated using the fixed sample size test methods described in Appendix B, Sequential test method shall employ simple random sampling.

A.10.4 Stratification Procedure. The following example illustrates the procedure for tasks which would be classified as corrective maintenance. Preventive maintenance or servicing tasks should not be combined with corrective maintenance tasks for the purpose of task stratification. For system level demonstration of maintainability requirements, the procedure should be applied to each contract end item equipment and through appropriate techniques, the achievement of system maintainability requirements may be demonstrated. Maintenance tasks may be performed concurrently or serially provided that provision has been made to record the expended maintenance time for each maintenance task. The requirement to be demonstrated shall determine the manner in which the data shall be analyzed. The following, Table I, illustrates the application of this procedure to a radar equipment consisting of: Antenna, Receiver/Transmitter, Frequency Tracker, Radar Set Control, and Drift Angle Indicator:

a. column 1 - Identify the major units which comprise the equipment.

b. column 2 - Subdivide each unit to the functional level at which maintenance for the demonstration is to be performed in accordance with the approved maintenance plan. This level may be an assembly, module, printed circuit card or piece part.

c. columns 3 & 4 - For each functional level of maintenance identified in Column 2, identify in Column 3 the type of maintenance task or tasks to be performed and in Column 4 the estimated mean maintenance time for the task. The maintenance task time shall include the time to perform each element of maintenance time as defined in MIL-STD-721B. The maintenance tasks and estimated maintenance time would be derived from a maintenance engineering analysis, a maintainability prediction effort, or from historical data. The same maintenance task, such as "remove and replace" of a module may result from different faults within the module. Column 3 would identify the maintenance task and not the fault or failure which results in the occurrence of the task.

d. column 5 - Determine the failure rate ($F/10^6$ hr.) for each module, printed circuit card, etc., for which the maintenance task was identified in Column 3. The failure rates used shall be the latest available from the associated reliability program. If there is no reliability program, the failure rates may be selected or extrapolated from sources approved by the procuring activity.

e. Column 6 - Determine the quantity of items in each major unit associated with each task in Column 3.

f. column 7 - Determine the duty cycle for each item associated with each task in Column 3 (e.g., operating time of a receiver to the operating time of the radar; engine operating hours to aircraft flight hours).

g. Column 8 - Group together the maintenance tasks identified in Column 3 which have both:

(1) Similar maintenance actions. NOTE: A maintenance action is an element of a maintenance task. Although the estimated mainteneince time for different maintenance tasks may be similar, the actions may be different, that is, one task may involve significant diagnostics and another involve minimum diagnostics but significant access time.

(2) Similar estimated maintenance times. The maintenance times in each group shall be within a range that shall not exceed the smallest value in the group by more than 50 percent.

Task grouping shall be limited to within major units identified in column 1.

h. column 9 - Determine the total failure rate for each task grouping identified in Column 8. The total failure rate is equal to the sum of the products of Columns 5 x 6 x 7 for all tasks within the group.

i. column 10 - Determine the relative frequency of occurrence for each task grouping by dividing the sum of the total failure rate (sum of Column 9) into the individual total failure rate for each group.

j. column 11 - Fixed Sample - A sample of maintenance tasks equal to at least four times the sample size specified for the selected test method (Appendix B) or as specified by the procuring activity, shall be allocated among the task groups in accordance with the relative frequency of occurrence of the task group. Example: Assume the test method to be employed requires that a sample of 50 maintenance tasks be demonstrated, a sample of 200 tasks (4 x 50) shall be allocated among the task groups as follows:

Group 1 - .177 X 200 = 35 tasks;

Group 2 - .178 X 200 = 36 tasks;

Group 3 - .016 X 200 = 3 tasks;

Group 7 - .013 x 200 = 3 tasks,

This allocation is shown in Column 11. The maintenance tasks allocated to each group shall be randomly selected and identified from the population of maintenance tasks applicable to that group. The total number of maintenance tasks which must be identified for the equipment must be equal to or greater than four tires the demonstration sample size (i.e., greater than 4 x 50 = 200 for this example) in order that the number of tasks identified with each group is sufficient such that the allocation of tasks to each group (i.e., 35 tasks for Group 1; 36 tasks for Group 2, etc.) maybe randomly selected from the population of tasks identified as applicable to that group. The maintenance tasks which have been randomly selected shall not be returned to the sample pool, When a task group consists of more than one module or assembly, etc., such as group 2 of Table 1, the maintenance tasks assigned to the group (Column 11, 36 tasks for this example) shall be allocated to the modules, assemblies, etc., within the group in accordance with the relative frequency of occurance of maintenance for each module, etc., within the group. The procedure would be the same as that used to determine the relative frequency of occurrence of the task groups (Column 10) but would be applied to the

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modules, etc., within the group. This is illustrated below with the allocation shown included in Table I, Column 11, Group 2.

Group 2	Total Failure Rate	Relative Freq. of Occ.	Demonstration Population Allocation
A-IF-A	23	.217	7.8 ≈8 (.217 x 36 = 7.8)
B-IF-B	21	.198	7.4 ≈7
C-Amplifier	21	.198	7.1 ≈7
D-Modulator	18	.170	6.0 ≈6
E-Power Supply	23	.217	7.8 » 8
	106	1.000	36

k. Column 12 - The maintenance tasks to be demonstrated (50 tasks for this example) shall be allocated among the task groups in accordance with the relative frequency of occurrence of maintenance for the group.

Example:

Group 1: .177 x 50 = 8.85 ≈9 tasks;

Group 2: .178 X 50 = 8.90 ≈9 tasks;

Group 3: .016 x 50 = .80 ≈1 task,

Group 7: .013 X 50 = .65 » 1 task

If a task group consists of more than one module, assembly, etc., such as group 2, Table I, the maintenance tasks to be demonstrated from the group (column 12, 9 tasks for this example) shall be allocated to the modules, assemblies, etc., within the Group in accordance with the relative frequency of occurrence of maintenance for each module, etc., within the grow. This is illustrated below with the sample allocation shown included in Table I, column 12.

Group 2	Relative Freq. of Occurrence	Demonstration Sample Size
IF-A	.217	1.95 ≈2 (.217 X 9 = 1.95)
IF-B	.198	1.78 ≈2
Amplifier	.198	1.78 ≈2
Modulator	.170	1.53 ≈1
Power Supply	.217	1.95 ≈ 2
		9 total

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TABLE II

FAILURE MODE SELECTION

1 Maintenance Task	2 Failure Mode	3 Effect	4 Relative Frequency of Occurrence (Percent)	5 Cumulative Range (Percent)
Receiver Remove/ Replace	1. Component out of tolerance	1. Noise	.20	0 - .199
	2. Component shorted/open	2. Receiver Inoperative	.35	.200 - .549
	3. Tuning failure	3. Cannot change frequency	.45	.550 - 1.00

NOTE: This table is for illustration only and does not reflect real failure modes, etc.

APPENDIX B

TEST METHODS AND DATA ANALYSIS

B.10 Scope.

B.10.1 Purpose - This appendix contains test methods and criteria for demonstrating the achievement of specified quantitative maintainability requirements: Some of the test methods included are identical to test methods contained in previous versions of this Standard. Table IA indicates the correspondence of the test methods included in this Standard to those included in past versions.

B.10.2 Application - The following matrix (Fig. B-1) summarizes the major characteristics of each test method as well as the quantitative requirements which must be specified for each test method. The data analysis method included with each test method provides the decision criteria for acceptance or rejection of the item being demonstrated.

B.10.3 Sample Size - Each of the test plans contained in this appendix includes an equation or other directions for determining a minimum sample size of maintenance tasks. Any departure from the minimum sample size requirement can affect the statistical validity of the test procedures. Some of the test plans in the appendix require a prior estimate of the variance of the distribution of interest for the calculation of sample size. Such prior estimates, subject to government approval, can be obtained from data on similar equipment provided similarities in maintainability design, skill levels of maintenance personnel, test equipment, manuals and the maintenance environment are considered in the estimation process. Equations for predicting the variance when prior estimates are not available are presented in DDC document AD-869396, Maintainability Prediction and Demonstration Techniques, Vol. II, cited in para. B.10.6, which can be used, provided the information needed for the prediction is available. The 85th - 95th upper confidence bound on the predicted or estimated variance shall be used to insure preservation of the desired risk values. Average observed values of the variance have ranged from $\sigma^2 = .5$ to $\sigma^2 = 1.3$.

B.10.4 Task Selection - Selection of tasks to be sampled when employing fault simulation will be made in accordance with Appendix A of this standard. The Procuring Activity shall have the option of surveillance over and/or participating in the random selection of tasks comprising the demonstration population (Column 11 of Table I) down to and including the specific faults to be simulated. This shall occur at a specific conference at a time established by the contractor, consistent with the Maintainability Program Plan schedule. In the event that tasks so chosen can result in events detrimental to safety of personnel or property, appropriate redesign action must take place; in the event that secondary failures result, they will be documented and their impact on item maintainability assessed. A report of such findings shall be made to-the procuring activity. Care must be exercised in selecting and sampling tasks to insure that a true simple random sample is obtained when sequential tests are employed. Departures from simple random sampling, such as proportionate stratified sampling, can effect the validity of the test procedures presented herein, however, this effect is considered minimal for the sample sizes required by the test procedures which are not sequential tests. Simple random sampling shall be used for sequential tests.

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B.10.5 Test Selection - In general, the test index to be demonstrated is the primary consideration in selecting a test procedure. Considerable savings in sample size can be obtained by use of sequential test procedures in preference to fixed sample tests. As a general rule, however, the sequential test should be used only when prior knowledge (e.g., from the prediction) indicates that the equipment may be much better (or worse) than the specified values.

B.10.5.1 A cross-reference of the test methods in MIL-STD-471A to those which were numbered differently in the original issues, MIL-STD-471 (and Notice 1) are listed in Table IA.

TABLE IA, TEST METHOD CROSS-REFERENCE LIST

MIL-STD-471A		MIL-STD-471
Test Method 8	→	Test Method 1
Test Method 9	→	Test Method 2
Test Method 4	→	Test Method 3
Test Method 10	→	Test Method 4
Test Method 11	→	Test Method 6

Test Method	Test Index	Assumptions	Sample Size	Sample Selection	Spec. Requirement
1-A	Mean	Log Normal Dist. Prior Knowledge of Variance	See Test Method	Natural Occurring Failures or Appendix A	H_0, H_1, α, β(1)
1-B		No Distribution Assumption, Prior Knowledge of Variance	"	"	"
2	Critical Percentile	Log Normal Dist. Prior Knowledge of Variance	"	"	"
3	Critical Maint. Time or Manhours	None	"	"	"
4	Median	A Specific Var. Log Normal	20	"	ERT
5	Chargeable Maint. Down-time/Flight (2)	None	See Test Method	Natural Occurring Failures	ORR or A $\frac{NCMDT}{NOF}$, $\frac{DDT}{NOF}$, α, β
6	Manhour Rate (3)	None	"	"	Manhour Rate Δ MR
7	Manhour Rate (4)	None	"	Natural Occurring Failures or Appendix A	μ_R, α

(1) See B.10.7 for definitions of α, β, H_0, H_1
(2) Test Method 5 is an indirect method for demonstrating operational ready rate (ORR) or Availability (A).
(3) Test Method 6 is intended for use with aeronautical systems and subsystems.
(4) Test Method 7 is intended for use with ground electronic systems where it may be necessary to simulate faults.

Fig. B-1 TEST METHOD MATRIX

Test Method	Test Index	Assumptions	Sample Size	Sample Selection	Spec Requirement
8	Mean and Percentile ------------------ Dual Percentile	Lognormal --------- None	See Test Method	Natural Occurring or Simple Random Sampling	Mean, M_{max} ---------- Dual Percentile
9	Mean (Corrective Task Time, Prev. Maint. Time, Down-time) ------------------ M_{max} (90 or 95 percentile)	None	30 minimum	Natural Occurring or Appendix A	μ_c, μ_{pm}, $\mu_{p/c}$ M_{max_c}
10	Median (Correct Task Time, Prev. Maint. Task Time) M_{max} (95 percentile) Corrective Maint. Task Time, Preventive Maint. Task Time	None	50 minimum	Natural Occurring or Appendix A	$\tilde{M}_{ct}$, $\tilde{M}_{dt}$, M_{max_c}, $M_{max_{pm}}$
11	Mean (preventive maint. task time) M_{max} (preventive maintenance task time, at any percentile)	None	All possible tasks	All	μ_{pm} $M_{max_{pm}}$

Fig. B-1- TEST METHOD MATRIX (CONTINUED)

The justification for use of the log-normal assumption for corrective maintenance times is based on extensive analysis of field data which have shown that the log-normal distribution provides a good fit to the data. However, in those cases where it is suspected that the log-normal assumption does not hold (e.g., equipments with a high degree of built-in diagnostics) then a distribution-free method should be employed to insure preservation of specified risks.

B.10.6 References - Details and additional references for the test plans (1, 2, 3) presented in this appendix can be found in RADC Technical Report 69-356 (AD 869 396), Volume II, entitled: "Maintainability Prediction and Demonstration Techniques." Copies of this document may be obtained from the Defense Documentation Center, Cameron Station, Alexandria, VA 22314.

B.10.7 List of Symbols - The following symbols and notations are common the test methods 1 - 3 contained in this appendix:

X = the random variable which denotes the maintence characteristics of interest (e.g., X can denote corrective maintenance time, preventive maintenance time, fault location time, manhours per maintenance task, etc.).

X_i = the ith observation or value of the random variable X.

n = the sample size

$\bar{X}$ = the sample mean (i.e., $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} (Xi)$

$\sigma^2 = E\left[(\ln X-\theta)^2\right]$ = the true variance of 1nX

μ = E(X) = the true mean of X.

d^2 = Var(X) $E\left[(X-\mu)^2\right]$ = the true variance of X.

$\hat{d}^2$ = the sample variance of X (i.e., $\hat{d}^2 = \frac{1}{n-1}\sum_{i=1}^{n} (Xi-\bar{X}^2)) =$
$\frac{1}{n-1}\left(\sum_{i=1}^{n} Xi^2 - n\bar{X}^2\right)$

$\tilde{d}^2$ = the prior estimate of the varianceof the maintenance time

X_p = the (1-p)th percentile of X (i.e.,X.05 = 95th percentilw of X)

$\tilde{M}$ = X_{50} = the median of X.

Y = 1n X = the naturallogarithm of X.

$\bar{Y}$ = the sample mean of Y

θ = E(1n X) = the true mean of 1n X.

$\tilde{\sigma}^2$ = the prior estimate of the variance of the logarithm of maintenance times

s^2 = the sample variance of in X.

Z_p = the standardized normal deviate exceeded with probability p (i. e.,

$$\int_{Z_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

Z_α $Z_{(1-\beta)}$ = standardized normal deviate exceeded with probabilities α and (1- β) respectively.

α = the producer's risk, the probability that the equipment will be rejected when it has a true value equal to the desired value (H_0).

β = the consumer's risk; the probability that the equipment will be accepted when it has a true value equal to the maximum tolerable value (H_1).

H_0 = the desired value specified in the contract or specification and is expressed as a mean, critical percentile, critical maintenance time.

H_1 = the maximum tolerable value. Note: $H_0 < H_1$.

When X is a log-normally distributed random variable:

$$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left(\ln x - \theta\right)^2}, \quad 0 < x < \infty$$

If Y = in X, the probability density of Y is normal with mean θ and σ^2 variance

$$Y \sim N(\theta, \sigma^2)$$

<u>Properties of the log-normal distribution:</u>

mean = $\mu = e^{\left(\theta + \frac{\sigma^2}{2}\right)}$

variance = $\sigma^2 = e^{\left(2\theta + \sigma^2\right)}\left(e^{\sigma^2} - 1\right)$

median = $\tilde{M} = e^{\theta}$

mode = $M = e^{\left(\theta - \sigma^2\right)}$

(1-p)th percentile = $X_p = e^{\left(\theta + Z_p \sigma\right)}$

Table of standardized normal deviates:

P	Z_p
.01	2.33
.05	1.65
.10	1.28
.15	1.04
.20	.84
.30	.52

B.10.8 List of Symblos. The following symbols are common to the test methods 4, 8 - 11 contained in this Appendix:

X_{ci} = Maintentenance downtime per corrective maintenance task (of the i^{th} task).

X_{pmi} = Maintenance downtimeper preventive maintenance task (of the i^{th} task).

n_c = Number of corrective maintenance tasks sampled.

n_{pm} = Number of preventive maintenance tasks occurring

β = Consumer's risk.

ϕ = That value, corresponding to risk, which is obtained from a table of normal distribution for a one-tail test.

f_c = Number of expected corrective maintenance tasks occurring during a representative operating time (T).

f_{pm} = Number of expected preventive maintenance tasks occurring during a representative operating time (T).

T = Item representative operating time period.

D_t = Total maintenance downtime in the representative operating time (T).

$\bar{X}_c, \bar{X}_{pm}, \bar{X}_{p/c}$ = Mean downtimes of sample. (Corrective, Preventive, and combined Corrective/Preventive Maintemnce Times.)

M'_{max_c} = Sample calculated maximum corrective maintenance downtime.

μc = Specified mean corrective maintenance time.

μpm = Specified mean preventive maintenance time.

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$\mu_{p/c}$ = Specified mean maintenance time. (Taking both corrective and preventive maintenance time into account.)

M_{max} = A requirement levied in terms of a maximum value of a percentile of task times (i.e., 95% of all corrective task times must be less than 60 minutes) usually taken as the 90th or 95th percentile.

M_{max_c} = Specified M_{max} of corrective maintenance downtimes.

$M_{max_{pm}}$ = Specified M_{max} of preventive maintenance downtimes.

θ_c = $E(\ln X_c)$ = Expected value of the logarithms of corrective maintenance tasks.

$\text{Log } X_{ci}$, $\text{Log } X_c$ = Log to the base 10 of X_{ci}, X_c.

$\ln X_{ci}$, $\ln X_c$ = Natural logs of X_{ci}, X_c.

$\widetilde{M_{ct}}$ = Median value of corrective maintenance tasks.

$\widetilde{M_{pm}}$ = Median value of preventive maintenance tasks.

TEST METHOD 1

B.20 General - This test provides for the demonstration of maintainability when the requirement is stated in terms of both a required mean value (μ_1) and a design goal value (μ_0) (or when the requirement is stated in terms of a required mean value (μ_1) and a design goal value (μ_0) is chosen by the contractor). The test plan is subdivided into two basic procedures, identified herein as Test Plan A and Test Plan B. Test A makes use of the lognormal assumption for determining the sample size, whereas Test B does not. Both tests are fixed sample tests, (minimum sample size of 30), which employ the Central Limit Theorem and the asymptotic normality of the sample mean for their development.

B.20.1 Assumptions - Test A - Maintenance times can be adequately described by a lognormal distribution The variance, σ^2, of the logarithms of the maintenance times is known from prior information or reasonably precise estimates can be obtained. Test B - No specific assumption concerning the distribution of maintenance are necessary. The variance d^2 of the maintenance times is known from prior information or reasonably precise estimates can be obtained.

B.20.2 Hypotheses - H_0: Mean = μ_0 (1-1)

H_1 : Mean = μ_1, ($\mu_1 > \mu_0$) (1-2)

Illustration: H_0: $\mu_0 = 3_0$ min.

H_1: $\mu_1 = 45$ min.

B.20.3 Sample Size - For a test with producer's risk α and consumer's risk β, the sample size for Test A is given by:

$$n = \frac{(Z_\alpha \mu_0 + Z_\beta \mu_1)^2}{(\mu_1 - \mu_0)^2} \quad (e^{\hat{\sigma}^2} \qquad (1\text{-}3)$$

where $\hat{\sigma}^2$ is a prior estimate of the variance of the logarithms of maintenance times. The sample size for Test B is given by:

$$n = \left(\frac{Z_\alpha + Z_\beta}{\frac{\mu_1 - \mu_0}{\hat{d}}} \right)^2 \qquad (1\text{-}4)$$

where $\hat{d}^2$ is a prior estimate of the variance of the logarithms of maintenance Z_α and Z_β are standardized normal deviates.

B.20.4 Decision Procedure. Obbtain a random sample of n maintenance times, $X_1, X_2, \ldots, X_n$, and compute the sample mean,

(1-5)

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and the sample variance

$$\hat{d}^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right) \tag{1-6}$$

Test A: Accept if $\bar{X} \leq \mu_0 + Z_\alpha \frac{\hat{d}}{\sqrt{n}}$ (1-7)

Test B: Accept if $\bar{X} \leq \mu_0 + Z_\alpha \frac{\hat{d}}{\sqrt{n}}$ (1-8)

Reject otherwise.

B.20.5 Discussion - By the central limit theorem, the sample mean $\bar{X}$ is approximately normal for large n with mean E(X) and variance Var $(\bar{X})$. In Test A under the log-normal assumption Var $\bar{X} = \frac{d^2}{n}$ where $d^2 = e^{(2\theta + \tilde{\sigma}^2)}(e^{\tilde{\sigma}^2} - 1) = \mu^2(e^{\tilde{\sigma}^2} - 1)$ thus the sample size n can be computed using a prior estimate of $\tilde{\sigma}^2$. In Test B, a prior estimate of d^2 is assumed to be available to calculate the sample size. A critical value C is chosen such that $\mu_0 + Z_\alpha\sqrt{\text{Var}\ \bar{X}} = C = \mu_1 - Z_\beta\sqrt{\text{Var}\ \bar{X}}$. If $\mu = \mu_0$. Then $P(\bar{X} > C) = \alpha$ and if $\mu = \mu_1$, then $P(\bar{X} \leq C) = \beta$.

B.20.6 Example - It is desired to test the hypothesis that the mean corrective maintenance time is equal to 30 minutes against the alternate hypothesis that the mean is 45 minutes $\alpha = \beta = .05$.

Then $H_0: \mu_0$ = 30 minutes.

$H_1: \mu_1$ = 45 minutes.

Test A: Under the log-normal assumption with Prior estimate of $\tilde{\sigma}^2 = .6$ the sample size using equation 1-3 is:

$$n_c = \frac{[1.65(30) + 1.65(45)]^2}{(45-30)^2}(e^{.6} - 1) = 56$$

Test B: Under the distribution-free case with a prior estimate of $\hat{d}^2$ = 900, (or d = 30), the sample size using equation 1-4 is:

$$n_c = \left[\frac{3.29}{\left(\frac{45-30}{30}\right)}\right]^2 = 43$$

B.20.7 O.C. Curve - The OC curve for Test B for this example is given in Figure B-3. It gives the probability acceptance for values of the mean maintenance time from 20 to 60 minutes. The OC curve for Test A for this example is given in Figure B-2. It gives the probability of acceptance for various values of the mean maintenance time. Thus, if the true value of μ is 40 minutes, then the probability that a demonstration will end in acceptance is 0.21 as seen from Fig. B-2.

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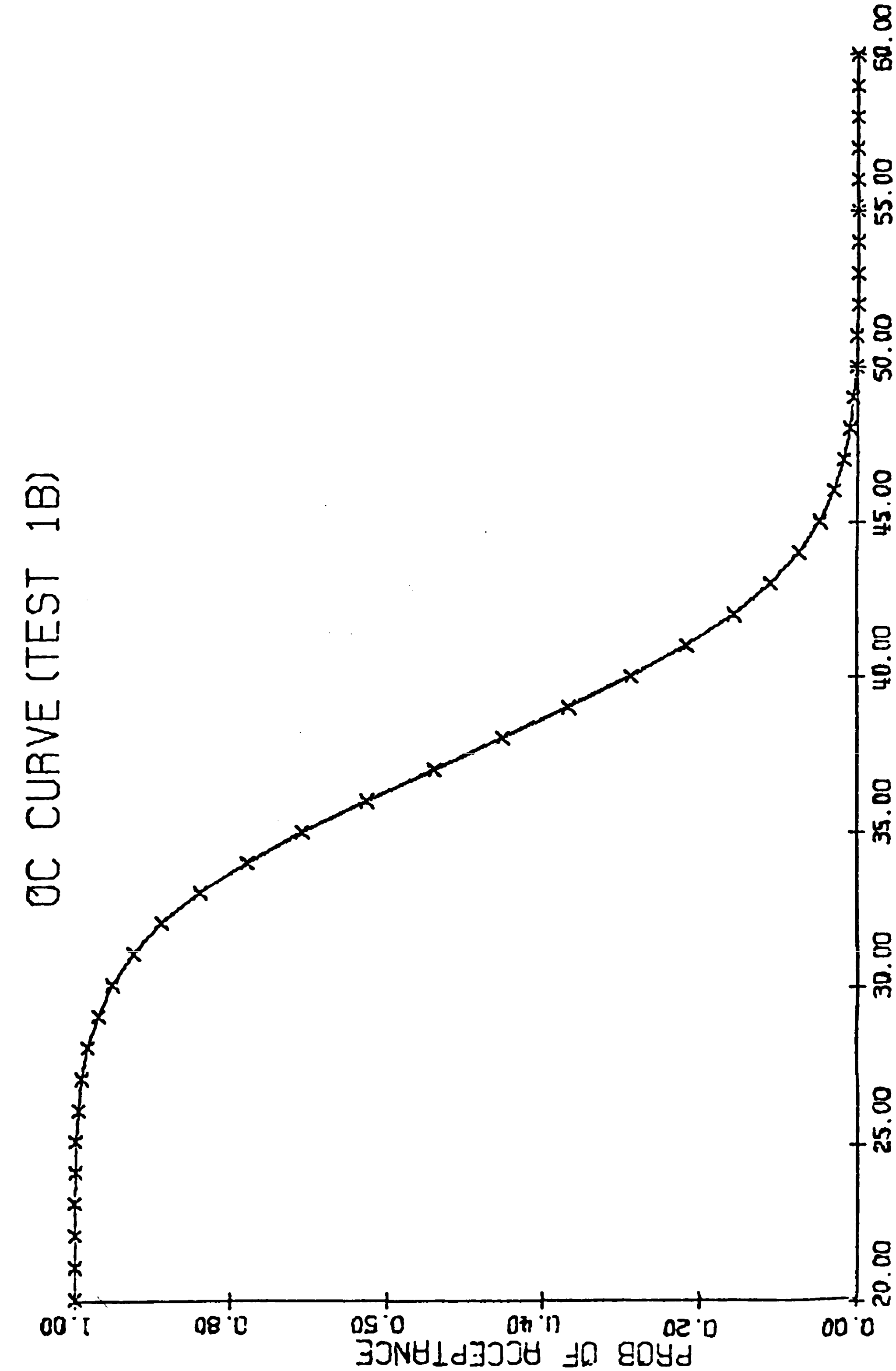
ØC CURVE (TEST 1B)
PROB OF ACCEPTANCE
1.00
0.80
0.60
0.40
0.20
0.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
55.00
60.00
MEAN

FIGURE B-3

TEST METHOD 2

TEST ON CRITICAL PERCENTILE

B.30 General - This test provides for the demonstration of mainability when the requirement is stated in terms of both a required critical percentile value (T_1) and a design goal value (T_0) [or when the requirement is stated in terns of a required percentile value (T_0) and a design goal value (T_0) is chosen by the contractor]. If the critical percentile is set at 50 percent, then this test method is a test of the median. The test is a fixed sample size test. The dexision criterion is baed upon the asymptotic normality of the maximum likelihood estimate of the percentile value.

B.30.1 Assumption - Maintenance times can be adequately described by a log-normal distribution. The variance σ^2 of the logarithms of the maintenance times is known from prior information or reasonably precise estimates can be obtained.

B.30.2 Hypotheses - H_0: (1-p)th percentile, $X_p = T_0$ (2-1)

or $P[X>T_0] = p$

H_1: (1-p)th percentile, $X_p = T_1$ (2.2)

or $P[X>T_1] = P, (T_1>T_0)$

Illustration: H_0: 95th percentile $= X_p = X_{.05} = 1.5$ hours =

T_0: $\ln T_0 = .4055$

H_1: 95th percentile $= X_p = X_{.05} = 2$ hours =

T_1: $\ln T_1 = .6932$

B.30.3 Sample Size - To meet specified a and b risks, the sample size to be used is given by the formula

$$n = \left(\frac{2 + Z_p^2}{2}\right) \tilde{\sigma}^2 \left(\frac{Z_\alpha + Z_\beta}{\ln T_1 - \ln T_0}\right)^2$$

(Round up to next integer) (2-3)

where

$\tilde{\sigma}^2$ is a prior estimate of σ^2, the variance of the logarithms of the maintenance times.

Z_p is the standatised normal deviate corresponding to the (1 - p) nomal deviate percentile.

B.30.4 Decision Procedure - Compute

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} \ln X_i \qquad (2\text{-}4)$$

$$s^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} (\ln X_i)^2 - n\bar{y}^2\right] \qquad (2\text{-}5)$$

$$X^* = \ln T_o + Z_\alpha S\left[\frac{1}{n} + \frac{Z_p^2}{2(n-1)}\right]^{1/2} \qquad (2\text{-}6)$$

Accept if $\bar{Y} + Z_p S \leq X^*$ (2-7)

Reject otherwise.

B.30.5 Discussion - This test is based upon the fact that under the log-normal assumption, the (1-p)th percentile value is given by $X_p = e^{(\theta + Z_p\sigma)}$. Taking logarithms gives $\ln X_p = \theta + Z_p\sigma$, and using maximum likelihood estimates for the normal parameters θ and σ, the (1-p)th percentile maximum likelihood estimate is $\ln\hat{X}_p = \bar{Y} + Z_p S\sqrt{\frac{n-1}{n}}$. $\ln X_p$ is approximately normal. To meet the producer's and consumer's risk requirements, a critical value X* is chosen for the sample estimate of the (1-p)th percentile X_p. Note $\bar{Y} = \hat{\theta}$ an estimate for θ .

B.30.6 Example - The following hypotheses are to be tested at $\alpha=\beta=.10$

H_0: 95th percentile = $X_{.05}$ = 1.5 hours = T_0; $\ln T_0$ = .4055

H_1; 95th percentile = $X_{.05}$ = 2.0 hours = T_1; $\ln T_1$ = .6932

A prior estimate of $\tilde{\sigma}^2$ is equal to 1.0 using equation 2-3.

$$n_c = \left(\frac{2 + (1.65)^2}{2}\right)(1)\frac{(2.56)^2}{(\ln 2.0 - \ln 1.5)^2}$$

or

n_c = 187

The critical value X^* " is given by equation 2-5

$$X^* = \ln T_o + Z_\alpha S\left[\frac{1}{n} + \frac{Z_p^2}{2(n-1)}\right]^{1/2}$$

$$= \ln 1.5 + 1.28S\left[\frac{1}{187} + \frac{(1.65)^2}{372}\right]^{1/2}$$

or

X^* =.4055 + 0.1437S

B.30.7 OC Curve - The OC curve for Test Method 2 for this example is given in Figure B-4. It gives the probability of acceptance for various values of the 95th percentile of the maintenance time distribution. If the true value of $X_{0.05}$ is 1.7 hours, then the probability that a demonstration will end in acceptance is 0.57 as seen from Figure B-4.

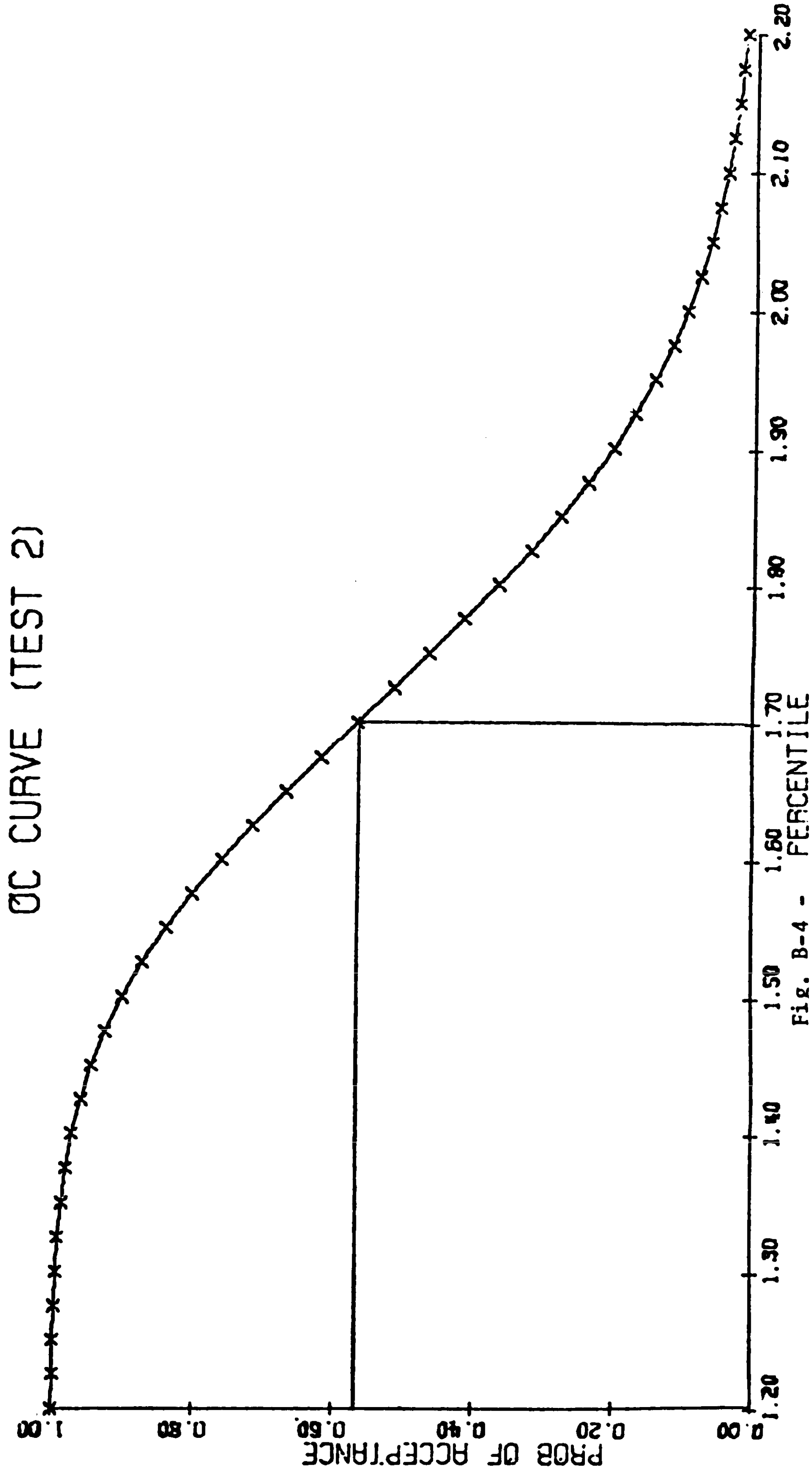

Fig. B-4 - PERCENTILE

TEST METHOD 3

TEST ON CRITICAL MAINTENANCE TIME OR MANHOURS

B.40 General - This test provides for the demonstration of maintainability when the requirement is specified in terms of both a required critical maintenance time (or critical manhours) (X_{p_1}) and a design goal value (X_{p_0}) (or when the requirement is stated in terms of a required critical maintenance time (X_{p_1}) and a design goal value (X_{p_0}) is chosen by the contractor). The test is distribution-free and is applicable when it is desired to establish controls on a critical upper value on the time or manhours to perform specific maintenance tasks. In this test both the null and alternate hypothesis refer to a fixed time and the percentile varies. It is different from Test Method 2 where the percentile value remains fixed and the time varies.

B.40.1 Assumptions - No specific assunmption is necessary concerning the distribution of maintenance time or manhours.

B.40.2 Hypothesis - H_0: $T = X_{p_0}$ $(p_1 > p_0)$ (3-1)

H_1: $T = X_{p_1}$ (3-2)

For specified α and β:

Illustration - H_0: 30 min. = $X_{0.05}$ = 50th percentile (median)

H_1: 30 min. = $X_{0.05}$ = 25th percentile

B.40.3 Sample Size, n, and Acceptance Number, c - The normal approximation to the binomial distribution is employed to find n and c when p_0 is not a small value. Otherwise, the Poisson approximation equations for n and c are as follows:

For $0.20 \leq p_0 \leq 0.80$ $(P_1 = 1 - Q_1)$

$$n = \left[\frac{Z_\beta\sqrt{P_1Q_1} + Z_\alpha\sqrt{P_0Q_0}}{P_1 - P_0}\right]^2$$ (Use next higher integer value.) (3-3)

$$c = n\left[\frac{Z_\beta P_0\sqrt{P_1Q_1} + Z_\alpha P_1\sqrt{P_0Q_0}}{Z_\alpha\sqrt{P_0Q_0} + Z_\beta\sqrt{P_1Q_1}}\right]$$ (Use next lower integer value.) (3-4)

For $p_0 < 0.20$

For this case n and c can be found from the following two equations:

$$\sum_{r=0}^{c} \frac{e^{-np_0}(np_0)^r}{r!} \geq 1-\alpha \qquad (3\text{-}5)$$

$$\sum_{r=0}^{c} \frac{e^{-np_1}(np_1)^r}{r!} \leq \beta \qquad (3\text{-}6)$$

Table B-I provides sampling plans for various a and b risks and ratios p_1/p_0 when $p_0<0.20$.

B.40.4 Decision Procedure. Random samples of maintenance times are taken, yielding n observations $X_1, X_2, \ldots X$. The number of such observations exceeding the specified time T is counted. This number is called r.

$$\text{Accept } H_0 \text{ if } r \leq c. \qquad (3\text{-}7)$$

$$\text{Reject } H_0 \text{ if } r > c. \qquad (3\text{-}8)$$

B.40.5 Discussion. In the development of the decision criteria and sample size, equations for this test, the normal or Poisson approximation to the binomial distribution is used.

B.40.6 Example. A median value of 30 minutes is considered acceptable whereas if 30 minutes is the 25th percentile then this is considered unacceptable. The following hypotheses result: (a=b= .10)

H_0: 30 minutes = $X_{0.05}$ = 50th percentile median

H_1: 30 minutes = $X_{0.75}$ = 25th percentile

Then $Z\alpha = Z\beta = 1.28$, $p_0 = .50$, $p_1 = .75$ using equations 3-3 & 3-4.

$$n = (1.28)^2\left[\frac{\sqrt{(.75)(.25)} + \sqrt{(.50)(.50)}}{(.25)}\right]^2 \simeq 23$$

and,

$$c = 23\left[\frac{(1.28).5\sqrt{(.75)(.25)} + 1.28\,(.75)\sqrt{(.50)\,(.50)}}{1.28\sqrt{(.50)(.50)} + 1.28\sqrt{(.75)(.25)}}\right] \simeq 14$$

B.40.7 OC Curve - The OC curve for Test Method 3 for this example is given in Figure B-5. It gives the probability of acceptance for values of probability p, varying from 0.3 to 1.0. Here X_p is (1-p) th percentile. Thus, if the true value of the given critical maintenance time is 40th percentile, i.e., if the value of p is 0.6, then the probability that a demonstration will end in acceptance is 0.61 as seen from Fig. B-5.

TEST METHOD 4

TEST ON THE MEDIAN (ERT)

B.50 General - This method provides for demonstration of maintainability the requirement is stated in terms of an equipment repair time (ERT) median, which will be specified in the detailed equipment specification.

B.50.1 Assumption - This method assumes the underlying distributlon of corrective maintenance task times is lognormal.

B.50.2 Sample Size - The sample size required is 20. This sample size must be used to employ the equation described in this test method.

B.50.3 Task Selection and Performance - Sample tasks shall be selected in accordance with the procedure outlined in Appendix "A". The duration of each shall be recorded and used tocompute the following statistics:

$$\text{Log } MTTR_G = \frac{\sum_{i=1}^{n_c} (\text{Log } X_{ci})}{n_c} \qquad (4\text{-}1)$$

$$S = \sqrt{\frac{\sum_{i=1}^{n_c} (\log X_{ci})^2}{n_c} - (\log MTTR_G)^2} \qquad (4\text{-}2)$$

All logarithms will be to the base 10

Where: $MTTR_G$ is the measured geometric mean time to repair. It is the eauivalent to the $\bar{M}_{ct}$ ' used in other plans included in this document.

B.50.4 Decision procedure - The equipment under test will be considered to have met the maintainability requirement (ERT) when the measured geometric mean-time-to-repair (MTTRG) and standard deviation(S) as determined in 50.3 satisfies the following expression:

Accept if $\log MTTR_G \leq \log ERT + .397(S)$ (4-3)

where: log ERT = logarithm of the equipment repair time

$\log MTTR_G$ = the value determined in accordance with para. 50.3

S = the value determined in accordance with para. 50.3

B.50.5 Discussion - The value of equipment repair time (ERT) to be specified in detailed equipment specification should be determined using the following espression:

$$\text{ERT (Specified)} = 0.37 \; ERT_{max} \qquad (4\text{-}4)$$

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where:

ERT_{max} = the maximum value of ERT that should be accepted no more than 10 percent of the time.

0.37 = a value resulting from application of "student's t" operating characteristics that assures a 95 percent probability that an equipment having an acceptable ERT will not be rejected as a result of the maintainability test when the sample size is 20, and assuming a population standard deviation (σ) of 0.55.

B.50.5.1 Derivation of Criteria - The following are brief explanations of the derivations of various criteria specified herein, and are intended for information purposes only. The acceptance criterion, $\log MTTR_G \leq \log ERT + 0.397(S)$; assures a probability of .95 of accepting an equipment or system as a result of one test when the true geometric mean-time-to-repair is equal to the specified equipment repair time (that is, a probability of 0.05 of rejecting an equipment or systems having a true MTTRG equal to the specified ERT). This was derived by using conventional methods for establishing acceptance criteria. The conventional methods for determining acceptance based on the measured mean of a small sample (that is, sample size less than 30), and when the true standard deviation (σ) of the population can only be estimated, is to compare the measured mean with the desired mean using the expression:

$$t = \frac{(\bar{x} - \bar{x}_o)}{S} \sqrt{n_c - 1}$$

where:

$$S = \sqrt{\frac{\Sigma_i (x_i - \bar{x})^2}{n_c}}$$ or the standard deviation of the sample;

$\bar{x}$ = the sample or measured mean

$\bar{x}_o$ = the specified or desired mean

n_c = the sample size

x_i = the value of one measurement of the sample

The decision to accept the product will be made when the test results give a value of t, as calculated from the above expression numerically less than or equal to a value of t obtained from "student's t" distribution tables at the established level (that is, 0.99, 0.95, 0.90, and so forth) of acceptance and the appropriate sample size. The "student's t" distribution tables (for a single tailed area) give a value of t = 1.729 at the 0.95 acceptance level when the sample size is 20 (that is, 19 degrees of freedom). The table for single tailed area is used since only values of $MTTR_G$ greater than the specified ERT are critical. An equipment with any value of $MTTR_G$ lower than the specified ERT is acceptable. To apply the expression for "t" to the maintainability test, let X_0 = log ERT (specified), $\bar{x}$ log $MTTR_G$ (measured), S = the measured standard deviation of the logarithms of the sample of measured repair time, and n_c = the sample size of 20. The measured $MTTR_G$ is then compared with the desired ERT by calculating the value of t using the expression below:

$$t = \frac{(\log MTTR_G - \log ERT)}{S} \sqrt{19}$$

The equipment under test can be acceptable if the value of t calculated from the expression above is equal to or less than + 1.729 (the value of t from the "student's t" distribution tables at an acceptable level of .95 when the sample size is 20). Therefore, the equipment should be accepted when:

$$\sqrt{19}\,\frac{(\log MTTR_G - \log ERT)}{S} \leq +1.729.$$

Upon rearranging and simplifying this expression, the acceptance criterion is obtained as shown below:

$$\log MTTR_G - \log ERT \leq \frac{1.729(S)}{\sqrt{19}}$$

$$\log MTTR_G \leq \log ERT + .397(S)$$

(NOTE: Reference - "Introduction to Mathematical Statistics," P. Heel, J. Wiley and Sons, Inc., 2nd Edition, 1954, Pp. 222-229)

TEST METHOD 5

TEST ON CHARGEABLE MAINTENANCE DOWNTIME PER FLIGHT

B.60 General - Because of the relatively small size of the demonstration fleet of aircraft and administrative and operational differences between it and fully operational units, operational ready rate or availability cannot be demonstrated directly. However, a contractual requirement for chargeable downtime per flight can be derived analytically from an operational requirement of operational ready rate or availability. This chargeable downtime per flight can be thought of as the allowable time (hours) for performing maintenance given that the aircraft has levied on it a certain availability or operational readiness requirement. The requirement for chargeable downtime per flight will be established using the procedure in B.60.3. Chargeable downtime per flight can then be demonstrated using the procedures in B.60.5.

B.60.1 Definitions - The following definitions apply to this test method:

A = Availability - A measure of the degree [expressed as a probability) to which an aircraft is in the operable and committable state at the start of the mission, when the mission is called for at an unknown (random) point in time. In this standard, availability is considered synonymous with operational readiness. The aircraft is not considered to be in an operable and committable state when it is being serviced and is undergoing maintenance (see MIL-STD-721B).

TOT = Total Active Time in Hours.

Active Time = That time during which an aircraft is assigned to an organization for the purpose of performing the organizational mission. It is time during which:

1. The aircraft is flying or ready to fly.
2. Maintenance is being performed.
3. Maintenance is delayed for supply or administrative reasons.

DUR = Daily Utilization Rate - The number of flying hours per day.

AFL = Average Flight Length - Flying hours per flight.

NOF = Number of Flights per Day.

TEST METHOD 8

TEST ON A COMBINED MEAN/PERCENTILE REQUIRMENT

B .90 General - This test provides for the demonstration of maintainability when the specification is couched in terms of a dual requirement for the mean and either the 90th or 95th percentile of maintenance times when the distribution of maintenance time is lognormal.

B.90.1 assumptions - For use as a dual mean and 90th or 95th percentile requirement the mean must be greater then 10 and less than 100 units of time; the ratio of the 90th percentile maximum value to the value of the mean must be less than two; the ratio of the 95th percentile maximum value to the value of the mean must be less than three.

Maxmum Ratio of Percentile to Mean

90th Percentile Value 2
95th Percentile Value 3
Distribution assumptions are as defined in B.90.

B.90.2 Discussion - The test method actually demonstrates the 61st percentile value of maintenance time in combination with either the 90th or 95th percentile values of maintenance time rather than the mean value of maintenance time in combination with either the 90th or 95th percentile values of maintenance time. However, because of the particular characteristic of the lognormal distribution once a 61st percentile value of maintenance time less than X_1 and a 90th or 95th percentile value less than X_2 has been demonstrated, for all practical purposes a mean value of less than approximately X_1 and a 90th or 95th percentile value less than X_2 have likewise been demonstrated.

A dual requirement on maintainability, assuming a lognomal distribution of repair times, of a maximum value of the in conjunction with either the maximum value of the 90th or 95th percentile of repair time (to be referred to as M_{max}) results in the definition of various combinations of θ's and σ's which are acceptable to the dual requirement. (A complete technical description of a lognormal distribution is provided by knowledge of θ & σ, hence, all possible lognormal distributions acceptable to the requirements are defined through definition of all possible acceptable values of θand σ.)) See Figure B-8A which defines the acceptable combinations of θ& σ f or a Mean of 30 minutes and a 95th percentile (M_{max}) of 60 minutes.

For the lognormal distribution it is also possible to structure a dual requirement made up of the maximum values of two percentiles (for example, the 61st percentile of repair time shall be a maximum of 30 minutes and the 95th percentile of repair time shall be a maximum of 60 minutes). This dual requirement also results in the definition of various combinations of acceptable values of θand σ. See Figure B-9B. If a dual percentile requirement could be structured such that the set of acceptable values of θand σ defined were almost identical to the set of values of θand σ(defined for a given dual Mean and percentile requirement then a demonstration of that dual percentile requirement would in reality also demonstrate the attainment of the dual Mean and M_{max} requirement. For this particular instance it has been found that under the assumptions listed above, almost identical acceptable values of θ and σ are provided for a combined Mean and M_{max} requirement and a combined 61st percentile (where the value of the 61st percentile is taken equal to the specified value of the Mean) and M_{max} requirement. See Figure B-8B which defines the values of θ and σ acceptable to a dual 61st percentile (where the value of the 61st percentile is taken equal to a specified mean of 30 minutes) and 95th percentile (where the maximum value of the 95th percentile, M_{max}, is given as 60 minutes) and Figure B-8C which is the superimposition of Figure B-8A on Figure B-8B.

Therefore, tests performed to demonstrate the attainment of both the percentiles in question actually demonstrates the attainment of values of θ & σ which are almost identically acceptable to a dual requirement of the Mean and M_{max}. It follows then that an accept decision relative to both percentiles would also approximately signify an accept decision for a dual Mean and M_{max} requirement.

Since both percentiles can be considered independent for practical purposes, the same samples can be used for demonstrating both percentiles, therefore, if desired the tests may be run simultaneously.

B.90.3 Procedure - Sample tasks shall be selected with respect to the procedure defined for variable sample/sequential tests. The same sample tasks may be used simultaneously in the demonstration of both the Mean and M_{max} requirements. Table 1*, Table 2*, Table 3* (which are based upon the sequential probability ratio test of proportion) define the accept/reject criteria for the values of the required mean, M_{max} (when defined as the maximum 90th percentile value), M_{max} (when defined as the maximum 95th percentile value), respectively. The number of observations greater than and less than the required values of the Mean and M_{max} shall be cumulated separately and compared to the decision values shown in the tables applicable to the two requirements. When one plan provides an accept decision, attention to that plan shall be discontinued. The second plan shall continue until a decision is reached. The equipment shall be rejected when a decision to reject on either plan has occurred regardless of the status of the other plan. The equipment shall be accepted only when an accept decision has been reached on both plans. If no accept or reject decision has been made after 100 observations, the following rule shall apply:

NOTE : *Tables 1, 2 & 3 are appropriate to Test Plans A_1, B_1 and B_2, respectively.

Plan A_1 - Accept only if 29 or less observations are more than the value of the required Mean.

Plan B_1 - Accept only if 5 or less observations are more than M_{max_C}.

Plan B_2 - Accept only if 2 or less observations are more than M_{max_C}.

It is recognized and accepted that truncation will somewhat modify probability of acceptance characteristics as described in the following section.

B.90.4 <u>The OC Curve</u> - The operating characteristic curve for the test procedure may be determined by mapping the probability of acceptance for various selected points on a diagram of the acceptable and unacceptable regions such as Figure B-8D. (Note that any point can be identified uniquely by the coefficient of Q, where Q = in (required Mean), on the ordinate and the coefficient of $\sqrt{Q}$ on the abcissa - let the coefficient of Q be denoted as (C) and the coefficient of $\sqrt{Q}$ be denoted as (K) - for example, point B on Figure B-8D can be uniquely located at C = 3/4, K = .4). Each point is also representative of a particular lognormal distribution possessing unique percentiles for the values given for μ_1 (required maximum value for Mean) and M_{max}, respectively.

The probability of acceptance relative to any point is equal to the compound probability of passing the percentile test relative to μ_1 (Test A1) and passing the percentile test relative to M_{max} (Test B1 or B2).

Let P_{A1}, P_{B1}, P_{B2} be the probability of passing test A_1, B_1, B_2, respectively for any given unique combination of θ and σ (a particular point).

P_{A1}, P_{B1}, P_{B2} may be determined by calculating Y_{A1}, Y_{B1}, Y_{B2} from the following equations:

$$Y_{A1} = \frac{\sqrt{Q}\,(1-C)}{K} \qquad (7\text{-}1)$$

$$Y_{B1} = Y_{B2} = \frac{\ln M_{max} - CQ}{K\sqrt{Q}} \qquad (7\text{-}2)$$

and entering Figure B-8E (for Test A1) with the calculated value of Y_{A1} and Figure B-8F (for Test B_1) or Figure B-8G (for Test B_2) with the calculated value of Y_{B1} or Y_{B2}. The corresponding value of probability of acceptance, P_{A1}, and P_{B1} or P_{B2} (whichever of the B tests are appropriate) is read <u>from each</u> Figure and P_{A1} and the appropriate P_{B1} or P_{B2} value are multiplied. e result of this multiplication is the probability of acceptance of a unit having a particular θ and σ characteristic defined by (C)and (K).

Repeating the above for a number of points as in Figure B-8H defines an operating characteristic map relative to a given dual requirement. Note that probabilities of acceptance always decrease as the point is located upward or to the right and always increase as the point in consideration is located downward or to the left on the figure. Hence, sufficient knowledge of test characteristics can be generated by evaluating relatively few points.

TABLE 1

Plan (A_1)
Observations Exceeding the Value of the Mean
(or 61st percentile value)

# of Tasks OBSR (N)	Accept	Reject	# of Tasks OBSR (N)	Accept	Reject
5		5	55	12	↕
6		6	56	13	20
7		↕	57	↕	21
8		6	58	13	↕
9		7	59	14	21
10		↑	60	↑	22
11		↓	61	↓	↕
12	0	7	62	14	22
13	↕	8	63	15	23
14	0	↕	64	↕	↑
15	1	8	65	15	↓
16	↑	9	66	16	23
17	↓	↑	67	↕	24
18	1	↓	68	16	↕
19	2	9	69	17	24
20	↕	10	70	↑	25
21	2	↕	71	↓	↑
22	3	10	72	17	↓
23	↕	11	73	18	25
24	3	↕	74	↕	26
25	4	11	75	18	↕
26	↑	12	76	19	26
27	↓	↑	77	↕	27
28	4	↓	78	19	↕
29	5	12	79	20	27
30	↕	13	80	↑	28
31	5	↕	81	↓	↑
32	6	13	82	20	↓
33	↕	14	83	21	28
34	6	↕	84	↕	29
35	7	14	85	21	↕
36	↑	15	86	22	29
37	↓	↑	87	↑	30
38	7	↓	88	↓	↕
39	8	15	89	22	30
40	↕	16	90	23	31
41	8	↕	91	↕	↑
42	9	16	92	23	↓
43	↑	17	93	24	31
44	↓	↑	94	↕	32
45	9	↓	95	24	↕
46	10	17	96	25	32
47	↕	18	97	↑	33
48	10	↕	98	↓	↑
49	11	18	99	25	↓
50	↕	19	100	26	33
51	11	↕			
52	12	19			
53	↑	20			
54	↓	↕			

TABLE 2

Plan (B_1)

Observations Exceeding M_{max} - 90 Percentile

# of Tasks OBSR (N)	Accept	Reject	# of Tasks OBSR (N)	Accept	Reject
2		2	52		4
3		↑	53		5
4			54		↑
5			55		
6			56		
7			57		
8			58		
9			59		
10			60		
11			61		
12		↓	62		
13		2	63	↓	
14		3	64	1	
15		↑	65	2	
16			66	↑	
17			67		
18			68		
19			69		
20			70		
21			71		↓
22			72		5
23			73		6
24			74		↑
25			75		
26	0		76		
27	↑		77		
28			78		
29			79		
30			80		
31			81		
32		↓	82		
33		3	83	↓	
34		4	84	2	
35		↑	85	3	
36			86	↑	
37			87		
38			88		
39			89		
40			90		
41			91		↓
42			92		6
43			93		7
44	↓		94		↑
45	0		95		
46	1		96		
47	↑		97		
48			98		
49			99	↓	↓
50			100	3	7
51		↓			

TABLE 3

Plan (B_2)

Observations Exceeding M_{max} - 95 Percentile

# of Tasks OBSR (N)	Accept	Reject	# of Tasks OBSR (N)	Accept	Reject
2		2	52		
3			53		
4			54		
5			55		
6			56		
7			57	0	
8			58		
9			59		
10			60		
11			61		
12			62		
13			63		
14			64		
15			65		
16			66		
17			67		
18			68		
19			69		3
20			70		4
21			71		
22			72		
23			73		
24			74		
25			75		
26			76		
27		2	77		
28		3	78		
29			79		
30			80		
31			81		
32			82		
33			83		
34			84		
35			85		
36			86		
37			87		
38			88		
39			89		
40			90		
41			91		
42			92		
43			93		
44			94		
45			95		
46			96		
47			97		
48			98	0	
49			99	1	
50			100	1	4
51					

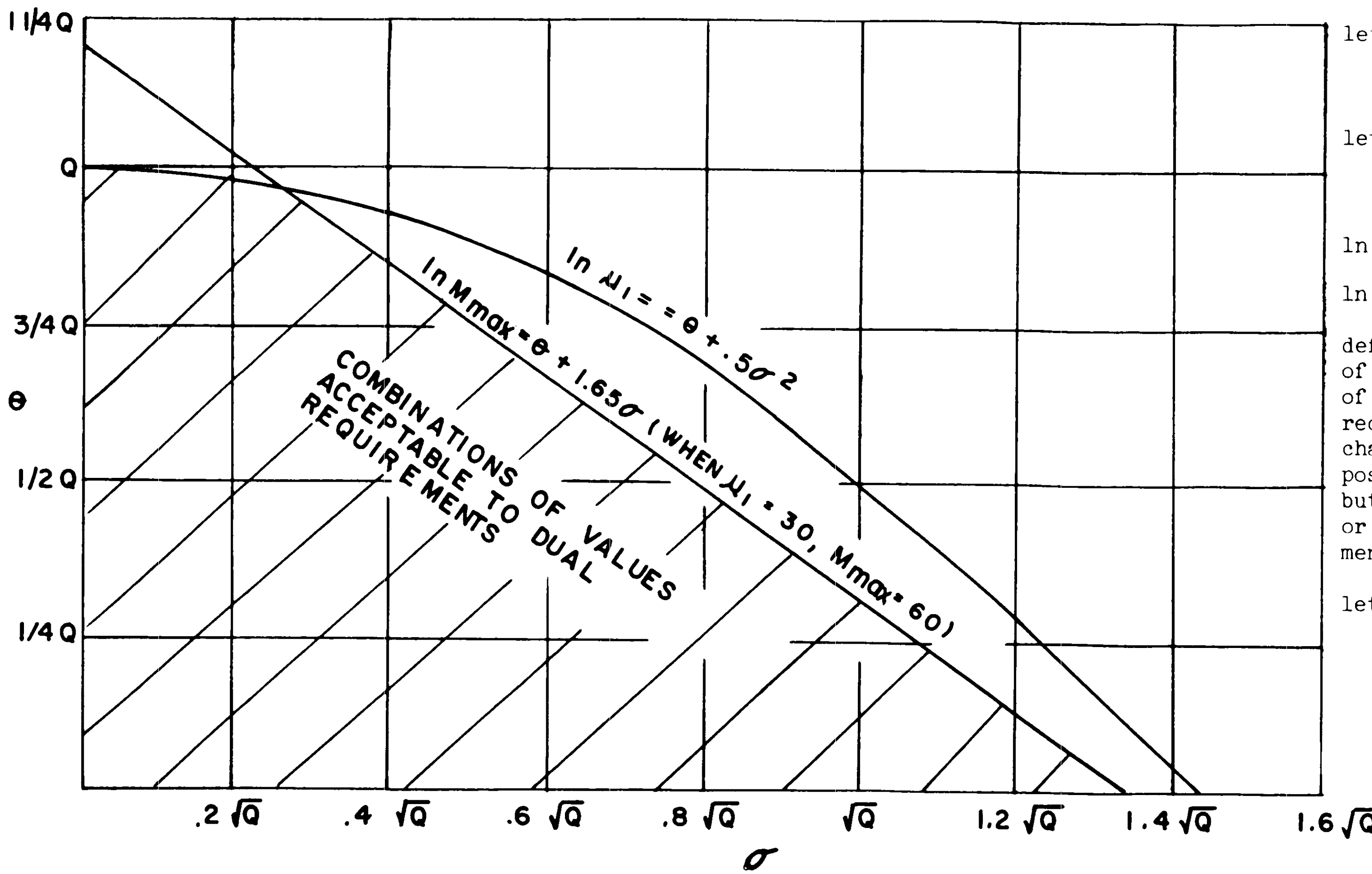


Figure B-8A

let μ_1 = required value of mean (maximum allowable)

let X_{p1} = maximum value of 95th percentile = M_{max}

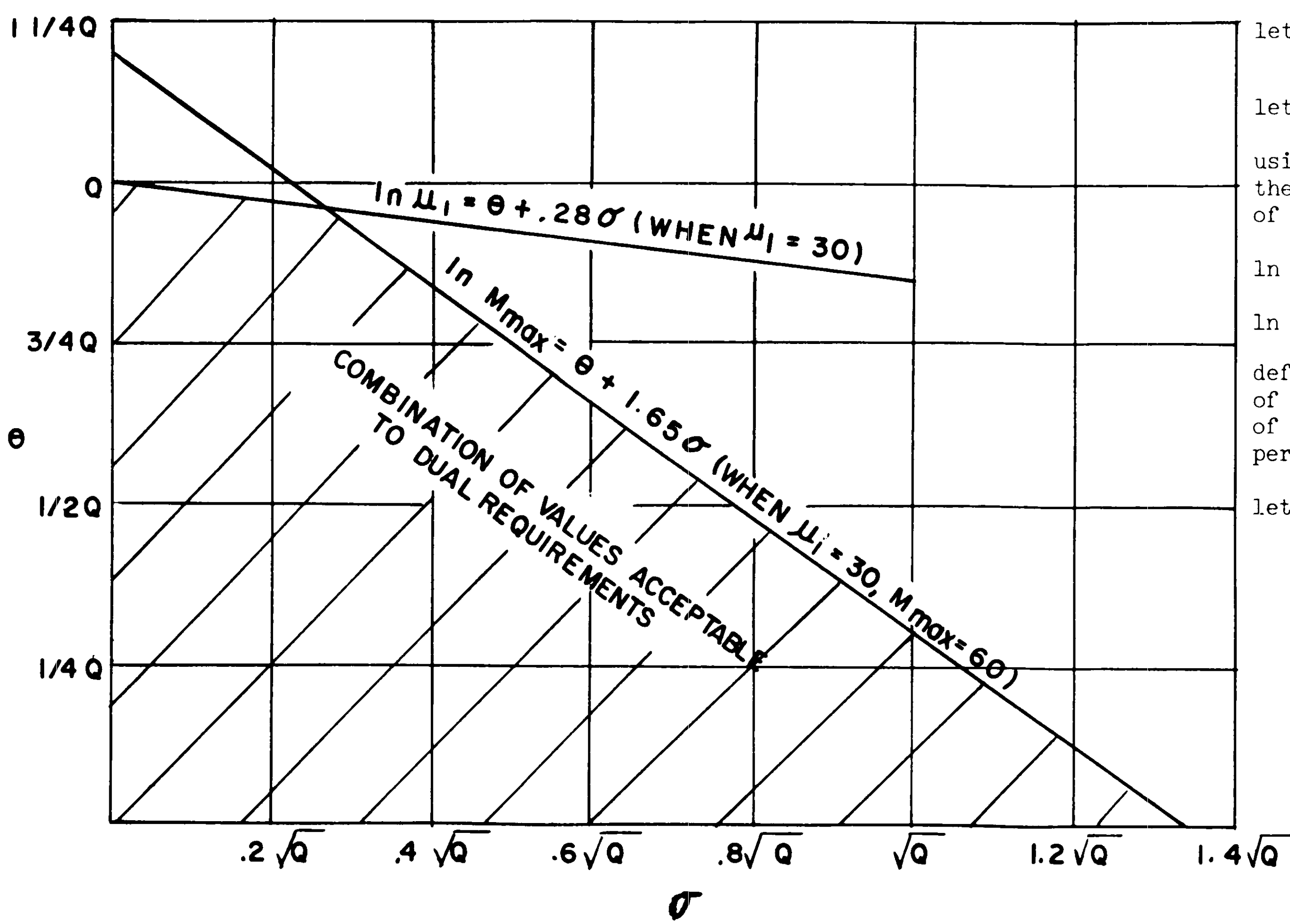
$\ln \mu_i \geq \theta + .5\sigma^2$

$\ln M_{max} \geq \theta + 1.65\sigma$

define all possible values of θ & σ which are capable of meeting or bettering the requirement (defines the characteristics of all possible lognormal distributions capable of meeting or bettering the requirements).

let $\ln \mu_1 = Q$

FIGURE B-8A



let μ_1 = required value of mean (max. allowable)

let X_{p1} = maximum value of 95th percentile = M_{max} using the required value for the mean as the maximum value of the 61st percentile.

$\ln \mu_1 \geq \theta + .28\,\sigma$

$\ln M_{max} \geq \theta + 1.65\,\sigma$

define all possible values of θ & σ which are capable of meeting the dual percentile requirement.

let $\ln \mu_1 = Q$

FIGURE B-8B

Figure B-8B

SUPERIMPOSITION OF FIGURES B-8A AND B-8B

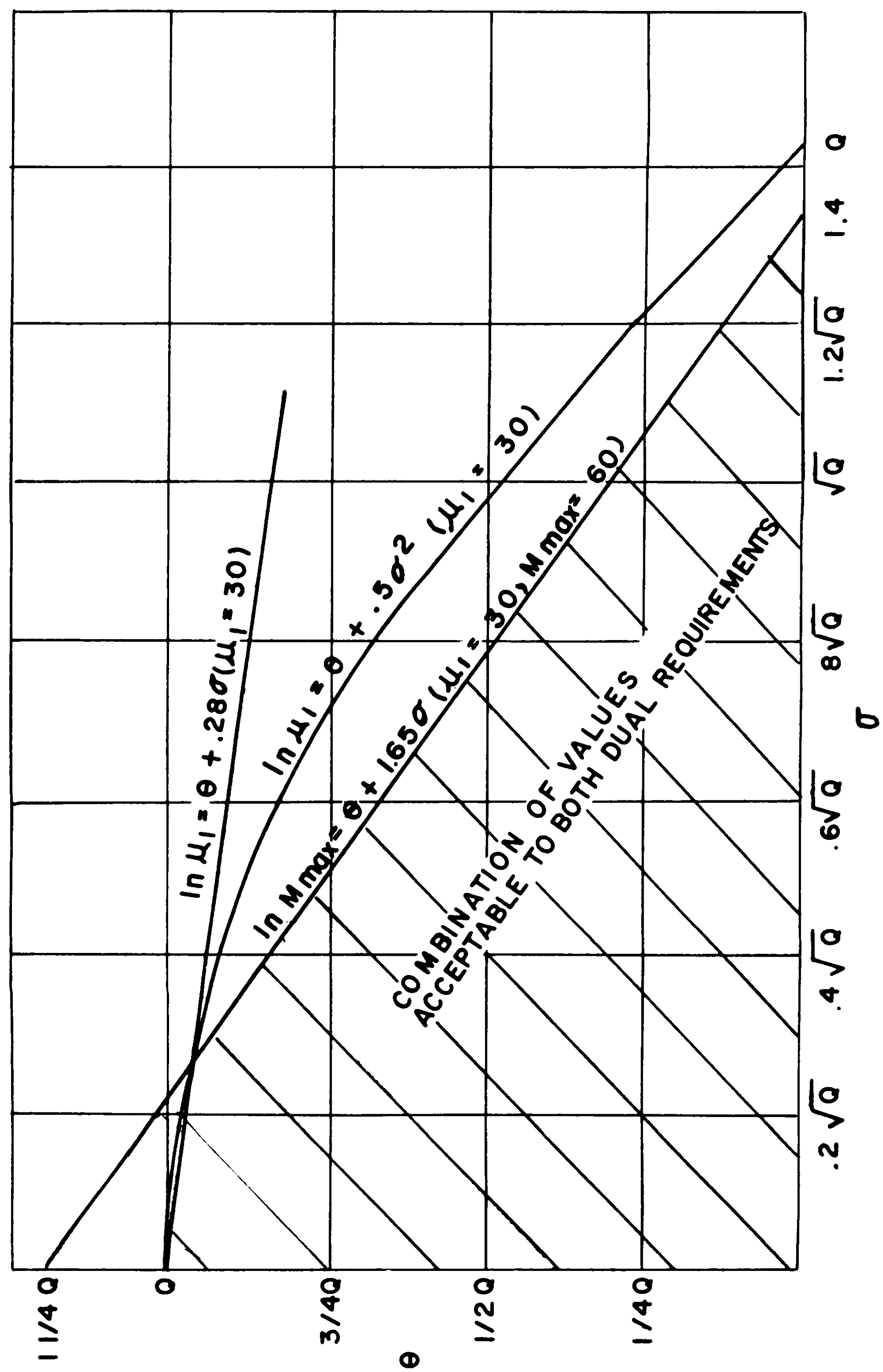


Figure B-8C

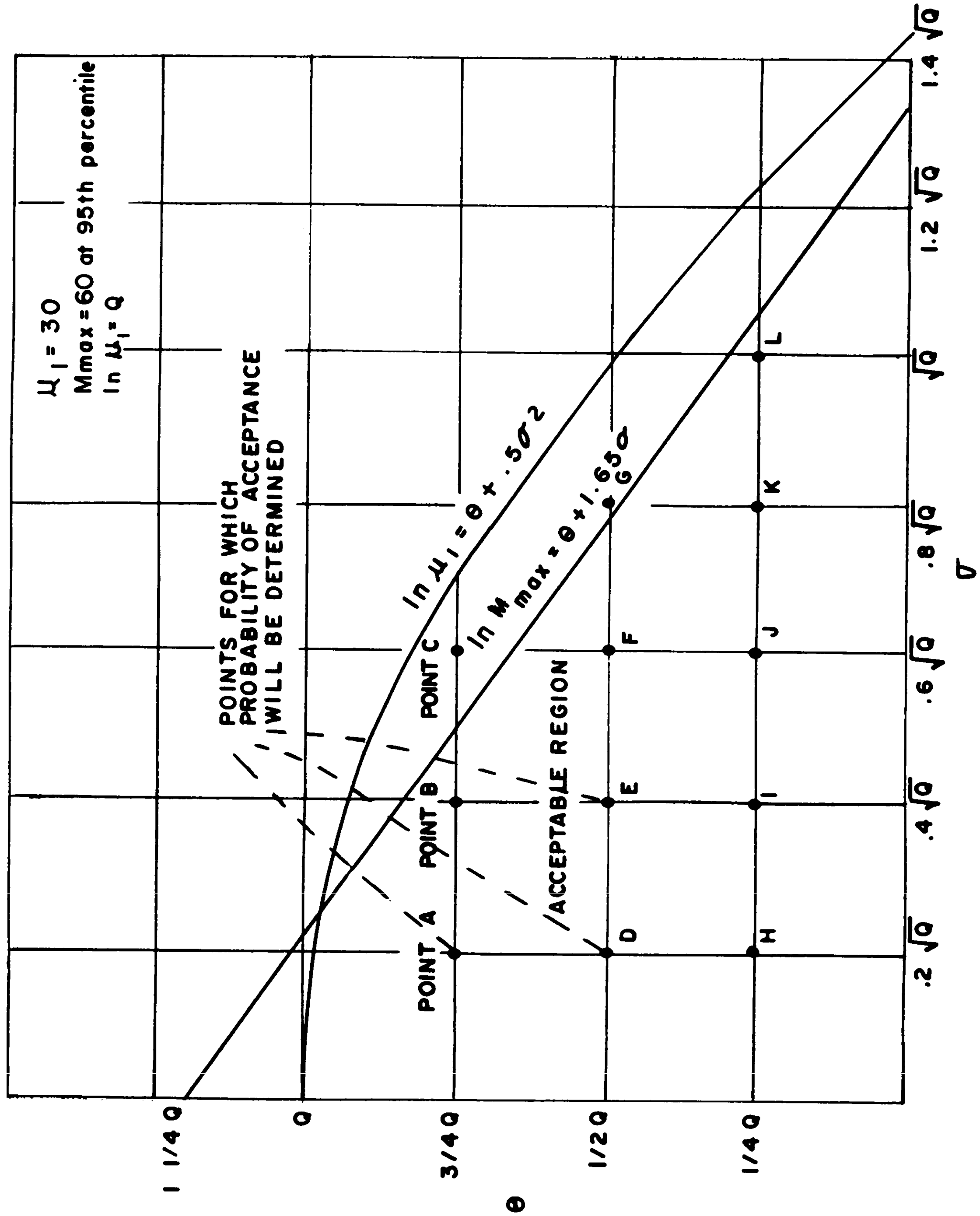

μ1 = 30
Mmax = 60 at 95th percentile
ln μ1 = Q
POINTS FOR WHICH PROBABILITY OF ACCEPTANCE WILL BE DETERMINED
ln μ1 = θ + .5σ2
ln M max = θ + 1.65σ
ACCEPTABLE REGION
POINT A
POINT B
POINT C
D
E
F
G
H
I
J
K
L
1 1/4 Q
Q
3/4 Q
1/2 Q
1/4 Q
θ
.2 √Q
.4 √Q
.6 √Q
.8 √Q
√Q
1.2 √Q
1.4 √Q
σ

Figure B-8D

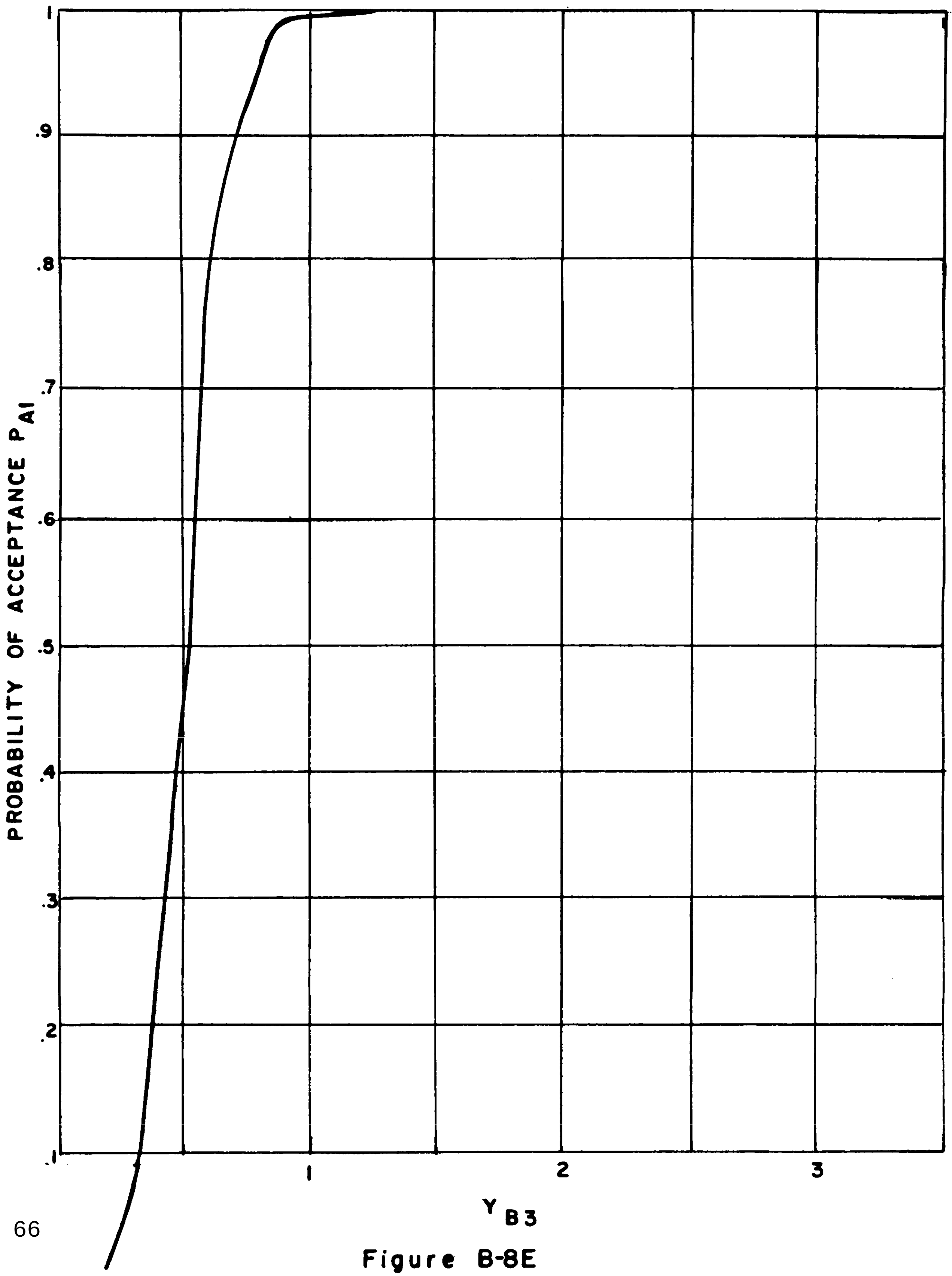


Figure B-8E

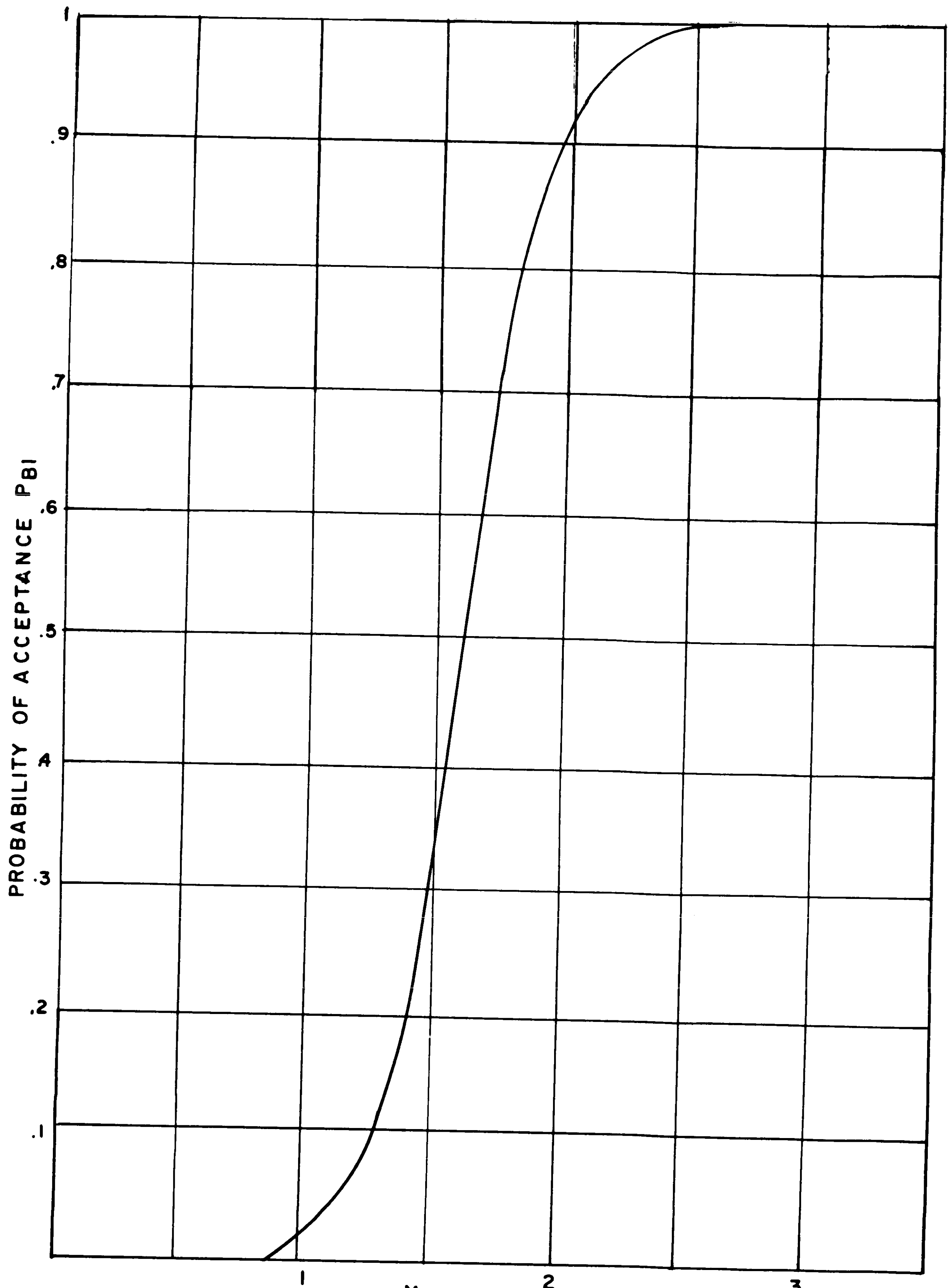


Figure B-8F

PROBAILITY OF ACCEPTANCE P_{B2}

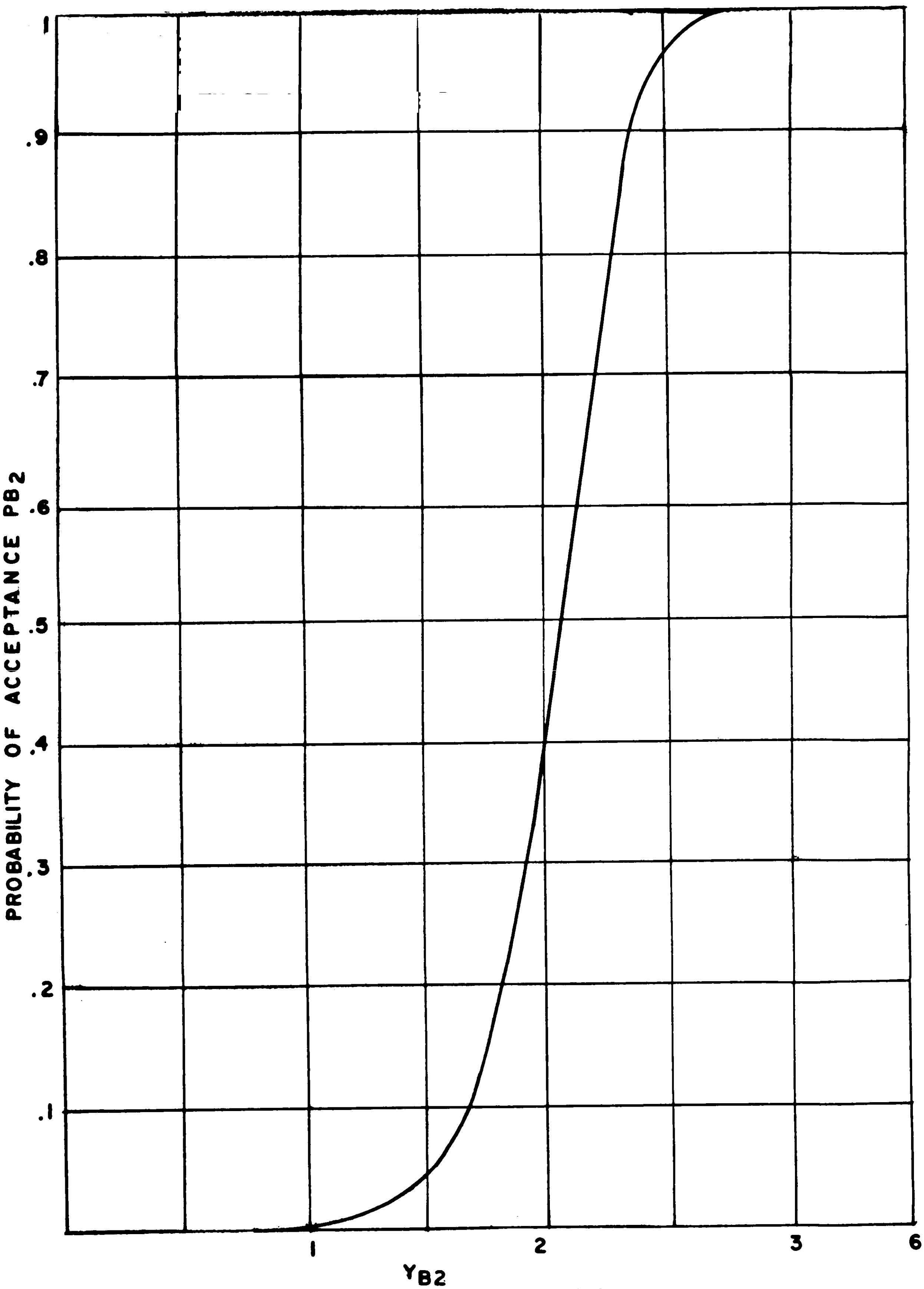


Figure B-8G

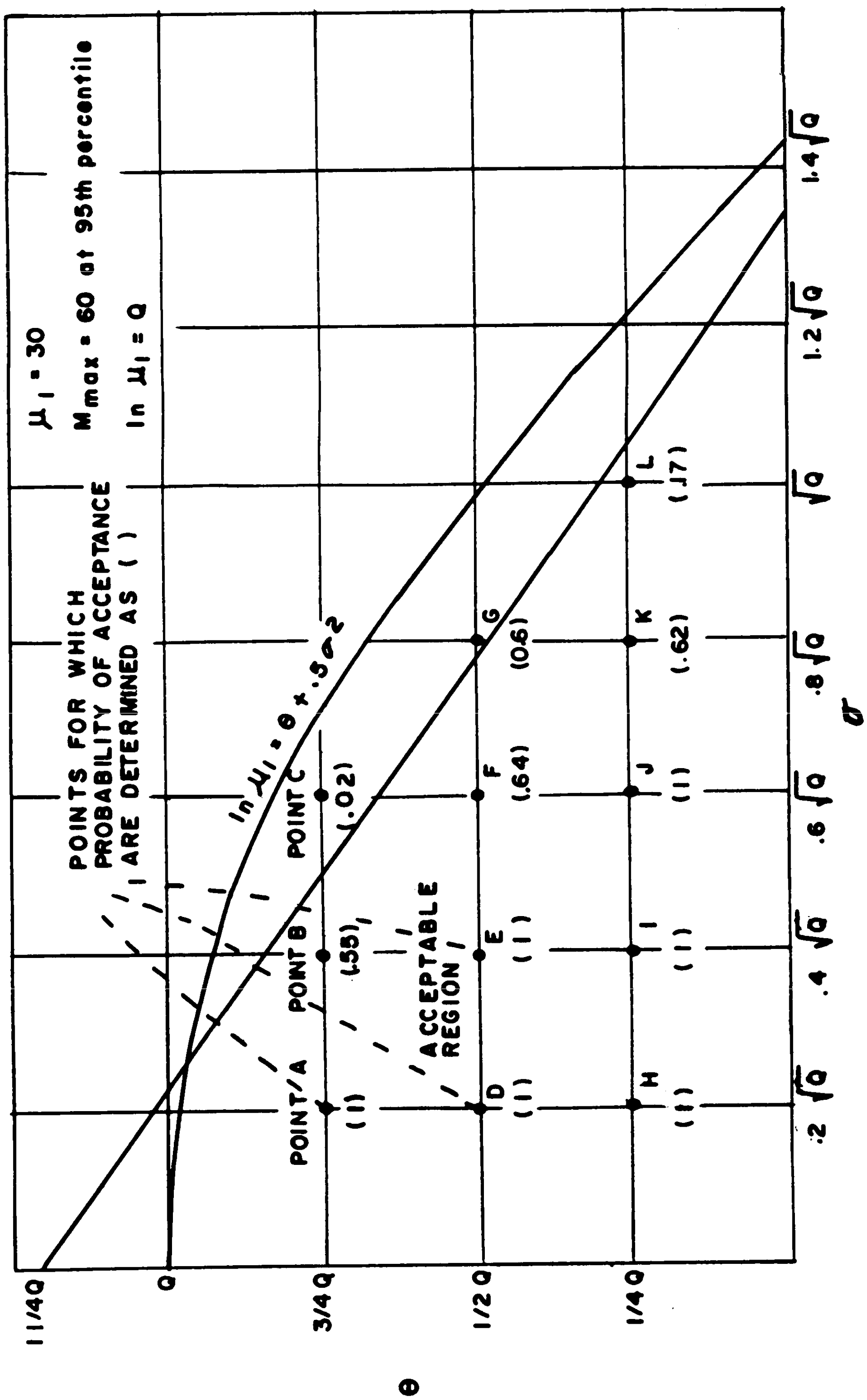

μ₁ = 30
M max = 60 at 95th percentile
ln μ₁ = Q
POINTS FOR WHICH PROBABILITY OF ACCEPTANCE ARE DETERMINED AS ( )
ln μ₁ = θ + .5σ²
ACCEPTABLE REGION
POINT A
POINT B
POINT C
D
E
F
G
H
I
J
K
L
(1)
(.55)
(.02)
(.64)
(.6)
(.62)
(.17)
1 1/4 Q
Q
3/4 Q
1/2 Q
1/4 Q
.2 √Q
.4 √Q
.6 √Q
.8 √Q
√Q
1.2 √Q
1.4 √Q
σ
θ

Figure B-8H

TEST FOR MEAN MAINTENANCE TIME (CORRECTIVE PREVENTATIVE COMBINATION OF CORRECTIVE AND PREVENTATIVE) AND M_{MAX}

B100. General - This method is applicable to demonstration of the following indices of maintainability: Mean Corrective Maintenance Time (μ_c), Mean Preventive Maintenance Time (μ_{pm}), Mean Maintenance Time (includes preventive and corrective maintenance actions) ($\mu_{p/c}$), and M_{max} (percentile of repair time).

B100.1 Conditions of Use - The procedures of this method for demonstration of μ_c, are based on the Central Limit Theorem. No information relative to the variance (d^2) of maintenance times is required. It may therefore be applied whatever the form of the underlying distribution, provided the sample size is adequate. The minimum sample size is set at 30. The actual sample size (if greater than 30 are required) shall be determined for each equipment to be demonstrated, and shall be approved by the procuring activity.

The procedure of this method for demonstrating M_{max} is valid for those cases where the underlying distribution of corrective maintenance task times is lognormal.

B100.2 Quantitative Requirements - Application of this plan requires identification of the index or indices of interest and specification of quantitative requirements for each. When demonstration involves μ_c or μ_{pm}, or a combination of both, consumer's risks will be specified. When demonstration involves M_{max_c}, the percentile point which defines the specified value of M_{max_c} will be specified. A minimum sample size of 30 corrective maintenance tasks is required for demonstration of corrective maintenance indices. A minimum sample of 30 preventive maintenance tasks is required where demonstration of preventive maintenance indices by sampling is permitted and is to be accomplished by this method.

B100.3 Task Selection and Performance - Sample tasks shall be selected in accordance with the procedure outlined in Appendix "A." The duration of each shall be recorded and used to compute the following statistics:

$$\bar{X}_c = \frac{\sum_{i=1}^{n_c} X_{ci}}{n_c}$$

$$\bar{X}_{pm} = \frac{\sum^{n_{pm}} X_{pmi}}{n_{pm}}$$

$$D_t = f_c \ \bar{X}_c + f_{pm} \ \bar{X}_{pm}$$

$$\bar{X}_{p/c} = \frac{f_c \ X_c + f_p \ X_{pm}}{f_c + f_{pm}}$$

$$M'_{max_c} = \underset{(\text{Base } e)}{\text{Antilog}} \left[\frac{\sum_{i=1}^{n_c} \ln X_{ci}}{n_c} + \psi \sqrt{\frac{\sum_{i=1}^{n_c} (\ln X_{ci})^2 - \frac{\left(\sum_{j=1}^{n_c} \ln X_{ci}\right)^2}{n_c}}{n_c - 1}} \right]$$

Where ψ is the value of the independent variable log-normal function which corresponds to the percentile point at which M_{max_c} has been establish. For the two most common percentile points, 90% and 95%, ψ is 1.282 and 1.645 respectively.

B.100.4 Accept/Reject Criteria - 4 table of the normal distribution function shall be consulted for values of ∅ (for a one-tail test) which corresponds to the specified livel of consumer risk β. The following table provides values of ∅ which correspond to the most commonly used values of β.

TABLE V
∅ vs. β

∅	β
0.84	20%
1.04	15%
1,28	10%
1.65	5%

Accept/reject criteria shall be computed for each specified index in accordance with the following sections:

B.100.4.1 Test for Mean Corrective Maintenance Time (μ_c) - The accept/reject value for μ_c is: $\bar{X}_c \not+ \frac{\phi \hat{d}_c}{\sqrt{n_c}}$

$\hat{d}_c$ = Std. deviation of sample of corrective maintenance tasks.

Accept if μ_c (specified)

Reject if μ_c (specified)

B.100.4.2 Test for Mean Preventive Maintenance Time (μ_{pm}) - The accept/ value for μ_{pm} is: $\bar{X}_{pm} \not+ \frac{\phi \hat{d}_{pm}}{\sqrt{n_{pm}}}$

$\hat{d}_{pm}$ = Std. deviation of preventive maintenance tasks.

Accept if μ_{pm} (specified) $\bar{X}_{pm} \not+ \frac{\phi \hat{d}_{pm}}{\sqrt{n_{pm}}}$

Reject if μ_{pm} (specified) $\bar{X}_{pm} \not+ \frac{\phi \hat{d}_{pm}}{\sqrt{n_{pm}}}$

B.100.4.3 Test for the Mean of all Maintenance Actions ($\mu_{p/c}$) - The accept/ reject value of $\mu_{p/c}$ is:

$$\bar{X}_{p/c} \not+ \phi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 \not+ n_c (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c \not+ f_{pm})^2}}$$

If μp/c (specified) $\bar{X}_{p/c} \not+ \phi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 \not+ (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c \not+ f_{pm})^2}}$ Accept

If $\mu_{p/c}$ (specified) $\bar{X}_{p/c} \not+ \phi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 \not+ n_c (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c \not+ f_{pm})^2}}$ Reject

B.100.4.4 Test for M_{max} - The accept/reject value for M_{max_c} is:

$$M'_{max_c} = \underset{(\text{Base } e)}{\text{Antilog}} \left[\frac{\sum_{i=1}^{n_c} (\ln X_{ci})}{n_c} \not+ \psi \sqrt{\frac{\sum_{i=1}^{n_c} (\ln X_{ci})^2 - \frac{\left(\sum_{i=1}^{n_c} \ln X_{ci}\right)^2}{n_c}}{n_c - 1}} \right]$$

Accept if M_{max_c} (specified) $\geq M'_{max_c}$

Reject if M_{max_c} (specified) $< M'_{max_c}$

Test Method 10

TESTS FOR PERCENTILES AND MAINTENANCE TIME (CORRECTIVE PREVENTATIVE MAINTENANCE)

B110. General - This method employs a test of proportion to demonstrate achievement of $\tilde{M}_{ct}$, $\tilde{M}_{pm}$, M_{max_c} and $M_{max_{pm}}$ when the distribution of corrective and preventive maintenance repair times is unknown.

B110.1 Conditions of Use - This method is intended for use in cases where no information is available on the underlying distribution of maintenance task times. The plan holds the confidence level at 75% or 90% as may be desired and requires a minimum sample size (N) of 50 tasks.

B110.2 Quantitative Requirements - Application of this method required specification of $\tilde{M}_{ct}$, $\tilde{M}_{pm}$, $M_{max_{ct}}$ (95th percentile) or $M_{max_{pt}}$ (95th percentile) and selection of 75% or 90% confidence level.

B110.3 Task Selection and Performance - Sample tasks shall be selected in accordance with the proceudres of Appendix "A." The duration of each task will be compared to the required value(s) of the specified index or indices ($\tilde{M}_{ct}$, $\tilde{M}_{pm}$, $M_{max_{ct}}$ and $M_{max_{pm}}$) and recorded as greater than or lesser than each index.

B110.4 Accept/Reject Criteria - The item under test shall be accepted when the number of observed task times which exceed the required value of each specified index is less than or equal to that shown in the Table (B-10A or B-10B) corresponding to each index for the specified confidence level.

B110.4.1 Test for the Median - Table B-10A below is a test of the median for corrective and preventive maintenance tasks. The acceptance level is shown for two confidence levels and a sample size (N) of 50 tasks.

Table B-10A

Acceptance Table for $\widetilde{M}_{ct}$ or $\widetilde{M}_{pm}$

Sample size =

Confidence Level	
75%	90%
Acceptance Level	
22	20

B110.4.2 Test for M_{max_c} and $M_{max_{pm}}$ - Table B-10B is a test for M_{max_c} and $M_{max_{pm}}$ at the 95th percentile. The acceptance level is shown for two confidence levels and a samle size (N) of 50 tasks.

Table B-10B

Acceptance Table for $M_{max_{ct}}$ or $M_{max_{pm}}$

Sample size = 50

Confidence Level	
75%	90%
Acceptance Level	
1	0

NOTE: Reference - "Introduction to Statistical Analysis" by Dixon & Massey, Page 230, McGraw-Hill Company, 2nd Edition, 1957.

Test Method 11

TEST FOR PREVENTATIVE MAINTENANCE TIMES

B.120 General - This method provides for maintainability demonstration when the specified index involves µpm and/or $M_{max_{pm}}$ and when all possible preventive maintenance tasks are to be performed.

B.120.1 Conditions of use - All possible tasks are to be performed and no allowance need be made for underlying distribution.

B.120.2 Quantitative requirements - Application of this plan requires quantitative specification of the index or indices of interest. In addition, the percentile point defining $M_{max_{pm}}$ must be stipulated when $M_{max_{pm}}$ is of interest.

B.120.3 Task selection and performance - All preventive maintenance tasks will be perfomed. The total population of PM tasks will be defined by properly weighing each task in accordance with relative frequency of occurrence as follows: Select the particular task for which the equipment operating time to task performance is greatest and establish that time as the reference period. Determine the frequency of occurrence (f_{pm}) of all other tasks during the reference period. Where the frequency of occurrence of a given task is a fractional number, the frequency shall be set at the nearest integer. The total population of tasks consists of all tasks with each repeated in accordance with its frequency of occurrence during the reference period.

B.120.4 Accept/reject criteria -

B.120.4.1 Test for μ_{pm} the mean shall be computed as follows:

$$\mu_{pm}\,(\text{Actual}) = \frac{\sum_{i=1}^{k} f_{pmi}\,(X_{pmi})}{\sum_{i=1}^{k} f_{pmi}}$$

Where: f_{pmi} is the frequency of occurrence of the i^{th} task in the reference period.

k is the number of different PM tasks.

Σf_{pmi} is the total number of PM tasks in the population.

Accept if: μ_{pm} (required) $\geq \mu_{pm}$ (actual)

Reject if: μ_{pm} (required) $< \mu_{pm}$ (actual)

B.120.4.2 Test for $M_{max_{pm}}$ - The PM tasks shall be ranked by magnitude (lowest to highest value). The equipment shall be accepted if the magnitude of the task time at the percentile of interest is equal to or less than the required value of $M_{max_{pm}}$.