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MILITARY HANDBOOK

DESIGN OF ELECTRICAL EQUIPMENT WITH SMALL STRAY MAGNETIC FIELDS

(METRIC)

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FOREWORD

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1. SCOPE

1.1 General. This handbook discusses the theory and establishes standard design requirements for the minimization of small stray magnetic fields on Naval ships.

1.1 Application. This handbook serves as a reference on the theory of stray magnetic fields and provides the design requirements for equipment which should have small stray magnetic fields. The principles and procedures given in this handbook should be considered during the initial design phases for equipment required to have small stray magnetic fields,

2. APPLICABLE DOCUMENTS

This section is not applicable to this handbook.

3. GENERAL DESIGN CONSIDERATION

3.1 Introduction. Magnetic fields are caused by the presence of ferro-magnetic material and by the passing of a current through a conductor. Some Navy applications require that the magnetic field caused by a current in a wire be minimized. This handbook is directed primarily to the design of circuits and equipment for minesweepers where a small stray magnetic field is of the utmost importance . However, other ships that require a small stray magnetic field can apply the principles set forth herein in the design of electric circuits and equipment. This design is particularly important for direct current (de), electric propulsion installations in surface ships or submarines. While it is not necessary in many ships to push refinements in design as far as it is necessary in minesweepers, disregard of the design principles discussed herein can result in stray magnetic fields that are unacceptably large. This handbook first discusses the general design considerations for reducing stray magnetic fields. Next comes a thorough discussion of current loops and how they generate stray magnetic fields. Many current loop configurations in the form of dipole moments arc shown along with related equations and contour plots. The principles of design of equipment, which will have small stray magnetic field, are discussed in general Next comes a discussion of specific design of magnetic minesweeping equipment, other than the minesweeping generator, having small stray magnetic field. The minimum stray magnetic field design of the magnetic minesweeping generators is discussed in a section of its own. Finally, there is a discussion of battery arrangements as used on xninesweepers/minehunters and as used on submarines. Appendix A guides the reader through the derivations of the magnetic field equations contained in the handbook. Appendix B lists all of the equations found in the handbook by equation number, brief description, and the page number on which the equation is found.

3.2 Definition of a stray magnetic field. A stray magnetic field is an unwanted magnetic field generated by a current, in particular a direct current The stray magnetic field produced by an electric circuit or equipment is the change in magnetic field outside of the equipment that is caused by changing the current in the circuit or equipment from zero to a value different from zero, with the following restrictions.

(a) If this magnetic field can be contained within an enclosure or a volume and does not impact magnetic silencing, it is not a stray field.
(b) If the magnetic field is desirable, it is not a stray field.

3.2.1 Measuring stray magnetic field. The stray magnetic field at a point can be calculated in simple cases (when no iron or other magnetic material is present). and can be measured in all cases. It can be measured as follows:

(a) Measure the magnetic field at the point in question when the current in the circuit or equipment is different from zero.
(b) Measure the magnetic field at the same point when the current in the circuit or equipment is zero, all other conditions being the same.
(c) Subtract the second measured value from the first.

3.3 Design for smallest stray magnetic field.

3.3.1 General. In order to design electric circuits and equipment with small stray magnetic fields, it is necessary to be able to estimate how much stray magnetic field will be produced by different circuits and equipment. Only when this can be done is it possible to make a rational choice of the arrangement that will give the smallest stray magnetic field.

3.3.2 The effect of a ferromagnetic material on a magnetic field.

3.3.2.1 The magnetic field of a ferromagnetic material. Ferromagnetic materials have a substantially higher relative magnetic permeability than nonferromagnetic materials. The induced magnetic field of a ferromagnetic material is proportional to the relative magnetic permeability of the material. Therefore, the higher the relative magnetic permeability of the material, the greater the induced magnetic field of this material will be.

3.3.2.2 The effects of ferromagnetic materials on the magnetic field of a conductor. The magnetic field created by the current in a conductor is shown on figure 1, with the current going into the page. If there is some ferrous material near to this conductor, the magnetic field of the conductor will be distorted. A magnetization will be induced into the ferromagnetic material, which is proportional to the current in the conductor and to the relative magnetic permeability of the ferromagnetic material. The magnetic field caused by the ferromagnetic material in the presence of a current carrying conductor is shown on figure 2. As figure 2 clearly indicates, the two fields tend to cancel each other above the conductor and tend to reinforce each other below the conductor. Bear in mind that the more magnetic field lines that are shown, the stronger the magnetic field will be.

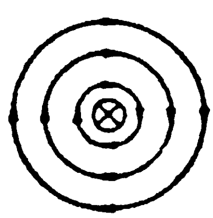


FIGURE 1. Magnetic field of a conductor

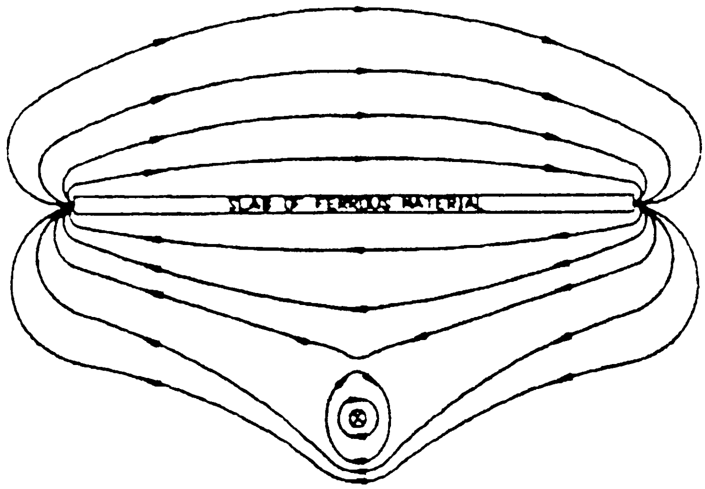


FIGURE 2. Distortion of a magnetic field by the introduction of a ferrous material.

3.3.2.3 The effect of ferromagnetic material on a solenoid. When a ferromagnetic material is introduced in a current loop, the resultant magnetic field will equal the magnetic field of the current loop (see figure 3) plus the magnetic. field of the material. In most cases, the magnetic field of a ferromagnetic material is much greater than the magnetic field of the current loop. Therefore, if we surround a magnetic material with multiple current loops, creating a solenoid, the magnetic material will be strongly magnetized by the solenoid, and the magnetic field will be very much greater than it would be if the magnetic material were removed (see figure 4).

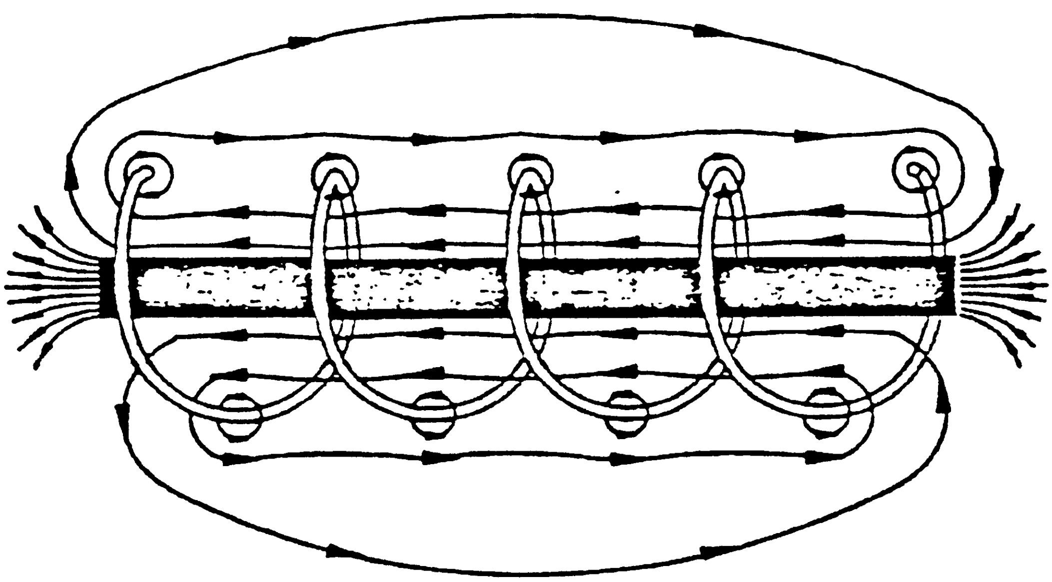


FIGURE 3. Effect of ferromagnetic material on a magnetic field.

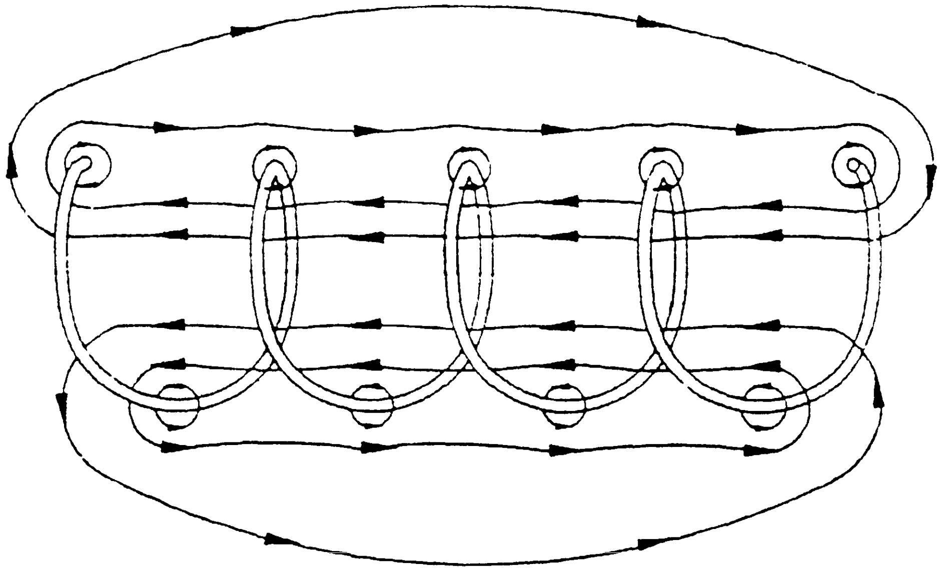


FIGURE 4. Magnetic field of a solenoid.

3.3.2.4 Avoid enclosure of magnetic material in current loops. If we consider a current loop around a bulkhead or a mast (see figure 5) made of magnetic material, we will find that the magnetic material will be strongly magnetized by the current, and the magnetic field strength will be very much greater than it would be if the magnetic material were removed. Situations such as those shown on figure 6 (A) must also be avoided. Here we have a bulkhead that is not enclosed by a current loop but is located between two current loops of the same polarity. By drawing hypothetical conductors at the ends of the current loops (see figure 6 (B)), we see that the bulkhead is, in effect, enclosed by two current loops, a small one and a large one. The small loop will produce a greater field at the bulkhead than the large loop, hence, the effect will be much the same as if the bulkhead were enclosed in a current loop.

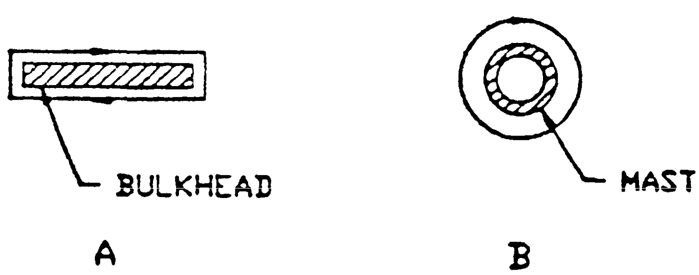


FIGURE 5. Current loop enclosing (A) a bulkhead and (B) a mast

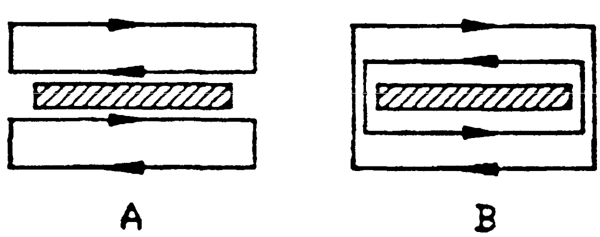


FIGURE 6. A bulkhead with two current loops (A) of the same polarity on either side of it and (B) completely enclosing it.

3.3.2.5 Magnetic material between two current loops of opposite polarity Now suppose we have a steel bulkhead symmetrically located between two equal and opposite current loops (see figure 7). An end view will look like the bulkhead shown in which 1, 2, 3, and 4 represent the conductors carrying current in the long direction of the current loops. The arrows diverging from point P represent roughly, in magnitude and direction, the magnetic fields that would be produced at this point by the currents in the four conductors if no magnetic material were present Note that the vertical component vanishes along the centerline of the bulkhead and that the horizontal component is not large. Magnetizing a bulkhead in the direction of its length or width, either of which can be considerable, will give rise to a substantial stray magnetic field. Magnetizing the bulkhead in the direction of its thickness, which is small, should not give rise to a very large magnetic field. We should expect, therefore, that magnetic material symmetrically placed between two equal and opposite current loops (see figure 7 (A)) will be less harmful than if it were between two current loops of the same polarity (see figure 6 (A)). Note, however, that there will be a vertical component if the steel bulkhead is not symmetrically located between the equal and opposite current loops but is closer to one than the other. For this reason, it is desirable that magnetic material be kept as far as possible from the current loops in the batteries or cable runs, and that in no case should it be disposed as shown on figures 6 (A) and 7 (A).

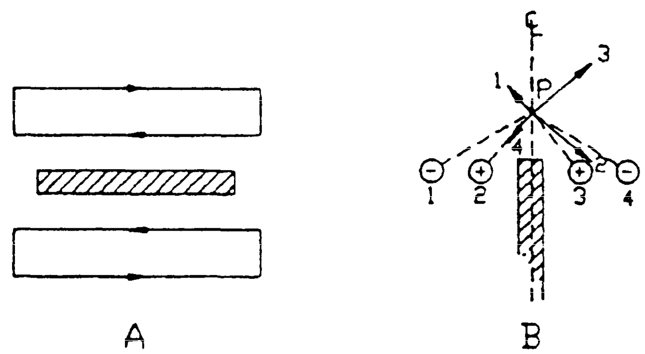


FIGURE 7. Steel bulkhead flanked by two current loops, (A) top view. (B) end view.

3.3.3 Shielding. Cable runs through a compartment will be a source of stray magnetic field during normal operation. In the optimum configuration to reduce stray magnetic field, cable runs will be configured with a number of cables per run that is a positive power of 2 (i.e. 2, 4, 8, ... cables). Of those cables half of them will carry the current in one direction and the remaining cables will carry current in the opposite direction. However, even where the optimum configuration for cable runs is employed, some stray field will occur. This occurs because of the fact that all of the cables in a run will not simultaneously be energized with the same magnitude of current. In these instances, we would consider shielding the stray field. This procedure involves enclosing the cable run in a structure made of ferromagnetic material. If we choose a material with a large relative permeability, then the magnetic signature of the enclosure would not change appreciably with the variation of the stray field within it.

3 .3.3.1 Shielding effectiveness. The shielding effectiveness of a ferromagnetic material is a function of the magnetic permeability of the material. We can determine the shielding effectiveness, in decibels (dB), from the following:

$$S = -20 \log_{10} \left| \frac{B}{B_o} \right| \qquad [3\text{-}1]$$

where:

S =- shielding effectiveness.

B = the magnetic field of the source inside the shielded volume measured at point P outside the shielded volume (see figure 8).

B_o = the magnetic field of the source at the point P if a shield were not present (see figure 8).

3.3,3.1.1 Using relative permeability to approximate shielding effectiveness. We know that relative magnetic permeability can be approximated from the magnetic fields as follows:

$$\mu_r = \frac{B_o}{B} \qquad [3\text{-}2]$$

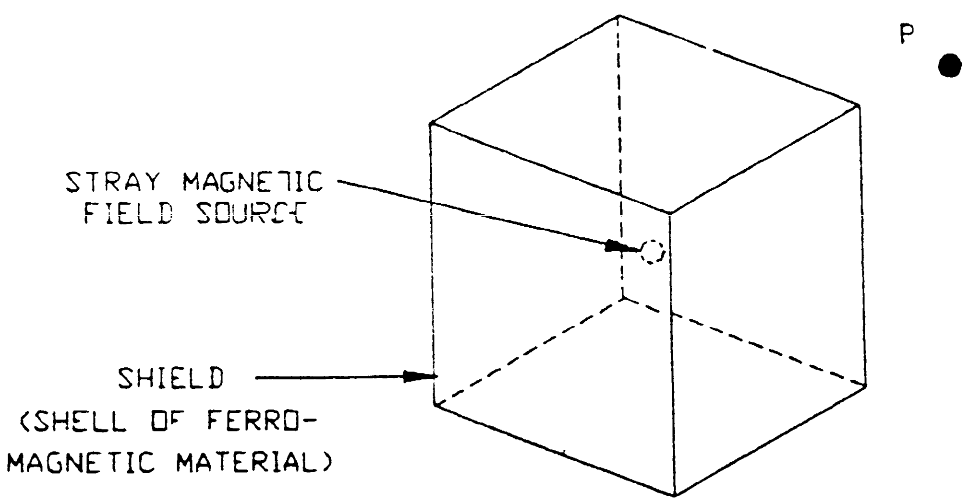


FIGURE 8. Shielding with ferromagnetic materials.

where.

μ_r = the relative permeability of the material.

Because of this relationship we can determine the approximate shielding effectiveness of a material simply by knowing its relative permeability. Therefore, the shielding effectiveness, in decibels, could be approximated by the following:

$$S = -20 \log_{10} \left| \frac{1}{\mu_r} \right| \qquad [3\text{-}3]$$

3.3.4 Calculating stray magnetic field. When iron and other magnetic materials are absent, the magnetic field produced by any arrangement of current-carrying conductors can be computed to any desired degree of accuracy by the expenditure of sufficient time and effort. Alternately, any arrangement of conductors can be replaced by one or more current loops that together produce the same magnetic field as the original arrangement of conductors. The advantage of this procedure is that the magnetic field produced by current loops can be calculated to an adequate degree of accuracy by reasonably simple formulas.

3.3.5 Theory of desire for small stray magnetic field. The theory of designing electric circuits and equipment with small stray magnetic fields is based upon calculating the magnetic fields produced by different combinations of current loops, and picking out the combinations that give a small stray magnetic field. In view of the importance of being able to calculate the magnetic field produced by current loops. this handbook is arranged as follows:

(a) Formulas for calculating the magnetic field produced by single current loops are contained in 4.2. Derivations are not given because they are not essential for use of the formulas.
(b) Formulas for computing the magnetic field produced by various combinations of two, four, six, or eight current loops of equal strength are contained in 4.3. Appendix A is included, however, to outline a method of deriving these formulas so that a user who needs a formula for a combination not covered in 4.3 can derive a formula to meet his needs.
(c) Section 5 is devoted to a discussion of general principles of designing electric circuits and equipment for small stray magnetic fields.
(d) Section 6 is devoted to the application of these principles to specific items of equipment.
(e) Appendix B provides a list of the equations used in the handbook and the pages on which they can be found.

3.3.6 Organization of handbook. Parts of this handbook will not be of equal interest to all readers. The designer of a disconnect switch will not be vitally interested in the material that deals with the design of generators, nor will the designer of generators be vitally interested in the material that deals with the design of disconnect switches. Nonetheless, it is highly desirable that each user of this handbook make himself generally familiar with its contents so that he will be able to pick and choose the material he needs to best advantage. Each user of the handbook should do the following:

(a) Thoroughly master sections 3, 4 (excluding 4.3), and 5.
(b) Be sufficiently familiar with paragraph 4.3 to know where to find the formulas he may need in the course of his work.

3.4 Illustration and measurement of stray magnetic field.

3 4.1 The effect of conductor shapes on stray magnetic fields.

3 4.1.1 Introduction. As stated earlier, a stray magnetic field is an unwanted field produced by a current source. In 3.4,1.2 and 3.4.1.3 we will review four conductor shapes that are the most basic sources of 2 stray magnetic field. The magnetic field due to these sources is dependent upon the following factors:

(a) Shape and orientation of the conductor.
(b) Magnitude and direction of the current.
(c) Radial distance from the source point to the point of interest of the magnetic field.

The mathematical formulas for the magnetic fields generated by current sources can be derived from the law of Biot-Savart (also known as Ampere's law for the current element). The actual derivation of the formulas is beyond the scope of this handbook. Therefore, for the four basic conductor shapes discussed in 3.4.1.2 and 3.4.1.3 only the final form of the formula (and not the derivation) will be given For this explanation we are concerned with the shape of the conductor in order to illustrate the variations in the magnetic fields.

3.4.1.1.1 Right-hand rule. The right-hand rule is a method of determining the direction of a vector after a cross product has been computed. In this handbook we will be concerned only with the direction of the resultant vector and not with the q ethod of computing the cross product. For a current loop the right-hand rule is used as follows:

(a) Curl the fingers of your right hand around the coil in the direction of the current.
(b) Your extended thumb will then point in the direction of the resultant vector.

3.4.1.2 Straight wire. First let us consider the most basic of all conductor shapes, a straight wire. We will see that there are two distinct expressions for the magnetic field produced by a straight wire. The difference between the expressions is based upon the effect of varying the length of the wire. We will consider the cases of a wire of infinite length (see 3.4.1.2.1) and a wire of finite length (see 3.4.1.2.2).

3.4.1.2.1 Infinitely long straight wire. A current in an infinitely long straight wire will produce a magnetic field which curls around the wire perpendicular to the direction of the current at all points in space. Thus, a current in the x direction will produce a magnetic field with y and z components, a current in the y direction will produce x and z components, and a current in the z direction will produce x and y components. As an example, we will assume that the infinitely long straight wire lies along the z axis (see figure 9), The formula for the magnitude of the magnetic field (B) of the wire at some point in space is as follows:

$$B = \mu_0 I / 2\pi r \qquad [3\text{-}4]$$

where:

μ_o = permeability of free space.
I = current.
r = distance perpendicular from the wire to the point in space.

Notice that the magnetic field is proportional to the current in the wire and is inversely proportional to the distance from the source to the point in space An increase in the current would cause an increase in the magnetic field and an increase in the distance from the source would cause a decrease in magnetic field

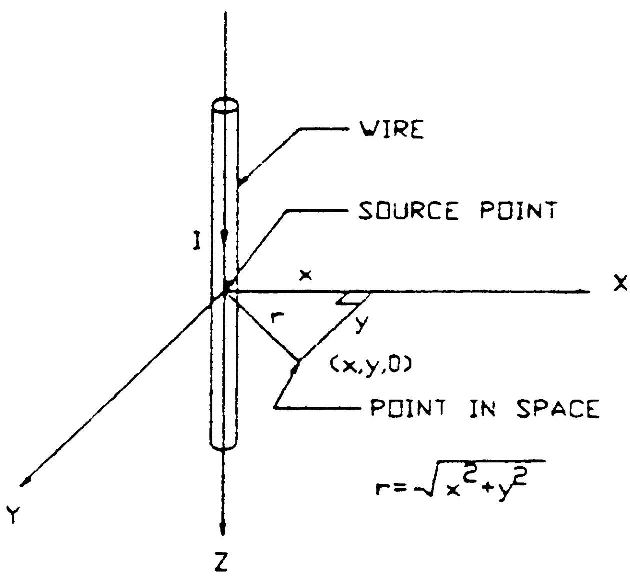


FIGURE 9. Infinitely long straight wire along the z axis.

3.4.1.2.2 Finite length straight wire. The previous subparagraph dealt with a magnetic field source that was a mathematical model rather than a physical model. We can have an infinitely long straight wire only in a mathematical sense. In practical applications we can use the formula for the infinitely long wire in cases where the ratio of r (the distance to the point at which the field is measured from the source) to the length of the wire approaches zero. Here, we will consider a finite length of straight wire along the z axis with the direction of the current in the positive z direction (see figure 10), a case where the radial distance to the length of the wire is not zero As in the infinite lenght case, the magnetic field produced by a finite length of wire will curl around the wire perpendicular to the direction of the current at all points in space. However, unlike the infinite length case, the magnetic field will be a function of the angles θ_1 and θ_2 (see figure 10). The formula for the magnitude of the magnetic field (B) measured at point (x, y, z) due to a finite length of wire is:

$$\vec{B}(x,y,z) = \frac{\mu_o I (x\vec{i} - y\vec{j})}{4\pi(x^2 + y^2)} (\sin\theta_2 - \sin\theta_1)$$

where:

μ_o = permeability of free space.
I = current.
$>$ = denotes a vector quantity,
$\vec{i}$, $\vec{j}$ = unit vectors in x and y directions, respectively.

$$\sin\theta_1 = \frac{z_1}{[x^2 + y^2 + z_1^2]^{1/2}} \qquad [3\text{-}6]$$

$$\sin\theta_2 = \frac{z_2}{[x^2 + y^2 + z_2^2]^{1/2}} \qquad [3\text{-}7]$$

3.4.1.3 Loops. Next we will see the effect on the stray magnetic field when we bend the wire into the two distinctive (but basic) loops. Again, the difference between the expressions is based upon the effect of two different geometric shapes, a rectangular (see 3.4.1.3.1) or a circular (see 3.4 1.3.2) loop of wire.

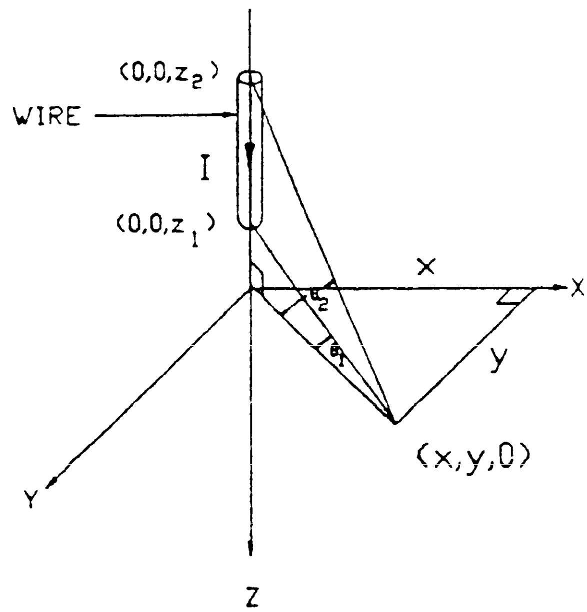


FIGURE 10. Finite straight wire along the z axis.

3.4.1.3.1 Square loop. Assume a square loop lies in the xy plan with each side of length L (see figure 11) (note that the case of a rectangular loop is addressed in 4.2.2). The magnetic field at the center of the square loop is equal to four times that caused by a single side. If we use the finite straight wire equation and set r = L/2, we obtain the following square loop equation for the magnitude of the magnetic field (B):

$$B = 2\sqrt{2}\mu_o \quad I/(\pi L) \qquad [3\text{-}8]$$

where:
μ_o = permeability of free space.
I = current.
L = length of wire segment per side of loop.

The direction of B and that of the current in the loop will follow the right hand rule (see 3.4.1.1.1). For the example giver. above with the current in the clockwise direction, the magnetic field will be in the positive z direction The example given above is for the simple case of a square loop with the field measured at the center of the loop, Rectangular loop equations for the field measured at any point in space are given in 4.2,2.

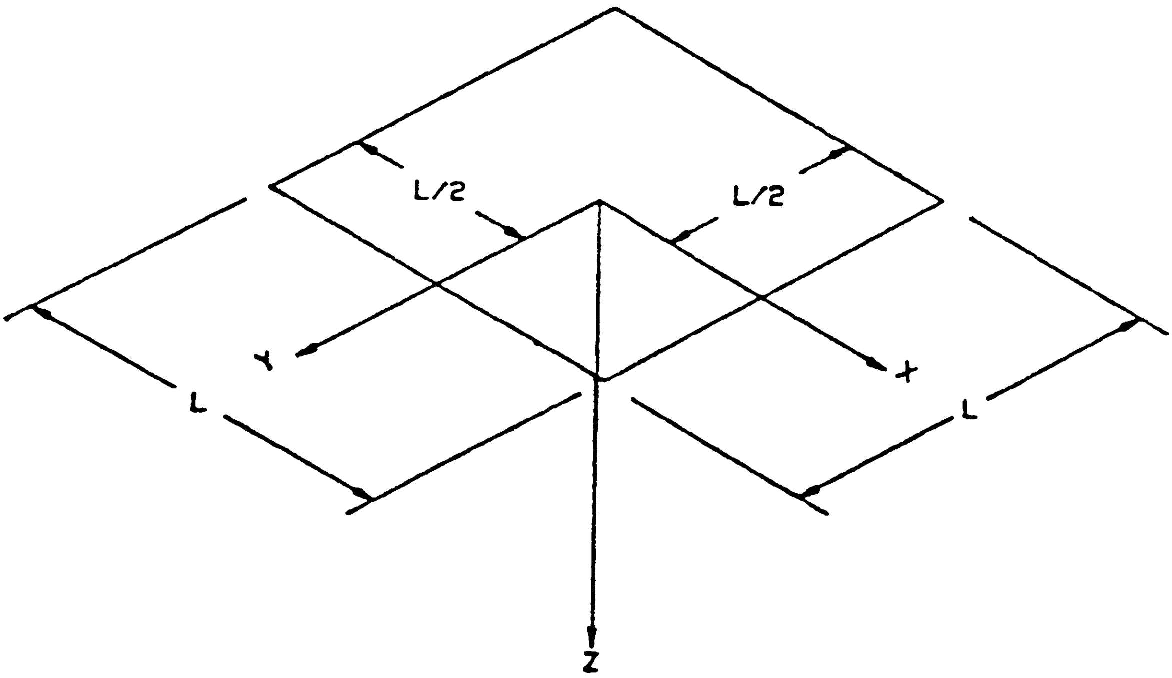


FIGURE 11. Rectangular current loop in the xy plane.

3.4.1.3.2 Circular loop. Assume the loop lies in the xy plane with its center at the origin (see figure 12). The magnetic field (B) measured at the origin for a loop of radius a, which as in the rectangular case represents the maximum value, is:

$$B = \mu_o I/2a \qquad [3\text{-}9]$$

and, the magnetic field at any point along the z axis is:

$$B = \mu_o I a^2 \left[2(z^2 + a^2)^{3/2}\right]^{-1} \qquad [3\text{-}10]$$

where:
μ_o = permeability of free space.
I = current.
a = radius of the loop.
z = length along the z axis.

The direction of B and that of the current in the loop will follow the right hand rule (see 3.4.1.1.1). As in the example of 3 .4.1.3.1, the magnetic field will be in the positive z direction if the current is in the clockwise direction.

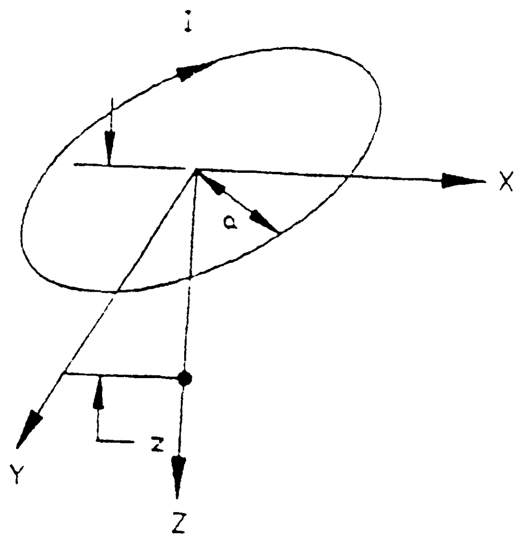


FIGURE 12. Circular current loop in the xy plane.

3.4.1.4 Summary. In each of the conductor shapes discussed there have been similarities in the expressions for the stray magnetic fields While the formulas are different, there are common elements:

(a) The strength of the magnetic field is proportional to the current. it increases as the current increases and decreases as the current decreases.

(b) The magnetic field strength is inversely proportional to the radial distance from the source.

3.4.1.5 Examples. The following examples of measurement will illustrate principles set forth in the preceding explanation.

3.4.1.5.1 A simple experiment. We can illustrate the stray magnetic field by use of a simple experiment. At a point far removed from magnetic materials and electric currents, we place a magnetometer that measures vertical, north-south, and east-west components of the magnetic field The initial readings of the magnetometer will give us the three components of the earth's magnetic field at the point where the magnetometer is placed. Next, we wind a coil of wire on a form and connect one terminal to a dry cell battery, making sure that all materials used in the wire, form, and battery are nonmagnetic. As long as only one terminal of the coil is connected to the battery, the current in the coil is zero and the magnetometer will show no change in the magnetic field even if the coil is brought close to the magnetometer. Now we connect both terminals of the coil to the battery so that current exists in the coil. The magnetometer will show that the magnetic field has changed. If this change is unwanted it is the stray magnetic field from the coil. If we increase the current, the stray magnetic field will increase. If we move the magnetometer around, we will find that the stray magnetic field is large near the coil, is smaller at greater distances, and is too small to measure at still greater distances. Now we place the magnetometer at a convenient distance from the coil, disconnect one terminal so that the current in the coil is zero, and place an iron core in the coil. The magnetometer shows that the magnetic field changes when the iron core is placed in the coil. The change is not, however, the stray magnetic field. It occurs because the iron has a permanent magnetization and is magnetized by induction due to its presence in the earth's magnetic field. The permanent and induced magnetizations produce a magnetic field around the iron. Leave the iron core in the coil and measure the change to the magnetic field that occurs when the battery is connected, thus applying current to the coil. This change is the stray magnetic field. Experiments made with and without the iron core in the coil will show that the stray magnetic field is different in the two cases. Other experiments with different coils and circuits will show that the stray magnetic field depends upon the following:

(a) The arrangement and number of current-carrying conductors.
(b) The magnitude of the current.
(c) The distance from the circuit or coil.
(d) Whether or not magnetic materials are present in the vicinity of the circuit or the magnetometer.

3.4.1.5.2 A large-scale experiment. Suppose that we place the magnetometer at the bottom of a ship channel. With no ships or other movable sources of magnetic field around, the magnetometer will read the earth's magnetic field at the bottom of the ship channel. We will call this magnetic field A. Next we bring up a ship having a generator aboard and moor it over the magnetometer. Suppose that initially the generator is not running and that no electric currents exist anywhere in the ship. The magnetic field at the bottom of the channel will be different from field A because of the permanent and induced magnetization in the iron of the generator and other magnetic material in the ship. We will call this magnetic field B. Next, we bring to the ship a source of electric current, which is arranged such that it has no stray magnetic field. We connect this source of current to degaussing coils that are arranged to produce a magnetic field that is, as nearly as possible, equal and opposite to B - A (B minus A). When the degaussing coils are energized, we will call this magnetic field C. Finally, with the degaussing coils still energized, we start the generator and apply current to it and its connected circuits and equipment. The stray magnetic field from the generator and its connected circuits and equipment will be superimposed upon the existing field C, and w1ll cause the resultant field to be different from C. We will call this magnetic field D. We thus have four magnetic fields: A for no ship, B for a ship with iron but no current (even in the degaussing coils), C for a ship with iron and current in the degaussing coils but nowhere else, and D for a ship with current in the degaussing coils and other electric circuits and equipment. The differences between these fields can be identified as follows:

(a) B - A is the undegaussed magnetic field of the ship.
(b) C - A is the degaussed magnetic field of the ship.

(c) D - C is the stray magnetic field of the ship.
(d) D - A is the total magnetic field of the ship.

3.5 How a magnetic mine works. A magnetic mine has a device to detect the change in the earth's magnetic field that occurs when a ship approaches the mine. If the change in the earth's magnetic field is large enough to actuate the detecting device, it fires the mine and destroys or damages the ship.

3.6 How a magnetic mine can be defeated. A magnetic mine will be defeated if there is no change in the magnetic field when a ship approaches a mine, that is, if A- C - D. Expressed somewhat differently, this condition is that the ship degaussed magnetic field, C - A, the ship stray magnetic field, D - C, and the ship total magnetic field, D - A, are all equal to zero. Where this is the case, the ship is "magnetically invisible" it cannot be detected by the mechanism in a magnetic mine, and therefore can pass close to the mine in safety. It is particularly important to make the magnetic field of a minesweeper as small as possible because minesweepers must venture into minefield and perhaps pass directly over magnetic mines before sweeping them. The safety of the minesweeper and of every man aboard depends upon how well the men who design and build the ship and its machinery do their part in making the ship magnetic field as small as possible.

3.7 How a ship magnetic field can be made equal to zero. Since a ship magnetic field is caused either by magnetic material or electric currents, or both, in principle it is very easy to make the magnetic field equal to zero. All that we need to do is the following:

(a) Use nothing but nonmagnetic materials in the construction of the ship.
(b) Make it impossible for electric currents to flow anywhere in the ship.

3.8 Practical ways of minimizing. ship magnetic fields. Many things that are very simple to do in principle are by no means simple to do in practice. Take, for example, a minesweeper that is to be used for sweeping magnetic mines. The two conditions that would make its magnetic field equal to zero are given in the preceding paragraphs, but we can use neither for the minesweeper. We cannot eliminate electric current because we need large currents for the minesweeper to do its job of sweeping magnetic mines. We cannot eliminate all magnetic materials because the most feasible way of obtaining the large currents needed is to make use of generators that require magnetic materials for their construction. We can, therefore, hardly hope to make the magnetic field of a minesweeper equal to zero. We can, nevertheless, make it very small if we approach this objective as follows:

(a) Use magnetic materials only where they are indispensable. The smaller the amount of magnetic materials, the more nearly field B is equal to field A, and the less the degaussing installation has to do.
(b) Put in degaussing coils to make the degaussed magnetic field, C - A, as nearly equal to zero as possible

(c) Design all electric equipment and circuits (except degaussing coils that are intentionally designed to produce a magnetic field) so that the stray magnetic field caused by current in the equipment and circuits is as small as possible. The total magnetic field of the ship, D - A, will then be as nearly equal to zero as we can make it.

When these things have been done, we have as close an approximation as we can attain to the desired condition, A - C - D, which protects a ship against detection by a magnetic mine.

4. MAGNETIC FIELDS PRODUCED BY CURRENT LOOPS

4.1 General information. In this section, we will introduce the concept of stray magnetic fields, specifically those produced by current loops. The stray magnetic field produced by a current loop is defined to be the change in magnetic field outside the loop that is caused by changing the current in the loop from zero to a value different from zero, other conditions remaining the same. We will investigate the magnetic field produced by simple current loops and give mathematical expressions that can be used to calculate the magnetic field at any point in space

4.1.1 Current loop. A current loop is a closed electric conductor. It may have one or more turns of any size or shape and be arranged in any way. A simple current loop is a closed conductor making one turn in a single plane, or alternately making a number of turns that are in the same plane or in parallel planes and so close together that, to a first approximation, they can be considered to be physically coincident in space. More complicated current loops can be resolved into a combination of simple current loops.

4.1.1.1 Coil. A coil is a particular type of simple current loop characterized by a relatively large number of turns wound closely together, such as the shunt field coil for a dc motor or generator.

4,1.2 Small and large current loops.

4.1.2.1 Example using small and large loop formulas. Formulas are given hereinafter for computing the magnetic field produced by small simple current loops of any shape and by large current loops of rectangular shape It is assumed in both cases that no magnetic materials are around, The large current loop formulas give correct values for the magnetic field at points both close to and far from the loop. The small current loop formulas give a good approximation to the correct values for points which are far from the loop, and a poor approximation to the correct values at points which are close to the loop. This is illustrated by the following example:

(a) Take a horizontal rectangular loop 1.0 meter wide and 2.0 meters long with its center at the origin of coordinates and its long axis parallel to the x axis. Suppose that the current is 1000 amperes.

(b) Now use the large loop formula (see equation 4-35) to compute the magnetic field at different points on a line parallel to and 0.5 meter below the x axis. The values computed by the large loop formula are shown in curve A of figure 13. They are equal (within the acceptable errors of measurement) to the values which would be measured by a magnetometer

(c) Now use the small loop (or dipole) formula (see equation 4-29) to compute the magnetic field at different points on the same line parallel to the x axis and 0.5 meter below it. The results computed by the small loop formula are shown in curve B of figure 13. It is immediately apparent in this case that the small loop (or dipole) formula gives a very poor approximation relative to the correct values shown in curve A.

(d) Now use both the large and small loop formulas to compute the vertical component of the magnetic field on a line parallel to the x axis but 10.0 meters below it. The values computed by the large and small loop formulas agree to within a few percent.

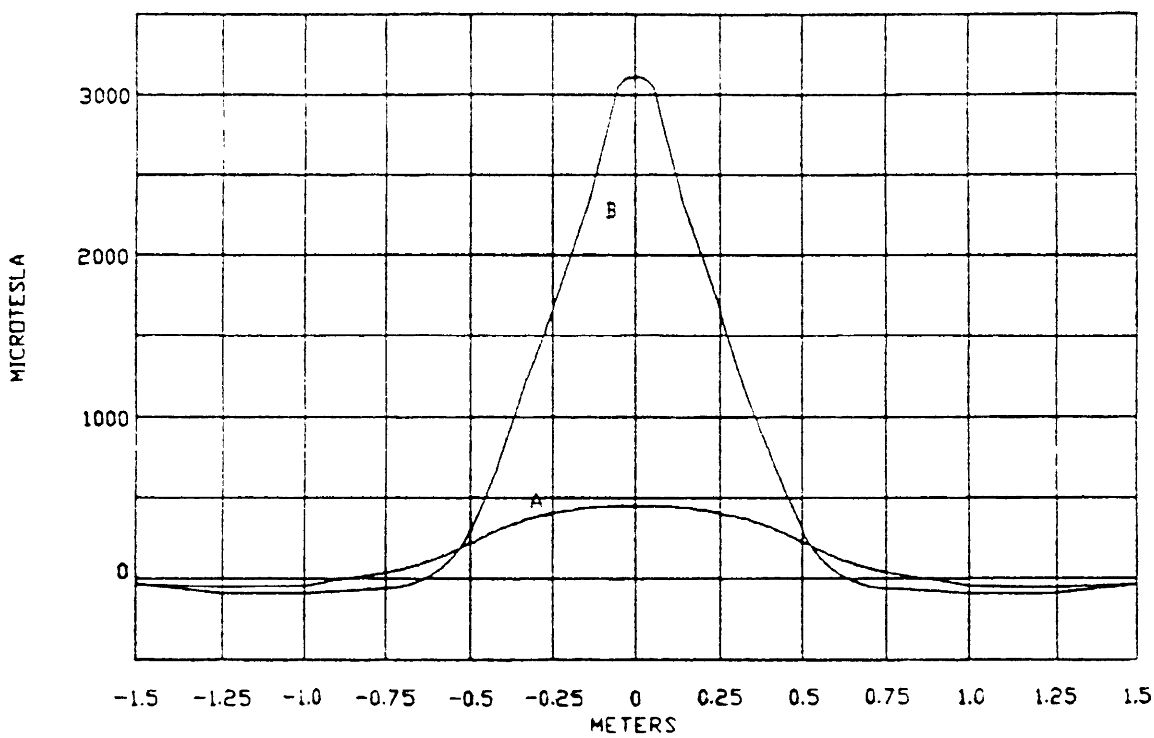


FIGURE 13 Comparison of large loop (A) and small loop (B) formulas.

4.1.2.2 Comparison of small and large loop formulas. As the above example illustrates, at 10.0 meters below a loop 2.0 meters long and 1.0 meter wide, the small loop formula gives a good approximation to the correct value: at 0.5 meter below the loop, it gives a very poor approximation. There is no sharp dividing line between small and large loops: however, the following example may be used as a guide. Suppose it is desired to calculate the magnetic field produced by a current loop at points on a plane z meters below the center of the loop, then the following would apply.

(a) If the greatest distance between any two points on the loop is less than z/10, the small loop formulas will give results correct to a degree of approximation which is usually adequate fcr stray field calculations . The smaller the loop, as compared to z, the better the approximation.

(b) If the greatest distance between any two points on the loop is more than z/10, the approximation will not be as good, becoming worse as the loop becomes larger. Note, however, that the example given above shows a good approximation even when the greatest distance between any two points on the loop is slightly more than z/5.

4.1.2.3 Small loop formulas. The small loop formulas are much easier to use than the large loop formulas and will give results as needed This is because, in almost all cases, the greatest distance between any two points on the loop is less than z/10 (usually considerably less). The formulas for large rectangular loops, given in 4.2.2, are included for the sake of completeness so as to be available in the rather unusual case of having to calculate the magnetic field close to a rectangular loop.

4.1.3 Coordinate axes. Throughout this handbook, a rectangular (Cartesian) coordinate system is used. The coordinate axes are as shown on figure 14

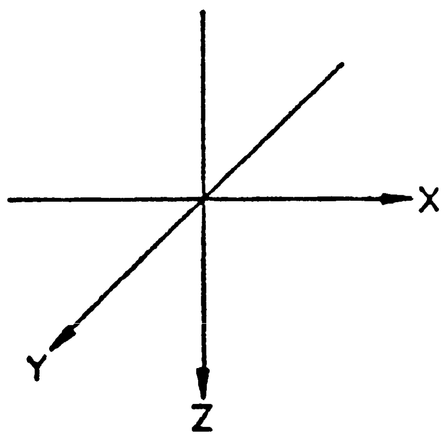


FIGURE 14. Coordinate axes.

4.1.4 Conventions on sign.

(a) A component of magnetic moment or of magnetic field along a particular coordinate axis is positive when in the same direction as the positive direction of the axis.

(b) A magnetic moment with its axis on a line passing through the origin 01 coordinates is positive if the positive direction of the moment is away from the origin.

4 1.5 Notation Throughout this section (except as otherwise indicated) the following notations will be used:

A = area enclosed by a coil or by a turn of a coil, in meters-squared.
a,b,c = the X, y, and z coordinates of a dipole, in meters. May also be used with subscripts as a_1, b_1, c_1 for coordinates of one dipole and a_2, b_2, c_2 for the coordinates of another.
B = magnetic field (magnetic flux density).
I = current, in amperes,

$$k^2 = \frac{x^2}{z^2} + \frac{y^2}{z^2} = \frac{x^2 + y^2}{z^2}$$

M = magnetic moment, in ampere meters-squared.
N = number of turns in a current loop or a coil.
r = radius, in meters, of a circle on which dipoles are arranged in a circular array.
x,y,z = x,y,z coordinates, in meters, of the point at which the magnetic field is computed.

4.2 Magnetic field produced by single current loop.

4.2.1 Small current loops

4.2.1.1 Samll current loops and magnetic dipoles. A bar magnet consists of a north pole and a south pole with the direction of positive magnetic flux (field) lines being from the south pole to the north pole (see figure 15 (A)). A magnetic dipole is the equivalent of an infinitely small current loop where one face of the loop behaves like the north pole of a bar magnet and the other face behaves like the south pole of a bar magnet. The magnetic field produced by a small simple current loop is practically independent of its shape and nearly equal to the magnetic field produced by a magnetic dipole that is situated at the right position, is oriented in the right direction, and has the right strength. The direction and intensity of field strength lines are dependent on the direction and amount of current, with the direction of the field in accordance with the right hand rule (see 3.4 1.1.1). Figure 15 (B) shows the field lines of a magnetic dipole model of a current loop where the current is in a clockwise direction. The dipole, insofar as producing the same magnetic field as a small current loop, will:

(a) Be located at the center of the loop.
(b) be oriented perpendicular to the plane of the loop with its positive direction in the direction of the magnetic field which the loop produces at its center.
(c) Have a magnetic moment that is produced in accordance with 4.2 2 1.

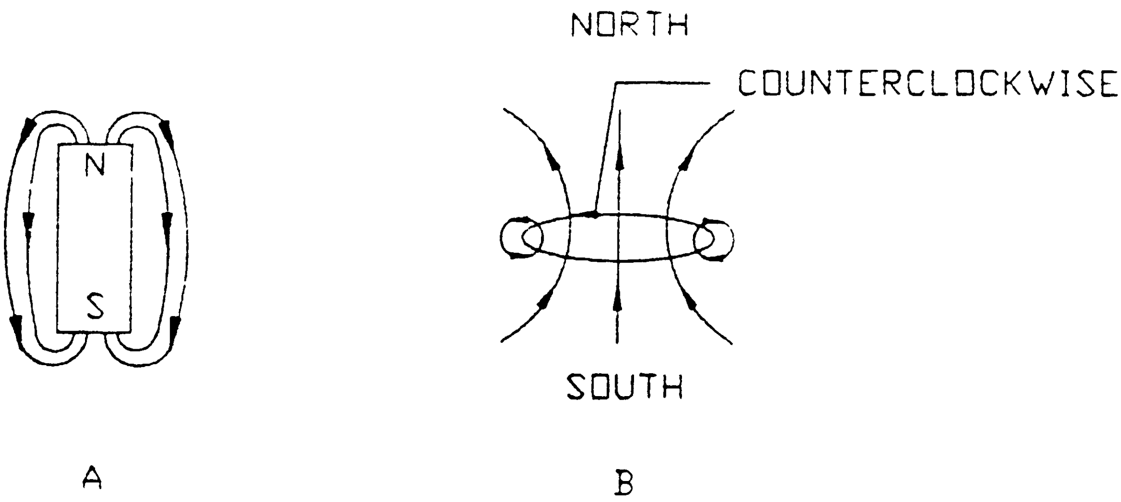


FIGURE 15, Magnetic dipole model.

4.2.1.1.1 Representations of current loops In the figures used later in this handbook, a current loop will be represented as follows:

(a) In some cases by drawings which show the conductors in the current loop; and
(b) In other cases by a small arrow for the dipole that is equivalent to the current loop insofar as producing the same magnetic field is concerned.

4.2 1.2 Computation of magnetic dipole moment The magnetic dipole moment is a vector whose magnitude is the product of the area enclosed by the loop, the number of turns in the loop, and the current in the loop. The direction of the vector, determined by the right hand rule, is normal to the plane of the loop.

4.2.1.2.1 Current loops of equal area. For current loops in which each turn encloses an equal area, the magnetic dipole moment is calculated as follows:

$$M = A N I \qquad [4\text{-}1)$$

where :
M = Magnetic moment of the current loop and its equivalent dipole in ampere meters squared (Am^2)
N = Number of turns.
A = Area enclosed by each turn in meters-squared.
I = Current in the loop.

4.2.1.2.2 Current loops of differing areas, For current loops with a small number of turns enclosing different areas, the magnetic moment is calculated as follows:

$$M = \sum_{i=1}^{N} A_i I_i = A_1 I_1 + A_2 I_2 + \ldots + A_N I_N \qquad [4\text{-}2]$$

where:

A_i = Area of loop.
I_i = Current in loop.

4.2.1.2.2.1 Example. If we have a current of 10 amperes, the magnetic moment of the four turn loop shown on figure 16 is:

$$M = (7m)(5m)(2)(10A) + (5m)(3m)(1)\ (10A) + (3m)(1m)(1)(10A)$$
$$M = 880\ Am^2$$

4.2.1.2.2.2 Magnetic moment of loops of different areas. For loops with a large number of turns enclosing different areas, the magnetic moment can be computed just as above but the amount of work involved will be considerable because of the large number of turns. An alternate procedure that involves less work and will give a good approximation is as follows:

(a) Divide the loop into a number of sections such that all turns in a section enclose approximately equal areas.
(b) Multiply the number of turns in each section by the current and average area enclosed by the turns in the section to obtain AND for the section.
(c) Add all of the values of ANI for the different sections to obtain the total magnetic moment for the coil.

4.2.1.2.3 Vector character of magnetic moment. As mentioned in 4.2.1.2, the magnetic moment is a vector having a magnitude equal to ANI and having a directional orientation. Given the orientation of the vector, it can be resolved into

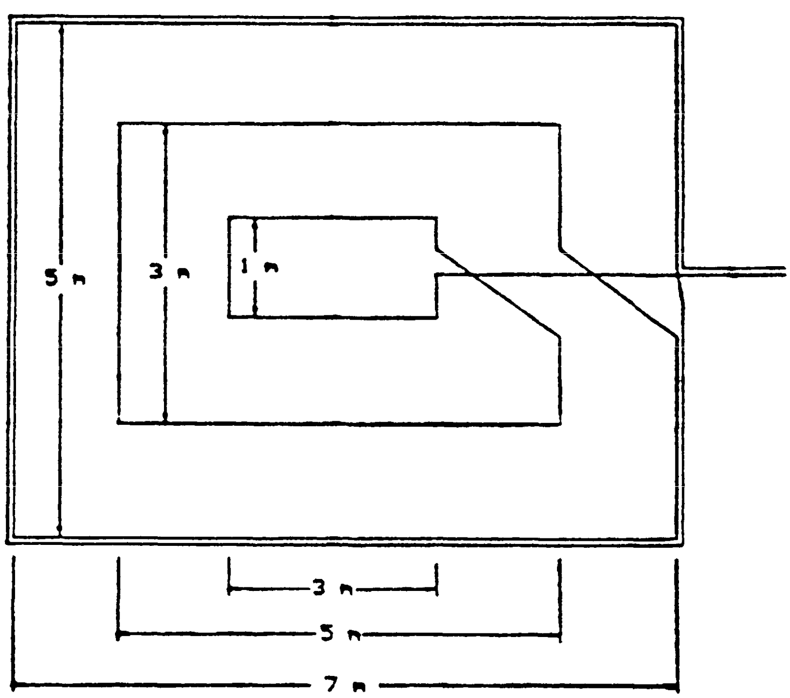


FIGURE 16. Four turn coil.

components on the coordinate axes. In the rectangular coordinate system, the vector can be resolved into x, y, and z components that will be designated as M_x, M_y, and M_z. Each component can be represented by a dipole parallel to that particular coordinate axis, that is, the M_x component could be represented by a dipole parallel to the x axis.

4.2 1.2.3.1 Vertical component The vector component of greatest importance in the design of minesweepers is the vertical (M_z) component. The reason for our interest in the vertical component is that if we design the electric circuits and equipment such that the vertical component of their stray magnetic field is small at all points on a horizontal reference plane below the equipment, then we can be sure that all three components of the stray magnetic field will be small at all points below the reference plane.

4.2.1.2.4 Magnet ic moment of an inclined current loop. The magnetic moment of an inclined current loop can be calculated as follows:

(a) Compute the magnetic moment of the equivalent dipole.
(b) Resolve the magnetic q oment into its M_x, M_y, and M_z components.
(c) Calculate the magnetic fields produced by the X, Y, and Z dipoles using the corresponding magnetic moment component from (b), and add the results. The sum will be the magnetic field produced by the inclined current loop. Note that the inclined current loop is first replaced by its equivalent dipole and that this is, in turn, replaced by X, Y, and Z dipoles. The reason for proceeding in this way is that it enables us to calculate the magnetic field from any small, simple current loop by making use of the formulas for X, Y, and Z dipoles. It is not necessary to have special formulas for inclined current loops.

4.2.1.2.5 Magnetic fields of noncoplanar current loops. Consider the noncoplanar current loop ABCDEF shown on figure 17 (A). Part of this loop is in a horizontal plane and part in a vertical plane. Imagine that the loop is opened at points C and F and that two hypothetical conductors are introduced between C and F to make two loops as shown on figure 17 (B), and that the same current exists in the horizontal loop ABCF and the vertical loop CDEF as in the original loop ABCDEF of figure 17 (A). The two hypothetical conductors between C and F of figure 17 (B) carry equal currents. Since they are purely hypothetical conductors, they can be imagined to be as close together as we wish, Their magnetic fields cancel completely if we assume them to be coincident; hence, the magnetic field produced by the two loops ABCF and CDEF will be exactly the same as that produced by the single loop ABCDEF. We can, "therefore, figure the fields of the two loops separately and add them to obtain the field of the original noncoplanar loop.

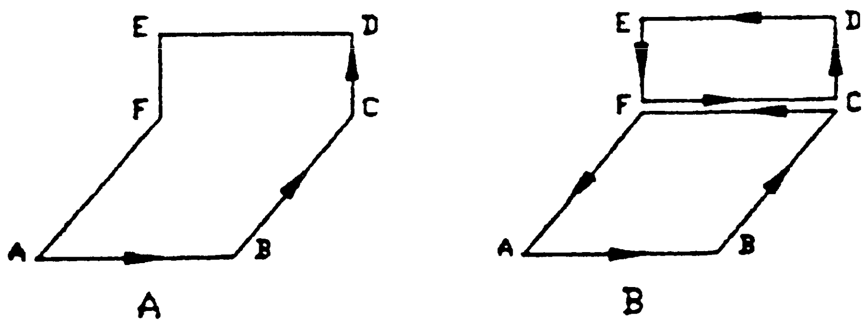


FIGURE 17. Noncoplanar current loops.

4.2.1.2.5.1 Splitting current loops into smaller loops. The same method of adding two hypothetical conductors that are assumed to be coincident in space, and that carry equal currents in opposite directions, can be used to split any current loop into two smaller loops. Each of these can, in turn, be split into two, and so on until the original loop is split into as many smaller loops as we desire By adding the magnetic fields produced by these, we obtain the magnetic field produced by the original loop.

4.2.1.3 Magnetic field contours. The vertical component of a magnetic field at different points on a horizontal reference plane can be represented graphically by a contour map. We can construct a contour map as follows: draw a curve (or curves) through all points of the reference plane at which the vertical component of magnetic field is zero; draw another curve (or curves) through all points where the vertical component of magnetic field is 1 microtesla; and, similarly, for points where the vertical component is 2, 3, 4 microtesla, and so on. Smaller intervals should be used when necessary. Note that two contours never intersect because the vertical component of magnetic field at any point has a single definite value; hence, only a single contour can go through the point. Note also that values of magnetic field which are greater than or equal to zero are shown as solid lines while those values of magnetic field which are less than zero are shown as dashed lines. Figures 18 through 20 are examples of contour maps. Figure 18 is for an X dipole (equivalent to a small current loop in the yz plane); figure 19 is for a Z dipole (equivalent to a small current loop in the xy plane); and figure 20 is for a dipole that makes an angle of 60 degrees with the z axis (equivalent to a small current loop in a plane that passes through the y axis and makes an angle of 60 degrees with the xy plane). In each case:

(a) The magnetic moment of the current loop and its equivalent dipole iis 2835 Am^2.
(b) The number alongside each contour gives the vertical component of magnetic field, in microtesla, that the current loop or its equivalent dipole produces on a plane 3 meters below the center of the loop or dipole.
(c) The positive direction of the vertical component is taken to be downward.
(d) The x and y coordinates are given in meters.

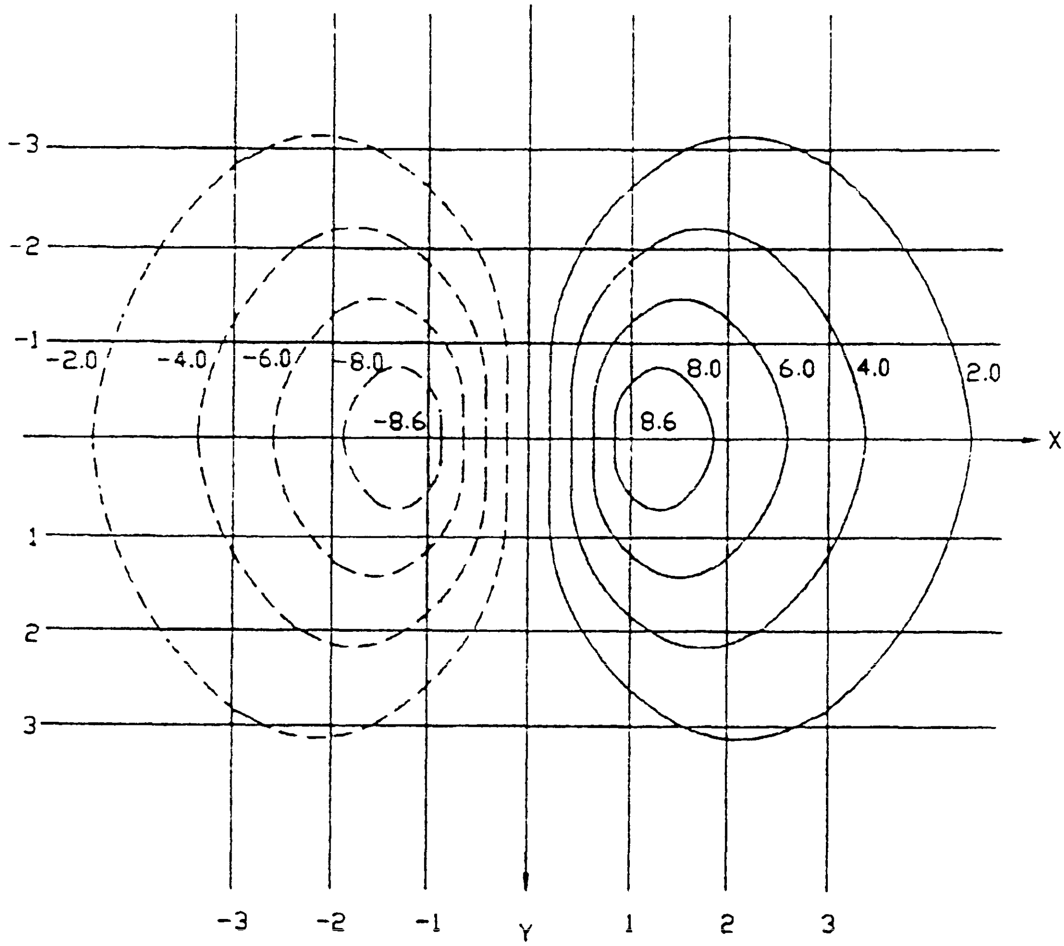


FIGURE 18. Contour map of the vertical component of the magneic field produced by an X dipole.

4.2.1.4 Magnetic field of a dipole.

4.2.1.4.1 Magnetic field of an X dipole. Consider a dipole centered at the origin of the coordinate system and aligned in the positive x direction. This is an X dipole. We will assign the magnetic moment of the X dipole the value of M. Because this is an X dipole it has only an M_x component.

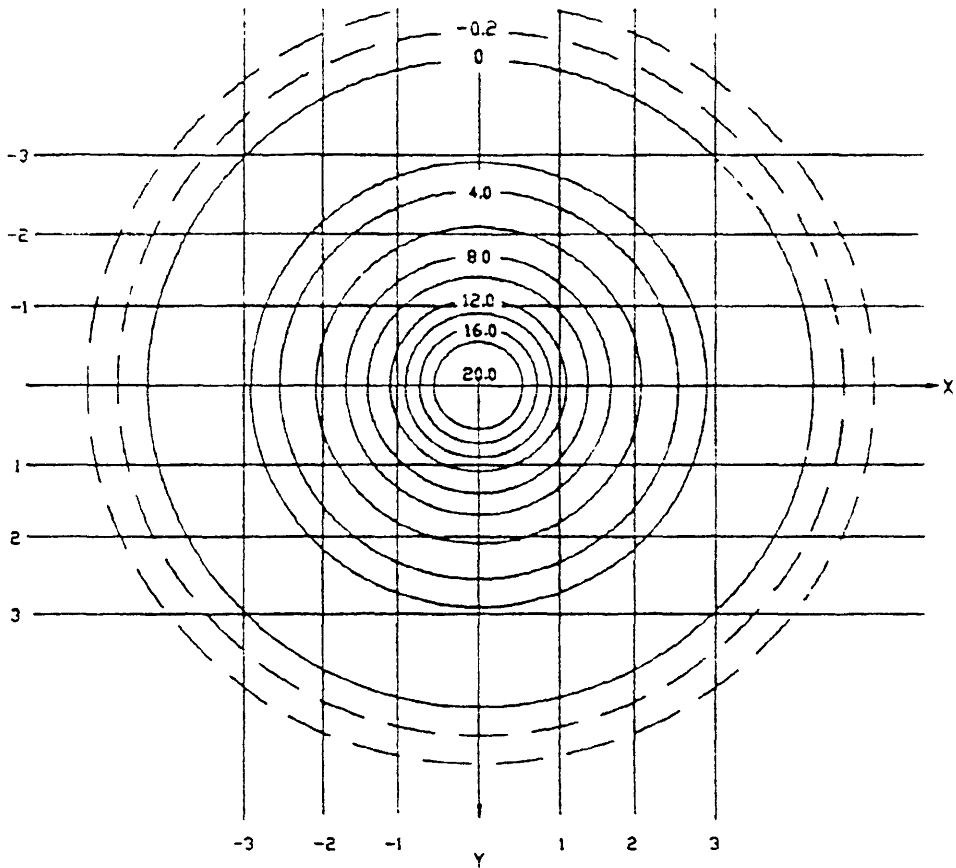


FIGURE 19. Contour map of the vertical component of the magnetic field produced by a Z dipole.

4.2.1.4 .1.1 The x component of the magnetic field of an X dipole. The X component of the magnetic field measured at point (x,y,z) produced by an X dipole is as follows:

$$B_x(x,y,z) = \frac{\mu_o M_x \ (2x^2 - y^2 - z^2)}{4\pi(x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}3]$$

where:

B_x = the x component of the magnetic field in tesla.
M_x = the x componet of the magnetic moment in ampere meters-squared.
μ_o = the permeability of free space, $4\pi \times 10^{-7}$ Henries/meter.

The maximum value occurs when

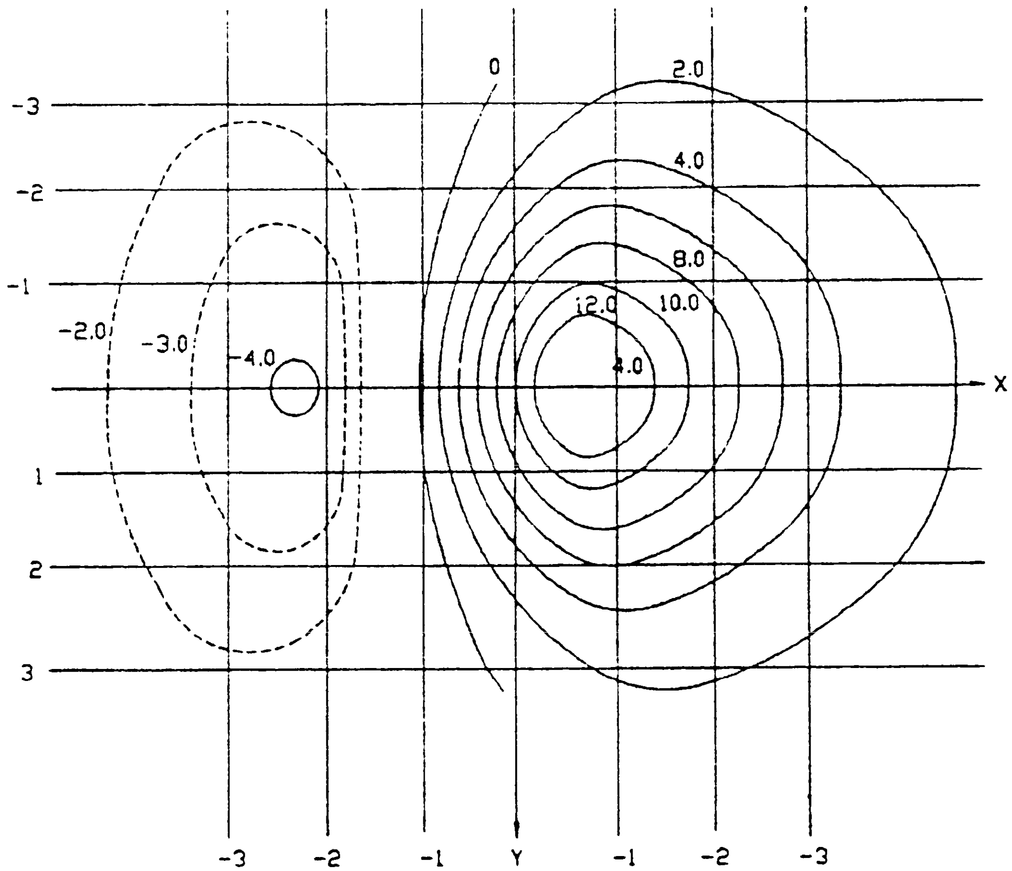


FIGURE 20. Contour map of the vertical component of the magnetic field produced by a dipole inclined 60 degrees to the z axis.

$$x = \pm\left[(3/2)(y^2 + z^2)\right]^{1/2} \qquad [4\text{-}4]$$

On the plane y = 0, the maximum value occurs when:

$$x = \pm\ (3/2)^{1/2} z \qquad [4\text{-}5]$$

On the plane z = 0, the maximum value occurs when:

$$x = \pm\ (3/2)^{1/2} y \qquad [4\text{-}6]$$

4.2.1.4.1.2 The v component of the magnetic field of an X dipole. The y component of the magnetic field measured at point (x,y,z) produced by an X dipole is as follows:

$$B_y(x,y,z) = \frac{\mu_o M_x\ 3xy}{4\pi(x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}7]$$

where:

B_y = the y component of the magnetic field in tesla.

The maximum value occurs when:

$$x = \pm (4y^2 - z^2)^{1/2} \qquad [4\text{-}8]$$

On the plane y = 0, the maximum occurs at complex roots.

On the plane z = 0, the maximum occurs when:

$$x = \pm 2y \qquad [4\text{-}9]$$

4.2.1.4.1.3 The z component of the magnetic field of an X dipole. The z component of the magnetic field at point (x,y,z) produced by an X dipole is as follows;

$$B_z(x,y,z) = \frac{\mu_o M_x \; 3xz}{4\pi \; (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}10]$$

where:
B_z = the z component of the magnetic field in tesla.

The maximum value occurs when:

$$x = \pm (4z^2 - y^2)^{1/2} \qquad [4\text{-}11]$$

On the plane y = 0, the maximum value occurs when:

$$x = \pm z/2 \qquad [4\text{-}12]$$

On the plane z = 0, the maximum occurs at complex roots.

4.2.1.4.1.4 Total magnetic field produced by an X dipole, To calculate the total magnetic field produced by an X dipole at point (x,y,y) we sum the contribution from each component. The total vector would be the M_x component multiplied by the x unit vector plus the M_y component multiplied by the y unit vector plus the M_z component multiplied by the z unit vector. Our concern will generally be with the vertical (z) component. Therefore, we will deal with components individually rather than collectively, and we will not compute the total magnetic field vector for the X, Y, or Z dipoles.

4 2.1 4.2 The magnetic field produced by a Y dipole Consider a dipole centered at the origin and aligned in the positive y direction. this is a Y dipole, As in the previous example, we will assign the dipole a value of M and will have only an M_y component.

4.2.1.4.2.1 The x component of the magnetic field of a Y dipole. The x component of the magnetic field measured at point (x,y,z) produced by a Y dipole is as follows:

$$B_x(x,y,z) = \frac{\mu_o M_y(-3xy)}{4\pi\ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}13]$$

The maximum value occurs when:

$$y = \pm\ (4x^2 - z^2)^{1/2} \qquad [4\text{-}14]$$

On the plane x = 0, the maximum occurs at complex roots.

On the plane z = 0, the maximum value occurs when:

$$y = \pm\ 2x \qquad [4\text{-}15]$$

4.2.1,4.2,2 The y component of the magnetic field of a Y dipole. The y component of the magnetic field measured at (x,y,z) produced by a Y dipole is as follows:

$$B_y(x,y,z) = \frac{\mu_o M_y(-x^2 + 2y^2 - z^2)}{4\pi\ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}16]$$

The maximum value occurs when:

$$y = \pm\ [(3/2)(x^2 + z^2)]^{1/2} \qquad [4\text{-}17]$$

On the plane x = 0, the maximum value occurs when:

$$y = \pm\ (3/2)^{1/2} z \qquad [4\text{-}18]$$

On the plane z = 0, the maximum value occurs when:

$$y = \pm\ (3/2)^{1/2} x \qquad [4\text{-}19]$$

4,2.1.4.2.3 The z component of the magnetic field of a Y dipole. The z component of the magnetic field measured at point (x,y,z) produced by a Y dipole is as follows:

$$B_z(x,y,z) = \frac{\mu_o M_y\ 3yz}{4\pi\ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}20]$$

The maximum value occurs when:

$$y = \pm\ (4z^2 - x^2)^{1/2} \qquad [4\text{-}21]$$

On the plane x = 0, the maximum value occurs when:

$$y = \pm 2z \qquad [4\text{-}22]$$

On the plane z = 0, the maximum occurs at complex roots.

4.2.1.4.3 The magnetic field Produced by a Z dipole. Consider a dipole centered at the origin and aligned in the positive z direction. This is a Z dipole . As in the two previous examples, we will assign the dipole a value of M and it will have only an M_z component.

4.2.1.4.3.1 The x component of the magnetic field of a Z dipole. The x component of the magnetic field measured at point (x,y,z) produced by a Z dipole is as follows

$$B_x(x,y,z) = \frac{\mu_o M_z \ 3xz}{4\pi \ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}23]$$

The maximum value occurs when

$$z = \pm (4x^2 - y^2)^{½} \qquad [4\text{-}24]$$

On the plane x = 0, the maximum occurs at complex roots

On the plane y = 0, the maximum occurs when:

$$z = \pm 2x \qquad [4\text{-}25]$$

4.2.1 4.3.2 The v component of the magnetic field of a Z dipole. The y component of the magnetic field measured at point (x,y,z) produced by a Z dipole is as follows:

$$B_y(x,y,z) = \frac{\mu_o M_z \ 3yz}{4\pi \ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}26]$$

The maximum value occurs when:

$$z = \pm (4y^2 - x^2)^{½} \qquad [4\text{-}27]$$

on tne plane x = 0, the maximum value occurs when.

$$z = \pm 2y \qquad [4\text{-}28]$$

On the plane y = 0, the maximum value occurs at complex roots.

4.2.1.4.3.3 The z component of the magnetic field of a Z dipole. The z component of the magnetic field measured at point (x,y,z) produced by a Z dipole is as follows.

$$B_z(x,y,z) = \frac{\mu_o M_z (x^2 + y^2 - 2z^2)}{4\pi \ (x^2 + y^2 + z^2)^{2.5}} \qquad [4\text{-}29]$$

The maximum value occurs when:

$$z = \pm [(3/2)(x^2 + y^2)]^{½} \qquad [4\text{-}30]$$

On the plane x = 0, the maximum value occurs when:

$$z = \pm (3/2)^{½} y \qquad [4\text{-}31]$$

On the plane y = 0, the maximum value occurs when:

$$z = \pm (3/2)^{½} x \qquad [4\text{-}32]$$

4.2.2 Large current loops.

4.2.2.1 General. Most of the current loops that we need to consider in stray field work are small compared to the distance at which it is desired to compute the stray magnetic field. In such cases. the small loop or dipole formulas can be used; however, the dipole formulas give highly incorrect values for large loops. Although it is seldom necessary to compute the magnetic field produced by a large loop, formulas for large rectangular current loops are given below to cover the rare case when it is necessary to compute the field that they produce. Note that the formulas are for large rectangular loops only, and do not hold for large loops of other shapes. It is assumed in both cases (large and small loops) that no magnetic materials are nearby. The large current loop formulas give correct values for the magnetic field at points both close to and far from the loop. The small current loop formulas give a good approximation to the correct values for points that are far from the loop, and a poor approximation to the correct values at points that are close to the loop. This is illustrated in the example of 4.1.2. Note that in the formulas that follow the permeability of free space (μ_o) and the factor of 4π are left out and that the magnetic field is measured in microtesla. This is because μ_o divided by 4π is 10^{-7} and 10^{-7} tesla is equal to 0.1 microtesla.

4.2.2.2 Horizontal rectangular loop. Consider a horizontal rectangular loop consisting of N-number of turns with its center at the origin of a rectangular coordinate system having the horizontal x and y axes parallel to the sides of the rectangular loop, as shown on figure 21. Let I be the current, in amperes; 2a and 2b the dimensions of the loop parallel to the x and y axes, respectively, in meters; and (x,y,z) the coordinates, in meters, of the point at which it is desired to measure the magnetic field (note that here a and b do not follow the notation of 4.1.5). The three components of the magnetic field, B_x, BY, and B_z, at point (x,y,z), in microtesla, are as follows:

$$B_x(x,y,z) = \frac{0.1\ IN\ z}{(a-x)^2 + z^2}\left[\frac{b+y}{[(a-x)^2 + (b+y)^2 + z^2]^{1/2}} + \frac{b-y}{[(a-x)^2 + (b-y)^2 + z^2]^{1/2}}\right] - \frac{0.1\ IN\ z}{(a+x)^2 + z^2}\left[\frac{b+y}{[(a+x)^2 + (b+y)^2 + z^2]^{1/2}} + \frac{b-y}{[(a+x)^2 + (b-y)^2 + z^2]^{1/2}}\right] \qquad [4-33]$$

$$B_y(x,y,z) = \frac{0.1\ IN\ z}{(b-y)^2 + z^2}\left[\frac{a+x}{[(a+x)^2 + (b-y)^2 + z^2]^{1/2}} + \frac{a-x}{[(a-x)^2 + (b-y)^2 + z^2]^{1/2}}\right] - \frac{0.1\ IN\ z}{(b+y)^2 + z^2}\left[\frac{a+x}{[(a+x)^2 + (b+y)^2 + z^2]^{1/2}} + \frac{a-x}{[(a-x)^2 + (b+y)^2 + z^2]^{1/2}}\right] \qquad [4-34]$$

$$B_z(x,y,z) = \frac{0.1\ IN\ (a-x)(b-y)}{[(a-x)^2 + (b-y)^2 + z^2]^{1/2}}\left[\frac{1}{(a-x)^2 + z^2} + \frac{1}{(b-y)^2 + z^2}\right] + \frac{0.1\ IN\ (a-x)(b+y)}{[(a-x)^2 + (b+y)^2 + z^2]^{1/2}}\left[\frac{1}{(a-x)^2 + z^2} + \frac{1}{b+y)^2 + z^2}\right]$$

$$+ \frac{0.1\ IN\ (a+x)(b-y)}{[(a+x)^2 + (b-y)^2 + z^2]^{½}} \left[\frac{1}{(a+x)^2 + z^2} + \frac{1}{(b-y)^2 + z^2} \right]$$

$$+ \frac{0.1\ IN\ (a+x)(b+y)}{[(a+x)^2 + (b+y)^2 + z^2]^{½}} \left[\frac{1}{(a+x)^2 + z^2} + \frac{1}{(b+y)^2 + z^2} \right]$$

[4-35]

4.2.2.2.1 Narrow horizontal rectangular loop. For a narrow horizontal rectangular loop, where b is small compared to both a and the distance from the center of the loop to the point where the field is to be computed, the equation in 4.2.2.2 becomes difficult to use because terms of nearly equal magnitude must be subtracted. We can use the formula for the vertical component of the magnetic field for this case, as follows:

$$B_x(x,y,z) = \frac{0.2\ IN\ z}{(a-x)^2 + z^2} \left[\frac{b}{[(a-x)^2 + y^2 + z^2]^{½}} \right]$$

$$- \frac{0.2\ IN\ z}{(a+x)^2 + z^2} \left[\frac{b}{[(a+x)^2 + y^2 + z^2]^{½}} \right]$$

[4-36]

$$B_y(x,y,z) = \frac{0.2\ IN\ (a+x)byz}{(y^2 + z^2)^2[(a+x)^2 + y^2 + z^2]^{½}} \left[\frac{(a+x)^2 + 3y^2 + 3z^2}{(a+x)^2 + y^2 + z^2} \right]$$

$$- \frac{0.2\ IN\ (a-x)byz}{(y^2 + z^2)^2[(a-x)^2 + y^2 + z^2]^{½}} \left[\frac{(a-x)^2 + 3y^2 + 3z^2}{(a-x)^2 + y^2 + z^2} \right]$$

[4-37]

$$B_z(x,y,z) - \frac{0.2\ IN\ (a+x)\ b}{(y^2 + z^2)^2[(a+x)^2 + y^2 + z^2)]^{½}} \left[2z^2 - y^2 - \frac{z^2(a+x)^2}{(a+x)^2 + y^2 + z^2} \right]$$

$$+ \frac{0.2\ IN\ (a-x)\ b}{(y^2 + z^2)^2[(a-x)^2 + y^2 + z^2)]^{½}} \left[2z^2 - y^2 - \frac{z^2(a-x)^2}{(a-x)^2 + y^2 + z^2} \right]$$

[4-38]

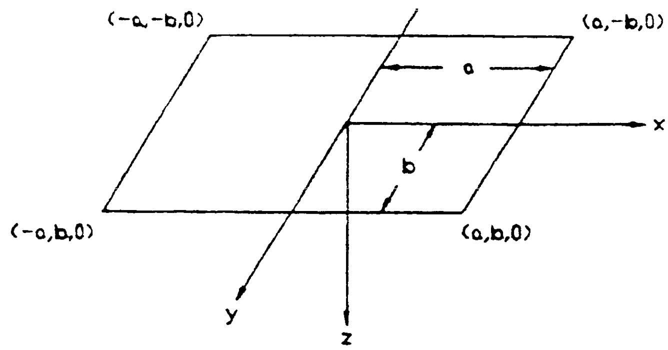


FIGURE 21. Horizontal rectangular loop.

4.2 2 3 Vertical rectangular loop Consider a N-turn vertical rectangular loop with its center at the origin of a rectangular coordinate system. Let 2c be the height of the loop in the direction of the vertical (z) axis, and 2b its length in the direction of the horizontal (y) axis (note that here b and c do not follow the notation of 4.1.5). Then the vertical component of magnetic field, in microtesla, for current of I amperes at a point (x,y,z), in meters, is as follows:

$$B_x(x,y,z) = \frac{0.1\ IN\ (b-y)(c-z)}{[x^2+(b-y)^2+(c-z)^2]^{1/2}}\left[\frac{1}{x^2+(b-y)^2}+\frac{1}{x^2+(c-z)^2}\right]$$

$$+\frac{0.1\ IN\ (b-y)(c+z)}{[x^2+(b-y)^2+(c+z)^2]^{1/2}}\left[\frac{1}{x^2+(b-y)^2}+\frac{1}{x^2+(c+z)^2}\right]$$

$$+\frac{0.1\ IN\ (b+y)(c-z)}{[x^2+(b+y)^2+(c-z)^2]^{1/2}}\left[\frac{1}{x^2+(b+y)^2}+\frac{1}{x^2+(c-z)^2}\right]$$

$$+\frac{0.1\ IN\ (b+y)(c+z)}{[x^2+(b+y)^2+(c+z)^2]^{1/2}}\left[\frac{1}{x^2+(b+y)^2}+\frac{1}{x^2+(c+z)^2}\right] \quad [4\text{-}39]$$

$$B_y(x,y,z) = \frac{0.1\ IN\ x}{x^2+(b-y)^2}\left[\frac{c+z}{[x^2+(b-y)^2+(c+z)^2]^{1/2}} + \frac{c-z}{[x^2+(b-y)^2+(c-z)^2]^{1/2}}\right]$$

$$- \frac{0.1\ IN\ x}{x^2 + (b+y)^2} \left[\frac{c+z}{[x^2 + (b+y)^2 + (c+z)^2]^{1/2}} + \frac{c-z}{[x^2 + (b+y)^2 + (c-z)^2]^{1/2}} \right] \quad [4\text{-}40]$$

$$B_z(x,y,z) = \frac{0.1\ IN\ x}{x^2 + (z-c)^2} \left[\frac{b+y}{[x^2 + (b+y)^2 + (c-z)^2]^{1/2}} + \frac{b-y}{[x^2 + (b-y)^2 + (c-z)^2]^{1/2}} \right]$$

$$- \frac{0.1\ IN\ x}{x^2 + (z+c)^2} \left[\frac{b+y}{[x^2 + (b+y)^2 + (c+z)^2]^{1/2}} + \frac{b-y}{[x^2 + (b-y)^2 + (c+z)^2]^{1/2}} \right] \quad [4\text{-}41]$$

4.2.2.3.1 Narrow vertical rectangular loop. For a narrow vertical rectangular loop, where c is small compared to both b and the distance from the center of the loop to the point at which the field is to be computed, the equation in 4.2.2.3 becomes difficult to use because terms of nearly equal magnitude must be subtracted. The formula that can be used for this case is as follows:

$$B_x(x,y,z) = \frac{0.2\ IN\ (b+y)\ c}{(x^2 + z^2)^2 [x^2 + (b+y)^2 + z^2]^{1/2}} \left[2x^2 - z^2 - \frac{x^2(b+y)^2}{x^2 + (b+y)^2 + z^2} \right]$$

$$+ \frac{0.2\ IN\ (b-y)\ c}{(x^2 + z^2)^2 [x^2 + (b-y)^2 + z^2]^{1/2}} \left[2x^2 - z^2 - \frac{x^2(b-y)^2}{x^2 + (b-y)^2 + z^2} \right] \quad [4\text{-}42]$$

$$B_y(x,y,z) = \frac{0.2\ IN\ x}{x^2 + (b-y)^2}\left[\frac{c}{x^2 + (b-y)^2 + z^2]^{\frac{1}{2}}}\right] - \frac{0.2\ IN\ x}{x^2 + (b+y)^2}\left[\frac{c}{[x^2 + (b+y)^2 + z^2]^{\frac{1}{2}}}\right] \qquad [4\text{-}43]$$

$$B_z(x,y,z) = \frac{0.2\ IN\ (b+y)\ cxz}{(x^2 + z^2)^2(x^2 + (b+y)^2 + z^2)^{\frac{1}{2}}}\left[\frac{3x^2 + (b+y)^2 + 3z^2}{x^2 + (b+y)^2 + z^2}\right] + \frac{0.2\ IN\ (b-y)\ cxz}{(x^2 + z^2)^2(x^2 + (b-y)^2 + z^2)^{\frac{1}{2}}}\left[\frac{3x^2 + (b-y)^2 + 3z^2}{x^2 + (b-y)^2 + z^2}\right] \qquad [4\text{-}44]$$

4.3 Magnetic field produced by two or ore small current loops.

4 3.1 General.

4.3.1.1 Principle of superposition. In a region that is free from magnetic materials, the stray magnetic field produced by a current. loop is precisely the same, regardless of whether the loop under consideration is the one and only loop or whether it is merely one of a multitude of loops that are all producing a magnetic field in the same region, Where two or more current loops are concerned, we can find the resultant field they produce at a point as follows:

(a) Calculate the magnetic field produced by each loop just as if it were the only loop involved.
(b) Add the magnetic fields produced by all of the loops to find the resultant field. Since the magnetic field is a vector quantity, this addition must be done vectorially. In other words, simply superimpose the magnetic field produced by the different loops.

4.3.1.1.1 Simple example of superposition. One of the simplest applications of the principle of superposition is furnished b) a two-turn loop in which the turns are very close together. This can be looked upon as being simply two single-turn loops of the same shape and size in the same position in space. The magnetic field at any point in space is found by adding the magnetic fields they produce, or, since each produces the same field for the same current, by doubling that produced by one loop. Similarly, the magnetic field produced by a loop with N turns of the same size and shape and in the same position in space is N times the magnetic field produced by one turn, This is the reason for the N factor in the expression ANI for magnetic moment of a current loop (see 4.2.1.2.1).

4.3.1.2 Principle of compensation. By the principle of compensation we shall mean the use of two or more adjacent current loops with magnetic moments of such magnitude and polarity that the resultant magnetic field produced by some Of the current loops is nearly equal and opposite to the resultant magnetic field produced by the remaining current loops. As a consequence, the magnetic field of some of the current loops compensates that of the others and the net or total magnetic field for the whole group can be substantially less than the magnetic field produced by one of the current loops alone.

4.3.1.2.1 Compensation with two current loops Two current loops furnish the simplest example of compensation. Suppose that the current loops are of exactly the same size and shape, are in exactly the same place, and carry exactly equal but opposite currents. In this idealized case, compensation is perfect and the resultant magnetic field is zero. Unfortunately , this is better than we can hope to realize in practice. We can, in principle at least, have two current loops of exactly the same size and shape carrying exactly equal and opposite currents , but we cannot have them in exactly the same place. They must be separated. If they are widely separated, there is no compensation and we have only two single current loops. If on the other hand, they are close together, the resultant magnetic field produced by the two loops is proportional to the magnetic moment of each times the distance between their centers To obtain the smallest possible stray magnetic field, the following conditions must be satisfied:

(a) The magnetic q oments of the current loops must be equal and opposite.
(b) The magnetic moment of each current loop must be as small as possible.
(c) The separation between the two loops must be as small as possible.

These conditions should always be satisfied when applying the principle of compensation.

4.3 1.2.2 Compensation with more than two current loops Consider a case with more than two current loops. Start out with a pair of two equal and opposite current loops, and then bring another pair close to it that produces a magnetic field opposite to that of the first pair. The resultant magnetic field of the four loops will be smaller than from either opposing pair alone even if the magnetic moments of all the individual current loops are equal. We can go still further and bring up another array of four loops that is opposed to the first. The magnetic field from the eight loops will be smaller than that from four. Obviously, this process can be continued as far as we like, doubling the number of current loops and decreasing the stray magnetic field at each step. In the interest of simplicity in the design and construction of equipment, this process should obviously be carried no further than is necessary to make the stray magnetic field as small as required. In many cases, one pair of two opposing current loops will be enough. We will, however, encounter four loops or two pairs in the folded type of disconnect switch boxes or contactor panels, and more than four loops in multipolar generators and motors.

4.3.1.2.3 Vector character of compensation. We have, up to now, tacitly assumed that the magnetic moments of the current loops under consideration were all parallel. Now, consider a number of current loops grouped close together without requiring that their magnetic moments be parallel. For compensation we must have:

$$\Sigma M_x = 0 \qquad [4\text{-}45]$$
$$\Sigma M_y = 0 \qquad [4\text{-}46]$$
$$\Sigma M_z = 0 \qquad [4\text{-}47]$$

where:

ΣM_x, ΣM_y, and ΣM_z = the sums of the x, y, and z components, respectively, of the magnetic moments of the current loop.

4.3.1.2.4 Example of compensation. What can be done by using the principle of compensation is shown in table I. This gives the maximum value of the vertical component of stray magnetic field on horizontal planes 6, 9, 12, and 15 meters below the centers of the following three different arrangements of current loops dipoles.

(a) A single X dipole, figure 22 (A), with M = 9300 ampere meters-squared.
(b) An opposing pair of X dipoles, figure 22 (B), each with M = 9300 ampere meters-squared and with a separation of 0.3 meter between centers.
(c) Four X dipoles, figure 22 (C), arranged at the corners of a horizontal square 0.3 meter on a side to make two pairs of opposing dipoles with M = 9300 ampere meters-squared for each dipole.

TABLE I. The effects of compensation on magnetic field.

Depth below dipoles, in meters	Maximum value of vertical component, in microtesla		
	One dipole 1/	Two dipoles 2/	Four dipoles 3/
6	3.52	0.615	0.0366
9	1.04	.121	.0048
12	0.44	038	.0011
15	.23	.016	.0004

1/ Computed from formula in equation 4-10 with x = z/2, y = 0.
2/ Computed from formula in equation 4-51 with x = 0, y = 0.
3/ Computed from formula of equation 4-78 with x = 0, y = ± 0.408z.

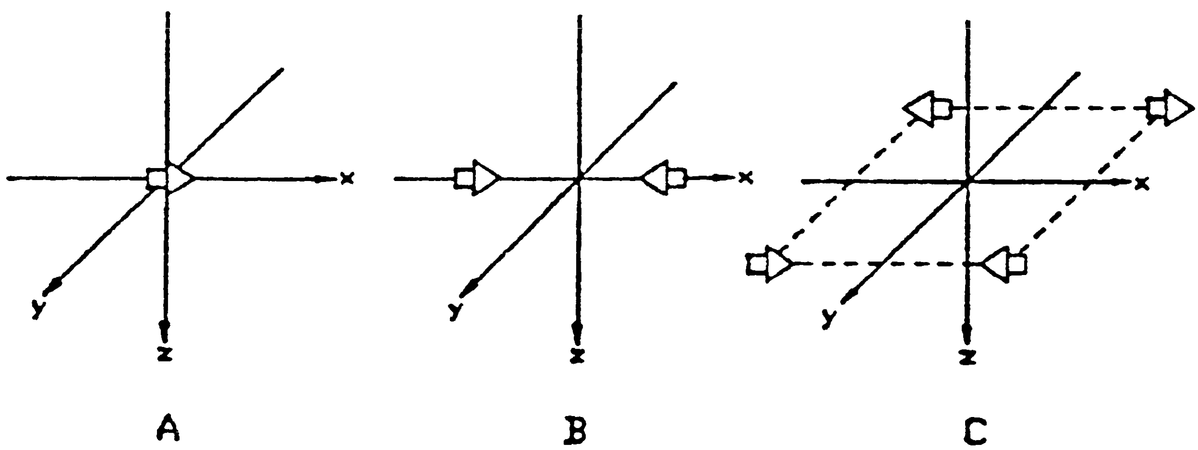


FIGURE 22. Three different dipole arrangements.

The preceding table shows clearly the decrease in stray magnetic field that is obtained by going from a single uncompensated current loop to an opposing pair, and the still greater decrease that is obtained by going from one opposing pair to two opposing pairs. Note also that the decrease is greater with increasing depth, as shown in table II.

TABLE II. The effects of increasing depth on magnetic field.

Depth below dipoles, in meters	Relative values of magnetic field		
	One dipole	Two dipoles	Four dipoles
6	100	17.5	1.04
9	100	11.6	0.46
12	100	8.7	0.25
15	100	7.0	0.17

4.3.2 General information on formulas.

4.3.2.1 Convenience. In order to estimate the stray magnetic field from an array of current loops, we must be able to calculate the magnetic field produced by several current loops This can always be done by calculating the magnetic field produced by each loop individually and adding the results. Such a process is slow and time consuming, however, and furnishes no immediate answer to the two questions of primary interest, namely, the position of the point or points where the vertical component of magnetic field has its maximum value, and the magnitude of the maximum value. Rather than calculating the magnetic field individually for each dipole in a group, it is more convenient to use the approximate formulas given in 4.3,3 when they are applicable. Note, however, that they are limited to symmetrically arrays of current loops or dipoles which all have magnetic moments of equal magnitude. When the dipoles are not arranged symmetrically or are not of equal strength, the formulas do not apply and it will be necessary to estimate the magnetic field in other ways.

4.3.2.2 Validity. The formulas given in 4.3.3 are valid only when iron and other magnetic material are absent.

4.3.2.3 Accuracy. The formulas given in 4.3.3 are approximations based upon the assumption that each of the current loops is small enough to be replaced by its equivalent dipole. This means that the maximum distance between any two points on the largest loops is less than z/10 where z is the distance from the center of the array of loops to the horizontal plane on which it is desired to compute the vertical component of magnetic field. Checks made for a number of cases indicate that the results given by the formulas for the maximum value of the vertical component on a plane at a distance z below the center of the array will be accurate, when the above conditions are satisfied to within a few percent. The percentage accuracy will be poor at points where the vertical component is nearly zero, but as we are primarily interested in the maximum values, this is of little importance.

4.3.2.4 Formulas for dipole arrays. The information provided with the formulas in 4.3.3 will include:

(a) A sketch showing the array of dipoles considered and a table listing the magnetic q oment and the rectangular coordinates on the center of each dipole in the array. In the sketch, each current loop is represented by a small arrow which gives the position and direction of the equivalent dipole.
(b) Formulas giving the components of magnetic field produced by the array of dipoles at a point with rectangular coordinates x, y, and z. By holding z constant and varying x and y, the vertical component can be computed at all points on a plane at a distance z below the center of the array. A similar procedure can be used to calculate the x and y components.
(c) The values of x and y at the points where the vertical component of the magnetic field reaches its greatest magnitude on a plane at distance z below the center of the array. In many cases, the vertical component of magnetic field will have a number of maxima or minima (in the mathematical sense) of different magnitudes. The values given for x and y are for the points where the vertical component of magnetic field is greater than any of the others.
(d) A formula giving the maximum value of the vertical component of magnetic field on a plane at distance z below the center of the array.

4.3.2.5 Contour maps of dipole arrays.

4.3.2.5.1 Two dipole arrays. A contour map showing the vertical component of the magnetic field, in microtesla, on a plane 3 meters below the center of a specific array is provided for each array with only two dipoles (see 4.3.3.1 through 4.3.3.5). The microtesla values on the contour map are for dipole arrays with M = 2835 Am^2, a - 0.3 m, and z = 3.0 m.

4.3.2.5.2 Arrays with more than two dipoles. A partial contour map showing the following is provided for each array with more than two dipoles:

(a) The contour lines for zero vertical component of magnetic field on a plane at a distance z meters below the center of the array.
(b) The regions on this plane where the vertical component of magnetic field is positive (shaded) and the regions where it is negative (unshaded).
(c) The points where the vertical component of the magnetic field has its greatest magnitude on the plane at distance z below the center of the array. These points are indicated by crosses.

4.3.2.5.3 Computation of contour maps. For the arrays with four or more dipoles, the partial contours shown were constructed from the approximate formulas in 4.3.3.6 through 4.3.3.26. For the arrays with two dipoles, the magnetic field produced by each dipole was calculated separately and the contour map was constructed to show the algebraic sum of the two fields. This procedure gives rigorously correct results given by the approximate formula for the magnetic field produced by two dipoles. For this reason, the maximum values of the stray magnetic field calculated from the formulas will not agree exactly with the maximum values shown on the contours for arrangements with two dipoles. For the same reason, the contours shown for the arrays of 4.3.3.3 and 4.3.3.4 are not exactly alike even though the approximate formulas for the two arrays are exactly the same. It should also be noted that the difference between the exact and approximate results are fairly small even though z, the distance from the center of the array to the plane of the contour, is only five times the distance between the two dipoles. For larger values of z, the exact and approximate values will agree even more closely.

4.3.2.6 Dimensions. In all formulas given in 4.3.3, the following will apply:

(a) Components of magnetic field are in microtesla.
(b) Magnetic moments are in ampere meters-squared,
(c) Distances are in meters.
(d)

$$k^2 = \frac{x^2}{z^2} + \frac{y^2}{z^2} \qquad [4\text{-}48]$$

4.3.2.7 Y dipoles. Formulas are given for X dipoles and Z dipoles but not for Y dipoles. The reason is that Y dipoles can be changed to X dipoles by rotating the coordinate axes so that separate formulas are not needed. For a similar reason, formulas are not given for Z dipoles on the y axis since this case can be reduced to Z dipoles on the x axis by rotation of the coordinate axes. There are a number of other cases in which two different arrays can be changed from one to the other by a rotation of the coordinate axes about the z axis. In all such cases, a formula is needed and given for only one array.

4.3.2.8 Radial dipole arrays. Formulas are given in 4.3.3.18 through 4.3.3.26 for the magnetic field produced by circular arrays of four, six, and eight dipoles equally spaced on a circle of radius r in the xz or xv plane. The magnetic moments are along the radii drawn from the origin of the coordinate system and are alternately plus and minus; plus for a moment directed away from the origin and minus for a moment directed toward the origin. It will be recognized that these dipoles represent in spatial arrangement and magnetic polarity the field coils of four-, six-, and eight-pole dc generators or motors. For the formulas given in 4.3.3.18 through 4.3.3.23, the axes of the machines are horizontal (along the y axis) For the formulas given in 4.3.3.24 through 4.3.3.26, the axes of the machines are vertical (along the z axis). Two cases are considered for each number of dipoles:

(a) Case 1. The dipoles are oriented so that the z axis (see 4.3 3 18, 4.3.3.20 and 4.3.3.22) or the y axis (see 4.3.3.24 and 4.3.3.26) is in line with one pair of diametrically opposite dipoles.

(b) Case 2. The dipoles are oriented so that the z axis (see 4.3.3.19, 4.3.3.21 and 4.3.3.23) or the y axis (see 4.3.3.25) bisects the angle between two adjacent dipoles.

4.3.3 Dipole configurations. Formulas and contour maps for common dipole configurations are given in 4.3.3.1 through 4.3.3.26. Parallel dipole configurations are covered in 4.3.3.1 through 4.3.3.17. Circular dipole configurations are covered in 4.3.3.18 through 4.3.3.26.

4.3.3.1 Two X dipoles on the x axis. Figure 23 shows the configuration and contour map for the magnetic field of two X dipoles on the x axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the two dipoles are also provided.

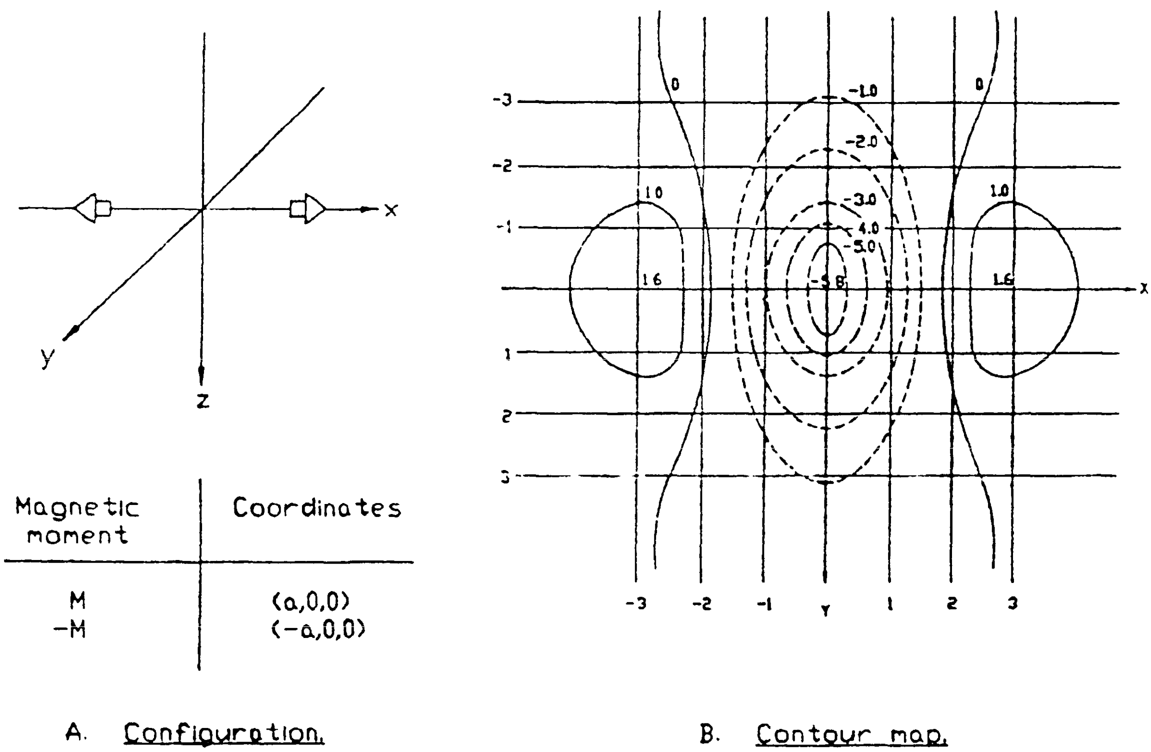


FIGURE 23. Two X dipoles on the x axis.

$B_x = -0.2Maz^{-4}(-6x^3z^{-3} + 9xy^2z^{-3} + 9xz^{-1})(1 + k^2)^{-3.5}$ [4-49]		
Maximum value = $0.220Maz^{-4}$	Maxima occur at:	$x = \pm 0.131z$ $y = 0$
$B_y = -0.6Maz^{-4}(-4x^2yz^{-3} + y^3z^{-3} + yz^{-1})(1 + k^2)^{-3.5}$ [4-50]		
Maximum value = $0.172Maz^{-4}$	Maxima occur at:	$x = 0$ $y = \pm 0.500z$
$B_z = -0.6Maz^{-4}(1 - 4x^2z^{-2} + y^2z^2)(1 + k^2)^{-3.5}$ [4-51]		
Maximum value = $0.6Maz^{-4}$	Maxima occur at:	$x = 0$ $y = 0$

FIGURE 23. Two X dipoles on the x axis. - Continued

4.3.3.1.1 Example of the use of parallel dipole configuration equations. In the example that follows, we shall use equation 4-51 to illustrate a method of calculating the magnetic field due to an array of parallel dipoles. The method consists of the steps that follow.

(a) First, we q ust identify the number of coils for which we want to calculate the magnetic field and identify the type of dipole (X, Y, or Z) represented by each. For our example, we shall consider the case of two X dipoles along the x axis Remember:

(1) A coil in the yz plane is represented by an X dipole.
(2) A coil in the xz plane is represented by a Y dipole.
(3) A coil in the xy plane is represented by a Z dipole.

(b) Second, we determine the distance between each dipole and the origin of the coordinate axes. In our example, both dipoles are at a distance "a".

(c) Third, we determine the magnetic moment (M) of each dipole. We use equation 4-1 (M - ANI) to determine the magnitude of M. Next, we determine the direction of M (+ or -) by using the right hand rule (see 3 4.1.1.1): curling the fingers of the right hand around the loop in the direction of the current, the extended thumb points in the direction of M. If that direction points in a positive direction with respect to the coordinate axes, M is positive. If that direction is negative, M is negative. In our example, one is positive and one is negative.

(d) Next, we make a sketch of the dipole configuration and compare it to those of 4.3.3.1 through 4.3.3.17 to find a match. If a partial match exists; for example, if our configuration consists of two X dipoles along tbe x axis and also two X dipoles along the y axis, we would use superposition to determine the total field. If there is no match or no partial match between the configuration we seek and those of the handbook, then we must use appendix A to derive our equations. For our example, we use the equations of 4.3.3,1, specifically equation 4-51 since we are interested in the z component of magnetic field. Equation 4-51 is as follows:

$$B_z = -0.6\ Maz^{-4}(1 - 4x^2z^{-2} + y^2z^{-2})(1 + k^2)^{-3\ 5}$$

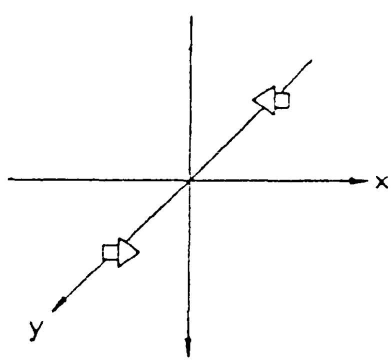
For A = 2 0 m^2, N = 5 turns, I = 900 A, a = 1.0 m, x = 0, y = 0, z = 10.0 m, the z component of magnetic field is:

$$B_z = -0.6\ ANI\ az^{-4}(1 - 0 - 0)(1 + 0 + 0)^{-3\ 5}$$

$$B_z = (-0.6)(2.0)(5)(900)(1.0)(10.0)^{-4} = -0.54\ \mu T$$

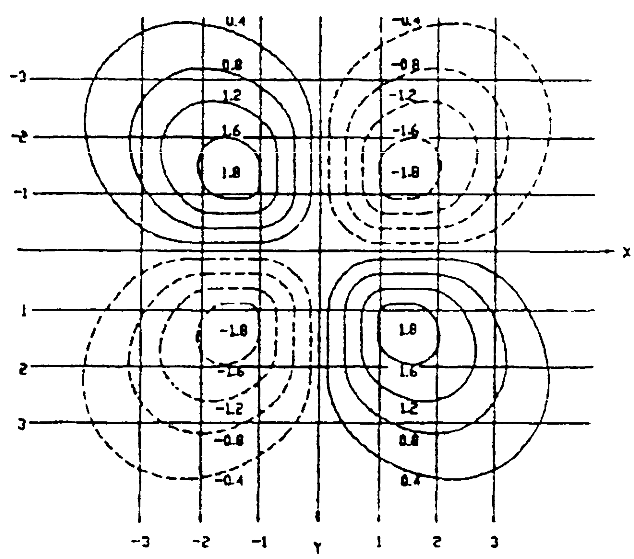
These steps can be followed to calculate the magnetic field due to any parallel dipole configuration.

4,3.3.2 Two X dipoles on the y axis. Figure 24 shows the configuration and contour map for the magnetic field of two X dipoles on the y axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the two dipoles are also provided,



Magnetic moment	Coordinates
M	(0,b,0)
-M	(0,-b,0)

A. Configuration.



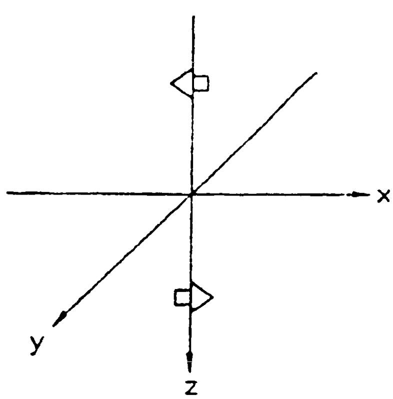
B. Contour map.

FIGURE 24. Two X dipoles on the y axis.

$B_x = -0.6Mbz^{-4}\,(y^3z^{-3} - 4x^2yz^{-3} + yz^{-1})(1 + k^2)^{-3.5}$		[4-52]
Maximum value = $0.172Mbz^{-4}$	Maxima occur at: $x = 0$ $y = \pm\,0.500z$	
$B_y = -0.6Mbz^{-4}(x^3z^{-3} - 4xy^2z^{-3} + xz^{-1})(1 + k^2)^{-3.5}$		[4-53]
Maximum value = $0.172Mbz^{-4}$	Maxima occur at: $x = \pm\,0.500z$ $y = 0$	
$B_z = 3Mbxyz^{-6}(1 + k^2)^{-3.5}$		[4-54]
Maximum value = $0.185Mbz^{-4}$	Maxima occur at: $x = \pm\,0.447z$ $y = \pm\,0.447z$	

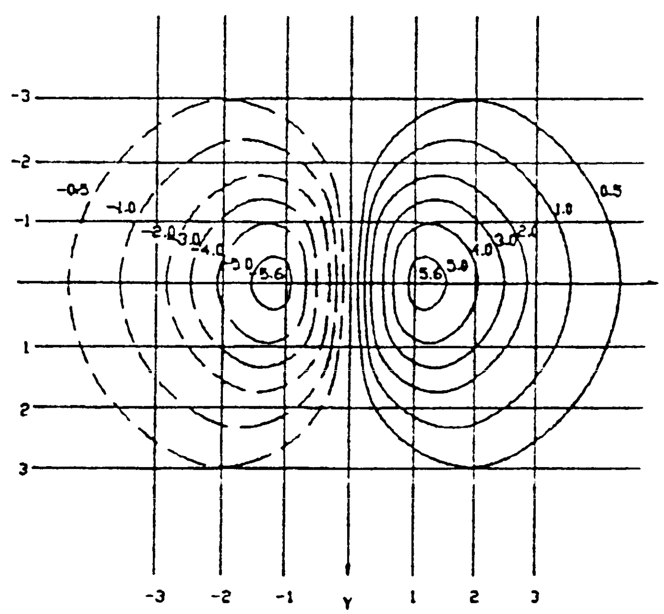
FIGURE 24. Two X dipoles on the y axis - Continued.

4,3.3.3 Two X dipoles on the z axis. Figure 25 shows the configuration and contour map for the magnetic field of two X dipoles on the z axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the two dipoles are also provided.



Magnetic moment	Coordinates
M	(0,0,c)
−M	(0,0,−c)

A Configuration.



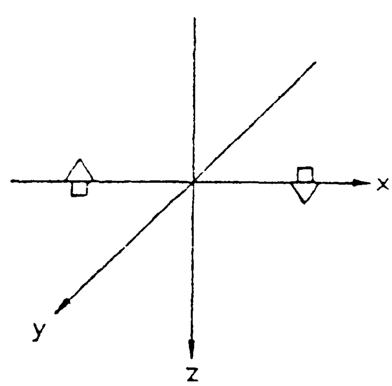
B. Contour map.

FIGURE 25. Two X dipoles on the z axis.

$B_x = -0.6Mcz^{-4}(-4x^2z^{-2} + y^2z^{-2} + 1)(1 + k^2)^{-3.5}$		[4-55]
Maximum value = $-0.6Mcz^{-4}$	Maxima occur at: $x = 0$, $y = 0$	
$B_y = -3Mcxyz^{-6}(1 + k^2)^{-3.5}$		[4-56]
Maximum value = $0.185Mcz^{-4}$	Maxima occur at: $x = \pm 0.447z$, $y = \pm 0.447z$	
$B_z = 0.6Mcxz^{-5}(4 - k^2)(1 + k^2)^{-3.5}$		[4-57]
Maximum value = $0.549Mcz^{-4}$	Maxima occur at: $x = \pm 0.389z$, $y = 0$	

FIGURE 25. Two X dipoles on the z axis - Continued.

4.3.3.4 Two Z dipoles on the x axis. Figure 26 shows the configuration and contour map for the magnetic field of two Z dipoles on the x axis. The coordinates of each dipole and the magnetic field equations for each component of the total field of the two dipoles are also provided.



Magnetic moment	Coordinates
M	(a,0,0)
-M	(-a,0,0)

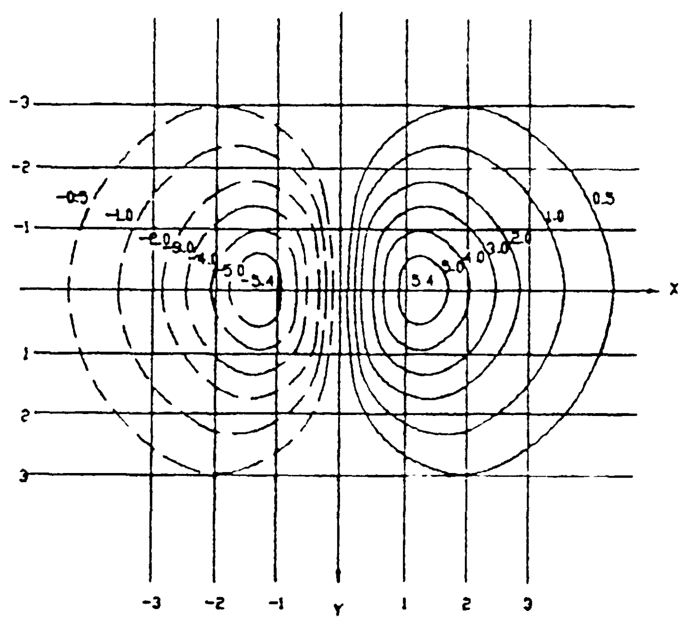
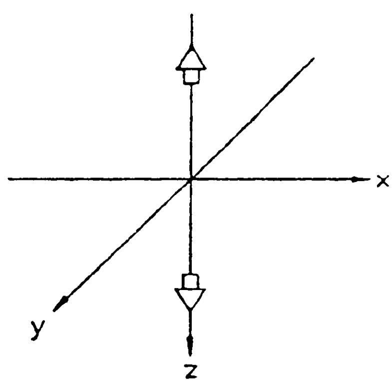


FIGURE 26. Two Z dipoles on the x axis.

$B_x = -0.6Maz^{-4}\ (-4x^2z^{-2} + y^2z^{-2} + 1)(1 + k^2)^{-3.5}$		[4-58]
Maximum value = $-0.6Maz^{-4}$	Maxima occur at: $x = 0$ $y = 0$	
$B_y = 3Maxyz^{-6}(1 + k^2)^{-3.5}$		[4-59]
Maximum value = $0.030Maz^{-4}$	Maxima occur at: $x = \pm\ 0.103z$ $y = \pm\ 0.103z$	
$B_z = 0.6Maxz^{-5}(4 - k^2)(1 + k^2)^{-3.5}$		[4-60]
Maximum value = $0.549Maz^{-4}$	Maxima occur at: $x = \pm\ 0.389z$ $y = 0$	

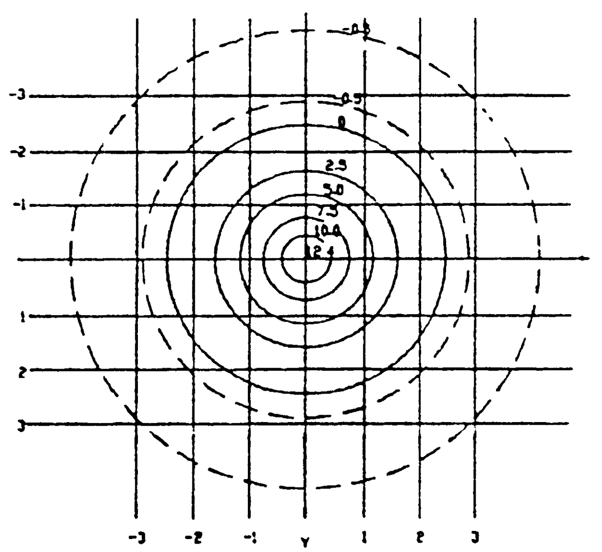
FIGURE 26 Two Z dipoles on the x axis - Continued.

4.3.3.5 Two z dipoles on the z axis. Figure 27 shows the configuration and contour map for the magnetic field of two Z dipoles on the z axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the two dipoles are also provided.



Magnetic moment	Coordinates
M	(0,0,c)
-M	(0,0,-c)

A. Configuration.



B. Contour map.

FIGURE 27. Two Z dipoles on the z axis.

$B_x = -0.6Mcxz^{-5}(k^2 - 4)(1 + k^2)^{-3.5}$		[4-61]
Maximum value = $0.549Mcz^{-4}$	Maxima occur at: $x = \pm 0.390z$, $y = 0$	
$B_y = -0.6Mcyz^{-5}(k^2 - 4)(1 + k^2)^{-3.5}$		[4-62]
Maximum value = $0.549Mcz^{-4}$	Maxima occur at: $x = 0$, $y = \pm 0.390z$	
$B_z = 0.6Mcz^{-4}(2 - 3k^2)(1 + k^2)^{-3.5}$		[4-63]
Maximum value = $1.2Mcz^{-4}$	Maxima occur at: $x = 0$, $y = 0$	

FIGURE 27 Two Z dipoles on the z axis - Continued

4.3.3.6 Four x dipoles on the x axis - case 1. Figure 28 (A) shows the configuration and contour map for the magnetic field of four X dipoles on the x axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.

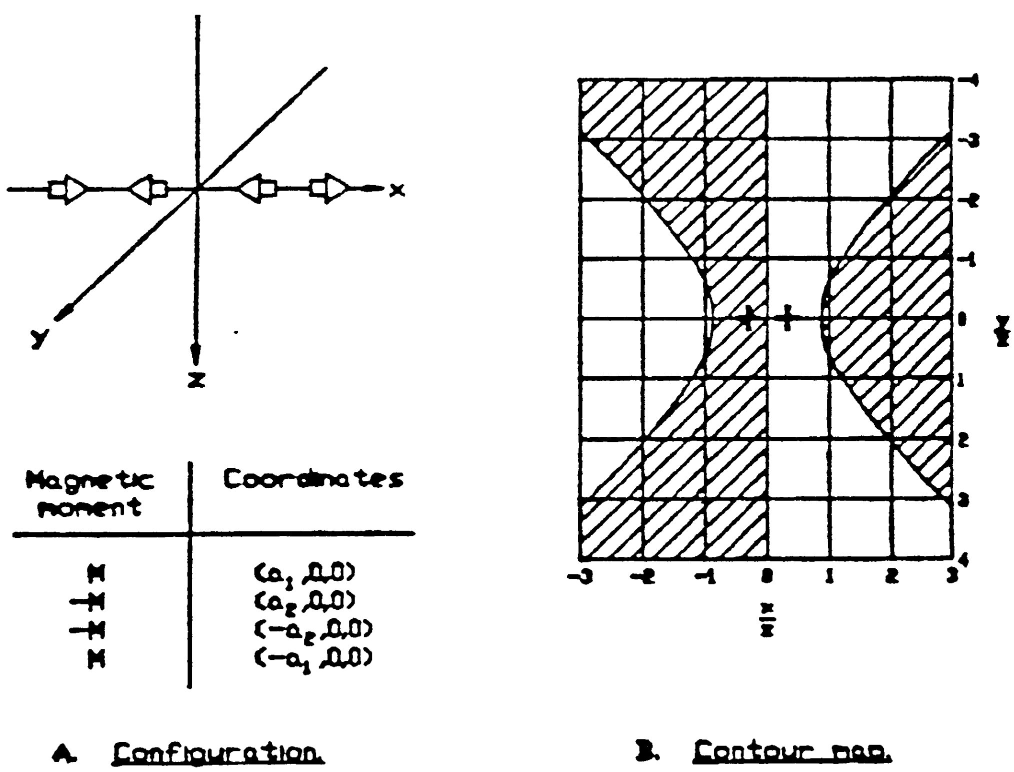


FIGURE 28 (A). Four X dipoles on the x axis - case 1.

$$B_x = 0.3M(a_1^2 - a_2^2)z^{-5}(8x^4z^{-4} + 3y^4z^{-4} - 24x^2y^2z^{-4} - 24x^2z^{-2} + 6y^2z^{-2} + 3)(1 + k^2)^{-4.5} \quad [4\text{-}64]$$

Maximum value = $0.9M(a_1^2 - a_2^2)z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

$$B_y = 1.5M(a_1^2 - a_2^2)z^{-5}(4x^3yz^{-4} - 3xy^3z^{-4} - 3xyz^{-2})(1 + k^2)^{-4.5} \quad [4\text{-}65]$$

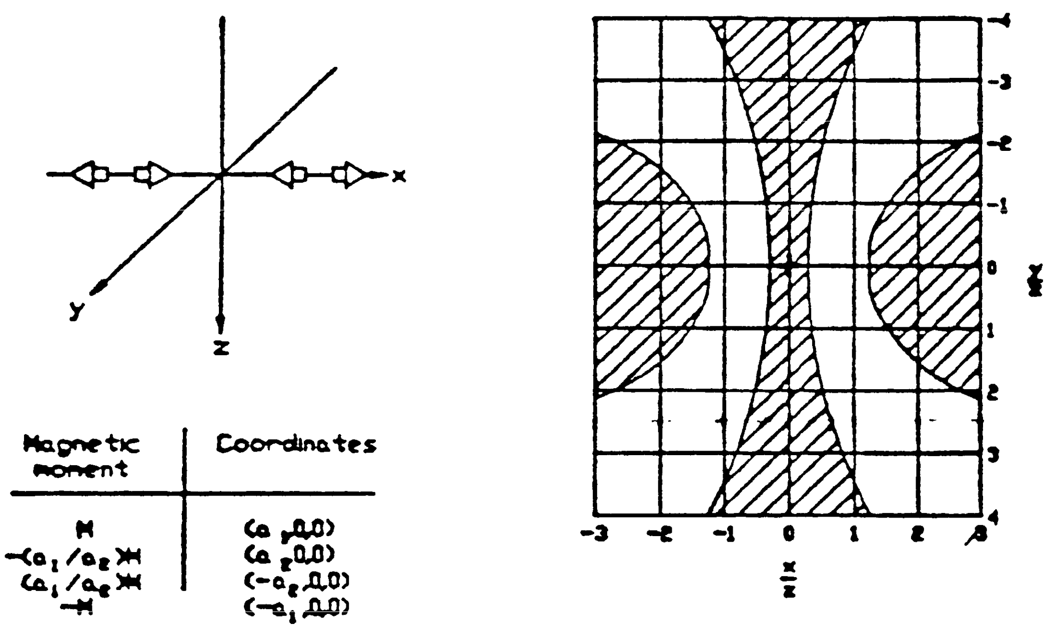
Maximum value = $0.023M(a_1^2 - a_2^2)z^{-5}$ Maxima occur at: $x = \pm 1.300z$, $y = \pm 0.450z$

$$B_z = 1.5M(a_1^2 - a_2^2)xz^{-6}(-4x^2z^{-2} + 3y^2z^{-2} + 3)(1 + k^2)^{-4.5} \quad [4\text{-}66]$$

Maximum value = $0.806M(a_1^2 - a_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.300z$, $y = 0$

FIGURE 28 (A). Four X dipoles on the x axis - case 1 - Continued.

4.3.3.7 Four X dipoles on the x axis - case 2. Figure 28 (B) shows the configuration and contour map for the magnetic field of four X dipoles on the axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



A. Configuration   B. Contour map

FIGURE 28 (B). Four x dipoles on the x axis - case 2.

$$B_x = 0.5M(a_1^3 - a_1a_2^2)z^{-6}(-8x^5z^{-5} - 15xy^4z^{-5} + 40x^3y^2z^{-5} + 40x^3z^{-3} - 30xy^2z^{-3} - 15xz^{-1})(1 + k^2)^{-5.5} \quad [4\text{-}67]$$

Maximum value = $0.005M(a_1^3 - a_1a_2^2)z^{-6}$ Maxima occur at: $x = \pm 1.91z$, $y = 0$

$$B_y = 1.5M(a_1^3 - a_1a_2^2)z^{-6}(-y^5z^{-5} - 8x^4yz^{-5} + 12x^2y^3z^{-5} - 2y^3z^{-3} - 12x^2yz^{-3} - yz^{-1})(1 + k^2)^{-5.5} \quad [4\text{-}68]$$

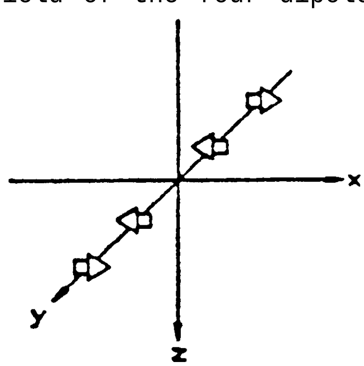
Maxima occur at complex roots

$$B_z = 1.5M(a_1^3 - a_1a_2^2)z^{-6}(8x^4z^{-4} - y^4z^{-4} - 12x^2y^2z^{-4} - 12x^2z^{-2} + 2y^2z^{-2} + 1)(1 + k^2)^{-5.5} \quad [4\text{-}69]$$

Maximum value = $1.5M(a_1^3 - a_1a_2^2)z^{-6}$ Maxima occur at: $x = 0$, $y = 0$

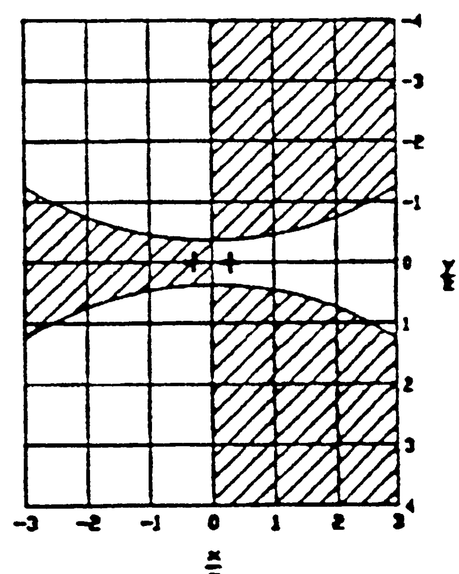
FIGURE 28 (B). Four X dipoles on the x axis - case 2 - Continued.

4.3.3.8 Four X dipoles on the y axis. Figure 29 shows the configuration and contour map for the magnetic field of four X dipoles on the y axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



Magnetic moment	Coordinates
M	$(0,b_1,0)$
−M	$(0,b_2,0)$
−M	$(0,-b_2,0)$
M	$(0,-b_1,0)$

A. Configuration



B. Contour map

FIGURE 29. Four X dipoles on the y axis.

$$B_x = 0.3M(b_1^2 - b_2^2)z^{-5}(-4x^4z^{-4} - 4y^4z^{-4} + 27x^2y^2z^{-4} - 3x^2z^{-2} - 3y^2z^{-2} + 3)(1 + k^2)^{-4.5} \quad [4-70]$$

Maximum value = $0.075M(b_1^2 - b_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.642z$, $y = \pm 0.642z$

$$B_y = 1.5M(b_1^2 - b_2^2)z^{-5}(-3x^3yz^{-4} + 4xy^3z^{-4} - 3xyz^{-2})(1 + k^2)^{-4.5} \quad [4-71]$$

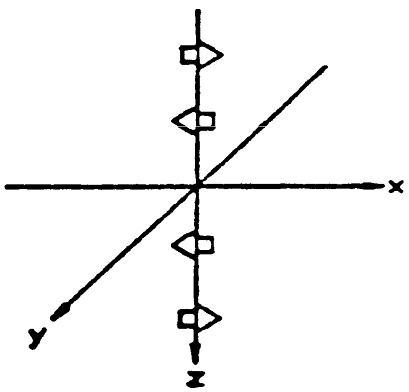
Maximum value = $0.023M(b_1^2 - b_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.450z$, $y = \pm 1.300z$

$$B_z = 1.5M(b_1^2 - b_2^2)xz^{-6}(x^2z^{-2} - 6y^2z^{-2} + 1)(1 + k^2)^{-4.5} \quad [4-72]$$

Maximum value = $0.360M(b_1^2 - b_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.408z$, $y = 0$

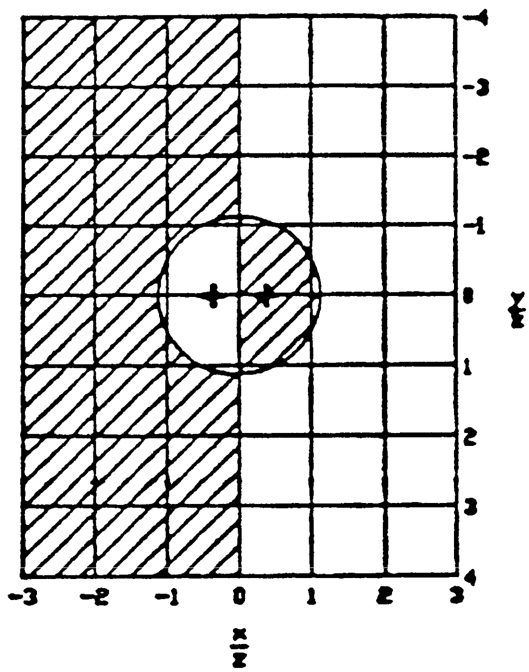
FIGURE 29. Four X dipoles on the Y axis - Continued.

4.3.3.9 Four X dipoles on the z axis. Figure 30 shows the configuration and contour map for the magnetic field of four X dipoles on the z axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



Magnetic moment	Coordinates
M	$(0,0,c_1)$
-M	$(0,0,c_2)$
-M	$(0,0,-c_2)$
M	$(0,0,-c_1)$

A. Configuration.



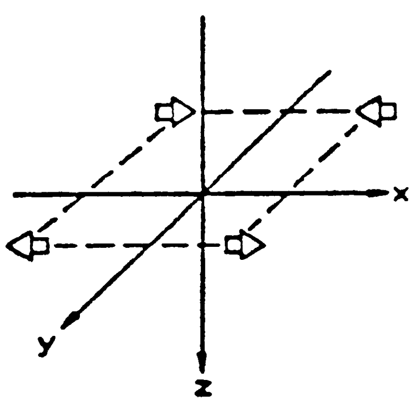
B. Contour map.

FIGURE 30. Four X dipoles on the z axis.

$$B_x = 0.3M(c_1^2 - c_2^2)z^{-5}(-4x^4z^{-4} + y^4z^{-4} - 3x^2y^2z^{-4} + 27x^2z^{-2} - 3y^2z^{-2} - 4)(1 + k^2)^{-4.5}$$	[4-73]
Maximum value $= 0.218M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.471z$, $y = 0$	
$$B_y = 1.5M(c_1^2 - c_2^2)xyz^{-7}(6 - k^2)(1 + k^2)^{-4.5}$$	[4-74]
Maximum value $= 0.395M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.362z$, $y = \pm 0.362z$	
$$B_z = 1.5M(c_1^2 - c_2^2)xz^{-6}(4 - 3k^2)(1 + k^2)^{-4.5}$$	[4-75]
Maximum value $= 1.140M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = \pm 0.319z$, $y = 0$	

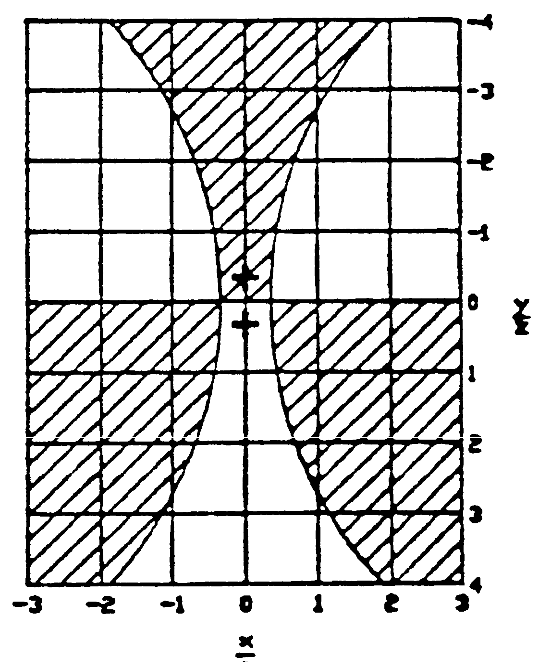
FIGURE 30. Four X dipoles on the z axis - Continued.

4.3.3.10 Four X dipoles on the xy plane. Figure 31 shows the configuration and contour map for the magnetic field of four X dipoles on the xy plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



Magnetic moment	Coordinates
M	(a,b,0)
-M	(-a,b,0)
-M	(-a,-b,0)
M	(a,-b,0)

A. Configuration



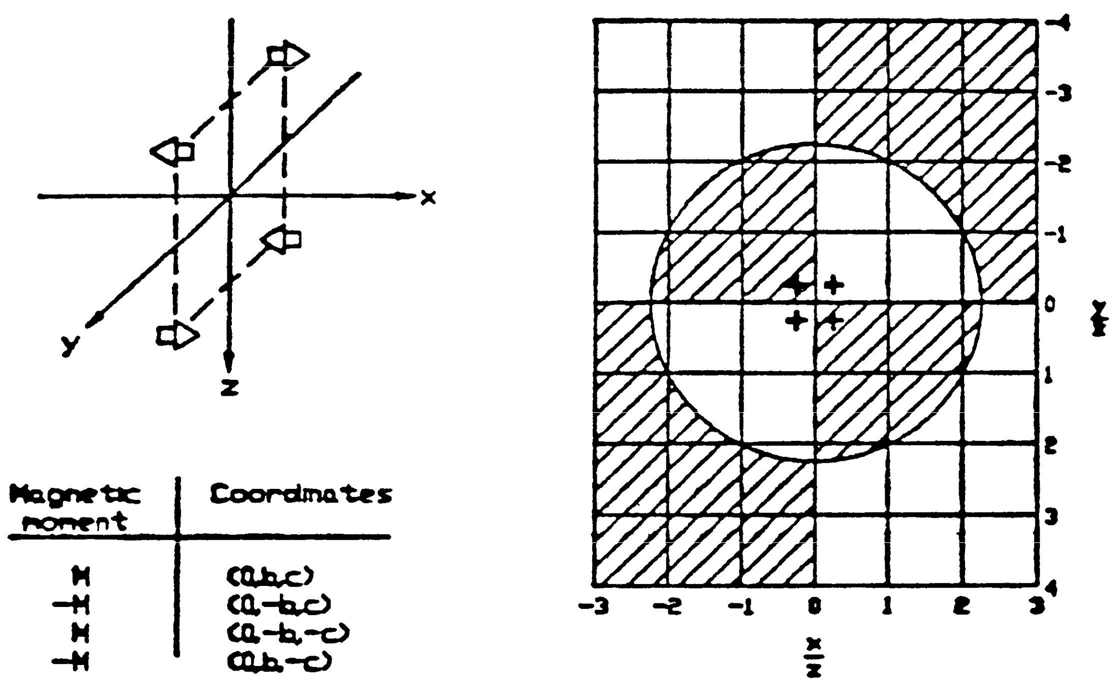
B. Contour map

FIGURE 31. Four X dipoles on the xy plane.

$B_x = 6Mabz^{-5}(4x^3yz^{-4} - 3xy^3z^{-4} - 3xyz^{-2})(1 + k^2)^{-4.5}$	[4-76]
Maximum value = $0.093Mabz^{-5}$ Maxima occur at: $x = \pm 1.300z$, $y = \pm 0.450z$	
$B_y = 1.2Mabz^{-5}(1 - 4k^4 - 3k^2 + 27x^2y^2z^{-4})(1 + k^2)^{-4.5}$	[4-77]
Maxima occur at complex roots	
$B_z = 6Mabyz^{-6}(-6x^2z^{-2} + y^2z^{-2} + 1)(1 + k^2)^{-4.5}$	[4-78]
Maximum value = $1.427Mabz^{-5}$ Maxima occur at: $x = 0$, $y = \pm 0.408z$	

FIGURE 31. Four X dipoles on the xy plane - Continued.

4.3.3.11 Four X dipoles on the yz plane. Figure 32 shows the configuration and contour map for the magnetic field of four X dipoles on the yz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided



A. Configuration. B. Contour map.

FIGURE 32. Four X dipoles on the yz plane.

$B_x = 6Myz^{-7}(6x^2z^{-2} - y^2z^{-2} - 1)(1 + k^2)^{-4.5}$	[4-79]
Maxima occur at complex roots	
$B_y = 6Mxz^{-7}(-x^2z^{-2} + 6y^2z^{-2} - 1)(1 + k^2)^{-4.5}$	[4-80]
Maxima occur at complex roots	
$B_z = 6Mbcxyz^{-7}(6 - k^2)(1 + k^2)^{-4.5}$	[4-81]
Maximum value = $1.580Mbcz^{-5}$ Maxima occur at: $x = \pm 0.366z$, $y = \pm 0.366z$	

FIGURE 32. Four X dipoles on the yz plane - Continued.

4.3.3.12 Four X dipoles on the xz plane. Figure 33 shows the configuration and contour map for the magnetic field of four X dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.

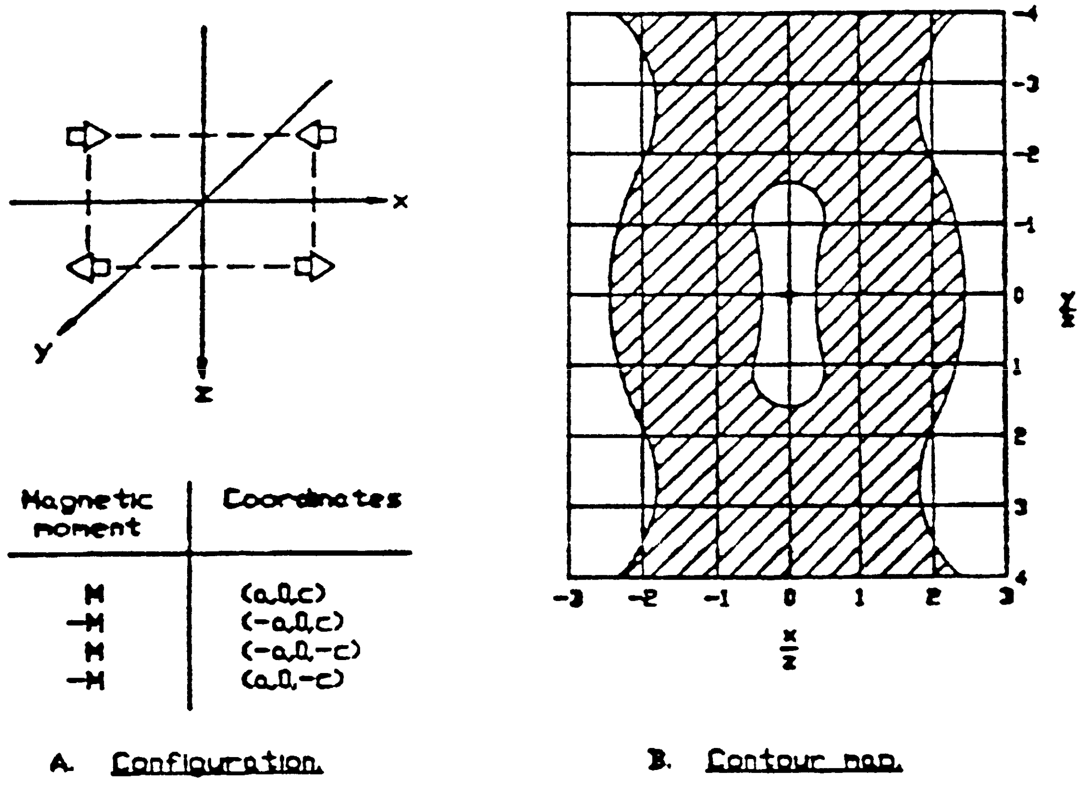


FIGURE 33. Four X dipoles on the xz plane.

$$B_x = 6Macxz^{-6}(4x^2z^{-2} - 3y^2z^{-2} - 3)(1 + k^2)^{-4.5}$$ Maximum value = $0.359Macz^{-5}$ Maxima occur at: $x = \pm 1.187z$, $y = 0$	[4-82]
$$B_y = 6Macyz^{-6}(6x^2z^{-2} - y^2z^{-2} - 1)(1 + k^2)^{-4.5}$$ Maxima occur at complex roots	[4-83]
$$B_z = -1.2Macz^{-5}(4x^4z^{-4} - y^4z^{-4} + 3x^2y^2z^{-4} - 27x^2z^{-2} + 3y^2z^{-2} + 4)(1 + k^2)^{-4.5}$$ Maximum value = $-4.8Macz^{-5}$ Maxima occur at: $x = 0$, $y = 0$	[4-84]

FIGURE 33. Four X dipoles on the xz plane - Continued.

4.3.3.13 Four Z dipoles on the x axis. Figure 34 shows the configuration and contour map for the magnetic field of four Z dipoles on the x axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.

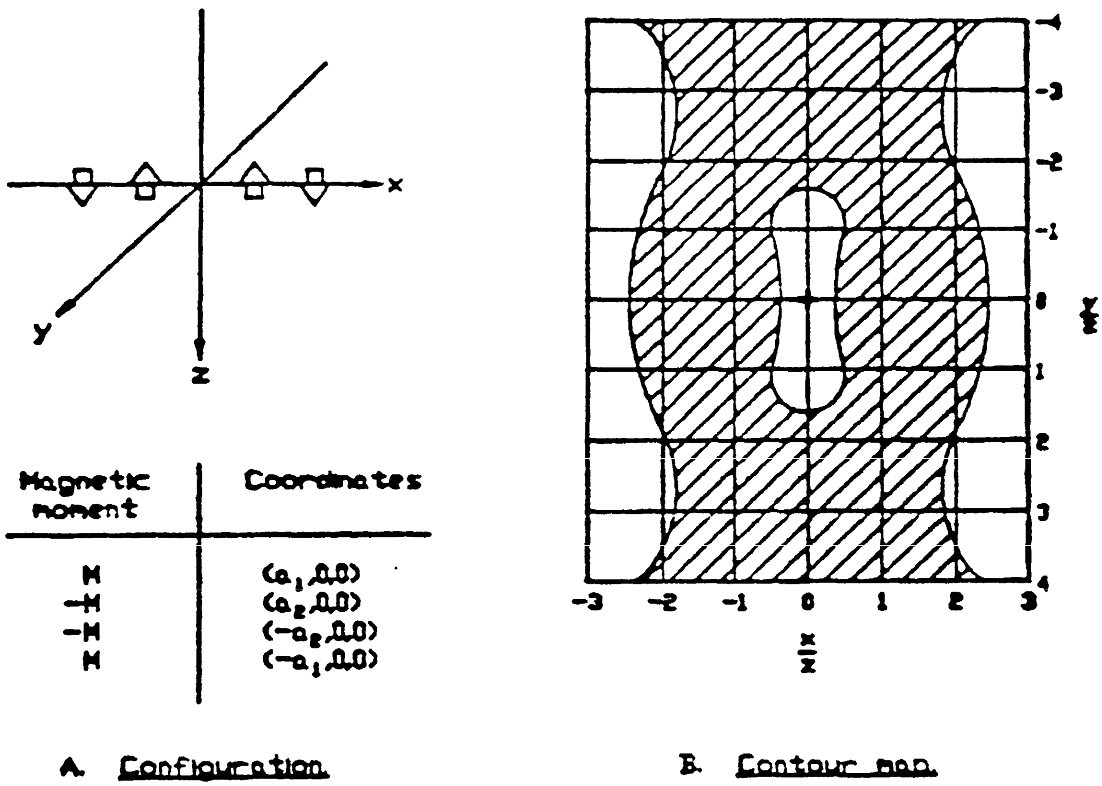


FIGURE 34. Four z dipoles on the x axis.

$$B_x = 1.5M(a_1^2 - a_2^2)xz^{-6}(4x^2z^{-2} - 3y^2z^{-2} - 3)(1 + k^2)^{-4.5} \qquad [4\text{-}85]$$

Maximum value = $0.090M(a_1^2 - a_2^2)z^{-5}$ Maxima occur at: $x = \pm 1.187z$, $y = 0$

$$B_y = 1.5M(a_1^2 - a_2^2)yz^{-6}(6x^2z^{-2} - y^2z^{-2} - 1)(1 + k^2)^{-4.5} \qquad [4\text{-}86]$$

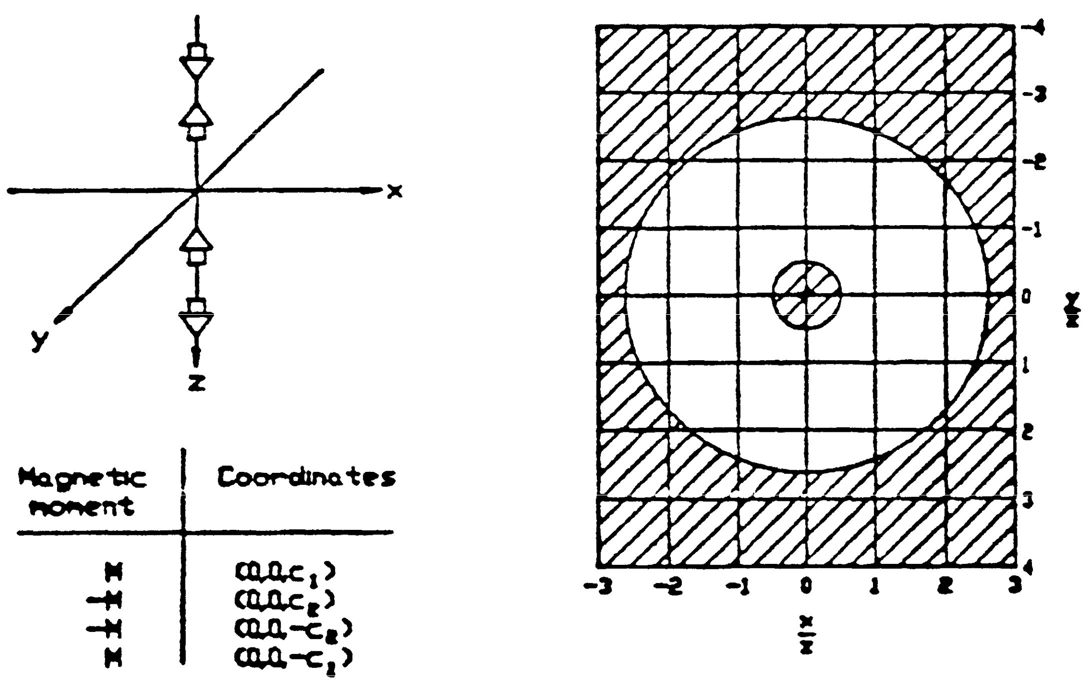
Maxima occur at complex roots

$$B_z = -0.3M(a_1^2 - a_2^2)z^{-5}(4x^4z^{-4} - y^4z^{-4} + 3x^2y^2z^{-4} - 27x^2z^{-2} + 3y^2z^{-2} + 4)(1 + k^2)^{-4.5} \qquad [4\text{-}87]$$

Maximum value = $-1.2M(a_1^2 - a_2^2)z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

FIGURE 34 Four Z dipoles on the x axis - Continued.

4.3.3.14 Four Z dipoles on the z axis - case 1. Figure 35 (A) shows the configuration and contour map for the magnetic field of four Z dipoles on the z axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



A. Configuration. B. Contour map.

FIGURE 35 (A). Four Z dipoles on the z axias - case 1.

$$B_x = 1.5M(c_1^2 - c_2^2)xz^{-6}(4 - 3k^2)(1 + k^2)^{-4.5} \qquad [4\text{-}88]$$

Maximum value $= 1.140M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = \pm\ 0.317z$, $y = 0$

$$B_y = 1.5M(c_1^2 - c_2^2)yz^{-6}(4 - 3k^2)(1 + k^2)^{-4.5} \qquad [4\text{-}89]$$

Maximum value $= 1.140M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = 0$, $y = \pm\ 0.317z$

$$B_z = -0.3M(c_1^2 - c_2^2)z^{-5}(8 - 24k^2 + 3k^4)(1 + k^2)^{-4.5} \qquad [4\text{-}90]$$

Maximum value $= 2.4M(c_1^2 - c_2^2)z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

FIGURE 35 (A). Four Z dipoles on the z axis - case 1 - Continued.

4.3.3.15 Four Z dipoles on the z axis - case 2. Figure 35 (B) shows the configuration and contour map for the magnetic field of four Z dipoles on the z axis. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.

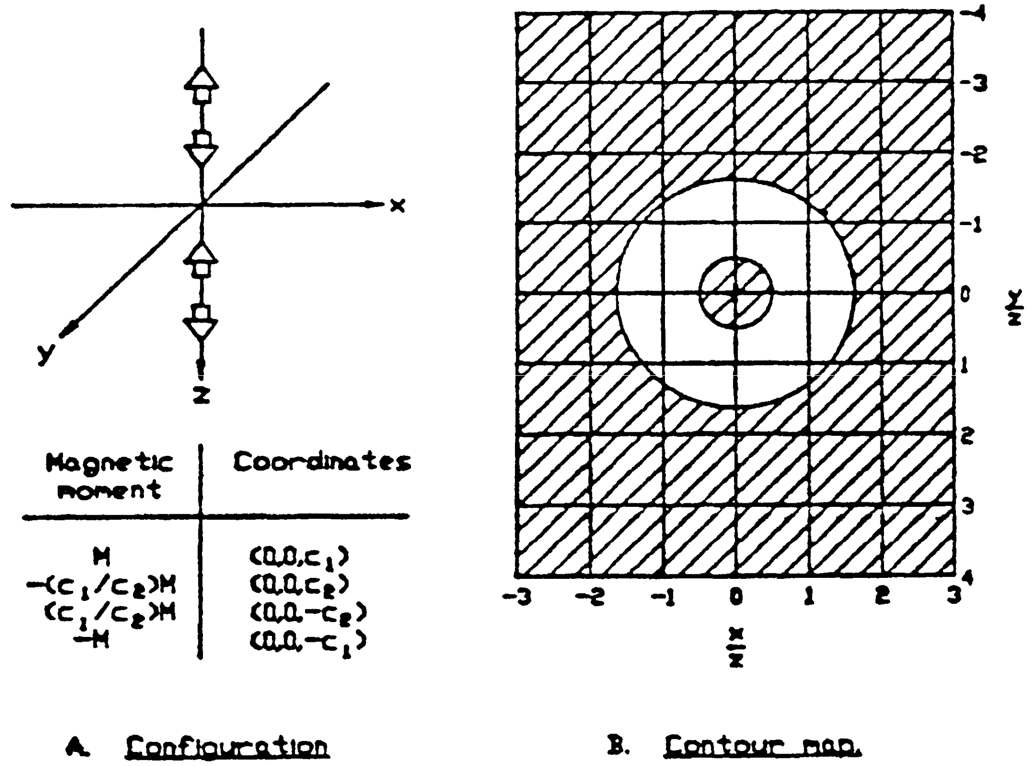
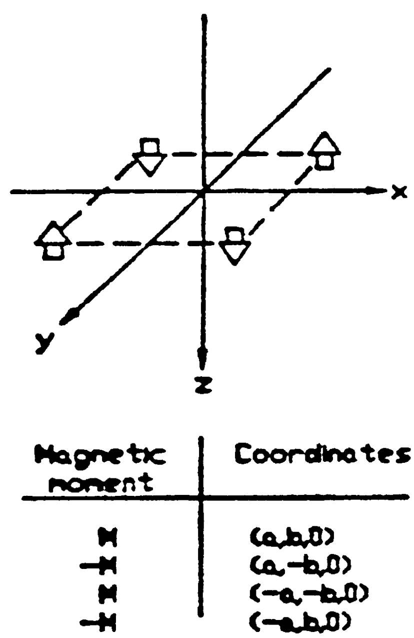


FIGURE 35 (B). Four z dipoles on the z axis - case 2.

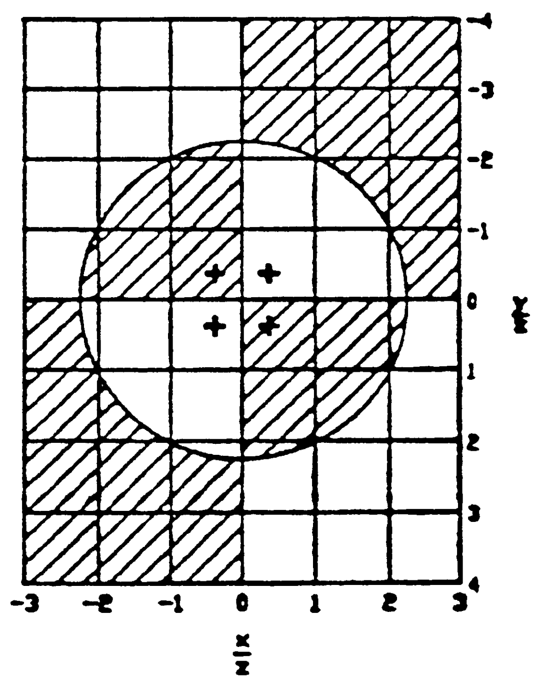
$B_x = 1.5M(c_1^3 - c_1c_2^2)xz^{-7}(12k^2 - k^4 - 8 - 2x^2y^2z^{-4})(1 + k^2)^{-5.5}$	[4-91]
Maximum value $= 0.00012M(c_1^3 - c_1c_2^2)z^{-6}$ Maxima occur at: $x = \pm 3.148z$, $y = 0$	
$B_y = 1.5M(c_1^3 - c_1c_2^2)yz^{-7}(12k^2 - k^4 - 8 - 2x^2y^2z^{-4})(1 + k^2)^{-5.5}$	[4-92]
Maximum value $= 0.00012M(c_1^3 - c_1c_2^2)z^{-6}$ Maxima occur at: $x = 0$, $y = \pm 3.148z$	
$B_z = 0.5M(c_1^3 - c_1c_2^2)z^{-6}(8 - 40k^2 + 15k^4)(1 + k^2)^{-5.5}$	[4-93]
Maximum value $= 4M(c_1^3 - c_1c_2^2)z^{-6}$ Maxima occur at: $x = 0$, $y = 0$	

FIGURE 35 (B). Four Z dipoles on the z axis - case 2 - Continued.

4.3.3.16 Four Z dipoles on the xy plane. Figure 36 shows the configuration and contour map for the magnetic field of four Z dipoles on the xy plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



A. Configuration.



B. Contour map.

FIGURE 36. Four Z dipoles on the xy plane.

$B_x = 6Mabz^{-5}(6x^2yz^{-3} - y^3z^{-3} - yz^{-1})(1 + k^2)^{-4.5}$		[4-94]
Maximum value = $1.428Mabz^{-5}$	Maxima occur at:	$x = 0$ $y = \pm\, 0.408z$
$B_y = 6Mabz^{-5}(-x^3z^{-3} + 6xy^2z^{-3} - xz^{-1})(1 + k^2)^{-4.5}$		[4-95]
Maximum value = $1.428Mabz^{-5}$	Maxima occur at:	$x = \pm\, 0.408z$ $y = 0$
$B_z = 6Mabxyz^{-7}(6 - k^2)(1 + k^2)^{-4.5}$		[4-96]
Maximum value = $1.581Mabz^{-5}$	Maxima occur at:	$x = \pm\, 0.366z$ $y = \pm\, 0.366z$

FIGURE 36. Four Z dipoles on the xy plane - Continued.

4.3.3.17 Four Z dipoles on the yz plane. Figure 37 shows the configuration and contour map for the magnetic field of four Z dipoles on the yz plane. The coordinates of each dipole and the magnetic field equations for each component of combined field of the four dipoles are also provided.

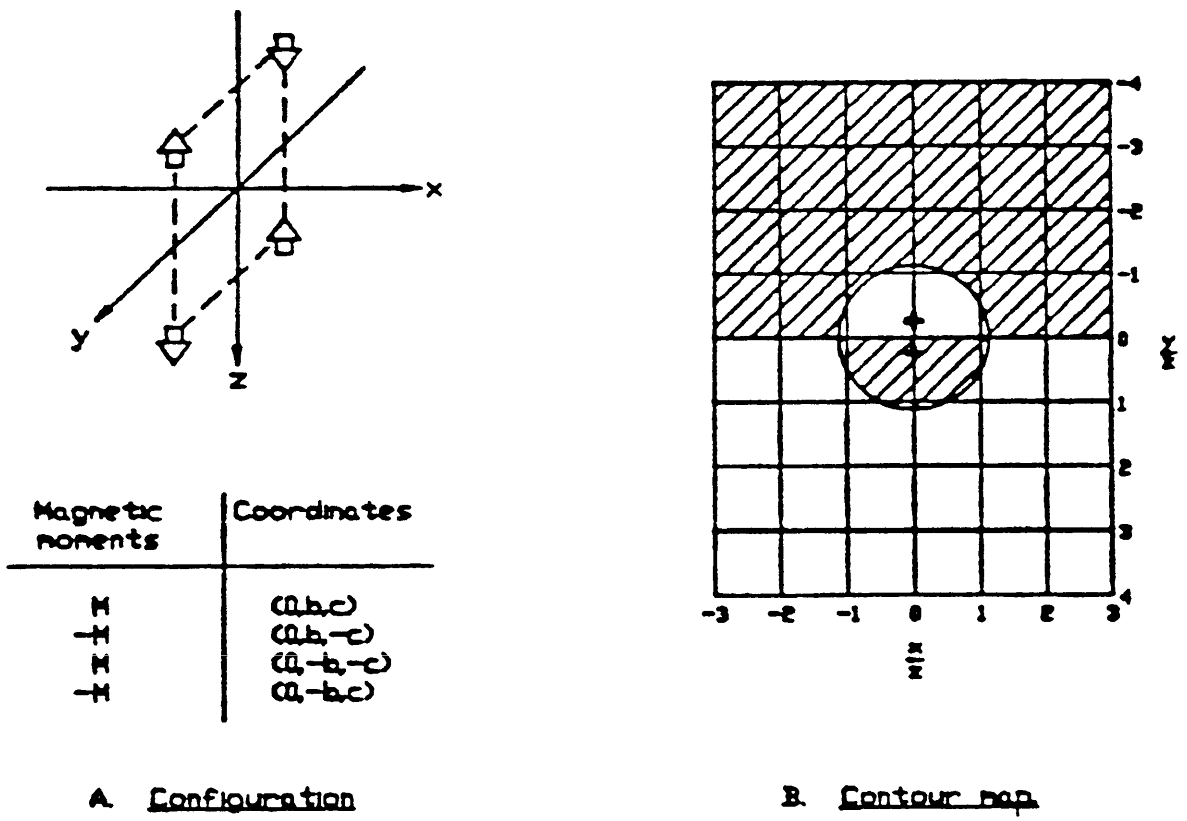


FIGURE 37. Four Z dipoles on the yz plane.

$$B_x = 6Mbcz^{-5}(-x^3yz^{-4} - xy^3z^{-4} + 6xyz^{-2})(1 + k^2)^{-4.5} \quad [4\text{-}97]$$

Maximum value = $0.038Mbcz^{-5}$ Maxima occur at: $x = \pm 2.162z$, $y = \pm 2.162z$

$$B_y = 1.2Mbcz^{-5}(x^4z^{-4} - 4y^4z^{-4} - 3x^2y^2z^{-4} - 3x^2z^{-2} + 27y^2z^{-2} - 4)(1 + k^2)^{-4.5} \quad [4\text{-}98]$$

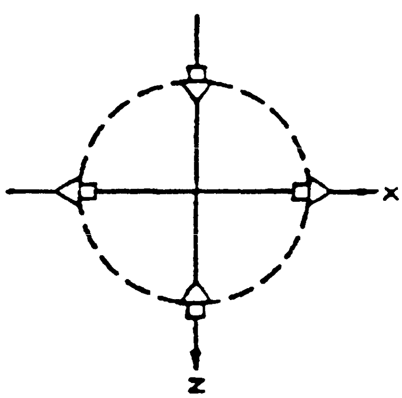
Maximum value = $-4.8Mbcz^{-5}$ Maxima occur at: $x = 0$, $y = 0$

$$B_z = 6Mbcyz^{-6}(4 - 3k^2)(1 + k^2)^{-4.5} \quad [4\text{-}99]$$

Maximum value = $4.573Mbcz^{-5}$ Maxima occur at: $x = 0$, $y = + 0.319z$

FIGURE 37. Four Z dipoles on the yz plane - Continued.

4.3.3.18 Four radial dipoles on the xz plane - case 1. Figure 38 (A) shows the configuration and contour map for the magnetic field of four radial dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided,



Magnetic moment	Coordinates
M	(r,0,0)
-M	(0,0,r)
M	(-r,0,0)
-M	(0,0,-r)

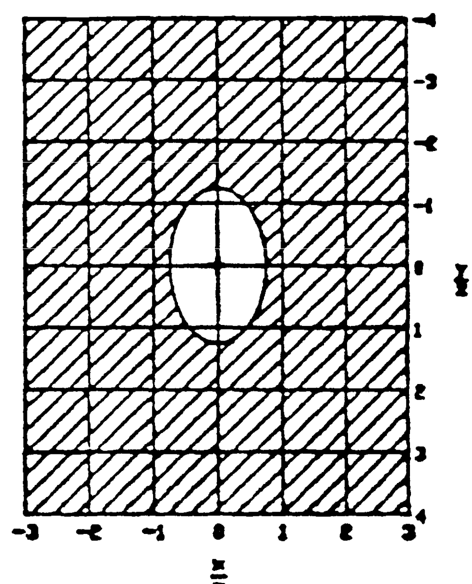


FIGURE 38 (A). Four radial dipoles on the xz plane - case 1.

$$B_x = 0.6Mrz^{-4}(-x^2z^{-2} + y^2z^{-2} - 1)(1 + k^2)^{-3.5} \qquad [4\text{-}100]$$

Maximum value = $0.013Mrz^{-4}$ Maxima occur at: $x = 0$, $y = \pm 1.340z$

$$B_y = 0.6Mrz^{-4}(2y^3z^{-3} - 3x^2yz^{-3} - 3yz^{-1})(1 + k^2)^{-3.5} \qquad [4\text{-}101]$$

Maximum value = $0.024Mrz^{-4}$ Maxima occur at: $x = 0$, $y = \pm 1.694z$

$$B_z = -06Mrz^{-4}(-2y^2z^{-2} - 7x^2z^{-2} + 3)(1 + k^2)^{-3.5} \qquad [4\text{-}102]$$

Maximum value = $-1.8Mrz^{-4}$ Maxima occur at: $x = 0$, $y = 0$

FIGURE 38 (A). Four radial dipoles on the xz plane - case 1 - Continued,

4.3.3.18.1 Example of the use of radial dipole configuration equations. In the example that follows, we shall use equation 4-102 to illustrate a method of calculating the magnetic field due to a radial array of dipoles. Basically, we will follow the procedures listed in 4.3.3.1.1. The only change from 4.3.3.1.1 would occur in the case of a dipole not lying along one of the coordinate axes. This situation will be considered in step (c).

(a) Identify the number of coils for which we want to calculate the magnetic field.
(b) Determine the distance between each dipole and the origin of the coordinate axes. In our example, each dipole is at a distance "r".
(c) Use equation 4-1 (M = ANI) to calculate M for each dipole. To determine the direction of M, we shall follow the rules listed below.

(1) If all dipoles lie along a coordinate axis, we follow the rules of 4.3.3.1.1(c).
(2) If some dipoles do not lie along a coordinate axis then the dipole is positive if it points toward the origin, and negative if it points away.

(d) Next, we make a sketch of the dipole configuration and compare it to those of 4.3.3.18 through 4.3.3.26. As in 4.3.3.1, if a partial match occurs, use superposition to construct a complete match. If no match occurs, use appendix A to derive the necessary equations . For our example, we use the equations of 4.3.3.18, specifically equation 4-102 since we are interested in the z component of magnetic field. Equation 4-102 is as follows:

$$B_z = -0.6Mrz^{-4}(-2y^2z^{-2} - 7x^2z^{-2} + 3)(1 + k^2)^{-3.5}$$

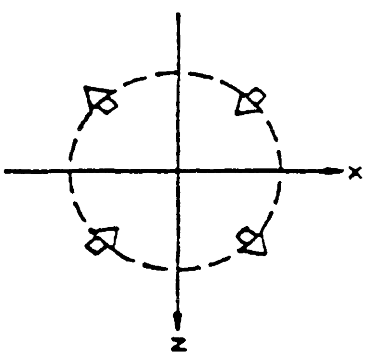
For $A = 2.0\ m^2$, $N = 5$ turns, $I = 900$ A, $r = 1.0$ m, $x = 0$, $y = 0$, $z = 10.0$ m, the z component of magnetic field is·

$$B_z = -0.6\ ANI(1.0)(10^{-4})(0 - 0 + 3)(1 + 0 + 0)^{-3\ 5}$$

$$B_z = (-0.6)(2.0)(5)(900)(1.0)(10^{-4})(3) = -1.62\ \mu T$$

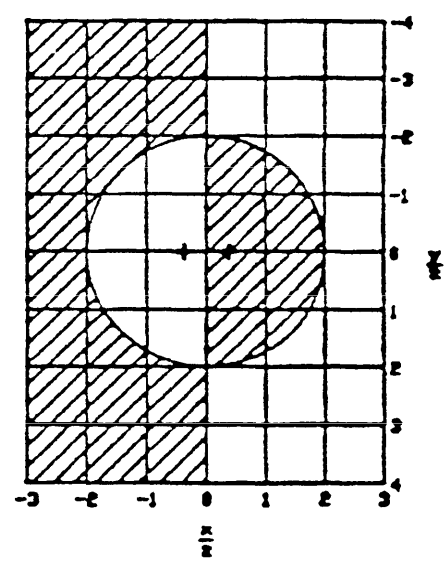
These steps can be followed to calculate the q agnetic field due to any radial array of dipoles,

4,3.3.19 Four radial dipoles on the xz plane - plane - case 2. Figure 38 (B) shows the configuration and contour map for the magnetic field of four radial dipoles on the xz plane, The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



Magnetic moment	Coordinates
M	(.707r,0,.707r)
-M	(-.707r,0,.707r)
M	(-.707r,0,-.707r)
-M	(.707r,0,-.707r)

A. Configuration.



B. Contour map.

$$B_x = 1.2Mrxz^{-5}(4 - k^2)(1 + k^2)^{-3\ 5} \quad [4\text{-}103]$$

Maximum value = $1.097Mrz^{-4}$ Maxima occur at: $x = \pm\ 0.389z$, $y = 0$

$$B_y = -0.6Mrz^{-4}(2 - 3k^2)(1 + k^2)^{-3.5} \quad [4\text{-}104]$$

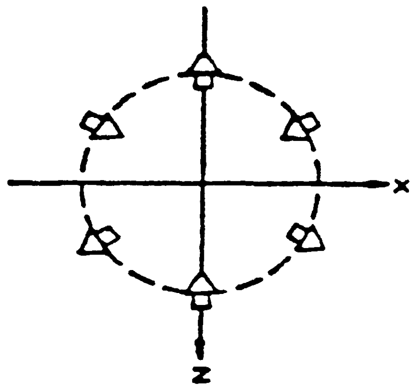
Maximum value = $-1.155Mrz^{-4}$ Maxima occur at: $x = 0$, $y = \pm\ 0.088z$

$$B_z = 1.2Mrxz^{-5}(4 - k^2)(1 + k^2)^{-3.5} \quad [4\text{-}105]$$

Maximum value = $1.097Mrz^{-4}$ Maxima occur at: $x = \pm\ 0.389z$, $y = 0$

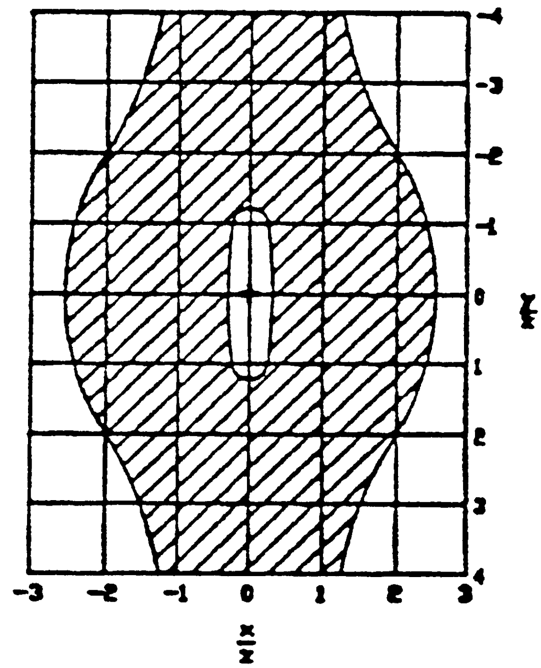
FIGURE 38 (B). Four radial dipoles on the xz plane - case 2.

4.3.3.20 Six radial dipoles on the xz plane - case 1. Figure 39 (A) shows the configuration and contour map for the magnetic field of six radial dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the six dipoles are also provided.



Magnetic moment	Coordinates
M	(.866r,0,.500r)
-M	(0,0,r)
M	(-.866r,0,.500r)
-M	(-.866r,0,-.500r)
M	(0,0,-r)
-M	(.866r,0,-.500r)

A. Configuration



B. Contour map.

$$B_x = 1.125Mr^2z^{-5}(15x^3z^{-3} - 6xy^2z^{-3} - 13xz^{-1})(1 + k^2)^{-4.5} \quad [4\text{-}106]$$

Maximum value = $2.668Mr^2z^{-5}$ Maxima occur at: $x = \pm 0.304z$, $y = 0$

$$B_y = 1.125Mr^2z^{-5}(-21x^2yz^{-3} - 7yz^{-1})(1 + k^2)^{-4.5} \quad [4\text{-}107]$$

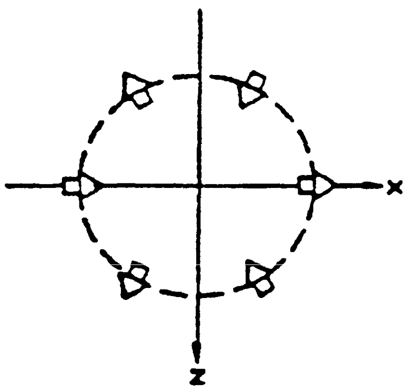
Maximum value = $1.638Mr^2z^{-5}$ Maxima occur at: $x = 0$, $y = \pm 0.354z$

$$B_z = -1.125Mr^2z^{-5}(3x^4z^{-4} + 3x^2y^2z^{-4} - 21x^2z^{-2} - 3y^2z^{-2} + 4)(1 + k^2)^{-4.5} \quad [4\text{-}108]$$

Maximum value = $-4.5Mr^2z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

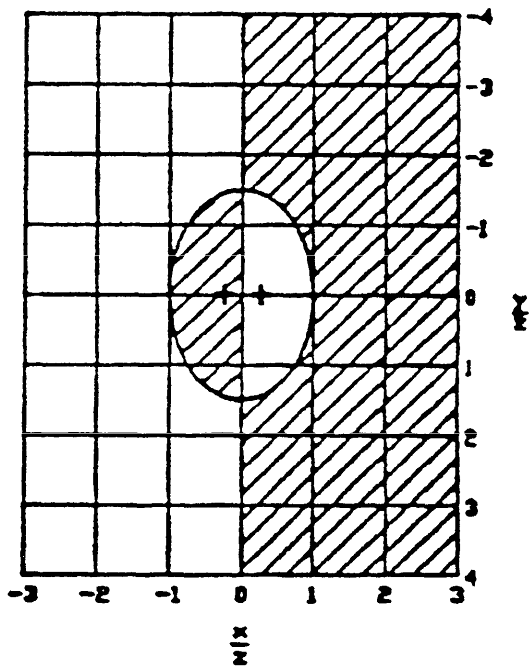
FIGURE 39 (A). Six radial dipoles on the xz plane - case 1.

4.3.3.21 Six radial dipoles on the xz plane - case 2. Figure 39 (B) shows the configuration and contour map for the magnetic field of six radial dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the six dipoles are also provided.



Magnetic moment	Coordinates
M	(r,0,0)
-M	(.500r,0,.866r)
M	(-.500r,0,.866r)
-M	(-r,0,0)
M	(-.500r,0,-.866r)
-M	(.500r,0,-.866r)

A. Configuration.



B. Contour map.

$$B_x = -1.125Mr^2z^{-5}(4x^4z^{-4} - 3x^2y^2z^{-4} - 21x^2z^{-2} + 3y^2z^{-2} + 3)(1 + k^2)^{-4.5} \quad [4\text{-}109]$$

Maximum value = $-3.375Mr^2z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

$$B_y = 1.125Mr^2xyz^{-7}(7x^2z^{-2} - 21)(1 + k^2)^{-4.5} \quad [4\text{-}110]$$

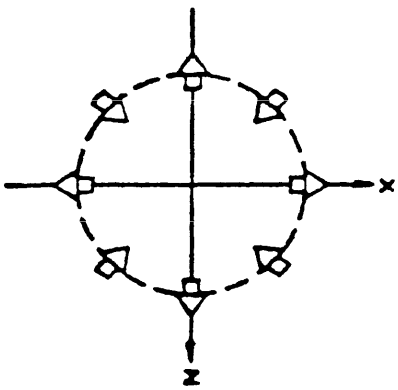
Maximum value = $1.026Mr^2z^{-5}$ Maxima occur at: $x = \pm 0.400z$, $y = \pm 0.400z$

$$B_z = -1.125Mr^2xz^{-6}(-13x^2z^{-2} - 6y^2z^{-2} + 15)(1 + k^2)^{-4.5} \quad [4\text{-}111]$$

Maximum value = $3.171Mr^2z^{-5}$ Maxima occur at: $x = \pm 0.315z$, $y = 0$

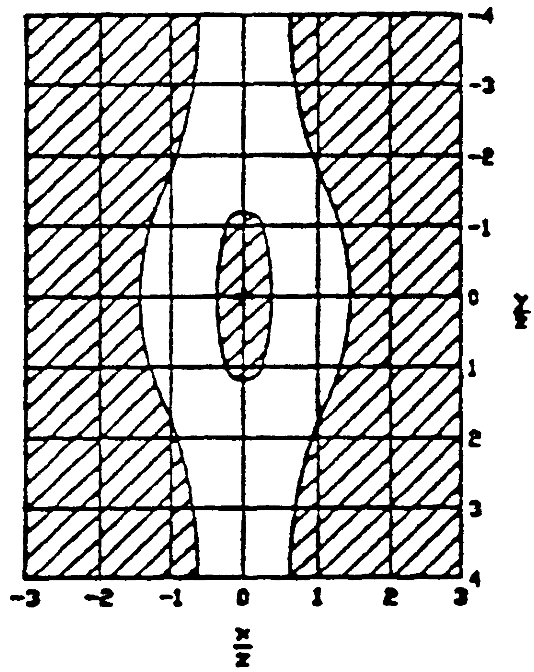
FIGURE 39 (B). Six radial dipoles on the xz plane - case 2.

4.3.3.22 Eight radial dipoles on the xz plane - case 1. Figure 40 (A) shows the configuration and contour map for the magnetic field of eight radial dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the eight dipoles are also provided.



Magnetic moment	Coordinates
M	(r,0,0)
-M	(.707r,0,.707r)
M	(0,0,r)
-M	(-.707r,0,.707r)
M	(-r,0,0)
-M	(-707r,0,-.707r)
M	(0,0,-r)
-M	(.707r,0,-.707r).

A. Configuration.



B. Contour map.

$$B_x = 0.25Mr^3z^{-6}(-35x^5z^{-5} + 28x^3y^2z^{-5} + 322x^3z^{-3} - 84xy^2z^{-3} - 147xz^{-1})(1 + k^2)^{-5.5} \quad [4\text{-}112]$$

Maximum value = $0.126Mr^3z^{-6}$ Maxima occur at: $x = \pm 1.683z$, $y = 0$

$$B_y = 0.25Mr^3z^{-6}(-63x^4yz^{-5} + 378x^2yz^{-3} - 63yz^{-1})(1 + k^2)^{-5.5} \quad [4\text{-}113]$$

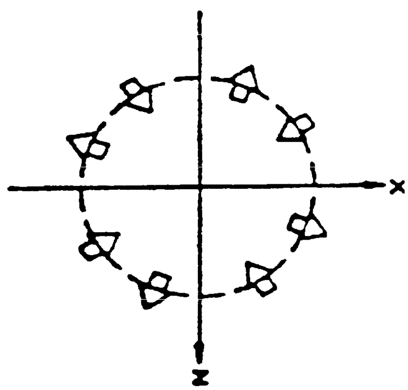
Maximum value = $2.948Mr^3z^{-6}$ Maxima occur at: $x = 0$, $y = \pm 0.316z$

$$B_z = 0.25Mr^3z^{-6}(147x^4z^{-4} + 84x^2y^2z^{-4} - 322x^2z^{-2} - 28y^2z^{-2} + 35)(1 + k^2)^{-5.5} \quad [4\text{-}114]$$

Maximum value = $8.750Mr^3z^{-6}$ Maxima occur at: $x = 0$, $y = 0$

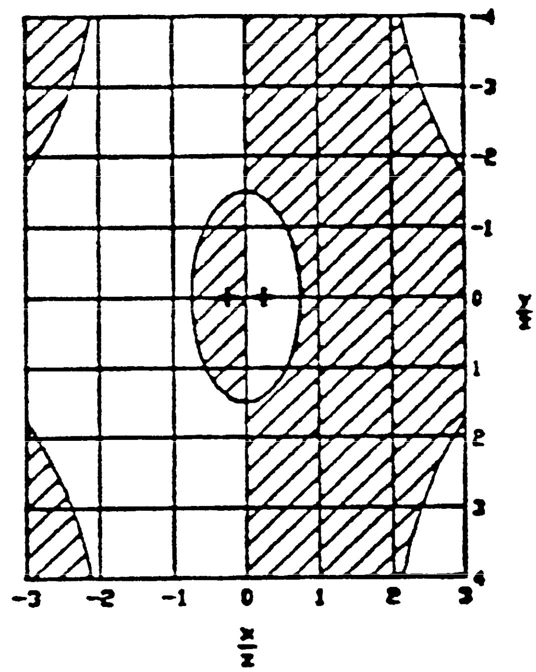
FIGURE 40 (A), Eight radial dipole on the xz plane - case 1.

4.3.3.23 Eight radial dipoles on the xz plane - case 2. Figure 40 (B) snows the configuration and contour map for the magnetic field of eight radial dipoles on the xz plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the eight dipoles are also provided.



Magnetic moment	Coordinates
M	(.924r,0,.383r)
-M	(.383r,0,.924r)
M	(-.383r,0,.924r)
-M	(-.924r,0,.383r)
M	(-.924r,0,-.383r)
-M	(-.383r,0,-.924r)
M	(.383r,0,-.924r)
-M	(.924r,0,-.383r)

A. Configuration.



B. Contour map.

$$B_x = 1.5Mr^3z^{-6}(-34x^4z^{-4} + y^4z^{-4} + 9x^2y^2z^{-4} + 65x^2z^{-2} - 5y^2z^{-2} - 6)(1 + k^2)^{-5.5} \qquad [4\text{-}115]$$

Maximum value = $-9Mr^3z^{-6}$ Maxima occur at: $x = 0$, $y = 0$

$$B_y = 63Mr^3z^{-6}(-x^3yz^{-4} + xyz^{-2})(1 + k^2)^{-5.5} \qquad [4\text{-}116]$$

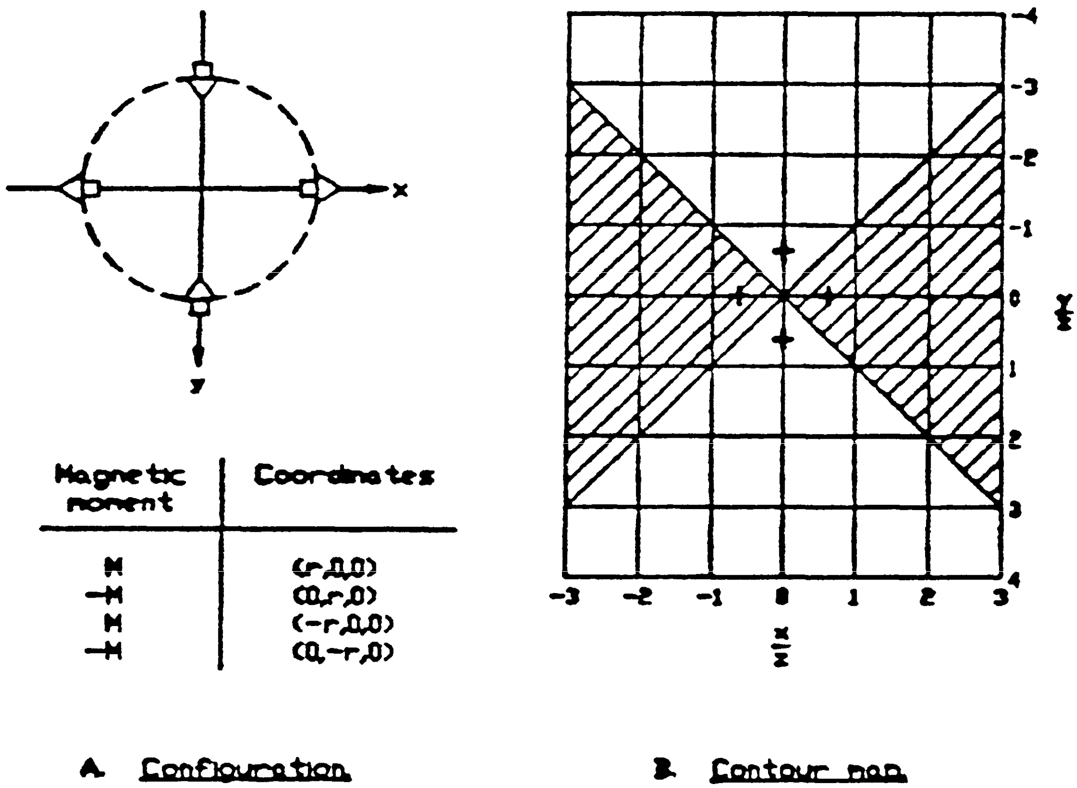
Maximum value = $2.081Mr^3z^{-6}$ Maxima occur at: $x = \pm 0.311z$, $y = \pm 0.311z$

$$B_z = -7Mr^3xz^{-7}(x^4z^{-4} + x^2y^2z^{-4} - 11x^2z^{-2} - 3y^2z^{-2} + 6)(1 + k^2)^{-5.5} \qquad [4\text{-}117]$$

Maximum value = $6.677Mr^3z^{-6}$ Maxima occur at: $x = \pm 0.263z$, $y = 0$

FIGURE 40 (B). Eight radial dipoles on the xz plane - case 2.

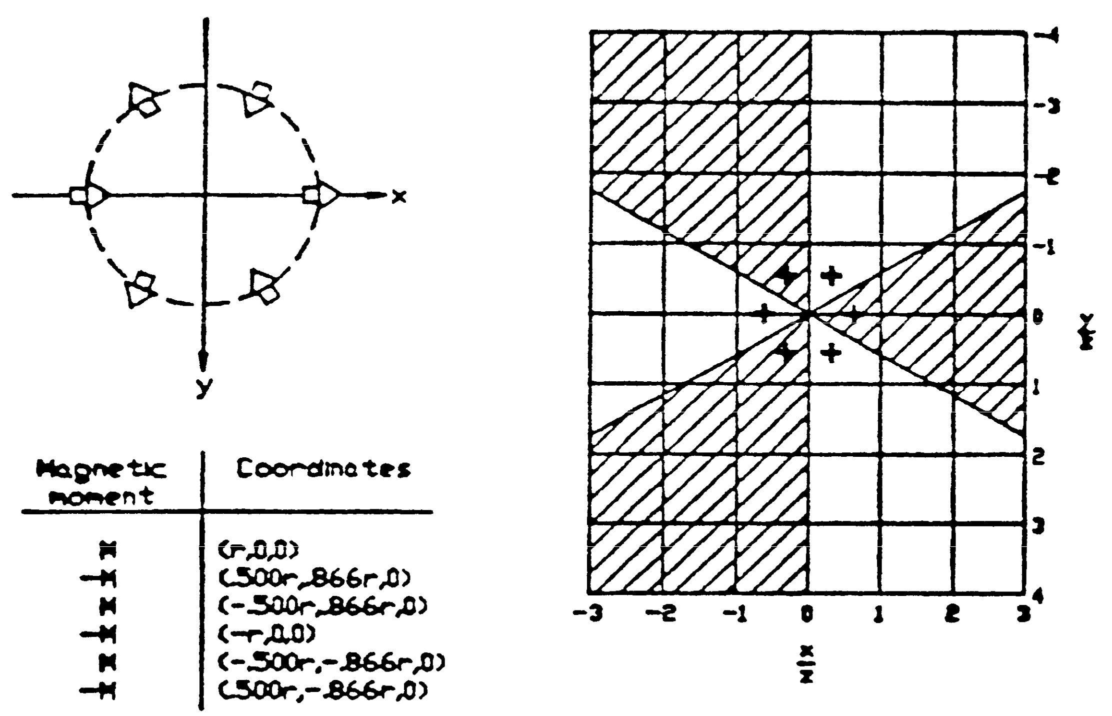
4.3.3.24 Four radial dipoles on the xy plane. Figure 41 shows the configuration and contour map for the magnetic field of four radial dipoles on the xy plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the four dipoles are also provided.



$B_x = 0.6Mrz^{-4}(-3x^3z^{-3} + 7xy^2z^{-3} + 2xz^{-1})(1 + k^2)^{-3.5}$		[4-118]
Maximum value = $0.231Mrz^{-4}$	Maxima occur at:	$x = \pm 0.318z$ $y = 0$
$B_y = 0.6Mrz^{-4}(3y^3z^{-3} - 7x^2yz^{-3} - 2yz^{-1})(1 + k^2)^{-3.5}$		[4-119]
Maximum value = $0.231Mrz^{-4}$	Maxima occur at:	$x = \pm 0.318z$ $y = 0$
$B_z = 3Mrz^{-4}(x^2z^{-2} - y^2z^{-2})(1 + k^2)^{-3.5}$		[4-120]
Maximum value = $0.369Mrz^{-4}$	Maxima occur at:	$x = 0$ $y = \pm 0.632z$

FIGURE 41. Four radial dipoles on the xy plane.

4.3.3.25 Six radial dipoles on the xy plane. Figure 42 shows the configuration and contour map for the magnetic field of six radial dipoles on the xy plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the six dipoles are also provided



A. Configuration. B. Contour map.

$$B_x = 0.225Mr^2z^{-5}(4x^4z^{-4} - y^4z^{-4} + 3x^2y^2z^{-4} - 27x^2z^{-2} + 3y^2z^{-2} + 4)(1 + k^2)^{-4.5} \quad [4\text{-}121]$$

Maximum value = $0.9Mr^2z^{-5}$ Maxima occur at: $x = 0$, $y = 0$

$$B_y = 1.125Mr^2z^{-5}(13x^3yz^{-4} - 15xy^3z^{-4} + 6xyz^{-2})(1 + k^2)^{-4.5} \quad [4\text{-}122]$$

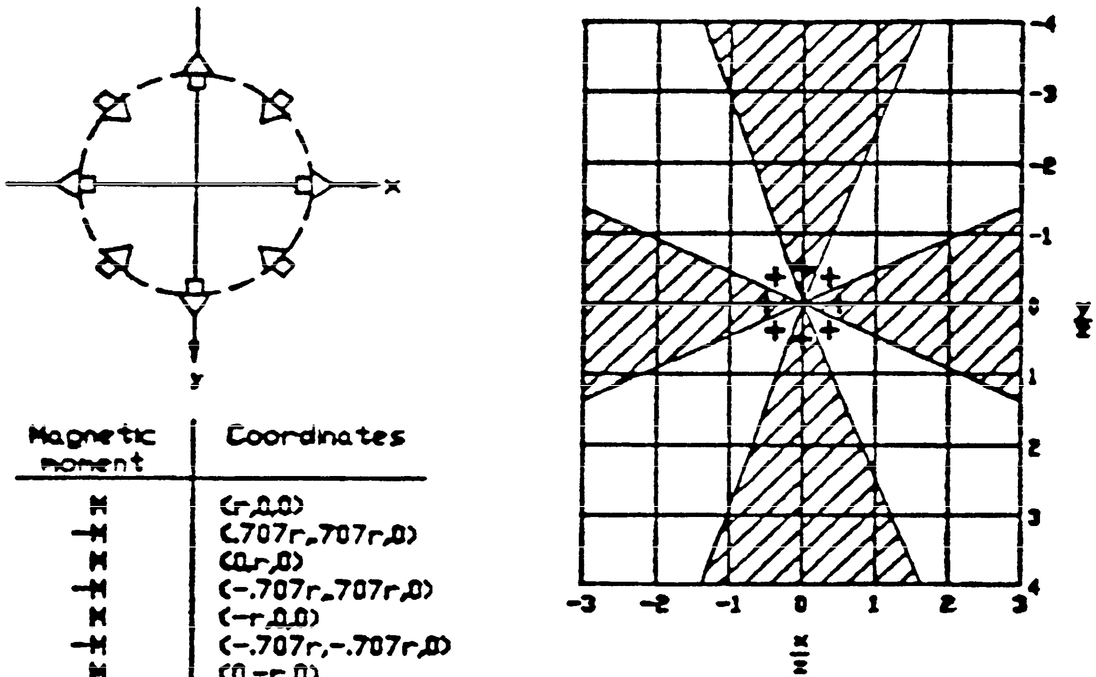
Maximum value = $0.360Mr^2z^{-5}$ Maxima occur at: $x = \pm 0.550z$, $y = \pm 0.320z$

$$B_z = 7.875Mr^2xz^{-6}(x^2z^{-2} - 3y^2z^{-2})(1 + k^2)^{-4.5} \quad [4\text{-}123]$$

Maximum value = $0.448Mr^2z^{-5}$ Maxima occur at $x = \pm 0.707z$, $\pm 0.353z$; $y = 0$, $\pm 0.613z$

FIGURE 42. Six radial dipoles on the xy plane.

4.3.3.26 Eight radial dipoles on the xy plane. Figure 43 shows the configuration and contour map for the magnetic field of eight radial dipoles on the xy plane. The coordinates of each dipole and the magnetic field equations for each component of the combined field of the eight dipoles are also provided.



Magnetic moment	Coordinates
M	(r,0,0)
-M	(.707r,.707r,0)
M	(0,r,0)
-M	(-.707r,.707r,0)
M	(-r,0,0)
-M	(-.707r,-.707r,0)
M	(0,-r,0)
-M	(.707r,-.707r,0)

A. Configuration B. Contour map

$$B_x = 0.25Mr^3xz^{-7}(-5x^4z^{-4} + 9y^4z^{-4} + 4x^2y^2z^{-4} + 46x^2z^{-2} - 66y^2z^{-2} - 12)(1 + k^2)^{-5.5} \quad [4\text{-}124]$$

Maximum value = $0.171Mr^3z^{-6}$ Maxima occur at: $x = \pm 0.966z$, $y = 0$

$$B_y = 0.25Mr^3yz^{-7}(147x^4z^{-4} - 35y^4z^{-4} + 322x^2y^2z^{-4} - 84x^2z^{-2} + 28y^2z^{-2})(1 + k^2)^{-5.5} \quad [4\text{-}125]$$

Maximum value = $0.156Mr^3z^{-6}$ Maxima occur at: $x = 0$, $y = \pm 0.395z$

$$B_z = 15.73Mr^3z^{-6}(x^4z^{-4} + y^4z^{-4} - 6x^2y^2z^{-4})(1 + k^2)^{-5.5} \quad [4\text{-}126]$$

Maximum value = $0.427Mr^3z^{-6}$ Maxima occur at: $x = \pm 0.755z, \pm 0.535z, 0$; $y = 0, \pm 0.535z, \pm 0.755z$

FIGURE 43. Eight radial dipoles on the xy plane.

4 .3.4 Summary of formulas for circular arrays

4 .3.4.1 Circular arrays in the xz plane . Table III summarizes formulas for the maximum value of the vertical component of the magnetic field produced on a plane z meters below the center of a circular array of dipoles in the xz plane. Values are given for two cases:

(a) Case 1. Dipoles so oriented that the z axis is in line with one pair of diametrically opposite dipoles.
(b) Case 2. Dipoles so oriented that the z axis bisects the angle between adjacent dipoles.

4.3.4.2 Circular arrays in the xy plane. Table III summarizes formulas for the maximum value of the vertical component of the magnetic field produced on a plane z meters below the center of a circular array of dipoles in the xy plane. Rotating the array in the xy plane will not change the value of the maximum vertical component on a plane below the array, it will merely change the position or the positions w-here the maximum value occurs .

TABLE 111. Maximum values of vertical component.

Number of dipoles in the array	Array in xz plane		Array in xy plane
	Case 1	Case 2	
4	$-1.8Mrz^{-4}$	$1.097Mrz^{-4}$	$0.369Mrz^{-4}$
6	$-4.5Mr^2z^{-5}$	$3.171Mr^2z^{-5}$	$0.448Mr^2z^{-5}$
8	$8.750Mr^3z^{-6}$	$6.677Mr^3z^{-6}$	$0.427Mr^3z^{-6}$

4.3 4.3 Advantages. It is apparent from table III that it is of some advantage, from the standpoint of small stray magnetic field, to have machines with horizontal axes built such that the z axis bisects the angle between two adjacent main field poles. The advantage to be gained becomes less as the number of poles is increased, but since a machine can be built as easily this way as with two main poles on the vertical axis, it is considered desirable to have the vertical axis bisect the angle between two adjacent main field poles.

4.3.4.4 Vertical axis machines. Table III also indicates that a very substantial reduction of stray magnetic field from the field coils can be obtained by building generators with a vertical axis such that the main field poles are in the xy instead of the xz plane. Unfortunately, this kind of construction introduces mechanical problems. The use of a sufficient number of main field poles gives a small stray magnetic field even for generators with horizontal axes; hence, it is not necessary to go to a vertical axis construction.

4.3.5 Different arrays of dipoles producing. the same vertical magnetic field. A study of the formulas given in 4.3.3 shows that there are a number of cases in which two different arrays of dipoles produce the same vertical component of magnetic field to the degree of approximation which is considered here If we consider the formulas of 4.3.3 to consist of three basic components as follows

$$B = CF(x,y,z) \qquad [4\text{-}127]$$

where: B = the vertical component of the magnetic field.
c = a constant factor which does not change in value when x, y, or z is changed.
F(x,y,z) = an equation whose value is a function of x,y, or z.

If we consider the array of 4.3.3.3, we find:

$$C = 0.6Mc \qquad [4\text{-}128]$$
$$F(x,y,z) = xz^{-5}(4 - k^2)(1 + k^2)^{-3.5} \qquad [4\text{-}129]$$

Now consider other arrays for which F(x,y,z) is identical. We can always make C the same for other arrays by proper choice of the magnetic moments and the distances separating the dipoles. When we do this, the arrays will produce the same vertical component of magnetic field at all points in space for which the formulas are valid, and to the degree of approximation used herein. Examples of different arrays which have the same F(x,y,z) as 4 3.3.3 are 4.3.3.4 and 4.3.3.19.

4.3.5.1 Other arrayS with similar equations. Other arrays with the same function equation are listed below by equation.

(a) $F(x,y,z) = yz^{-6}(-6x^2z^{-2} + y^2z^{-2} + 1)(1 + k^2)^{-4.5}$ [4-130]

(1) The array of 4.3.3.8 rotated bodily through a right angle about the z axis such that the dipoles arc moved to positions on the x axis and brought into parallelism with, the y axis.
(2) The array of 4.3.3.10.

(b) $(x,y,z) = yz^{-6}(4 - 3k^2)(1 + k^2)^{-4.5}$ [4-131]

(1) The array of 4.3.3.9 rotated bodily through a right angle about the z axis such that the dipoles are parallel to the y axis.
(2) The array of 4.3.3.17,

(P) $F(x,y,z) = xyz^{-7}(6 - k^2)(1 + k^2)^{-4.5}$ [4-132]

(1) The array of 4.3.3.11.
(2) The array of 4.3.3.16.

(d)
$$F(x,y,z) = z^{-5}(4x^4z^{-4} - y^4z^{-4} + 3x^2y^2z^{-4} - 27x^2z^{-2} + 3y^2z^{-2} + 4)(1 + k^2)^{-4.5} \qquad [4\text{-}133]$$

(1) The array of 4.3.3.12.
(2) The array of 4.3.3.13.

4.3.5.2 Compensation using arrays of dipoles producing the same vertical magnetic field. Take two different arrays of dipoles which produce the same vertical magnetic field. By opposing one against the other, they compensate and give zero magnetic field to the degree of approximation considered here. Note, however, that this does not mean absolutely zero field. The formulas given above are based upon a power series expansion for the stray magnetic field from a single dipole or from arrays of dipoles. The only term used in the approximate formulas is the first term which does not vanish, At a sufficient distance from the arrays, the higher order terms are small, in comparison to the first term which does not vanish, and can be neglected When two different arrays give the same magnetic field to the degree of approximation considered, all that is meant is that they both have the same first term in their series expansion, but the higher order terms do not cancel and we are left with a magnetic field different from zero.

4.3.6 Calculation of magnetic field using Taylor's series expansion. The magnetic field produced by an array of an even number of dipoles grouped closely about the origin of the coordinate system where the distance from the dipoles to the origin is small (one-tenth or less) when compared to the radial distance to the point where it is desired to compute the magnetic field component can be determined by expanding a function, F(x,y,z), in a Taylor series (see appendix A). The Taylor series expansion for a function of three variables is given as follows:

$$\begin{aligned} F(x,y,z) = {} & f(a,b,c) + (x-a)f_x(a,b,c) + (y-b)f_y(a,b,c) + (z-c)f_z(a,b,c) \\ & + \frac{1}{2!}\Big[(x-a)^2 f_{xx}(a,b,c) + 2(x-a)(y-b)f_{xy}(a,b,c) \\ & + 2(x-a)(z-c)f_{xz}(a,b,c) + 2(y-b)(z-c)f_{yz}(a,b,c) \\ & + (y-b)^2 f_{yy}(a,b,c) + (z-c)^2 f_{zz}(a,b,c)\Big] \\ & + \text{higher order terms} \qquad [4\text{-}134] \end{aligned}$$

4.3.7 Location and magnitude of maximum. Suppose we want to determine at what points on a plane z meters below the center of the array the vertical component of the magnetic field will have its maximum value, and what this maximum value will be. To do this, it is convenient to introduce variables C and 6 and redefine x and y as follows:

$$x = Cz\cos\theta \qquad [4\text{-}135]$$
$$y = Cz\sin\theta \qquad [4\text{-}136]$$

We will demonstrate how the redefined x and y values are used to determine the magnitude and location of the maximum by solving equation 4-81 for its maximum. By making this change in variables, the equation becomes:

$$B_z = 6MbcC^2z^{-5}\sin\theta\cos\theta(6 - C^2)(1 + C^2)^{-4.5} \qquad [4\text{-}137]$$

or by using the trigonometric identity (see table XXIII of appendix A)

$$2\sin\theta\cos\theta = \sin 2\theta \qquad [4\text{-}138]$$

we obtain:

$$B_z = 6MbcC^2z^{-5}\sin 2\theta(6 - C^2)(1 + C^2)^{-4.5} \qquad [4\text{-}139]$$

This equation will have a maximum value when $\sin 2\theta = \pm 1$ and when C has a value which maximizes $C^2(6 - c^2)(1 + c^2)^{-4.5}$. The maximum value of $\sin 2\theta$ will occur when θ - 45 degrees, 135 degrees, 225 degrees, or 315 degrees. To find the maximum value of C we must first differentiate equation 4-131 with respect to C and set the derivative equal to zero. The result is:

$$5C^5 - 46C^3 + 12C = 0 \qquad [4\text{-}140]$$

The values of C which satisfy this equation are:

$$C = 0,\ C = \pm 0.517,\ C = \pm 2.99$$

For these values of C, equation 4-137 will have either a maximum, minimum, or a point of inflection. A little consideration shows that it has a minimum value of zero at C = 0, a maximum value of 0.0528 at C = $\pm$ 0.517 and a minimum value (maximum value of absolute magnitude) of -0.00009 at C = $\pm$ 2.99. The only values in which we are interested are C = $\pm$ 0.517 and θ = 45 degrees, 135 degrees, 225 degrees, or 315 degrees, at which points the vertical component of magnetic field has the greatest magnitude anywhere on a plane z meters below the center of the array, Making use of the preceding values of C and substituting them into the equations relating C and x and C and y yields:

Maximum value at: $x = \pm 0.366z$ **and** $y = \pm 0.366z$

$$B_z = 1.582Mbcz^{-5} \qquad [4\text{-}141]$$

5. PRINCIPLES OF DESIGNING EQUIPMENT AND CIRCUITS FOR MINIMUM STRAY MAGNETIC FIELD

5.1 General The preceding section shows that:

(a) The magnetic field produced by a current loop is proportional to its magnetic moment, ANI, where A is the area enclosed by the loop, N is the number of turns, and I is the current in the loop.

(b) The magnetic field will not be equal to zero unless A = O, N = O, or I - 0, none of which can be achieved in practice.
(c) The conditions found on minesweepers are such that there will inevitably be some current loops that individually produce a magnetic field larger than can be tolerated.

5.2 Principles of design for small stray magnetic field. Some of the individual current loops will inevitably produce a larger stray magnetic field than can be tolerated. Such fields must be compensated for by adding other fields that are, as nearly as possible, equal and opposite. This is the principle of compensation (see 4.3.1.2). It is so general and has so many ramifications that there are advantages to splitting it into a number of smaller and more specific principles. Doing this, and adding a number of other principles that do not fall under the general heading of compensation, we arrive at the following list of principles that have to be kept in mind when designing equipment for small stray magnetic field:

(a) The principle of simplicity (see 5.3).
(b) The principles of minimum magnetic moment and minimum separation (see 5.4).
(c) The principle of zero magnetic moment (see 5.5).
(d) The principle of series compensation (see 5.6).
(e) The principle of parallel compensation (see 5.6 and 5.7)
(f) The principle of self compensation (see 5.8).
(g) The principle of mutual compensation (see 5.9).
(h) The principle of compatibility (see 5.10).
(i.) The principle of minimum disturbance by magnetic material (see 5.11).
(j) The principle of final check (see 5.12).

5.3 The principle of simplicity. The principle of simplicity states that the methods used to ensure small stray magnetic fields should be as simple as possible and should do the following:

(a) Make maximum use of the conductors that are needed in a piece of equipment for it to perform its function; and
(b) Require minimum addition of extra conductors and components that have no purpose other than to reduce the stray magnetic field.

5.3.1 Extreme example. One conceivable way of getting a small stray magnetic field from a generator is as follows:

(a) Take a generator that has been designed and built with no regard for small stray magnetic field.
(b) Measure the stray magnetic field produced by the field current of the generator. Vary the current and take a number of readings.
(c) Measure the stray magnetic field produced by the armature current of the generator. Vary the current and take a number of readings.
(d) Install two sets of compensating coils, which we will call stray magnetic field degaussing coils. Each set would consist of three coils, one in each of three mutually perpendicular planes. One

set would be used to compensate the stray magnetic field produced by the field current of the generator; the other, the stray magnetic field produced by the armature current of the generator.

(e) Install some regulatory system that would respond to changes in the generator field and armature currents and automatically adjust the stray magnetic field degaussing coil currents so as to compensate the stray magnetic field from the generator

We may have to use this arrangement to compensate for generators that have already been built, installed in ships, and found to have large stray magnetic fields, but which cannot be modified without a large expenditure of time and money. This arrangement should, however, be scrupulously avoided in new design. It is completely contrary to the principle of simplicity.

5.3.2 Example of extreme simplicity. Consider a minesweeping cable run vith four single-conductor cables. If the cable arrangements are such that the polarities are as follows:

+ -
+ -

then the stray magnetic fields generated by the cable run will be large; if, however, the cables are rearranged to change the polarities as follows:

+ -
- +

then the resulting stray magnetic field will be small. In the first case, we have two current loops that produce stray magnetic fields that add; in the second case, we have two current loops that produce stray magnetic fields that oppose. The stray magnetic field is significantly reduced by simply changing the connection of the cables. The cable run's performance is not affected by the change, and no additional equipment was required to reduce the stray magnetic field. This is an excellent example of the principle of simplicity. In many cases, extreme simplicity cannot be attained; but the example given illustrates the goal to be approached as closely as possible.

5.3.3 Example of simplicity in field poles of generators and motor. Another illustration of the principle of simplicity is furnished by the cancellation of magnetic fields produced by the field poles of dc generators and motors. For machines with four or more poles, the magnetic fields produced by the poles cancel to an extent that increases with increasing number of poles No additional coils need be added for the sole purpose of compensation. This is accomplished by the field poles themselves, which are indispensable for the operation of the machine. To be sure, we may have to use more poles than would be needed if stray magnetic field is not a consideration, but this is an increase in the number of elements of the same kind rather than the addition of elements of a different kind. It illustrates the application of the principle of simplicity in a case where we are unable to attain the extreme simplicity of a cable run with four cables

5.4 Principles of minimum magnetic moment and minimum separation. AS shown in the formulas given in 4.3, the magnetic field produced by two equal and opposite current loops or dipoles is proportional to the magnetic moment of each, times the distance between them. Therefore, the magnetic moments of individual current loops or dipoles must be as small as possible, and the loops muse be as close together as possible These are the principles of minimum magnetic moment and minimum separation. respectively. Individually and together they point the way to a reduction in stray magnetic field, For example, suppose we have two equal and opposite dipoles a short distance apart.

(a) If we halve the magnetic moment of each dipole and leave the separation unchanged, we halve the magnetic field.
(b) If we halve the separation and leave the magnetic moments unchanged, we halve the magnetic field.
(c) If we halve both the magnetic moment and the distance, the magnetic field is reduced to one fourth of its original value,

5.5 Principle of zero net magnetic moment. The arrays of dipoles considered in 4.3 show that the resultant or net magnetic moment for each array is zero. This is the best overall condition for a small stray magnetic field from any number of dipoles. Expressed mathematically, the principle of zero net magnetic moment requires the following:

$$\Sigma M_x = 0 \qquad [5\text{-}1]$$
$$\Sigma M_y = 0 \qquad [5\text{-}2]$$
$$\Sigma M_z = 0 \qquad [5\text{-}3]$$

where ΣM_x, ΣM_y, and ΣM_z denote the sums of the x, y, and z components, respectively, of the magnetic moments of the current loops or dipoles under consideration.

5.5.1 Application to equipment. Consider a relatively large piece of equipment, for example, one that fills a cube 1.5 meters on a side. Suppose that there are large but equal and opposite current loops on two opposite faces of the cube. The net magnetic moment is zero, but the stray magnetic field will be large because of the large magnetic moment of each loop and the large separation between the loops. Zero net q agnetic q oment alone is not enough to ensure a small stray magnetic field. The principle of zero net magnetic moment must be applied together with the principles of minimum moment and minimum separation. In other words , to ensure a small stray magnetic field for a piece of equipment we must meet the following conditions:

(a) We must have zero net magnetic moment for the whole equipment
(b) We must have zero net magnetic moment for each of a number of subgroups of loops. The loops in each subgroup should be relatively close together and the combination of all the loops in all the subgroups represents all the loops in the equipment. For example, one subgroup might contain all the current loops in a single plane or in two parallel planes very close together; while another subgroup might include all the loops in a particular cubic meter of the equipment, and so forth.

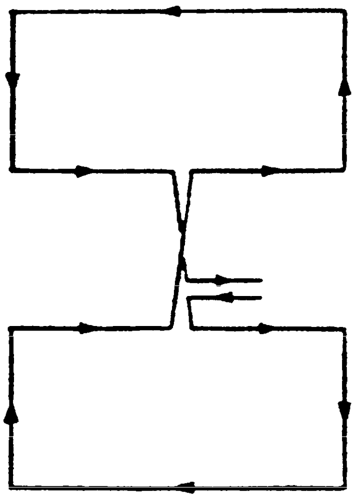
5.5.2 Limitation of the principle. Let us consider the piece of equipment of 5.5.1. Divide the 1.5 meter cube into 1000 small cubes, each 15 centimeters on a side. Now suppose that the net magnetic moment for the current loop in each of the small cubes is zero. The principle of zero magnetic moment is therefore satisfied for the whole piece of equipment and for each of the small cubes. Furthermore because of the small size of the cubes, it is to be expected that the magnetic moments of the individual current loops each contains will be small, and it is certain that they will be quite close together. Despite this, it is not certain that the stray magnetic field generated by this piece of equipment will be small. For instance, if we suppose that in each of the small cubes the current loops are exactly alike and are oriented in the same way, the stray magnetic field generated by the equipment would then be approximately 1000 times as large as the stray magnetic field generated by one of the small cubes. While the stray magnetic field of one of the small cubes may indeed be negligible, the magnitude of the stray magnetic field generated by the entire equipment could be too large to be tolerated. Although the preceding example is a rather unlikely situation, it is possible. Application of the principles of minimum magnetic moment, minimum separation, and zero net magnetic moment does not guarantee that the stray magnetic field generated by the equipment will be small. It is, therefore, necessary to make a final check to be completely certain,

5.6 Principles of series and parallel compensation. Series compensation is illustrated on figure 44 (A), parallel compensation on figure 44 (B).

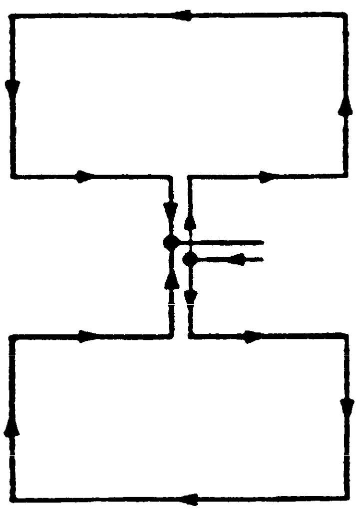
5.6.1 Comparison of series and Parallel compensation. Suppose that on figure 44 (A) and figure 44 (B) the current loops are coplanar, have the same size, shape and number of turns, and carry the same current. When these conditions are satisfied, the two loops will generate exactly the same stray magnetic field, and there is no difference between series and parallel compensation, Nonetheless, series compensation is preferred because of the greater likelihood that the conditions for compensation will be satisfied (see 5.6.1.1).

5 6.1.1 Inequality of current division

5.6.1.1.1 Series compensation. There is no problem with unequal current division in series compensation. Let us consider two opposing loops connected for series compensation. The current will, at all times, be the same in both loops; if the current changes in one loop it must change in the other. There is the possibility that insulation will deteriorate so that the current can leak out of the circuit and cause the currents in the two loops to differ. However, if even a few amperes leak out of the loop the insulation will deteriorate so rapidly and will be so badly damaged that the circuit cannot be used. When two loops are connected in series, we can be sure either that the current is the same in both loops or that, if it differs by a significant amount, insulation failure will take the circuit out of use,



A. SERIES COMPENSATION



B. PARALLEL COMPENSATION

FIGURE 44. Series and parallel compensating current loops.

5.6.1.1.2 Parallel compensation. Conditions are different with two loops connected for parallel compensation. Suppose that the two loops are of equal area, number of turns, and resistance. They will divide the total current equally and compensation should be satisfactory. The difficulty with parallel compensation is that so many things can happen to disturb this satisfactory situation. One loop may heat up more than the other, its resistance will become higher, and the current in it will be less than the other loop. Contact resistance may change over the course of time, and, since the total resistance of each loop will be low, a change in contact resistance may be enough to create an appreciable inequality of current division. In the extreme case, one loop could develop an open circuit, force the other to carry the total current, and completely destroy compensation. Furthermore, this condition might exist for a considerable period of time before it becomes apparent. In view of this, it is apparent that series compensation is preferred to parallel compensation whenever series compensation can be employed. When parallel compensation is used, either because series compensation is impractical, or because parallel compensation has other advantages which justify its use, the precautions given in 5.7 should be observed.

5.7 Use of parallel compensation. There are a number of cases where parallel compensation must be used. A minesweeping generator has a number of parallel circuits that are inherent in the construction of the machine. Parallel compensation is also needed for minesweeping cable runs. In other cases it has advantages that justify its use even when series compensation might be possible. It should be kept in mind that while parallel compensation has drawbacks when compared to series compensation, it has advantages as well and can be used effectively if the precautions given below are observed.

5.7.1 Symmetry Take two circuits that are connected in parallel so that the stray magnetic field from one circuit compensates that from the other. These circuits should be as symmetrical as possible in all respects. Loop areas and turns should be equal so that equal currents are needed for compensation. Resistances should be equal so that currents will be equal. Time constants should be equal so that currents in the two circuits rise and fall together at the beginning and end of a sweep pulse. If this condition is not satisfied, we will have compensation when the currents in the two circuits reach their steady-state values, but will lack compensation during the transients at the beginning and end of a sweep pulse. Careful design and workmanship are needed to make sure that the currents in the two parallel circuits are equal when the equipment is built, and that everything possible has been done to make sure that this condition will not change in the course of time.

5.7.2 Precaution to minimize the effects of unequal current division. In addition to doing everything possible to ensure equal current division between parallel circuits, the equipment should be designed and built to minimize the effects of unequal current division if it occurs. For example, cable runs are twisted for this purpose (see 6.2.3); the folded design of disconnect switches minimizes the bad effect of unequal current division (see 6.4.8); and proper design of generator brush rigging gives current loops that have zero net magnetic moment regardless of whether the total armature current of the generator is or is not equally divided between the brush sets connected in parallel (see 7,3).

5.8 Principle of self compensation. The principle of self compensation means simply that each individual piece of equipment should be compensated by itself. The principle should be applied to each minesweeping generator, disconnect switch box, contactor panel, circuit breaker, or other component used in the construction of the larger items of equipment. The principle of self compensation should be applied to all items, large and small, because if each item is individually compensated so that its stray magnetic field is small , then the stray magnetic field from all the equipment on the ship will be small, regardless of whether some or all of the equipment is in operation

5.8.1 Orientation. Some equipment, such as minesweeping generators, large contactor panels, and switchboards are designed and built for installation with only one orientation. A minesweeping generator, for example, is built for installation with its axis 'horizontal, and cannot be stood on end and installed with its axis vertical. Smaller items of equipment, however, may be installed in several different ways depending on the available space and the arrangement of other equipment on the ship. For all such equipment, the principle of self compensation should be applied so that the stray magnetic field below the equipment will be small regardless of its orientation. Normally, the vertical component of the magnetic field at a given distance below. the equipment will vary with the orientation of the equipment. It might be extremely difficult or even impossible to design the equipment so that the stray field is independent of orientation. Fortunately, this is not necessary. All that is necessary is to make sure that the largest stray magnetic field for any orientation is small enough to be acceptable. If this objective is attained, the ship designer and shipbuilder will have greater freedom in arranging the equipment.

5.9 Principle of mutual compensation. The principle of mutual compensation is applicable when there are two or more pieces of the same equipment installed on the ship. This involves connecting the pieces of equipment so that the stray magnetic field of one opposes that of the other.

5.9.1 Utility. The utility of the principle of mutual compensation can be illustrated by considering the case of a minesweeper with two minesweeping generators. The first objective is to use the principle of self compensation to make the stray magnetic field of each generator as small as possible. Having done this , there are still two generators, which can be used either alone or in parallel and which have stray magnetic fields different from zero. These two generators can be connected so that their stray magnetic fields add, or, in accordance with the principle of mutual compensation, so that they oppose. When the two generators are used in parallel, we get approximately twice the magnetic field of one in the first case, and perhaps less Than the magnetic field of one in the second case. This has been observed on a number of minesweepers. Therefore, there is a substantial improvement obtained by using the principle of mutual compensation. At the same time, the principle of mutual compensation is secondary to the principle of self compensation. The most important concept is that of making the stray magnetic field from each generator as small as possible. This is necessary to minimize the stray magnetic field when only one generator is in use, as well as when both are in use. If each generator, individually, has a large stray magnetic field, the two together could have a stray magnetic field that would be too large even if we use the principle of mutual compensation Mine - sweeping generators are large machines: they cannot be placed very close together.

Consequently, mutual compensation is not overly effective. It helps, somewhat, but not enough to bring a large reduction of stray magnetic field so that two generators with large magnetic fields could be connected in opposition with the resulting stray magnetic field small enough to be acceptable.

5.10 Principle of compatibility. The principle of compatiblity means simply that each item of minesweeping equipment should be designed in such a manner that its terminals are arranged for easy, convenience, and stray-magnetic. field-free connection to the cable run that connects it to the rest of the system. For instance, the minesweeping current will, as a minimum, go from the minesweeping generator through a cable run to the terminal box and out to the sweep tail. It may also pass through a disconnect switch or contactor panel. These interconnections must be easily made and not generate a large stray magnetic field.

5.10.1 Justification. Without the principle of compatibility, it would be possible to have minesweeping generators, terminal boxes, and other equipment that generate small stray magnetic fields but whose terminals are arranged such that a large stray magnetic field may be set up at the connection of the equipment to the cable run. This situation would have to be corrected by the shipbuilder through the use of a specially designed and constructed adapter to allow interconnection of equipment without the generation of large stray magnetic fields. Obviously, this case can be avoided if the principle of compatibility is considered during the design of the minesweeping equipment (see 6.3).

5.11 Princip le of minimum disturbance by magnetic material.

5.11.1 Minimization of magnetic material. The use of magnetic material aboard a minesweeper should be kept to a minimum The greater the amount of magnetic material, the larger the undegaussed magnetic signature of the ship will be. Therefore, the degaussing system on the minesweeper will have to work much harder. In addition, magnetic material placed near current carrying conductors may produce dangerously large stray magnetic fields. Suppose, for example that the positive and negative legs of a circuit pass on opposite sides of a long iron tie rod or stanchion. This forms a current loop enclosing a long magnetic core. The stray magnetic field of this loop will be much larger than that of a loop of the same area that does not enclose a magnetic core (see 3.3.2).

5.11.2 Arrangement of conductors with respect to magnetic material. While the use of magnetic material on a minesweeper should be kept Co a minimum (see 3.3.2), there are some instances where magnetic material must be used. For example , motors and generators require the use of magnetic circuits to operate properly, In these machines, the arrangement of conductors with respect to the magnetic material is fixed by the need to design the equipment to function properly. Specifically, the conductors in the field coils must form coils that enclose the magnetic field poles, and the armature conductors must form coils around the armature core. However, with careful design, motors and generators can produce only small stray magnetic fields. Current carrying conductors and magnetic material must not be arranged in such a way as to enhance the magnetic field produced by the conductors Suppose for example, that the positive and negative armature leads of a generator are passed through two holes drilled in the iron voke of the generator The stray magnetic field will be increased for two reasons : first, drilling holes in the yoke will destroy its magnetic symmetry; and, second, the iron between the positive and negative leads forms a magnetic core for a current loop. To avoid these problems, the armature leads, as well as any other leads, should be taken out through the nonmagnetic end bells of the machine The general rules for arrangement of conductors are as follows:

(a) Never arrange conductors and magnetic materials in such a way that the magnetic material forms a core for a current loop unless this arrangement is dictated by the necessity of designing equipment so that it will perform its intended function.
(b) If it is absolutely necessary to pass a circuit through a magnetic enclosure or other magnetic material, the plus and minus legs of the circuit must be as close together as possible and both must pass through the same hole in the magnetic material.
(c) Care must be taken in the arrangement of conductors and magnetic material to avoid any possibility that the magnetic material will cause an unnecessary increase in the stray magnetic field generated by the current in the conductors (see 3.3.2.2).

5.12 Principle of final check.

5.12.1 Need for final check. The principles that have been discussed so far are guidelines that muse be followed in order to design and construct a piece of equipment having a small stray magnetic field. They do not, however, automatically ensure that the stray magnetic field will be small. For example, the principle of zero net magnetic moment can be applied to the entire space occupied by a piece of equipment and to each of a large number of smaller regions into which this space can be subdivided, and still have a stray magnetic field that is too large to be acceptable (see 5.5.2), This possibility requires that a final check be made after a tentative design is completed. The design should be rejected unless it checks out satisfactorily.

5 12.2 Nature of the check. Ideally, the check calculations should be so complete and accurate that they give an estimated value for the stray magnetic field to within a few percent of the values that would be measured if the equipment were built and tested. This goal should be attainable for relatively simple equipment which has no magnetic material used in its construction, but may not be attainable for complicated equipment, particularly for equipment in which magnetic material is used. For such equipment, a somewhat simplified check will have to be used. This will involve a check made without taking into account the effect of iron, followed by the best estimate that can be made OIL the effect of the iron. The steps in detail are as follows:

(a) First make a careful analysis of the complete equipment to resolve it into all the current loops it contains. This step is basic to all the rest.
(b) If a suitable computer is available for use, compute the vertical component of magnetic field produced by each current loop at a specified point on a plane at the desired distance below the equipment, and add the results for the different loops to find the total for the whole equipment. The stray magnetic field from each loop must be calculated to many significant figures in order to

give an algebraic sum which will be accurate to a few significant figures. The large volume of calculations makes calculation without the use of a computer impractical.

(c) If a suitable computer is not available, the check will have to be made using a simpler method, as follows:

1. Divide the total nunber of current loops into a number of subgroups, each subgroup containing two, four, six, of eight loops of equal magnetic moment with a symmetrical arrangement and polarity so as to give a small magnetic field. This can always be done for equipment that has been designed with opposing current loops so that the net magnetic moment is zero. If the net magnetic moment is almost but not quite equal to zero, it should be possible to proceed this way: Suppose that there are four loops symmetrically arranged, and make an array like that of 4.3.3.10, except that the magnetic moments are 100, -100, 100, and -95 instead of -100 for the loops. At the -95 loop acid two hypothetical loops, one with a magnetic moment of +5 and one with a magnetic moment of -5. Now divide all the loops into two subgroups: one consisting of a symmetrical array of four loops with magnetic moments of 100, -100, 100, -100, and the other consisting of a single loop of magnetic moment +5. In one way or another, the complete group of loops in the equivalent can be broken down into a number of subgroups.
2. Compute the maximum value of the vertical component of magnetic field produced by each subgroup of current loops. This can be done using the formulas of 4.3 or, if necessary, by deriving new formulas for subgroups of a kind not covered by 4.3.
3. Suppose, to be specific, that there are six subgroups and that the maximum values of vertical component of magnetic field (without regard to algebraic sign) are Bz_1, Bz_2, Bz_3, Bz_4, Bz_5, and Bz_6. The worst possible case would be for all six maxima to occur at the same place and to have the same sign. In this case, the magnitude of the total stray magnetic field would be:

$$Bz_1 + Bz_2 + Bz_3 + Bz_4 + Bz_5 + Bz_6$$

If this value is small enough to be acceptable, the design can be considered satisfactory.
4. If the value computed above is too large to be acceptable, it does not mean that the design is unsatisfactory. It may be that the maxima for the different subgroups occur at fairly widely spaced locations, or that some of the maxima are of a different algebraic sign than others. It is very likely that the correct value for the maximum for the whole equipment will be less than the sum of the absolute values of the maxima of the subgroups, and may even be less than the maximum for any one of the subgroups. This possibility should be checked before concluding that the design is unsatisfactory.
5. If a careful check shows that the calculated value of stray magnetic field is too large to be acceptable, the design should be changed.

(d) The preceding calculations have been made on the assumption that no iron is present. This will be the case for certain types of equipment, and for these the calculated values should be in good agreement with those measured after the equipment is built. For other equipment, such as generators and motors, iron will inevitably be present. Very little is known about the quantitative effect of iron. For several small motors, the measured stray magnetic field is approximately twice as large as the stray magnetic field calculated using the assumption that no iron is present. It is to be expected, therefore, that with iron present the stray magnetic field will be several times larger than that calculated on the assumption of no iron. The design of the equipment must make allowances for this.

5.13 Objectives of desire. The ultimate objective in designing equipment for minesweepers is to develop a minesweeper that has a very small stray magnetic field during all conditions of operation, This objective should be met with certainty and as simply as possible. To this end there are two intermediate objectives :

(a) Each piece of equipment should have an extremely small stray magnetic field (principle of compensation) .
(b) The different pieces should fit together easily and naturally with no chance that stray magnetic fields will be caused by the connections between them (principle of compatibility).

When these intermediate objectives are attained, the task of the shipbuilder is simplified and it is likely that the ultimate objective will be reached.

6. DESIGN OF MAGNETIC MINESWEEPING EQUIPMENT

6.1 General. In this section, we will consider the design of magnetic minesweeping equipment that is used to carry minesweeping current. The equipment. listed below will be considered in some detail. Their designs will serve as examples for the design of other equipment not considered here.

(a) The minesweeping cable runs.
(b) Disconnect switch boxes, contactor panels, or other equipment that may be connected in the cable run between the generator and the terminal box.
(c) The terminal box at which connection is made to the minesweeping cable.
(d) The minesweeping cable.

6.1.1 Interfacing equipment In addition to the equipment that carries minesweeping current, there is also a great deal of other equipment on a minesweeper carrying currents that are considerably smaller (except perhaps for short internals of time) than the minesweeping current. Such equipment includes

(a, Switchboards and distribution system.
(b) Motors.
(c) Storage batteries.
(d) Galley ranges.

6.2 Cable runs. We will consider the arrangement of power cables so that we may reduce to a minimum the stray magnetic field produced by the current in these cables. The design considerations discussed herein are applicable to shipboard installation of dc and alternating current (at) power cables aboard minesweepers and other special installations where stray magnetic fields must be reduced to a minimum

6.2.1 Cable runs modeled by current loops. Suppose we have a long cable run from a generator to a motor that consists of a single conductor cable in the positive leg and a single conductor cable in the negative leg. We can model the cable run by current loops (see figure 45). If we introduce hypothetical conductors between A and B and between C and D, we break the cable run into a small current loop from the generator to AB, a long current loop from AB to CD, and a small current loop from CD to the motor. The design considerations for cable runs should govern the long current loop from AB to CD.

6.2.1.1 Single current loop. With only one single-conductor cable in the positive and negative legs of the long cable run discussed in 6.2.1, there will be only one current loop. This will not be a small current loop (see 4.5) since the distance to the point where we are interested in the magnetic field may be less than the length, or large dimension, of the cable run. The magnetic field of the current loop can be calculated by using the formulas for large loops (see 4.2.2). Even if the cables forming the single current loop cable run are placed tightly together so that the geometric interior of the current loop has no area, the stray magnetic field can be appreciable.

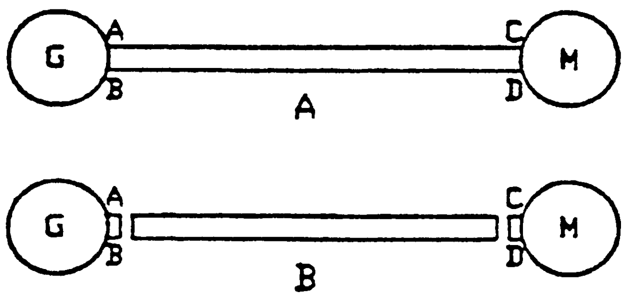


FIGURE 45. Current loops in cable runs.

6.2.1.2 Mulitiple current loops. Minesweeping currents are so large that two or more single conductor cables are used in each leg of a minesweeping cable run to provide adequate current-carrying capacity. With such an arrangement, the cable run will have two or more current loops Depending upon how the cables are connected, the magnetic fields from these loops will add or subtract.

6.2.2 Power cables and cable configurations. Power cables and cable configurations will be in accordance with 6.2.2.1 through 6.2.2.2.2.

6,2.2.1 Dc power cables. Design considerations for dc power cables will be in accordance with 6.2.2.1.1 through 6.2.2.1.5.

6.2.2.1.1 Use of single-conductor cables. For best results, we will use an even mulitple of parallel, single-conductor cables in the positive and negative branches of a dc cable. Quadded cables consisting of four single-conductor cables will be preferred for dc power cable connections. We will avoid cable runs consisting of only one or an odd multiple of single-conductor cables in the positive and negative branches of a dc circuit. This is because an odd number of current loops makes magnetic field compensation by arrangement of the conductors impracticable and results in a large magnetic field.

6.2.2.1.2 Use of double-conductor cables. If four single-conductor, quadded cable is unavailable or not practicable, we may use double-conductor cable in a dc cable run. It will be twisted such that the cable's conductors are transposed at regular intervals by the twist of the conductors to form the lay of the cable or the conductors are concentric and the size of the cable is small enough to be handled conveniently for the selected application.

6.2.2.1.3 Arrangement of cables for opposing current loops. We will arrange parallel, single-conductor cables in the positive and negative branches of a dc cable run in a manner that will result in opposing (subtractive) current loops, so that their magnetic fields are opposed and self-compensating. Figure 46 illustrates typical cable arrangements that are preferred and nonpreferred and gives their respective magnetic flux density values.

6.2.2,1.4 Equal current division among cables connected in parallel. We will select and install parallel, single-conductor cables in the positive and negative branches of a dc cable run so that the resistance of each branch circuit in parallel is as nearly equal as possible in order for the circuit current to divide equally among all of the conductors connected in parallel. Figure 47 illustrates the effects of unequal current division in cables connected in parallel.

6.2.2.1.5 Unequal current division in double-conductor cables. We will not connect two or more double-conductor cables in parallel dc cable runs since unequal division of current between conductors connected in parallel can give rise to a magnetic field of objectionable magnitude, If we have a positive conductor in one cable carrying 1,100 amperes (A) and the negative conductor carrying 900 A, while in the other cable the positive conductor carries 900 A and the negative carries 1,100 A, we have an arrangement that is equivalent to two single conductor cables each carrying 200 A. The center-to-center distance between the cables will be considerable because of the size of the cables, and the magnetic field produced by such a division of current will be considerable. This is the reason why double conductor cables connected in parallel should not be used in minesweeping cable runs.

6.2.2.2 AC newer cables. Design considerations for ac power cables will be in accordance with 5.2.2.2.1 through 6.2.2.2.2.

6.2.2.2.1 Phase conductors in a common cable. We need not be concerned with special cable arrangements if all phase conductors of an ac circuit are in the same cable.

6.2.2.2.2 Phase conductors in separate cable . If we find it necessary to install ac phase conductors in separate cables, these cables will not be grouped on the same cable hangers with cables carrying dc and they will be as close together as practicable throughout the entire length of the cable run Closed loops of magnetic material around a phase conductor cable and the placement of any magnetic q aterial between the phase conductor cables will be avoided.

6.2.3 Spiraling the cable run to minimize the effects of unequal current division. A dc circuit cable run array consisting of multiple, single-conductor cables will be spiraled (twisted) on itself from one cable hanger to the next Even when the greatest of care is exercised to ensure equal current division, absolute equality is difficult to obtain and maintain in low resistance parallel circuits such as those involved in minesweeping cable runs. Unequal heating of the cables, for example, will cause differences in the temperatures and resistances of the conductors connected in parallel and corresponding differences in the currents they carry. Spiraling the cable run on itself provides a way to minimize the effects of such inequality in current division. Cable runs less than 1.5 meters long need not be spiraled.

A. Typical preferred cable arrangements.

Cable arrangement	Max magnetic flux density - nanotesla
(+)(-)(-)(+)	2.0
(+)(-) (-)(+)	1.0

B. Typical non-preferred cable arrangements.

Cable arrangement	Max magnetic flux density - nanotesla
(+)(-)(-)(+)(-)(+)	90
(+)(+)(-)(-)	570
(+)(-)(+)(-)	280
(+)(-) (+)(-)	280
(+)(+) (-)(-)	150
(+)(+)(+)(-)(-)(-)	850
(+)(-)(+)(-)(+)(-)	280
(+)(-)(+) (-)(+)(-)	50
(+)(+)(+) (-)(-)(-)	150
(+)(-)	310

NOTES :

1. Cable arrangement figures illustrate the cross-sectional views of the single conductor cables in a cable run "+" indicates positive conductor cables connected in parallel. '-" indicated negative conductor cables connected in parallel. For a given arrangement, if all "+" are changed to "-" and all "-" are changed to "+", the same indicated maximum flux density will result.

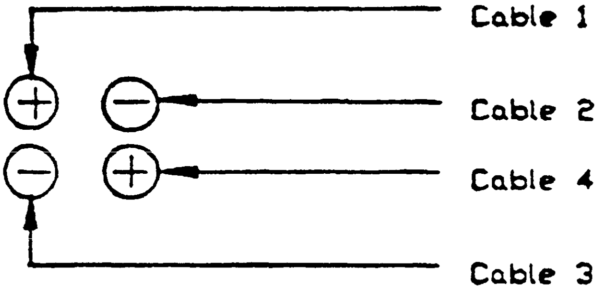
The indicated maximum flux density is the vertical component calculated on a plane 7. 5 meters above or below the cable run based upon cables with an outside diameter of 40 mm that are touching

FIGURE 46. Arrangement of single-conductor dc cables for opposing current loops.

each other so that the center-to-center distance between adjacent cables is 40 mm. The length of the cable run is 18 meters and the direct current magnitude is 2,000 A (1,000 A per cable for cable runs with two cables in each branch and 667 A per cable for cable runs with three cables in each branch). No magnetic materials are assumed to be present anywhere in the region around the cable run and the total current is assumed to be equally divided between conductors connected in parallel.

FIGURE 46. Arrangement of single-conductor dc cables for opposing current loops - Continued.

Cable arrangement:

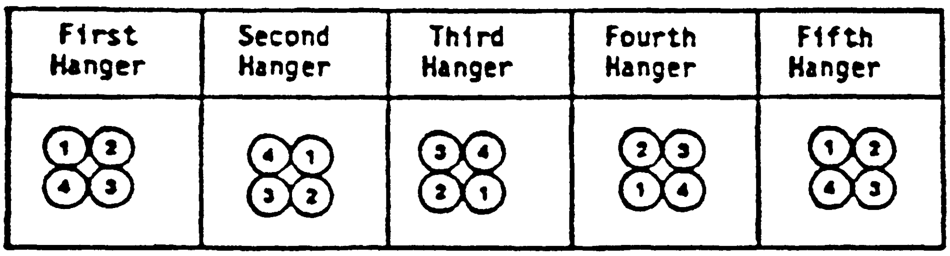


Current division among conductors (amperes)				Maximum magnetic flux density (nanotesla)
Cable 1	Cable 2	Cable 3	Cable 4	
1,000	1,000	1,000	1,000	1
2,000	2,000	0	0	280
0	1,000	1,000	2,000	150
900	900	1,100	1,100	30

FIGURE 47. Effects of unequal current division in single - conductor cables connected in parallel.

6 .2.3.1 Spiraling a four cable quad arrangement. If we have a preferred quad arrangement (see 6.2.2.1.3) with current unequally divided so that the two upper cables each carry 900 A, and two lower cables each carry 1,100 A, the resultant magnetic field will be considerable and directed upward (taking the plus sign to indicate a current going into the paper) at all points along the length of the cable run. The result, as pointed out in 6,2.2.1.3, is a magnetic field much greater than it would be if each cable carried 1,000 A. If we spiral or twist the quad array of cables from one cable hanger to the next, the configuration at five consecutive cable hangers shall be as shown on figure 48. If the magnetic field caused by an unequal current division is directed upward at the first cable hanger, it will be directed to the right at the second; down at the third; to the left at the fourth; up at the fifth, and so on. The cable run is thus broken up into a number of short lengths of opposing polarity so that their net effect will be small.

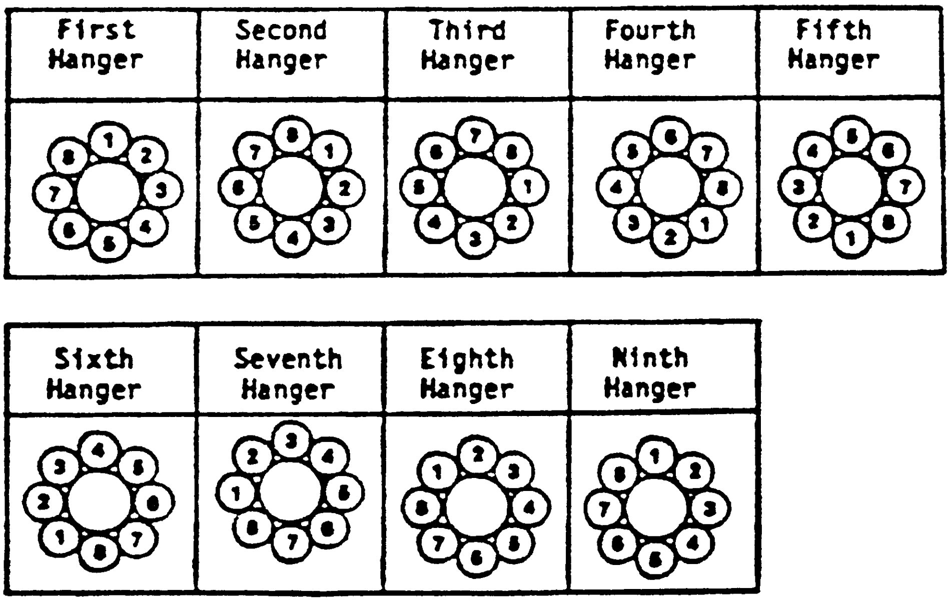
6.2.3.2 Spiraling eight cables in a circular array. We can minimize the magnetic field of an eight cable arrangement by forming a circular array of eight cables spiraled around a central core (see figure 48). Just as in the case of the twisted quad, the direction of the magnetic field caused by any inequality in current division will turn through 45 degrees from one cable hanger to the next. The result is that the magnetic field produced by any inequality in current division will be less if the cable is spiraled than if it is not. It is immediately apparent that the same conclusion holds for a circular array of any even number of cables, the cables being alternately plus and minus.



A. Four cable array configuration at five consecutive cable hangers (cross-sectional views)

NOTE:
1, 2, 3, and 4 denote each of the four cables in a quad array. Each cable shifts 90 degrees from one cable hanger to the next.

FIGURE 48. Typical spiraling arrangements for dc circuit cable array



B. Eight cable circular array configuration at nine consecutive cable hangers (cross-sectional views)

FIGURE 48. Typical spiraling arrangements for dc circuit cable array - Continued.

6.2.3.3 Sprialing eight cables in a rectangular array. If we have eight cables that are arranged in a double quad (see figure 49) the magnetic field will be small, provided that the currents in all cables are of equal magnitude. If we laid the cable run straight and the current is unequally divided, the magnetic field generated may be appreciable. Spiraling the cable run would minimize the field; however, it would be very difficult, if not entirely impractical, to spiral a double quad array of eight cables on itself. It will do us no good to spiral the right-hand and left-hand quads separately so as to end up with two twisted quads lying side by side because the total current may be divided in such a way that the sum of the currents in the four right-hand cables may be plus 200 A, for example, and the sum of the currents in the four left-hand cables may be minus 200 A. The two twisted quads lying side by side would then be roughly equivalent (so far as stray magnetic field is concerned) to two single conductor cables each carrying 200 A and spaced center to center by a distance approximately equal to the separation between the centers of the two quads. This can give rise to a magnetic field of considerable magnitude; consequently, two twisted quads do not furnish a solution to the problem that arises when eight cables are necessary to carry the current from one generator.

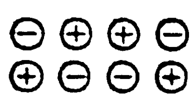
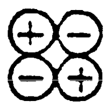
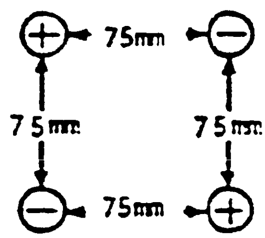
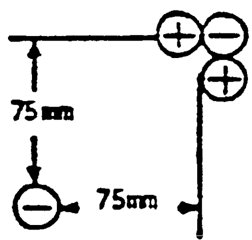


FIGURE 49. Double quad rectangular array.

6.2.3.4 Other numbers of cables. Any even number of cables spiraled about a central core with the cables alternately plus and minus will produce a small stray magnetic field. For convenient connection to three-terminal equipment the number of cables will be an integral multiple of four because one-half of the cables must be connected to the central terminal and one-fourth of the cables to each of the two outer terminals. An adapter between the cable run and the equipment will be required if the number of cables is not an integral multiple of four. This does not mean that the number of cables in the cable run must be an integral multiple of four. It is conceivable that generator rating and available cable sizes might be so related that four cables vould not be enough, six just right, and eight more than enough. In such a case it would be desirable not to throw out six cables immediately in favor of eight, but to make a fairly detailed comparison between a six cable run with adapters and an eight cable run without adapters. We might discover that the overall advantage might be with the six cable run.

6.2.4 Cable spacing. Equality between two opposing current loops can be disturbed by inequality of current division, a condition that we have already considered, or by inequality in cable spacing. Suppose we have a quad array in which the upper two cables are spaced farther apart than the lower two. In this case the upper and lower current loops will not be of equal strength, and the principle of zero net magnetic moment (see 5 5) will not be satisfied even if the loop currents are equal. If we want to avoid the magnetic field that would be produced by opposing loops of unequal strength, we must arrange the cables in a quad arrangement so that the cables are pressed tightly together all along the cable run, with their centers at the corners of a square In a circular array of cables arranged about a central core, we must have the cables equally spaced. This condition will be automatically satisfied if the cables are all of the same diameter, the central core is of the right diameter, and the cables are tightly clamped against the core. Figure 50 illustrates the typical effects of excessive cable separation in a quad arrangement.

Cable arrangement	Maximum magnetic flux density (nanotesla)
	0.42
	2.62
	31.4

NOTE:

1. The indicated maximum flux density is calculated at a distance of 7.5 meters based upon cables with an outside diameter of 50 millimeters (mm). The length of the cable run is 1.25 meters and the dc magnitude is 1,500 A (750 A per cable).

FIGURE 50. <u>Typical effects of excessive cable separation in a quad arrangement.</u>

6.2.5 <u>Concentric cable</u> Concentic cable has an inner conductor with its center on the axis of an annular or cylindrical conductor that carries current in the reverse direction. The stray magnetic field from such a cable should be very small. In principle, we would think concentric cable would be ideal for minesweeping cable runs. There are, however, problems in using it for large currents:

(a) Only one concentric cable should be used to carry the full output of one minesweeping generator.
(b) The problem of unequal current division between. parallel conductors results if two or more concentric cables are used in parallel

(c) A single concentric cable co carry the current from the largest minesweeping generator will be large in diameter, difficult to manufacture, and inconvenient to install.

Therefore, the present view is that the minesweeping cable runs now in use are satisfactory and that we do not need to use concentric cable.

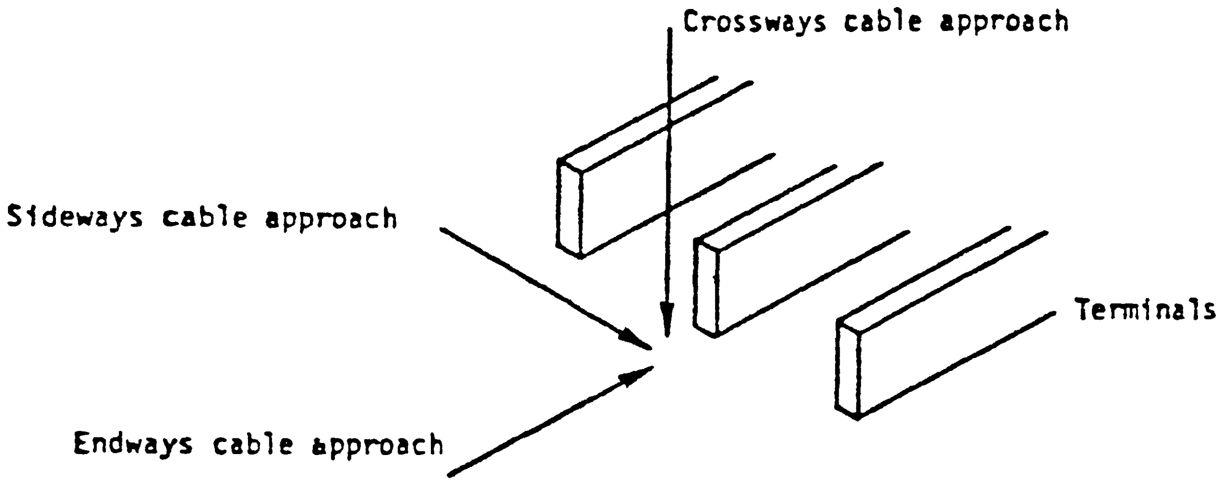
6.3 Connections between equipment and cable runs. The design considerations for connections between equipment and cable runs will be in accordance with 6 3.1 through 6.3.4.7.

6.3.1 Compatibility of cable runs and terminal. In accordance with the principle of compatibility (see 5.10), we will arrange terminals on equipment for ready connection to the cable runs that take current to and from the equipment. In the absence of a terminal arrangement to which the cable run can be conveniently connected, we will use adapters between the cable run and the equipment .

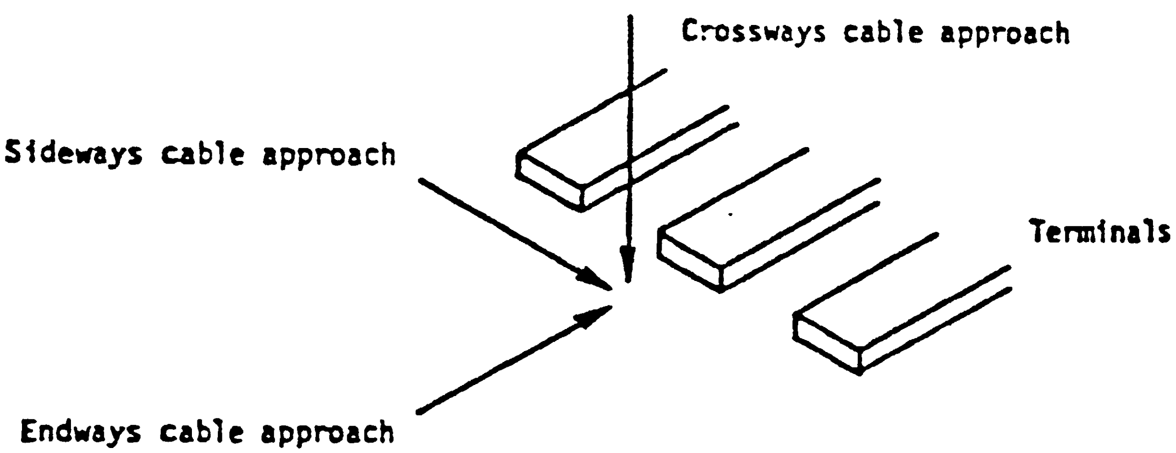
6.3.2 Arrangement of terminals and approach by cable run. Terminals for cable runs will be equally spaced, parallel terminals in the same plane arranged edgewise (see 6.3.4.1) or flat (see 6,3,4.3) with the cable run approaching the terminals endways (see 6.3.4.3), crossways (see 6.3,4.2), or sideways (see 6.3.4.5). Edgewise terminals are preferred since the center-to-center distance between terminals can be made smaller than for flat terminals, which results in smaller current loops. Edgewise terminals can use the same type of cable lugs as endways and crossways connections.

6.3.3 Cable runs. We will usually have four, single-conductor cables in a cable run. However, on occasion there will be eight single-conductor cables in a cable run, and it is conceivable that it may be advantageous to use a different number in other cases (see 6.2.6.5).

6.3 4 Three-terminal arrangements. We will make cable connections to three-terminal arrangements in accordance with 6.3.4.1 through 6.3.4.7. We will orient the cable run approach either endways, crossways or sideways (see figure 51). If the terminals rotate into some other plane, or in some other orientation in the same plane, the endways, crossways, and sideways directions for cable connections will rotate with the terminals.



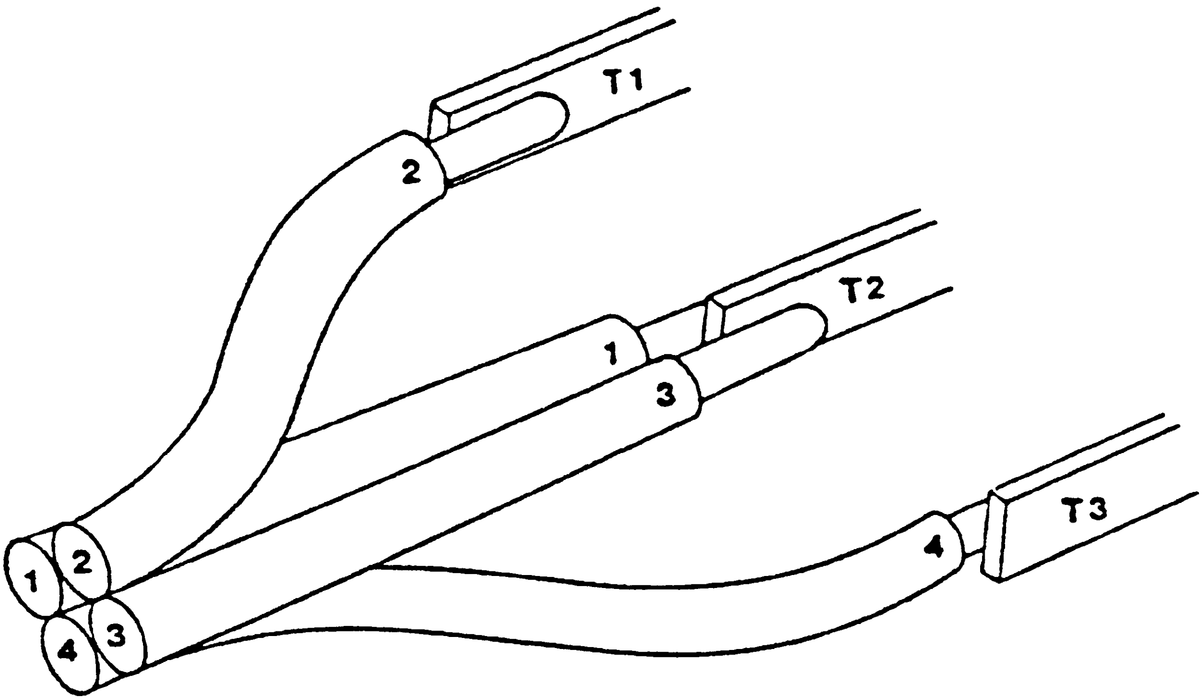
A. EDGEWISE ARRANGEMENT



B. FLAT ARRANGEMENT

FIGURE 51. Edgewise and flat three-terminal arrangements.

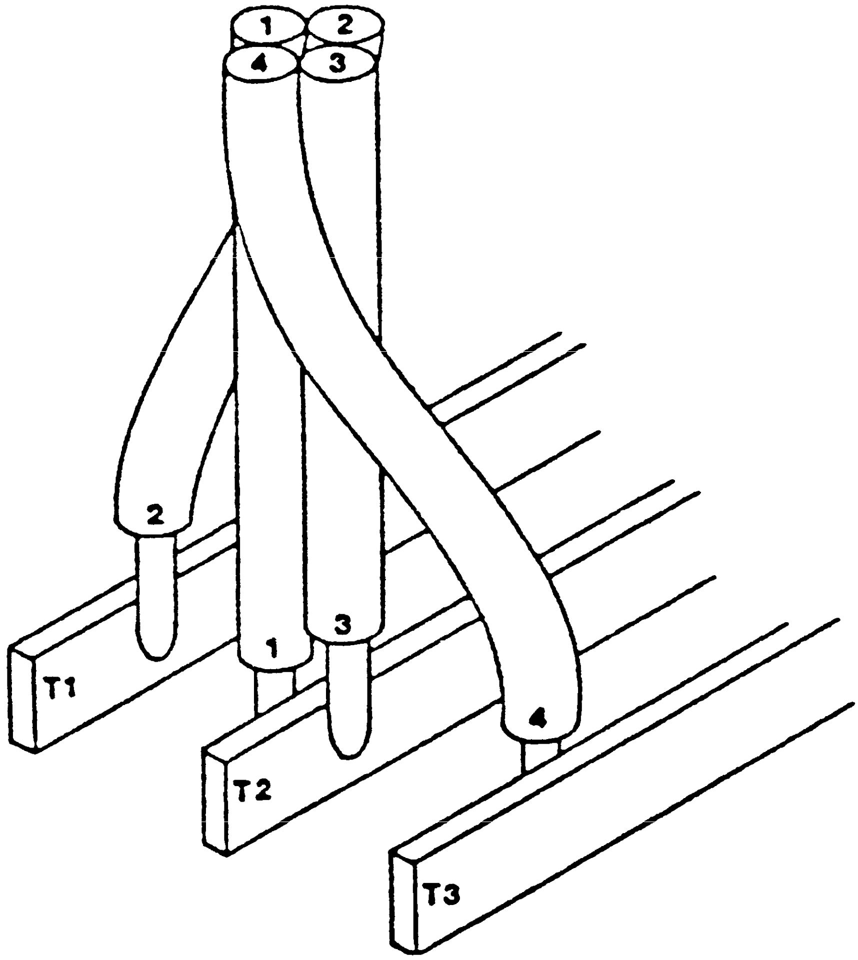
6.3.4.1 Four-cable connection with endwavs approach to edgewise terminals. We will connect a four, single-conductor cable run approaching endways to edgewise terminals in accordance with figure 52.



NOTES:
1. The cable run will approach the terminals with its axis in the line with the horizontal axis of the center terminal, T2.
2. The two cables 1 and 3, which lie in a horizontal plane, will be bolted to the opposite sides of terminal T2.
3. Cable 2 will be bent to the left and slightly down to connect to terminal T1. Cable 4 will be bent to the right and slightly up to connect to terminal T3.
4. The area outlined by cables 1 and 4 equals the area of the cable run outlined by cables 2 and 3.
5. The polarity of the current in the loop made by cables 1 and 4 is opposite to the polarity of the current in the loop made by cables 2 and 3.

FIGURE 52. Connection of an endways cable run with four cables to edgewise terminals.

6.3.4.2 Four-cable connection with crossways approach to edgewise terminals. We will connect a four, single-conductor cable run approaching crossways to edgewise terminals in accordance with figure 53.

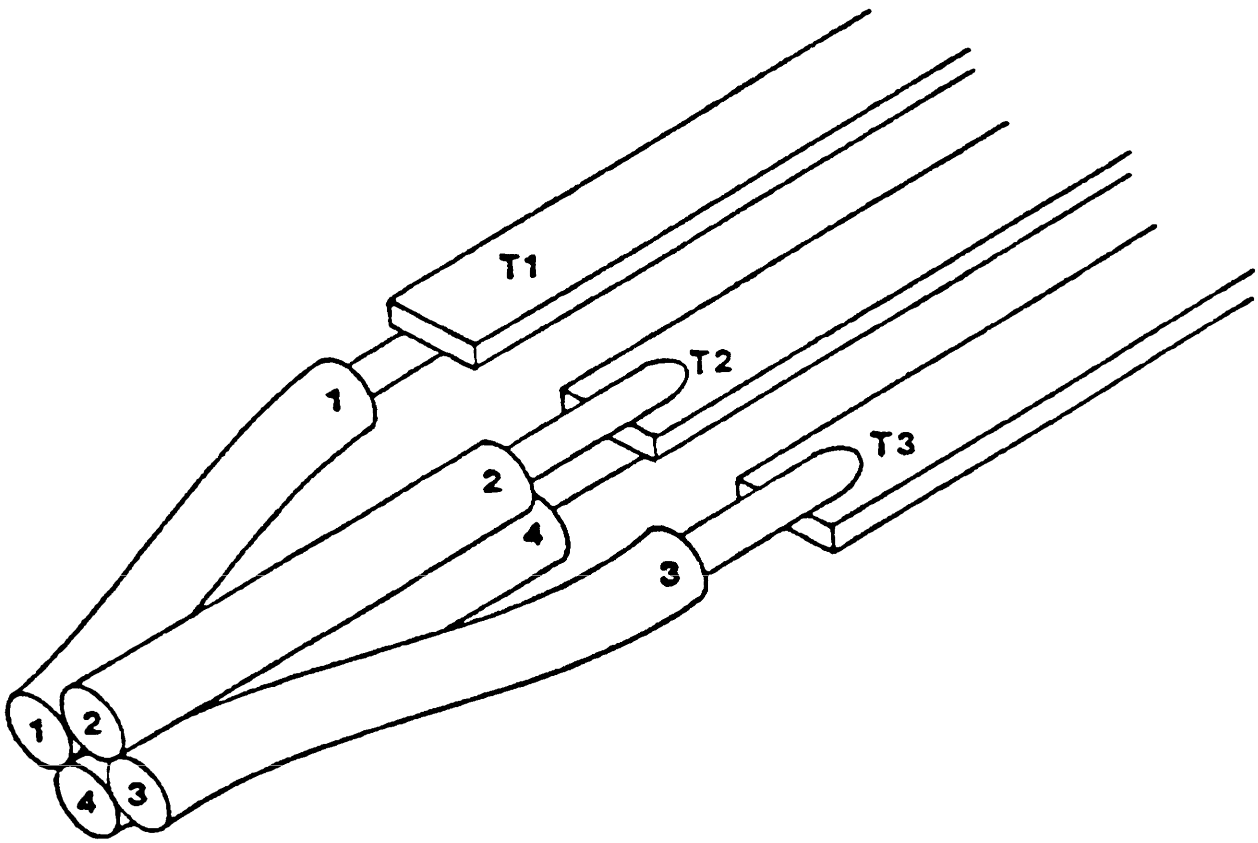


NOTES:

1. This figure can be obtained from figure 52 by rotating through 90 degrees the cable run (or terminals) about an axis through the points of attachment of the cables to the terminals when these points are on a line perpendicular to the terminals.
2. The cable run will approach the terminals with its axis in line with the vertical axis of the center terminal, T2.
3. The two cables 1 and 3, which lie in a vertical plane, wiil be bolted to the opposite sides of terminal T2
4. Cable 2 will be bent to the left and slightly forward to connect to terminal T1. Cable 4 will be bent to the right and slightly back to connect to terminal T3.
5. The area outlined by cables 1 and 4 equals the area outlined by cables 2 and 3.
6. The polarity of the current in the loop made by cables 1 and 4 is opposite to the polarity of the current in the loop made by cables 2 and 3.

FIGURE 53 Connection of a crossways cable run with four cables to edgewise terminals.

6.3.4.3 Four-cable connection with endways approach to flat terminals We will connect a four, single-conductor cable run approaching endways to flat terminals in accordance with figure 54. Cable lugs used for edgewise terminals can be used with flat terminals.

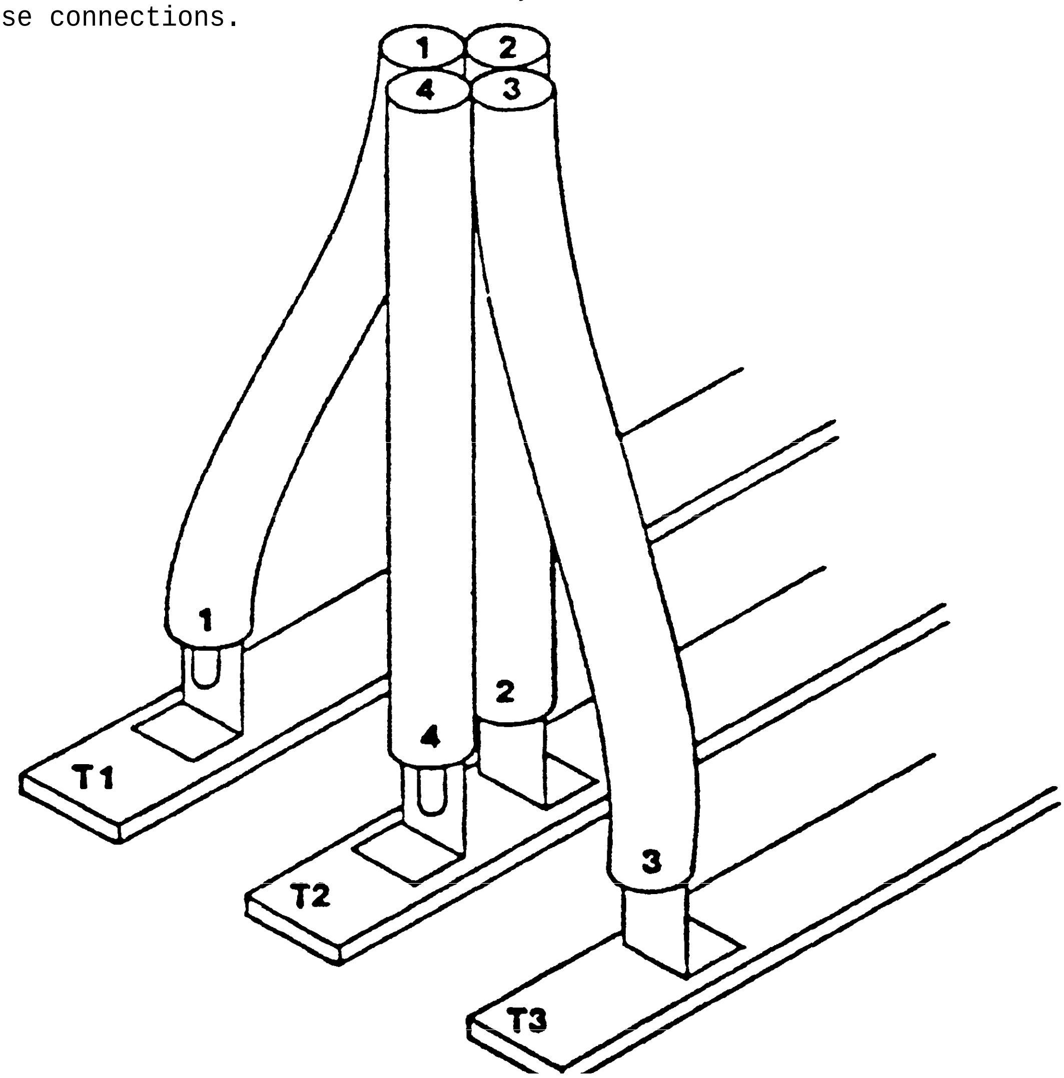


NOTES:

1. The cable run will approach the terminals with its axis in line with horizontal axis of the center terminal T2,
2. The two cables 2 and 4, which lie in a vertical plane, will be bolted to the upper and lower sides of terminal, T2.
3. Cable 1 will be bent to the left and slightly down to be bolted to the under side of terminal T1. Cable 3 will be bent to the right and slightly up to the upper side of terminal T3.
4. The area outlined by cables 1 and 4 equals the area outlined by cables 2 and 3.
5. The polarity of the current in the loop made by cables 1 and 4 is opposite to the polarity of the current in the loop made by cables 2 and 3.

FIGURE 54. Connection of an endways cable run with four cables to flat terminals.

6.3.4.4 Four-cable connection with crossways approach to flat terminals. We will connect a four, single-conductor cable run approaching crossways to flat terminals in accordance with figure 55. With flat terminals, different types of cable lugs will have to be used for crossways connections rather than those used with edgewise connections.



NOTES:

1. The cable run will approach the terminals with its axis in line with vertical axis of the center terminal T2.
2. The two cables 2 and 4, which lie in a vertical plane, will be bolted to the upper side of terminal, T2.
3. Cable 1 will be bent to the left and slightly forward to be bolted to the upper side of terminal T1. Cable 3 will be bent to the right and slightly back to the upper side of terminal T3.
4. The area outlined by cables 1 and 4 equals the area outlined by cables 2 and 3.
5. The polarity of the current in the loop made by cables 1 and 4 is opposite to the polarity of the current in the loop made by cables 2 and 3.

FIGURE 55. Connection of a crossways cable run with four cables to flat terminals.

6.3,4.5 Four-cable connection with sideways approach to edgewise or flat terminals. We will avoid a four, single-conductor cable run approaching sideways to edgewise or flat terminals since we will need special bus bar adapters to make a connection that will result in a minimum stray magnetic field,

6 3 4.6 Eight-cable connection. We will connect an eight, single-conductor cable run approaching endways. crossways. or sideways to flat terminals in accordance with figures 56, 57, and 58, respectively. In each case, the positive and negative cables will be located symmetrically about the axis of the cable run. The general scheme shown is also applicable to edgewise terminals.

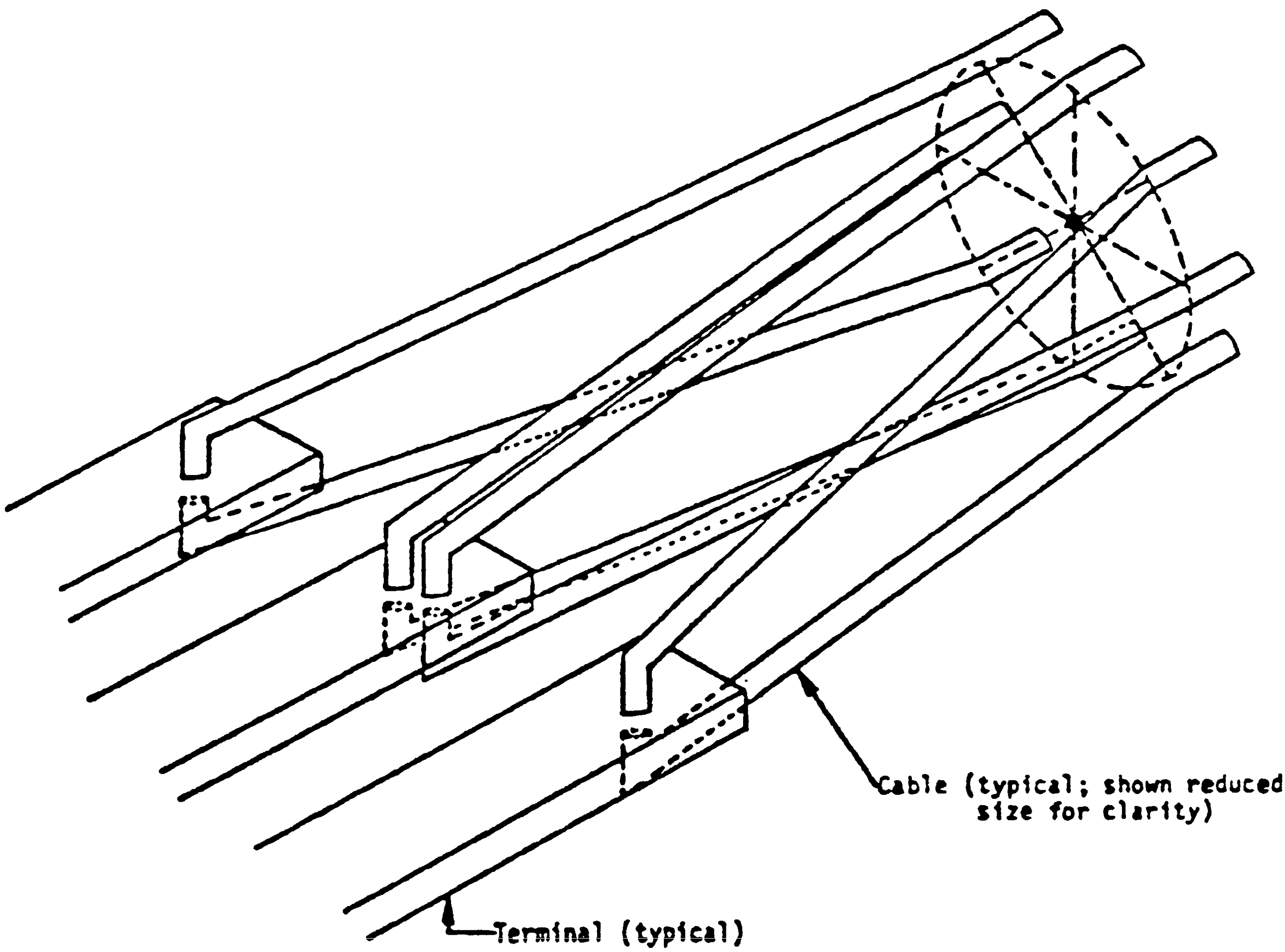


FIGURE 56. Connection of an endways cable run with eight cables to flat terminals.

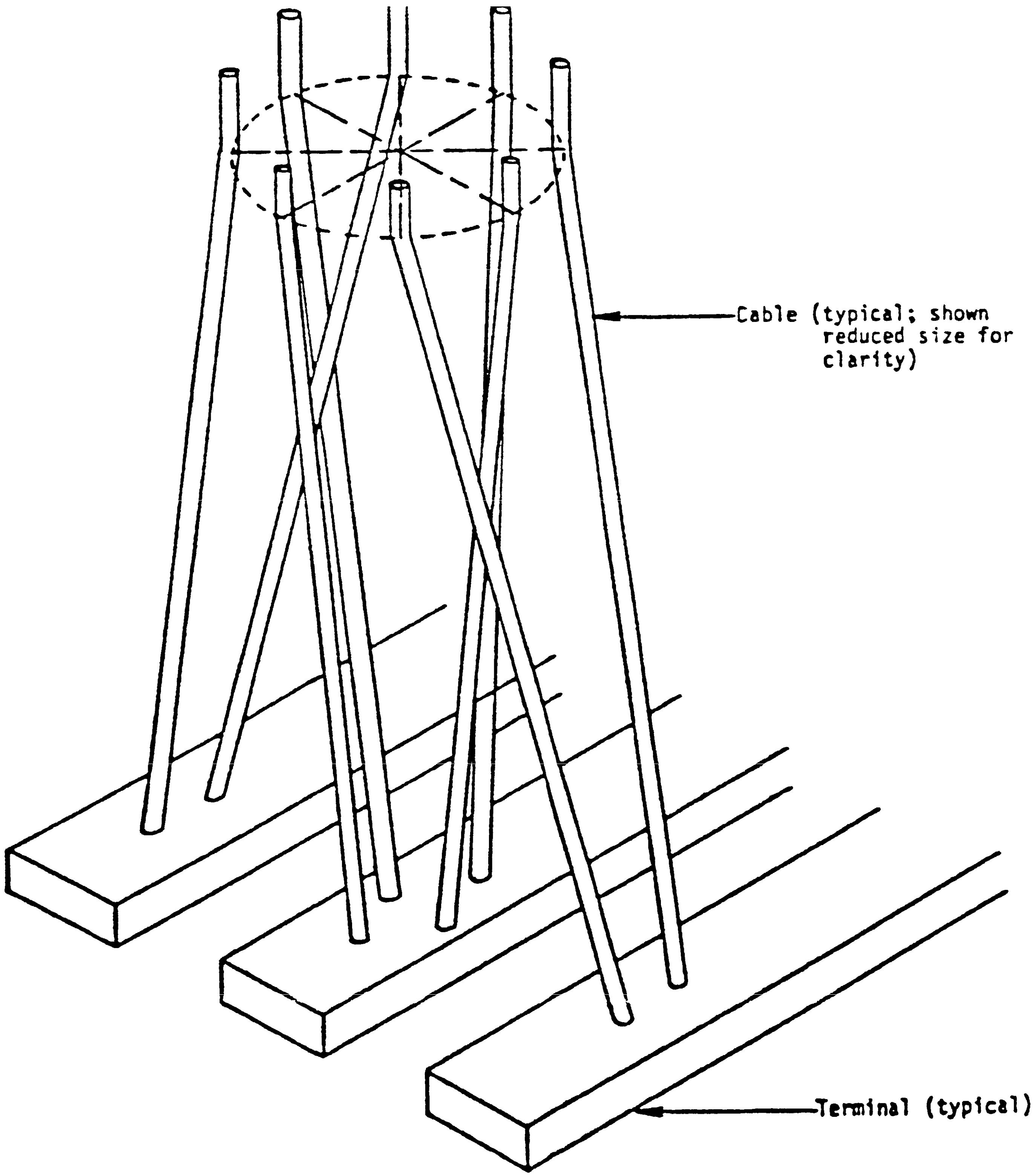


FIGURE 57. Connection of a crossways cable run with eight cables to flat terminals.

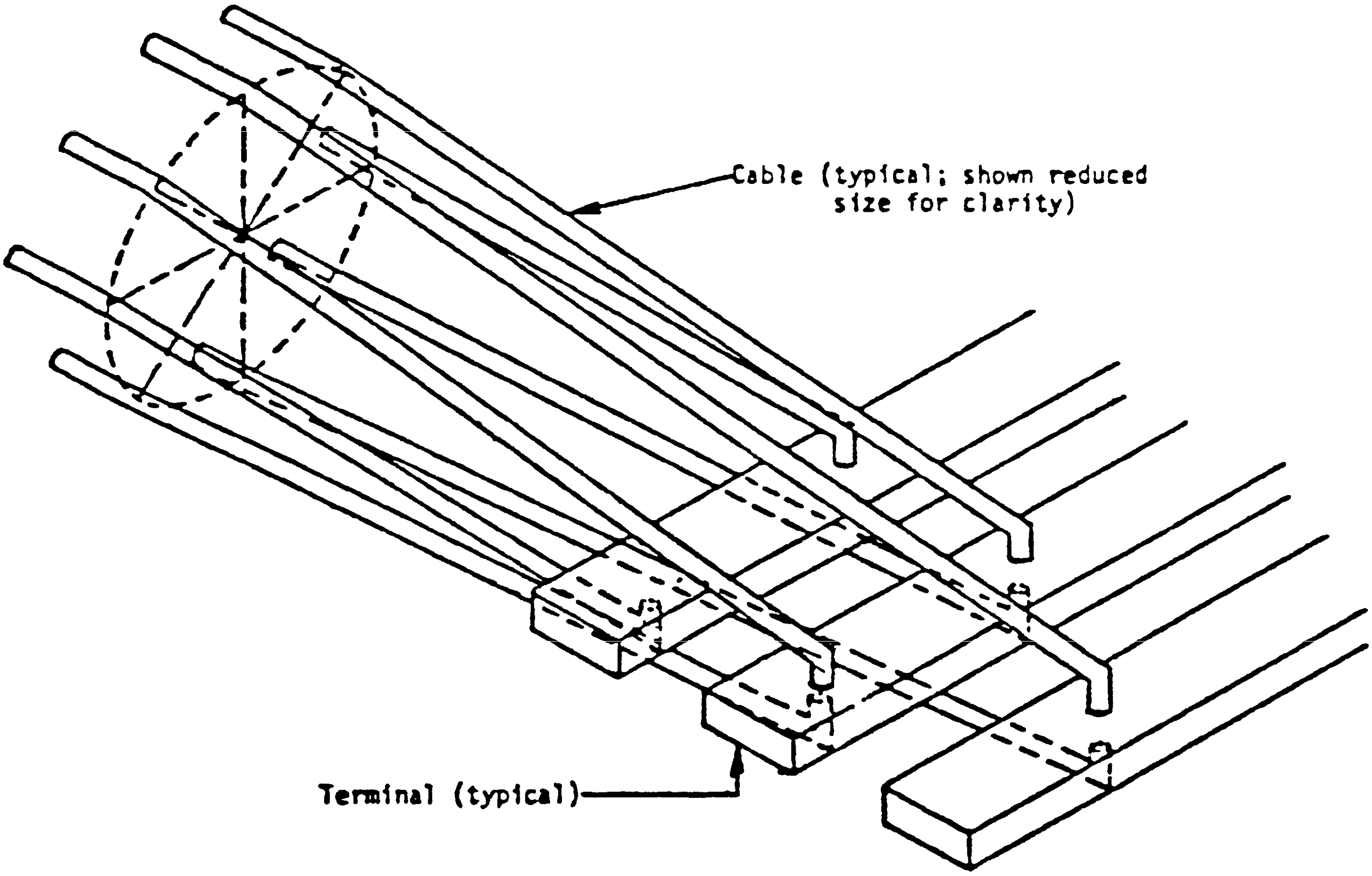


FIGURE 58. Connection of a sideways cable run with eight cables to flat terminals.

6.3.4.7 General connection method for six cables   We will make endways or crossways connections to three terminals for a cable run consisting of six single-conductor cables about a central core as follows (see figure 59):

(a) Draw in every other cable and connect it to a single conductor that is on an extension of the cable axis. This single conductor will then be connected to the central terminal of the equipment.
(b) Connect the remaining three cables to a conducting ring that is concentric with the cable axis.
(c) Connect two diametrically opposite points on this ring to the two outside terminals of the equipment.

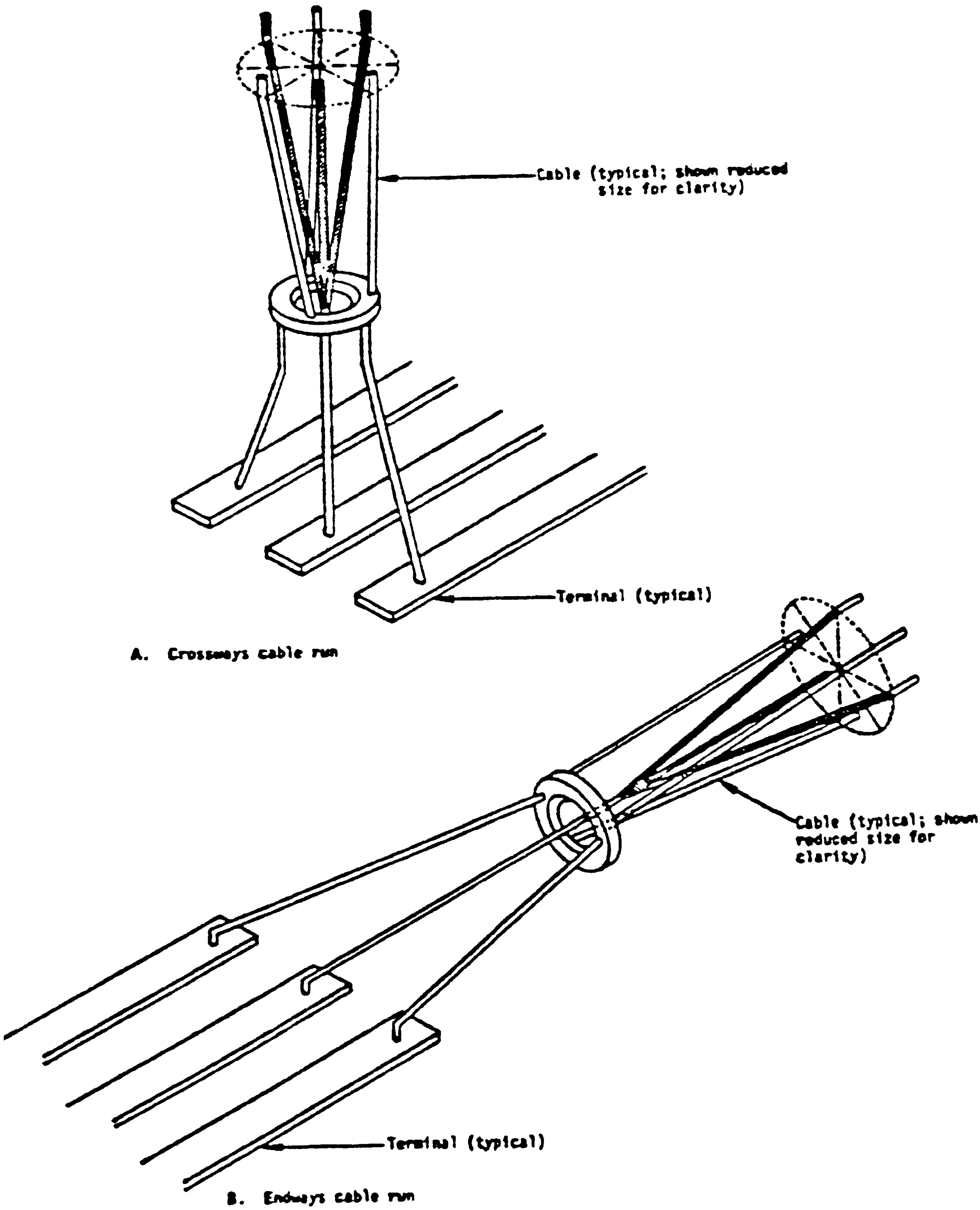
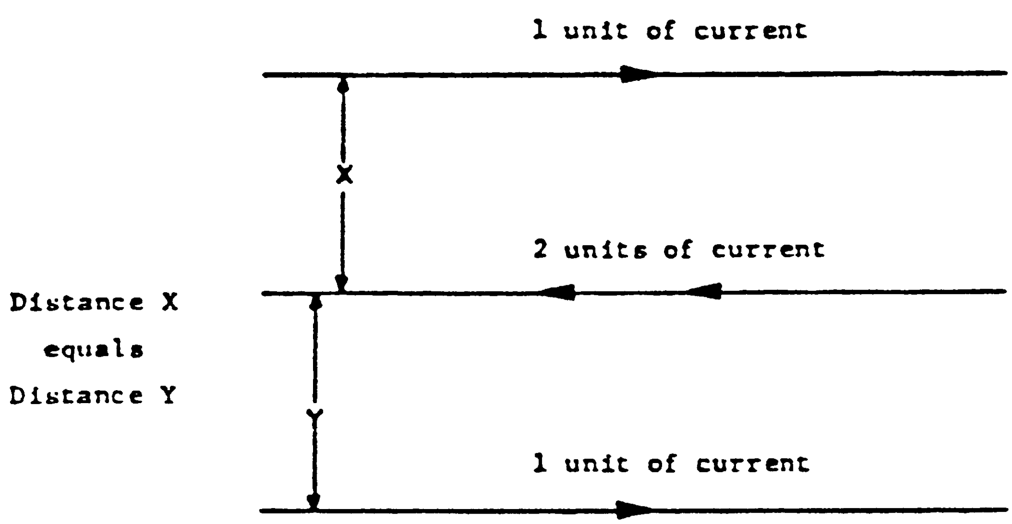


FIGURE 59. Connection to an endways and crossways cable run with six cables.

This arrangement will create current loops of equal area and opposite polarity, causing the stray magnetic field to be small. A simple modification of the above procedure can accommodate a sideways cable run.

6.4 Disconnect switch boxes and contactor panels. We will consider here the design of disconnect switch boxes and contactor panels that may be connected in a magnetic minesweep cable run between a magnetic minesweep generator and a mine-sweep cable terminal box. The design considerations may also be applied to other control equipment used in dc power circuits such as switchboards, power panels and motor controllers. The arrangement of conductors and devices for disconnect switch and contactor panels will be in accordance with 6.4.1 through 6.4.12.4.

6.4.1 Basic conductor arrangement. The basic arrangement of conductors in disconnect switch boxes, contactor panels, switchboards, power panels and motor controllers will be a central conductor carrying two units of current in one direction sandwiched between two symmetrically placed conductors each carrying one unit of current in the opposite direction (see figure 60). The basic conductor arrangement is compatible with the three-terminal arrangements for magnetic minesweep quad cable runs in accordance with 6.3.4.



NOTE: In all figures, quantity of arrows indicates relative magnitude of current. The direction of the l rrwa indicates direction of current.

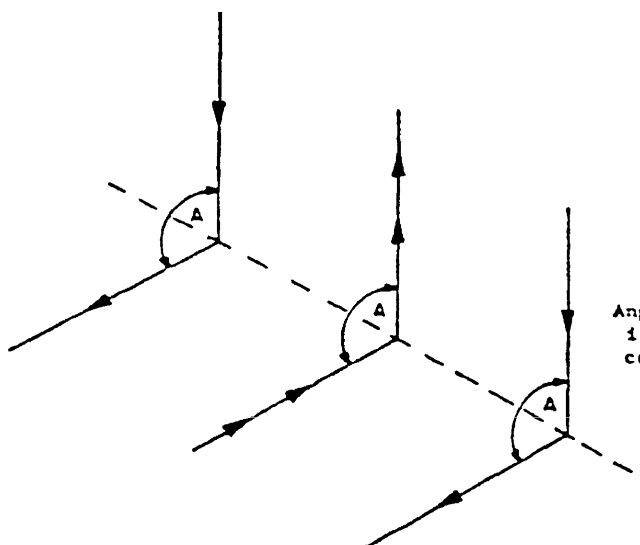
FIGURE 60. Basic arrangement of conductors in disconnect switch boxes and contactor panels.

6 4.2 Conductor bends. Conductor bends can be one of two types:

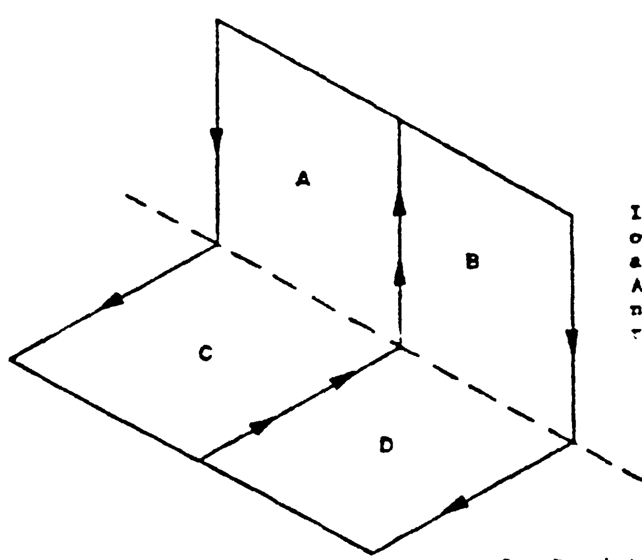
(a) The conductors lie in different planes on each side of the bend.
(b) The conductors lie in the same plane on each side of the bend

6.4.2.1 Conductors lying in different planes on each side of the bend, We can bend the basic conductor arrangement so that the conductors lie in different planes on each side of the bend (see figure 61). If the line of the bend is perpendicular to the original plane of the cables and the two current loops have

magnetic moment is satisfied. The
the angle of the bend.



A. Conductor bend in two planes.



B. Resulting current loops.

FIGURE 61 Bend for conductors lying in different planes on each side of the bend.

6.4.2.2 Conductors lying in the same plane on each side of the bend. We can also bend the basic conductor arrangement so that the conductors lie in the same plane both before and after the bend (see figure 62). The conductor arrangement creates opposing current loops of equal area so that the condition of zero net magnetic moment is satisfied. This arrangement is difficult to accomplish in practice. Consequently , we preferred the use of conductor bends lying in different planes on each side of the bend for power circuit arrangements of disconnect switch boxes and contactor panels.

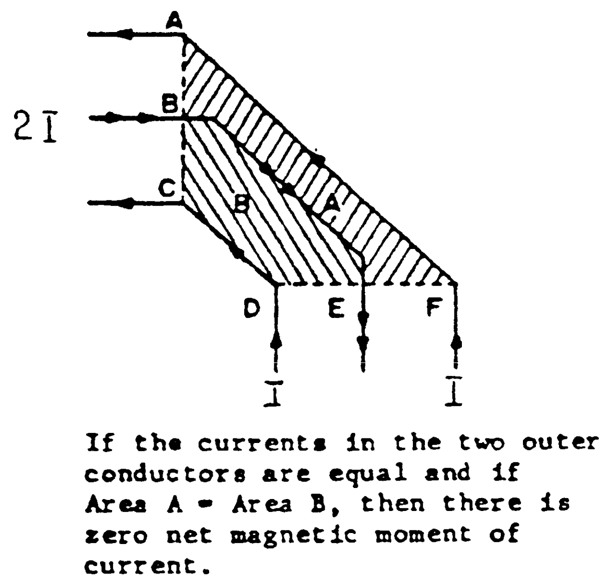


FIGURE 62. Bend for conductors lying in the same plane on each side of the bend.

6.4.3 Parallel compensation. The basic conductor arrangement we have considered makes use of parallel compensation (see 5.6.1.1.2). Zero net magnetic moment in parallel compensation is dependent upon equality of current division between two (or more) conductors in parallel. Absolute equality of current division is difficult to ensure in low resistance circuits; hence, in order to be safe it will be necessary to add refinement to the basic conductor arrangement, which will guarantee that the magnetic field will not be greatly increased if nominally equal currents actually depart somewhat from exact equality.

6.4.4 Current loops in power circuits, We will keep the size of current loops created by the power circuits as small as possible to minimize the magnetic moments of the current loops. The distance between these current loops will also be minimized. Conductor and device separation will be kept to the minimum.

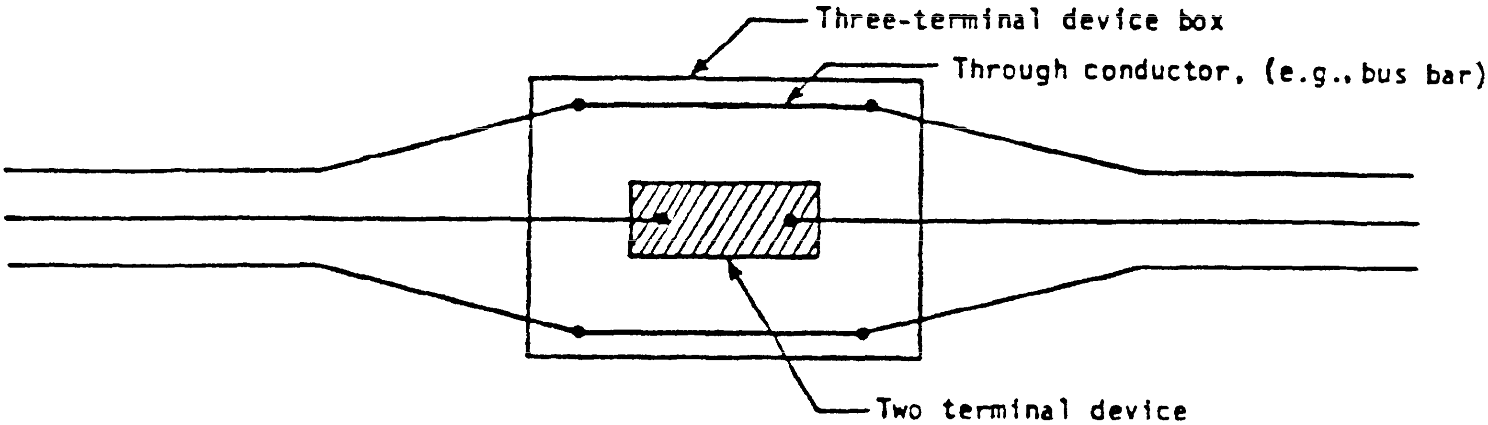
6.4.5 Terminals. Three terminals are necessary for connection of a basic conductor arrangement. Most of the equipment we will use will be three terminal devices; however, some are two terminal devices. A generator needs only one set of three terminals. A disconnect switch or contactor panel needs two sets of three terminals, one set to connect the incoming cable run and one set to connect the outgoing cable run.

6 4.6 Shunt box. The design considerations for shunt box connections will be in accordance with 6.4.6.1 through 6.4.6.3.

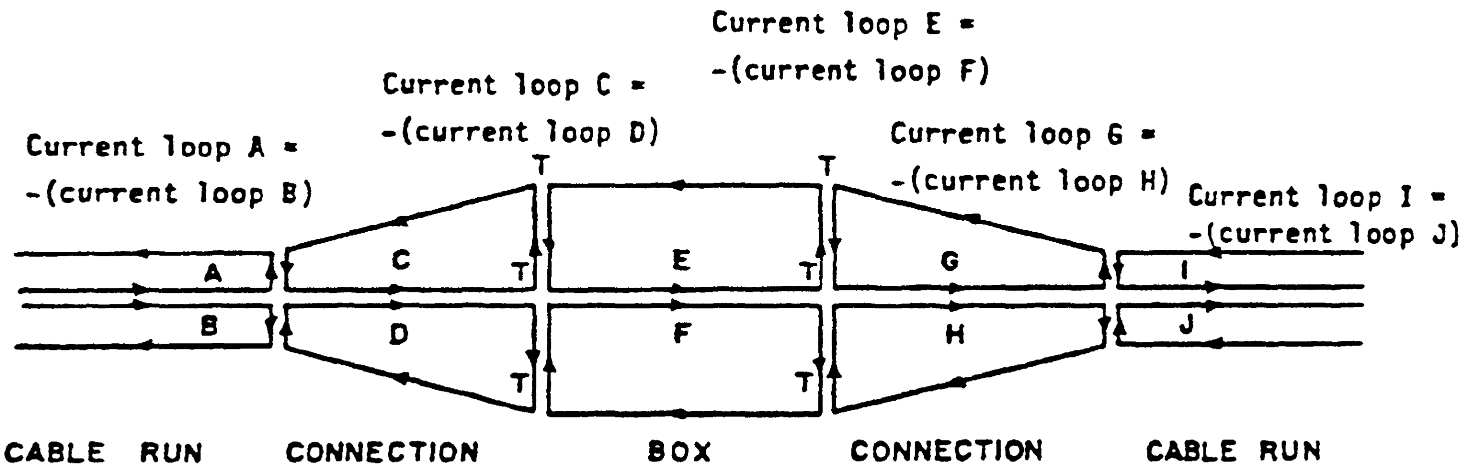
6.4.6.1 Description of connection. Consider the problem that arises if we have to connect an ammeter shunt in a minesweeping cable run. An obvious solution is to cut the two positive cables, connect the shunt in the cut, and spread the two negative cables so that one goes around each side of the shunt (see figure 63). For a more elaborate arrangement, put the shunt in a box with three terminals sticking out of each end of the box for connection to the cable runs This shunt box is a simple example of three-terminal equipment that evolves naturally out of the quad cable run.

6.4.6.2 Current loops. The current loops associated with the shunt box (see figure 63) are divided into three groups as follows:

(a) Loops A, B, I, and J are caused by the cable runs to and from the shunt box.
(b) Loops C, D, G, and H start where the cables depart from the quad array and end where connections are made to the terminals, T, of the box. These loops are caused by the connection between the cable run and the box.
(c) Loops E and F are caused by the components and bus work in the box itself.



A. Circuit for a two-terminal device in a three-terminal box.

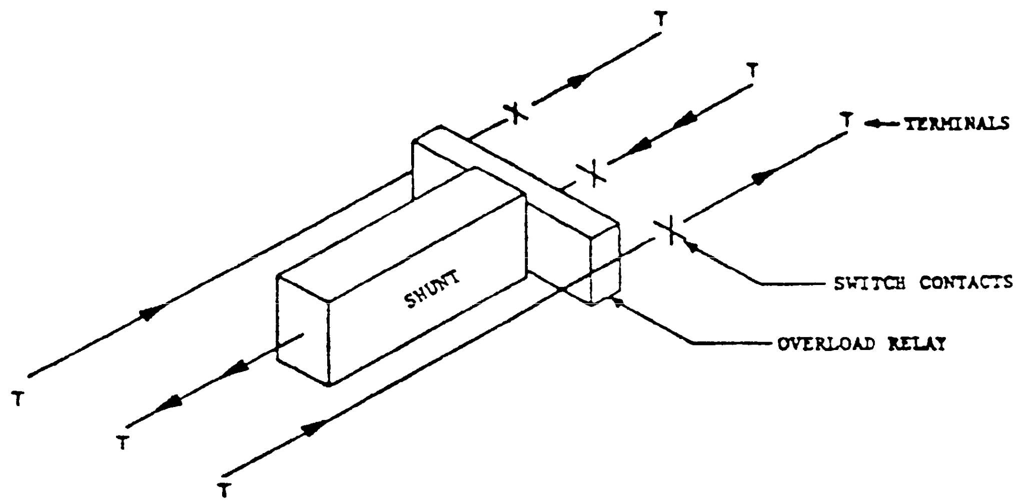


B. Resulting current loops for circuit in A.

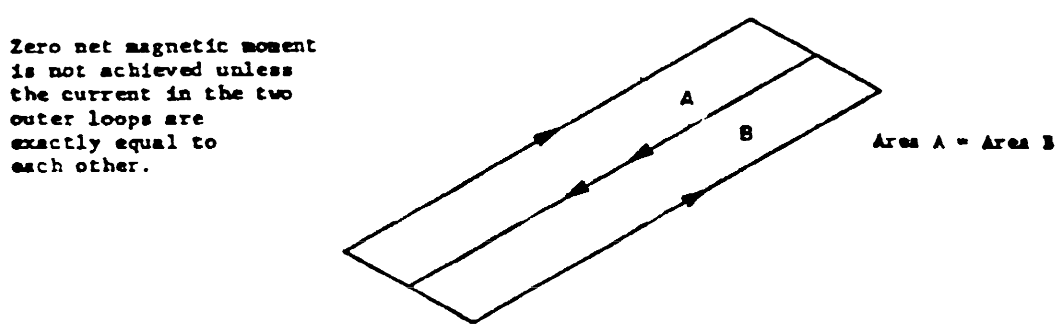
FIGURE 63 Conversion of a two-terminal device to a Three- terminal device and resulting current loops.

6 4.6.3 Cancellation of magnetic field. With the two outer conductors arranged symmetrically with respect to the center conductor and with equal currents in the outer conductors, loop A is equal in strength to loop B and of opposite polarity. The same is true of loops C and D, E and F. G and H, and I and J. The resultant magnetic field should be small.

6.4.7 Straight-through disconnect switch box. We will not use the straight-through arrangement for a disconnect switch box (see figure 64). The folded (see 6.4.8) and criss-cross (see 6.4.9) arrangements are preferred. The one serious disadvantage associated with the simple straight-through arrangement is that there is no provision to minimize the effect of unequal current division between the two outer conductors.



A. Circuit for disconnect switch box with a straight-through conductor arrangement.



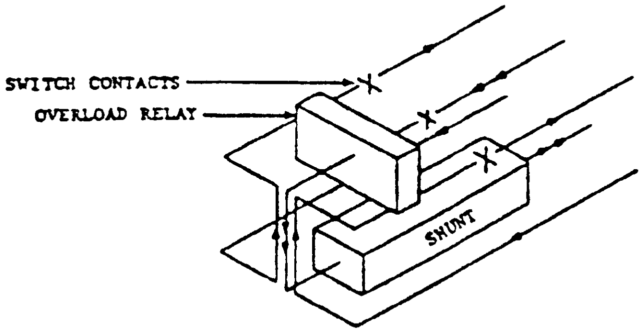
B. Resulting current loops for circuit in A.

FIGURE 64. Straight - through arrangement (not to be used) for a disconnect switch box.

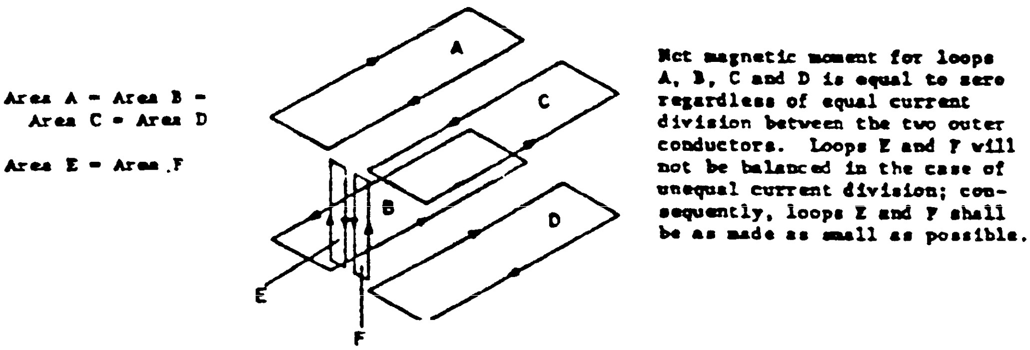
6 4.8 Folded arrangement, The folded arrangement for disconnect switch boxes and contactor panels will be the preferred arrangement. We will use this arrangement in all cases except where special circumstances make it indispensable or highly advantageous to have a disconnect switch box or contactor panel with incoming and outgoing terminals at opposite ends of the box or panel. The folded arrangement inherently requires the incoming and outgoing terminals to be at the same end of the box or panel. Actual separation of conductors will be the minimum required to meet the applicable electrical creepage and clearance distances specified for the disconnect switch box.

6.4.8.1 Folded arangement for disconnect switch box. The design considerations for the folded arrangement for a disconnect switch box will be in accordance with 6.4.8.1.1 through 6.4.8.1.4.

6.4.8.1.1 Description of connection. Using a straight through arrangement, we will spread the overload relay and shunt apart longitudinally, and then bend the bus work between them through two right angles so that the components are folded upon each other (see figure 65).



A. Circuit for disconnect switch box with a folded arrangement.



B. Resulting current loops for circuit in A.

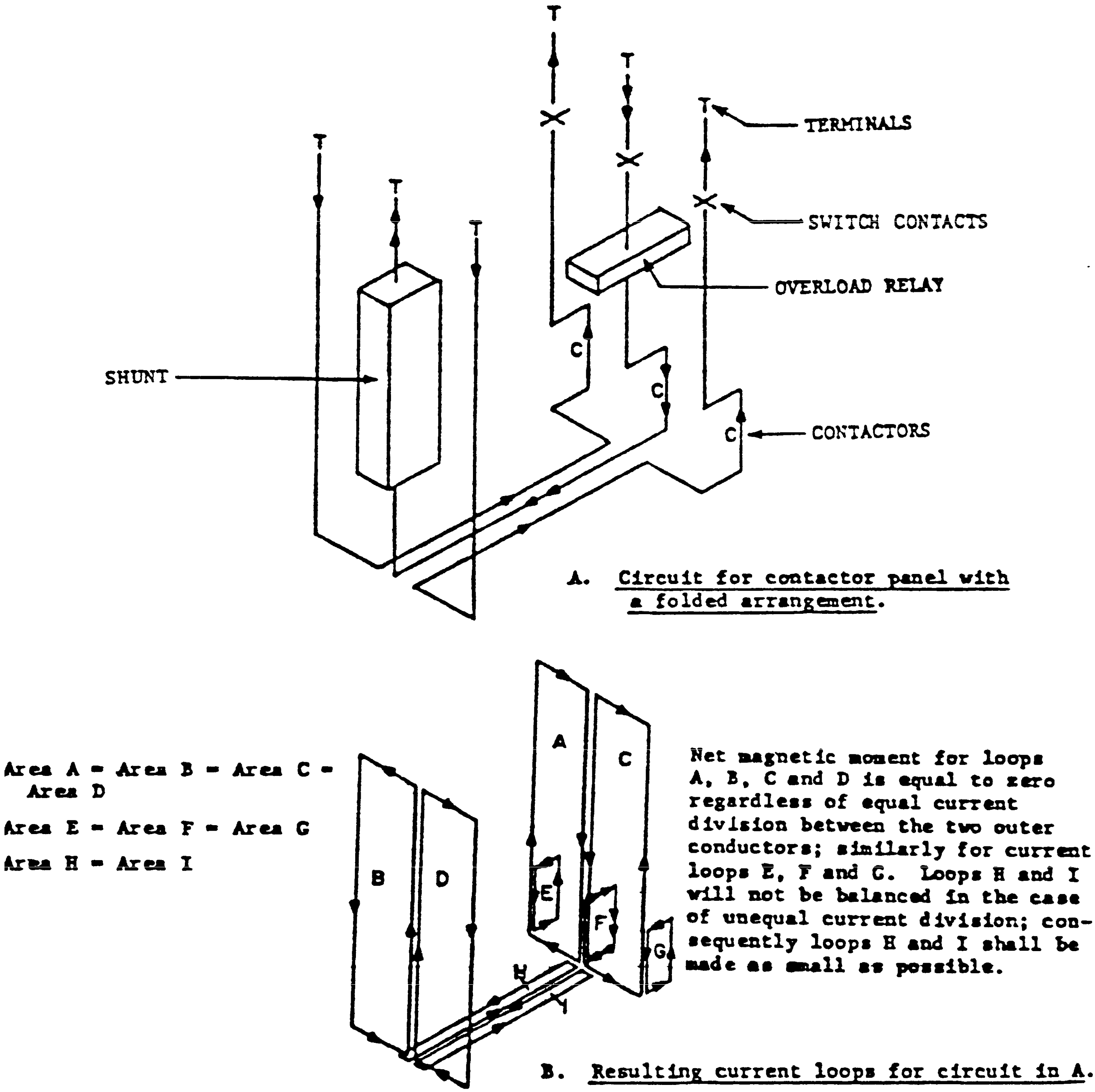
FIGURE 65. Folded arrangement for a disconnect switch box.

6.4.8.1.2 Advantages. The folded arrangement is a refinement of the straight-through arrangement in that the effect of unequal current division between parallel circuits is almost eliminated. The folded arrangement (see figure 65) has six current loops, A, B, C, D, E, and F. We will design the disconnect switch box so that loops A, B, C, and D are all equal in area and the area of E is equal to the area of F.

6.4 8.1.3 Equal current loops. If the current is the same in the outer conductors of the disconnect switch box, loops A, B, C, and D will be of equal strength, A and D producing a magnetic field directed down, B and C a magnetic field directed up. The resultant field from these four loops will be small. Loops E and F will be of equal strength and opposite polarity. The resultant field from these two loops will be small. Therefore, the resultant magnetic field from all six loops together will be small

6.4.8.1.4 Unequal current loops. Suppose that the current is unequally divided between the two outer conductors. Loops A, B, C, and D will no longer be of equal strength, but A will still be equal to B since the same current flows in both, and C will be equal to D The resultant field of loops A, B, C, and D will be small because A is equal and opposite to B, and C is equal and opposite to D Loops E and F will, however, no longer be equal and opposite. Their resultant field will be approximately equal to that of a single loop with an area equal to that of loop E or F, and a current equal to the difference between the currents in the two outer conductors. However, the area of loops E and F in the folded arrangement is much less than the area of the two loops in the straight-through arrangement. Therefore, the effect of unequal current division will be correspondingly less than for the straight-through arrangement.

6.4.8.2 Folded arrangement for contactor panel. We will form the contactor panel equipment into a folded arrangement (see figure 66). The design should be such that current loops A, B, C and D are equal in area and as small and as close together as feasible. If the current division is equal or unequal the external magnetic field will be small No matter how we divide the current, loop A is equal and opposite to loop B since they are of equal area and carry the same current. The same is true of loops C and D. The net magnetic moment for loops A, B, C, and D is, therefore, equal to zero regardless of whether the current is equally divided between the two outer conductors or not. The sum of the currents in loops E and G is always equal to the current in loop F no matter how we divide the current between loops E and G. Therefore, the net magnetic moment for loops E, F, and G is equal to zero whether the current is equally divided between the two outer conductors or not. Loops H and I, however, will not be balanced in the case of unequal current division, but the areas of the loops can be made very small by putting the bus bars across the bottom of the contactor panel very close together. If this is done, the magnetic field from the contactor panel should be small for either equal or unequal division of current between the outer conductors .



A. Circuit for contactor panel with a folded arrangement.

B. Resulting current loops for circuit in A.

FIGURE 66. Folded arrangment for a contactor panel.

6.4 8.2.1 Single-pole contactor. We do not discuss single-pole contractors because if a single-pole contactor is used, it will be put in the center conductor and the two outer conductors will be shaped to simulate the other two poles of a three-pole contactor.

6.4.9 Flexibility of folded arrangement. If we look at a folded dlsconnect switch box from the side (see figure 67), the conductors in the box will look like the three sides of a rectangle shown to the right of the terminals T. On figure 67 each T indicates three terminals in a line perpendicular to the plane of the paper so that only one is visible. We see in 6.3.4.1 that cable runs approaching and leaving the box in any of the directions shown can be readily connected to the three terminals. Furthermore, the incoming and outgoing cable runs do not have to be either horizontal or vertical but can be at any angle. In addition, all of the arrangements can be rotated through 90 degrees (or any other angle) in the plans of the figure so that there is a considerable degree of flexibility in connecting the folded arrangement to cable runs. For this reason it is considered unlikely that there will be any need for anything but the folded arrangement of disconnect switch boxes or contactor panels. The only exception would appear to be in the case of special circumstances, which are considered in 6.4.10.

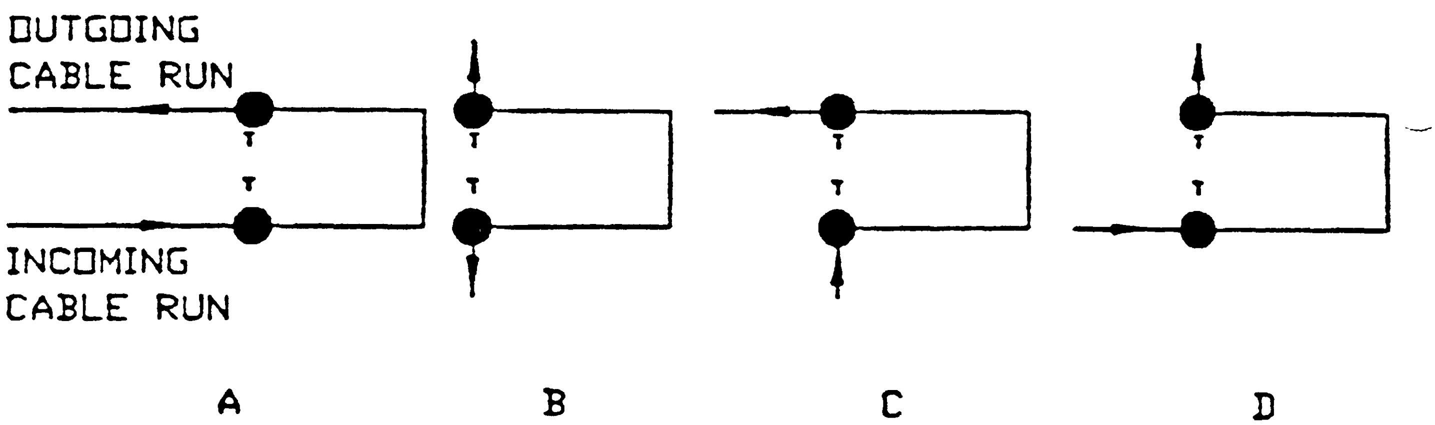


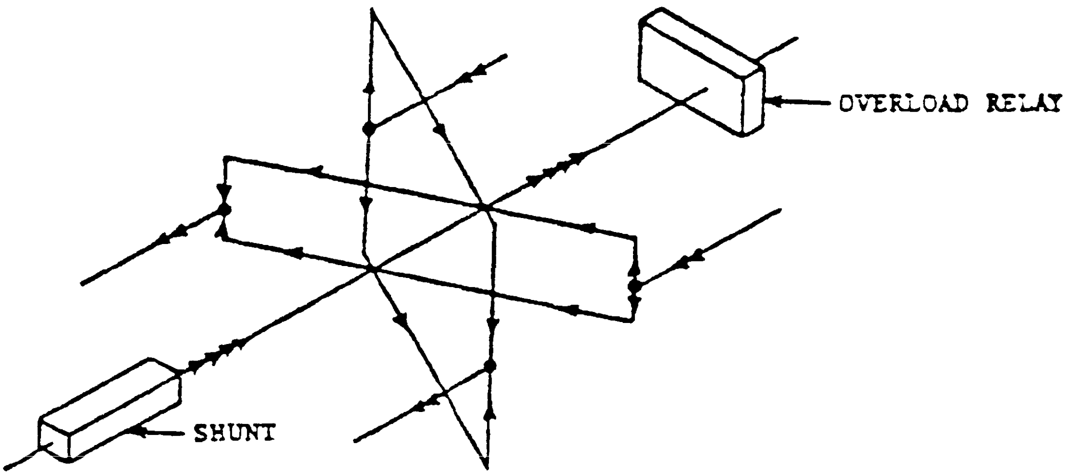
FIGURE 67. Various ways in which cable runs can be connected to a folded disconnect switch.

6.4.10 The crisscross arrangement In the folded arrangement we will have the connections to the terminals for the incoming and outgoing cable runs at the same end of the disconnect switch box or contactor panel In some cases that will be an advantage. In other cases it will be of no particular disadvantage because of the flexibility in connecting cable runs to the folded arrangement. There may also be cases, however, in which special circumstances make it indispensable or highly advantageous to have a disconnect switch box or contactor panel with incoming and outgoing terminals at opposite ends In such cases, but only in such cases, the crissscross arrangement may be used

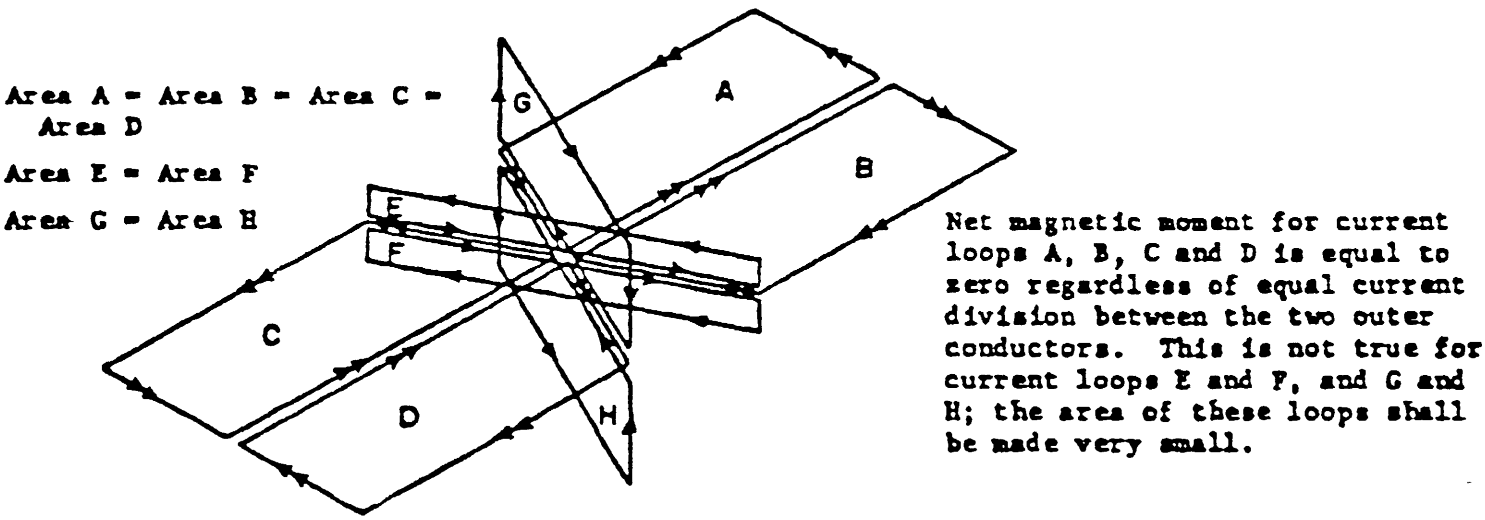
6.4.10.1 The crisscross. We start out with a straight-through arrangement of a disconnect switch box. We move the shunt and overload relay apart a short distance longitudinally, and between them place a crisscross (see figure 68). In actual construction the conductors should be as close together as is consistence with the provisions of adequate insulation between conductors of different potential.

6.4.10.2 Current loops. If we use the crisscross arrangement we break the two current loops of the straight-through arrangement into eight current loops (see figure 68). In the case of unequal current division, the balance of loops E, F, G and H will be distributed. However, the area of loops E, F, G and H can be made very small by using wide and thin conductors for the crossovers and arranging them with their flat sides close together and separated only by the insulation that will be needed where conductors at different potentials cross each other. Note also that the current carried by each conductor in a crossover is only one-fourth the total current so that the difference in current, if any, should be only a small number of amperes. For these reasons, the magnetic field produced by loops E to H should be small. We will leave these loops out of further consideration.

6.4.11 Difference between folded and crisscross arrangements. Although the four loops A to D in the folded arrangement (see figure 66) and the four loops A to D in the crisscross arrangement (see figure 68) have zero net magnetic moment for both equal and unequal current division, the four loops of the folded arrangement are arranged to give a better cancellation of magnetic field than the four loops of the crisscross arrangement. In the folded arrangement we have two pairs of equal and opposite current loops arranged so that the magnetic field of one pair is opposed to the magnetic field of the other pair. In the crisscross arrangement we have two pairs of equal and opposite current loops, but in this case the loops are arranged so that the magnetic field of one pair adds to the magnetic field of the other pair. The difference is brought out by the comparison given in 6.4.12.



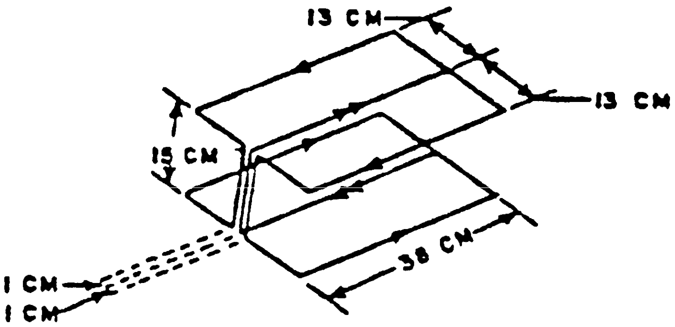
A. Partial circuit for disconnect switch box with a crisscross conductor arrangement.



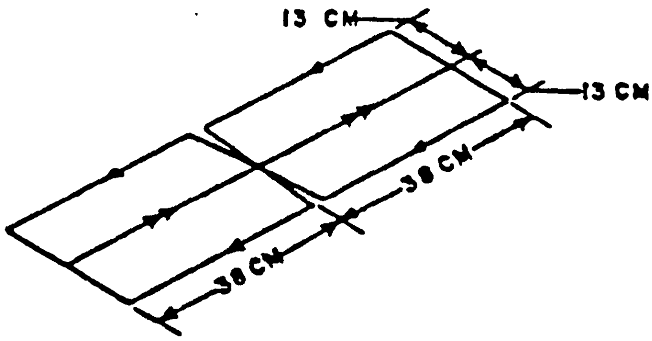
B. Resulting current loops for circuit in A.

FIGURE 68. Crisscross arrangement for a disconnect switch box.

6.4.12 Comparison of folded and crisscross arrangements. We have compared the magnetic fields produced by folded and crisscross arrangements (see figure 69) for disconnect switch boxes in 6.4.12.1 through 6.4.12.4. The dimensions are arbitrarily chosen. In all cases, the total current is taken to be 1000 A. The current loops are modeled as rectangular in shape and the areas are calculated as if they were rectangular. In the crisscross arrangement, the four current loops in the crisscross (loops E to H) have been omitted since these can be made so small in area that their magnetic field should be negligible.



A. Folded arrangement.



B. Crisscross arrangement.

FIGURE 69. Comparable folded and crisscross arrangements.

6.4.12.1 Comparison for equal current division between outer conductors. Fol- current equally divided between the two outer conductors (that is, each outer conductor carries 500 A while the center conductor carries 1,000 A), we find that the maximum values of the vertical component of the magnetic field in a plane 6 meters below the centers of the two arrangements are as indicated in table IV.

TABLE IV. Equal division of current (500 amperes in each outer conductor).

Arrangement type	Flux density (nanoteslas)
Folded	0.060
Crisscross	1.220

6.4.12 2 Comparison for unequal current division between outer conductors. For current unequally divided between the two outer conductors (that is, one outer conductor carries 600 A and the other 400 A while the center conductor carries 1,000 A), we find that the maximum values of the vertical component of the magnetic field in a plane 6 meters below the centers of the two arrangements are as indicated in table V.

TABLE V. Unequal division of current (600 amperes in one outer conductor and 400 amperes in the other outer conductor) .

Arrangement type	Flux density (nanoteslas)
Folded	0.700
Crisscross	1.300

6.4.12.3 Comparison for zero current in one outer conductor and 1.000 amperes in the other outer conductor. For the extreme case of current inequality, which would arise if an open circuit developed so that one of the two outer conductors carries 1,000 A and the other carries none, we find that the maximum values of the vertical component of the magnetic field in a plane 6 meters below the centers of the two arrangements are as indicated in table VI.

TABLE VI. Zero current in one outer conductor and 1.000 amperes in the other outer conductor.

Arrangement type	Flux density (nanoteslas)
Folded	3.000
Crisscross	4.000

6.4.12.4 Conclusions. It is apparent from tables IV, V, and VI that the stray magnetic field of the folded arrangement is the preferred arrangement because it has tne smaller field for all conditions

6.5 Design of induction clutches, We use various types of clutches or couplings on minesweepers to transmit torque. If the clutch is of the mechanical or hydraulic type, no electric currents will be involved and no stray magnetic field problems arise. If, however, the clutch is of an electrical or magnetic type, stray magnetic field problems arise and we must design the equipment to keep the stray magnetic field as small as possible. Induction clutches are also known as electromagnetic or eddy current clutches or couplings.

6.5.1 Typical induction clutch. A typical induction clutch design for application in which a low stray magnetic field is not required is shown on figure 70. Because of its excessive stray field, normal or traditional induction clutch design is never used on minesweepers. One shaft, the shaft of the inner rotating member, for example, is connected to the engine, the other shaft to the driven equipment. The inner member has a single doughnut shaped dc supplied field coil concentric with the shafts, and a magnetic circuit for the flux set up by current in the field coil. If we look at the upper half of the machine, the path of magnetic flux will be up to the right of the field coil, into the finger shown solid in the figure, across the air gap into the drum that is made of magnetic material, tangentially in the drum for a short distance, then across the air gap into the finger shown dotted, down on the left side of the field coil, and finally from left to right inside the field coil. The interdigitated fingers make it possible for a single field coil to excite a multitude of field poles on the inner member, These poles are long in the direction of the machine axis, narrow in the tangential direction, and alternately of north and south polarity.

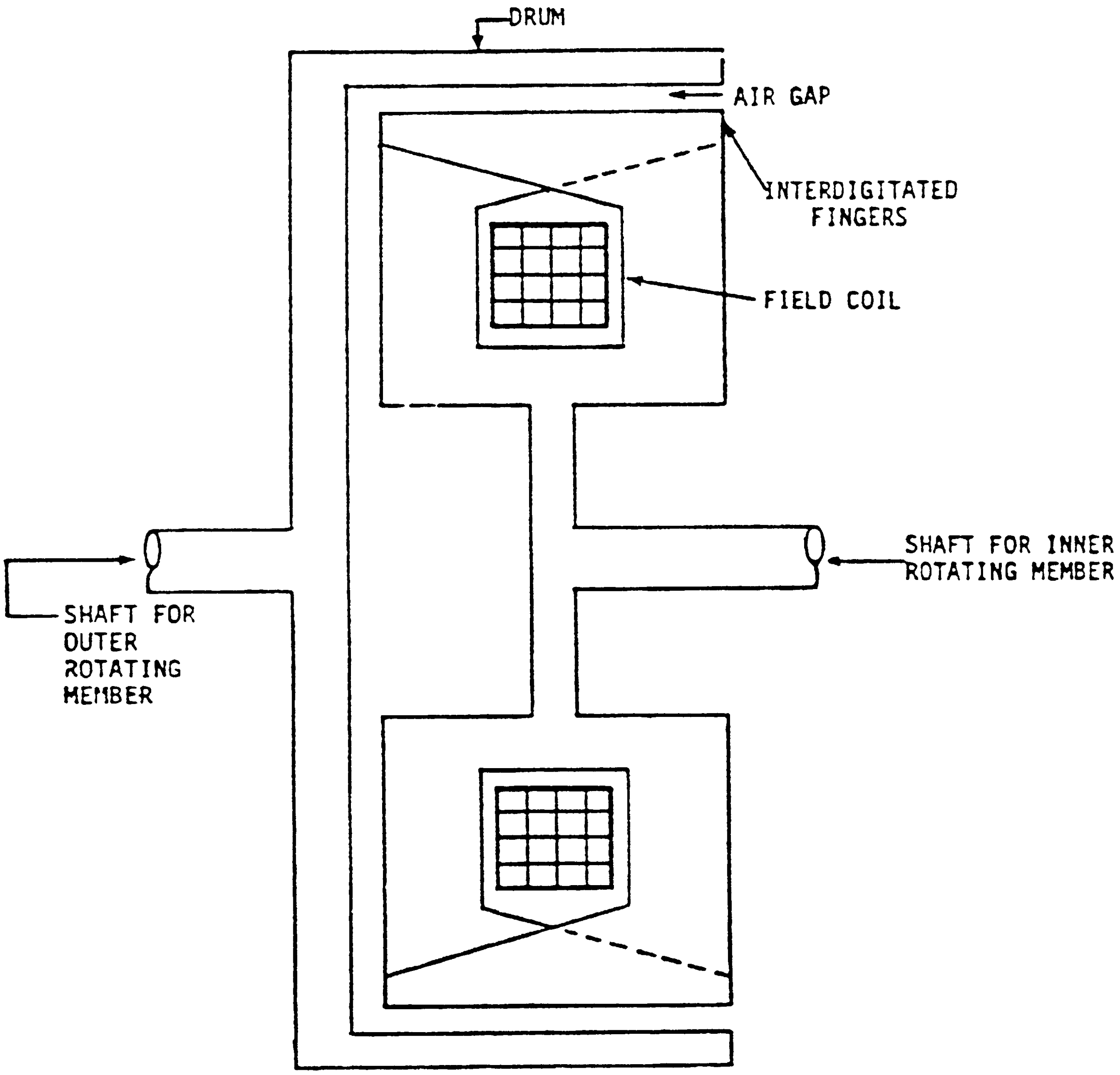


FIGURE 70. Typical induction clutch design for applications in which a low stray magnetic field is not required (longitudinal section).

6.5.1.1 Operation. Suppose the inner member is rotated by the engine. The magnetic field sweeping by the drum on the outer member induces electric currents in the drum, and the drum is dragged around. The device thus operates in essentially the same way as a three-phase induction motor. In the induction motor, three-phase alternating current in stationary windings sets up a rotating magnetic field. In the induction clutch, electromagnets excited by dc are mechanically rotated to set up a rotating magnetic field we can vary the slip between the inner and outer members of the clutch by changing the field current. Thus , such a clutch can be used to obtain either a constant speed output from a variable speed input or a variable speed output from a constant speed input.

6.5.1.2 Disadvantages . The advantages of the typical induction clutch construction do not apply for small stray magnetic field applications. There is one field coil that may be of considerable size, a large number of turns, and a substantial field current. The induction clutch will set up a considerable stray magnetic field. This form of construction must never be used in any induction clutch that is to be installed on a minesweeper.

6.5.2 Induction clutch for small stray magnetic field. We will need a series of long and narrow magnetic poles that are alternately north and south. We can get these by fixing pole pieces (on the outside of the inner member or the inside of the outer member) and winding field coil around each pole piece. A typical design with the pole pieces on the outside of the inner member of the induction clutch is shown on figure 71. Connections to the field coils should be made as for dc generator shunt field coils (see 6.5.2.1).

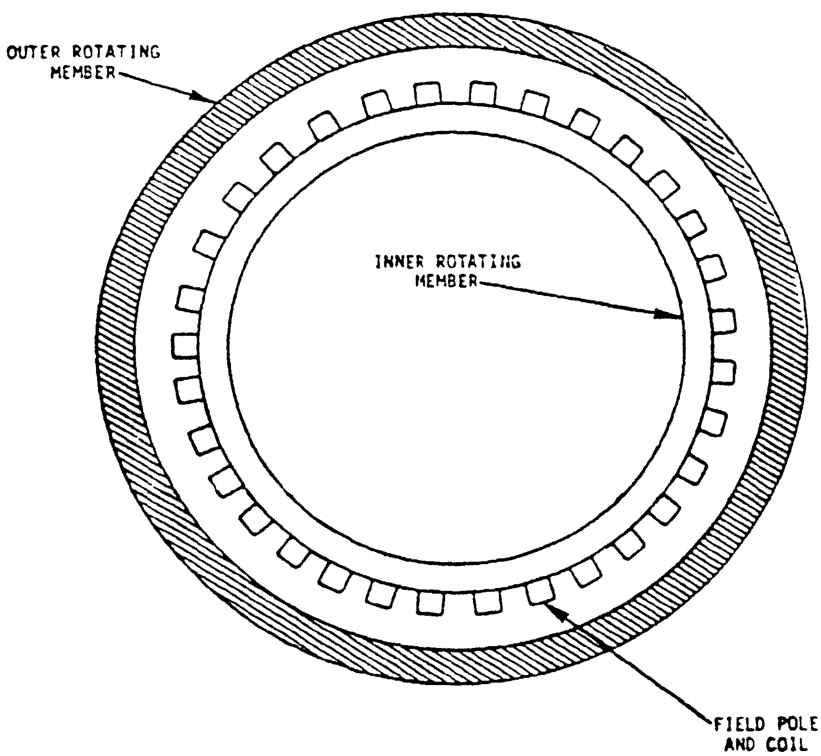


FIGURE 71. Typical induction clutch design with multiple field poles for low stray magnetic field (end view).

6.5.2.1 Field coils. We will make the field coil connections at the same end of the clutch assembly and arrange them as close together as possible. The connections will be arranged to prevent a single concentric turn from being formed around the shaft of the machine. Figure 72 illustrates a typical interconnection for an 8-pole field.

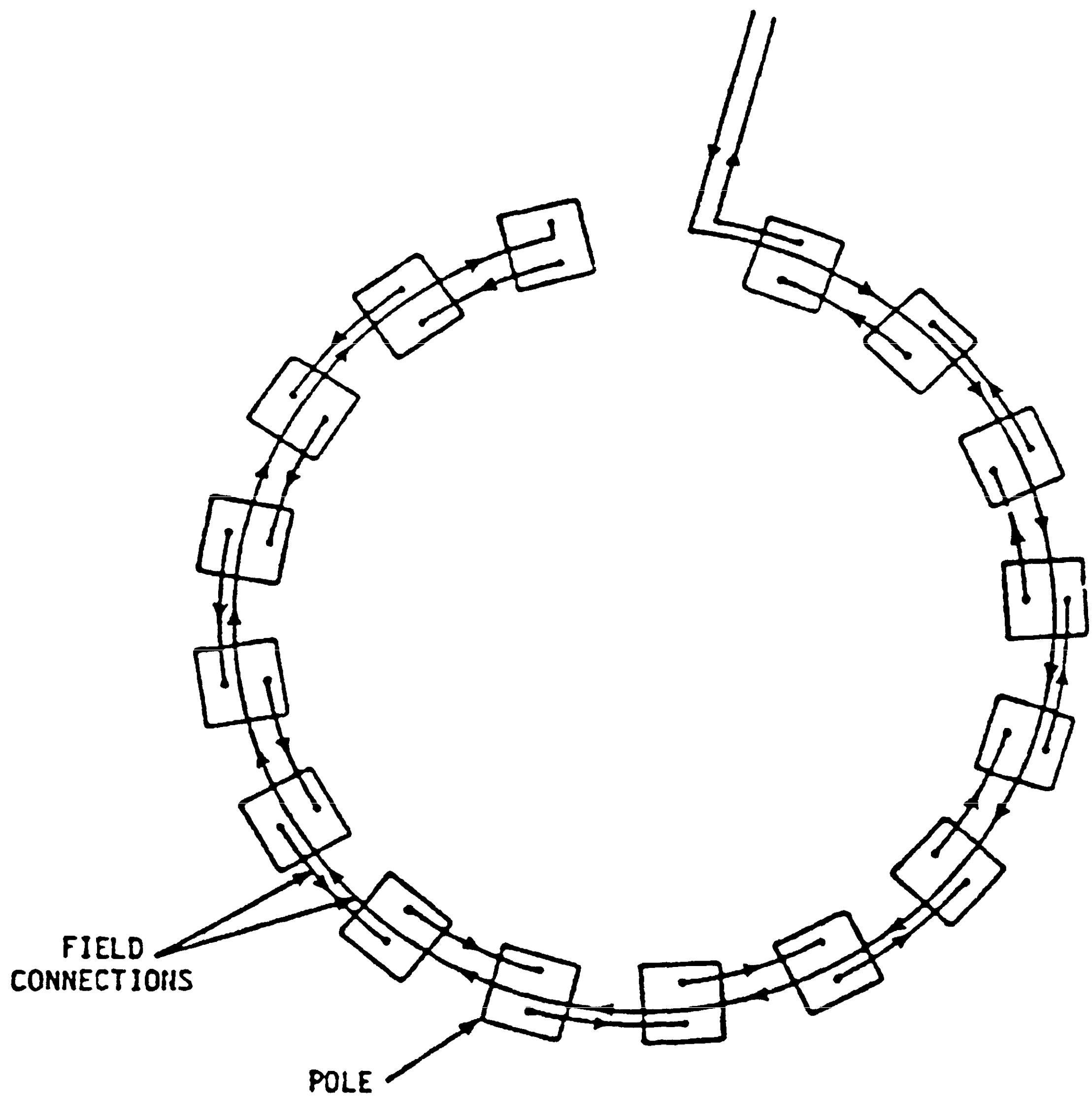


FIGURE 72. Typical inter-pole connections for an induction clutch design with low stray magnetic field.

6.5.2.2 Advantages of the form. The small stray magnetic field form will have a much smaller field than the typical induction clutch design for the following two reasons:

(a) First, each of the small field coils in the small stray magnetic field form will have a much smaller magnetic moment than the single field coil in the typical induction clutch design. This will give us a smaller stray magnetic field.

(b) Second, and even more important, is the large number of poles. If we take coils uniformly spaced on a circle of radius R with coils alternately north and south, the stray magnetic field at a distance decreases as the number of poles increases, even when the radius of the circle and the magnetic moments of the individual coils remain the same. The induction clutch will have a considerable number of field coils, probably sixteen or more. With so many field coils, the stray magnetic field will be small

7. DESIGN OF MINESWEEPING GENERATORS

7.1 General.

7.1.1 Introduction. Strong magnetic fields inside a dc motor or generator are essential for the operation of the machine. These magnetic fields, being inside the machine, obviously cannot affect a magnetic mine outside the machine and are not a source of danger. What is dangerous, so far as magnetic nines are concerned, is the external magnetic field or the stray magnetic field outside the machine. This is the difference between the magnetic field when current is flowing and when current is not flowing in any circuit or part of the machine. This stray magnetic field is zero when all currents are zero. Its magnitude, when currents are flowing, depends upon the magnitudes of currents and the nature of the particular circuits in which they are flowing.

7.1.1.1 Desire objectives. Depending upon the size and design of a machine, its external or stray magnetic field may be either relatively large or quite small; large enough to actuate a mine many feet from the machine or small enough to be undetected by a mine quite close to the machine. One of the primary objectives in designing a minesweeping generator is to make the stray magnetic field as small as possible.

7.1.1.2 Application to machines other than minesweeeping generators. The discussion in this paragraph is specifically in terms of the design of minesweeping generators. These must be designed with the greatest of care because they are large machines and are used in minesweepers that may pass directly over magnetic mines. Therefore, they must produce only the smallest stray magnetic field possible in order to minimize the danger of firing a mine under the minesweeper. Note, however, that the same principles must be used in the design of any \ motor or generator for which small stray magnetic field is an important consideration.

7.1.2 Cause of stray magnetic field. Electric current is the cause of stray magnetic field. In a minesweeping generator there are two principal circuits, the field circuit and the armature circuit. Current in either one or in both can give rise to stray magnetic field. Both must be designed to make this effect. as small as possible. The field circuit is considered in 7.2; the armature circuit, which carries a much larger current, in 7.3.

7.1.3 Essential features in generator desire. Theoretical studies show that there are certain essential features that must be incorporated in minesweeping generators designed for minimum stray magnetic field. The necessity for some of these features has been amply verified by experience. The necessity for others has not yet been tested by experience but is firmly based upon theoretical studies. The essential features are listed below. Details concerning them and the reasons why they are essential are given in the rest of this section in the paragraphs to which references are given.

7.1.3.1 Frame. Minesweeping generator frames will have the following features:

(a) The frame will be either solid, with no joints, or laminated (see 7.2.3.4 (a))
(b) The weld in the frame will be at a main pole (see 7.2.3.4 (b)).
(c) The frame will be machined inside and outside, to ensure uniform cross section throughout (see 7.2.3.4 (c)).
(d) The material of the frame will be magnetically homogeneous throughout (see 7.2.3.4 (d)).
(e) The outside of the magnetic material in the frame will be, as nearly as possible, a smooth surface of revolution with its axis coincident with the axis of the generator. There will be no magnetic feet or other major projections of magnetic material on the outside of the frame (see 7.2.3.4 (e)).
(f) Current carrying leads are not to be taken through the frame. They will go through some part of the generator enclosure that is nonmagnetic (see 7.2.3.4 (f)).

7.1.3.2 Number of field poles. The machine must have an adequate number of field poles in accordance with 7.2.3.5 and 7.2.3.6.

7.1.3.3 Symmetry and uniformity. The following aspects of symmetry and uniformity will be applied:

(a) Air gaps will be uniform (see 7.2.3.4 (g)).
(b) In machines that have commutating poles, there will be as many commutating poles as main poles (see 7.2.3.4 (h)).
(c) Coils of the same type, such as shunt field coils and commutating pole coils, will be of the same size and have exactly the same number of turns (see 7.2.3.4 (i)).

7.1.3.4 Wiring around the frame Wiring around the frame will have the following features:

(a) Connections to the shunt field coils will have no net turns around the shaft and no uncompensated current loops (see 7.2.3.7).
(b) End connections to the commutating coils and the compensating winding will have no net turns around the shaft and no uncompensated current loops (see 7.4.3 to 7.4.5 and 7.5.2.1 to 7.5.2.4).

7.1.3.5 Lap -wound armature. Lap-wound armatures will have equalizer connections in the form of rings of uniform cross section throughout (see 7.3.2.9).

7.1.3.6 Brush collector rigns. Brush collector rings will have the following features:

(a) The brush collector rings will be complete rings, concentric with the axis of the machine, and of uniform cross section throughout the entire circumference (see 7.3.2.18).
(b) There will be either: (1) three brush collector rings equally spaced in the direction parallel to the axis of the machine, with the center ring carrying full positive (or negative) current and each of the two outer rings carrying one-half of the negative (or positive) current (see 7.3.2.25); or (2) two concentric rings, one larger than the other, mounted in the same plane perpendicular to the axis of the machine (see 7.3.2.26).
(c) The current take-off points for the brush collector rings will be either:

  (1) In line with the axis of the machine for machines with three brush collector rings, or
  (2) In a plane passing through the axis of the machine for machines with two concentric brush rings in the same plane (see 7.3.2.26).

7.1.3.7 Connections for brush collector rings. Connections from brush collector rings of a machine to the associated circuit will be arranged with a central conductor carrying full current and two symmetrically placed flanking conductors , each carrying half current, all so arranged as to avoid unbalanced current loops (see 7.3.3.1 through 7.3.3.6).

7.1.3.8 Double-armature machines. Machines with two armatures on the same shaft will be designed in accordance with the principle of mutual compensation (see 5 9), The two armatures will be as nearly alike as possible and have their connections so arranged that the magnetic field of one is in opposition to that of the other.

7.1.4 Desirable features in generator desire. In addition to the features listed above, which are considered essential, there are a number of others that are less well established, which may not make much difference, but theoretical studies indicate they will be desirable.

7.1.4.1 Angular position of takeoff Points. It is probably better to have the takeoff point from one brush collector ring at the same point where one set of brushes is connected to the ring (see 7.3.2.29).

7.1.4.2 Number of commutator bars. It is better to have the number of commutator bars equal to an integral multiple of the number of poles (see 7.3.2.30).

7 1 4.3 Brush rigging. The brush rigging should be so designed as to force a well-defined current path from a set of brushes to the brush collector ring. It will be desirable for this current path to be in a plane that passes through the axis of the machine (see 7.3.2.28)

7.1.4.4 Position of brush collector rings. The axial distance from the brush collector rings to the commutator risers should be so chosen as to minimize the effect of unequal current division between different sets of brushes (see 7.3.2.28).

7.2 Field circuit design.

7.2.1 Appliability. Minesweeping generators are separately-excited machines. The following discussion will, therefore, be specifically in terms of the field circuit for a separately excited machine. Note, however, that similar considerations apply to the shunt field circuit of a shunt-wound motor or generator.

7.2.2 Sources of stray magnetic field. The field circuit consists of two separate stray magnetic field sources: the field coils and the connections to the field coils. Design guidelines to minimize the stray magnetic field produced by the field coils are described in 7.2.3.1 through 7.2.3.5. The design guidelines to minimize the stray magnetic field produced by connections to the field coils are described in 7.2.3.

7.2.3 Minimization of stray magnetic field produced by the field coils. The field coils set up the useful or working flux that is essential for the operation of the generator. In a large generator, the field coils are of respectable size, have a considerable number of turns, and carry an appreciable field current. One field coil alone will set up a stray magnetic field that is larger than can be tolerated for a minesweeping generator. All field coils together set up a stray magnetic field that is smaller (except for a two-pole machine) than the stray magnetic field from a single coil. We will use the term ‘polar leakage field" to designate the stray magnetic field that is set up by the field coils alone The polar leakage field depends upon the following:

(a) The magnetic moment of each field coil.
(b) The number and orientation of the field coils.
(c) The distance from the field coils to the center of the machine.
(d) The effect of iron in the machine.
(e) The effect of nonuniformity in pole strengths, air gaps, and magnetic circuit.

7.2.3.1 Number and orientation of field coils. In order to avoid the consideration of the effect of iron, consider four sets of coils; one each with two, four, six, and eight coils held in nonmagnetic supports in the position that they would occupy in two-, four-, six-, and eight-pole generators with one pair of poles arranged vertically. Let us also compare these to a single coil. The five arrays are shown on figure 73, where each coil is represented by a short arrow in the direction of the coil's magnetic moment. Assume that the magnetic moment of each coil is 93 square meter ampere turns (1000 square foot ampere turns) and that each set of coils is arranged in a circle with a radius of 0,3 meter (1 foot). From the formula given in 4.3, the maximum values of the vertical component of magnetic field, in nanotesla, for distances 6, 9, and 12 meters (20, 30, and 40 feet) below the center of the arrays will be as shown in table VII.

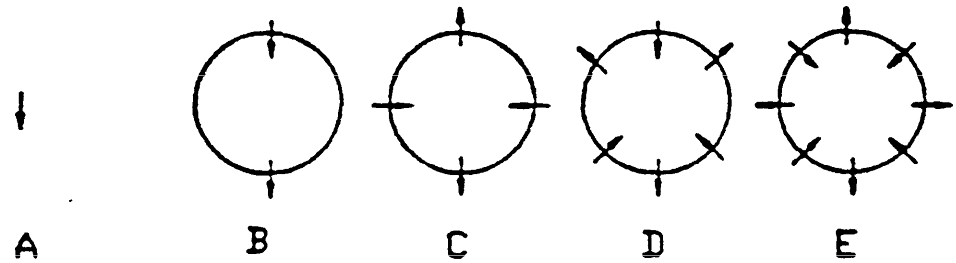


FIGURE 73. Arrays of coils.

TABLE VII. Magnetic field values of coils with a constant magnetic moment.

	Maximum value of vertical component of magnetic field (nanoteslas)		
	D = 6 meters	D = 9 meters	D = 12 meters
1 coil	86.11	25.51	10.76
2 coils	172.22	51.03	21.52
4 coils	35.37	7.35	2.42
6 coils	4.84	0.64	0.15
8 coils	0.47	0.04	0.01

The comparison given above shows clearly that the stray magnetic field decreases as we go from two to eight poles. The comparison, however, fails to show the full extent of improvement to be expected by increasing the number of poles. Start out with a two-pole generator. We would expect that a four-pole generator of the same capacity would need only about half the magnetic moment, a six-pole machine only a third, and an eight-pole machine only a fourth. Table VIII compares arrays with magnetic moments of 93, 46.5, 31, and 23.25 square meter ampere turns (1000, 500, 333, and 250 square foot ampere turns) for two-, four-, six-, and eight-pole machines, respectively.

TABLE VIII. Magnetic field values of coils with varying magnetic moment.

	Maximum value of vertical component of magnetic field (nanoteslas)		
	D = 6 meters	D = 9 meters	D = 12 meters
2 coils (93)	172.22	51.03	21.52
4 coils (46.5)	19.38	3.83	1.21
6 coils (31)	1 61	0.21	0.05
8 coils (23.25)	0.12	0104	0.002

The actual reduction in stray magnetic field will be somewhere between the values given in tables VII and VIII. It is clear in any event that we get a substantial reduction of polar leakage field by increasing the number of poles and that the improvement is greater with increasing depth. For example, from table VII we see that at 6 meters the ratio of the eight-coil to the two-coil field is O 00273, at 12 meters it is only 0.00046, substantially less than at 6 meters. It must be kept in mind, however, that the improvement, which has been calculated for an increase in number of poles, is based upon a number of assumptions that may not be realized in practice. These are as follows:

(a) Exact symmetry of coil arrangement.
(b) Exact quality of coil magnetic moment.
(c) Complete absence of iron or other magnetic materials. We will consider the effect of departure from these idealized conditions later. The preceding discussion has been based upon comparison of generators in which one pair of main field coils is on the vertical line. A little consideration will show that much the same results would be obtained by considering generators so oriented that the vertical line splits the angle between adjacent main field poles.

7.2.3.2 Effect of radius. The formulas in 4.3 give the magnetic fields that will be produced by arrays of coils (arranged on the circumference of a circle of radius r) on planes z meters below the centers of the arrays. These arrays are arranged just like the field coils of a generator. The formulas are valid only when iron and other magnetic materials are absent and r/z is small, not more than 0.1. When these conditions are satisfied, inspection of the formulas of 4.3 shows that we have the stray magnetic field varying with r as follows:

Number of coils in array	Stray magnetic field varies as power of r given below
2	0
4	1
6	2
8	3

Except for a two-pole machine, for which the polar leakage field is independent of r to a first approximation, stray magnetic field will increase as r is increased, if other factors remain the same. In the interest of small stray magnetic field, r should be small rather than large. This does not mean, however, that it is necessary to go to unconventional generator designs of smaller radius and greater length than are customary. So far as we know it should be possible to have small stray magnetic field from generators of normal proportions. Abnormally long and small diameter designs are usually unnecessary, and abnormally short and large diameter designs should be avoided because they will tend to increase stray magnetic field.

7.2,3.3 Effect of iron. Our considerations up to now have been based upon the assumption of an air core (no iron present). The reason for making this assumption is that we can calculate the magnetic field that will be produced by an array of field coils in an air core, but we cannot as yet calculate it when iron is present. Nonetheless, an iron core is present in a machine and even though we cannot calculate its effect we must make some allowance for it. The assumption of no iron being present is unrealistic for a generator because it has an iron core. The field coils have iron poles inside them; the poles are arranged outside the iron armature, and are in turn surrounded by the iron frame or yoke of the machine. At first sight it might appear that the iron yoke will carry the magnetic flux from pole to pole, keep it from escaping outside the machine, and cause the stray magnetic field to be less than it would be if no iron were present. The limited data we have on the effect of iron indicates chat this expectation is incorrect. For three small two-pole motors, the measured polar leakage field is about twice as much as the polar leakage field calculated on the assumption no iron is present. The available data on machines with six or more poles indicate that for these, the measured magnetic field is much more than twice as great as the magnetic field calculated on the assumption that no iron is present. As yet we have too little information to warrant any sweeping generalization, except that much remains to be learned about the effect of iron.

7.2.3.4 Uniformity and symmetry. The decrease in the polar leakage field that is obtained by the use of a large number of poles is dependent upon a high degree of uniformity and symmetry in the construction of the machine. We can see this by referring back to table VII in 7.2.3 1. At 12 meters the calculated stray magnetic field for eight coils, each with a magnetic moment of 93 square meter ampere turns (1,000 square foot ampere turns), is only 0.00007/0.102 - 0.07 percent as much as the stray magnetic field from a single coil of the same magnetic moment. Hence, if we start out with an array of eight exactly equal coils and then slightly strengthen one, even a very slight degree of unbalance will lead to a stray magnetic field many times what it was before and cause us to lose part of the advantage that would otherwise be gained by the use of eight poles The following features must be incorporated in minesweeping generators to ensure uniformity and symmetry:

(a) Frame construction The frame of the machine consists of two parts: the magnetic frame and nonmagnetic frame support. The magnetic frame must be either of one piece construction or a radially laminated construction such that the flux density in the outer 25 percent of the frame thickness will remain in the linear portion of the dc B-H tune of the magnetic material used throughout the operating range of the machine.

(b) Location of weld. The magnetic frame of the machine is usually made by rolling a steel slab into a ring and welding the ends of the slab together The weld should be located at the centerline of the top main field pole for vertically oriented poles, and at the center of one of the two highest field poles for machines with the vertical equally dividing the angle between the two highest field poles. Since the main flux splits at the centerline homogeneity at the weld will have a minimum effect if the weld is located on the centerline of a main field pole.

(c) Machining After the frame has been welded into a ring and seasoned, it should be machined inside and outside and on the ends to ensure uniform cross section throughout. If the ring is machined only on the inside, there will inevitably be differences in the thickness of the frame at different points and loss of symmetry,

(d) Material. The material of which the frame is made must be magnetically homogeneous to avoid lack of symmetry.

(e) Outside of frame. The outside of the frame must be as magnetically smooth and symmetrical as possible. Magnetic feet and other major projections of magnetic materials must be absent. Pole bolt heads should be recessed, and should preferably be cap screws that almost completely fill the holes drilled in the frame to recess the heads.

(f) No leads through frame. No current carrying leads will pass through the magnetic frame or other magnetic parts of the machine. These inevitably introduce an element of asymmetry. The effects will be less if the leads go through nonmagnetic parts of the machine enclosure ,

(g) Air gaps. Main pole air gaps must be as nearly equal as possible. Commutator pole air gaps must be a nearly equal as possible. However, main pole air gaps may be different than the commutator pole air gaps.

(h) Number of commutating poles. Minesweeping generators will have as many commutating poles as main poles. Some small motors or generators are built, however, with only half as many commutating poles as main poles. Such a construction is unacceptable for any motor or generator, whatever its size, that is intended for installation on a mint-sweeper. lf a machine for installation on a minesweeper has any commutating poles at all, it must have as many as there are main poles.

(i) Coils. Coils of the same kind (shunt field coils, commutating field coils, and so forth) will be of the same size and shape and have the same number of turns. Some engine-starting motors have series field coils with different number of turns on the different poles . Such a construction is unacceptable for motors or generators intended for use on a minesweeper.

Even when care is used to make a machine as uniform and symmetrical as possible, there will inevitably be slight departures from this desired condition. It is believed that the equalizer connections on the armature will operate to minimize the effects of certain sources of nonuniformity. These connections and their function are discussed in detail in 7.3.2.9. Here, it will suffice to say chat they function to strengthen weak poles and weaken strong poles, hence, to smooth out and equalize the main field flux distribution. They should, therefore, tend to minimize the effects of such causes of nonuniformity as difference in air gaps and field coils that are not of exactly equal magnetic q oment because of manufacturing tolerances. On the other hand, it is possible that the equalizer connections would not do much, if anything, to minimize the effects of lack of symmetry caused by the presence of magnetic feet outside the frame of the generator.

7.2.3.5 Minimum number of poles in minesweeping generators. We have seen in 7.2.3.1 that the polar leakage field is decreased by increasing the number of field poles, other things remaining the same. This raises the question of what is the minimum number of field poles permissible in minesweeping generators that must have a low stray magnetic field. The answer is to be obtained only by a detailed study of each particular case, taking into account the size of the generator and the stringency of the magnetic field limits it has to meet. The following guidelines are to be kept in mind:

(a) Two poles are never enough.
(b) Four and six poles are doubtful Until we have more information than we have at present, neither four nor six poles should be used for minesweeping generators.
(c) Eight poles should be enough, but for the largest machines more would be better.

7.2.3.6 Minimum number of poles in other generators and in motors. The problem is even more complicated for these than for minesweeping generators because of the great range in size involved. For a small machine, the magnetic moments of the field coils and their distances from the center of the machine will be less than for a large machine. Both effects will tend to give a smaller stray magnetic field and it should not be necessary to have as many poles as on a minesweeping generator.

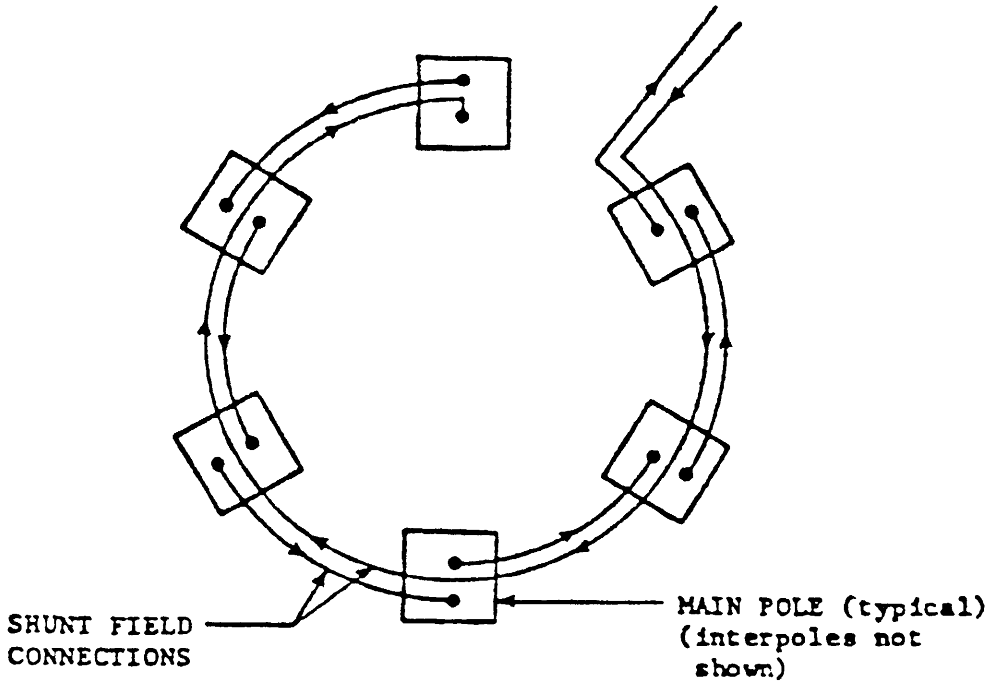
(a) Two-pole motors or generators should never be used on minesweepers except in very small sizes, 100 watts input or output or less. Even for the very small machines it would be better to have four poles. The two-pole machine is completely different from machines with four or more poles. In the two-pole machine, the magnetic fields produced by the two field poles are necessarily additive.

In machines with four or more poles the magnetic fields produced by the individual poles cancel each other and leave only a residual field that is much smaller than the field from one pole alone .

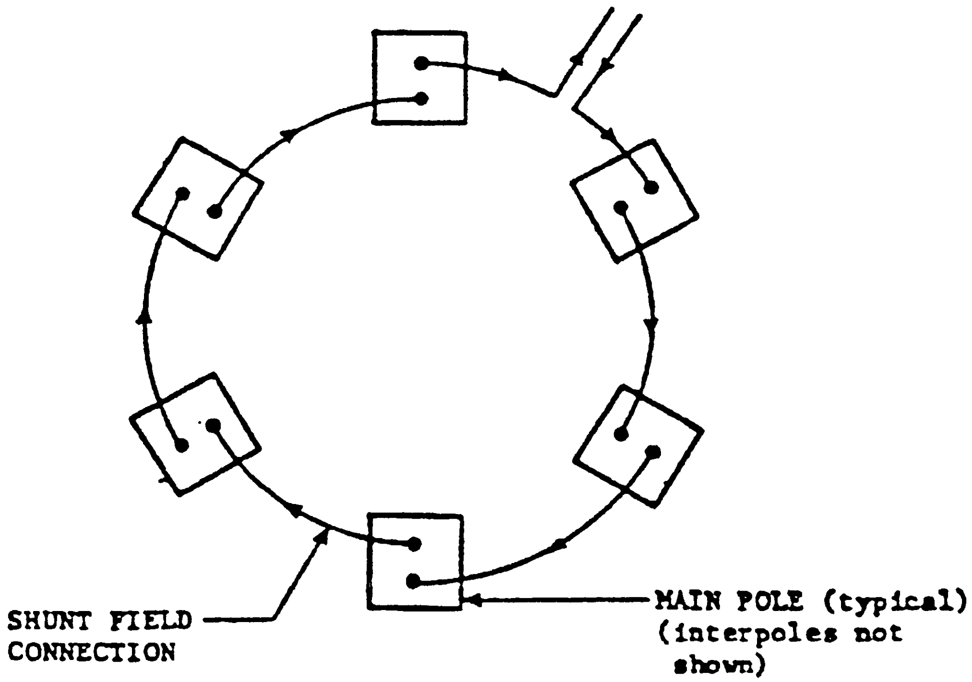
(b) Rotary amplifier type exciters that have more than two physical poles but only two main magnetic poles should never be used on minesweepers . These will have the same stray magnetic field characteristics as other two-pole machines, and since they are necessarily machines of some size, will be highly objectionable.

(c) Four poles will probably be enough for most dc motors and generators except for the minesweeping generators. Each individual case must be carefully studied and more poles must be used if there is any doubt that four are enough.

7.2.3.7 Minimization of stray magnetic field Produced by the connections to the field coils. Figure 74 shows the preferred and non-preferred methods of making connections to the field coils. The advantage of the preferred method is that there is no turn around the shaft of the machine, as there is in the non-preferred method. The turn around the shaft would be a large current loop that would produce a large stray magnetic field Therefore, the preferred method must be used. The field connections must all be on the same end of the machine and should be as close together as possible.



A. Preferred method.



B. Non-preferred method.

FIGURE 74. Preferred and non-preferred methods of making of making connections to the field coils.

7.3 Armature circuit.

7.3.1 General. The armature current in a minesweeping generator is large. It flows in complicated circuits in the machine and, unless these circuits are carefully designed, can produce a large stray magnetic field. The armature current flows in the following:

(a) Armature winding.
(b) Commutator risers.
(c) Commutator bars.
(d) Brush rigging.
(e) Connections from the brush collector rings to the commutating winding, the compensating winding and the machine's terminals to an external circuit.
(f) Commutating winding.
(g) Compensating winding

These circuits will be grouped and treated as follows:

(a) The armature winding, commutator risers, commutator bars, brush rigging, and the brush collector rings will be grouped together and then divided into the various arrays of current loops that will give the same magnetic field as the current actually flowing in the machine.
(b) The connections from the brush collector rings to the commutating winding, compensating winding, and external circuit will then be discussed,
(c) Finally, the commutating winding and the compensating winding will be discussed.

7.3.2 The armature circuit from the armature winding to the brush collector - - We will confine our attention to a four-pole generator in our consideration of the armature circuit from the armature windings to the brush collector rings. This does not mean four poles are enough for a minesweeping generator. We can use a four-pole machine as an example because it has enough poles to illustrate the points that are involved. The use of a larger number of poles would complicate our equations and discussion without adding anything essential to our insight into the principles of designing a generator having a small stray magnetic field.

7.3.2.1 Simplifying assumptions. A number of simplifying assumptions are made in order to bring our problems down to manageable proportions even for the fairly simple case of a four-pole generator. These assumptions include the following:

(a) The armature has a lap winding but no equalizer connections (also known as armature cross connections).
(b) The number of commutator bars is an integral multiple of the number of poles.
(c) At all times, each set of brushes makes contact. with only one commutator bar.
(d) Lath set of brushes touches the commutator bar with which it makes contact at only one point.

(e) The connection from a set of brushes to the brush collector ring is directed radially outward from the point of contact with a commutator bar to the point of connection to the brush collector ring.

Some of these conditions could be satisfied in an actual machine and some could not. It is possible that none will be satisfied in a particular minesweeping generator. We are, therefore, considering an idealized machine for the purpose of obtaining a simplified over-all picture of the stray magnetic fields that will be produced by the flow of armature current from the armature winding to the brush collector rings, inclusive, Having done this, we can then proceed to examine what modifications of this simplified picture would be made necessary by dropping the simplifying assumptions.

7.3.2.2 Developed view of a machine. Consider the armature of the machine and the brush collector rings. The armature winding and the commutator are on the surfaces of two coaxial cylinders The brush collector rings are circles concentric with the cylinders. Imagine that the whole structure is slit lengthwise, laid out flat, and that the parts of greater diameter than the commutator are simultaneously shortened so that equal lengths in the direction between points 5 and 8 of figure 75 correspond to equal angular spread between points when in the positions actually occupied in the machine. This procedure gives us the developed view shown on figure 75. In this view, all commutator bars are shown of short length except those with which the brushes make contact at points 6, 8, 9 and 10. All bars not touched by a brush carry no current and require no consideration in a discussion of current paths. To complete the circuit and permit current flow in the armature, commutator risers, commutator bars, and brush collector rings, the two brush collector rings are connected between points 11 and 12. These are the points at which the armature current leaves one brush collector ring and returns to the other. They will be referred to as the takeoff points. Note that the takeoff points have an angular spread of b-a degrees. It will appear later that there will be a longitudinal component of magnetic moment unless this angular spread is zero.

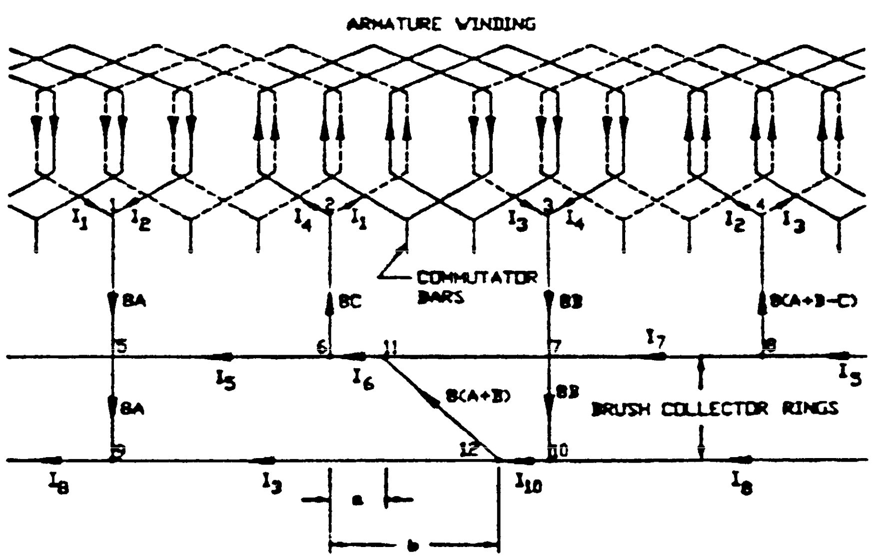


FIGURE 75. Developed view of armature winding commutator and brush collector rings.

7.2.2.3 Determination of currents. The straightforward way of determining the currents in the armature winding and other parts of the circuit would be to assume that e_1, e_2, e_3, and e_4 are the unidirectional voltages induced in the four parts of the armature winding between adjacent sets of brushes; assume values for the resistances of the four sets of brushes and other elements in the circuit; and then proceed to calculate the currents. In the ideal case of a perfectly symmetrical generator with exactly equal poles, all four Induced voltages will be equal in magnitude and alternately plus and minus in sign if voltage is counted positive in one direction around the armature. In an actual generator the induced voltages may not all be equal because of differences in pole strength, and they should , therefore, be assumed to be unequal. Note, however, that the four induced voltages cannot all be chosen arbitrarily but are subject to the condition that:

$$e_1 + e_2 + e_3 + e_4 = 0 \qquad [7\text{-}1]$$

This equation says that if we start at any point on the armature winding and proceed around it until we come back to the starting point, the total induced voltage will be zero. We can see this in a number of ways. Perhaps the simplest is as follows. The flux-cutting conductors of the armature winding are represented by twelve sets of two adjacent vertical lines in figure 75. The two lines represent two conductors or coil sides in the same armature slot; they cut the same magnetic flux; hence, the same voltage will be induced in each, The voltage induced in one will be in a clockwise direction around the coils of the armature winding, in the other, in the reverse direction. Since the two induced voltages are equal and opposite, the net voltage is zero. The same will be true for all the other pairs of flux-cutting conductors and the net induced voltage all around the armature winding will be zero. We can, therefore, assume arbitrary values for only three voltages, for example e_1, e_2, and e_3. The fourth voltage will be e_4 - - e_1 - e_3 - e_2. If we now proceed in a straightforward manner to solve for the currents, we will ultimately come up with expressions for the currents in terms of e_1, e_2, e_3, and resistances. It can be done this way, but a considerable amount of algebra is involved and the results are in a less immediately usable form than if an alternative procedure is used. This is to start out by assuming the currents in three of four brush sets (only three currents can be chosen arbitrarily, as shown herein) and then solve for all other currents in terms of these three. This is the procedure that will be used.

7.3.2.4 Currents in commutator bars. In the ideal case, each of the four sets of brushes would take the same current and the currents in the four commutator bars they touch (those between points 1-9, 2-6, 3-10, and 4-8) would be equal in magnitude and have the directions shown on figure 75. In actual machines, however, the different sets of brushes do not take the same current. To take this into account, we will assume that the currents in the four commutator bars touched by the brushes are as shown on figure 75. Three of these four currents, 8A, 8B, and 8C, can be chosen arbitrarily. The fourth current q ust then be 8(A + B - C) to make the current returning to the armature winding equal to the current leaving it. The current output of the machine is 8A + 8B, which is the current in the connection between points 11 and 12.

7.3.2.5 Currents in armature winding. Let I_1, I_2, I_3, and I_4 be the currents in the armature winding. These currents can be calculated as follows: The sum of the currents to point 1 must be equal to the sum of the currents away hence, $I_1 + I_2 = 8A$. Applying the same consideration to points 2, 3, and 4 we have the four following equations:

$$I_1 + I_2 = 8A \qquad [7-2]$$
$$I_1 + I_4 = 8C \qquad [7-3]$$
$$I_3 + I_4 = 8B \qquad [7-4]$$
$$I_2 + I_3 = 8(A + B - C) \qquad [7-5]$$

Here are four equations in four unknowns, but only three of them are independent. One more equation is needed. It is obtained as follows: Start from point 2 and follow current I_1 up and to the right. It ultimately ends at point 1. Doing the same for other points, we arrive at the simplified layout of armature currents on figure 76 in which the points 1, 2, 3, and 4 are shown on a circle in the position they actually have in the machine rather than the positions shown in the developed view of figure 75. Now start at point 2, proceed counterclockwise, and write the equation saying that the total potential drop is zero starting from any point and returning to the same point. This equation is:

$$R(I_1 - I_2 + I_3 - I_4) - E = 0 \qquad [7-6]$$

where:
- R = resistance of that part of the armature winding through which current I_1 flows. This will also be equal to the resistances through which I_2, I_3, and I_4 flow.
- E = generated emf in the armature winding counted positive in a counterclockwise direction from point 2 all the way around and back to point 2.

However, E is equal to zero. The generated emf in one direction is equal to that in the other with the result that net emf is zero. Putting E = 0 in equation 7-6, it reduces to:

$$I_1 - I_2 + I_3 - I_4 = 0 \qquad [7-7]$$

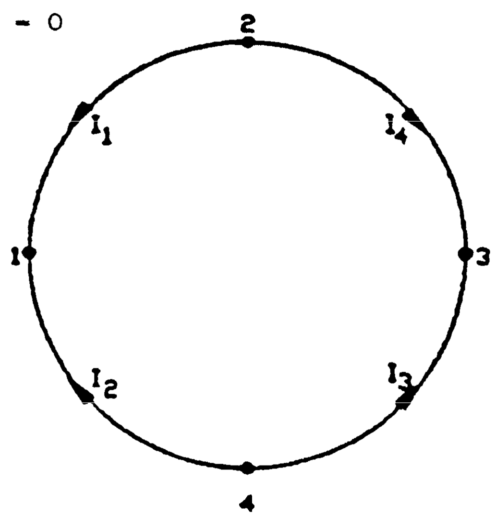


FIGURE 76. Currents in armature winding.

The equation added to any three of the four equations 7-8 to 7-11 enables us to find the four armature currents. They are as follows:

$$I_1 = 2A - 2B + 4C \qquad [7\text{-}8]$$
$$I_2 = 6A + 2B - 4C \qquad [7\text{-}9]$$
$$I_3 = 2A + 6B - 4C \qquad [7\text{-}10]$$
$$I_4 = -2A + 2B + 4C \qquad [7\text{-}11]$$

For the simple armature winding shown on figure 75 the currents in the armature are uniquely determined, as shown above, when the currents in the commutator bars are known or are assumed. We will proceed with the discussion using these values for the currents in the armature winding, and later on (see 7.3.2.9) consider the modifications that must be made to take into account the equalizer connections that an actual lap wound armature will have.

7.3.2.6 Currents in brush collector rings. As pointed out in 7.1.3, the brush collector rings will be complete rings of uniform cross section throughout. When this condition is satisfied, the resistance of a brush collector ring between any two points is directly proportional to the angular separation between the points. Consider the brush collector ring that passes through points 5, 6, 7, and 8 of figure 75, and write the equation that states that the total potential drop around the ring is zero. Since there is no generated emf, the total potential drop is the sum of the products obtained by multiplying each current by the resistance through which it flows, hence:

$$180\, rI_5 + arI_6 + (180 - a)rI_7 = 0 \qquad [7\text{-}12]$$

where:

r = resistance of one degree arc of the brush collector ring.
a = angular spread in degrees between points 6 and 11 of figure 75.

Now divide the above equation by 90 r, and let a/90 = c. The result is:

$$2I_5 + cI_6 + (2 - c)\, I_7 = 0 \qquad [7\text{-}13]$$

This equation is used in connection with equations for any two of the three junction points, 6, 11, and 8, to obtain the three currents in the brush collector ring. The values obtained are shown on figure 77. The currents in the other ring are obtained in a similar fashion and are also shown on figure 89. In the expression for these currents, d = b/90.

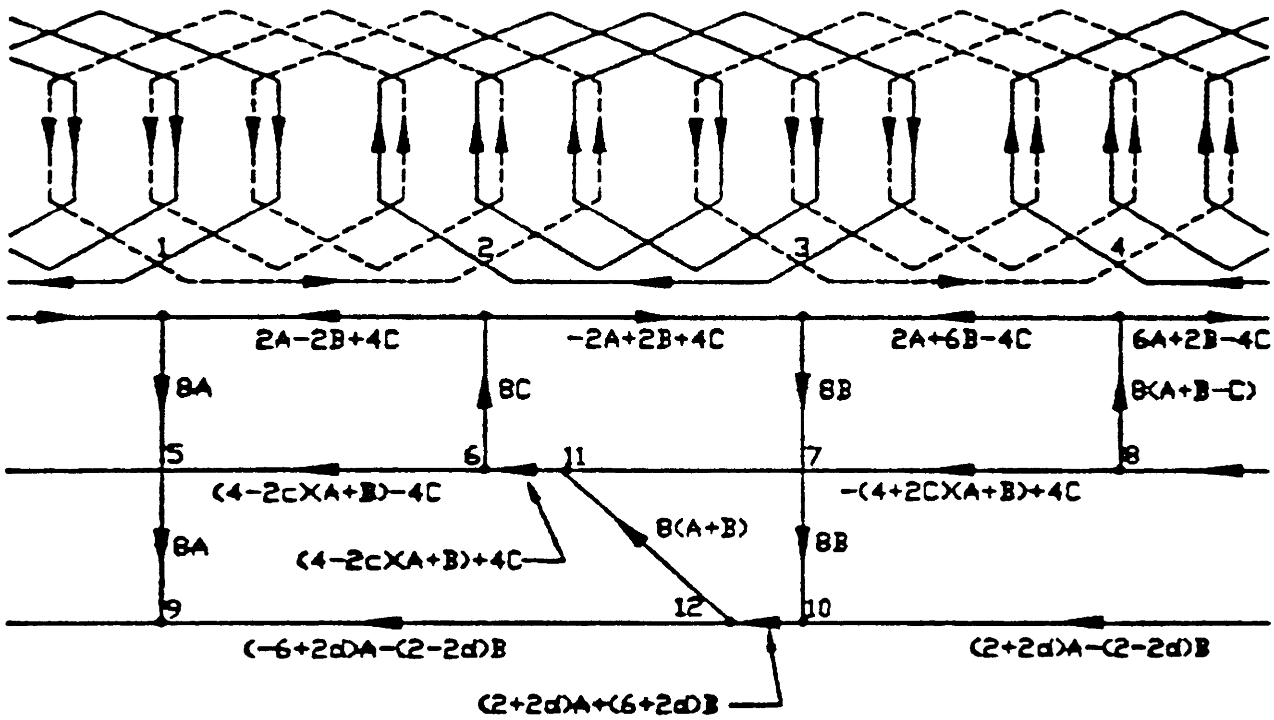


FIGURE 77. Currents the conductors in the armature circuit of a four-pole generator with two brush collector ring.

7.3.2.7 Isolation of armature currents. We now have the values of the currents in all the conductors visible on figure 75 and have determined them for the general case in which the four sets of brushes do not take equal currents. It must be remembered, however, that the conductors shown on figure 75 are not the only ones with which we are concerned. In addition to these, there are other conductors that will be perpendicular to the plane of the paper, and therefore will be invisible in the developed view of figure 75. These invisible conductors are the commutator risers that connect the commutator to the armature winding at points 1, 2, 3, and 4, and the connections from the brushes to the brush collector rings at points 6, 8, 9, and 10. The invisible conductors will be considered later, For the time being, we will confine our attention to the currents in the conductors that are visible on figure 75. Even with this restriction, it is not easy to visualize the magnetic field that will be produced by the conductors that are visible on figure 75. To understand this magnetic field, we turn our attention from the currents in the conductors to the system of current loops that will give the same current distribution and produce the same magnetic field. We then break the system of current loops down into several groups that are individually small enough to digest. The first step in this process is the transition from figure 75 to figure 77. Comparison of these figures shows that they are the same everywhere except along a line connecting points 1, 2, 3, and 4, Figure 75 shows that there is no direct (straight line) connection between points 1 and 2; hence, no current flowing directly between these points. On figure 77, however, there is a direct connection between these points The current in this connection is immediately adjacent to an equal and opposite current, so the net current, which flows directly between points 1 and 2, is zero, just as on figure 75. Similalar considerations apply to the connections that run directly between points 2 and 3, 3 and 4, and 4 and 1. The configurations shown on figure 75 and 77 thus have the same current distribution at all places and will produce the same magnetic field. The advantage of figure 77 is that there is no electrical connection between the upper and lower parts of the figure. We can, therefore, separate the two parts and study each by itself.

7.3 2.8 Magnetic field produced by armature winding. Let us study the armature winding, the upper part of figure 77, in more detail. The armature winding has four loops, one of which is shown by itself on figure 78. It is a three-turn loop that will produce a magnetic field directed down, The adjacent loops will produce magnetic fields that are directed up. In the actual machine, the four loops of the armature winding are not in a plane as shown on the upper part of figure 77, but are wrapped around the surface of the cylindrical armature core. Looked at from the end of the machine, we have four loops carrying currents as shown on figure 79. In this figure, the positive direction for currents is taken to be in the counterclockwise direction around the circle, Note that figure 79 is not intended to show the correct orientation of the four loops with respect to the main field poles of the machine but only with respect to each other. If the currents taken by the four sets of brushes are equal, that is, if 8A - 8B - 8C = 8(A + B - C), the currents in the loops shown in end view on figure 79 will be 4A, -4A, 4A and -4A. We thus have equal loops of plus and minus polarity arranged symmetrically on the armature core, Their net magnetic moment will be equal to zero . If, however, the brush currents are unequal, the currents in the loops in the armature winding will also be unequal, and the net magnetic moment of the loops will not be equal to zero. Thus, in a machine having a simple lap-wound armature of the kind we have considered, the net magnetic moment produced by currents in the armature winding will be zero if, and only if, the currents taken by the different sets of brushes are all equal.

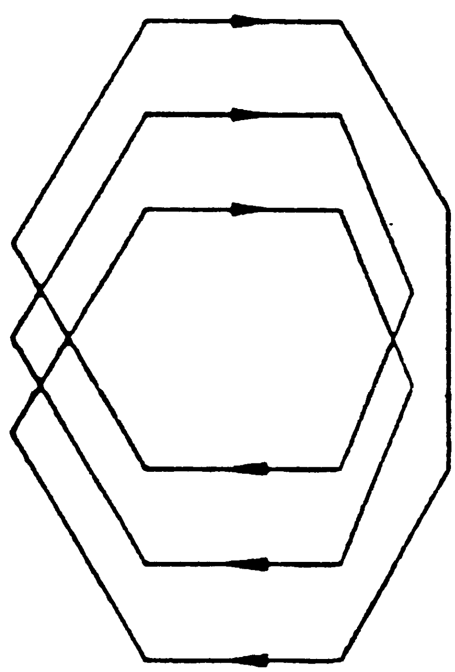


FIGURE 78. One loop of the armature winding.

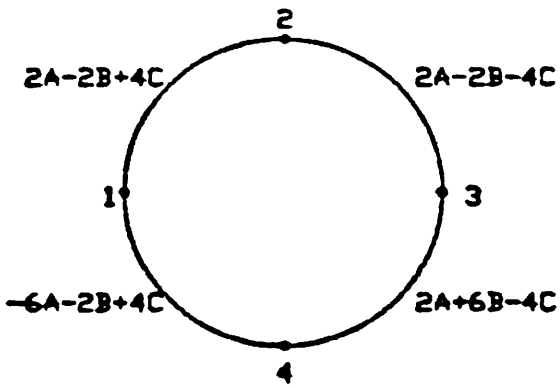


FIGURE 79. End view showing currents in the current loops of the armature winding.

7.3.2.9 Effect of equalizer connections. So far, we have been considering a lap wound armature without equalizer connections, which are also known as armature cross connections. An actual lap wound armature will have equalizer connection in the form of conduction rings at the front or back end of the armature winding. One ring is connected to all points on the armature winding that are theoretically at the same potential, zero potential, for example. Another ring is connected to all points that are 20 volts above the zero potential, and so on. In an ideal machine, these equalizer connections would be unnecessary, as all the pole strengths end brush resistances would be exactly equal. However, this ideal, highly symmetrical machine would be all but impossible to realize. We will therefore, use two simplified cases to illustrate the effect of equalizer connections on practical machines. These two cases are as follows:

(a) A machine that is completely symmetrical except that the field poles are not of exactly equal strength.
(b) A machine that is completely symmetrical except that the brush resistances are unequal.

For the first case let us again consider a four pole machine. If the pole strengths are not exactly equal, the voltages induced in the four quarters of the armature winding will not be equal and the four brush currents will not be equal, even though the brush resistances are all equal. Now let us add equalizer connections. In machines designed and built for small stray magnetic fields, the equalizer connections should be complete rings. of uniform cross sections, concentric to the shaft, which are connected to the appropriate points on the armature. The equalizing connections provide current paths for alternating currents that flow in such direction as to magnetize the weak poles, demagnetize the strong poles, and equalize the flux distribution and induced voltages around the machine. Although they cannot give perfect equalization, since this would reduce the induced voltages that cause current to flow in them to zero, equalizing connections are highly effective, as shown by the following two examples reported in the technical literature. In one case, d fourteen-pole generator with equalized armature operated reasonably well for several months before it was discovered that two of the main field-poles were reversed. The equalizer connections had equalized the flux distribution to permit operation of the machine. In the other case, one of the bearings on a generator slipped, allowing the armature to almost touch the field poles. With the equalized armature that was on this generator, there was no evidence of magnetic pull or unbalancing, except possibly a slight sparking at the commutator. We can conclude, therefore, that with an equalized armature winding, inequalities of field pole strength will be almost completely neutralized.

Let us now consider a four-pole generator with exactly equal pole strengths but unequal brush resistances. This will cause the brush currents to be unequal We can see this most easily by assuming that one of the two positive brush sets is completely disconnected. We then have one of the positive brush sets taking full output current while the other positive brush set takes none. Obviously, the equalizer connections on the armature car, do nothing to change this situation. Therefore, we see that while the equalizer connections can equalize the flux distribution and generated voltages, they cannot ensure equality of current division between the brush sets. It remains, therefore, to consider the effect that this inequality of current division will have. For a nonequalized armature winding, we have already seen (7.3.2.8) that the four loops in the armature winding will have a net magnetic moment different from zero when the brush currents are not equal in magnitude. This net magnetic moment will give rise to a stray magnetic field. Now consider an equalized armature winding. Suppose first that the field poles are exactly equal strength N, S, N, S. Now suppose that one N pole is made a little stronger and that the diametrically opposite N pole is made weaker by the same amount. This gives N+n, S, N-n, S. We see that this is the superposition of four poles of O. The function of the equalizer winding is to permit alternating currents to flow in such direction as to eliminate the two-pole field and replace it with the four-pole field, This is what the equalizing winding does when an initially symmetrical four-pole field is distorted by the superposition of a two-pole field caused by strengthening one N pole and weakening the diametrically opposite N pole. Now consider this situation Suppose we cut off the shunt field current, drive the armature at rated speed, lift the negative brushes, and from an external source, send current into the armature through one of the positive brush sets and take it away through the other positive brush sets (that would have to be reconnected to do this). The current through the armature will set up a two-pole field. Even though the armature is rotating, the two-pole field it creates will be stationary in space, just as the armature reaction of any dc armature is stationary in space. The situation thus appears to be precisely the same as if the two-pole field were set up by strengthening one field pole and weakening the diametrically opposite pole. The equalizer connections equalize such a two-pole field; they should also equalize fields caused by current in the armature . We conclude, therefore that with an equalized armature, the four-pole armature will set up a field with N, S, N, S poles of very nearly equal strength and the net magnetic moment of the four loops of the armature will be very nearly equal to zero regardless of whether the brush currents are equal or not.

7.3.2.10 <u>Wave windings</u>. We have considered up to now only lap wound armatures , initially without and then with equalizer connections. Wave wound armatures have no equalizer connections because the winding itself functions as an equalizer. It would appear, therefore, that a wave winding will act in the same way as a lap winding so far as stray magnetic field is concerned.

7.3.2.11 Symmetry. One of the simplifying assumptions (see 7.3.2.1) was that the number of commutator bars is an integral multiple of the number of poles. This condition is obviously necessary to give a completely symmetrical machine. It can be satisfied by lap windings, but not by wave windings, for a machine with four or more poles. In the interest of symmetry, therefore, it would be preferable to use a lap wound armature having a number of commutator bars equal to an integral multiple of the number of poles. We do not know how much difference in stray magnetic field there will be between two generators that are identical except that the armature of one has a number of commutator bars equal to an integral multiple of number of poles, while the armature of the other does not. It may be that the difference is too small to be appreciable, but in light of our present knowledge, it would appear that the advantage will be with the machine for which this condition is satisfied.

7.3.2.12 Need for uniformity of construction. It might appear from the preceding discussion that we can build a machine without taking pains to have poles of equal strength and leave it to the equalizer connections to equalize the pole strengths. It may be that we can do this without paying any penalty in greater stray magnetic field than from a machine carefully built to have poles of equal strength. As of now, however, we do not know that we can. We have no experimental tests to prove that inequality of pole strength does not increase stray magnetic field if the armature has equalizer connections. Until we have such proof, we must have our minesweeping generators built with extreme care to ensure equal pole strength, Even so, there will be unavoidable deviations from exact quality, and the equalizer connections must be relied upon to minimize their effects.

7.3.2.13 Summary of current loops in the armature winding.

(a) The equalizer connections will cause the current loops in the armature winding to be very nearly equal in strength regardless of whether the brush currents are equal or not.
(b) Such equal current loops, at least eight or more in a minesweeping generator, will give a small stray magnetic field.
(c) Most minesweeping generators have a compensating or pole-face winding. This is wound in slots in the pole faces, carries armature current, and produces a magnetic field that is opposed to that produced by current in the armature winding, The resultant field produced by both the armature winding and the pole-face winding should be very small.
(d) For these reasons, the armature winding and the currents in it should not be a major source of stray magnetic field. This is fortunate as it is not immediately apparent how they could be changed, The construction of the armature with the armature winding, commutator, and commutator risers is dictated by the necessity of having a generator chat will function properly.

7.3.2.14 The rest of the armature circuit. Let us now turn our attention to the lower part of figure 77. As pointed out in 7.3.2.7 this is incomplete in that it does not show the commutator risers and the connections from the brushes to the brush collector rings. In order to bring these into view, let us change the lower part of figure 77 into the developed isometric drawing shown on figure 80. Here we see the commutator risers and connections from brushes to brush collector rings that were invisible on figure 77. Corresponding points in the two figures are identified by the same number on both figures. The current in each conductor on figure 80 is shown beside the conductor. Remember that figure 80 is a developed and isometric view of a configuration that is actually bent around in a circle so that the right- and left-hand ends of the conductors sloping downward from the left meet and connect together For this reason, the current that approaches point 1 from the left is the same as the current that leaves point 4 and proceeds from left to right. The value is shown only at one end in order to avoid duplication.

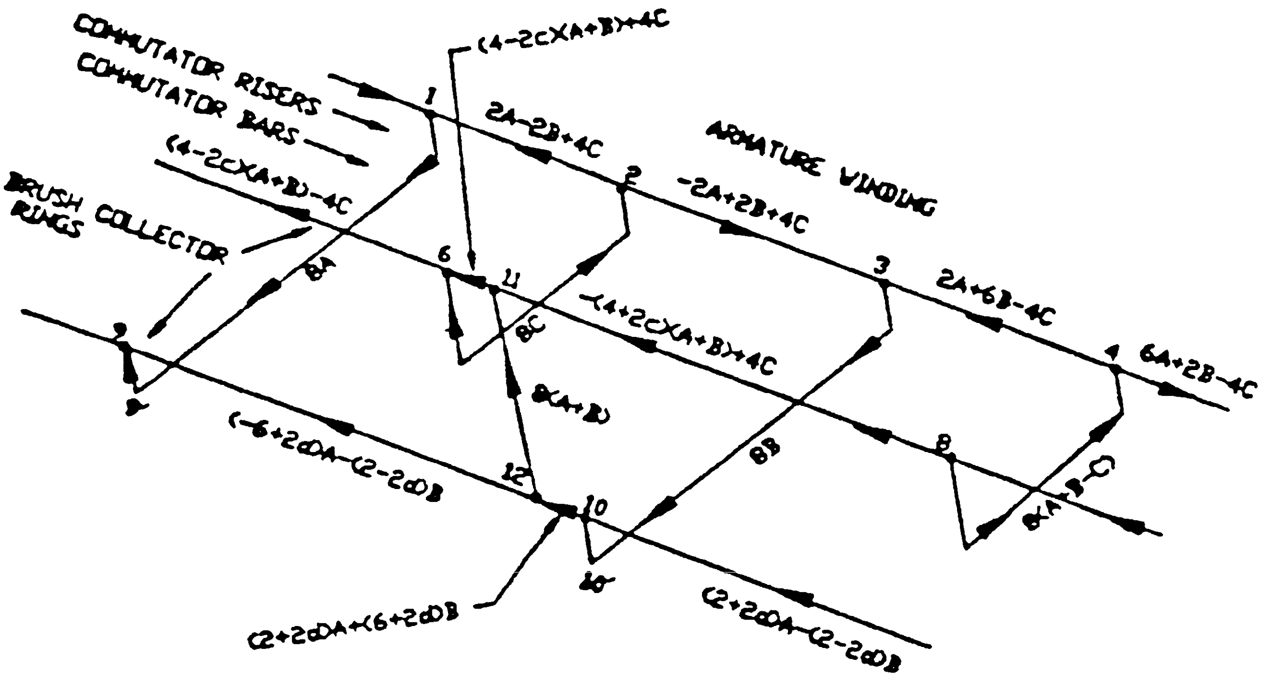


FIGURE 80. Developed isometric view of currents in the armature circuit.

7.3.2.15 Resolution into current loops. The next step is to break the circuit of figure 80 down into various groups of current loops that can be considered individually. The resolution of a circuit into two or more current loops can be represented in a drawing in at least two different ways. Start out with the circuit of figure 81 (A). Its resolution into two current loops can be represented in a drawing by showing the two loops completely separated as on figure 81 (B). This was the procedure used in 7.3.2,7 to permit splitting figure 77 into two distinct parts, which could then be separated and studied individually. Here, and in various other cases, it is an advantage to do it this way. On the other hand, the scheme of figure 81 (B) has the disadvantage that a drawing has to show two lines along the boundary that separates two current loops, or three lines in the case of the three loops with a common boundary, a case that we will soon encounter. Two, or still worse, three lines along the common boundary between loops make for complicated drawings. To avoid this we can use the scheme shown on figure 81 (C) with only a single line along the common boundary between loops, regardless of whether two, three, or more loops abut along this boundary. The loop currents are represented by numbers or symbols placed near the centers of the loops with a curved arrow over the expression for the current to indicate the direction of the loop current. Now use the scheme of figure 81 (C) to resolve the circuit shown on figure 80 into current loops. The result is shown on figure 82. We have three sets of loops in vertical planes between lines 1-4 and 1′-4′, lines 6-8 and 6′-8′, and lines 9-10 and 9′-10′; two sets of loops in a horizontal plane between lines 1′-4′ and 6′-8′ and lines 5′-8′ and 9′-1O′; and a single vertical loop 11-11′ -12′ -12. We must also have a loop represented by line 13-14 as we will see later. It remains to find the loop currents for the loops on figure 82. Start out with loop 1-1′ -2′-2. The only current from point 2 to 1 is the loop current, hence, the loop current must be equal to the current shown from point 2 to 1 on figure 80, namely, 2A - 2B + 4C. The loop currents for the other loops between lines 1-4 and 1′ -4′ are found in the same way. These loop currents must give the correct values for the currents in the commutator risers between points 1 and 1′, 2 and 2′, 3 and 3′, and 4 and 4′. A little consideration of figures 81 and 81 shows that they do. The next step is to find the loop current for loop 1-5′-6′-2′. Along line 1′ -2′ this loop abuts against loop 1-1′ -2′-2. Furthermore , there is zero current in line 1′ -2′ . To satisfy this condition, the loop current must be 2A - 2B + 4C for loop 1′-5′-6′ -2′. The loop currents for the remaining loops between lines 1′ -4′ and 5′-8′ are found similarly and are shown on figure 82.

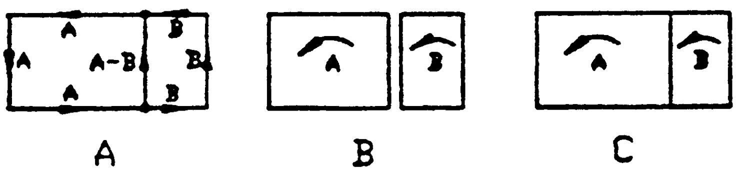


FIGURE 81. <u>Resolution of circuit into current loops</u>.

We now move down to line 5′-8′. We have three loops abutting along line 6′-11′ : two horizontal loops, 2′-6′-11′-7′ -3′ and 11′ -6′-5′-9′ -12′, and one vertical loop, 6-6′-11′ -11 must be (4-2c)(A+B)+4C to give the right current along line 6-11. We now know the current loops in two of the three loops abutting along line 6′-11′ . The current in the third loop must be so chosen as to make the current along line 6′-11′ equal to zero. This condition gives (6-2c)A+(2-2c)B for the loop current in loop 11-6′-5′-9′ -12. A similar procedure is used to find the loop currents in the other loops between lines 5′-8′ and 9′-10′ . The next step is to find the loop current in loop 9-9′ -12′ -12′. Two loops meet along line 9′-12′; namely, 9-9′-12′-12 and 5′-9′-12′-11′-6′ With loop current directions as shown on the figure, it is immediately apparent that the loop currents in these two abutting loops must be equal and opposite to make the current along 9′-12′ equal to zero. In a similar way, we find the loop currents in the other vertical loops between lines 9-10 and 9′-10′. There is one more loop needed to complete the loop system. This is represented on figure 82 by the line 13-14. A straight line does not look much like a loop; but it must be remembered that in the actual machine this line will be bent around in the form of a circle and have its ends connected. Line 13-14 thus represents a circular loop having the same diameter as the brush collector rings . Let x be the current in this loop. To find x we have the relation that x plus the current in loop 9-9′ -12'-12 must be equal to the current shown in line 12-9 of figure 80. This gives the equation:

$$x - (6 - 2c)A - (2 - 2c)B = (-6 + 2d)A - (2 - 2d)B \qquad [7\text{-}14]$$

where:

$$x = (2d - 2c)(A + B) \qquad [7\text{-}15]$$

A careful comparison of figures 81 and 82 shows that the current loops of figure 82 everywhere give exactly the same currents that are shown in the conductors on figure 80. They will, therefore, produce exactly the same magnetic field. The first step, finding the magnetic field produced by the currents shown on figure 80, has now been completed. We have replaced the currents by a system of current loops that will produce exactly the same magnetic field. The next step is to find the magnetic field produced by the current loops.

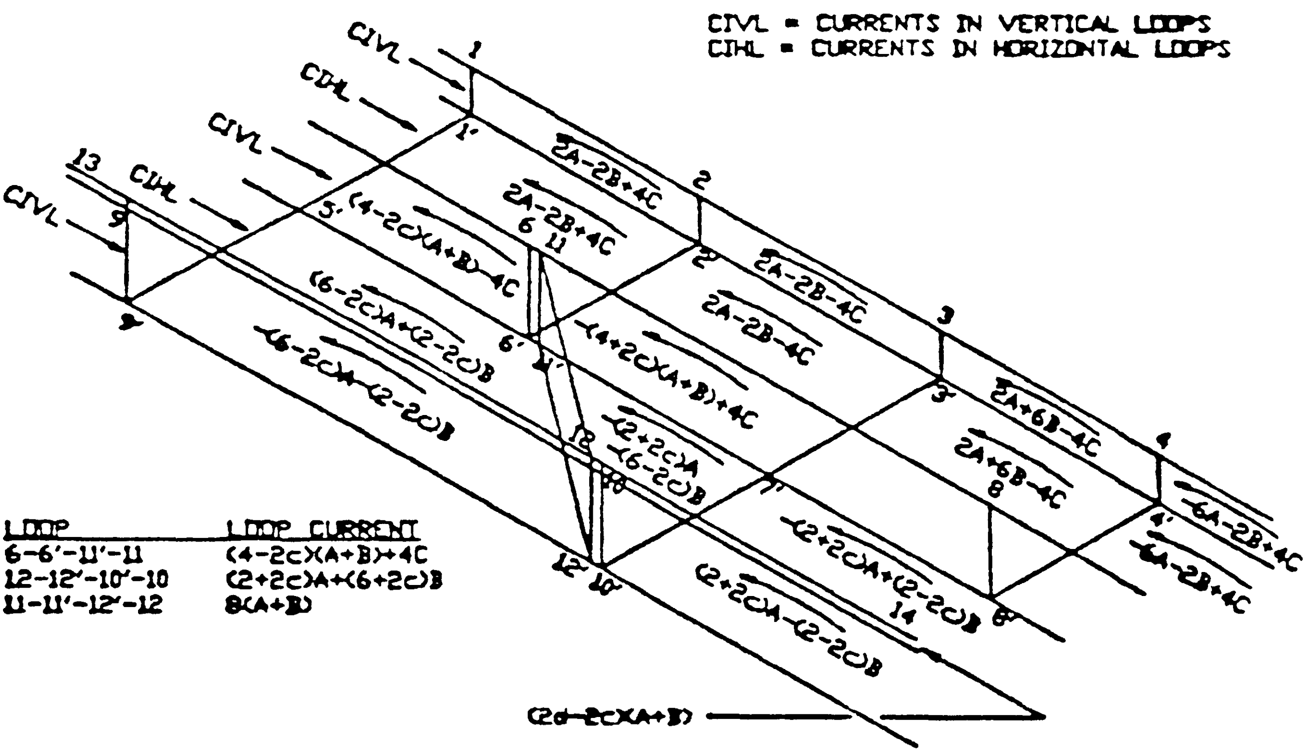


FIGURE 82. Developed isometric view of current loops,

7.3.2.16 Grouping the current loops. For further study it is convenient to divide the system of current loops shown on figure 82 into groups that are individually small enough to be assimilated. These groups are:

(a) Group 1, The four current loops between lines 1-4 and 1′-4′. These loops are in the plane of the commutator risers.
(b) Group 2. The three current loops between lines 6-8 and 6′-8′ . These loops are in the plane of one of the brush collector rings.
(c) Group 3 The three current loops between lines 9-10 and 9′-10′ . These loops are in the plane of the other brush collector rings.
(d) Group 4. The single current loop represented by line 13-14 of figure 82. This is a circular loop having a diameter equal to that of the brush collector rings.
(e) Group 5. The four current loops between lines 1′-4′ and 5′-8′. These loops are wrapped around the cylindrical surface of the commutator.
(f) Group 6. The three current loops between lines 5′-8′ and 9′-10′. Like those in group 5, these loops are also wrapped around the cylindrical surface of the commutator.
(g) Group 7. The single loop 11-11′ -12′ -12.

We now have the system of current loops broken down into a number of groups. It remains to study these groups individually; to identify those which produce only a small stray magnetic field and those which, on the contrary, may produce a large stray magnetic field; and to deduce from this study some ways to design generators with only small stray magnetic fields.

7.3.2.17 Group 1. The four loops in group 1 are in the plane of the commutator risers. If we imagine that figure 82 bent around to bring the loops into the configurations they have in the machine, the loops in group 1 will appear as shown on figure 83 when they are viewed from the commutator end of the machine The four loops are of equal areas; hence, their magnetic moments will be proportional to the loop currents . The algebraic sum of the loop currents is zero, so, the net magnetic moment perpendicular to the plane of the commutator risers is zero. The loops in group 1 thus satisfy principle (c) (see 5.2), the principle of zero net magnetic moment, for all the loops in the same plane. Note also that this condition is satisfied regardless of whether 8A, 8B, 8C, 8(A + B - C) are equal or not, that is, regardless of whether the four sets of brushes take equal currents or not. We thus have the principle of zero net magnetic moment satisfied at all times for the loops of group 1. It is not dependent upon equality of current division between brushes. It is not to be inferred, however, that the four loops of group 1 will give a zero stray magnetic field simply because the net magnetic moment is equal to zero. Suppose we assume for a moment that A - B - C. Reference to figure 83 shows that each loop will have a current 4A and that the polarities will be alternately positive and negative. Now the dipoles that represent the four current loops of figure 83 have the same configuration as the four dipoles shown in 4.3.3.11 for b = c. These produce a magnetic field that is different from zero and can be calculated by the formula given there, It is a smaller magnetic field than would be produced by one dipole alone, or by two opposing dipoles, but it is still greater than zero and can be decreased. It can be decreased by going to a larger number of poles. Consider two generators with the same output current and voltage; one with four poles and one with eight poles. The armatures will be approximately the same size. In the four-pole machine there will be four current loops in group 1. In the eight-pole machine there will be eight. Each will have half the area, half the current, and one-fourth the magnetic q oment of the group 1 loops in the four-pole machine. In addition, they will be closer together. Their stray magnetic field will be much less than that of the corresponding loops in the four-pole machine. We thus see that increasing the number of poles is not only advantageous in reducing the polar leakage field from the field coils, but is also advantageous in reducing the magnetic field from the group 1 loops in the armature circuit. It is to be noted that in going from four to eight poles we are making use of two general principles of design: principle no. 2, reducing the strength of the individual current loops (see 5.4), and principle no. 3, bringing the opposing current loops closer together (see 5.4). The final conclusion is that in a generator with an appropriate number of poles, the loops in group 1 will not be a significant source of stray magnetic field.

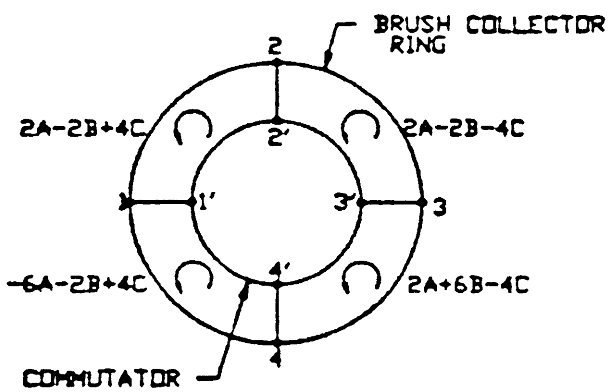


FIGURE 83. Loops in group 1.

7.3.2.18 Groups 2 and 3. Now consider the current loops in group 2, the three loops in the plane of the brush collector ring closest to the commutator. These loops will appear as shown on figure 84 when viewed from the commutator end of the machine. At points 6′ and 8′ the brushes rest on the commutator, at points 6 and 8, they are connected to the brush collector ring. Point 11, at which a connection is made to the other brush collector ring, is at an angle of "a" degrees from the uppermost brush connection, point 6, These loops have areas proportional to their angular spread, 180, a, and 180 - a degrees, or 2, c and 2 - c where c - a/90. Since the magnetic moments are equal to the products of currents and areas, we have the following table:

TABLE IX. Current, areas, and magnetic moments,

Current	Area	Magnetic moment
$(4 - 2c)(A + B) - 4C$	2	$(8 - 4c)(A + B) - BC$
$(4 - 2c)(A + B) + 4C$	c	$(4c - 2c^2)(A + B) + 4cC -$
$-(4 + 2c)(A + B) + 4C$	$2 - c$	$(-8 + 2c^2)(A + B) + (8 - 4c)C$
		Total 0

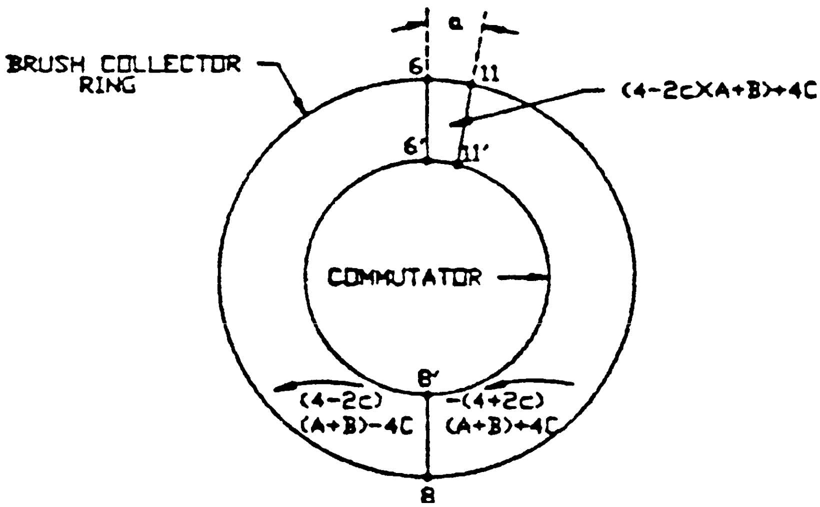


FIGURE 84. Loops in group 2.

we see, therefore, that the net magnetic moment for group 2 is zero. We see that this condition is satisfied regardless of whether A - B - C or not; that is, regardless of whether the brushes take equal currents or not. This is obviously a desirable situation since we have a balance that is not disturbed if the brushes should take unequal currents. We get this desirable feature by using brush collector rings that are complete rings of uniform cross section throughout the whole circumference, This is the reason why such brush collector rings are a necessity in a minesweeping generator built to have a small stray magnetic field Similar considerations applied to the three loops in group 3 show that for these loops the net magnetic moment is also zero and that this balance is independent of equality of current division between the four brushes. We thus see that the current loops in groups 2 and 3 satisfy the principle of zero net magnetic moment. Further discussion of these groups will be deferred to 7.3.2.29 where they will be taken up again in connection with the problem of finding the best position for the takeoff points, points 11 and 12 of figure 80. These are the points from which current leaves one brush collector ring, and the points at which it returns to the other ring.

7.3.2.19 Group 4. Group 4 is a single loop in the form of a circle coincident with the brush collector ring farthest from the armature. In the machine we have been considering the angular spread between the takeoff points is b - a degrees. Let b/90 = d, and a/90 = c. The current in the single loop in group 4 is (2d - 2c)(A + B). The preceding conclusion has been reached as a result of thorough study of the current loops involved. It can also be derived faster by a more intuitive approach With the takeoff points separated by an angular spread of b - a degrees, we have the full output current making a fraction of a turn equal to (b - a)/360. We can consider this the equivalent of one complete turn carrying a current equal to (b - a)/360 times the total current. The output current for the machines is 8(A + B), hence:

$$\text{Loop current} = 8\ (A + B)(b - a)/360 \qquad [7\text{-}16]$$

$$= (2d - 2c)(A + B) \qquad [7\text{-}17]$$

since we have b/90 - d and a/90 - c

This expression can be written in terms of the output current, I = 8(A + B) , as follows:

$$\text{Loop current} = (d - c)\ 1/4 \qquad [7\text{-}18]$$

In a minesweeping generator the brush collector rings are several meters in diameter and the armature current is large If (b - a)/360 is an appreciable fraction, the stray magnetic field from the single loop in group 4 will be large, and must be eliminated.

7.3.2.20 Elimination of group 4. There is one and only one practical way of eliminating group 4. The magnetic moment of the single loop in this group is as follows:

$$M = A\ (d - c)\ I/4 \qquad [7\text{-}19]$$

where:

A = area of the loop that has the same diameter as the brush collector rings.

d - c = angular spread between the takeoff points expressed as a fraction of 90 degrees.

I = output current of the machine.

The only ways in which we can make M = 0 are to make A = 0, or d - c = 0. We cannot make the area enclosed by the brush collector rings equal to zero. We can make I = 0 but we then have a generator with zero output current. The remaining alternative, d - c = 0, is the only one that we can use. It means that there is zero angular spread between the takeoff points on the two brush rings, or that the two takeoff points are in line with the axis of the machine. Therefore, this condition must be satisfied in any dc generator or motor designed for minimum stray magnetic field.

Since we must have the takeoff points in line with the axis of the machine, it would be futile to spend any more time in consideration of figure 82 that shows the current loops for a machine in which this condition is not satisfied. We must change figure 82 to correspond to a machine in which the zero angular spread condition is satisfied. All that is necessary to do this is to move the takeoff points into alignment with the axis of the machine. The result of the change is shown on figure 85. Comparison of figures 82 and 85 shows that we have the same groups of current loops in each, except for the elimination of group 4 from figure 85. Furthermore, groups 1 and 2 are exactly the same in both figures. Therefore. it is unnecessary to retrace our steps to reconsider these groups. A little consideration will show that the same conclusion holds for group 3. Group 4 has been eliminated, so only 5, 6, and 7 remain to be considered.

7.3.2.21 Group 5. Group 5 consists of the four current loops between lines 1′-4′ and 5′-8′ of figure 85. These loops are all of the same size. The algebraic sum of the currents is zero; hence, for four loops of this kind in the same plane, the net magnetic moment is zero. Furthermore, this condition is always satisfied regardless of whether the brushes carry equal currents or not, Note, however, that while the four loops in group 5 are in a plane on figure 85, they are not in a plane in the machine, Here they are wrapped around the cylindrical surface of the commutator

Each loop will have a shape similar to that of the four pieces that would be obtained by cutting off both ends of a cylindrical tin can and slitting the open-ended cylinder, which remains in four pieces, by four equally spaced cuts parallel to the axis of the cylinder. Looked at from the commutator end of the machine the four loops will appear as shown on figure 86 where the loop currents are shown alongside the loops. Now suppose that each current loop is divided into a large number of very small and narrow current loops of the shape that would be obtained by drawing on the cylindrical surface of a tin can two very closely spaced lines parallel to the axis of the cylinder. Consider the particular infinitesimal loop of this kind that is shown in end view by the line segment hi on figure 86. Its width in the plane of the paper is hi = $r d\theta$; where r is the radius of the commutator and $d\theta$ is the infinitesimal angle that hi subtends at the center of the commutator. Its length perpendicular to the plane of the paper is W, where W is the distance between lines 1′-4′ and 5′-8′ of figure 85. The current is 2A - 2B - 4C, since the infinitesimal loop is in the sector where this is the loop current. Hence, the magnetic moment of the infinitesimal loop is:

$$dM = rW\,(2A - 2B - 4C)\,d\theta \qquad [7\text{-}20]$$

A comparison of figure 85 and 86 shows that the magnetic moment of the infinitesimal current loop will be directed outward for a positive loop current The horizontal component of magnetic moment is:

$$dM_x = rW\,(2A - 2B - 4C)\cos\theta\,d\theta \qquad [7\text{-}21]$$

We find the total horizontal component of magnetic moment for the sector from point 3′ to point 2′ by integrating with respect to θ from 0 to 90 degrees, We find the horizontal component for other sectors in the same way, then add them to get the total for all four. We find the vertical component in much the same way, except that - $\sin\theta$ is used instead of $\cos\theta$. (Note: - $\sin\theta$ is used instead of $\sin\theta$ because the positive direction is taken to be down.) Doing this we find for the total horizontal and vertical components of magnetic moment from all four loops in group 5:

$$M_x = 8r\ W(A + B - 2C) \qquad [7\text{-}22]$$

$$M_y = -8r\ W(A - B) \qquad [7\text{-}23]$$

For both components of net magnetic moment to be zero, we must have A - B - C. This means that all four sets of brushes carry the same current. It would be desirable to have zero net magnetic moment regardless of whether or not the brushes carry equal current, but we do not have this situation for the current loops in group 5. We see, however, in 7.3.2.28 that there are other current loops that can be arranged to produce a compensating field. These other loops and group 5 individually do not have zero net magnetic moment unless the total armature current is equally divided between the brush sets, but taken together the net magnetic moment is zero regardless of whether the brush currents are equal or unequal.

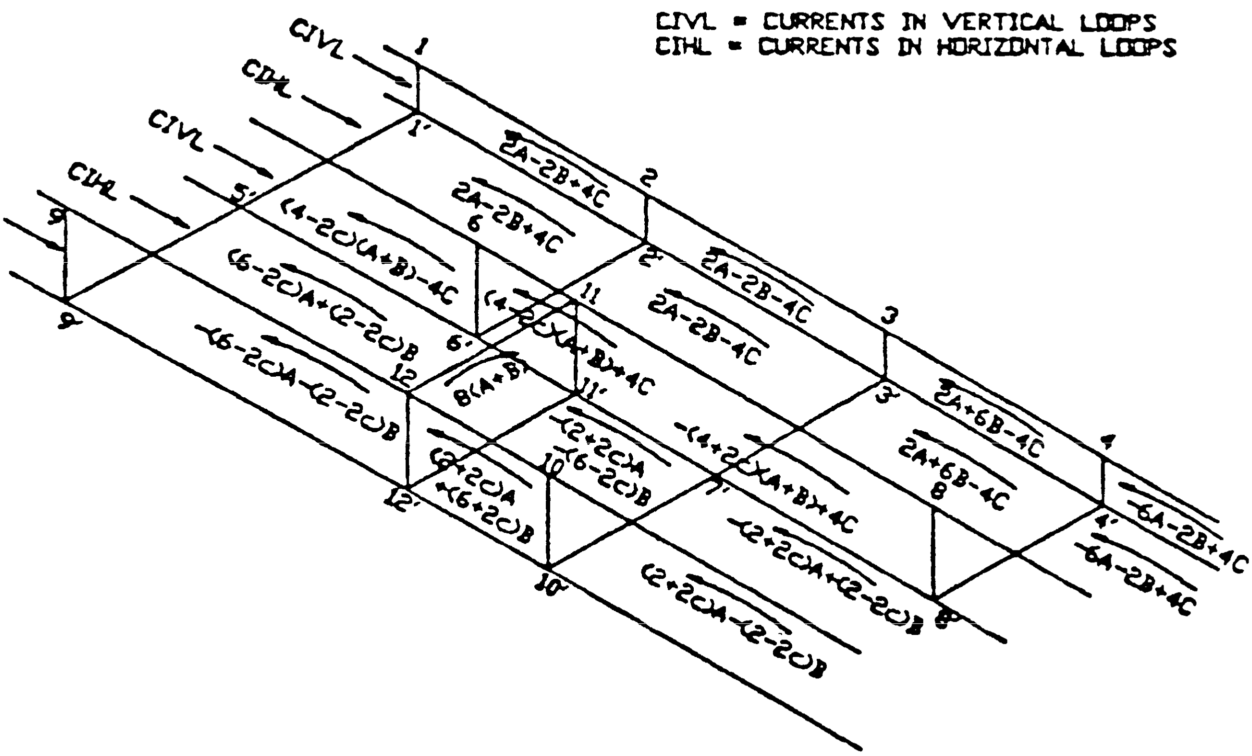


FIGURE 85. Developed isometric view of current loops for zero angular spread between takeoff points.

7.3.2.22 Group 6. Group 6 consists of the three loops between lines 5′ -8′ and 9′ -10′ of figure 85. Looked at from the commutator end of the machine, they will appear as shown on figure 87. We can find the net horizontal and vertical components of magnetic moment using the same method that was used for group 5. The results for group 6 are:

$$M_x = -8r\,S\,(A + B)\cos a \qquad [7\text{-}24]$$
$$M_y = -8r\,S\,(A + B)\sin a - 8r\,S\,(A - B) \qquad [7\text{-}25]$$

where S is the distance between lines 5′ -8′ and 9′-10′ of figure 85. Here we have a group of loops that does not give zero net magnetic moment even if A - B. When A = B, the total net magnetic moment is:

$$M_t = (M^2x + M^2y)^{1/2} = 8r\,S\,(A + B) \qquad [7\text{-}26]$$

and is perpendicular to the radius vector drawn from the center of the circle to point 12′.

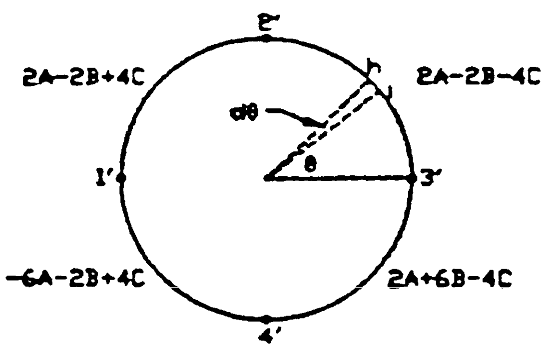


FIGURE 86. Loops in group 5.

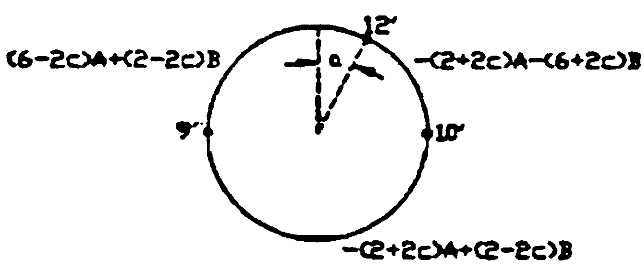


FIGURE 87. Loops in group 6.

We have now encountered three different types of groups of current loops:

(a) Groups 1, 2, and 3. For each of these three groups the magnetic moment is zero regardless of whether the current is equally divied between the brush sets or not; that is, regardless of the values of the four brushes currents, 8A, 8B, 8c, and 8 (A + B -C).

(b) Group 5. For this group the net magnetic moment is not equal to zero for all values of the four brush currents, but is equal to zero for the norm of machine behavior; namely, four equal brush currents, 8A- 8B - 8C - 8(A + B - C).

(c) Groups 4 and 6. For these two groups the net magnetic moment is not equal to zero when the brush currents are unequal, and is also not equal to zero when the four brush currents are equal. Group 4 was eliminated by putting the takeoff points in line with the axis of the machine but this change did not remedy group 6. Something additional must be done about this group. Before going into the question of what can be done, however, it will be advantageous to consider group 7.

7.3,2.23 Group 7. This consists of a single loop, 11′-12′ -12-11, of figure 85. The current is 8(A + B), that is, the output current of the generator. Its dimension in the direction of the axis of the machine is S, the separation between the two collector rings. Its dimension in the radial direction is R - r, where R is the radius of the brush collector rings and r is the radius of the commutator. Its magnetic moment is 8(R - r)S(A + B), and will not be equal to zero except for zero output current. A little consideration will show that the magnetic moment of the loop in group 7 is precisely the same direction as the net magnetic moment for group 6 when A - B. Thus, for normal machine behavior, we have:

Magnetic moment of group 6: 8rS(A + B)

Magnetic moment of group 7: 8(R - r)S(A + B)

Total for groups 6 and 8: 8RS(A + B)

Thus for the two groups together we have a magnetic moment equal to the full output current of the machine times the radius of the brush collector rings times the distance between them. For a minesweeping generator the current is large and the brush collector rings are of fairly considerable diameter The spacing between the brush collector rings is small, on the order of centimeters, but even so we are left with a magnetic moment large enough to produce an objectionable stray magnetic field. Therefore, groups 6 and 7 must be compensated.

7.3.2.24 Compensation of groups 6 and 7. We have seen before that with an arrangement like the one shown on figure 88, we always have an uncompensated current loop, but that with an arrangement like the one shown on figure 100, we have two equal and opposite current loops. A little consideration will show that in a generator with two brush collector rings we will have the situation shown on figure 88, and that this situation is responsible for the uncompensated magnetic moment from groups 6 and 7.

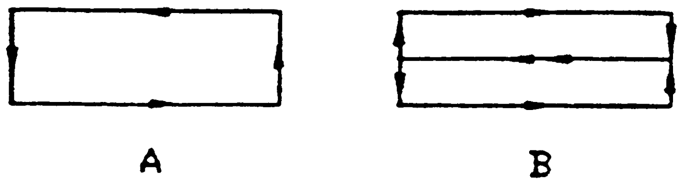


FIGURE 88. Uncompensated and compensated current loops.

One remedy is to provide three brush collector rings, the center rings to carry the same current as the positive brush collector ring in a two-ring machine, and the two outer loops each to carry one-half the current in the negative brush collector ring of the two-ring machine. Figure 89 shows a developed flat view of the three-ring machine with currents indicated in the armature winding, commutator bars, brush collector rings, and the connections from the takeoff point 12 on the center ring to the two takeoff points 11 and 13 on the outer rings. In calculating these currents it is assumed that the two outer rings each take the same current. We can carry through the analysis without making this assumption, but it would complicate the expressions for the currents and would lead to no other conclusion than what we already know; namely, that for best compensation the currents in the outer brush rings must be equal. Now go from the developed flat view of figure 89 to the developed isometric view of figure 90, which shows the loop currents. A comparison of these two figures will show that the loop currents will give in each conductor precisely the current that is shown on figure 89. Now compare the two developed isometric views, figure 81 for the two-ring machine and figure 90 for the three-ring machine. This comparison will show that:

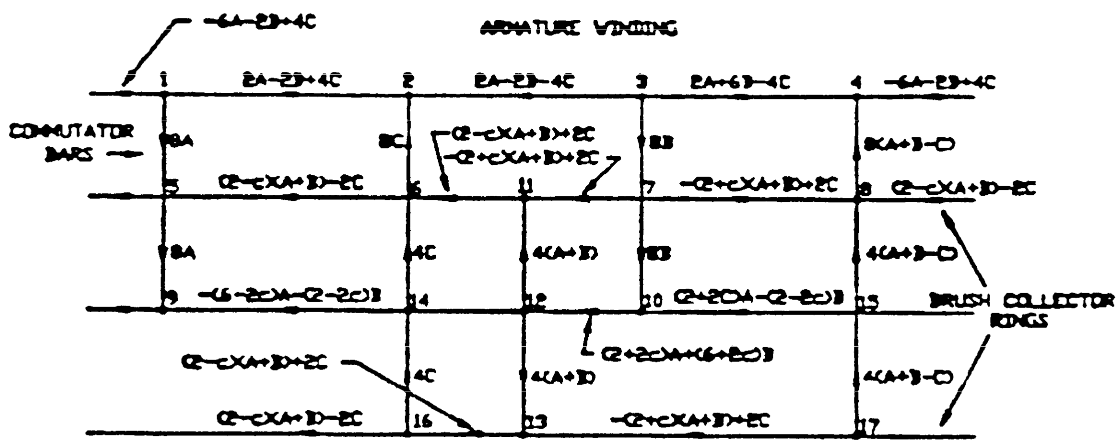


FIGURE 89. Developed view of currents in a machine with three brush collector rings.

(a) The current loops in groups 1, 2, and 5 are precisely the same in both .
(b) The current loops in group 3 for the two-ring machine have been split into two exactly equal halves in the three-ring machine. These two halves together will be very nearly equal to the single group 3 of the two-ring machines,
(c) Group 4 has been eliminated in a three-brush ring machine by having the three takeoff points, 11, 12, and 13, in line with the axis of the machine.

It follows, therefore, that what has been said about groups 1, 2, 3, 4, and 5 in the two-ring machine is equally valid for the corresponding groups in the three-ring machine. It is, therefore, unnecessary to consider these groups further.

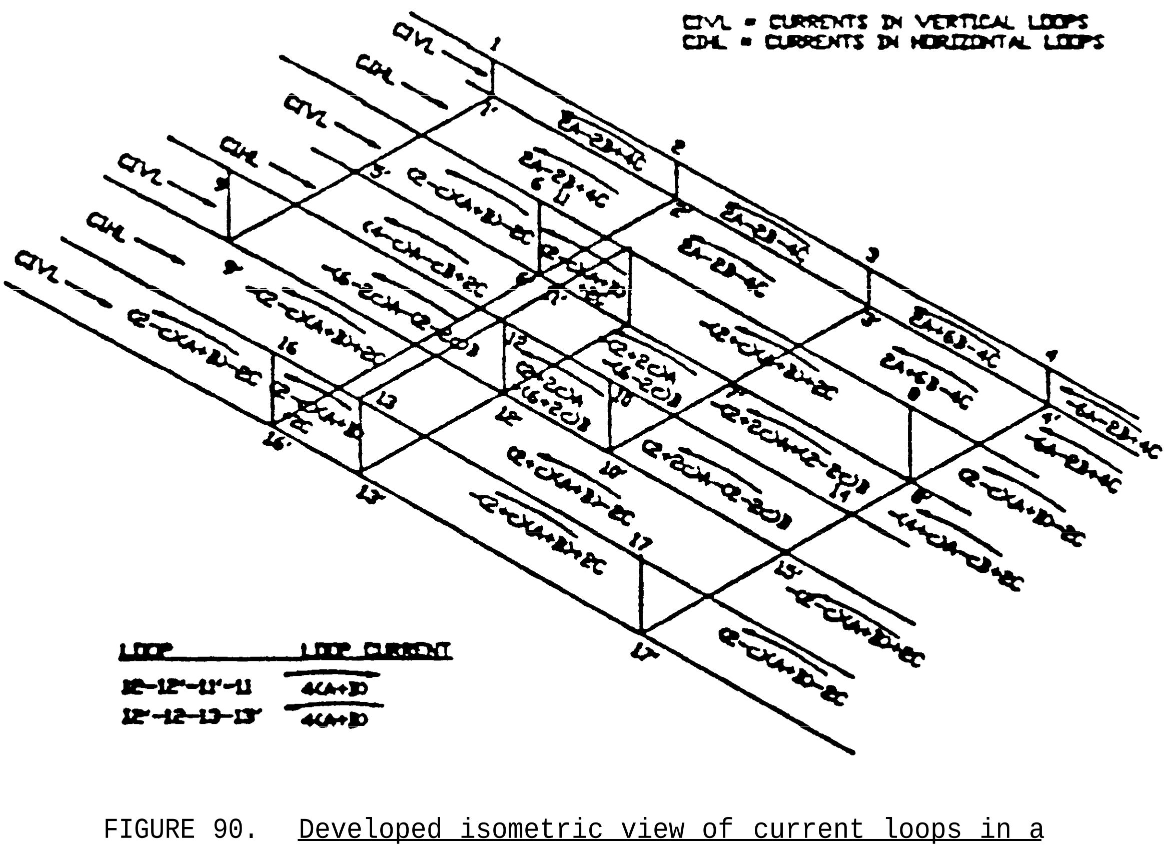


FIGURE 90. Developed isometric view of current loops in a machine with three brush collector rings.

7.3.2.25 Group 6 in the three ring machine. In the two-ring machine, group 6 consisted of three current loops between lines 5′ -8′ and 9′-10′ of figure 82. In the three-ring machine, group 6 is split into two subgroups:

Subgroup 6A: Five current loops between lines 5′-8′ and 9′-10′ of figure 90.

Subgroup 6B: Three current loops between lines 9′-10' and 16′-17′ of figure 90.

(a) Subgroup 6A. These loops are curved around the cylindrical surface of the commutator. Looked at end on, they will appear as shown on figure 91, We can compute the net horizontal and vertical components of magnetic moment as in 7.3.2.22:

$$M_x/rS = 4(A + B - 2C) - 4(A + B)\cos a \qquad [7\text{-}27]$$

$$_{z}/rS = 8(A - B) - 4(A + B)\sin a \qquad [7\text{-}28]$$

where:
r = the radius of the commutator
s = the axial separation between the center ring and the ring on each side of it.

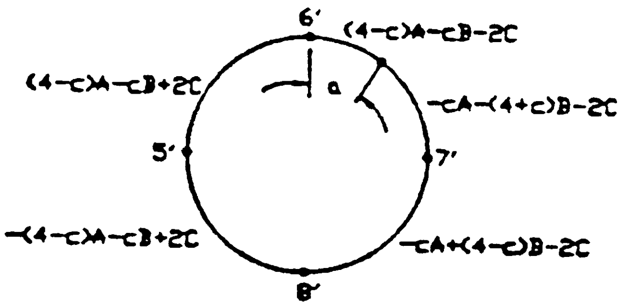


FIGURE 91. Loops in subgroup 6A.

(b) Subgroup 6B. Looked at end on, the three loops in this subgroup will appear as on figure 92. The net horizontal and vertical components of magnetic moment are:

$$M_x/rS = 4(A+B - 2C) + 4(A + B)\cos a \qquad [7\text{-}29]$$

$$M_{y/rS} = 4(A + B)\sin a \qquad [7\text{-}30]$$

(c) Total for group 6. Adding the results attained above for subgroups 6A and 6B, we have the following for group 6.

$$M_x/rS = 8(A+B-2C) \qquad [7\text{-}31]$$

$$L/rS = 8(A - B) \qquad [7\text{-}32]$$

Note that the horizontal and vertical components of the net magnetic moment of group 6 will both be equal to zero if the brush currents are equally divided, that is, A - B = C. Note also the difference between two- and three-brush collector rings for group 6.

For normal machine behavior (brush currents equal) the group 6 loops have zero net magnetic moment for three brush rings, but a net magnetic moment equal to 8rS(A + B) for two brush rings (see 7.3.2.23). This is the reason why three brush collector rings should be used. It is true that even with three brush collection rings the group 6 loops do not give a zero net magnetic moment when the brush currents are not equal. In this respect they are like group 5 loops (see 7.3.2.21). We shall see later (see 7.3.2.28) how we can compensate for this and arrive at a condition of zero net magnetic moment regardless of whether the brush currents are equal or not.

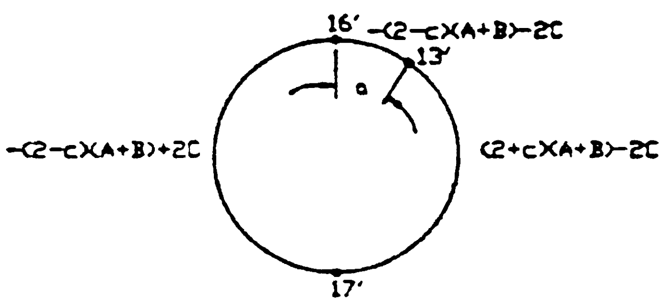


FIGURE 92 Loops in subgroup 6B.

7.3.2 26 Two concentric brush collector rings. The usual construction for dc generators has two brush collector rings of the same diameter separated by a small distance in the axial direction of the machine. We have seen in 7.3.2.23 that this construction leads to current loops wrapped around the commutator (group 6), which have a net magnetic moment equal to rSI square meter ampere turns, where r is the radius of the commutator, S is the axial distance between the brush collector rings, and I = 8(A + B) is the total armature current, We have also seen in 7.3.2.25 that the use of three brush collector rings of the same diameter equally spaced in the axial direction of the machine splits group 6 into two subgroups and that the total net magnetic moment for the two subgroups is equal to zero for equal division of brush currents. We can do the same job by using two concentric brush collector rings, one slightly larger than the other, in the same plane perpendicular to the axis of the machine. The use of three brush collector rings splits the group 6 loops into two subgroups that are in opposition. The use of two concentric brush collector rings in the same plane eliminates the group 6 loops completely by shrinking their width to zero. Figure 93 is a developed isometric view that gives the currents in the conductors of a machine with two concentric brush collector rings. The drawing shows the takeoff points, 11 and 12, on the same radius in view of the natural expectation that we will have a fractional turn around the shaft and a large stray magnetic field if they are at any other location. The current loops shown on figure 94 will give the current distribution shown on figure 93. Inspection of this figure shows that the current loops between lines 1-4 and 1′-4′ are precisely the same as the group 1 loops considered in 7.2.2.17, and that the current loops between lines 1′-4′ and 9′-8′ are the same as group 5 loops considered in 7.2.2 21. We are left with group 8, loops between lines 6-8 and 9-10 of figure 94, and group 9, loops between lines 9-10 and 9′-10′ of figure 94. The three loops in group 8 all have the same width in the vertical direction (radial direction in the machine) and lengths proportional to the angles they span. as illustrated by the following table:

TABLE X. Current and magnetic moment for group B.

Current	Length proportional to	Magnetic moment proportional to
$(4 - 2c)(A + B) - 4C$	2	$(8 - 4c)(A + B) - 8C$
$(4 - 2c)(A + B) + 4C$	c	$(4c - 2c^2)(A + B) + 4cC$
$-(4 + 2c)(A + B) + 4C$	$2 - c$	$(-8 + 2c^2)(A + B) + (8 - 4c)C$
		Total 0

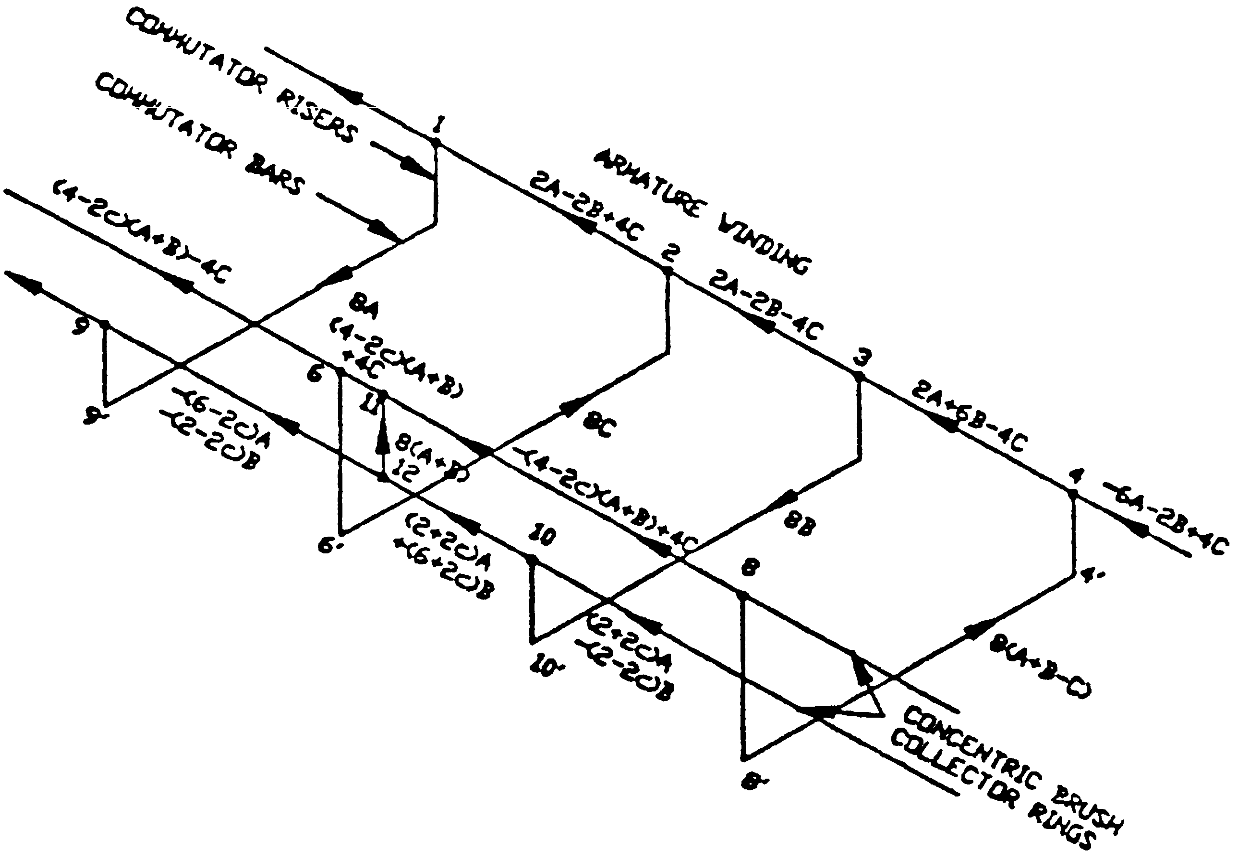


FIGURE 93. Developed isometric view showing currents for a machine with two concentric brush collector rings.

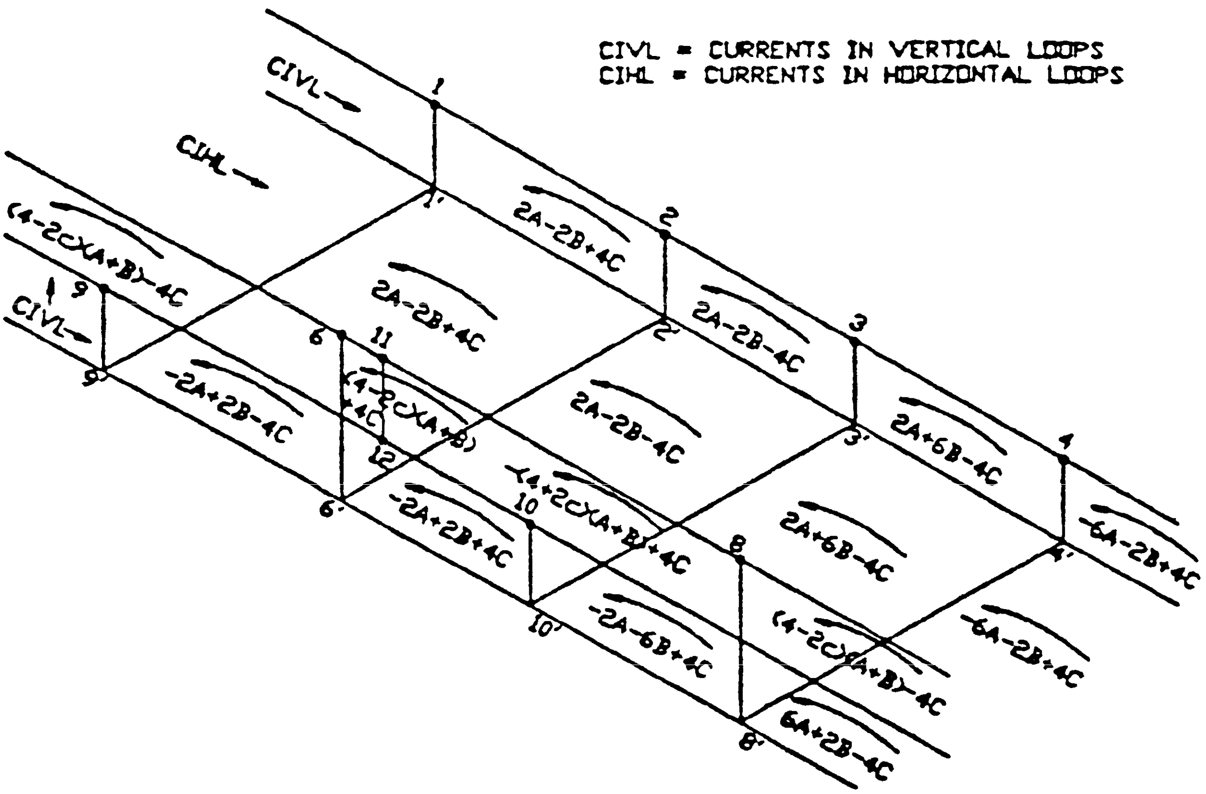


FIGURE 94. Developed isometric view showing current loops for a machine with two concentric brush collector rings.

The group 8 loops thus have zero net magnetic moment. This condition is satisfied regardless of whether the brush currents are equally divided or not. A similar table for the group 9 loops shows that they satisfy the same condition,

7.3.2.27 Comparison of the three ring and concentric two ring constructions . We can summarize the results for the three brush collector ring and the concentric two brush collector ring constructions as follows:

(a) For both constructions we have groups of loops in planes perpendicular to the axis of the machine. The net magnetic moment of each of these groups of loops is zero regardless of whether the brush currents are equal or not. These groups are:

(1) For the three-brush collector ring construction, the loops in the plane of the commutator risers, and the loops in the planes of the three brush collector rings.
(2) For the concentric two-brush collector ring construction, the loops in the plane of the commutator risers, and the loops in the common plane of the two concentric brush collector rings

(b) For both constructions, we have groups of loops wrapped around the commutator. The net magnetic moment of each of these groups is zero when the brush currents are equal, and different from zero when this condition is not satisfied. These groups are as follows:

(1) For the three-brush collector ring construction.

TABLE XI. Three-brush collector ring construction.

Group	See paragraph	Components of net magnetic moment	
		Horizontal	Vertical
5	7.3.2.21	8rW(A + B - 2C)	-8rW(A - B)
6	7.3.2.25	8rS(A + B - 2C)	-8rS(A - B)
Total		8r(W + S) (A + B - 2C)	-8r(W + S) (A - B)

(2) For the concentric two-brush collector ring construction:

TABLE XII. Two-brush collector ring construction.

Group	See paragraph	Components of net magnetic moment	
		Horizontal	Vertical
8	7.3.2.21	8rW(A + B - 2C)	-8rW(A - B)

In both constructions, we have a net magnetic moment that is not equal to zero unless the brush currents are equal. We shall see in 7.3.2.28 how, by proper design of the brush rigging and proper positioning of the brush collector rings, we can arrive at a net magnetic moment equal to zero regardless of whether the brush currents are equal or not. There appears to be no clear cut superiority of either the three-ring or two-ring construction insofar as small stray magnetic field is concerned.

7,3.2.28 Best position for brush rings. Two of the simplifying assumptions made in 7.2.1 were that each set of brushes makes contact with a commutator bar at one point, and that the connection to the brush collector ring is made along a line that is directed radially outward through this single point of contact. It is obviously impossible to have a single point of contact. Each set of brushes touches a commutator bar over a considerable part of its length. A commutator bar is so narrow that the assumption of line contact will be a good approximation of physical reality. We will also assume that current passes radially outward through a brush then along a conductor parallel to the commutator bars until it comes to the plane of a brush collector ring, and then passes radially outward to the brush collector ring. Current can be forced to flow in such a path by proper design of the brush rigging. Now look at a longitudinal horizontal section through half of the armature and brush collector rings. The heavy line on figure 95 shows the current path from the armature winding to the positive brush collector ring for our initial assumption of point contact between a brush set and a commutator bar. For our new assumption of line contact, the current paths will be as shown on figure 96. We obtain this current distribution from that shown on figure 95 by adding a number of small current loops of which loop 1-2-3-4 on figure 96 is a representative.

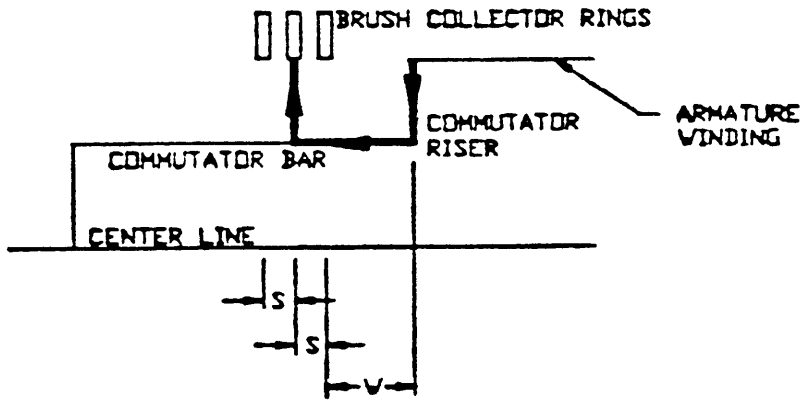


FIGURE 95. Longitudinal section through half of machine for point contact between brushes and commutator bars.

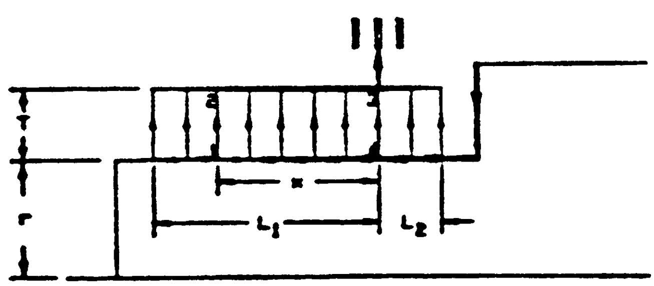


FIGURE 96. Longitudinal section through half of machine for line contact between brushes and commutator bars.

These loops will be in a plane that passes through the axis of the machine. We shall assume that the current taken by the brush set for an infinitesimal element of length dx is $idx/(L_1+L_2)$, where i is the total current taken by the brush set and L_1 and L_2 are the distances shown, Look at the current that is taken from an element of length at a distance x to the left of the central brush collector ring. It forms a loop 1-2-3-4 having a length x and width T that will have a magnetic moment directed down into the plane of the paper. its magnetic moment is as follows:

$$dM - Tixdx/(L_1 + L_2) \qquad [7\text{-}33]$$

Integrating this expression to find the total or net magnetic moment for all the current loops gives:

$$0.5\ Ti(L_1^2 - L_2^2)/(L_1 + L_2) - 0.5\ Ti(L_1 - L_2) \qquad [7\text{-}34]$$

By making $L_1 - L_2$, that is, by putting the central brush collector ring at half the length of the brush set, the magnetic moment is q ade zero. At first sight, this seems to be the thing to do. Further consideration shows that we can do better. We can choose T, L_1, and L_2 in such a way that the loops in the brush rigging do not have zero magnetic moments but oppose the net magnetic moment that we have from the loops on the commutator when the four brush sets do not take equal currents. Now look at the commutator end of the machine. Figure 97 shows four brush sets numbered 1 to 4 at positions corresponding to points 1, 2, 3, and 4 of figure 90, and the direction and magnitude of the currents in the four commutator bars that are directly under the brush sets. Further consideration will show that the magnetic moments of the loops such as 1-2-3-4 of figure 96 will have the directions shown on figure 97 for L_1 greater than L_2, and will be reversed in sign if L_2 is less than L_1. The magnetic moment will be zero for L_1 - $_2$, and will be reversed in sign if L_1 is less t-ban L_2. The magnetic moment at each set of brushes is obtained from equation 7-34 by substituting in the appropriate value for the currents, as shown in the following table:

TABLE XIII. Magnetic moments at brush sets.

Brush set	Magnetic moment for loops at brush sets	
	Horizontal component (positive to tight)	Vertical component (positive down)
1	0	$4T(L_1 - L_2)A$
2	$4T(L_1 - L_2)C$	0
3	0	$-4T(L_1 - L_2)B$
4	$-4T(L_1 - L_2)(A + B - 2C)$	0
Total	$-4T(L_1 - L_2)(A + B - 2C)$	$4T(L_1 - L_2)(A - B)$

Referring back to 7.3.2.27, we see that for the three-brush collector ring construction we have horizontal and vertical components of net magnetic moment for the loops around the commutator as follows.

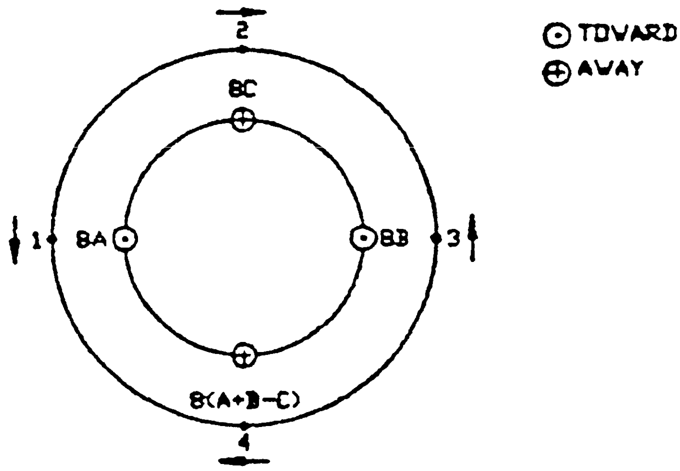


FIGURE 97. End view of machine .

Horizontal (positive to right)

$$8r(W + S)(A + B - 2C) \qquad [7\text{-}35]$$

Vertical (positive down)

$$-8r(W + S)(A - B) \qquad [7\text{-}36]$$

Add these to the results for the loops at the brush sets and we get:

Horizontal (positive to right)

$$[8r(W + S) - 4T(L_1 - L_2](A + B - 2C) \qquad [7\text{-}37]$$

Vertical (positive down)

$$[4T(L_1 - L_2) - 8r(W + S)](A - B) \qquad [7\text{-}38]$$

We can make both components equal to zero by designing the machine such that:

$$T(L_1 - L_2) = 2r(W + S) \qquad [7\text{-}39]$$

This condition should be the objective of the designer. When this condition is satisfied, the net magnetic moment for the loops on the commutator plus the loops at the four sets of brushes is equal to zero regardless of whether the currents taken by the brush sets are equal or not. For a machine with two concentric brush collector rings, similar consideration shows that the condition to satisfy is:

$$T(L_1 - L_2) = 2rW \qquad [7\text{-}40]$$

In equations 7-39 and 7-40, T is the distance from the center of the current in the commutator bar to the center of the current in the conductor that collects current from the brushes and conducts it axially to the junction with the conductor that leads the current radially outward to the brush collector ring. W is the axial distance between the midpoint of the commutator risers and the midpoint of the brush collector ring closest to the commutator risers. S is the axial distance between the midpoints of two adjacent brush collector rings. While the equations given above are based upon the assumption that the loops in the brush rigging are rectangular, it is not essential that they should be Loop areas have to be right but the shape is immaterial. Of course, different formulas will have to be used if the loops are not rectangular.

7.3.2.29 Best position for takeoff points. We saw in 7.3.2.20 that the takeoff points, the points from which current is taken from the positive brush ring (or rings) and returned to the negative brush ring (or rings), must have zero angular spread between them to avoid a fraction of a turn around the shaft. Apart from this, we have run into nothing to indicate that any particular angular position is better than any other. There are, however, two considerations, symmetry and spatial location, which indicate a preferred position. Consider the three brush collector ring construction. Suppose that the three rings are so close together that they can be considered as being in the same plane; that the positive takeoff point is at the same point where one set of brushes is connected to the center brush collector ring (assumed positive); and that the brush sets carry equal currents. For this condition the loops in the plane of the center brush collector ring will be as shown on figure 98 (A). The loops in the planes of the outer rings, added together, will be as shown on figure 98 (B); and the sum of the loops in the planes of all three rings will be as shown on figure 98 (C). On figure 98 (C) we have a perfectly symmetrical arrangement of current loops. It is true that this perfectly symmetrical condition holds only when the four brush sets take equal currents. Nonetheless , equal brush currents must be considered normal machine behavior and, for this reason, we conclude that any position of the takeoff points that gives the perfect symmetry of figure 98 (C) is a preferred position. Note that this position is with the takeoff point on one ring at exactly the same place where one set of brushes is connected to the ring. Since there are a number of points where brush sets are connected to a brush collector ring, there are just as many positions for the takeoff points that should be equally good so far as symmetry is concerned. There is, however, another reason for choosing the highest of these positions. Starting from the takeoff points, we have connections to the machine and the terminals. These connections, even when carefully designed, will be a source of stray magnetic field and it is desirable to have them as high as possible. We conclude then that one takeoff point should be at the same point there a brush set is connected to a brush collector ring (or rings). This will put the other takeoff point midway between the points of connection of two adjacent brush sets to the other brush collector ring (or rings). Furthermore , the takeoff points should be as high up on the brush collector rings as possible.

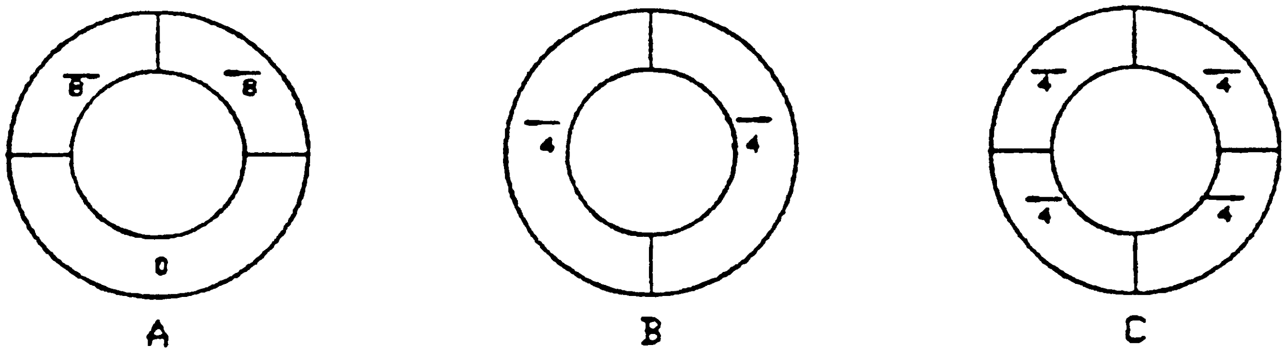


FIGURE 98. End view of loops in the plane of the brush collector rings.

7.3.2.30 Discussion of simplifying assumptions. In 7.3.2.1 we made a number of simplifying assumptions about the machine considered. It is now time to consider what effect these simplifying assumptions have upon the conclusions reached.

(a) Equalizer connections. We started out by considering a lap wound armature without equalizer connections. Any actual armature for a minesweeping generator will have such connections. Their effect has been considered in 7.2.3.4 and 7.3.2.9. The conclusion reached is that equalizer connections on the armature will help *to* reduce stray magnetic fields.

(h) Number of commutator bars. We assumed that the number of commutator bars was equal to an integral multiple of the number of poles . This condition can be satisfied with a lap winding. It is believed that machines in which it is satisfied, other conditions being the same, will have a smaller stray magnetic field than machines in which it is not. The differences, however, will probably not be large. Test results would be needed to obtain conclusive information on this point.

(c) Contact with one commutator bar. We have carried through the treatment on the assumption that at each instant a brush set touches only one commutator bar. Such an assumption is needed to make the treatment neat and simple. It does not appear that any modifications of our conclusions are necessitated by the fact that in an actual machine each brush set will be in contact with several commutator bars.

(d) Point contact. We started out with the assumption that each brush set made contact with a commutator bar at only a single point. This assumption was later discarded and replaced with the more realistic assumption of the line contact (see 7.3.2.28). Hence , the initial assumption of point contact does not impose any limitations on the validity of our results.

(e) Radial connection. We assumed initially that the connections from commutator bar to a brush collector ring are made by a line conductor that extends radially outward from the point of contact with the commutator bar. We then discarded this assumption (see 7.3.2 28) in favor of a more realistic assumption that tile connection is made in a radial plane passing through the axis of the machine and the commutator bar . The initial assumption thus does not invalidate our conclusions.

7.3.2.31 Conclusions on armature current from winding to brush collector rings. In a properly designed machine, the armature current from the armature winding to the brush collector rings should have only a small stray magnetic field, regardless of whether the current is equally divided between the different sets of brushes or not.

7.3.3 Connections from brush collector rings.

7.3.3.1 The problem. From the brush collector rings the armature current must go to the following:

(a) The commutating pole and compensating windings or to the commutating pole winding alone if the machine has no compensating windings.
(b) The machine terminals.

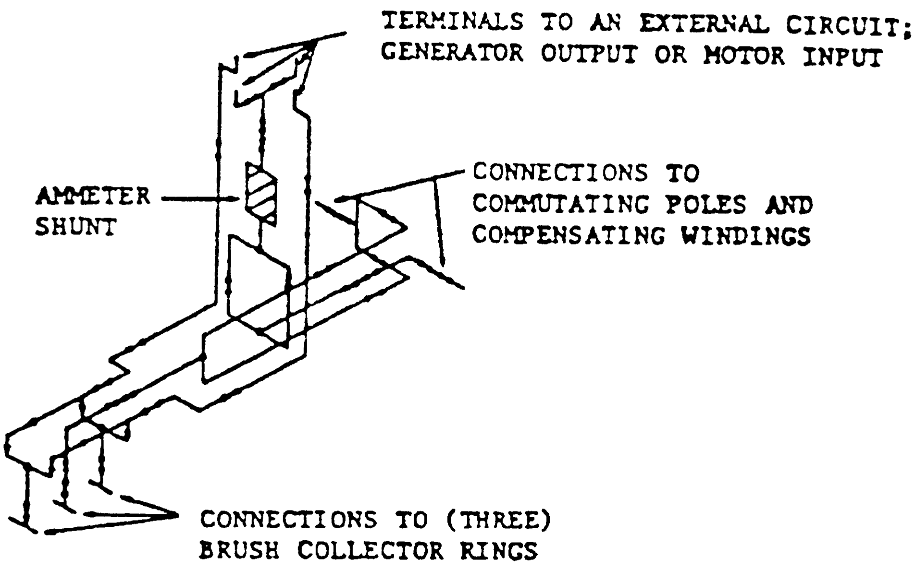
The problem is to make the necessary connections in a way that will give rise to only a very small stray magnetic field. Since these connections carry the whole armature current, which is considerable in large minesweeping generators , it is apparent that they must be carefully designed to attain this objective,

7.3.3.2 General principles of solution. The connections can be designed co have a small stray magnetic field. as follows:

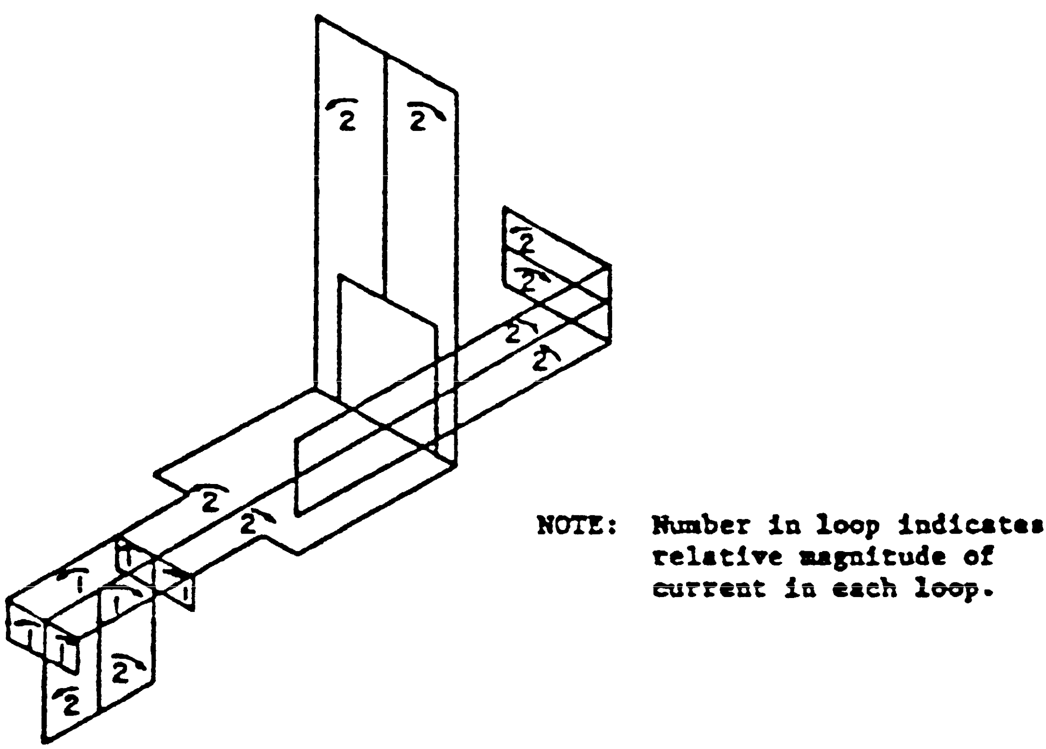
(a) Use three conductors, a central conductor carrying full armature current in one direction and two outer conductors that are symmetrically arranged on opposite sides of the central conductor with each carrying half current in the reverse direction.
(b) Keep the conductors as close together as possible, taking into consideration the requirement for insulation between them, This will keep all current loops small and minimize the disturbance caused by departures from exact equality of current division between the outer conductors.

For each machine, the designer must consider the physical arrangement of the different parts, the space available, and innumerable details that will vary from machine to machine. It is impossible to suggest designs here that will be applicable for any and all conditions. The best that can be done is to give two acceptable designs, one for the three-brush collector ring construction and one for the concentric two-brush collector ring construction. These are to be taken as very simple examples illustrating the general principles involved rather than as designs for specific machines

7.3,3.3 Connections for three-brush collector ring construction. Figure 99 is an example of connections that could be used for a three-brush collector ring machine. The arrows indicate the direction of the current in each conductor. Figure 99 gives the corresponding current loops. We can see that for every current loop there is in the same plane a closely adjacent loop of the same magnetic moment but opposite polarity. The total stray magnetic field should be small.



A. Connections for a machine with three brush collector rings.



B. Resulting current loops for the connections in A.

FIGURE 99. Connections and resulting current loops for a machine with three brush collector rings.

7.3.3.4 Connections for concentric two-brush collector ring construction. Figure 100 is an example of connections that could be used for a machine with two concentric brush collector rings. It is apparent that this arrangement can be readily adapted for use on a machine with three-brush collector rings.

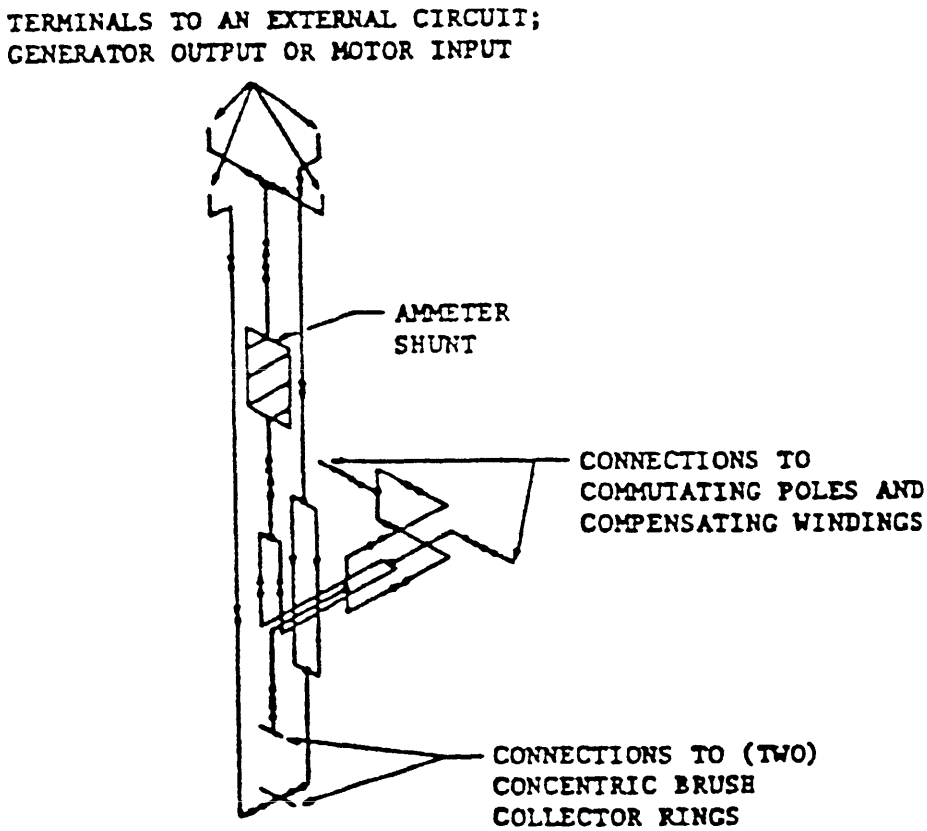


FIGURE 100. Connections for a machine with two concentric brush collector rings.

7.3.3.5 Terminals. The machine should have either of the following:

(a) Four terminals arranged at the comers of a square with positive terminals at the ends of one diagonal and negative terminals at the ends of the outer diagonal; or

(b) Three terminals equally spaced along a line with a positive (or negative) terminal at the center and two negative (or positive) terminals on the outside.

With the four-terminal arrangement there is one terminal for connection to each of the cables in the quadded minesweeping cable run. With the three-terminal arrangement, two diagonally opposite cables in the cable run are connected to the center terminal; the other two cables are connected to the outside terminals (see 6 3.4)

7 .3.3.6 Conditions for good desire. The designer has the problem of fitting to his machine connections that embody the principles illustrated on figures 99 and 100. After the design has been completed, it should be checked to see if it satisfies the following conditions.

(a) The current loops are individually as small as they can be made, taking into account the physical dimensions of the conductors and the necessity of spacing to provide room for insulation.
(b) For each current loop there is in the same plane an immediately adjacent loop, which is as close as possible to the first loop, and of the same magnetic moment but opposite polarity.
(c) Parallel circuits are and will remain of equal resistance to ensure equal division of current between the circuits.

Unless all of these conditions are satisfied, the design is not as good as it should be, If all are satisfied, the design is as good as possible.

7.4 Commutating pole or interpole winding.

7.4.1 General. Commutating poles are used to improve commutation. They are located midway between the main field poles and are frequently referred to as interposes . Some dc generators have interposes but no compensating or pole-face winding. This was the case for all of our World War II minesweeping generators and is also the case for a few of the minesweeping generators built for the minesweepers of today. Other dc generators have interposes and a compensating or pole-face winding as well. This is the case for most minesweeping generators built for the minesweepers of today. Current in the interpole circuit is through the interpole coils and the connections to the coils. The coils themselves (see 7.4.2) produce only a small stray magnetic field; hence, the problem involved in the design of an interpole circuit is to arrange the connections so that they also will produce only a small stray magnetic field. On figures 101 to 102, the interpole circuit is shown completely closed on itself. It is tied into the rest of the armature circuit by cutting one of the connections between coils and connecting to the brush collector rings and terminals at the points marked "CONNECTION TO COMMUTATING POLES AND COMPENSATING WINDING" on figures 99 (A) and 100. Only the interpoles are shown on figures 101 to 103. The main poles are omitted because they are not connected in the interpole circuit.

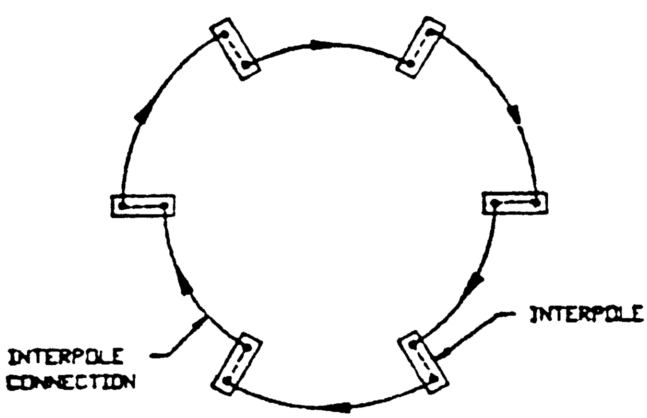


FIGURE 101. Very bad arrangement of connections to interpoles,

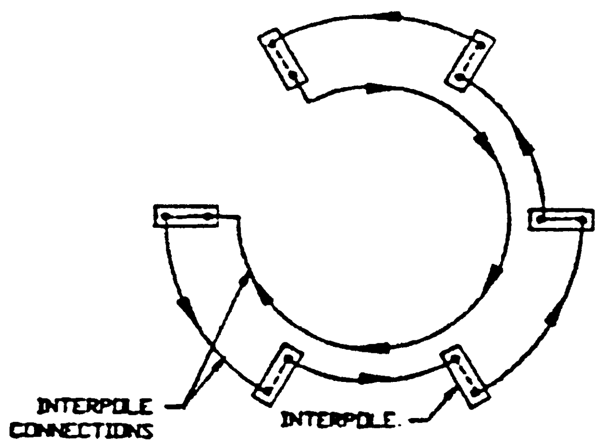
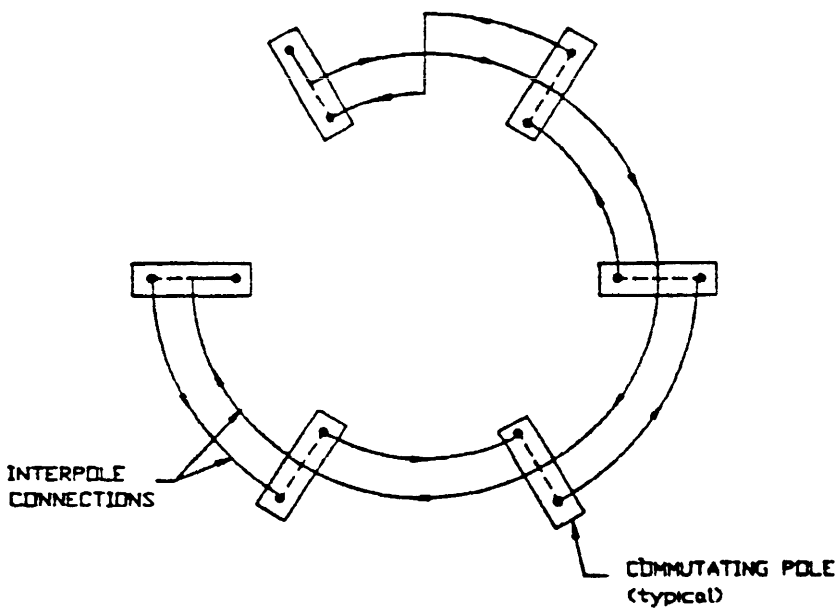


FIGURE 102. Bad arrangement of connections to interpoles.



NOTE: The compensating circuit is shown completely closed on itself. It is tied to the rest of the armature circuit by cutting one of the connections between coils and connecting to the brush collector rings and terminals at points indicated on figures 99 and 100. Only the compensating poles are shown; the main poles are omitted since they are not connected in the commutating circuit

FIGURE 103. Satisfactory arrangement of connections to interpoles (commutating poles).

7.4.2 Number of interpoles. If there are any interpoles at all, there must be as many as main poles in machines designed for small stray magnetic field. This is the usual construction for machines of any appreciable size. When this condition is satisfied, the interposes will produce a stray magnetic field of essentially the same pattern as the main field poles. These, as we have seen before, should produce only a small stray magnetic field if the proper number of field poles is used. The same will be true of the interposes if there are as many of them as there are main field poles.

Some small machines are built with only half as many interpoles as main poles. This gives an unsymmetrical arrangement of interposes as compared to main poles, and the stray magnetic field will be larger than for an equal number of interposes and main poles. Machines that do not have as many interposes as main poles should not be used on minesweepers, no matter what the size of the machine may be. The only exception would be very small machines that have no interpoles at all,

7.4.3 Very bad arrangement of connections. Figure 101 shows an extremely bad arrangement of connections to the interpoles. The whole armature current encloses a large loop around the shaft. The armature current is large for minesweeping generators and large dc generators and motors used for ship propulsion, and the stray magnetic field produced by such a loop can be enormous. During World War II, a stray magnetic field of more than 4 microtesla was measured 18.6 meters (61 feet) below the waterline. Such a stray magnetic field is far too large for minesweepers or other vessels. Interpole connections such as shown on figure 101 should never be used on machines for shipboard use.

7.4.4 Bad arrangement of connections. Figure 102 shows an arrangement of connections that do not make a complete turn around the shaft, but that are still bad, though not as bad as those on figure 101. The trouble with this arrangement is that the outgoing and return conductors are not close together, but far apart. They enclose a loop of considerable area between them, which will inevitably give rise to an objectionable stray magnetic field.

7.4.5 Satisfactory arrangement of connections. Figure 103 shows a satisfactory arrangement of interpole connections. Connections must be at the same end of the machine and as close together as possible, not separated as shown in the drawing for the sake of clarity. Note also that with connections arranged this way, the current loops they form are in pairs with the two loops in a pair of the same magnitude but opposite polarity. The net magnetic moment for the loops will thus be zero.

7.5 Compensating or pole-face windings

7.5.1 General.

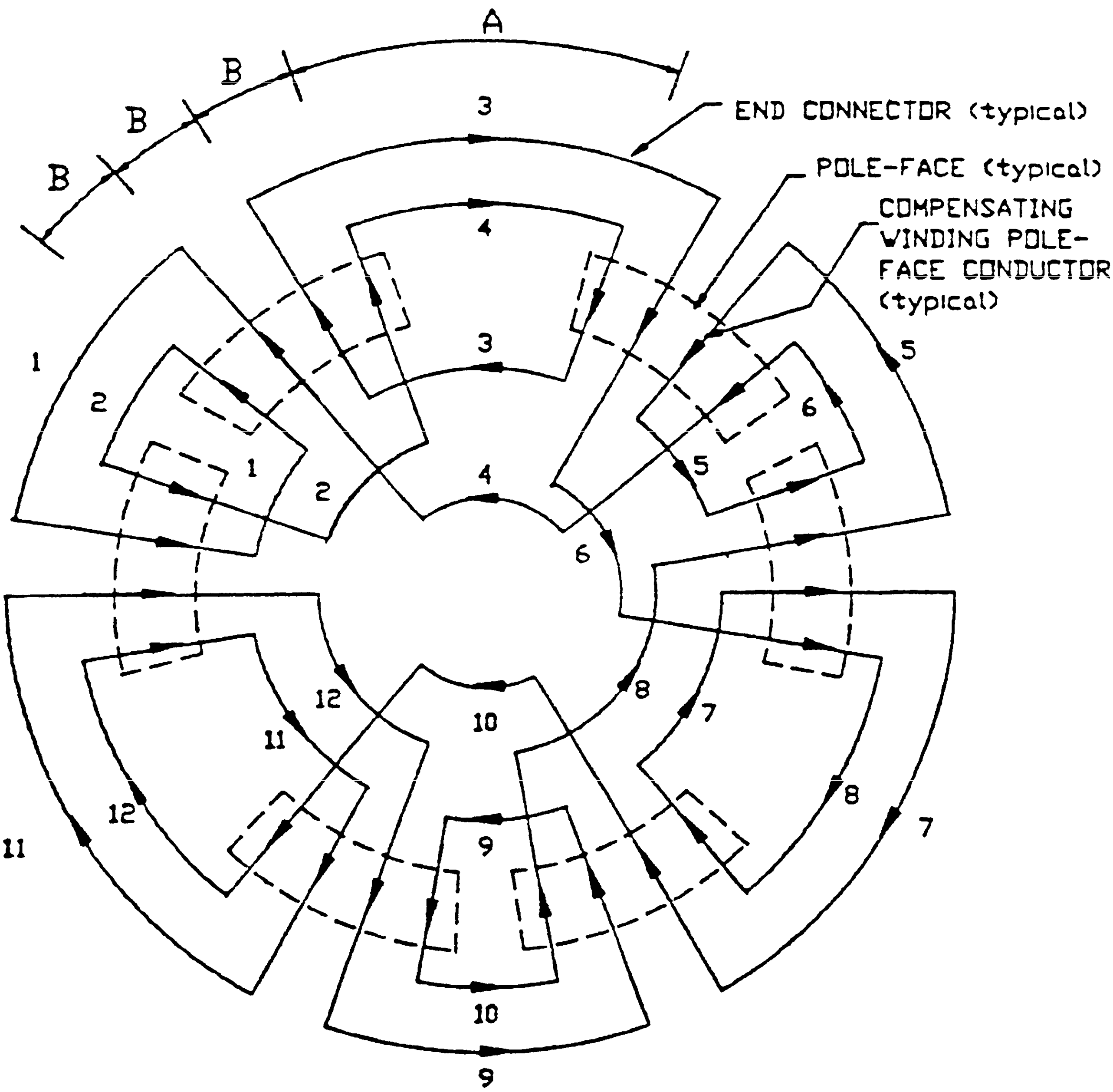
7.5.1.1 Use. Compensating windings are used on large dc motors and generators, including many of the present day minesweeping generators, to compensate armature reaction and improve machine performance. A compensating winding is used on many motors and generators in which a small stray magnetic field is of no consequence whatsoever; hence, it is not a feature that is added to compensate or cancel other magnetic fields in order to arrive at a small stray magnetic field from the machine.

7.5.1.2 Description. A compensating winding consists of the following:

(a) Conductors embedded in slots in the faces of the main field poles, the slots being parallel to the axis of the machine.
(b) End connectors to conduct the current to and from the pole-face conductors . The end connectors give rise to turns around the shaft and large stray magnetic fields unless they are properly arranged.

7.5.2 Schematic representation.

7.5.2.1 Scheme used. On figures 104 to 106, which show different compensating windings, it is supposed that we are looking straight into one end of the machine and that the outer end is made smaller in diameter. The compensating conductors, which are axial in the machine, will then show up as radial lines on the figures. The end conductors are represented by arcs of circles. On the figures, the end connectors at one end of the machine are shown inside; those at the other end of the machine are shown outside the compensating conductors.



NOTE: The compensating circuit is shown completely closed on itself. It is tied to the rest of the armature circuit by cutting one of the connections between poles and connecting to the brush collector rings and terminals at points indicated on figures 99 and 100.

FIGURE 104. Compensating winding for a six-pole machine with four shots per pole face.

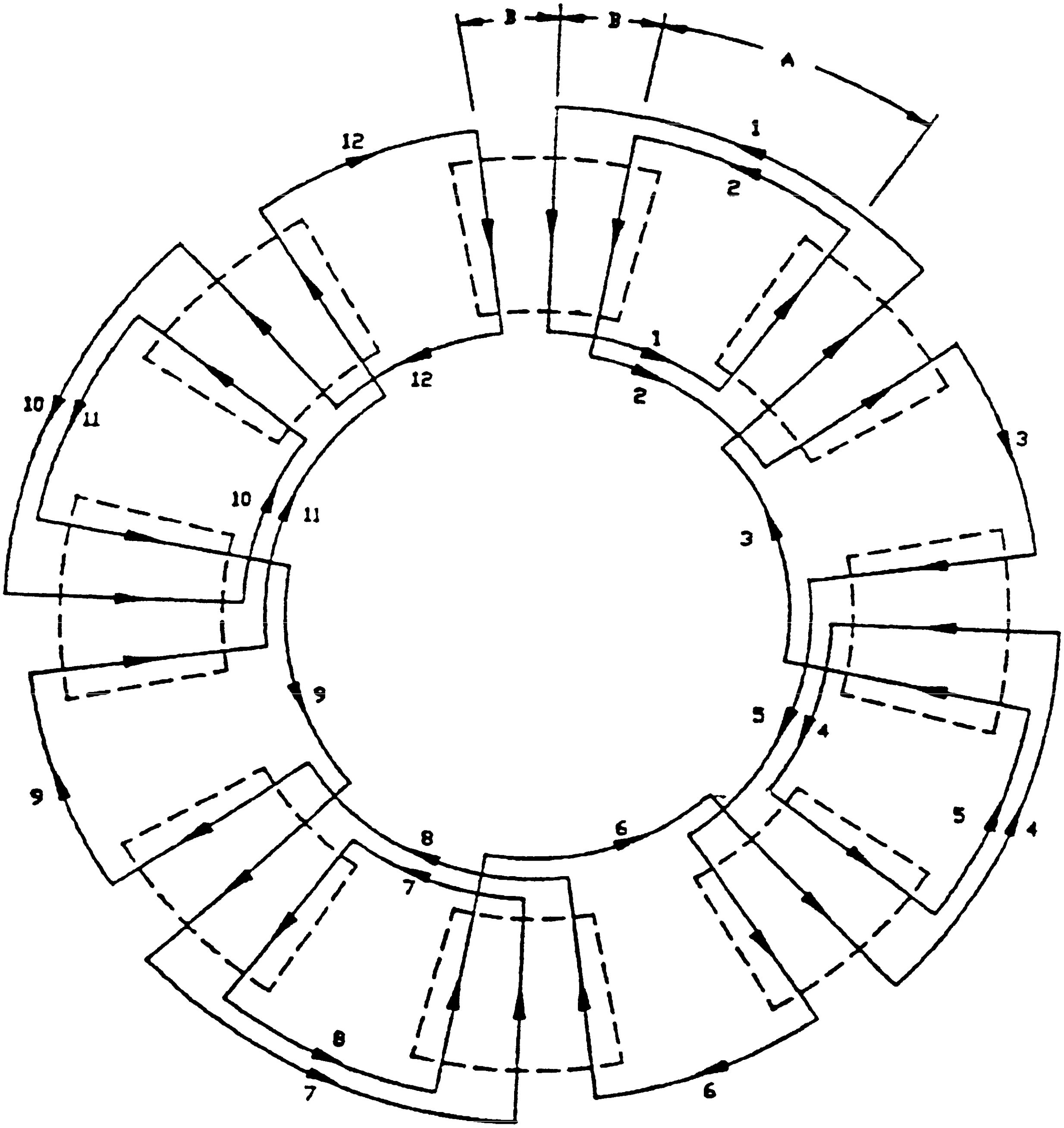
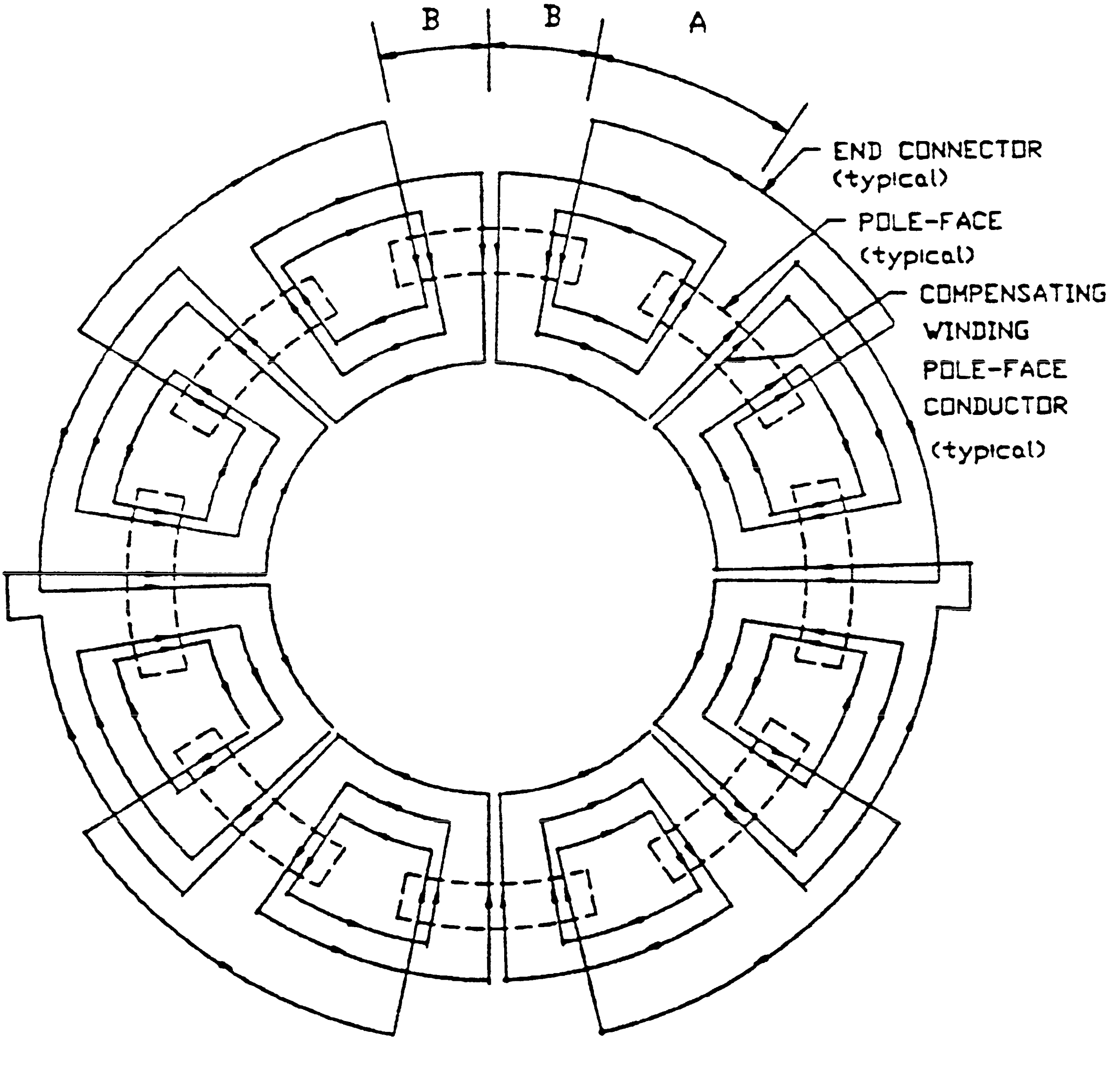


FIGURE 105. Single series circuit compensating winding for an eight-pole machine with three slots per pole face.



NOTE: The two parallel parts of the compensating circuit are show completely closed on itself. It is tied to the rest of the armature circuit by cutting one of the connections between poles for each of the parallel parts and connecting to the brush collector rings and terminals at points indicated on figures 99 and 100.

FIGURE 106. Parallel circuit compensating winding for an eight-pole machine with three slots per pole face.

7.5.2.2 Connections to rest of circuit. For the sake of simplicity, figures 104 to 106 show the compensating winding as a circuit closed on itself and do not show the connection to the leads that bring current to and take it from the winding. We can visualize this connection by supposing that a cut is made in one of the end connectors and that the ends of the connectors on the two sides of the cut are connected to the rest of the armature circuit at the points marked "CONNECTION TO COMMUTATING POLE AND COMPENSATING WINDING" OR figures 89 and 90.

7.5.2.3 Interpoles. The interpoles, which are connected in series with the compensating winding, are also not shown for the sake of simplicity. We can visualize the connection to the interpoles as follows: Make a cut through one of the end connectors of the compensating winding where it passes close to an interpole, and connect the ends of the connector on the two sides of the cut to the interpole coil.

7.5.2.4 Separation. It is to be specifically noted that the end connectors of the compensating winding are shown quite widely separated for the sake of clarity in the diagrams. In the actual machine they should be arranged as close together as possible.

7.5.3 Even number of slots per pole face.

7.5.3.1 Tabulation of angles. Figure 104 shows a schematic diagram of a compensating winding for a six pole machine with four slots per pole face, If the winding is traced through starting from any point and returning to the same point, it will be seen that the winding is a single series circuit and that the same current flows in each of the pole-face conductors and end connectors. Table XIV lists each end connector and gives the angle through which currents flow in it, either in a clockwise or counterclockwise direction.

TABLE XIV. End connectors and current angles for six pole machine,

Inner connectors	
Clockwise	Counterclockwise
1. A + B	3. A + B
2. A + 3B	4. A + 5B
5. A + B	7. A + B
6. A + 5B	8. A + 5B
9. A + B	11. A + B
10. A + 5B	12. A + 3B
Total 6A + 16B	Total 6A + 16B
Net angle = 0	

TABLE XIV. End connectors and current angles for six pole machine - Continued.

Outer connectors	
Clockwise	Counterclockwise
3. A + 2B 4. A 7. A + 2B 8. A 11. A + 2B 12. A	1. A + 2B 2. A 5. A + 2B 6. A 9. A + 2B 10. A
Total 6A + 6B	Total 6A + 6B
Net angle = 0	

7.5.3.2 Balance at both ends. Inspection of table XIV shows that the sum of the clockwise angles is equal to the sum of the counterclockwise angles, both for the inner connections, which represent the end connectors at one end of the machine, and the outer connectors, which represent the end connectors at the other end of the machine. There are, therefore, zero net turns around the shaft at each end of the machine. It is to be expected, therefore, that the stray magnetic field caused by the compensating winding will be small, provided that the end connectors at each end of the machine are as close together as possible. This expectation has been verified by tests on a machine that has a compensating winding of this kind and gives a small stray magnetic field. It is vitally important that the clockwise and counterclockwise angles balance at each end of the machine. It is not enough to have a clockwise angle at one end of the machine balanced b) an equal counterclockwise angle at the other end of the machine. The cancellation of magnetic field caused by this combination is not good enough to be satisfactory. This will be discussed in more detail in 7.5,4,2. The compensating winding of figure 116 satisfies the condition of balance at each end individually. A little consideration will show that a single series circuit compensating winding can always be laid out to satisfy this condition when the number of slots per pole-face is even.

7.5.4 Odd number of slots per pole face.

7.5.4.1 Tabulation of angles. Figure 105 shows a pole-face winding for an eight-pole generator with three slots per pole face. The clockwise and counterclockwise angles are given in table XV for both the inner and outer connectors .

TABLE XV. End connectors and current angles for eight pole machines.

Inner connectors	
Clockwise	Counterclockwise
1. A + B 2. A + 2B 4. A + B 5. A + 4B 7. A + B 8. A + 4B 10. A + B 11. A + 4B	3. A + 3B 6. A + 3B 9. A + 3B 12. A + B
Total 8A + 18B	Total 4A + 10B
Net angle = 4A + 8B clockwise	
Outer connectors	
Clockwise	Counterclockwise
3. A 6. A 9. A 12. A	1. A + 2B 2. A 4. A + 2B 5. A 7. A + 2B 8. A 10. A + 2B 11. A
Total 4A	Total 8A + 8B
Net angle = 4A + 8B counterclockwise	

7.5.4.2 Unbalanced at each end. The compensating winding of figure 105 gives a clockwise angle of 4A + 8B at one end of the machine and a counter-clockwise angle of 4A + 8B at the other end. An angle of 4A + 8B is half a turn, so that at each end of the machine we have full armature current flowing through half a turn. This is equivalent, so far as stray magnetic field is concerned, to a full turn with half current. We have in effect, therefore, at one end of the machine a full turn with half current and at the other end a full turn with half current in the reverse direction. The magnetic fields produced by these turns are in opposition and will cancel to a certain extent. The turns are too far apart, however, for the cancellation to be good enough. This has been verified by tests on a machine with a compensating winding of this kind.

7.5.4.3 Series circuit winding in odd number of slots. It may be possible to lay out a single series circuit compensating winding in such a way that the clockwise and counterclockwise angles will balance at each end, as was the case for the winding discussed in 7.5.3 for a machine with an even number of slots per pole face. Unfortunately, however, we do not yet know how to lay out such a winding and, moreover, it seems probable that this cannot be done. Consequently, in all cases where a small stray magnetic field is an important consideration, a single compensating winding must not be used if the number of slots per pole is odd,

7.5.5 Winding for an odd number of slots per pole face.

7.5.5.1 Conditions for use. If possible, minesweeping generators should be designed with compensating windings in an even number of slots per pole face. If, however, the characteristics of the machine are such as to make an odd number of slots either indispensable or highly desirable, they can be used provided the number of poles is an integral multiple of four.

7.5.5.2 Example for an eight-pole machine. An example of a suitable compensating winding for an eight-pole machine with three slots per pole face is shown on figure 106. There are two conductors per slot instead of only one (or several conductors in parallel) as in the windings discussed previously. The compensating winding has two parallel circuits, each of which carries only one-half the armature current. One parallel circuit is to the right of the vertical center line; the other is to the left. Connections to the leads that bring current to and take it from the parallel circuits are not shown but can be made on the vertical centerline where the two halves are close together. Inspection of the figure and preparation of tables similar to tables XIV and XV will show that each half of the winding is balanced on itself at each end of the machine. Cancellation of magnetic fields is not dependent upon equality of current division between the halves and the stray magnetic field should be small. No machines with compensating windings like this have yet been tested for stray magnetic fields but the results should be good. This assumes, however, that the end connectors are arranged as close together as possible (see 7.5.6)

7.5.5.3 Extension to other cases. Although the winding shown on figure 106 is for the specific case of an eight-pole machine with three slots per pole face, further consideration will show that the basic scheme can be used for any generator with eight poles and any odd number of slots per pole face. It can also be used for any odd number of slots and any number of poles that is an integral multiple of four If the number of poles is not an integral multiple of four, six poles for example, then each half of the compensating winding will not be completely balanced on itself. For this reason, the basic scheme of the winding shown on figure 106 should not be used except for generators with four (too few for minesweeping generators), eight, twelve, or sixteen poles.

7.5.6 Loops in transverse Planes. The end connectors at each end of the machine will form loops in a plane perpendicular to the axis of the machine, These loops should be kept small by keeping the end connectors close together. Furthermore , these loops should have positive and negative magnetic moments so that the sum of the magnetic moments at each end of the machine is zero.

7.5.7 Summary on compensating windings.

(a) The preferable arrangement is to have compensating windings in an even number of slots per pole face. A single series compensating winding that carries full armature current can be used in this case.
(b) If an odd number of slots must be used. this can be done provided the number of poles is an integral multiple of four. Two parallel windings are used in this case, each carrying half armature current.
(c) The combination of an odd number of slots per pole face and a number of poles that is not an integral multiple of four should not be used in minesweeping generators.
(d) End connections should be so arranged that the current loops in the planes of the end connectors at the two ends of the machine sum up to zero net magnetic moment at each end.

8. ARRANGEMENT OF STORAGE BATTERIES FOR LOW STRAY MAGNETIC FIELDS

8.1 Battery arrangements. In this section we consider the arrangement of storage batteries and the design considerations that will reduce the stray magnetic field produced by the current through che batteries and their associated connections.

8.1.1 Batteries on minesweepers. The design consideration for batteries on minesweepers will be as specified in 8.1.1.1 through 8.1.1.6.

8.1.1.1 Use of storage batteries. We use storage batteries on minesweepers to supply power for a number of purposes, such as starting small engines. It would be preferable to have all engines on minesweepers started nonelectrically because this would eliminate all stray magnetic field problems caused by electric starting. Pending the trial and approval of suitable nonelectrical starting methods, however, electric starting will be used for at least some of the engines on minesweepers. Even when electric starting of engines on minesweepers is eliminated, we will still use storage batteries for other purposes.

8.1.1.2 Types of battery trays. The three-cell, 6-volt battery tray arrangements covered by this section will be of the following types:

(a) Type I - When you look down on the top of the tray, with the cells oriented in the vertical direction, and the top cell terminals negative-positive from left to right, make the external negative battery connection at the top of the tray and the external positive battery connection at the bottom of the tray (see figure 107).
(b) Type II- When you look down on the top of the tray, with the cells oriented in the vertical direction, and the top cell terminals negative-positive from left to right, make the external positive battery connection at the top of the tray and the external negative battery connection at the bottom of the tray <see figure 107).

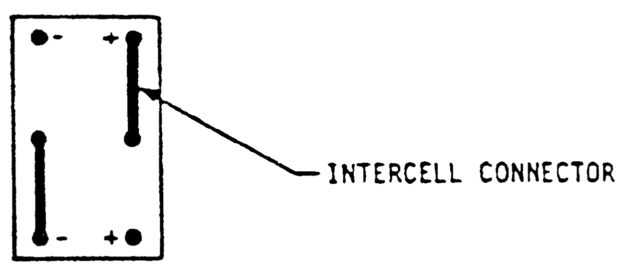
We can convert a type I tray to a type II, or vice versa by shifting intercell connectors .

8.1.1.3 Design considerations. We will incorporate the following design considerations in battery circuits in a manner that will create a minimum stray magnetic field:

(a) Battery circuit arrangements will be accomplished in as simple a manner as possible.
(b) Magnetic moments of equal and opposite current loops or dipoles created by the battery circuit will be as small as possible. The distance between these current loops or dipoles will be as small as possible.
(c) The resultant or net magnetic moment of all current loops or dipoles will be as close to zero as possible.

8.1.1.3.1 Utilization of series and parallel compensation in the arrangement of battery circuit. We will use series compensation (see figure 44) in the arrangement of the battery circuit in order to achieve a minimum net magnetic moment. Parallel compensation (see figure 44) will be used in the battery circuit in order to achieve a minimum net magnetic moment only when precautions have been taken to minimize the effects of unequal current division.

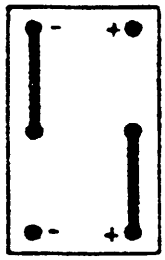
8.1.1.3.2 Positioning of battery connecting bus bars. Except for crossover points, the centerlines of the bus bars connected to the battery will be in the same horizontal plane as the centerlines of the intercell connectors. At cross-over points, the bus bar or cables will depart from this plane only as much as is needed, to provide room for insulation clearance.



TYPE I BATTERY TRAY ARRANGEMENT

NOTE: A type I tray can be converted to a type II tray, or vice versa, by shifting intercell connectors.

FIGURE 107. Type I and type II battery tray arrangements for three-cell, 6-volt battery trays.



TYPE II BATTERY TRAY ARRANGEMENT

FIGURE 107. Type I and type II battery tray arrangements for three-cell, 6-volt battery trays - Continued.

8.1.1.4 Battery tray arrangement. We can form the battery tray arrangements considered in 8.1.1.4.1 through 8.1.1.4.4 by connecting in series an even number of battery trays, each of which contains three storage battery cells. For minimum stray magnetic field, we must use an even number of trays.

8.1.1.4.1 Two-tray arrangement. The two-tray arrangement shown on figure 108 is completely closed on itself to isolate the current path in the battery from that in the circuit. We have eight current loops in a basic two-tray arrangement. Figure 108 shows the two horizontal current loops, A and B. Since they have equal areas, carry the same current, and are in series the battery arrangement will have a small stray magnetic field.

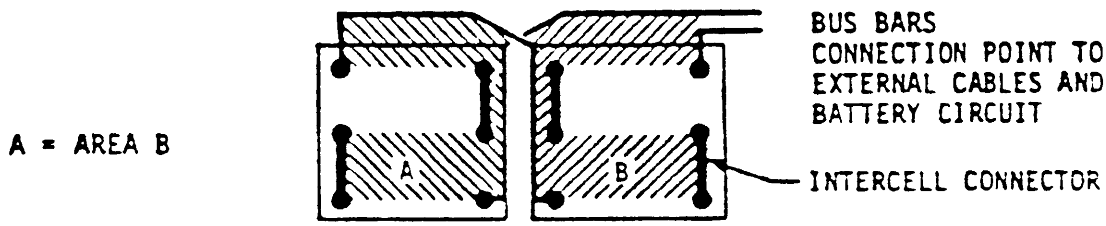


FIGURE 108. Two-tray arrangement.

8.1.1.4.1.1 Current loops. Figure 109 is an isometric view showing the current path in the cells, intercell connectors, and bus bars. The current entering a cell at one terminal does not pass directly in a straight line to the other terminals Instead it ducks down from the terminal to the plates connected to the terminals, passes through the electrolyte to the plates opposite polarity, and then up to the other terminal In its passage through the cell, the current is no longer confined to a relatively small metallic conductor but swells out to become a distributed current filling the cell. Nonetheless, the magnetic effects of this distributed current will be nearly the same, at distant points, as if the current were concentrated in a small conductor that extends down about half way into the cell from one terminal, crosses the cell horizontally, and then rises vertically to the other terminal. These hypothetical small conductors inside the cells are shown by the lines below the plus and minus terminals indicated on figure 109. The current loops that give the same current distribution are show-n on figure 110. We have two loops in a horizontal plane, A and B, that are also shown on figure 110 and six vertical loops C to H. The vertical loops are in opposing pairs, C and F, D and G and E and H, so that the net magnetic moment for the vertical loops is zero, just as it is for the two horizontal loops. The vertical loops, as well as the horizontal, must be taken into account in designing battery arrangements for small stray magnetic field, and must be arranged so that their net magnetic moment is zero.

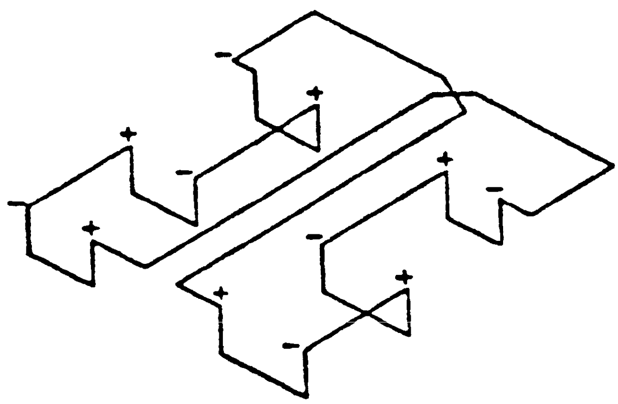


FIGURE 109. Isometric view showing current path.

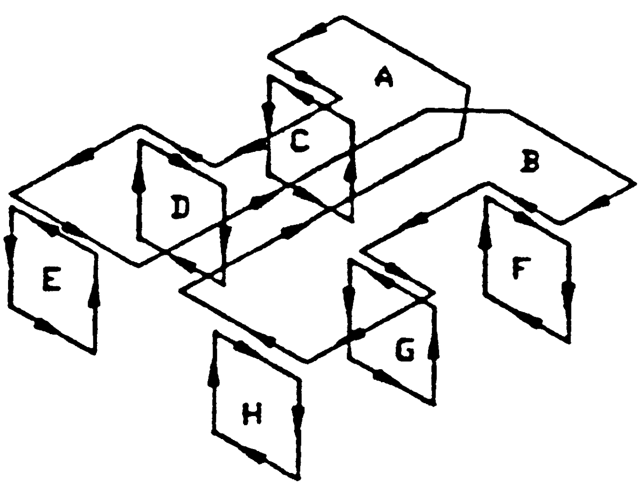


FIGURE 110. Battery current loops for two-trays.

8.1.1.4.2 Four-tray arrangements, A typical four-tray arrangement is shown on figure 111. In this arrangement we have three horizontal current loops. Area A plus area C must equal area B to create a minimum stray magnetic field. Four - trays may be arranged in series, rows or double stacked. Four-tray arrangements, except double stacked, must be in accordance with 8.1.1,3.2.

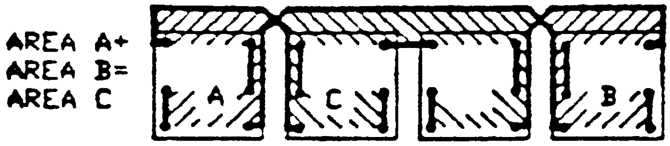


FIGURE 111. Four-trav arrangement.

8.1.1.4.3 Six-tray arrangement. A typical six-tray arrangement is shown on figure 112. In this arrangement we have four horizontal current loops. In order to satisfy the conditions of a minimum stray magnetic field:

(a) The area A must equal the area D.
(b) The area B must equal the area C.
(c) The area of B or C must equal the area of two times the area of A or D.

Six-tray arrangements must be in accordance with 8.1.1.3.2.

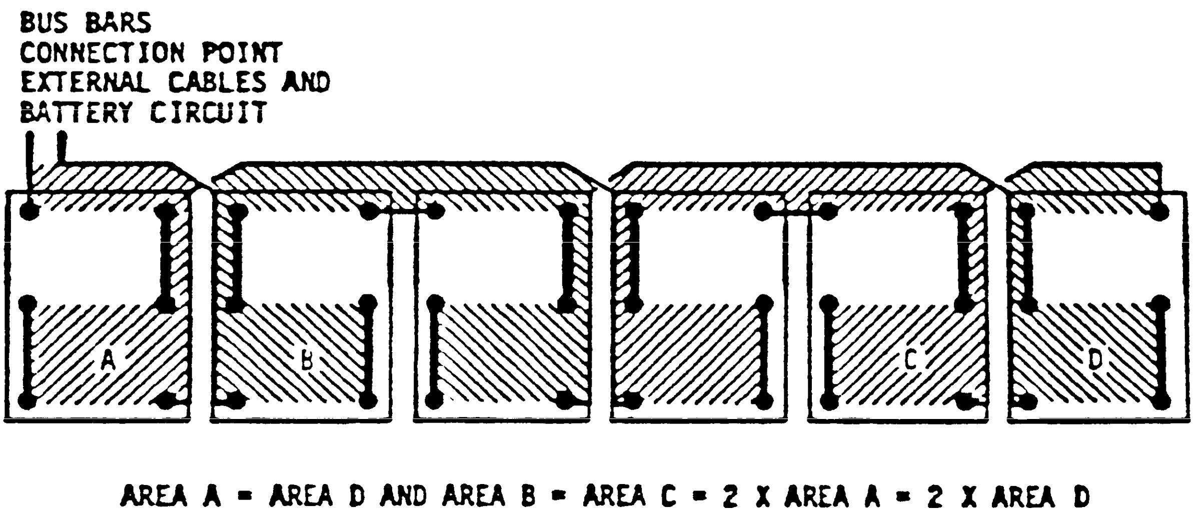


FIGURE 112. Six-tray arrangement.

8.1.1.4.4 Other tray arrangements . Other tray arrangements must be in accordance with 8.1.1.3 through 8.1.1.3.2.

8.1.1.5 Estimating magnetic fields. We will estimate the stray magnetic fields caused by arrangements of two, four and six trays by evaluating the basic unit (see figure 113). The dimensions we will use are approximately the dimensions of the standard Navy 6-volt battery that might be used as storage batteries on minesweepers. The arrangements for two, four and six trays (see figures 108, 111, and 112) are constructed from the basic unit and its mirror image in line CC′ . The determination of the magnetic field will be in accordance with 8.1.1 5.1 through 8.1.1 .5.3 .4. Dimensions are in millimeters

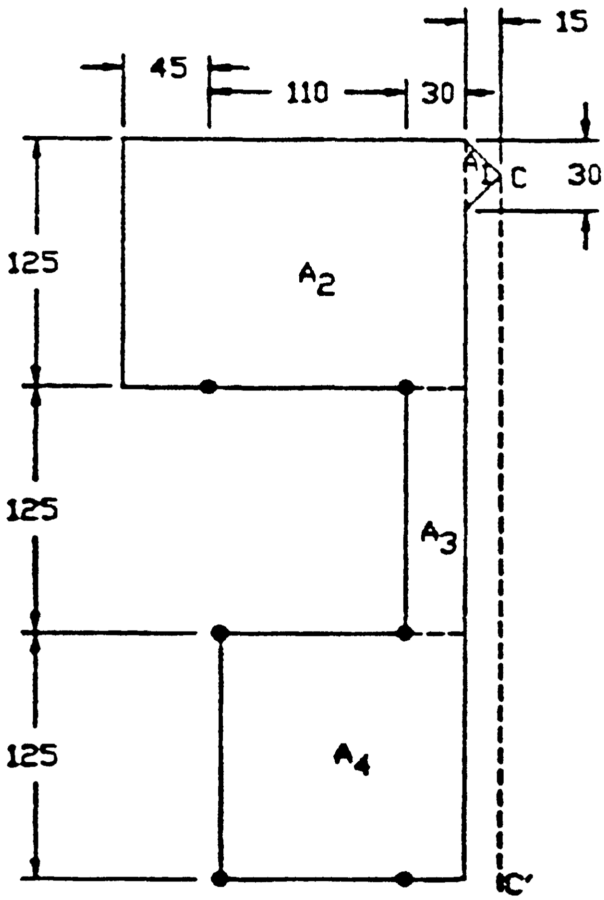


FIGURE 113. Basic battery unit.

8.1.1.5.1 Horizontal loops. We can model the horizontal loop of figure 108 with a vertical dipole located at the center of gravity of the loop. The dipole moment will be determined as follows:

(a) Find the areas in square millimeters of the four areas shown on figure 113 and the total area.
(b) Determine the distance from the center of gravity of each area co the line CC′ (moment arm).
(c) Determine the static moment about line CC′ for each of the four areas and determine the total static moment where the static moment is equal to the area multiplied by the moment arm.
(d) Find the distance to the center of gravity for the dipole using the total static moment and total area.

For a current of 1000 A, the magnetic moment will be 44.6 Am^2 and the dipole will be located 91.7 mm to the left of line CC′.

8.1.1.5.2 Vertical loops. We will model the vertical loops (C, D and E) of figure 110 by horizontal dipoles. The dipoles for each loop associated with the basic unit of 8.1.1.5 will be determined in accordance with 8.1.1.5.1. The arbitrary dimensions of each vertical loop will be 110 mm x 200 mm. For a current of 1000 A, the magnetic moment of each loop will be 22.0 Am^2 and the dipole will be located 100 mm below the plane of loop A.

8.1.1.5.3 Maximum vertical component. If we want to find an exact value for the maximum vertical component of the magnetic field at a point approximately 6 meters below the plane of the basic unit it would require considerable work. However, we can estimate by dividing the number of loops in the tray arrangement into three groups.

8.1.1.5.3.1 Two-tray arrangement. We have eight loops under consideration in a two-tray arrangement. The three groups will be in accordance with 8.1.1.5.3.1.1 through 8.1.1.5.3.1.3.

8.1.1.5.3.1.1 Group one. Our first group will consist of loops A and B (see figure 110). These two loops are equivalent to the vertical dipoles of 4.3.3.4 with M-44.6 Am^2, a = 91.7 mm, and Z = 6 meters. The maximum value of the vertical component using the formula in 4.3.3.4 will be 1.732 nanoteslas (nT).

8.1.1.5.3.1.2 Group two. Our second group will consist of loops C and F (see figure 110). These two loops, when rotated through 90 degrees about a vertical axis, are equivalent to the horizontal dipoles (see 4.3.3.3) with M = 22.0 Am^2, b = 100 mm and z = 5.9 meters, The maximum value of the vertical component using the formula in 4.3.3.2 will be 0,335 nT.

8.1.1.5.3.1.3 Group three. Our third group will consist of loops D, E, G and H (see figure 110). These four loops, when rotated through 90 degrees about a vertical axis, are equivalent to the horizontal dipoles of 4.3.3.10 with M = 22.0 Am^2, a - 62.5 mm, b - 100 mm, and z = 5.9 meters. The maximum value of the vertical component using the formula in 4,3 3,10 will be O 027 nT.

8.1.1.5.3.1.4 Limits on the maximum field. After we determine the maximum fields the next problem is to set limits on the field produced by the three groups acting together. The maximum value would be the sum of the three maxima, 1.732 + 0.335 + 0.027 = 2.095 nT. The minimum value would be the largest maxima minus the other two. The largest maxima would be from group one, which produced 1.732 nT. The minimum value would be 1.732 - 0.335 - 0.027 = 1.370 nT. The minimum would occur if the maximum values all occurred at the same point on the plane 6 meters below, and if the maximum for the first group was of one sign and the maxima for the other two groups were of the opposite sign. The precise value of the maximum vertical component that will be produced by the eight loops is not known, but will be bracketed between 1.370 nT and 2.095 nT.

8.1.1.5.3.2 Four-tray arrangements. With four trays we have four horizontal loops and twelve vertical loops, four in each of the three parallel vertical planes, which will be designated plane 1, plane 2, and plane 3. We will divide the sixteen loops into three groups in accordance with 8. 1. 1. 5. 3 .2. 1 through 8.1.1.5.3.2.3.

8. 1.1.5. 3 .2.1 Group one. Our first group will consist of the four horizontal loops (see figure 114). These are equivalent to the four vertical dipoles (see 4 3.3.13) with M = 44.6 Am^2, a_1 = 291.7 mm, a_2 = 108.3 mm, and z - 6 meters. The maximum value of vertical component is 0.505 nT.

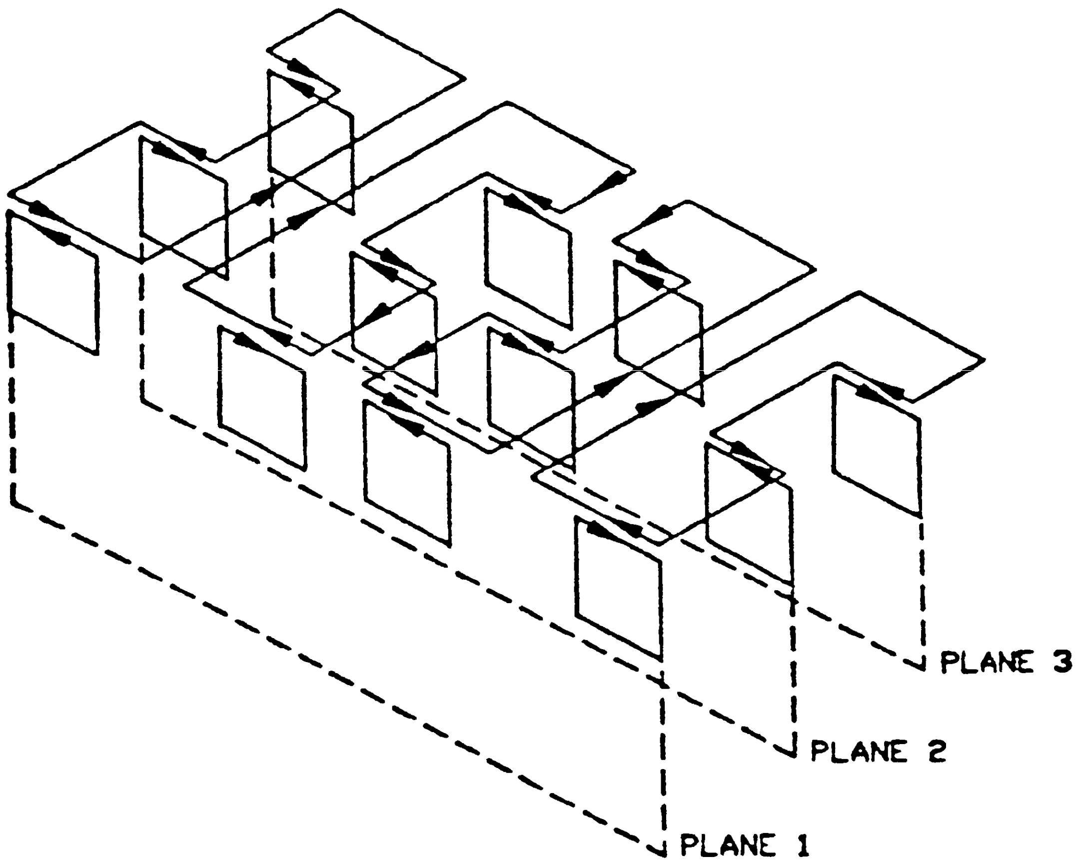


FIGURE 114. Battery current loops for four-trays.

8.1.1.5.3.2.2 Group two. Our second group will consist of the four vertical loops in plane 1 (see figure 114). These four loops, when rotated through 90 degrees about a vertical axis, are equivalent to the horizontal dipoles (see 4.3.3.7) with M- 22.0 Am^2, b_1 - 300 mm, b_2 = 100 mm, and z = 5.9 meters. The maximum value of vertical component is 0.088 nT.

8.1.1.5.3.2.3 Group three. Our third group will consist of the eight vertical loops in plane 2 and 3 (see figure 114). The formula for this array must be derived by using the methods given in appendix A. We have not shown the derivation of the formula due to its complexity; however, we know the maximum value of the vertical component given by the formula is O nT to the degree of approximation used here.

8.1.1.5 3.2.4 Limits on the maximum field. Tile maximum magnetic field produced by the three groups acting together would be the sum of the three maxima. 0.505 nT+ G 088 nT + O nT = 0.593 nT. The minimum -k'slut would be 0.505 nT - 0.088 nT - 0 nT = 0.417 nT.

8.1.1.5.3.3 Six-tray arrangement. With six trays we have six horizontal loops and eighteen vertical loops, six in each of three parallel vertical planes. We will divide the twenty four loops into three groups in accordance with 8.1.1.5.3,3.1 through 8.1.1.5.3.3. 3.

8.1.1.5.3.3.1 Group one. Our first group will consist of six horizontal loops. These are equivalent to six Vertical dipoles.

8.1.1.5.3.3.2 Group two our second group will consist of six vertical loops in one vertical plane.

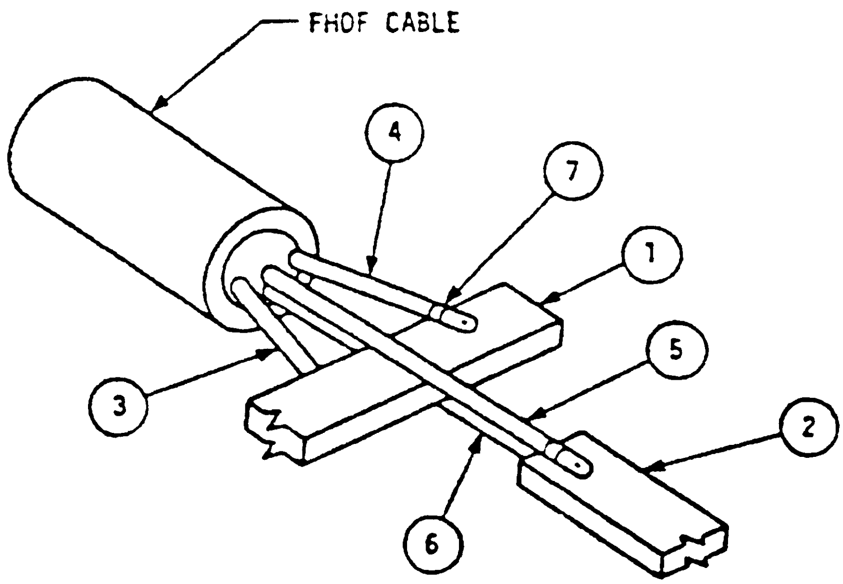
8.1.1.5.3.3.3 Group three. Our third group will consist of the remaining twelve loops in the remaining two vertical planes.

8.1.1.5.3.3.4 Limits on the maximum fields. The field for each group must be determined by deriving an equation using the methods given in appendix A. The limits on the field can be determined as demonstrated in 8.1 1.5.3.1 and 8.1.1.5.3.2.

8.1.1.5.3.4 Importance of equivalent loop areas. We will determine the maximum field values given in 8.1.1.5.3,1.4, 8.1.1.5.3.2.4 and 8.1.1.5.3.3.4 on the assumption that all horizontal loops are of exactly equal area and that all vertical loops are also of exactly equal area. Suppose the areas of the four horizontal loops in a four-tray arrangement are 44,600 square meters (m^2), 44,600 m^2, 44,600 q 2, and 60,000 q 2. The magnetic field produced by these will be the same as that produced by a group of four loops of 44,600 m^2 each plus a single loop of 15,400 m^2. The magnetic field for a group of four loops (see 8.1.1.5.3.2.1) is 0.505 nT. The single horizontal loop of 15,400 m^2 will, for a current of 1000 A, produce a field of almost twenty times the magnetic field of four loops of equal area. This illustrates the need for keeping loop area equal, Also , it is better to use bus bars rather than cable for connecting the trays together because bus bars can be bent into shape with more precision and are more likely to stay in shape.

8.1.1.6 External battery circuit connections, We will make external battery circuit connections in order to minimize the creation of stray magnetic fields. External battery circuit cable connections to the bus bar intercell connectors will be made at the bus bar termination points shown on figures 108, 111 and 112, in accordance with the method shown on figure 115.

8.1.2 Batteries in submarines. The design criteria for batteries in submarines will be in accordance with 8.1.2.1 through 8.1.2.5.



1 and 2. Bus Bars

3 and 4. Diagonally opposite conductors in 4-conductor cable. One to be connected above and one below bus bar 1 at points equidistant from the center line of bus bar 2.

5 and 6. Diagonally opposite conductors in 4-conductor cable. One to pass above and one below bus bar 1 and be connected to upper and lower side of bus bar 2.

7. Cable Lugs

FIGURE 115. Connections to the battery.

8.1.2.1 Batteries. The storage batteries used in submarines to supply power for underwater propulsion have more than 100 cells connected in series. The cells are large, roughly one-half meter square and a meter or more high. Battery currents are thousands of amperes. Because of the large size of the individual cells and the large currents that are possible, a submarine storage battery can give rise to large stray magnetic fields unless the batteries are arranged to minimize this effect.

8.1.2.2 Objectives of submarine battery arrangement. We have different objectives for the arrangement of batteries in submarines. In minesweepers we must have as small a stray magnetic field as possible at almost any cost. In submarines we must have simple arrangements that will give a small magnetic field, hut not necessarily as small as could be obtained by more complicated arrangements. Minesweepers have relatively small batteries that we must arrange to have as small a magnetic field as possible. Submarines, however, have steel hulls and degaussed magnetic fields that are large compared to the degaussed magnetic fields of wood hull minesweepers. It is sufficient to make sure that the stray magnetic field for submarines storage batteries is small compared to the degaussed field of the submarines. Therefore, it is unnecessary to arrange batteries in submarines so that their stray magnetic field is as small as that from batteries in minesweepers

8.1.2.3 Evaluation of submarine battery arrangement. We will describe and evaluate the small stray magnetic field of submarine batteries in 8.1.2.3.1 through 8.1.2.3 10.6,

8.1.2.3.1 Eight cell batteries. We will use a battery with eight cells arranged in two rows of four cells each for the example arrangement. The simplest way of connecting these in series is shown in the plan view of figure 116. The battery is shown short-circuited on itself in order to confine our attention to the current loops in the battery itself. In any actual installation, connection to the external circuit would be made at one end of the battery.

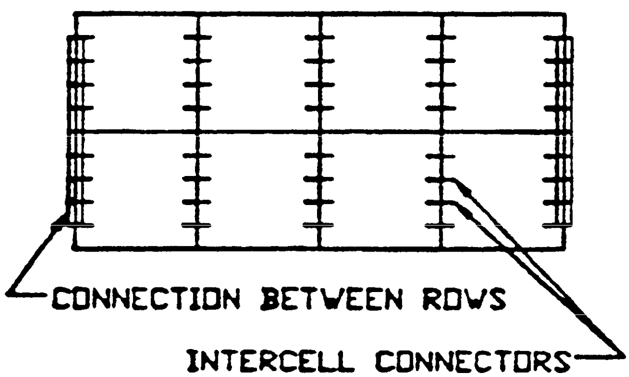


FIGURE 116. Plan view of series connection.

8.1.2.3.2 Current path. The current in the battery will be distributed over the cross section of the cells, but to a first approximation it can be considered as a concentrated current in a conductor along the center of a row. This gives the current path shown on figure 117. The two rows of cells give a single, horizontal current loop that will have a substantial stray magnetic field.

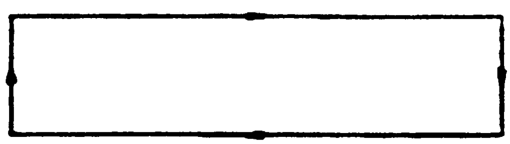


FIGURE 117. Current path in battery.

8.1.2.3.3 Reduction of stray magnetic field. We can substantially reduce the stray magnetic field by putting in two crossovers (see figure 118), one betweem the first and second cells in a row and the other between the third and fourth. This will break the single current loop (see figure 117) into three current loops (see figure 118). The end loops, 1 and 3, each have half the area of the center loop 2 and both have a magnetic polarity opposite that of the center loop. The net magnetic moment is thus equal to zero.

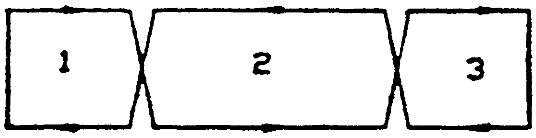


FIGURE 118. Current path for reduction of stray magnetic field

8.1.2.3.4 Comparative values of magnetic field. Suppose each cell is 0.5 square meter and that the current is 3000 A. In this case, the single loop (see figure 117) would have a magnetic moment of 3000 Am^2 for a current of 3000 A. If we consider figure 118, it is desirable to view loop 2 as being two equal loops of the same length and the same polarity: therefore, the circuit can be modeled as four loops with equivalent dipoles (see 4.3.3.12). The area of each loop will be $0.25m^2$ if the small changes in area caused by the crossovers are neglected and the magnetic moment of each will be 750 Am^2 for 3000 A. For these conditions, the maximum vertical component of magnetic field on a plane 15 meters below the loop will be:

Single current loop	177.78nT
Three current loops	0.59nT

8.1.2.3.5 Vertical current loops. Just as in the case of the batteries used in minesweepers, the current in submarine storage batteries does not pass directly from one terminal of a cell to the other, rather, it enters at one terminal, ducks down in the plates, passes through the electrolyte to the plates of opposite polarity, and then rises to the other terminal. The current path will thus be somewhat as shown on figure 119 The equivalent current loops are shown on figure 120. There are three current loops, 1 to 3, in the horizontal plane plus eight current loops, 4 to 11, in two vertical planes. We have already considered the horizontal loops. Reference to figure 120 shows that loops 4 and 7 are of one polarity, 5 and 6 of the opposite. These four loops are thus in opposition and should give only a small stray magnetic field, The same is true of loops 8 to 11. Furthermore, loops 4 to 7 are in opposition to loops 8 to 11; hence, the resultant stray magnetic field from the eight vertical loops should be quite small.

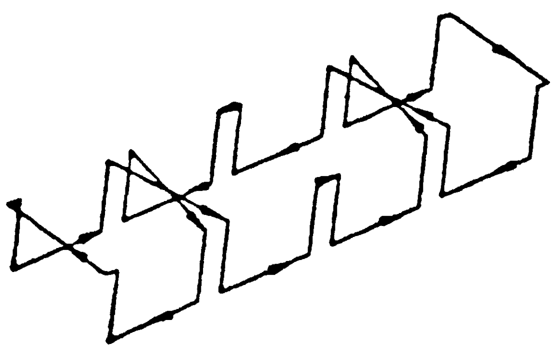


FIGURE 119. Isometric view of current path in a battery of eight submarine type cells.

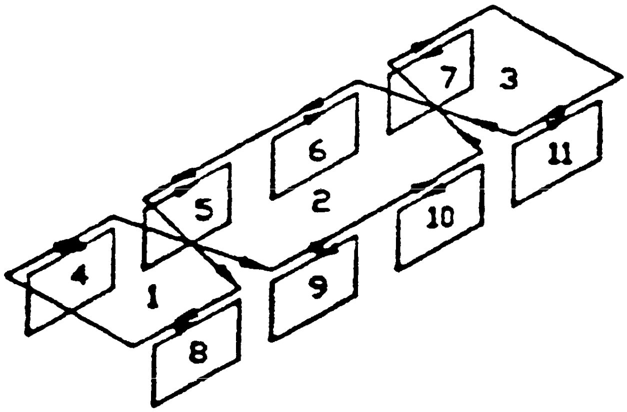


FIGURE 120. Current loops of a battery of eight submarine type cells.

8.1.2.3.6 Rows on different levels. In a submarine, the outboard rows of cells are higher than the inboard rows to conform to the shape of the submarine. Along the longitudinal centerline we may have two or more rows of adjacent cells on the same level. Farther out, adjacent rows of cells will be at different levels. Refer to figure 120 and suppose that the left-hand row of cells is raised bodily upward. Current loops 1 to 3 will be tilted out of the horizontal plane into an inclined plane, but their relative areas will remain unchanged and their magnetic fields will still be opposed. The vertical loops will remain vertical, and their magnetic fields will also be opposed. Thus the basic scheme used will give a small stray magnetic field regardless of whether the two rows of cells are on the same level or on different levels.

8.1.2.3.7 Preferred arrangement. If no restrictions were placed upon our choice, the preferred arrangement for submarine storage batteries would have 8 rows of cells on the same level, with 4N cells in each row, where N is an integral number, The rows would be grouped in four pairs of two adjacent rows, with cross-overs placed so that each pair of rows would give three current loops that are N-2N-N cells long. We would have a total of twelve current loops with polarities as shown on figure 121. The connections from row to row (not shown on figure 121) can be made all at one end of the battery, or partly at one end and partly at the other, in such a way as to give the horizontal loops the polarities shown on figure 121. The end connections should be as close together as possible to minimize the size of the current loops that they form.

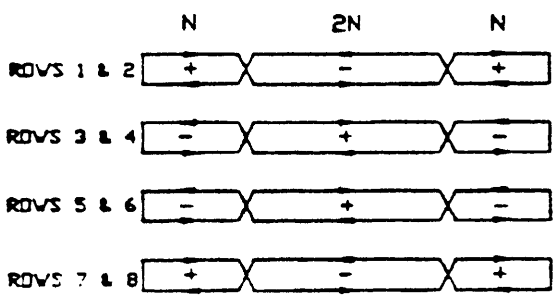


FIGURE 121. Schematic representation of Preferred arrangement for submarine storage battery.

8.1.2.3.8 Cancellation of magnetic field. The preferred arrangement inherently cancels the magnetic fields at four different levels as follows:

(a) Dipole level, Divide the middle loop of three horizontal loops formed by current flow in one pair of adjacent rows into two loops to give a total of four loops of equal magnitude. Model each of these loops with its equivalent dipole, which is permissible as a first approximation provided the loops are not too long, and the arrangement develops magnetic moments of the following signs:

Rows 1 and 2	+ - - +

The two dipoles on the left and the two dipoles on the right are equal and opposite; therefore, the arrangement cancels the magnetic field at the dipole level.

(b) Row level. With two pairs of opposing dipoles in a pair of rows, we have opposing magnetic fields, and the arrangement cancels the magnetic field at the row level.

(c) Two pairs of rows level. If we consider two pairs of rows, the configuration of their dipoles will be two sets of four dipoles eac , each set balanced in itself and two sets in opposition so that the magnetic field is cancelled at the two pairs of rows leve ,

Rows 1 and 2	+ - - +
Rows 3 and 4	- + + -

Note that this cancellation should be particularly effective because the two rows of dipoles are quite close together.

(d) Overall level. If we consider all the rows of the arrangement, the first two pairs of rows form a balanced set of eight dipoles and the last two pairs of rows do the same. These two sets will oppose and, therefore, the magnetic field is cancelled at the overall level.

Rows 1 and 2	+ - - +
Rows 3 and 4	- + + -
Rows 5 and 6	- + + -
Rows 7 and 8	+ - - +

(e) Vertical loops Similar cancellations will apply for the vertical loops shown on figure 120.

8.1.2.3.9 Advantages. The advantages of the preferred arrangement are its relative simplicity and the small magnetic field it will give. More complicated arrangements can be devised that will give a smaller magnetic field, but should be unnecessary unless for special reasons a small magnetic field is extremely important. Even in such cases, a careful study should be undertaken to make sure that the more complicated arrangement will actually give a worthwhile decrease in magnetic field

8.1.2.3.10 Modifications of preferred arrangement. We cannot use the preferred arrangement without modification unless the number of cells in the battery is an integral multiple of 32 (eight rows with 4N cells in each row), and there is sufficient space available to arrange a battery with eight rows all on the same level. Usually one or both of these conditions will not be satisfied and the preferred arrangement will have to be modified to fit the submarine.

8.1.2.3.10.1 Cells on different level. To conserve space, we arrange the battery in a submarine to conform to the shape of the hull. The central rows of cells are at the lowest level, and the outer rows at progressively higher levels. This will not result in quite as effective cancellation of magnetic field as if all rows were on the same level, but, as pointed out in 8.1.2.3.6 neither should it result in any great increase in magnetic field.

8.1.2.3.10.2 Number of cells. The preferred number of cells is an integral multiple of 32. Next in order of decreasing preference are integral multiples of 16, 8 and 4. An odd multiple of 2 is undesirable from the stray magnetic field standpoint. Take 126 cells, for example. The current loops in the battery will have a total length of 63 cells. By suitably positioned crossovers, we can give a plus polarity to current loops with a total length of 31 cells and a minus polarity to current loops with a total length of 32 cells. This creates a net magnetic moment corresponding to a current loop one cell long. At high rates of charge or discharge, this can give rise to a magnetic field large enough to be undesirable. For this reason, it is desirable that the total number of cells in the battery be a multiple of 4 if not a multiple of a higher power of 2. If the total number of cells in the battery is 4N, the total length of current loops will be 2N. This can be split by suitably placed crossovers into positive and negative loops that each have a total length of N cells. The net magnetic moment will be zero.

8.1.2.3.10.3 Arranging the battery. Space considerations may not permit a battery arrangement with eight rows of equal length. In such cases we must consider the best choice of number of rows, the grouping of the rows, and the number of cells in a row.

8.1.2.3.10.4 Number of rows. Our first choice for the number of rows is a multiple of 8; next, a multiple of 4; and last a multiple of 2. An odd number of rows should never be used except for extremely compelling reasons. An odd number of rows will inevitably result in a battery that is poorly arranged from the standpoint of small stray magnetic field.

8.1.2 3.10 5 Grouping of rows. It is preferable to have the battery arranged in groups of 8 adjacent rows of equal length; next in order of preference, groups of 4 adjacent rows of equal length. The reason for wanting to have groups of 4 adjacent rows of equal length is that with these we can have current loops as follows:

Loops in rows 1 and 2 --------- $+n_1$, $-n_2$, $+n_3$
Loops in rows 3 and 5 --------- $-n_1$, $+n_2$, $-n_3$

where n_1, n_2, and n_3 give the polarity and numbers of cells in the lengths of the current loops. Note that the net magnetic moment is zero whatever the values of n_1, n_2, n_3. Note that the current loops of rows 1 and 2 are quite close to the equal and opposite loops of rows 3 and 4, so that the cancellation of magnetic field should be good. If the battery has a total of 10 rows, for example, we will have one group of two rows left over. This is by no means fatal because by proper choice of n_1, n_2, and n_3, the three current loops formed by current flow in the two adjacent rows can be made quite small, but not as small as they would be if these three current loops were adjacent to three others that are equal and opposite. For this reason, battery arrangements with a total of 6, 10, 14, or 18 rows are less desirable from a stray magnetic field standpoint than battery arrangements with 4, 8, 12, or 16 rows. It may be necessary to use 6, 10, 14, or 18 rows because of space considerations.

8.1.2.3.10.6 Number of cells in a row. The preferred number is 4n where n is any integer. In this case the crossovers would be placed to form three loops n, 2n, and n cells long. Unfortunately, it is not always possible to have rows 4n cells long; hence, rows that are 4n + 1, 4n + 2, and 4n + 3 cells long must be considered. In such cases, the best positions of the crossovers are those that give current loops of the polarity and length (measured in number of cells) as follows:

$$4n+1 \text{ ----------- } +n,\ -2n,\ +(n+1)$$
$$4n+2 \text{ ----------- } +n,\ -(2n+1),\ +(n+1)$$
$$4n+3 \text{ ----------- } +(n+1),\ -(2n+1),\ +(n+1)$$

Note that it is undesirable if we have only two rows of cells with 4n + 1 or 4n + 3 cells in each row, then we have only three current loops and the net magnetic moment is not equal to zero. If, however, we have four adjacent rows of cells with 4n + 1 cells in each row, then we have six current loops with polarities and lengths as follows:

$$+n,\ -2n,\ +(n+1)$$
$$-n,\ +2n,\ -(n+1)$$

The net magnetic moment is zero. Similar considerations apply to rows 4n +3 cells long. With the arrangement suggested for rows 4n + 2 cells long, the net magnetic moment will be equal to zero whether there are only two rows of this length or four.

8.1.2.4 Cable runs. The batteries and cables carrying large currents in submarines must be so arranged that they do not form current loops that enclose steel bulkheads, masts, or other bodies of magnetic material (see 3.3.2.4). Cable runs from submarine storage batteries, and other cable runs in submarines that carry large currents, should be arranged as follows:

+ -		+ - - +
- +	+ - - +	- + + -

or in some equivalent fashion to minimize stray magnetic field. Twisting of the cable runs should not be necessary on submarines,

9. NOTES

9.1 Intended use. This handbook serves as a reference on the principles of small stray magnetic fields methods for determining the magnitude of stray magnetic fields are established and design guidelines are presented which may be implemented to reduce the stray magnetic fields on Naval ships.

9.2 Subject term (key word) listing.

Battery arrangements
Cable, concentric
Cable spiraling
Circuit, armature
Cluthes, induction
Coils, field
Collector rings, brush
Conductors
Dipoles
Dipoles, circular arrays of
Dipoles, radial
Dipoles, x, y, and z
Loops, current
Magnetic moment
Minesweeping
Shielding, ferromagnetic
Winding, armature

Preparing activity:
Navy - SH
(Project 1905-N015)

APPENDIX A

METHOD OF DERIVING MAGNETIC FIELD EQUATIONS USING TAYLOR'S SERIES EXPANSION

10. GENERAL

10.1 Scope. This appendix explains the Taylor's series expansion method for deriving equations for two or more magnetic dipoles, Tables of partial derivatives , which will be useful when deriving formulas for arrays of magnetic dipoles, are also provided. This appendix is not a mandatory part of this handbook. The information contained herein is for reference only.

20. APPLICABLE DOCUMENTS

This section is not applicable to this appendix

30. GENERAL INFORMATION

30.1 Magnetic field due to one X dipole. Consider a single X dipole located at point (a,b,c), The vertical (z) component of the magnetic field produced by this dipole at point (x,y,z) is given by the following expression:

$$B_z(x,y,z) = MF(x-a,y-b,z-c)$$

where:

M = Magnetic moment of the dipole in 10^{-7} ampere meters squared.

$F(x-a,y-b,z-c) = 3(x-a)(z-c)[(x-a)^2 + (y-b)^2 + (z-c)^2]^{-2.5}$

30.2 Calculations of magnetic field using Taylor's series expansion. When only a single X dipole is involved, the simplest way of computing the vertical component of the magnetic fiel at point (x,y,z) is to use the formula of 30 1. At times, however, it is necessary to find the magnetic field produced by an array of an even number of X dipoles grouped closely about the origin of the coordinate system so that the distance from the dipoles to the origin is small (one-tenth or less) as compared to the radial distance to the point where it is desired to compute the B_z component. In such cases it is advantageous to expand $F(x-a,y-b,z-c)$ in a Taylor's series. The Taylor's series expansion for a function of three variables is given as follows:

$$f(x+t,y+u,z+v) = f(x,y,z) + d_1[f(x,y,z)] + (1/2!)d_2[f(x,y,z)] + \ldots + (1/n!)d_n[f(x,y,z)]$$

$$= \sum_{n=0} \frac{d_n[f(x,y,z)]}{n!}$$

where:

$$d_n = \left[\frac{t\delta}{\delta x} + \frac{u\delta}{\delta y} + \frac{v\delta}{\delta z} \right]^n$$

For the expression of 30.1, the Taylor's series expansion is:

$$\begin{aligned} F(x-a,y-b,z-c) = F(x,y,z) &- (ap_x + bp_y + cp_z)F(x,y,z) \\ &+ (1/2)(ap_x + bp_y + cp_z)^2F(x,y,z) \\ &- (1/6)(ap_x + bp_y + cp_z)^3F(x,y,z) \\ &+ \text{higher order terms} \end{aligned}$$

Where p_x, p_y, and p_z indicate partial differentiation with respect to x, y, and respectively. The following notation is used to denote the results of partial differentiation:

$p_xF = F_x$	$p_x{}^2F = F_{xx}$	$p_x{}^3F = F_{xxx}$
$p_yF = F_y$	$p_xp_yF = F_{xy}$	$p_x{}^2p_yF = F_{xxy}$
$p_zF = F_z$	$p_y{}^2F = F_{yy}$	$p_xp_y{}^2F = F_{xyy}$
	$p_xp_zF = F_{xz}$	$p_y{}^3F = F_{yyy}$
	$p_yp_zF = F_{yz}$	$p_x{}^2p_zF = F_{xxz}$
	$p_z{}^2F = F_{zz}$	$p_xp_z{}^2F = F_{xzz}$
		$p_z{}^3F = F_{zzz}$
		$p_y{}^2p_zF = F_{yyz}$
		$p_yp_z{}^2F = F_{yzz}$
		$p_xp_yp_zF = F_{xyz}$

Tables XVI through XXII list equations for the partial derivatives of the magnetic fiel components of the X, Y, and Z dipoles. Table XVIII gives analytical expressions for the preceding functions. Notice that the above functions are given only up to the third partial, which represents order three. For our purposes, the contribution of those higher order terms will be negligible and, consequently, they will be ignored. However, if a specific application warrants the inclusion of other higher order terms, we can calculate those terms by taking the necessary partial derivatives of the third order equations.

40. SPECIFIC INFORMATION

40.1 Taylor's series approximation of the vertical component of the magnetic field due to four X dipoles. Consider an array of four X dipoles. The magnetic moments and coordinates of the dipoles are as follows:

Dipole	Magnetic moment	Coordinates of dipole
1	M	(0,b,c)
2	-M	(0,-b,c)
3	M	(0,-b,-c)
4	-M	(0,b,-c)

The vertical component of the magnetic field produced by each dipole at point (x,y,z) will be:

$$B_{z1}(x,y,z) = MF(x,y-b,z-c)$$

$$B_{z2}(x,y,z) = -MF(x,y+b,z-c)$$

$$B_{z3}(x,y,z) = MF(x,y+b,z+c)$$

$$B_{z4}(x,y,z) = -MF(x,y-b,z+c)$$

Using the equation from 30.1 and the abbreviations for the partial derivatives given in 30.2, the Taylor's series expansions are as follows:

$$B_{z1} = M[F - bF_y - cF_z + (1/2)b^2F_{yy} + bcF_{yz} + (1/2)c^2F_{zz} - (1/6)b^3F_{yyy}$$
$$- (1/2)b^2cF_{yyz} - (1/2)bc^2F_{yzz} - (1/6)c^3F_{zzz} + \text{higher order terms}]$$

$$B_{z2} = -M[F + bF_y - cF_z + (1/2)b^2F_{yy} - bcF_{yz} + (1/2)c^2F_{zz} + (1/6)b^3F_{yyy}$$
$$- (1/2)b^2cF_{yyz} + (1/2)bc^2F_{yzz} - (1/6)c^3F_{zzz} + \text{higher order terms}]$$

$$B_{z3} = M[F + bF_y + cF_z + (1/2)b^2F_{yy} + bcF_{yz} + (1/2)c^2F_{zz} + (1/6)b^3F_{yyy}$$
$$+ (1/2)b^2cF_{yyz} + (1/2)bc^2F_{yzz} + (1/6)c^3F_{zzz} + \text{higher order terms}]$$

$$B_{z4} = -M[F - bF_y + cF_z + (1/2)b^2F_{yy} - bcF_{yz} + (1/2)c^2F_{zz} - (1/6)b^3F_{yyy}$$
$$+ (1/2)b^2cF_{yyz} - (1/2)bc^2F_{yzz} + (1/6)c^3F_{zzz} + \text{higher order terms}]$$

Summing these values, the total magnetic field measured at point (x,y,z) is:

$$B_z = B_{z1} + B_{z2} + B_{z3} + B_{z4} = M\ 4bcF_{yz} + \text{higher order terms}$$

$$= M\ (4bc)(3)(-5x^3y - 5xy^3 + 30xyz^2)(x^2 + y^2 + z^2)^{4.5}$$

$$= 12\ (Mbc(-5x^3y - 5xy^3 + 30xyz^2)(x^2 + y^2 + z^2)^{-4.5}$$

In comparison, the equation for the vertical component of the magnetic field of a single X dipole located at point (a,b,c) and measured at point (x,y,z) is as follows:

In each case M is given in 10^{-7} Am^2. The above example illustrates the utility of the Taylor's series expansion. For approximate calculations, the higher order terms in the equation can be ignored because they will be small in comparison to the retained terms, provided that the radial distance from the dipoles to the point at which the magnetic field is measured is at least ten times greater than b or c. In order to compute the vertical component of the magnetic field produced by the array of four dipoles we no longer need to compute the field produced by each dipole separately and take the sum. Instead, a single formula gives the sum directly This has a number of advantages. By taking the equation and making use of

$$k^2 = \frac{x^2 + y^2}{z^2}$$

the equation becomes

$$B_z = 60 \text{ Mbc } xyz^{-7} (6 - k^2)(1 + k^2)^{-4.5}$$

Using this equation it is easy to see the general character of the vertical component of the magnetic field on a plane z meters below the center of the array. Inspection of this equation shows that the vertical component of the magnetic field is zero on the two straight lines, x = 0 and y = 0, and on a circle

$$k^2 = \frac{x^2}{z^2} + \frac{y^2}{z^2} = 6$$

The above advantages are bought at the price of using an approximation instead of an exact expression for the vertical component of the magnetic field. In most cases , however, the approximation is quite good and the advantages are well worth the price that is paid,

40.1.1 Example. Consider the array of four X dipoles given in 40.1. As an example, take M - 100,000 Am^2, a - 0 m, b - 0.500, c = 0.250 m and calculate the vertical component of the magnetic field at x - 3.66 m, y - 3.66 m, and z = 10.00 m. This is one of four places where the vertical component of the magnetic field reaches its greatest magnitude on a plane 10 m below the center of the array. The exact value is obtained by using the equation of 30.1 co compute the field produced by each dipole separately and adding the results to yield the total vertical component. This is then compared to the approximate value computed by using the equation of 40.1. The results are as follows:

(a) Exact value

B_{z1} = 70.12 mG

B_{z2} = -60.36 mG

B_{z3} = 52.40 mG

B_{z4} = -60.19 mG

B_z = 1.97 mG

(b) Approximate value

B_z = 1.98 mG

40.2 Using Taylor's series expansion to compute x and y components of magnetic field of an array of four X dipoles. The x and y components of the magnetic field produced by an array of four X dipoles are computed in a similar fashion to that of the vertical component. The difference lies in the specific table of partial derivatives to be used when computing the Taylor's series expansion. When computing the x component we follow the same procedure as mentioned in 40.1 with the exception that the D partials of table XVI would be used. For the v component, the same procedure mentioned above would be used except that the E partials table XVII would be used.

40.3 Taylor's series expansion for arrays of Y or Z dipoles.

40.3.1 Y dipoles, In computing the magnetic field produced by an array of Y dipoles, we follow a procedure similar co that mentioned in 40.1 or 40.2, depending upon the number of dipoles in the array and the component of the magnetic field desired. Tables XIX, XX, and XXI contain the equations of the partial derivatives of the x, y, and z components, respectively.

40.3.2 Z dipoles. In computing the magnetic field produced by an array of Z dipoles, we follow a procedure similar to that mentioned in 40.1 or 40.2, depending upon the number of dipoles in the array and the component of the magnetic field desired. Tables XVIII, XXI, and XXII contain the equations of the partial derivatives of the x, y, and z components, respectively.

40.4 Circular arrays of radial dipoles in the xz plane.

40.4.1 Single radial dipole. Consider an array of radial dipoles on a circle in the xz plane which has a radius equal to r and a center at the origin of the coordinate system. Pick out a particular dipole on a radial line from the origin that makes an angle θ with the positive x axis. Let M be the magnetic moment that will be along the radius. To find the vertical component of the magnetic field produced by this dipole we proceed as follows:

(a) Resolve the magnetic moment into two components, $M_x = M \cos\theta$ and $M_y = M \sin\theta$.

(b) Using M_x and the rectangular coordinates of the dipole, a = r cosθ, b = 0, and c = r sinθ, calculate the Taylor's series expansion for the vertical component of magnetic field produced by the z component of the magnetic moment,

(c) Using M_z and the rectangular components of the dipole, a = r cosθ, b = 0, - rsinθ, calculate the Taylor s series expansion. for the vertical component of the magnetic field produced by the z component of magnetic moment.

(d) Add the results obtained in the two preceding steps to obtain the total vertical component of the magnetic field produced by the radial dipole.

The result of carrying out the preceding steps and making some algebraic and trigonometric (see table XXIII) reductions is as follows:

$$
\begin{aligned}
B_z = {} & MF\cos\theta - (1/2)Mr[F_x\{1 + \cos(2\theta)\} + F_z\sin(2\theta)] \\
& + (1/8)Mr^2[F_{xx}\{3\cos\theta + \cos(3\theta)\} + 2F_{xz}\{\sin\theta + \sin(3\theta)\} \\
& \qquad + F_{zz}\{\cos\theta - \cos(3\theta)\}] \\
& - (1/48)Mr^3[F_{xxx}\{3 + 4\cos(2\theta) + \cos(4\theta)\} \\
& \qquad + 3F_{xxz}\{2\sin(2\theta) + \sin(4\theta)\} + 3F_{xzz}\{1 - \cos(4\theta)\} \\
& \qquad + F_{zzz}\{2\sin(2\theta) + \sin(4\theta)\}] \\
& + MJ\sin\theta - (1/6)Mr[J_x\sin(2\theta) + J_z\{1 - \cos(2\theta)\}] \\
& + (1/24)Mr^2[J_{xx}\{\sin\theta + \sin(3\theta)\} + 2J_{xz}(\cos\theta - \cos(3\theta)\} \\
& \qquad + J_{zz}\{\sin\theta - \sin(3\theta)\}] \\
& - (1/144)Mr^3[J_{xxx}\{2\sin(2\theta) + \sin(4\theta)\} + 3J_{xxz}\{1 - \cos(4\theta)\} \\
& \qquad + 3J_{xzz}\{2\sin(2\theta) - \sin(4\theta)\} \\
& \qquad + J_{zzz}\{3 - 4\cos(2\theta) + \cos(4\theta)\}] \\
& + \text{higher order terms}
\end{aligned}
$$

40.4.2 An array of radial dipoles. To find the vertical component of the magnetic field produced at point (x,y,z) by a circular array of four, six, or eight dipoles in the xz plane:

(a) Write out the Taylor's series expansion for each dipole, using the appropriate angle and sign for the moment of the dipole.

(b) Add the Taylor's series expansions.

(c) The lower terms in z^{-1} will cancel out. The first term that does not vanish will involve the partial derivatives and tables according to 1, 2, and 3 below. Refer to the tables for the values of the derivatives in terms of x, y, and z and substitute them into the first term that does not vanish, The result will be an approximate formula that gives the field produced by the array in terms of M, r, and x, y, and z.

(1) For the x component, the partial derivatives of D and F found in tables XVI and XVIII, respectively, are used.
(2) For the y component, the partial derivatives of E and I found in tables XVII and XXI, respectively, are used.
(3) For the z component, the partial derivatives of F and J found in tables XVIII and XXII, respectively, are used.

40.5 Circular arrays of radial dipoles in the XY plane.

40.5.1 Single radial dipole. Consider a single radial dipole in the xy plane at a distance r from the origin on a radial line that makes an angle ϕ with the positive x direction. For this dipole $a = r\cos\phi$, $b = r\sin\phi$, and $c = 0$.

(a) Resolve the magnetic moment into its x and y components:

$$M_x = M\cos\phi \qquad M_y = M\sin\phi$$

(b) Write out the Taylor's series expansion for each component and add them. The result of carrying out the preceding steps is as follows:

$$\begin{aligned}
B_z = {} & MF\cos\phi - (1/2)Mr[F_x(1 + \cos(2\phi)) + F_y\sin(2\phi)] \\
& + (1/8)Mr^2[F_{xx}(3\cos\phi + \cos(3\phi)) + 2F_{xy}(\sin\phi + \sin(3\phi)) \\
& \qquad + F_{yy}(\cos\phi - \cos(3\phi))] \\
& - (1/48)Mr^3[F_{xxx}(3 + 4\cos(2\phi)) + 3F_{xxy}(2\sin(2\phi) \\
& \qquad + \sin(4\phi)) + 3F_{xyy}(1 - \cos(4\phi)) \\
& \qquad + F_{yyy}(2\sin(2\phi) - \sin(4\phi))] \\
& + MI\sin\phi - (1/2)Mr[I_x\sin(2\phi) + I_y(1 - \cos(2\phi))] \\
& + (1/8)\,Mr^2[I_{xx}(\sin\phi + \sin(3\phi)) + 2I_{xy}(\cos\phi - \cos(3\phi)) \\
& \qquad + I_{yy}(3\sin\phi - \sin(3\phi))]
\end{aligned}$$

$$- (1/48)\ Mr^3\{I_{xxx}\{2\sin(2\phi) + \sin(4\phi)\} + 3I_{xxy}\{1 - \cos(4\phi)\} + 3I_{xyy}\{2\sin(2\phi) - \sin(4\phi)\} + I_{yyy}\{3 - 4\cos(2\phi) + \cos(4\phi)\}\}$$

+ higher order terms

40.5.2 An array of radial dipoles. To find the vertical component of the magnetic field produced at point (x,y,z) by a circular array of four, six, or eight dipoles in the xy plane:

(a) Write out the Taylor's series expansion for each dipole, using the appropriate angle and sign for the moment of the dipole
(b) Add the Taylor's series expansions.
(c) The lower terms in z^{-1} will cancel out. The first term that does not vanish will involve the partial derivatives and cables according to 1, 2, and 3 below. Refer to the tables for the values of the derivatives in terms of x, y, and z and substitute them into the first term that does not vanish. The result will be an appropriate formula that gives the field produced by the array in terms of M, r, and x, y, and z

(1) For the x component, the partial derivatives of D and G found in tables XVI and XIX, respectively, are used.
(2) For the y component, the partial derivatives of E and H found in tables XVII and XX, respectively, are used.
(3) For the z component, the partial derivatives of F and I found in tables XVIII and XXI, respectively. are used.

TABLE XVI. Partial derivatives for x magnetic field component of X dipole

D	=	$(2x^2-y^2-z^2)(x^2+y^2+z^2)^{-2.5}$
D_x	=	$(-6x^3+9xy^2+9xz^2)(x^2+y^2+z^2)^{-3.5}$
D_y	=	$(-12x^2y+3y^3+3yz^2)(x^2+y^2+z^2)^{-3.5}$
D_z	=	$(-12x^2z+3y^2z+3z^3)(x^2+y^2+z^2)^{-3.5}$
D_{xx}	=	$(24x^4-72x^2y^2-72x^2z^2+9y^4+18y^2z^2+9z^4)(x^2+y^2+z^2)^{-4.5}$
D_{xy}	=	$(60x^3y-45xy^3-45xyz^2)(x^2+y^2+z^2)^{-4.5}$
D_{yy}	=	$(-12x^4+81x^2y^2-9x^2z^2-12y^4-9y^2z^2+3z^4)(x^2+y^2+z^2)^{-4.5}$
D_{xz}	=	$(60x^3z-45xy^2z-45xz^3)(x^2+y^2+z^2)^{-4.5}$
D_{yz}	=	$(90x^2yz-15y^3z-15yz^3)(x^2+y^2+z^2)^{-4.5}$
D_{zz}	=	$(-12x^4-9x^2y^2+81x^2z^2+3y^4-9y^2z^2-12y^4)(x^2+y^2+z^2)^{-4.5}$
D_{xxx}	=	$(-120x^5+600x^3y^2+600x^3z^2-225xy^4-450xy^2z^2-225xz^4)(x^2+y^2+z^2)^{-5.5}$
D_{xxy}	=	$(-360x^4y+540x^2y^3+540x^2yz^2-45y^5-90y^3z^2-45yz^4(x^2+y^2+z^2)^{-5.5}$
D_{xyy}	=	$(60x^5-615x^3y^2+15x^3z^2+270xy^4+225xy^2z^2-45xz^4)(x^2+y^2+z^2)^{-5.5}$
D_{yyy}	=	$(270x^4y-615x^2y^3225x^2yz^2+60y^5+15y^3z^2-45yz^4)(x^2+y^2+z^2)^{-5.5}$
D_{xxz}	=	$(-360x^4z+540x^2y^2z+540x^2z^3-45y^4z-90y^2z^3-45z^5)(x^2+y^2+z^2)^{-5.5}$
D_{xzz}	=	$(60x^5+15x^3y^2-615x^3z^2-45xy^4+225xy^2z^2+270xz^4)(x^2+y^2+z^2)^{-5.5}$
D_{yyz}	=	$(90x^4z-765x^2y^2z+75x^2z^3+90y^4z+75y^2z^3-15z^5)(x^2+y^2+z^2)^{-5.5}$
D_{yzz}	=	$(90x^4y+75x^2y^3-765x^2y^2z-15y^5+75y^3z^2+90yz^4)(x^2+y^2+z^2)^{-5.5}$
D_{zzz}	=	$(270x^4z+225x^2y^2z-615x^2z^3-45y^4z+15y^2z^3+60z^5)(x^2+y^2+z^2)^{-5.5}$
D_{xyz}	=	$(-630x^3yz+315xy^3z+315xyz^3)(x^2+y^2+z^2)^{-5.5}$

TABLE XVII. Partial derivatives for y magnetic field component of X dipole.

$$E = 3xy(x^2+y^2+z^2)^{-2.5}$$

$$E_x = 3(-4x^2y+y^3+yz^2)(x^2+y^2+z^2)^{-3.5}$$

$$E_y = 3(x^3-4xy^2+xz^2)(x^2+y^2+z^2)^{-3.5}$$

$$E_z = 3(-5xyz)(x^2+y^2+z^2)^{-3.5}$$

$$E_{xx} = 3(20x^3y-15xy^3-15xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$E_{xy} = 3(-4x^4+27x^2y^2-3x^2z^2-4y^4-3y^2z^2+z^4)(x^2+y^2+z^2)^{-4.5}$$

$$E_{yy} = 3(-15x^3y+20xy^3-15xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$E_{xz} = 3(30x^2yz-5y^3z-5yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$E_{yz} = 3(-5x^3z+30xy^2z-5xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$E_{zz} = 3(-5x^3y-5xy^3+30xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$E_{xxx} = 3(-120x^4y+180x^2y^3+180x^2yz^2-15y^5-30y^3z^2-15yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$E_{xxy} = 3(20x^5-205x^3y^2+5x^3z^2+90xy^4+75xy^2z^2-15xz^4(x^2+y^2+z^2)^{-5.5}$$

$$E_{xyy} = 3(90x^4y-205x^2y^3+75x^2yz^2+20y^5+5y^3z^2-15yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$E_{yyy} = 3(-15x^5+180x^3y^2-30x^3z^2-120xy^4+180xy^2z^2-15xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$E_{xxz} = 3(-210x^3yz+105xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$E_{xzz} = 3(30x^4y+25x^2y^3-255x^2yz^2-5y^5+25y^3z^2+30yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$E_{yyz} = 3(105x^3yz-210xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$E_{yzz} = 3(-5x^5+25x^3y^2+25x^3z^2+30xy^4-255xy^2z^2+30xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$E_{zzz} = 3(105x^3yz+105xy^3z-210xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$E_{xyz} = 3(30x^4z-255x^2y^2z+25x^2z^3+30y^4z+25y^2z^3-5z^5)(x^2+y^2+z^2)^{-5.5}$$

TABLE XVIII Partial derivatives for z magnetic field component of X dipole and x magnetic field component of Z dipole.

$$F = 3xz(x^2+y^2+z^2)^{-2.5}$$

$$F_x = 3(-4x^2z+y^2z+z^3)(x^2+y^2+z^2)^{-3.5}$$

$$F_y = 3(-5xyz)(x^2+y^2+z^2)^{-3.5}$$

$$F_z = 3(x^3+xy^2-4xz^2)(x^2+y^2+z^2)^{-3.5}$$

$$F_{xx} = 3(20x^3z-15xy^2z-15xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$F_{xy} = 3(30x^2yz-5y^3z-5yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$F_{yy} = 3(-5x^3z+30xy^2z-5xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$F_{xz} = 3(-4x^4-3x^2y^2+27x^2z^2+y^4-3y^2z^2-4z^4)(x^2+y^2+z^2)^{-4.5}$$

$$F_{yz} = 3(-5x^3y-5xy^3+30xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$F_{zz} = 3(-15x^3z-15xy^2z+20xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$F_{xxx} = 3(-120x^4z+180x^2y^2z+180x^2z^3-15y^4z-30y^2z^3-15z^5)(x^2+y^2+z^2)^{-5.5}$$

$$F_{xxy} = 3(-210x^3yz+105xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$F_{xyy} = 3(30x^4z-255x^2y^2z+25x^2z^3+30y^4z+25y^2z^3-5z^5)(x^2+y^2+z^2)^{-5.5}$$

$$F_{yyy} = 3(105x^3yz-210xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$F_{xxz} = 3(20x^5+5x^3y^2-205x^3z^2-15xy^4+75xy^2z^2+90xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$F_{xzz} = 3(90x^4z+75x^2y^2z-205x^2z^3-15y^4z+5y^2z^3+20z^5)(x^2+y^2+z^2)^{-5.5}$$

$$F_{yyz} = 3(-5x^5+25x^3y^2+25x^3z^2+30xy^4-255xy^2z^2+30xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$F_{yzz} = 3(105x^3yz+105xy^3z-210xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$F_{zzz} = 3(-15x^5-30x^3y^2+180x^3z^2-15xy^4+180xy^2z^2-120xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$F_{xyz} = 3(30x^4y+25x^2y^3-255x^2yz^2-5y^5+25y^3z^2+30yz^4)(x^2+y^2+z^2)^{-5.5}$$

TABLE XIX. Partial derivatives for x magnetic field component of Y dipole.

G	=	$-3xy(x^2+y^2+z^2)^{-2.5}$
G_x	=	$-3(-4x^2y+y^3+yz^2)(x^2+y^2+z^2)^{-3.5}$
G_y	=	$-3(x^3-4xy^2+xz^2)(x^2+y^2+z^2)^{-3.5}$
G_z	=	$-3(-5xyz)(x^2+y^2+z^2)^{-3.5}$
G_{xx}	=	$-3(20x^3y-15xy^3-15xyz^2)(x^2+y^2+z^2)^{-4.5}$
G_{xy}	=	$-3(-4x^4+27x^2y^2-3x^2z^2-4y^4-3y^2z^2+z^4)(x^2+y^2+z^2)^{-4.5}$
G_{yy}	=	$-3(-15x^3y+20xy^3-15xyz^2)(x^2+y^2+z^2)^{-4.5}$
G_{xz}	=	$-3(30x^2yz-5y^3z-5yz^3)(x^2+y^2+z^2)^{-4.5}$
G_{yz}	=	$-3(-5x^3z+30xy^2z-5xz^3)(x^2+y^2+z^2)^{-4.5}$
G_{zz}	=	$-3(-5x^3y-5xy^3+30xyz^2)(x^2+y^2+z^2)^{-4.5}$
G_{xxx}	=	$-3(-120x^4y+180x^2y^3+180x^2yz^2-15y^5-30y^3z^2-15yz^4)(x^2+y^2+z^2)^{-5.5}$
G_{xxy}	=	$-3(20x^5-205x^3y^2+5x^3z^2+90xy^4+75xy^2z^2-15xz^4)(x^2+y^2+z^2)^{-5.5}$
G_{xyy}	=	$-3(90x^4y-205x^2y^3+75x^2yz^2+20y^5+5y^3z^2-15yz^4)(x^2+y^2+z^2)^{-5.5}$
G_{yyy}	=	$-3(-15x^5+180x^3y^2-30x^3z^2-120xy^4+180xy^2z^2-15xz^4)(x^2+y^2+z^2)^{-5.5}$
G_{xxy}	=	$-3(-210x^3yz+105xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$
G_{xzz}	=	$-3(30x^4y+25x^2y^3-255x^2yz^2-5y^5+25y^3z^2+30yz^4)(x^2+y^2+z^2)^{-5.5}$
G_{yyz}	=	$-3(105x^3yz-210xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$
G_{yzz}	=	$-3(-5x^5+25x^3y^2+25x^3z^2+30xy^4-255xy^2z^2+30xz^4)(x^2+y^2+z^2)^{-5.5}$
G_{zzz}	=	$-3(105x^3yz+105xy^3z-210xyz^3)(x^2+y^2+z^2)^{-5.5}$
G_{xyz}	=	$-3(30x^4z-255x^2y^2z+25x^2z^3+30y^4z+25y^2z^3-5z^5)(x^2+y^2+z^2)^{-5.5}$

TABLE XX. Partial derivatives for y magnetic field component of Y dipole.

H	=	$(-x^2+2y^2-z^2)(x^2+y^2+z^2)^{-2.5}$
H_x	=	$(3x^3-12xy^2+3xz^2)(x^2+y^2+z^2)^{-3.5}$
H_y	=	$(9x^2y-6y^3+9yz^2)(x^2+y^2+z^2)^{-3.5}$
H_z	=	$(3x^2z-12y^2z+3z^3)(x^2+y^2+z^2)^{-3.5}$
H_{xx}	=	$(-12x^4+81x^2y^2-9x^2z^2-12y^4-9y^2z^2+3z^4)(x^2+y^2+z^2)^{-4.5}$
H_{xy}	=	$(-45x^3y+60xy^3-45xyz^2)(x^2+y^2+z^2)^{-4.5}$
H_{yy}	=	$(9x^4-72x^2y^2+18x^2z^2-72y^2z^2+24y^4+9z^4)(x^2+y^2+z^2)^{-4.5}$
H_{xz}	=	$(-15x^3z+90xy^2z-15xz^3)(x^2+y^2+z^2)^{-4.5}$
H_{yz}	=	$(-45x^2yz+60y^3z-45yz^3)(x^2+y^2+z^2)^{-4.5}$
H_{zz}	=	$(3x^4-9x^2y^2-9x^2z^2-12y^4+81y^2z^2-12z^4)(x^2+y^2+z^2)^{-4.5}$
H_{xxx}	=	$(60x^5-615x^3y^2+15x^3z^2+270xy^4+225xy^2z^2-45xz^4)(x^2+y^2+z^2)^{-5.5}$
H_{xxy}	=	$(270x^4y-615x^2y^3225x^2yz^2+60y^5+15y^3z^2-45yz^4)(x^2+y^2+z^2)^{-5.5}$
H_{xyy}	=	$(-45x^5+540x^3y^2-90x^3z^2-360xy^4+540xy^2z^2-45xz^4)(x^2+y^2+z^2)^{-5.5}$
H_{yyy}	=	$(-225x^4y+600x^2y^3-450x^2yz^2-120y^5+600y^3z^2-225yz^4)(x^2+y^2+z^2)^{-5.5}$
H_{xxz}	=	$(90x^4z-765x^2y^2z+75x^2y^3+90y^4z+75y^2z^3-15z^5)(x^2+y^2+z^2)^{-5.5}$
H_{xzz}	=	$(-15x^5+75x^3y^2+75x^3z^2+90xy^4-765xy^2z^2+90xz^4)(x^2+y^2+z^2)^{-5.5}$
H_{yyz}	=	$(-45x^4z+540x^2y^2z-90x^2z^3-360y^4z+540y^2z^3-45z^5)(x^2+y^2+z^2)^{-5.5}$
H_{yzz}	=	$(-45x^4y+15x^2y^3+225x^2yz^2+60y^5-615y^3z^2+270yz^4)(x^2+y^2+z^2)^{-5.5}$
H_{zzz}	=	$(-45x^4z+225x^2y^2z+15x^2z^3+270y^4z-615y^2z^3+60z^5)(x^2+y^2+z^2)^{-5.5}$
H_{xyz}	=	$(315x^3yz-630xy^3z+315xyz^3)(x^2+y^2+z^2)^{-5.5}$

TABLE XXI. Partial derivatives for z magnetic field component of Y dipole and y magnetic field component of Z dipole.

$$I = 3yz(x^2+y^2+z^2)^{-2.5}$$

$$I_x = 3(-5xyz)(x^2+y^2+z^2)^{-3.5}$$

$$I_y = 3(x^2z-4y^2z+z^3)(x^2+y^2+z^2)^{-3.5}$$

$$I_z = 3(x^2y+y^3-4yz^2)(x^2+y^2+z^2)^{-3.5}$$

$$I_{xx} = 3(30x^2yz-5y^3z-5yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$I_{xy} = 3(-5x^3z+30xy^2z-5xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$I_{yy} = 3(-15x^2yz+20y^3z-15yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$I_{xz} = 3(-5x^3y-5xy^3+30xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$I_{yz} = 3(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)(x^2+y^2+z^2)^{-4.5}$$

$$I_{zz} = 3(-15x^2yz-15y^3z+20yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$I_{xxx} = 3(-210x^3yz+105xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$I_{xxy} = 3(30x^4z-255x^2y^2z+25x^2z^3+30y^4z+25y^2z^3-5z^5)(x^2+y^2+z^2)^{-5.5}$$

$$I_{xyy} = 3(105x^3yz-210xy^3z+105xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$I_{yyy} = 3(-15x^4z+180x^2y^2z-30x^2z^3-120y^4z+180y^2z^3-15z^5)(x^2+y^2+z^2)^{-5.5}$$

$$I_{xxz} = 3(30x^4y+25x^2y^3-255x^2yz^2-5y^5+25y^3z^2+30yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$I_{xzz} = 3(105x^3yz+105xy^3z-210xyz^3)(x^2+y^2+z^2)^{-5.5}$$

$$I_{yyz} = 3(-15x^4y+5x^2y^3+75x^2yz^2+20y^5-205y^3z^2+90yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$I_{yzz} = 3(-15x^4z+75x^2y^2z+5x^2z^3+90y^4z-205y^2z^3+20z^5)(x^2+y^2+z^2)^{-5.5}$$

$$I_{zzz} = 3(-15x^4y-30x^2y^3+180x^2yz^2-15y^5+180y^3z^2-120yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$I_{xyz} = 3(-5x^5+25x^3y^2+25x^3z^2+30xy^4-255xy^2z^2+30xz^4)(x^2+y^2+z^2)^{-5.5}$$

TABLE XXII. Partial derivatives for z magnetic field component of Z dipole

$$J = (-x^2-y^2+2z^2)(x^2+y^2+z^2)^{-2.5}$$

$$J_x = (3x^3+3xy^2-12xz^2)(x^2+y^2+z^2)^{-3.5}$$

$$J_y = (3x^2y+3y^3-12yz^2)(x^2+y^2+z^2)^{-3.5}$$

$$J_z = (9x^2z+9y^2z-6z^3)(x^2+y^2+z^2)^{-3.5}$$

$$J_{xx} = (-12x^4-9x^2y^2+3y^4+81x^2z^2-9y^2z^2-12z^4)(x^2+y^2+z^2)^{-4.5}$$

$$J_{xy} = (-15x^3y-15xy^3+90xyz^2)(x^2+y^2+z^2)^{-4.5}$$

$$J_{yy} = (3x^4-9x^2y^2-9x^2z^2-12y^4+81y^2z^2-12z^4)(x^2+y^2+z^2)^{-4.5}$$

$$J_{xz} = (-45x^3z-45xy^2z+60xz^3)(x^2+y^2+z^2)^{-4.5}$$

$$J_{yz} = (-45x^2yz-45y^3z+60yz^3)(x^2+y^2+z^2)^{-4.5}$$

$$J_{zz} = (9x^4+18x^2y^2+9y^4-72x^2z^2-72y^2z^2+24z^4)(x^2+y^2+z^2)^{-4.5}$$

$$J_{xxx} = (60x^5+15x^3y^2-615x^3z^2-45xy^4+225xy^2z^2+270xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{xxy} = (90x^4y+75x^2y^3-15y^5-765x^2yz^2+75y^3z^2+90yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{xyy} = (-15x^5+75x^3y^2+90xy^4-765xy^2z^2+75x^3z^2+90xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{yyy} = (-45x^4y+15x^2y^3+225x^2yz^2+60y^5-615y^3z^2+270yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{xxz} = (270x^4z+225x^2y^2z-615x^2z^3-45y^4z+15y^2z^3+60z^5)(x^2+y^2+z^2)^{-5.5}$$

$$J_{xzz} = (-45x^5-90x^3y^2-45xy^4+540x^3z^2+540xy^2z^2-360xz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{yyz} = (-45x^4z+225x^2y^2z+15x^2z^3+270y^4z-615y^2z^3+60z^5)(x^2+y^2+z^2)^{-5.5}$$

$$J_{yzz} = (-45x^4y-90x^2y^3+540x^2yz^2-45y^5+540y^3z^2-360yz^4)(x^2+y^2+z^2)^{-5.5}$$

$$J_{zzz} = (-225x^4z-450x^2y^2z+600x^2z^3-225y^4z+600y^2z^3-120z^5)(x^2+y^2+z^2)^{-5.5}$$

$$J_{xyz} = (315x^3yz+315xy^3z-630xyz^3)(x^2+y^2+z^2)^{-5.5}$$

TABLE XXIII. Trigonometric identities.

$2\cos^2\theta$	=	$1 + \cos(2\theta)$
$2\sin\theta\cos\theta$	=	$\sin(2\theta)$
$2\sin^2\theta$	=	$1 - \cos(2\theta)$
$4\cos^3\theta$	=	$3\cos\theta + \cos(3\theta)$
$4\cos^2\theta\sin\theta$	=	$\sin\theta + \sin(3\theta)$
$4\cos\theta\sin^2\theta$	=	$\cos\theta - \cos(3\theta)$
$4\sin^3\theta$	=	$3\sin\theta - \sin(3\theta)$
$8\cos^4\theta$	=	$3 + 4\cos(2\theta) + \cos(4\theta)$
$8\cos^3\theta\sin\theta$	=	$2\sin(2\theta) + \sin(4\theta)$
$8\cos^2\theta\sin^2\theta$	=	$1 - \cos(4\theta)$
$8\cos\theta\sin^3\theta$	=	$2\sin(2\theta) - \sin(4\theta)$
$8\sin^4\theta$	=	$3 - 4\cos(2\theta) + \cos(4\theta)$

APPENDIX B

LIST OF FORMULAS USED IN MIL-HDBK-XX43

10. GENERAL

10.1 Scope. This appendix provides a listing of all of the mathematical formulas used in MIL-HDBK-XX43. Included in the list are the equation number, a brief description of the equation, and the page number where it can be found. This appendix is not a mandatory part of this handbook. The information contained herein is for reference only.

20. APPLICABLE DOCUMENTS

This section is not applicable to this appendix.

30. GENERAL INFORMATION

This section is not applicable to this appendix.

40. SPECIFIC INFORMATION

40.1 List of equations. The equations used in MIL-HDBK-XX43 are listed in table XXIV.

TABLE XXIV. List of equations.

Equation number	Description of equation	Page number
3-1	Shielding effectiveness	6
3-2	Approximation of relative magnetic permeability from magnetic fields	6
3-3	Shielding effectiveness using relative magnetic permeability	6
3-4	Magnetic field of an infinitely long straight wire	8
3-5	Magnetic field of a finite length straight wire	9
3-6	$\sin\theta_1$	10
3-7	$\sin\theta_2$	10
3-8	Magnetic field of a rectangular loop	10
3-9	Magnetic field at the center of a circular loop	11
3-10	Magnetic field along the central axis of a circular loop	11
4-1	Magnetic dipole moment of current loops of equal area	19

TABLE XXIV List of equations - Continued.

TABLE XXIV. List of equations - Continued.

TABLE XXIV. List of equations - Continued.

TABLE XXIV List of equations - Continued.

TABLE XXIV. List of equations - Continued.

Equation number	Description of equation	Page number
4-83	The y component of the magnetic field of four X dipoles on the xz plane	53
4-84	The z component of the magnetic field of four X dipoles on the xz plane	53
4-85	The x component of the magnetic field of four Z dipoles on the x axis	54
4-86	The y component of the magnetic field of four Z dipoles on the x axis	54
4-87	The z component of the magnetic field of four Z dipoles on the x axis	54
4-88	The x component of the magnetic field of four Z dipoles on the z axis - case 1	55
4-89	The y component of the magnetic field of four Z dipoles on the z axis - case 1	55
4-90	The z component of the magnetic field of four Z dipoles on the z axis - case 1	55
4-91	The x component of the magnetic field of four Z dipoles on the z axis - case 2	56
4-92	The y component of the magnetic field of four Z dipoles on the z axis - case 2	56
4-93	The z component of the magnetic field of four Z dipoles on the z axis - case 2	56
4-94	The x component of the magnetic field of four Z dipoles on the xy plane	57
4-95	The y component of the magnetic field of four Z dipoles on the xy plane	57
4-96	The z component of the magnetic field of four Z dipoles on the xy plane	57
4-97	The x component of the magnetic field of four Z dipoles on the yz plane	58
4-98	The y component of the magnetic field of four Z dipoles on the yz plane	58
4-99	The z component of the magnetic field of four Z dipoles on the yz plane	58
4-100	The x component of the magnetic field of four radial dipoles on the xz plane - case 1	59
4-101	The y component of the magnetic field of four radial dipoles on the xz plane - case 1	59
4-102	The z component of the magnetic field of four radial dipoles on the xz plane - case 1	59

TABLE XXIV List of equations - Continued.

Equation number	Description of equation	Page number
4-103	The x component of the magnetic field of four radial dipoles on the xz plane - case 2	60
4-104	The y component of the magnetic field of four radial dipoles on the xz plane - case 2	60
4-105	The z component of the magnetic field of four radial dipoles on the xz plane - case 2	60
4-106	The x component of the magnetic field of six radial dipoles on the xz plane - case 1	61
4-107	The y component of the magnetic field of six radial dipoles on the xz plane - case 1	61
4-108	The z component of the magnetic field of six radial dipoles on the xz plane - case 1	61
4-109	The x component of the magnetic field of six radial dipoles on the xz plane - case 2	62
4-110	The y component of the magnetic field of six radial dipoles on the xz plane - case 2	62
4-111	The z component of the magnetic field of six radial dipoles on the xz plane - case 2	62
4-112	The x component of the magnetic field of eight radial dipoles on the xz plane - case 1	63
4-113	The y component of the magnetic field of eight radial dipoles on the xz plane - case 1	63
4-114	The z component of the magnetic field of eight radial dipoles on the xz plane - case 1	63
4-115	The x component of the magnetic field of eight radial dipoles on the xz plane - case 2	64
4-116	The y component of the magnetic field of eight radial dipoles on the xz plane - case 2	64

TABLE XXIV. List of equations - Continued

TABLE XXIV. List of equations - Continued.

TABLE XXIV. List of equations - Continued.

TABLE XXIV. List of equations - Continued.

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