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PERFORMANCE SPECIFICATION

WORLD MAGNETIC MODEL (WMM)

This specification is approved for use by all Departments and Agencies of the Department of Defense.

Comments, suggestions, or questions on this document should be addressed to the National Geospatial-Intelligence Agency (NGA), ATTN: Departmental Standardization Officer, 3838 Vogel Road, Arnold, MO 63010-6238, Mail Stop L66, or by e-mail to DepSO@nga.mil. Since contact information can change, you may want to verify the currency of this address information using the ASSIST Online database at https://assist.dla.mil.

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CONTENTS

1. SCOPE

1.1 Scope. This specification is designed to provide guidelines for the preparation and use of the World Magnetic Model (WMM) and the charts and grid tables derived from the model. This specification also provides guidelines for the preparation and use of computer programs and subroutines that compute the model magnetic field elements, namely the declination (*D*), inclination (*I*), total intensity (*F*), horizontal intensity (*H*), and grid variation (*GV*) at given locations and times.

1.2 Purpose. The purpose of this document is to specify the WMM and the products that are derived from the model.

2. APPLICABLE DOCUMENTS

2.1 General. The documents listed in this section are specified in sections 3, 4, or 5 of this specification. This section does not include documents cited in other sections of this specification or recommended for additional information or as examples. While every effort has been made to ensure the completeness of this list, document users are cautioned that they must meet all specified requirements of documents cited in sections 3, 4, or 5 of this specification, whether or not they are listed.

2.2 Government documents.

2.2.1 Specifications, standards, and handbooks. The following specifications, standards, and handbooks form a part of this document to the extent specified herein. Unless otherwise specified, the issues of these documents are those cited in the solicitation or contract.

INTERNATIONAL STANDARDIZATION AGREEMENTS

STANAG 7172 - Use of Geomagnetic Models

(Copies of these documents are available online at https://quicksearch.dla.mil)

NATIONAL GEOSPATIAL-INTELLIGENCE AGENCY

NGA.STND.0036_1.0.0_WGS84, Department of Defense World Geodetic System 1984: Its Definition and Relationships with Local Geodetic Systems, Version 1.0.0, 8 July 2014.

(Copies of these documents are available online at https://nsgreg.nga.mil).

2.2.2 Other Government documents, drawings, and publications. The following other Government documents, drawings, and publications form a part of this document to the extent specified herein. Unless otherwise specified, the issues of these documents are those cited in the solicitation or contract.

- Lemoine, F.G., S.C. Kenyon, J.K. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp, and T.R. Olson (1998), The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96, NASA Tech. Publ. TP 1998 206861, 575 pp. It is available at https://cddis.gsfc.nasa.gov/926/egm96/egm96.html .
- Chulliat, A., S. Macmillan, P. Alken, C. Beggan, M. Nair, B. Hamilton, A. Woods, V. Ridley, S. Maus and A. Thomson, 2015. The US/UK World Magnetic Model for 2015-2020: Technical Report, NOAA National Geophysical Data Center, Boulder, CO, doi: 10.7289/V5TB14V7. It is available at www.ngdc.noaa.gov/geomag/WMM.
- NIST Handbook of Mathematical Functions (ISBN 978-0-521-14063-8, paperback), on-line at https://dlmf.nist.gov/14.

Users needing information on the latest version of the WMM and documentation should contact the National Centers for Environmental Information (NCEI) at:

National Centers for Environmental Information (NCEI)
NOAA, Mail Code E/NE42
325 Broadway
Boulder, CO 80305
USA
e-mail: ncei.info@noaa.gov or geomag.models@noaa.gov
web-site: https://www.ngdc.noaa.gov/geomag/WMM/

or the British Geological Survey (BGS) at:

British Geological Survey
The Lyell Centre
Research Avenue South
Edinburgh, EH14 4AP
UK
e-mail: smac@bgs.ac.uk or wb@bgs.ac.uk
web-site:
https://www.geomag.bgs.ac.uk/research/modelling/WorldMagneticModel.html

2.3 Non-Government publications. The following documents form a part of this document to the extent specified herein. Unless otherwise specified, the issues of these documents are those cited in the solicitation or contract.

- Backus, G., R.L. Parker, and C. Constable, 1996. *Foundations of geomagnetism*. Cambridge University Press.
- Gradshteyn, I.S. and I.M. Ryzhik, 1994. *Table of integrals, series and products* (5th ed). Academic Press.
- Heiskanen, W. A. and Moritz, H., 1967. *Physical Geodesy*. W. H. Freeman and Company; San Francisco, California and London, UK.
- Langel, R.A., 1987. The main field, In Geomagentism, J.A. Jacobs (ed). Academic Press, NY, pp. 249-512.
- Pavlis, N. K., S. A. Holmes, S. C. Kenyon, and J. K. Factor, 2012. The development and evaluation of the Earth Gravitational Model 2008 (EGM2008), J. Geophys. Res., 117, B04406, doi:10.1029/2011JB008916.

(Non-Government standards and other publications are normally available from the organizations that prepare or distribute the documents. These documents also may be available in or through libraries or other informational services.)

2.4 Order of precedence. Unless otherwise noted herein or in the contract, in the event of a conflict between the text of this document and the references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

3. REQUIREMENTS

3.1 General description. The WMM is a spherical harmonic model of the Earth's main (mostly core generated) magnetic field and its secular (slow temporal) change. The main field portion is to degree and order 12 and consists of 168 Schmidt-normalized spherical harmonic coefficients. The secular change field is also specified to degree and order 12 and consists of 168 coefficients. For WMM2005 and earlier WMM's, the secular variation (SV) coefficients above degree 8 were set to zero. This was due to the lack of adequate data to generate this portion of the model. However for WMM2010, these same coefficients were non-zero (many of them) due to the exceptionally complete data-set available to build WMM2010. Software developers incorporating the WMM are directed to assume that these coefficients may be non-zero. This model characterizes only the long wavelength (>3200 km) portion of the earth's magnetic field. Medium and short wavelength features associated with the Earth's magnetic crust are not represented. A much higher degree and order model, such as the Enhanced Magnetic Model (EMM), would be required to model these features. Also not included in this model are seasonal, daily and other temporal fluctuations of the Earth's magnetic field that are driven by the Solar Wind and modulated by magnetospheric and ionospheric current systems such as the Ring Current, the North and South Polar Auroral Electrojet Currents, Solar-quiet (Sq) currents, and the Equatorial Electrojet Current. Magnetic fields generated by these current systems can be quite pronounced particularly in regions of high magnetic latitudes and during periods of high solar activity. The WMM, nevertheless, does represent approximately 95% of the earth's magnetic field. The world magnetic charts referenced in Section 3.10 are generated directly from the WMM and are indicative of the spatial resolution of the model. Starting with WMM2020, a Black out Zone region was added and is further discussed in sections 3.4.7 and 3.4.9. Amplification of these concepts and the details of the mathematical model are given in Appendix A.

3.2 Units of measure. The total intensity (*F*), horizontal intensity (*H*), northerly component (*X*), the easterly component (*Y*), and downward component (*Z*) are expressed in nanoTesla (nT) while the secular variation of these quantities are expressed in nanoTesla per year (nT/yr). The declination (*D*), inclination (*I*) and grid variation (*GV*) are expressed in decimal degrees and their secular variations are expressed in minutes of arc per year (min/yr). The main field WMM coefficients are expressed in nT. The secular variation WMM coefficients are expressed in nT/yr. The geodetic latitude and longitude are expressed in decimal degrees, while the altitude is expressed in kilometers (km). Time is referenced in decimal years (e.g., 15 May 2019 is 2019.367). Note that the day-of-year (DOY) of January 1st is zero and December 31st is 364 for non-leap year. For a leap year, DOY of December 31st is 365.

3.3 Sign conventions. The Earth's magnetic field components are referenced to a local Cartesian coordinate frame, which is oriented such that the positive X-axis points North, the positive Y-axis points East, and the positive *Z*-axis points downward, as defined by the normal to the ellipsoid. Consequently, the negative X-axis points South, the negative Y-axis points West, and the negative Z-axis points upwards. The latitude ranges between –90° to +90°, while the longitude ranges from –l80° to +180°. Without confusion, as an option, longitudes in the range 0° to 360° area acceptable.

3.4 Magnetic element accuracies.

3.4.1 Definition of accuracy. When comparing an observed magnetic element (for example, *D*, as given by an electronic compass) with the value predicted by the WMM, three errors come into play to account for the possible lack of exact agreement between the measured (observed) value, and the WMM supplied value: (i) measurement error, (ii) modeling error of commission, and (iii) modeling error of omission. These are explained in turn:

- Measurement error. This category includes all errors due to the measurement itself, including errors due to instrument inaccuracy and errors due to distortions of the magnetic field by man-made objects.

- Modeling error of commission. This category deals with the error of the WMM in representing the geophysical phenomena it claims to model, namely, the main field to degree and order 12 and its secular variation. It is dominated by the uncertainty of predicting the change of the field over the 5 year life span of the model. The commission error directly depends on the availability of global satellite magnetic measurements in the years preceding the release of the WMM.

- Modeling error of omission. This category includes all natural contributions to the magnetic field which are not included in the WMM. Primarily, these consist of crustal fields caused by magnetized rocks in the outer layer of the Earth, and disturbance fields caused by ionospheric and magnetospheric electric currents.

The WMM error estimates are Root Mean Square (RMS) differences between the WMM and an error-free measurement, thereby including the modeling error of commission and the modeling error of omission. They are calculated using values on a latitude/longitude grid on the WGS 84 ellipsoid surface, weighted by the cosine of the latitude over the 5 year life span of the model.

3.4.2 Northerly component (*X*). The northerly component values generated by the WMM and its software implementations.

3.4.3 Easterly component (*Y*). The easterly component values generated by the WMM and its software implementations.

3.4.4 Downward component (*Z*). The downward component magnetic field values generated by the WMM and its software implementations.

3.4.5 Horizontal intensity (*H*). The horizontal intensity values generated by the WMM and its software implementations will have errors smaller than 200 nT RMS at the WGS 84 ellipsoid surface worldwide at any time over the entire 5 year lifetime of the model. The horizontal component is used to define the blackout zone for the declination and grid variation; see sections 3.4.7 and 3.4.9.

3.4.6 Total Intensity (*F*). The total intensity values generated by the WMM and its software implementations will have errors smaller than 280 nT RMS at sea level worldwide at any time during the entire 5 year lifetime of the model.

3.4.7 Declination (*D*). The total declination value generated by the WMM and its software implementations will have errors smaller than 1° RMS at sea level worldwide at any time during the entire 5 year lifetime of the model. For the purpose of this requirement, the RMS error excludes a black-out zone defined as the area at the WGS 84 ellipsoid surface where the horizontal intensity is smaller than 2000 nT.

3.4.8 Inclination (*I*). The inclination values generated by the WMM and its software implementations will have errors smaller than 1° RMS at sea level worldwide at any time during the entire 5 year lifetime of the model.

3.4.9 Grid Variation (*GV*). The grid variation north (GV_N) and south (GV_S) values generated by the WMM and its software implementations will each have errors smaller than 1° RMS at sea level at any time during the entire 5 year lifetime of the model. For the purpose of this requirement, the RMS error is calculated over each polar region – the portion of the Earth where the latitude is above 55°N or below 55°S – excluding a blackout zone defined as the area at the WGS 84 ellipsoid surface where the horizontal intensity is smaller than 2000 nT.

3.5 Vertical and horizontal datums. The reference for longitudes and latitudes (the horizontal datum) and the reference for heights-above-the-ellipsoid (vertical datum) used by the WMM is the World Geodetic System 1984 (WGS 84). The reference ellipsoid for this datum is published in NGA.STND.0036_1.0.0_WGS84, Department of Defense World Geodetic System 1984: Its Definition and Relationships with Local Geodetic Systems, Version 1.0.0, 8 July 2014.

3.6 Time datum. Each edition of the WMM shall have a specified base year (epoch). Unless otherwise specified or approved, each edition of the WMM will have a valid operational service life limited to 5 years forward from that time. Barring unforeseen circumstances, a new WMM is released every 5 years, e.g. 2015, 2020, 2025 and the model epoch will be January 1 of that year. For example, in 2018 a special out of cycle update was released prior to WMM2020 focusing on 55 degrees north and above, resulting in new coefficients and a new technical report.

3.7 Vertical aspect of model validity. The Earth's magnetic field extends deep into the Earth and far out into space, thus a WMM is to meet the error specification in the region from 1 km below the WGS 84 ellipsoid surface to approximately 850 km above it. The error validation for the models have only been done for the ellipsoid surface so the vertical aspect is approximate. Even above 850 km, a WMM is a good model of the Earth's magnetic field, but without necessarily meeting the error specifications.

3.8 Software conformance. The software implementation of a WMM will conform to the model equations given in Appendix A over an interval of time extending 10 years beyond the epoch of the model. The WMM may have to be used beyond the intended 5 year design-life of the model. In such cases, the software implementation should continue to output values, but should display a warning.

3.9 Computer software. A procurement of a WMM by the U.S. or U.K. governmental sponsoring agencies will consist of one or more software implementations, in one or more programming languages and systems. The details are to be specified in the statement of work for such procurements. The WMM producer will provide a programmer's guide, an end-user's guide, or on-line help documentation as appropriate (such as an information or help email like geomag.models@noaa.gov), for each item of software delivered. For information on the format, the updated coefficient files, test values, and on-line calculators see https://www.ngdc.noaa.gov/geomag/WMM/soft.shtml and https://www.geomag.bgs.ac.uk/research/modelling/WorldMagneticModel.html.

3.10 World magnetic charts. As part of the delivery of a WMM, the producer will deliver a digital set of contour charts, one for each magnetic field element (*X, Y, Z, H, F, D, I,* and *GV*), one each for the main field and secular variation separately, and one each for the south polar, mid-latitudes, and north polar regions separately. The details of the file formats are to be specified in the procurement statement-of-work, but are to include the capability to display the charts on-screen or print them as charts, or process them further by a Geographic Information System. Contour maps for *GV* are desired for the polar regions only. The map projections recommended for the three regions are the South Polar Stereographic, Mercator, and North Polar Stereographic projections. For the information on the charts of the magnetic elements and available charts and shapefiles, see https://www.ngdc.noaa.gov/geomag/WMM/image.shtml

3.11 Product delivery. The main mode of delivery of the products associated with the WMM will be as downloads from the websites of NCEI and BGS.

3.12 Technical report. The producer will provide a technical report describing the data sets used to build the model and the procedures employed to build the model from the data sets. The technical report will further provide estimates of WMM omission errors due to crustal and disturbance fields which are not represented in the WMM.

3.13 Security classification. The WMM spherical harmonic coefficients, related computer software, charts, grids and other products derived from the model are UNCLASSIFIED.

4. VERIFICATION

4.1 Conformance inspection. The producer will provide test data with the delivered software to allow verification of correct installation of the software.

4.2 Technical review. NGA's subject matter experts will review the scientific efforts of the producer during the preparation of the technical report.

5. PACKAGING

5.1 Packaging. For acquisition purposes, the packaging requirements shall be as specified in the contract or order. When packaging of materiel is to be performed by DoD or in-house contractor personnel, these personnel need to contact the responsible packaging activity to ascertain packaging requirements. Packaging requirements are maintained by the Inventory Control Point's packaging activities within the Military Service or Defense Agency, or within the military service's system commands. Packaging data retrieval is available from the managing Military Department's or Defense Agency's automated packaging files, CD-ROM/DVD products, or by contacting the responsible packaging activity.

6. NOTES

This section contains information of a general or explanatory nature that may be helpful, but is not mandatory.

6.1 Intended use. This specification is intended to provide guidelines for the preparation and use of the World Magnetic Model to support DoD and MoD missions including mapping, charting, navigation, azimuth determination, and other military applications. The WMM is embedded in many military platforms and systems. Geomagnetic data and models belong to the general category of geospatial intelligence (GEOINT) for which NGA is the functional manager for DoD and the Intelligence Community.

The World Magnetic Model is considered Public Release. This specification may also be used by public, private, academic, scientific, and other domains for application of the World Magnetic Model.

6.2 WMM data. The WMM is primarily constructed from measurements of satellites in polar low-Earth orbits. These are complemented with data from the global network of ground magnetic observatories, operated by a wide variety of public and private agencies of domestic and foreign origin which adhere to the scientific standards and nomenclatures adopted by INTERMAGNET and the International Association of Geomagnetism and Aeronomy, IAGA.

6.3 Acquisition requirements. Acquisition documents should specify the following:

a. Title, number, and date of this specification.

6.4 Supersession data. This specification supersedes MIL-W-89500A, dated 30 October 2015.

6.5 Subject term (key word) listing.

geomagnetism
geophysics
magnetic field
nanoTesla
secular variation
spherical harmonic model

6.6 Abbreviations.

BGS	British Geological Survey
D	Declination (magnetic variation)
EGM	Earth Gravitational Model
EMM	Enhanced Magnetic Model
F	Total intensity
GV	Grid variation
H	Horizontal intensity
HAE	Height above Ellipsoid
I	Inclination
IAGA	International Association of Geomagnetism and Aeronomy
MoD	Ministry of Defence (UK)
NATO	North Atlantic Treaty Organization
NCEI	National Centers for Environmental Information (formerly National Geophysical Data Center, NGDC)
NGA	National Geospatial-Intelligence Agency
NOAA	National Oceanic and Atmospheric Administration
nT	nanoTesla
RMS	Root Mean Square
STANAG	STANdardization AGreement
WGS 84	World Geodetic System 1984
WMM	World Magnetic Model
X	North component of the magnetic field vector
Y	East component of the magnetic field vector
Z	Downward component of the magnetic field vector

6.7 International standardization agreement implementation. This specification implements NATO STANAG 7172, Use of Geomagnetic Models. When amendment, revision, or cancellation of this specification is proposed, the preparing activity must coordinate the action with the U.S. national point of contact for the international standardization agreement, as identified in the ASSIST database at https://assist.dla.mil.

6.8 Changes from previous issue. Marginal notations are not used in this revision to identify changes with respect to the previous issue due to the extent of the changes.

WMM DATA CONCEPTS

A.1 SCOPE

A.1.1 Scope. This appendix is a mandatory part of the specification.

A.2 DATA DESCRIPTION

A.2.1 Introduction. The Earth is like a giant magnet. At every location on or above the Earth, its magnetic field has a more or less well-known direction, which can be used as a reference frame to orient ships, aircraft, satellites, antennas, drilling equipment and handheld devices. At some places on the globe the horizontal direction of the magnetic field coincides with the direction of geographic north ("true" north), but in general this is not the case. The angular amount by which the horizontal direction of the magnetic field differs from true north is called the magnetic declination, or simply declination (D, see Figure A-1). This is the correction required to convert between a magnetic bearing and a true bearing. The main utility of the World Magnetic Model (WMM) is to provide magnetic declination for any desired location on the globe. In addition to the magnetic declination, the WMM also provides the complete geometry of the field from 1 km below the WGS 84 ellipsoid surface to 850 km above it. The magnetic field extends deep into the Earth and far out into space, but the WMM is not designed to meet the error specification there.

A.2.2 Sources of geomagnetism. The Earth's magnetism has several sources. All the sources will affect a scientific or navigational instrument but only some of them are represented in the WMM. The strongest contribution, by far, is the magnetic field produced by the Earth's liquid-iron outer core, called the "core field". Magnetic minerals in the crust and upper mantle make a further contribution that can be significant locally. Electric currents induced by the flow of conducting sea water through the ambient magnetic field make a further, albeit weak, contribution to the observed magnetic field. All of these are of "internal" origin. Deliberately excluded from the WMM by the data selection process and by other means are the so-called "disturbance fields". These are contributions arising from electric currents in the upper atmosphere and near-Earth space. Because the "external" magnetic fields so produced are time-varying, there is a further effect. They induce electric currents in the Earth and oceans, producing secondary internal magnetic fields, which are considered part of the disturbance field and are therefore not represented in the WMM.

A.2.3 Main field. The mathematical method of the WMM is an expansion of the magnetic potential into spherical harmonic functions to degree and order 12. The minimum wavelength resolved is $360° / \sqrt{12 \text{ x } 13} = 28.8°$ in arc-length, corresponding to 3200 km at the Earth's surface (see Sec. 3.6.3 of Backus et al, 1996). The WMM is a model of those internal magnetic fields that are not part of the disturbance field and have spatial wavelengths exceeding 28.8° in arc-length. This includes almost the entire core field and small additional contributions of long-wavelength crustal and oceanic fields. In this specification, the term "main field" refers to the portion of the Earth's magnetic field that is modeled by the WMM and exists at the epoch of the WMM. The error of the WMM in representing this part of the field is referred to as the WMM commission error. Neglect of other contributions to the field is summarily called the WMM omission error.

A.2.4 Secular Variation. The core field changes perceptibly from year to year. This effect, called secular variation (SV), is accounted for in the WMM by a linear model. (Specifically, a straight line is used as the model of the time-dependence of each coefficient of the spherical harmonic representation of the magnetic potential). Due to unpredictable non-linear changes in the core field, the values of both the main and SV coefficients have to be updated every five years. These non-linear changes are the primary source of the WMM commission error, provided that the main field and linear secular variation were accurately determined from satellite observations. If no such satellite observations are available to construct the model, the WMM commission error will be significantly larger, due to the inaccurate specification of the main field and secular variation coefficients at the beginning of the model epoch.

A.2.5 The magnetic elements. Figure A-1 shows the 7 magnetic elements (*X, Y, Z, H, F, D,* and *I*). *GV* is explained in the next paragraph:

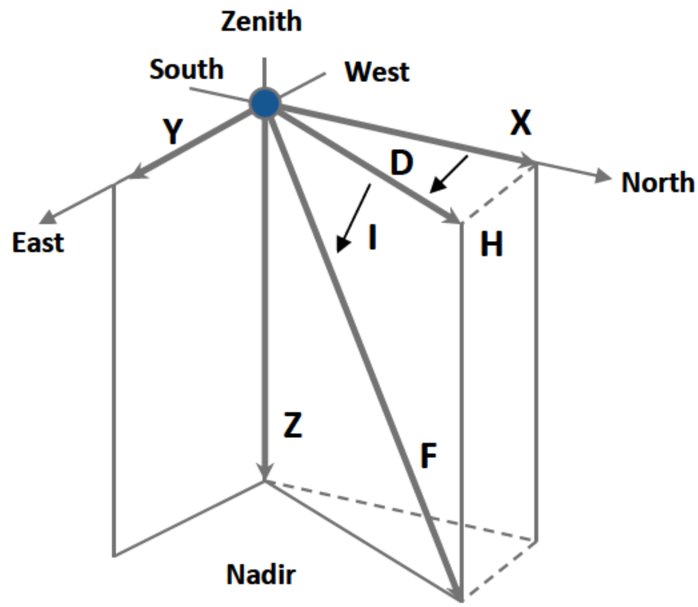

FIGURE A-1: Magnetic elements

A.2.6 Grid variation. In the polar regions, or near the rotation axis of the Earth, the angle *D* in degrees changes strongly with a change in the longitude of the observer, and is therefore a poor measure of the direction of the field. For this reason, the WMM will provide two auxiliary angles in degrees, called grid variation north (GV_N) and south (GV_S), for the direction of the field in the horizontal plane in each polar region. Their definitions are:

$$GV_N = D - \lambda \; for \; \varphi > 55° \qquad GV_S = D + \lambda \; for \; \varphi < -55° \tag{1}$$

where λ is the longitude and φ is the geodetic latitude.

The quantities GV_N and GV_S defined above are examples of a more general concept, namely grid variation (also called grid magnetic angle or grivation). At a location on the plane of a chosen horizontal grid coordinate system, grivation is the angle between grid north and magnetic north, i.e., the angle measured clockwise from the direction parallel to the grid's Northing's' axis to the horizontal component of the field at the observer's location. Grivation is useful for local surveys, where location is given by grid coordinates rather than by longitude and latitude. It is dependent on the map projection used to define the grid coordinates. In general, it is

$$GV_{grid} = D - C \tag{2}$$

where D is the magnetic declination and C is the "convergence-of-meridians" defined as the clockwise angle from the northward meridional arc to the grid Northing direction.

For example, large scale military topographic mapping routinely employs the Universal Transverse Mercator (UTM) grid coordinates for the map projection of the sheet, for the definition of a grid to overprint, and for a grivation calculation as defined above. Above 84°N and below 80°S, it employs the Universal Polar Stereographic (UPS) grid. For these two grids, the grid variation could be notated GV_{UTM} and GV_{UPS}, respectively.

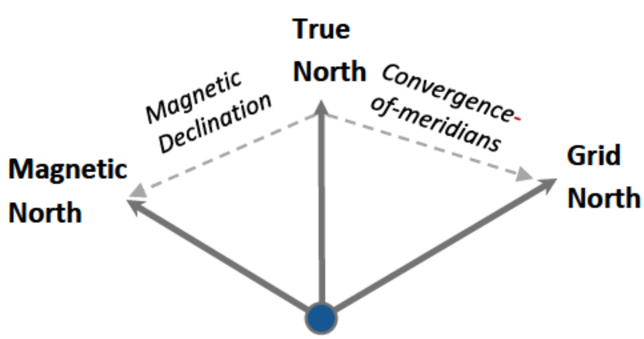

FIGURE A-2: Grid variation (GV_{grid})

A.2.7 Model equations. This section describes the representation of the magnetic field in the WMM and lists the equations needed to obtain the magnetic field elements for the desired location and time from the WMM coefficients. All variables in this section adhere to the following measurement conventions: angles are in radians, lengths are in meters, magnetic intensities are in nT and times are in years. End-user software may display these quantities in other units, which should be identified.

The main field $\mathbf{B_m}$ is a potential field and therefore can be written in geocentric spherical coordinates (longitude (λ), latitude (φ'), radius (r), and time (t)) as the negative spatial gradient of a scalar potential:

$$\boldsymbol{B_m}(\lambda, \varphi', r, t) = -\nabla V(\lambda, \varphi', r, t) \tag{3}$$

This potential can be expanded in terms of spherical harmonics:

$$\boldsymbol{V(\lambda, \varphi', r, t) = a \left\{ \sum_{n=1}^{N} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (g_n^m(t) \cos(m\lambda) + h_n^m(t) \sin(m\lambda)) \breve{P}_n^m(\sin \varphi') \right\}} \tag{4}$$

where N=12 is the degree of the expansion of the WMM, a = 6371200 meters is the geomagnetic reference radius, (λ, φ', r) are the longitude, latitude and radius in a spherical geocentric reference frame, and $g_n^m(t)$ and $h_n^m(t)$ are the time-dependent Gauss coefficients of degree n and order m describing the Earth's main magnetic field. For any real number, μ, $\breve{P}_n^m(\mu)$ are the Schmidt semi-normalized associated Legendre functions defined as:

$$\begin{aligned} \breve{P}_n^m(\mu) &= \sqrt{2\frac{(n-m)!}{(n+m)!}} P_{n,m}(\mu) \ if\ m > 0 \\ \breve{P}_n^m(\mu) &= P_{n,m}(\mu) \ if\ m = 0 \end{aligned} \tag{5}$$

Here, we use the definition of $P_{n,m}(\mu)$ commonly used in geodesy and geomagnetism (e.g., Heiskanen and Moritz, 1967, eq. 1-60; Langel, 1987, eq. 8). Sample functions, for geocentric latitude φ', are:

$$\begin{aligned} P_{3,0}(\sin\varphi') &= \frac{1}{2}(\sin\varphi')(5\sin^2\varphi' - 3) \\ P_{3,1}(\sin\varphi') &= -\frac{3}{2}(\cos\varphi')(1 - 5\sin^2\varphi') \\ P_{3,2}(\sin\varphi') &= 15(\sin\varphi')(1 - \sin^2\varphi') \\ P_{3,3}(\sin\varphi') &= 15(\cos^3\varphi') \end{aligned} \tag{6}$$

These $P_{n,m}(\mu)$ are related to the $P_n^m(\mu)$ defined in the NIST Handbook of Mathematical Functions (Section 14.2, page 352) or Gradshteyn and Ryzhik (1994, Chapter 8.7) by $P_{n,m}(\mu) = (-1)^m P_n^m(\mu)$.

The WMM will comprise two sets of Gauss coefficients to degree and order *N=12*. One set provides a spherical harmonic main field model at the epoch of the model in units of nT. The other set provides a predictive secular variation model for the 5 year period following the epoch of the model in units of nT/year.

A step by step procedure is provided below for computing the magnetic field elements at a given location and time (λ, φ, h_{MSL}, t), where λ and φ are the geodetic longitude and latitude, h_{MSL} is Mean Sea Level (MSL) height, and t is the time given in decimal years.

In the first step, the user is requested to provide the time, location, and MSL height at which the magnetic elements are to be calculated. The MSL height is then converted to height h above the WGS 84 ellipsoid by using an appropriate Earth Gravitational Model such as EGM96 (Lemoine et al, 1998) or EGM2008 (Pavlis et al, 2012). The details of this are outside the scope of this specification which is concerned solely with the geomagnetic model. The current implementations of WMM software from NCEI allows the user to enter height as above the MSL (EGM96) or above the WGS 84 ellipsoid. Alternately, and preferably, in this step, the user can directly provide the height h above the ellipsoid.

The geodetic coordinates (λ, φ, h) are then transformed into spherical geocentric coordinates (λ, φ', r) by recognizing that λ is the same in both coordinate systems, and that (φ', r) is computed from (φ, h) according to the equations:

$$\begin{aligned}
R_c &= \frac{A}{\sqrt{1 - e^2 \sin^2 \varphi}} \\
p &= (R_c + h)\cos\varphi \\
z &= (R_c(1 - e^2) + h)\sin\varphi \\
r &= \sqrt{p^2 + z^2} \\
\varphi' &= \arcsin\frac{z}{r}
\end{aligned} \qquad (7)$$

where A and e are constants pertaining to the WGS 84 ellipsoid:

$$A = 6378137\ m$$
$$\frac{1}{f} = f^{-1} = 298.257223563 \quad (8)$$
$$e^2 = f(2-f)$$

The above is standard fare in geodesy with A as the semi-major axis of the reference ellipsoid (WGS 84 here), f^{-1} as its inverse flattening, f as its flattening, and e as its first eccentricity. If x, y and z are geocentric Cartesian coordinates of the point of interest defined in the standard way (with $\varphi = 0$ if and only if $z = 0$ *etc*.), then we also have $p = \sqrt{x^2 + y^2}$. The symbol R_c stands for the radius of curvature of the east-west normal section through the point on the reference ellipsoid given by (λ, φ) of the point of interest.

In the second step, the Gauss coefficients $g_n^m(t)$ and $h_n^m(t)$ are determined for the desired time t from the model coefficients $g_n^m(t_0)$, $h_n^m(t_0)$, $\dot{g}_n^m(t_0)$, and $\dot{h}_n^m(t_0)$ as

$$\begin{aligned} g_n^m(t) &= g_n^m(t_0) + (t - t_0)\dot{g}_n^m(t_0) \\ h_n^m(t) &= h_n^m(t_0) + (t - t_0)\dot{h}_n^m(t_0) \end{aligned} \quad (9)$$

where the time is given in decimal years and t_0 is the reference epoch (base date) of the model. The quantities $g_n^m(t_0)$ and $h_n^m(t_0)$ are called the main field coefficients and the quantities $\dot{g}_n^m(t_0)$ and $\dot{h}_n^m(t_0)$ are called the secular variation coefficients.

In the third step, the field vector components X', Y' and Z' in topocentric coordinates (Z' points towards the origin, X' points north and Y' points east as shown in Figure A-3) are computed as

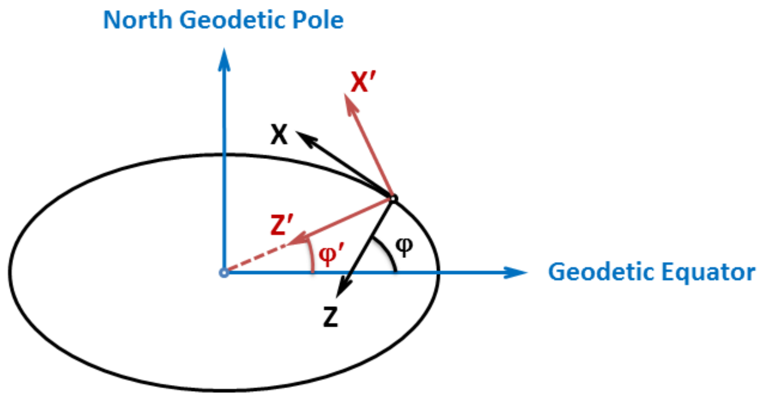

FIGURE A-3: Topocentric coordinate system

$$X^{'}(\lambda,\varphi',r) = -\frac{1}{r}\frac{\partial V}{\partial \varphi'}$$

$$= -\sum_{n=1}^{12}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left(g_n^m(t)\cos m\lambda + h_n^m(t)\sin m\lambda\right)\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'} \tag{10}$$

$$Y^{'}(\lambda,\varphi',r) = -\frac{1}{r\,cos\,\varphi'}\frac{\partial V}{\partial \lambda}$$

$$= \frac{1}{cos\,\varphi'}\sum_{n=1}^{12}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m\left(g_n^m(t)\sin m\lambda - h_n^m(t)cos\,m\lambda\right)\breve{P}_n^m(\sin\varphi') \tag{11}$$

$$\boldsymbol{Z^{'}(\lambda,\varphi',r) = \frac{\partial V}{\partial r}}$$

$$\boldsymbol{= -\sum_{n=1}^{12}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left(g_n^m(t)\,cos\,m\lambda + h_n^m(t)\sin m\lambda\right)\breve{P}_n^m(\sin\varphi')} \tag{12}$$

At this point, one can also compute the secular variation of the field components as

$$\dot{X}^{'}(\lambda,\varphi',r) = -\frac{1}{r}\frac{\partial \dot{V}}{\partial \varphi'} \tag{13}$$

$$= -\sum_{n=1}^{12}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}(\dot{g}_n^m \cos m\lambda + \dot{h}_n^m \sin m\lambda)\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'}$$

$$\dot{Y}'(\lambda,\varphi',r) = -\frac{1}{r\,cos\,\varphi'}\frac{\partial\dot{V}}{\partial\lambda}$$

$$= \frac{1}{cos\,\varphi'}\sum_{n=1}^{12}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m(\dot{g}_n^m \sin m\lambda - \dot{h}_n^m cos\,m\lambda)\breve{P}_n^m(\sin\varphi') \qquad (14)$$

$$\dot{\boldsymbol{Z}}'(\boldsymbol{\lambda},\boldsymbol{\varphi}',\boldsymbol{r}) = \frac{\boldsymbol{\partial\dot{V}}}{\boldsymbol{\partial r}}$$

$$= -\sum_{n=1}^{12}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}(\dot{g}_n^m\, cos\, m\lambda + \dot{h}_n^m \sin m\lambda)\breve{P}_n^m(\sin\varphi') \qquad (15)$$

For computational purposes, the derivative expression occurring in the above formulas for X' and $\dot{X}'$ should be replaced according to:

$$\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'} = (n+1)(\tan\varphi')\breve{P}_n^m(\sin\varphi') - \sqrt{(n+1)^2 - m^2}(\sec\varphi')\,\breve{P}_{n+1}^m(\sin\varphi') \qquad (16)$$

In the fourth step, the magnetic field vector components X',Y' and Z', are rotated into the ellipsoidal reference frame (see Figure A-3), using

$$X = X'\cos(\varphi' - \varphi) - Z' sin(\varphi' - \varphi)$$

$$Y = Y' \qquad (17)$$

$$Z = X'\sin(\varphi' - \varphi) + Z' cos(\varphi' - \varphi)$$

Similarly, the time derivatives of the vector components, $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ are rotated using

$$\dot{X} = \dot{X}'\cos(\varphi' - \varphi) - \dot{Z}' sin(\varphi' - \varphi)$$

$$\dot{Y} = \dot{Y}' \qquad (18)$$

$$\dot{Z} = \dot{X}'\sin(\varphi' - \varphi) + \dot{Z}'cos(\varphi' - \varphi)$$

In the 5th and last step, the magnetic elements *H, F, I,* and *D* are computed from the orthogonal components:

$$H = \sqrt{X^2 + Y^2}$$

$$F = \sqrt{H^2 + Z^2} \qquad (19)$$

$$I = \text{atan2}(Z, H)$$

$$D = \text{atan2}(Y, X)$$

where atan2 is the double argument version of arctangent such as provided in FORTRAN and C. In particular, $\text{atan2(y, x)} = \arctan\left(\frac{\text{y}}{\text{x}}\right)$, if $x > 0$. This produces a declination in the range of $-\pi$ to π and an inclination in the range of $-\pi/2$ to $\pi/2$. Software tools typically convert these angles to degrees for display, printing, or user-interfacing.

The secular variation of these elements is computed using

$$\dot{H} = \frac{\mathrm{X} \cdot \dot{\mathrm{X}} + \mathrm{Y} \cdot \dot{\mathrm{Y}}}{H}$$

$$\dot{F} = \frac{\mathrm{X} \cdot \dot{\mathrm{X}} + \mathrm{Y} \cdot \dot{\mathrm{Y}} + \mathrm{Z} \cdot \dot{\mathrm{Z}}}{F}$$

$$\dot{I} = \frac{\mathrm{H} \cdot \dot{\mathrm{Z}} - \mathrm{Z} \cdot \dot{\mathrm{H}}}{F^2} \qquad (20)$$

$$\dot{D} = \frac{\mathrm{X} \cdot \dot{\mathrm{Y}} - \mathrm{Y} \cdot \dot{\mathrm{X}}}{H^2}$$

$$G\dot{V} = \dot{D}$$

where $\dot{F}$, $\dot{D}$, and $G\dot{V}$are given in radians per year. Software tools typically convert these quantities to arc-minutes per year for display, printing, or user-interfacing.

Sample values for testing purposes can be found in previous editions of the WMM.

CONCLUDING MATERIAL

Custodians:
Army – AV
Navy – NO
Air Force – 09

Preparing Activity:
NGA – MP
(Project GINT-2019-001)

Review Activities:
Army – MI
Navy – CG, MC
Air Force – 33, 99
DIA – DI
DISA – DC1
NSA – NS

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