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MILITARY HANDBOOK

CALCULATION AND DESIGN OF MASTER GEARS

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F O R E W O R D

1. This military handbook is approved for use by the U.S. Army Tank-Automotive Command, Department of the Army, and is available for use by all Departments and Agencies of the Department of Defense.

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3. This handbook provides a convenient means for implementing uniform design practices and calculation methods for master gears used in the final acceptance of Tank-Automotive component part gears. It also serves as a textbook for specialized programs of instruction for centralized or local training, and as a preparedness measure for accelerated training of product assurance personnel during mobilization.

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CHAPTER 1
INTRODUCTION

1.1. Purpose. The purpose of this handbook is primarily for use as an instructional aid to the gage designer who is unfamiliar with both the theory and design of master gears. It also provides a means of producing specifications for these master gears which are consistent and reliable.

1.2. Scope. This handbook includes the instructions for the design, development, and calculation and for specifying requirements of master gears for the functional inspection of external and internal, spur and helical, parallel and askew axis, involute profile gears. Materials pertinent to the inspection of bevel and worm gears and related inspection equipment are also covered.

1.3 Application. This handbook is applicable to DOD product assurance personnel engaged in the preparation, maintenance and interpretation of final acceptance inspection equipment designs of inspection method control sheets (IMCs) for Tank-Automotive materiel and by gage designers engaged in the design of master gears.

1.4 Designer qualifications. While this handbook simplifies the task of developing a master gear for those not familiar with the required geometric hypotheses, the user must possess certain minimum qualifications. These qualifications are listed below:

a. Must be a capable gage designer, able to determine suitable body proportions with regard to fabrication and function.

b. Must be able to establish both the magnitude and direction of gage tolerances.

c. Must have knowledge of mathematics that allows interpretation and performance of algebraic and trigonometric formulae when proper values are given.

1.5 Supplementary publications. The designer must have access to a suitable table of natural trigonometric functions. To be suitable, the table should:

a. Interrelate functions of angles measured in degrees and radians.

b. Have tabular divisions no greater than 0.01 degrees.

c. Include tabular values of the involute function.

d. Be arranged for accurate linear interpolation.

e. Have an accuracy of at least seven significant figures in the portion of the table to be used. See selected bibliography for publications that meet the stated requirements and are recommended for use.

discussed in 5.7

1.6 Fundamental inspection concepts.

1.6.1 Inspection. Acceptance inspection of spur and helical involute gears can be effected by two different methods. A careful examination of the circumstances involved dictate which method (or possibly both) will be used. The methods are generally referred to as "Analytical Inspection" and "Functional Inspection"; the basic difference being that the first method determines each manufacturing error individually, while the latter views them compositely. Functional inspection, which is the primary consideration of this handbook, is performed by engaging the part gear in intimate (metal to metal) contact with a specifically designed conjugate mate called a master gear. The manufacturing errors are then displayed as variations in center distance between the two gears. A shortcoming in this concept exists since the center distance variations are caused by simultaneous errors in both the gear and master. This produces a distorted indication (either positive or negative) of the actual errors in the gear. It follows that inaccuracies in fabrication of the master gear can be costly.

1.6.2 Master gear. The master gear, to meet minimum requirements as an inspection tool, must contact the part gear over its entire active profile. The gears active profile, as the name implies, is defined as that portion of the tooth surface which is active during operation with its intended mate(s). Sometimes this feature is included as part of the gear specification. Unfortunately the nomenclature used for this feature is not consistent. The term most often encountered appears to be "contact diameter", which simply means that the profile is active from this diameter to the addendum diameter. Some other synonomus terms are "start of active profile", "last point of contact", "first point of contact", and, erroneously, "form diameter" 1/.

1.6.3 Active profile. When the active profile is not determined by the gear specification, it must be found by a mathematical interpretation, of mating characteristics. When the gear operates with more than one mate, the condition causing the greatest active profile, i.e. smallest contact diameter for external gears or largest contact diameter for internal gears will be used.

1.7 Handbook overview. This handbook has been structured and organized to cover the general elements of design and calculations of master gears. A description of the handbook contents on a chapter-by-chapter basis follows:

a. Chapter 2 Definitions. This chapter contains the meanings of basic symbols and subscripts used throughout the handbook. It also includes the definitions of terms used in master gear design.

1/ The form diameter for external gears should be less than the contact diameter by a suitable form clearance, conversely true for internal gears.

b. Chapter 3 Calculation sheets for master gear. This chapter includes the various types of calculation sheets for different types of gears. It also includes the explanation of how the motion is transmitted between two involute gears.

c. Chapter 4 Master gear drawing. This chapter provides instructions for the proper arrangement of the master gear specification drawings.

d. Chapter 5 Bevel gear inspection. This chapter includes the instructions for inspection and testing of bevel gears. It also includes the various types of errors in master gears and instruments to measure those errors.

e. Chapter 6 Inspection of worm gearing. This chapter contains the inspection procedures used to check worm gears.

f. Chapter 7 Recommendation on gear inspection equipment. This chapter includes the information on various inspection equipments used in checking gears.

g. Tables and figures. The tables and figures are combined and included at the end of the document.

h. Appendix Listing of qualified gear inspection equipment suppliers. The appendix includes the names of various companies manufacturing different type of gears. The list is included only as a guide.

CHAPTER 2
DEFINITIONS

2.1 Symbols. The following basic symbols are applicable gear terminology used in this handbook. When they are not accompanied by a subscript they normally relate to the basic pitch cylinder, i.e. N/P.

Symbol	Meaning
a	Addendum
b	Dedendum
c	Clearance
C	Distance, Center
d	Diameter (not concentric to axis of gear)
D	Diameter (concentric to axis of gear)
F	Width, Face
h	Depth
m	Ratio
M	Measurement
N	Number of Teeth
p	Pitch, Circular
P	Pitch, Diametral
r	Radius
R	Radius
s	Space Width, Arc
S	Space Width, Chordal
t	Tooth Thickness, Arc
T	Tooth Thickness, Chordal
y	Pitch point (see 2.3)
β	Angle, General
Δ	Difference, Variation
ϵ	Angle Roll

Symbol	Meaning
θ	Angle, Polar
λ	Angle, Lead
Σ	Angle, Shaft
ϕ	Angle, Pressure (see 2.4)
ψ	Angle, Helix

2.2 Subscripts. To restrict a basic symbol to a particular condition or location, the following subscripts may be applied.

b	Base
c	Contact
F	Form
i	Inside
n	Normal
N	Minimum
o	Outside
r	Operating
R	Root
t	Tooth
w	Wire
x	Maximum
1	First
2	Second

2.3 Pitch point. The pitch point (y) is the intersection of the line joining the centers and the path of contact. The pitch circles (r_1 and r_2) have their centers at the axes of rotation and contain the pitch point. The ratio of the pitch radii is always identical to the ratio of the number of teeth. Similarly as the center distance is varied, the pitch circles vary in magnitude but not in definition.

2.4 Pressure angle. The pressure angle (ϕ) is the angle between the line joining the centers of the gears and the normal to the path of contact. The pressure angle varies directly as the center distance (C) varies.

CHAPTER 3
MASTER GEAR DESIGN

3.1 General. This chapter covers the various types of calculation sheets and associated preparation instructions required to design master gears which are used to functionally inspect external and internal, spur and helical, parallel and askew axis, involute profile gears. A specialized knowledge of the kinematics of involute profile gearing is not required for the successful usage of this handbook. However, the following brief treatise will aid the user in understanding the geometric relationships encountered in master gear design.

3.2 Prime requisite for transmittal of motion between two involute gears. The prime requisite for proper transmittal of motion between any two involute gears of the same type on parallel axes, is that their base pitch (pb) be equal (see figures 1 & 2). This assumes, that the other gear features (such as tooth thickness, addendum diameter, etc.), are properly proportioned. Contact between the teeth occurs on a line tangent to the base diameters ($\overline{ad}$), and is limited to that portion of the line intercepted by the addendum circles of the gears ($\overline{bc}$). These line segments are known as "path of contact" and "active path of contact" respectively. The ratio of the active path of contact to the base pitch is referred to as the contact ratio (m). It can be established that if this ratio is greater than one, overlap will occur and transmittal of motion will be continuous, if less than one, transmission of motion will be sporadic.

3.3 Calculation sheets. The calculation sheets were prepared with the intent of producing a correct master gear regardless of the proportions or specifications of the part gear. However, it is reasonable to assume that unusual gear properties may subsequently incur undesirable master gear specifications. For this reason, the designer is directed to review the dimensional proportions of the master gear before finalization. The calculation sheets themselves make the designer aware of the critical areas and indicate suitable limitations. When these limitations are exceeded, the necessary alterations to normal design procedures follow as part of the instructions for use of the calculation sheets. For root clearance dimension mentioned on calculation sheets refer to table I.

3.4 Sets of calculation sheets. There are seven individual sets of calculation sheets intended to include all external or internal, spur or helical, parallel or askew axes, involute profile gear combinations normally encountered. The gear form identification chart (table II) identifies to the designer the proper set of calculation sheets to be used to design a specific gear combination. The succeeding paragraphs contain the instructions for preparing the calculation sheets.

3.5 Collection of data. Having the proper calculation sheets, the information required under the sub-heading "known data" is obtained from the product drawings for the gear and its mate(s) 1/. Occasionally, the

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1/ When contact radius of the gear is included as part of its specification, the mating gear(s) data is not required.

operating center distance (Cr) will not be specified on the gear drawings. In this event, the mounting distance given on the housing drawing will be substituted.

3.5.1 Determining tooth thickness. The tooth thickness dimension of the part gear is a feature necessary to complete the design of the master gear. When the tooth thickness is not included in the gear specification, it can be found from the dimension over (between) wires (see table III). If neither a wire measurement nor tooth thickness dimension is included on the product drawing, the drawing will be considered deficient and an Engineering Change Proposal (ECP) will be initiated to correct this deficiency.

3.5.2 Helical gear tooth profile. Helical gear tooth profiles are involutes in a transverse plane 2/ only. For simplicity of presentation, the geometric relationships required to develop the master gear are conducted in this plane. If the basic data of the product gear is offered in a normal plane 3/, it must be converted as indicated on the applicable calculation sheets.

3.5.3 Calculation of gear data. The formulae under the sub-heading "calculated gear data" are used to orient the known data of the gear and its mate(s) to terms directly applicable to subsequent master gear geometry. It is possible for the expressions which determine the line segment cd to yield a negative magnitude. If this should occur, the tooth profiles of the part gear must be undercut to avoid interference with its mate(s). This condition may be undesirable, and possibly can be avoided. A suitable inquiry should be directed to the Product Engineering Agency for verification of the intent of the undercut.

3.6 Trial pitch radius. A suitable trial pitch radius (R*) for the master gear is now determined by one of the three given expressions. The first two expressions make certain that the master gear has a contact pressure angle of at least five degrees (5°) and a contact radius at least five thousandths inches (.005) larger than the base radius. The third expression determines a size suitable to most gear rolling machines; however available equipment may preclude the use of this result.

3.7 Limitation on the pitch radius. The limitation of four inches maximum on the pitch radius of the master gear is intended as a guide only and may be exceeded if conditions warrant. The reverse may also be true where a size limitation less than four inches is necessary. In either event, with almost no exception, the pitch diameter requirement of the master can be reduced by varying the operating pressure angle. If the master gear is too large because of its contact pressure angle requirement, increase the operating pressure angle. If it is too large because of the clearance requirement between the addendum of the master and root of the gear, reduce the operating pressure angle.

2/ Any plane normal to the axis of rotation.
3/ Any plane normal to the helix of the pitch cylinder.

3.8 Designing master for internal gear. When designing a master for an internal gear, consideration of the possibility of tip fouling or trochoidal interference is necessary. The graph, figure 3, showing the minimum difference between the numbers of teeth for a specific pressure angle and addendum will serve as a guide. The possibility of fouling can also be verified by layout. If interference is present, the condition normally can be alleviated by varying the operating pressure angle.

3.9 Tooth thickness at outside radius. The tooth thickness at the outside radius of the master gear must be at least five thousandths inches (.005) to facilitate manufacture. If this limitation is violated by normal design procedures, there are two means of correction.

a. Increase the number of teeth of the master gear.

b. Decrease the operating pressure angle.

3.10 Face width of master gear. The face width of the master gear as determined by the calculation sheet is a minimum value only, and not necessarily a suitable magnitude. The master must also maintain a proper relationship between its diameter and width. The requirements of the intended gear rolling machine may also influence the face width.

3.11 Algebraic expression for wire diameter. Unfortunately, the (following) exact algebraic expression for determining the theoretical measuring wire diameter for helical gears is both involved and transcendental.

$$d_w^* = \frac{D_b \left[\text{inv arc tan}\left(\frac{d_w^* \cos \psi_b}{D_b} + \tan \emptyset \right) + \frac{\pi}{N} - \frac{t}{D} - \text{inv}\,\emptyset \right]}{\sec \psi_b}$$

As a substitute, the applicable calculation sheets have been provided with an approximate solution which has proven to be adequately reliable. The error in this solution is that it assumes the wire makes contact with the involute helicoid in a normal plane, but in reality, the wire contacts the flanks of adjacent teeth in a plane normal to the locus of a ball (equal in diameter to the wire) allowed to roll between the teeth of the gear 4/. The approximate solution is therefore least reliable as the difference between the helix of the pitch cylinder and the helix described by the ball increases.

3.12 Profile of askew axis helical (spiral) gears. When askew axis helical (spiral) gears operate at a center distance other than standard 5/, the determination of the active profile of the gear to be inspected usually

4/ Involutometry and trigonometry, by W.F. Vogel, Dr. Eng., Pages 294 thru 296.

5/ Standard center distance, here defined, is equal to the sum of the pitch radii of generation, which normally are those specified on the product drawings.

becomes a complex problem. The solution is based on a premise similar to that of parallel axis gearing; that is, that the path of contact is a common internal tangent to the base diameters of the gear and its mate(s), regardless of the center distance. The added consideration is that the path of contact does not lie in a transverse plane of either gear, but rather in a plane containing the pitch point (y), and perpendicular to the helices described on the pitch cylinders of both gears. Again, the pitch cylinders and the corresponding helix angles (ψ) vary directly as the center distance varies. Consequently the shaft angle (Σ) varies 6/. See figure 4.

Referring to FORM G of the calculation sheets, when the operating center distance (Cr) is not equal to the standard center distance (C1):

$$\tan \psi_{r1} = \frac{Y + \sqrt{Y^2 + 4XZ}}{2X}$$

Where

$$X = \frac{r_{b1} \sin \phi_n \sin \Sigma}{\sin \phi_1}$$

$$Y = C_r \sin \psi_1 \cos \phi_n \sin \Sigma - \frac{r_{b1} \sin \phi_n \cos \Sigma}{\sin \phi_1} - \frac{r_{b2} \sin \phi_n}{\sin \phi_2}$$

$$Z = C_r \sin \psi_1 \cos \phi_n \cos \Sigma$$

And

$$\psi_{r2} = \Sigma - \psi_{r1}$$

$$\phi_{r1} = \text{arc cos} \left(\frac{\tan \psi_1 \cos \phi_1}{\tan \psi_{r1}} \right)$$

$$\phi_{r2} = \text{arc cos} \left(\frac{\tan \psi_2 \cos \phi_2}{\tan \psi_{r2}} \right)$$

$$\overline{yc} = \frac{\left(\sqrt{r_{o2}^2 + r_{b2}^2} - r_{b2} \tan \phi_{r2} \right) \sin \phi_{r2}}{\sin \phi_{r1}}$$

$$\overline{cd} = r_{b1} \tan \phi_{r1} - \overline{yc}$$

6/ This discussion is restricted to the most common case - where the sum of the helix angles of the pitch cylinders equal the shaft angle, and the direction of the helices is common.

3.13 Substitution of trigonometric identities. When performing the indicated operations on any of the calculation sheets it may be desirable at times to substitute trigonometric identities for those functions specified. For an example, as an angle (Ø) nears zero degrees (0°), the cot Ø approaches infinity, and it is not normally tabulated to the required accuracy. In such an instance the cot 0 should be replaced by an identity more suitable (1/tan Ø or cos Ø/sin Ø). In addition, some of the acceptable trigonometric function tables do not include the secant, cosecant and cotangent functions in their tabulations, necessitating substitution of their respective co-functions. It is not the intent of this handbook to restrict the designer where restriction is not required, only to present a means to an end.

CHAPTER 4
MASTER GEAR DRAWING

4.1 General. This chapter provides instructions for the proper arrangement of the master gear specification drawings.

4.2 Interpretation. Lower case letters are used to present the instructional text. Nomenclature and characteristics required for drawing data presentation are depicted in capital letters. Dimensions, other than body dimensions, will be specified to the nearest ten thousandths of an inch.

4.3 Master gear drawing. The examples in figures 5 and 6 provide instructions for the proper composition and arrangement of the master gear specification drawing. The symbols in rectangles on the figures are to be replaced by their numerical values (represented by these symbols) from the calculation sheets. The symbols in triangles refer to the tables of allowable manufacturing errors included near the top of the illustrations.

4.4 Curve plotting. Upon completion of the calculation sheets, the designer shall plot a curve on the master gear drawing showing the relationship between the measurement over wires (tooth thickness) and the outside diameter. This relationship is considered to be a linear function requiring only two points for definition. The graph shown on figure 7 will be used to indicate this relationship, each space representing two ten thousandths of an inch (.0002).
Consider the following example where it was found that -

$M_w = 3.1743 + .0050$

$D_o = 3.1123$

$D_{ox} = 3.1209$

The co-ordinates of point A are D_o and M_w, the co-ordinates of point B are D_{ox} and $M_w + .005$. The slope of the line AB will vary as the operating pressure angle varies.

4.5 Bore diameter. The bore diameter should be established according to the requirements of the intended gear rolling machine. In the event that the machine to be used is not known or selected, a 1.250 inch diameter will satisfy most equipment, with 0.750 inch or 1.750 inch diameters being secondary choices. A bushing can be provided for a master gear having too large a bore; however, if the bore is too small, it is almost impossible to machine it larger without reducing the overall accuracy.

4.6 Holes to drive chart recorder. The 0.1875 inch diameter holes shown on each face of the illustration master gear are used by some of the more common machines to drive a chart recorder (see figures 5 and 6). If they are not applicable or if they interfere with other master gear features they are to be omitted.

4.7 Plane of data. When the basic data of the helical component gear is specified in a transverse plane, it is preferred that the diametral pitch and pressure angle of the master gear also be specified in a transverse plane.

CHAPTER 5
BEVEL GEAR INSPECTION

5.1 General. The successful manufacture of bevel gears, particularly precision gears and those requiring utmost quietness in operation, depends vitally on a most careful inspection procedure. Because of the complex configurations in bevel gears and the need for conjugate tooth action, it is necessary that diligent inspection be carried out throughout the whole manufacturing process.

5.2 Inspection procedure. The inspection procedure involves a progressive proving process known in the industry as "Testing". Testing is in addition to all basic inspections, such as for diameters, thicknesses, shoulders, bores, etc. Testing is a functional, rotational check and is introduced in the manufacturing process immediately following cutting the teeth in the green blank and continues thru all subsequent operations which in any way affect the trueness, shape or size of the finished gear. "Testing" therefore is an essential phase of bevel gear inspection.

5.3 Testing Bevel Gears.

5.3.1 Tooth shapes. Smooth, quiet-running bevel gears must have correct tooth shapes, uniform spacing of the teeth and teeth concentric with the bore or shank. Inaccuracies in any of these conditions produces noise and rapid wear on high-speed or heavily-loaded installations, and non-uniform motion in the case of precision instruments and control equipment. Testing machines provide a practical method for determining that the gears are right before assembly, and for precisely controlling the quality of the gears during all the stages of their manufacture.

5.3.2 Running test. In the testing machine, the gears are mounted in the operating position and run under power and light brake load, simulating operation under normal running conditions in their final mountings. Smoothness and quietness of operation, tooth bearing, tooth size, surface finish, runout, and tooth spacing errors are all checked by the running test.

5.3.3 Tooth bearing. Tooth bearing is an indication of correct tooth shape both up and down and lengthwise on the tooth profile. It is that portion of the gear tooth surface which makes contact with its mate and can readily be observed by painting the teeth with a marking compound and running the gears for a few seconds.

5.3.4 Bevel gear testing machines. Bevel gear testing machines are provided with all necessary adjustments to bring the axes of the gears into such a relationship that optimum tooth bearing is obtained. By means of these adjustments, it is possible to determine how the gears will operate in and out of position in their final mountings and to measure directly the slight adjustments which must be made in the cutting or grinding machine to locate the tooth bearing correctly. To insure resetting of the testing machines to duplicate perfectly the original proven settings for a particular pair of bevel gears, special set-up gages are employed. The use of set-up gages is a practical solution to obtain, on repeat orders, the same quality as produced originally.

5.4 Testing practice. The routine manufacture of bevel gears consists of first preparing a "trial or development pair" to be ultimately inspected and approved for operation in the final mounting. Following approval, several sets of master control gears are made which duplicate exactly the "developed pair". These hardened control gears, tailored to satisfy requirements at various operations as production progresses, are then used in the testing machines to control quality at all stages of manufacture. Production gears are tested to the control pinions and the pinions tested to the control gears. This method insures positive control of tooth bearing and makes possible manufacture of quality bevel gears on a production (non-selective) basis. Such gears when paired indiscriminately should show tooth bearing comparable to the tooth bearing obtained when run with control gears.

a. Control gears. Because the successful manufacture of bevel gears demands control at all stages of manufacture, several types of control gears are prepared as follows:

b. Master control gears. Used for checking inspection control gears only.

c. Inspection control gears. Used for checking finished production gears only.

d. Green production control gears. Used for checking gears in the green or unhardened state.

5.5 Final test. A final test usually is made on each pair after hardening, or in the case of curved-tooth bevel gears after subsequent lapping or grinding. Where adjustability is provided in the final mounting, the testing machine provides a rapid and easy means of determining, for each pair of gears, the mounting distances at which they operate best. In this case, the gears are matched in sets and usually wired together. It is common to mark the measured "mounting distance" and "backlash" values on the outer end of the teeth.

5.6 Bevel gear measuring instruments. On certain high precision gear work, such as for instruments and for aircraft, extremely close tolerances of tooth spacing and concentricity must be held. Checking of these values by sight and sound on a testing machine is not sufficient; they must also be inspected by direct measurement. Also in the course of manufacturing, individual elemental inspections must be made to prove setups, cutter action, machine performance, tooling, etc. For these purposes three inspection devices are employed; the composite error tester, the tooth spacing tester, and concentricity tester.

5.6.1 Composite error test. The composite error test is a gear rolling operation for determining size variations, runout, trueness errors, tooth spacing errors, tooth bearing, and backlash.

5.6.2 Trueness errors. Trueness errors are represented as slight oscillations in the indicator as each tooth goes in and out of mesh during rotation, in close engagement without load. These oscillations may or may not be uniform in magnitude and are caused by the modification in the tooth profile shape (necessary to localize the tooth bearings) and by tooth spacing errors. The oscillation caused by modification in the tooth profile shape is referred to as the profile wave. This will be approximately equal to the average value of all the individual oscillations around the gear.

5.6.3 Spacing errors or tooth defects. The spacing errors or tooth defects (thick or thin teeth) or presence of dirt or chips are indicated by sudden jumps of the indicator either up or down, on top of or in addition to the profile wave.

5.6.4 Runout. The runout is determined by rolling the gear with a mating member or "control gear" known to have no appreciable runout. Runout shows up as a gradual increase in the indicator reading up to a high point and a gradual decrease in the indicator reading to a low point, which is generally 180° around the circumference of the member being tested from the high point. Since the amount of the profile wave is the same for every tooth, it has nothing to do with runout. Therefore, it must be subtracted from the difference between the high and low reading to obtain the actual runout. Faulty arbors or improper setup may show up as runout in otherwise good gears.

5.6.5 Tooth spacing tester. This instrument measures the variation in spacing from tooth to tooth. Two contact points are employed, one a fixed stop, the other movable to actuate a dial indicator. In use, each tooth of the gear is brought to bear against the fixed stop with the movable point contacting the corresponding flank of the neighboring tooth. the dial indicator reading is noted and the operation is then repeated for all the teeth in the gear.

5.6.6 Concentricity tester. Concentricity of the pitch circle with the center about which the gear turns is measured by the concentricity tester. The runout is twice the eccentricity. This instrument employs two contact points 180° apart, one a fixed stop, the other actuating a dial indicator. The gear to be checked is mounted on a dead center held in the vise, and indicator readings are taken on diametrically opposite teeth with the contacts acting on corresponding sides of these teeth. The contact points are lifted while the gear is turned from tooth to tooth. the runout is equal to one-half the difference between the high and low readings, which makes this method of inspection extremely accurate.

NOTE

Because of the great amount of time required in inspecting gears by these last two devices, they are used primarily for periodic and not general production checking.

5.6.7 Special checking fixtures. Testing machines are preferred over special fixtures because they show how the gears will operate in service and they provide adjustments necessary for determining slight changes in setup of the cutting machine for control of tooth bearing.

5.6.8 Checking of tooth profile. The checking of the tooth profiles is not carried out on bevel gears for two principle reasons. First, the shape of a bevel gear tooth varies at every position from end to end, instead of being constant as in a spur gear. Hence, a different reading is obtained at every checking position along the gear tooth and even a slight variation in checking position from gear to gear will give incorrect results. Second, even if this check were practical for bevel gears, it would not show the true contact along the entire length of the tooth, as shown by a running test in a testing machine.

5.7 Recognized errors in straight, spiral, zerol and hypoid bevel gears. Fundamental errors for which tolerances are recognized.

Runout (see 5.7.1).
Pitch Variation (see 5.7.2).
Maximum Index Error (see 5.7.3).
Required Tooth Contact (see 5.7.4).
Backlash (see 5.7.5).

5.7.1 Runout of a bevel gear. Runout of a bevel gear is the difference between high and low readings of a dial indicator suitably arranged to denote the off-center relation of the axis of the tooth profiles with respect to the journals or the axis about which the gear rotates. It is twice the eccentricity. It includes and is inseparable from the effect of outside runout or wobble.

5.7.2 Pitch variation. Pitch variation of bevel and hypoid gears is the variation in spacing between corresponding sides of adjacent teeth, measurements preferably being taken near the middle of the tooth height at the midface.

5.7.3 Maximum index error. Maximum index error of a gear is the deviation between the theoretical and the actual displacement between teeth as measured near the pitch line.

5.7.4 Required tooth contact. Required tooth contact determination is best accomplished by means of a testing machine. Proper contact involves three factors; namely, position, shape, and area of contact.

5.7.5 Backlash. Backlash in gears is generally the play between mating teeth. For purposes of measurement and calculation, backlash is the amount by which a tooth space exceeds the thickness of an engaging tooth. When not otherwise specified, numerical values of backlash apply on the pitch circles.

5.8 Bevel gear mounting. Proper service from a pair of bevel gears (straight, zerol, spiral, or hypoid) can be obtained only when they are manufactured accurately and mounted in rigidly designed and carefully machined gear boxes. At the same time, it is impractical to construct a gear mounting which is entirely free from deflections under load. The effect of these deflections is to concentrate the load at one end or the other of the teeth with resultant rapid wear and eventual breakage. However, with the aid of a bevel gear testing machine, allowance for these small deflections can be made in the manufacture of the gears.

5.9 Running test. Since the most conclusive test of bevel gears is their operation under normal running conditions in their final mountings, testing machines are designed to provide a running test of the gears simulating these conditions. In these testing machines the gears are rigidly mounted in the correct operating position and run under power and light brake load. The smoothness and quietness of operation, the tooth bearing, the tooth size, the surface finish, and appreciable runout and tooth spacing errors are all checked by the running test.

CHAPTER 6
INSPECTION OF WORM GEARING

6.1 General. Conventional worm gearing (single or multiple start worms and throated worm wheels) can be inspected functionally by rolling with a master in the same manner as spur and helical gears.

6.2 Worms. Worms are usually inspected for size by the conventional over wire method. Outside diameter and root diameter are routine micrometer checks. Lead, pitch, and pressure angle can be inspected on conventional Illinois, Michigan Tool, and National Broach worm checking machines. A functional check is performed in a rolling setup, employing a master spur or helical gear to engage the worm in the same manner as a rolling check for helical or spur gears.

6.3 Worm wheels (throated). Worm wheels (throated) can be functionally inspected in a conventional rolling machine or special rolling fixture. In many instances the same machine or fixture will serve for both worm and wheel. Because the tooth contact of the worm with the throated worm wheel is a curved helical pattern of matching contour, it is important that the master worm be designed to a corresponding diameter and lead, and be positioned in the rolling fixture at operating center distance and axis position. In order, then, to obtain tight mesh contact; to register runout; conjugate action; etc., the allowed backlash in the worm wheel must be absorbed, not by closing center distance, but by adding the backlash allowance to the tooth thickness of the master worm. In this way the same operating center distance and pattern of contact (master worm to wheel) are preserved with the parts engaged in tight (metal to metal) mesh. Throated worm wheels can be checked elementally for spacing, helix, etc., but the process is slow and expensive and requires considerable preparation. The rolling check is simple, practical and discloses all the necessary information. At the same time, by using color compound, it provides evidence of the tooth bearing pattern as the worm rotates in the wheel.

6.4 Master worm. The master worm should be a near duplicate of the part worm with respect to outside diameter, pitch, lead, pressure angle, and number of starts. Outside diameter should be increased to include the allowable total indicator reading (T.I.R.) runout of the worm. When the worm wheel is a spur or helical gear, a rolling check with a master spur or helical gear is practical. A selected worm can also be used if the operating backlash does not cause interference when the parts are rolled together in tight mesh. Other types of worm and worm wheel designs, such as tapered, dual lead, hourglass, etc., introduce special inspection problems. In such cases, the manufacturer should be consulted.

CHAPTER 7
RECOMMENDATION ON GEAR INSPECTION EQUIPMENT

7.1 General. Inspection of precision gears requires not only reliable measuring equipment but careful training in its use. The modern gear inspection laboratory is equipped with ingenious and sensitive inspection machines and devices to prove functional behavior of gears and to measure their individual incremental errors. Most of today's gear inspection equipment is designed to operate in conjunction with automatic recorders to graphically chart indicator readings formerly done manually. The recorded chart reveals the true condition of the inspected surface much more accurately than numerical values. The recorded graph will reveal, by the nature of its continuous pattern, the cause of errors, whereas numerals representing indicator readings show only the magnitude of these errors. Graphic inspection not only supplies a continuous representation of the surface being checked but removes the element of doubt always present in manually posted readings. Many limits have become so close that reliable clues are needed to tell constantly where the errors come from. Graphic inspection charts form a permanent record for future use and by being easy to read and understand greatly facilitate communication between the inspector and the supplier.

7.2 Functional and analytical inspection. While functional inspection, using a master gear and rolling fixture or machine, supplies the best answer to gear behavior short of actual final assembly, it is not capable of identifying, specifically and minutely, the full range and magnitude of gear errors, as would be disclosed by analytical inspection instruments. For instance, the rolling check may show excessive runout for a particular gear. Analytical inspection on other machines may show runout to be caused by errors in tooth spacing, wobble due to faulty arbors, eccentricity due to oversize bore or eccentricity and oversize of OD of the gear blank. In other words the rolling check qualifies the gear generally, while analytical inspection methods identify particular contributory errors and their magnitude. Both inspection systems are vital in gear manufacture, the rolling check for fast inspection of the overall characteristics, and the analytical inspection machines to check individual elements such as size, spacing, involute, parallelism, runout, etc. Analytical inspection serves to locate and evaluate errors so that prompt, corrective measures can be taken.

7.3 Functional inspection equipment. Gear rolling devices should have free acting slides or pivot arms. Supporting posts for the gear and master should be stubby or close coupled to the slides or arms. High posts tend to give unreliable readings. Adjustable engagement pressure, master with gear, is essential. An adjusting knob, with numbered graduations, serves as one means to register variable contact pressure. Direct backlash readings are desirable for positive identification of backlash, and attachments are available on most instruments for direct readings. Backlash values may be interpreted from change in center distance. This is a fast and widely practiced method. A graphical chart recorder is a desirable feature for any rolling fixture. Most manufacturers of gear instruments supply rolling fixtures and machines following similar lines and of creditable performance, with and without chart recorders.

7.4 Involute profile inspection equipment. Involute profile inspection instruments for both spur and helical gears are available with and without graphical chart recorders. Various mechanical systems are in use to generate the involute action with respect to the contact stylus, but all machines end up with the same reliable result. Some systems, however, require special tooling to accommodate the work while others require only a change in settings. Involute profile inspection instruments are available for coarse and fine pitch gears.

7.5 Tooth spacing, runout, and parallelism inspection equipments. Tooth spacing, runout, parallelism checkers for spur and helical gears are also available with and without recorders.

7.6 Lead checkers. Lead checkers for helical and herringbone gears and worms are currently supplied in two types. In one, the lead of the gear is compared to a master lead groove on a cylinder mounted in the headstock on the work spindle. Several cylinders of varying standard leads are supplied with the instrument with special lead cylinders available. Leads slightly different than the standards can be checked by an adjusting means which adds or subtracts from any standard lead. The other type machine follows the true sine bar principle which is adjustable directly to any lead angle. Both machines are reliable and are obtainable with and without recorders.

7.7 Worm and worm gear inspection equipment. Worms and worm gears can be tested functionally on a rolling machine. A floor type machine, adjustable to suit a wide range of worm and gear sizes, is available with chart recorder. This machine is adjustable for worms up to 6" diameter and worm gears up to 18" diameter. The worm wheel is mounted on a vertical post, the worm lying in the horizontal or the specified angular plane. Worm axis position can be raised or lowered to engage the worm wheel at the correct throat center. Rotation of the worm is manual. The principle of checking is the same as used in the rolling fixtures for spur and helical gears, namely, functional errors are reported on the dial indicator as the change in center distance. In this machine mating parts can be matched together or worms and wheels can be qualified separately by running each against opposite masters or selected parts. Similar equipment is available for fine pitch gears - with recorder. Most gear instrument manufacturers offer specially designed worm and gear rolling fixtures to check composite error and backlash.

7.8 Multistart worms - inspect index. The equipment described under lead checkers in paragraph 7.6 is equipped to check accuracy of index.

7.9 Optical comparator projector. The optical comparator projector, supplied in both bench and floor pedestal types, is a practical and necessary instrument for checking root and tip shape on spur gears and worms of coarse pitch and for checking the entire tooth and root profile on fine pitch gears. It can also check helical gears where helix angle is slight or face width narrow. In some instances, magnification of nearly 50 to 1 can be achieved giving a clear indication of the actual contours projected on a background illustration of the proper magnification.

7.10 Over and between wires measurement. Highly sensitive and accurate measuring means are available for determining "over and between wires" measurement for external and internal gears. Similarly worms and helical gears can be checked regardless of whether pins or balls are employed. In instances of internal helicals and certain helical externals, the ball system requires special holding and handling means because of the general awkwardness of dealing with balls. The Pratt and Whitney Company for one, supplies a very high precision standard measuring machine for gears, screws, worms, etc. and the Electrolimit Comparator for both internal and external gears supplies readings compared to a pre-established gage block setting. Other companies such as Starret, Sheffield, etc., supply equally versatile and accurate instruments.

BIBLIOGRAPHY

Buckingham, E., "Manual of Gear Design, Section One," The Industrial Press, New York

Vogel, W.F., Dr. Eng., "Involutometry and Trigonometry," Excello Corporation, Troy, Michigan

TABLE I. Root Clearance

ROOT CLEARANCE (MIN)	
P	c_R (min)
3 thru 6	.004
above 6 thru 12	.003
above 12 thru 20	.002
above 20 thru 32	.001

NOTE: Root clearance dimensions are mentioned under "Root clearance" in Design calculation sheets (Reference paragraph 3.3).

TABLE II
Gear form identification chart

GEAR TYPE	MATES WITH	FORM	FIGURE
EXT SPUR	EXT SPUR	A	8
INT SPUR	EXT SPUR	B	9
EXT SPUR	INT SPUR	C	10
EXT HELICAL	EXT HELICAL	D 1/	11
INT HELICAL	EXT HELICAL	E	12
EXT HELICAL	INT HELICAL	F	13
EXT HELICAL	EXT HELICAL	G 2/	14

1/ Parallel axes gears.
2/ Askew axes gears.

TABLE III. Determining the arc tooth thickness of a gear from the dimension over (between) wires.

SPUR GEAR	HELICAL GEAR
EXTERNAL	EXTERNAL
$\sec \phi_w = \frac{M_w - d_w}{D_b}$ (when N is even)	$\sec \phi_w = \frac{M_w - d_w}{D_b}$ (when N is even)
$\sec \phi_w = \frac{M_w - d_w}{D_b \cos \frac{90°}{N}}$ (when N is odd)	1/ $\sec \phi_w = \frac{M_w - d_w}{D_b \cos \frac{90°}{N}}$ (when N is odd)
$t = D\left(\text{inv}\,\phi_w + \frac{\pi}{N} - \frac{d_w}{D_b} - \text{inv}\,\phi\right)$	$t = D\left(\text{inv}\,\phi_w + \frac{\pi}{N} - \frac{d_w \sin \phi}{D_b \sin \phi_n} - \text{inv}\,\phi\right)$
INTERNAL	INTERNAL
$\sec \phi_w = \frac{M_w + d_w}{D_b}$ (when N is even)	$\sec \phi_w = \frac{M_w + d_w}{D_b}$ (when N is even)
$\sec \phi_w = \frac{M_w + d_w}{D_b \cos \frac{90°}{N}}$ (when N is odd)	1/ $\sec \phi_w = \frac{M_w + d_w}{D_b \cos \frac{90°}{N}}$ (when N is odd)
$t = D\left(\text{inv}\,\phi + \frac{\pi}{N} - \frac{d_w}{D_b} - \text{inv}\,\phi_w\right)$	$t = D\left(\text{inv}\,\phi + \frac{\pi}{N} - \frac{d_w \sin \phi}{D_b \sin \phi_n} - \text{inv}\,\phi_w\right)$
	1/ These equations are valid only when d_w is spherical (a ball). Involutometry and trigonometry: W.F. Vogel, Dr. Eng; p. 296

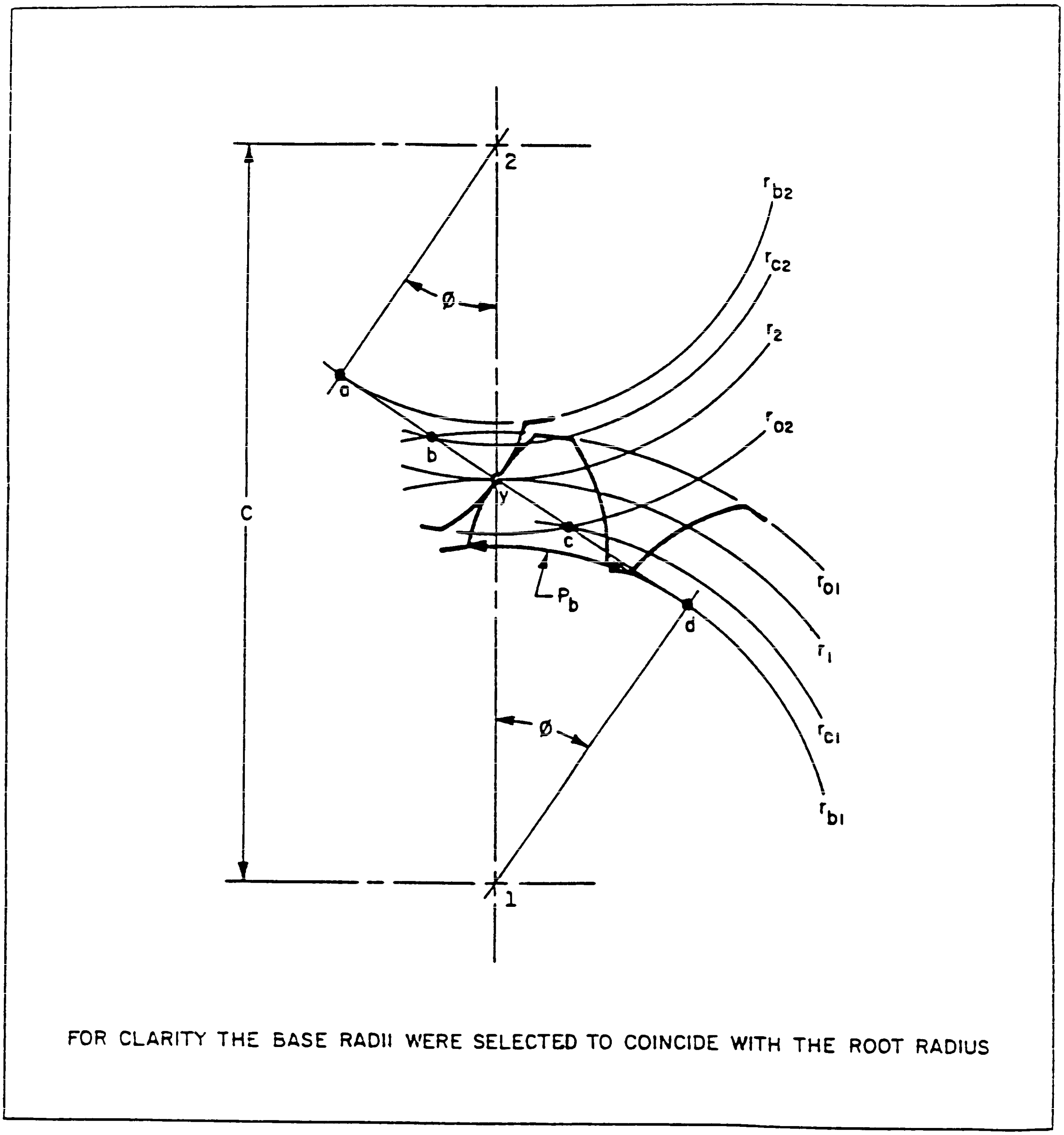
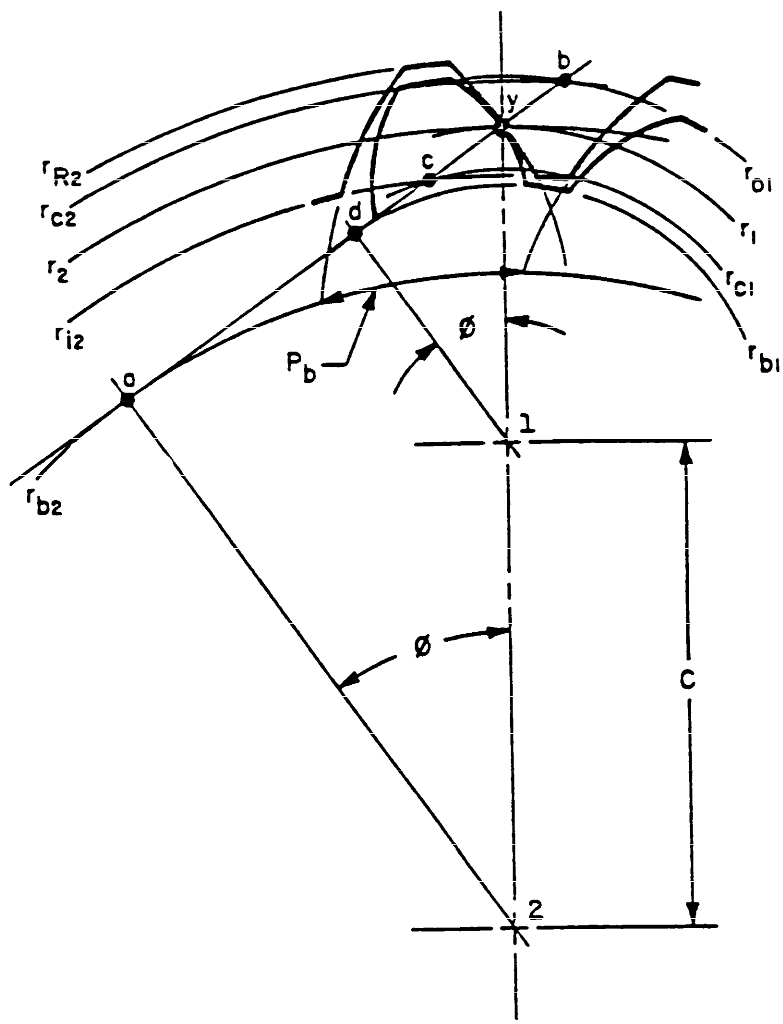


FIGURE 1. Path of contact diagram for external gear drive (Spur or helical).



FOR CLARITY THE BASE RADIUS WAS SELECTED TO COINCIDE WITH THE ROOT RADIUS

FIGURE 2. Path of contact diagram for internal gear drive (Spur or helical)

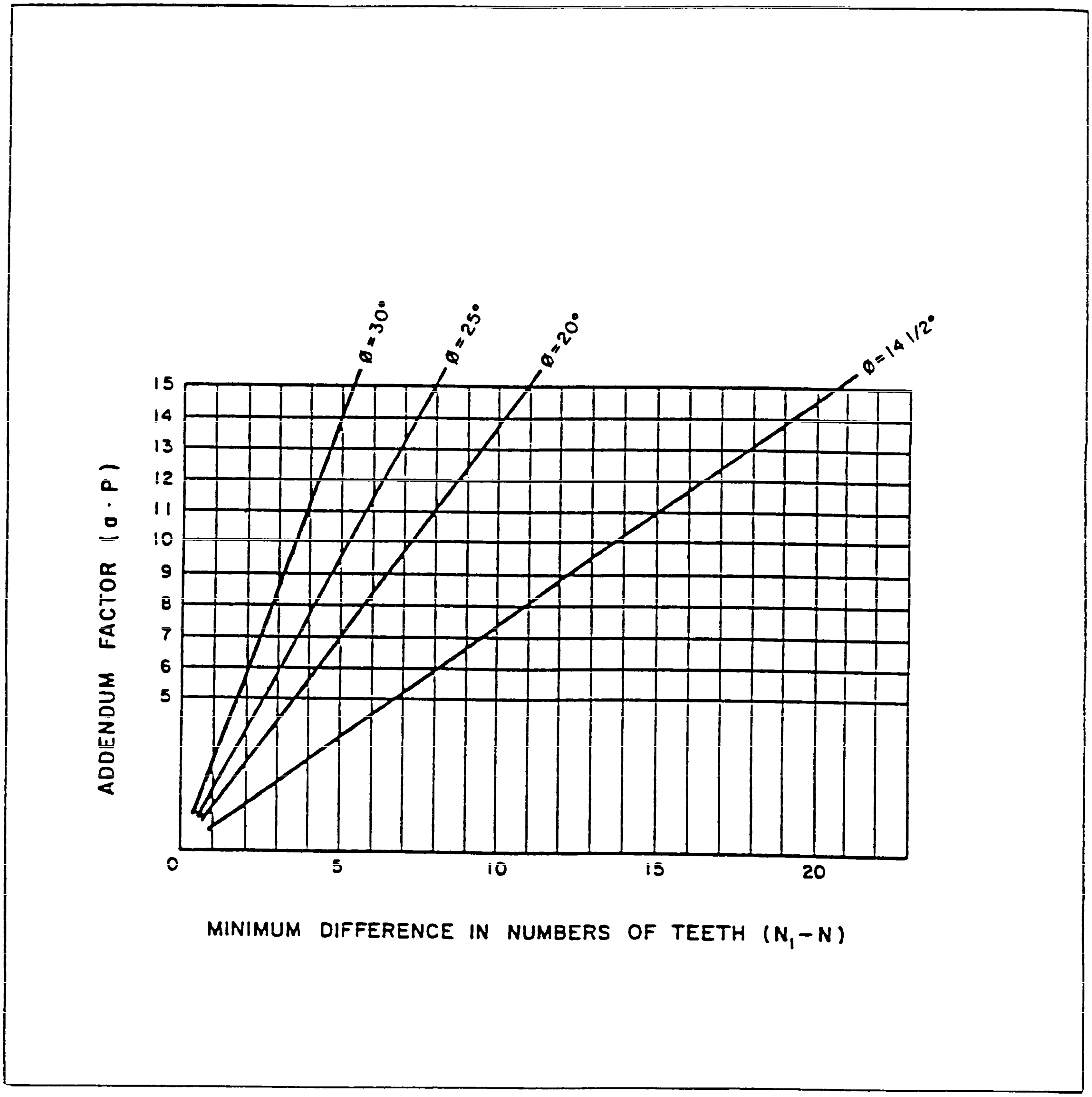


FIGURE 3. Minimum difference in numbers of teeth to avoid tip fouling or trochoidal interference (Internal gear, external master).

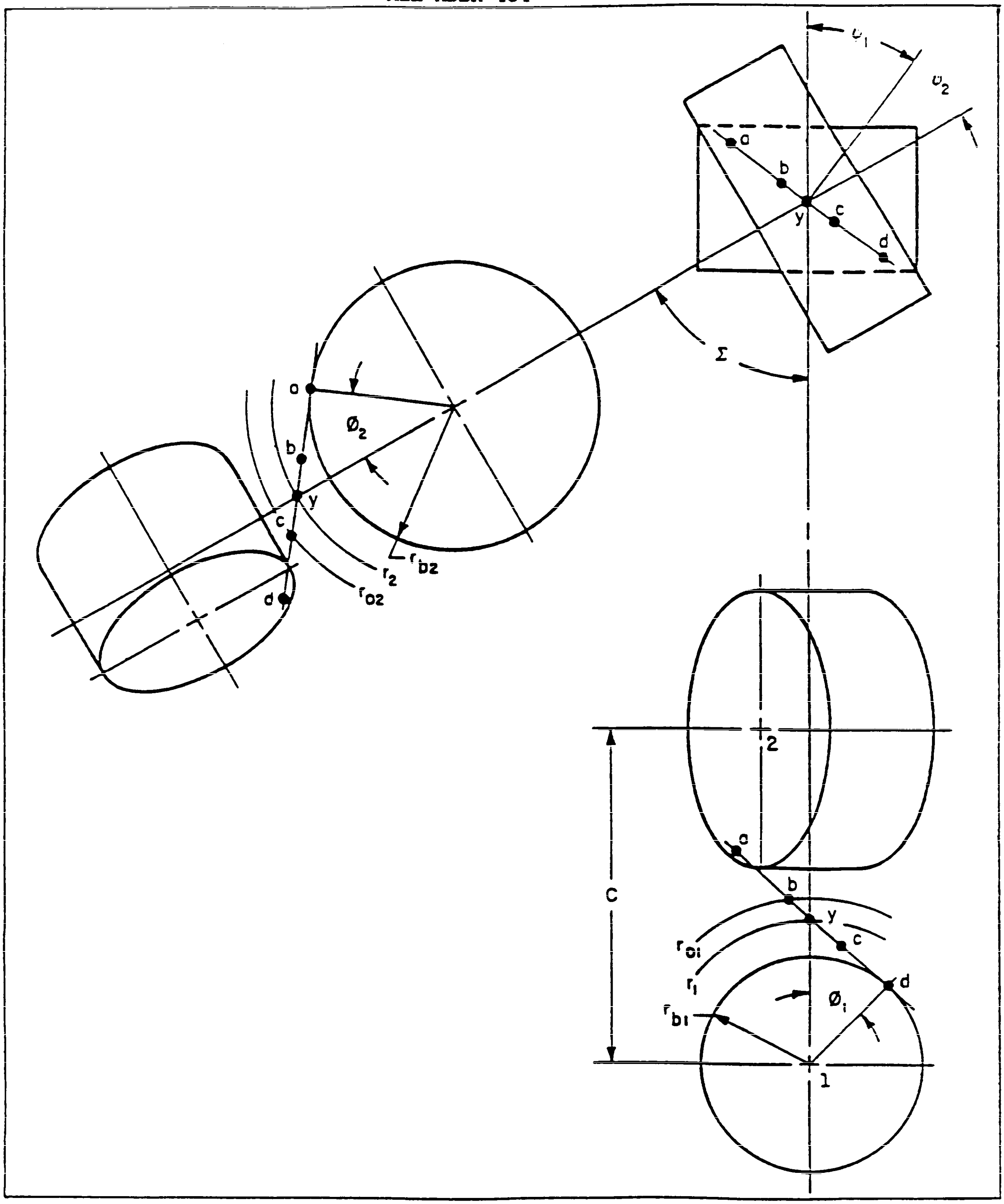


FIGURE 4. Path of contact diagram for askew axes helical (spiral) gear drive.

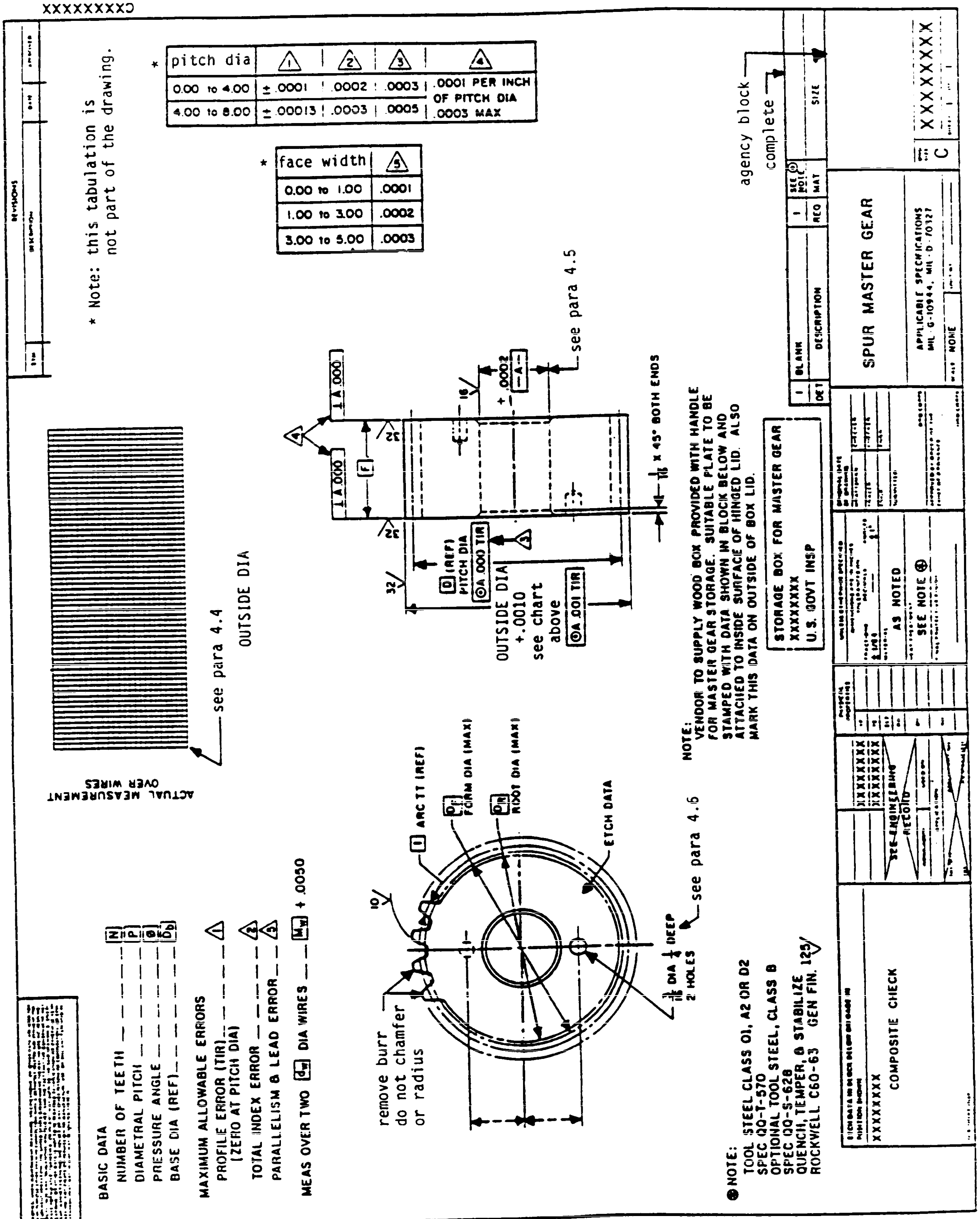


FIGURE 5. Illustration of master gear drawing (spur).

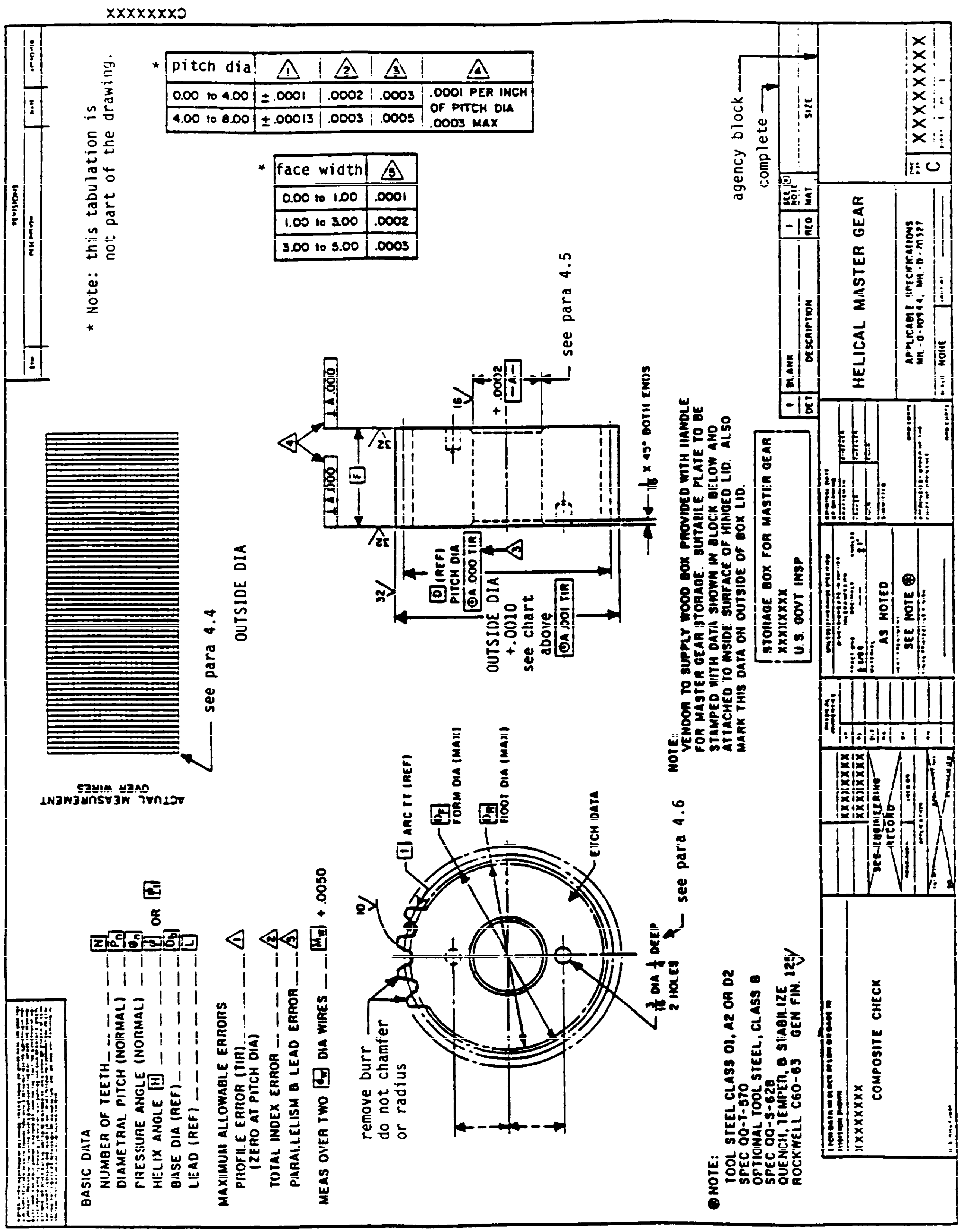


FIGURE 6. Illustration of master gear drawing (helical).

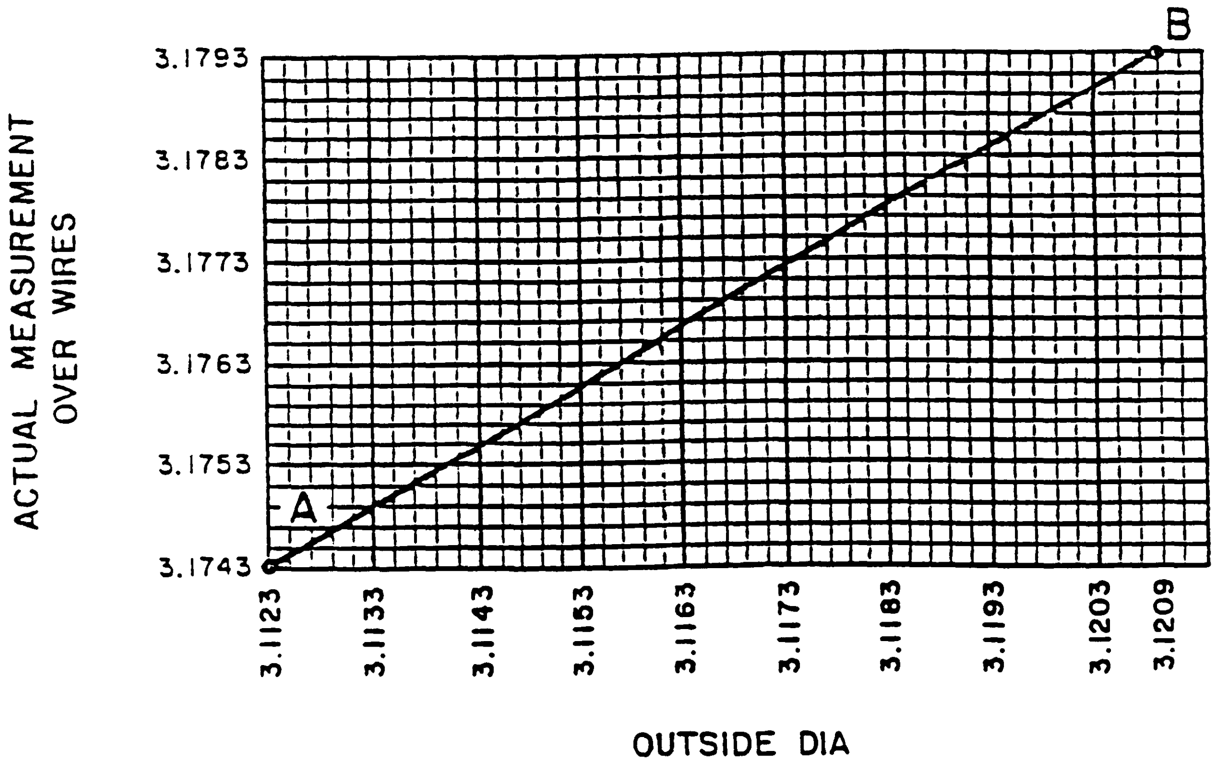


FIGURE 7. Relationship between measurement over wires (tooth thickness) and the outside diameter.

FORM A	Sheet 1 of 3	DWG #	
DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR - EXT MATE)		JOB #	
		GEAR #	
		& LEC	
		MATE #	
		MASTER #	

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		DIAMETRAL PITCH	P	
OUTSIDE RAD (MAX)	r_{o1}		PRESSURE ANGLE	$\phi°$	
CONTACT RAD[1]	r_{c1}		OPERATING CTR DIST (MIN)[1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE[1]		
ARC TOOTH THICKNESS (MAX)	t_1		NUMBER OF TEETH	N_2	
ARC TOOTH THICKNESS TOL	Δt_1		OUTSIDE RAD (MAX)	r_{o2}	
FACE WIDTH	F_1				

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos \phi$		
Line Segment $\overline{yb}$		$\overline{yb} = \sqrt{r_{o1}^2 - r_{b1}^2} - r_1 \sin \phi$		
Standard Center Distance[1]		$C_1 = \frac{N_1 + N_2}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 - When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 - When $C_r \equiv C_1$	$\overline{cd} = C_1 \sin \phi - \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos \phi}{2P}\right)^2}$		
	Method #3 - When $C_r \neq C_1$	$\sec \phi_r = \frac{C_r \sec \phi}{C_1}$ $\overline{cd} = C_r \sin \phi_r - \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos \phi}{2P}\right)^2}$		

CALCULATED MASTER DATA

Trial Pitch Radius (Select the Largest Value of R^* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\phi° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin \phi \cos \phi + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin \phi + .005 \cos \phi}{\sin^2 \phi}$		
	$R^* = 1.375 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$		N

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 8. Form A Design calculation sheets for spur master gear (Ext. gear - Ext. mate).

FORM A	DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR – EXT MATE)		Sheet 2 of 3
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \phi$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \phi - \overline{cd})^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \frac{\Delta t_1 \cot \phi}{2}$		
Root Clearance	$c_R = C + h_{t1} - R_o - r_{o1} - \Delta C$ LIMIT ($c_R \geqq c_R$ min, Table 1)		
ALTERNATE MASTER DATA			
When $c_R < c_R$ min, Substitute Alternate Master Data: Root Clearance (Min)	Obtain c_R min from Table 1		
Factor J	$J = (r_1 \sin \phi - \overline{cd}) \cos \phi$		
Factor H	$H = r_1 \cos^2 \phi + \overline{cd} \sin \phi + h_{t1} - r_{o1} - \Delta C - c_R \min$		
Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \phi$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \phi - \overline{cd})^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM			
Secant of Pressure Angle At Outside Radius	$\sec \phi_o = \frac{R_o}{R_b}$		
Inv ϕ_o	From suitable Table		
Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\phi - \text{inv}\,\phi_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D \cos \phi$		D_b
Form Diameter	$D_F = \sqrt{(D \sin \phi - 2\overline{y_D})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F

FIGURE 8. Form A Design calculation sheets for spur master gear (Ext. gear - Ext. mate) - Continued.

FORM A

DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR – EXT MATE)

Sheet 3 of 3

Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		t
Pressure Angle with Theoretical Meas Wires	$\widehat{\phi_w}^* = \widehat{\phi} + \frac{\pi}{N} - \frac{t}{D}$		
Tan ϕ_w^*	From suitable Table		
Meas Wire Dia - Theoretical	$d_w^* = D_b(\tan \phi_w^* - \tan \phi)$		
Meas Wire Dia - Actual	Select d_w as nearest standard to d_w^*		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv } \phi_w = \frac{t}{D} + \text{inv } \phi + \frac{d_w}{D_b} - \frac{\pi}{N}$		
Sec ϕ_w	From suitable Table		
Tan ϕ_w	From suitable Table		
Meas over Wires	$M_w = D_b \sec \phi_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{arc} \tan\left(\tan \phi_w - \frac{d_w}{D_b}\right)\right]$ LIMIT $(D_F < D_{cw} < D_o)$		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$		D_R
	$D_R = D_o - 2h_{t1}$		
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(r_1 \sin \phi + \overline{yb} - \overline{cd})P}{\pi \cos \phi}$ LIMIT $(m > 1.0)$		
CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER			
Secant of Maximum Pressure Angle with Meas Wire	$\sec \phi_{wx} = \frac{M_w + .005 - d_w}{D_b}$		
Inv ϕ_{wx}	From suitable Table		
Involute of Operating Pressure Angle (Max)	$\text{inv } \phi_{rx} = \frac{P[t_1 + D \text{ inv } \phi_{wx} - d_w \sec \phi] + N_1 \text{ inv } \phi}{N + N_1}$		
Tan ϕ_{rx}	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(C\cos \phi \tan \phi_{rx} - \overline{cd})^2 + D_b^2}$		D_{ox}

FIGURE 8. Form A Design calculation sheets for spur master gear (Ext. gear - Ext. mate) - Continued.

FORM B	Sheet 1 of 3	DWG #	
DESIGN CALCULATIONS FOR SPUR MASTER GEAR (INT GEAR - EXT MATE)		JOB #	
		GEAR # & LEC	
		MATE #	
		MASTER #	

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		DIAMETRAL PITCH	P	
INSIDE RAD (MIN)	r_{i1}		PRESSURE. ANGLE	$\phi°$	
CONTACT RAD[1]	r_{c1}		OPERATING CTR DIST (MAX)[1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE[1]		
ARC TOOTH THICKNESS (MAX)	t_1		NUMBER OF TEETH	N_2	
ARC TOOTH THICKNESS TOL	Δt_1		OUTSIDE RAD (MAX)	r_{o2}	
FACE WIDTH	F_1				

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos\phi$		
Line Segment $\overline{yb}$		$\overline{yb} = r_1 \sin\phi = \sqrt{r_{i1}^2 - r_{b1}^2}$		
Standard Center Distance[1]		$C_1 = \frac{N_1 - N_2}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 - When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 - When $C_r \equiv C_1$	$\overline{cd} = C_1 \sin\phi + \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		
	Method #3 - When $C_r \neq C_1$	$\sec\phi_r = \frac{C_r \sec\phi}{C_1}$ $\overline{cd} = C_r \sin\phi_r + \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		

CALCULATED MASTER DATA

Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\phi° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin\phi \cos\phi + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin\phi + .005 \cos\phi}{\sin^2\phi}$		
	$R^* = .5 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$ LIMIT (See para 3.8)		N

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 9. Form B Design calculation sheets for spur master gear (Int. gear - Ext. mate).

FORM B — DESIGN CALCULATIONS FOR SPUR MASTER GEAR (INT GEAR - EXT MATE) — Sheet 2 of 3

	Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
	Base Radius	$R_b = R \cos \phi$		
	Center Distance	$C = \frac{N_1 - N}{2P}$		
	Outside Radius	$R_o = \sqrt{(\overline{cd} - C \sin \phi)^2 + R_b^2}$		
	Center Distance Variation	$\Delta C = \frac{\Delta t_1 \cot \phi}{2}$		
	Root Clearance	$cR = r_{i1} + h_{t1} - C - R_o - \Delta C$ LIMIT ($cR \geqq cR$ min, Table 1)		
ALTERNATE MASTER DATA				
When $cR < cR$ min, Substitute Alternate Master Data	Root Clearance (Min)	Obtain cR min from Table 1		
	Factor J	$J = (\overline{cd} - r_1 \sin \phi) \cos \phi$		
	Factor H	$H = r_{i1} + h_{t1} - r_1 \cos^2 \phi - \overline{cd} \sin \phi - \Delta C - cR\ \text{min}$		
	Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
	Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$ LIMIT (See para 3.8)		N
	Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
	Base Radius	$R_b = R \cos \phi$		
	Center Distance	$C = \frac{N_1 - N}{2P}$		
	Outside Radius	$R_o = \sqrt{(\overline{cd} - C \sin \phi)^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM				
	Secant of Pressure Angle At Outside Radius	$\sec \phi_o = \frac{R_o}{R_b}$		
	Inv ϕ_o	From suitable Table		
	Arc Tooth Thickness At Outside Radius	$t_o = 2R_o \left(\frac{\pi - Pt_1}{N} + \text{inv}\, \phi - \text{inv}\, \phi_o \right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA				
	Outside Diameter	$D_o = 2R_o$		D_o
	Pitch Diameter	$D = \frac{N}{P}$		D
	Base Diameter	$D_b = D \cos \phi$		D_b
	Form Diameter	$D_F = \sqrt{(D \sin \phi - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F

FIGURE 9. Form B Design calculation sheets for spur master gear (Int. gear - Ext. mate) - Continued.

FORM B

DESIGN CALCULATIONS FOR SPUR MASTER GEAR (INT GEAR – EXT MATE)

Sheet 3 of 3

Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		t
Pressure Angle with Theoretical Meas Wires	$\widehat{\emptyset}_w{}^* = \widehat{\emptyset} + \frac{\pi}{N} - \frac{t}{D}$		
Tan $\emptyset_w{}^*$	From suitable Table		
Meas Wire Dia – Theoretical	$d_w{}^* = D_b(\tan \emptyset_w{}^* - \tan \emptyset)$		
Meas Wire Dia – Actual	Select d_w as nearest standard to $d_w{}^*$		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv } \emptyset_w = \frac{t}{D} + \text{inv } \emptyset + \frac{d_w}{D_b} - \frac{\pi}{N}$		
Sec $\emptyset_w$	From suitable Table		
Tan $\emptyset_w$	From suitable Table		
Meas over Wires	$M_w = D_b \sec \emptyset_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{arc} \tan\left(\tan \emptyset_w - \frac{d_w}{D_b}\right)\right]$ LIMIT $(D_F < D_{cw} < D_o)$		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$ $D_R = D_o - 2h_{t1}$		D_R
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(\overline{yb} + \overline{cd} - r_1 \sin \emptyset)P}{\pi \cos \emptyset}$ LIMIT $(m > 1.0)$		
CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER			
Secant of Maximum Pressure Angle with Meas Wire	$\sec \emptyset_{wx} = \frac{M_w + .005 - d_w}{D_b}$		
Inv $\emptyset_{wx}$	From suitable Table		
Involute of Operating Pressure Angle (Min)	$\text{inv } \emptyset_{rN} = \frac{P[d_w \sec \emptyset - D \text{ inv } \emptyset_{wx} - t_1] + N_1 \text{ inv } \emptyset}{N_1 - N}$		
Tan $\emptyset_{rN}$	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4[\overline{cd} - C\cos \emptyset \tan \emptyset_{rN}]^2 + D_b{}^2}$		D_{ox}

FIGURE 9. Form B Design calculation sheets for spur master gear (Int. gear - Ext. mate) - Continued.

FORM C	Sheet 1 of 3	DWG #	
DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR - INT MATE)		JOB #	
		GEAR # & LEC	
		MATE #	
		MASTER #	

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		DIAMETRAL PITCH	P	
OUTSIDE RAD (MAX)	r_{o1}		PRESSURE ANGLE	Ø°	
CONTACT RAD[1]	r_{c1}		OPERATING CTR DIST (MAX)[1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE[1]		
ARC TOOTH THICKNESS (MAX)	t_1		NUMBER OF TEETH	N_2	
ARC TOOTH THICKNESS TOL	Δt_1		INSIDE RAD (MIN)	r_{i2}	
FACE WIDTH	F_1				

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos \text{Ø}$		
Line Segment $\overline{yb}$		$\overline{yb} = \sqrt{r_{o1}^2 - r_{b1}^2} - r_1 \sin \text{Ø}$		
Standard Center Distance[1]		$C_1 = \frac{N_2 - N_1}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 - When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 - When $C_r \equiv C_1$	$\overline{cd} = \sqrt{r_{i2}^2 - \left(\frac{N_2 \cos \text{Ø}}{2P}\right)^2} - C_1 \sin \text{Ø}$		
	Method #3 - When $C_r \neq C_1$	$\sec \text{Ø}_r = \frac{C_r \sec \text{Ø}}{C_1}$ $\overline{cd} = \sqrt{r_{i2}^2 - \left(\frac{N_2 \cos \text{Ø}}{2P}\right)^2} - C_r \sin \text{Ø}_r$		

CALCULATED MASTER DATA

Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\text{Ø}° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin \text{Ø} \cos \text{Ø} + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin \text{Ø} + .005 \cos \text{Ø}}{\sin^2 \text{Ø}}$		
	$R^* = 1.375 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$		N

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 10. Form C Design calculation sheets for spur master gear (Ext. gear - Int. mate).

FORM C	DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR – INT MATE)		Sheet 2 of 3
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \emptyset - \overline{cd})^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \frac{\Delta t_1 \cot \emptyset}{2}$		
Root Clearance	$c_R = C + h_{t1} - R_o - r_{o1} - \Delta C$ LIMIT ($c_R \geqq c_R$ min, Table 1)		
ALTERNATE MASTER DATA			
When $c_R < c_R$ min, Substitute Alternate Master Data: Root Clearance (Min)	Obtain c_R min from Table 1		
Factor J	$J = (r_1 \sin \emptyset - \overline{cd}) \cos \emptyset$		
Factor H	$H = r_1 \cos^2 \emptyset + \overline{cd} \sin \emptyset + h_{t1} - r_{o1} - \Delta C - c_R$ min		
Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \emptyset - \overline{cd})^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM			
Secant of Pressure Angle At Outside Radius	$\sec \emptyset_o = \frac{R_o}{R_b}$		
Inv $\emptyset_o$	From suitable Table		
Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\emptyset - \text{inv}\,\emptyset_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D \cos \emptyset$		D_b
Form Diameter	$D_F = \sqrt{(D \sin \emptyset - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F

FIGURE 10. Form C Design calculation sheets for spur master gear (Ext. gear - Int. mate) - Continued.

FORM C — Sheet 3 of 3

DESIGN CALCULATIONS FOR SPUR MASTER GEAR (EXT GEAR – INT MATE)

Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		t
Pressure Angle with Theoretical Meas Wires	$\widehat{\phi}_w{}^* = \widehat{\phi} + \frac{\pi}{N} - \frac{t}{D}$		
Tan $\phi_w{}^*$	From suitable Table		
Meas Wire Dia - Theoretical	$d_w{}^* = D_b(\tan \phi_w{}^* - \tan \phi)$		
Meas Wire Dia - Actual	Select d_w as nearest standard to $d_w{}^*$		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv } \phi_w = \frac{t}{D} + \text{inv } \phi + \frac{d_w}{D_b} - \frac{\pi}{N}$		
Sec ϕ_w	From suitable Table		
Tan ϕ_w	From suitable Table		
Meas over Wires	$M_w = D_b \sec \phi_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{ arc } \tan\left(\tan \phi_w - \frac{d_w}{D_b}\right)\right]$ LIMIT $(D_F < D_{cw} < D_o)$		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$		D_R
	$D_R = D_o - 2h_{t1}$		
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(r_1 \sin \phi + \overline{yb} - \overline{cd})P}{\pi \cos \phi}$ LIMIT $(m > 1.0)$		

CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER

Secant of Maximum Pressure Angle with Meas Wire	$\sec \phi_{wx} = \frac{M_w + .005 - d_w}{D_b}$		
Inv ϕ_{wx}	From suitable Table		
Involute of Operating Pressure Angle (Max)	$\text{inv } \phi_{rx} = \frac{P[t_1 + D \text{ inv } \phi_{wx} - d_w \sec \phi] + N_1 \text{ inv } \phi}{N + N_1}$		
Tan ϕ_{rx}	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(C\cos \phi \tan \phi_{rx} - \overline{cd})^2 + D_b{}^2}$		D_{ox}

FIGURE 10. Form C Design calculation sheets for spur master gear (Ext. gear - Int. mate) - Continued.

FORM D	Sheet 1 of 4	DWO #	
DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR - EXT MATE)		JOB #	
		GEAR # & LEC	
		MATE #	
		MASTER #	

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		NORM. DIAMETRAL PITCH	P_n	
OUTSIDE RAD (MAX)	r_{o1}		NORM. PRESSURE ANGLE	ϕ_n°	
CONTACT RAD [1]	r_{c1}		OPERATING CTR DIST (MIN) [1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE [1]		
NORM. ARC TOOTH THK (MAX)	t_{n1}		NUMBER OF TEETH	N_2	
NORM. ARC TOOTH THK TOL	Δt_{n1}		OUTSIDE RAD (MAX)	r_{o2}	
HELIX ANGLE AND HAND	ψ		Sec ψ =	Tan ϕ_n =	
FACE WIDTH	F_1		Tan ψ =	Sin ϕ_n =	

CONVERSION TO TRANSVERSE PLANE

Diametral Pitch	$P = \frac{P_n}{\sec\psi}$		
Pressure Angle	$\phi^\circ = \text{arc tan}(\tan\phi_n \sec\psi)$		
Sin ϕ	From suitable Table		
Cos ϕ	From suitable Table		
Cot ϕ	From suitable Table		
Inv ϕ	From suitable Table		
Arc Tooth Thickness (Max)	$t_1 = t_{n1} \sec\psi$		
Arc Tooth Thickness Tol	$\Delta t_1 = \Delta t_{n1} \sec\psi$		

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos\phi$		
Line Segment $\overline{yb}$		$\overline{yb} = \sqrt{r_{o1}^2 - r_{b1}^2} - r_1 \sin\phi$		
Standard Center Distance [1]		$C_1 = \frac{N_1 + N_2}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 - When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 - When $C_r \equiv C_1$	$\overline{cd} = C_1 \sin\phi - \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		
	Method #3 - When $C_r \neq C_1$	$\sec\phi_r = \frac{C_r}{C_1 \cos\phi}$ $\overline{cd} = C_r \sin\phi_r - \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 11. Form D Design calculation sheets for helical master gear (Ext. gear - Ext. mate).

FORM D | DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE) | Sheet 2 of 4

CALCULATED MASTER DATA

Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \dfrac{\overline{yb}\cos 5°}{\sin(\phi° - 5°)}$		
	$K = \sqrt{\overline{yb}\sin\phi\cos\phi + .0025}$ $R^* = \dfrac{.1K + \overline{yb}\sin\phi + .005\cos\phi}{\sin^2\phi}$		
	$R^* = 1.375 + \dfrac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$		N
Pitch Radius	$R = \dfrac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R\cos\phi$		
Center Distance	$C = \dfrac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C\sin\phi - \overline{cd})^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \dfrac{\Delta t_1 \cot\phi}{2}$		
Root Clearance	$c_R = C + h_{t1} - R_o - r_{o1} - \Delta C$ LIMIT ($c_R \geqq c_R$ min, Table 1)		

ALTERNATE MASTER DATA

When $c_R < c_R$ min, Substitute Alternate Master Data	Root Clearance (Min)	Obtain c_R min from Table 1		
	Factor J	$J = (r_1\sin\phi - \overline{cd})\cos\phi$		
	Factor H	$H = r_1\cos^2\phi + \overline{cd}\sin\phi + h_{t1} - r_{o1} - \Delta C - c_R$ min		
	Trial Outside Radius	$R_o^* = \dfrac{H^2 + J^2}{2H}$		
	Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$		N
	Pitch Radius	$R = \dfrac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
	Base Radius	$R_b = R\cos\phi$		
	Center Distance	$C = \dfrac{N + N_1}{2P}$		
	Outside Radius	$R_o = \sqrt{(C\sin\phi - \overline{cd})^2 + R_b^2}$		

TEST FOR POINTED ADDENDUM

Secant of Pressure Angle At Outside Radius	$\sec\phi_o = \dfrac{R_o}{R_b}$		
Inv ϕ_o	From suitable Table		

FIGURE 11. Form D Design calculation sheets for helical master gear (Ext. gear - Ext. mate) - Continued.

FORM D	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE)		Sheet 3 of 4
Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\phi - \text{inv}\,\phi_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D\cos\phi$		D_b
Form Diameter	$D_F = \sqrt{(D\sin\phi - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F
Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		
Angle β (Degrees)	$\beta^\circ = \left(\frac{\pi - Pt}{N\sec^2\psi}\right)\frac{180^\circ}{\pi}$		
Sin β	From suitable Table		
Cos $(\phi_n^\circ + \beta^\circ)$	From suitable Table		
Meas Wire Dia - Theoretical	$d_w^* \approx \frac{D\sin\beta\sec\psi}{\cos(\phi_n^\circ + \beta^\circ)}$		
Meas Wire Dia - Actual	Select d_w as nearest standard to d_w^*		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv}\,\phi_w = \frac{d_w\sin\phi}{D_b\sin\phi_n} + \frac{t}{D} + \text{inv}\,\phi - \frac{\pi}{N}$		
Sec ϕ_w	From suitable Table		
Tan ϕ_w	From suitable Table		
Meas over Wires	$M_w = D_b\sec\phi_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec\,\text{arc tan}\left(\tan\phi_w - \frac{d_w\sin\phi_n}{D_b\sin\phi}\right)\right]$ LIMIT ($D_F < D_{cw} < D_o$)		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$ $D_R = D_o - 2h_{t1}$		D_R
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(F_1\tan\psi\cos\phi + r_1\sin\phi + \overline{yb} - \overline{cd})P}{\pi\cos\phi}$ LIMIT ($m > 1.0$)		
Lead of Helix	$L = \frac{\pi D}{\tan\psi}$		L
Direction of Helix	Opposite Hand of Gear		H

FIGURE 11. Form D Design calculation sheets for helical master gear (Ext. gear - Ext. mate) - Continued.

FORM D

DESIGN CALCULATIONS FOR
HELICAL MASTER GEAR
(EXT GEAR - EXT MATE)

Sheet 4 of 4

CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER

Secant of Maximum Pressure Angle with Meas Wires	$\sec \phi_{wx} = \dfrac{M_w + .005 - d_w}{D_b}$		
Inv ϕ_{wx}	From suitable Table		
Involute of Operating Pressure Angle (Max)	$\text{inv}\,\phi_{rx} = \dfrac{P\left(t_1 + D\,\text{inv}\phi_{wx} - \dfrac{d_w}{\cot\phi \sin\phi_n}\right) + N_1 \text{inv}\phi}{N + N_1}$		
Tan ϕ_{rx}	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(C\cos\phi \tan\phi_{rx} - \overline{cd})^2 + D_b^2}$		D_{ox}

FIGURE 11. Form D Design calculation sheets for helical master gear (Ext. gear - Ext. mate) - Continued.

FORM E	Sheet 1 of 4	DWG #	
DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (INT GEAR – EXT MATE)		JOB #	
		GEAR # & LEC	
		MATE #	
		MASTER #	

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		NORM. DIAMETRAL PITCH	P_n	
INSIDE RAD (MIN)	r_{i1}		NORM. PRESSURE ANGLE	ϕ_n°	
CONTACT RAD[1]	r_{c1}		OPERATING CTR DIST (MAX)[1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE[1]		
NORM. ARC TOOTH THK (MAX)	t_{n1}		NUMBER OF TEETH	N_2	
NORM. ARC TOOTH THK TOL	Δt_{n1}		OUTSIDE RAD (MAX)	r_{o2}	
HELIX ANGLE AND HAND	ψ		Sec ψ =	Tan ϕ_n =	
FACE WIDTH	F_1		Tan ψ =	Sin ϕ_n =	

CONVERSION TO TRANSVERSE PLANE

Diametral Pitch	$P = \frac{P_n}{\sec\psi}$		
Pressure Angle	$\phi^\circ = \arctan(\tan\phi_n \sec\psi)$		
Sin ϕ	From suitable Table		
Cos ϕ	From suitable Table		
Cot ϕ	From suitable Table		
Inv ϕ	From suitable Table		
Arc Tooth Thickness (Max)	$t_1 = t_{n1} \sec\psi$		
Arc Tooth Thickness Tol	$\Delta t_1 = \Delta t_{n1} \sec\psi$		

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos\phi$		
Line Segment $\overline{yb}$		$\overline{yb} = r_1 \sin\phi - \sqrt{r_{i1}^2 - r_{b1}^2}$		
Standard Center Distance[1]		$C_1 = \frac{N_1 - N_2}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 – When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 – When $C_r \equiv C_1$	$\overline{cd} = C_1 \sin\phi + \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		
	Method #3 – When $C_r \neq C_1$	$\sec\phi_r = \frac{C_r}{C_1 \cos\phi}$ $\overline{cd} = C_r \sin\phi_r + \sqrt{r_{o2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2}$		

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 12. Form E Design calculation sheets for helical master gear (Int. gear - Ext. mate)

FORM E	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (INT GEAR – EXT MATE)		Sheet 2 of 4
CALCULATED MASTER DATA			
Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\emptyset° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin \emptyset \cos \emptyset + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin \emptyset + .005 \cos \emptyset}{\sin^2 \emptyset}$		
	$R^* = .5 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$ LIMIT (See para 3.8)		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N_1 - N}{2P}$		
Outside Radius	$R_o = -\sqrt{(\overline{cd} - C \sin \emptyset)^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \frac{\Delta t_1 \cot \emptyset}{2}$		
Root Clearance	$c_R = r_{i_1} + h_{t_1} - C - R_o - \Delta C$ LIMIT ($c_R \geqq c_R$ min, Table 1)		
ALTERNATE MASTER DATA			
When $c_R < c_R$ min, Substitute Alternate Master Data			
Root Clearance (Min)	Obtain c_R min from Table 1		
Factor J	$J = (\overline{cd} - r_1 \sin \emptyset) \cos \emptyset$		
Factor H	$H = r_{i_1} + h_{t_1} - r_1 \cos^2 \emptyset - \overline{cd} \sin \emptyset - \Delta C - c_R$ min		
Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$ LIMIT (See para 3.8)		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N_1 - N}{2P}$		
Outside Radius	$R_o = -\sqrt{(\overline{cd} - C \sin \emptyset)^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM			
Secant of Pressure Angle At Outside Radius	$\sec \emptyset_o = \frac{R_o}{R_b}$		
Inv $\emptyset_o$	From suitable Table		

FIGURE 12. Form E Design calculation sheets for helical master gear (Int. gear - Ext. mate) - Continued.

FORM E	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (INT GEAR – EXT MATE)		Sheet 3 of 4
Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\phi - \text{inv}\,\phi_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D \cos \phi$		D_b
Form Diameter	$D_F = \sqrt{(D \sin \phi - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F
Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		
Angle β (Degrees)	$\beta° = \left(\frac{\pi - Pt}{N \sec^2 \psi}\right)\frac{180°}{\pi}$		
Sin β	From suitable Table		
Cos ($\phi_n° + \beta°$)	From suitable Table		
Meas Wire Dia - Theoretical	$d_w^* \approx \frac{D \sin \beta \sec \psi}{\cos(\phi°_n + \beta°)}$		
Meas Wire Dia - Actual	Select d_w as nearest standard to d_w^*		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv}\,\phi_w = \frac{d_w \sin \phi}{D_b \sin \phi_n} + \frac{t}{D} + \text{inv}\,\phi - \frac{\pi}{N}$		
Sec ϕ_w	From suitable Table		
Tan ϕ_w	From suitable Table		
Meas over Wires	$M_w = D_b \sec \phi_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{arc} \tan\left(\tan \phi_w - \frac{d_w \sin \phi_n}{D_b \sin \phi}\right)\right]$ LIMIT ($D_F < D_{cw} < D_o$)		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$		D_R
	$D_R = D_o - 2h_{t1}$		
Minimum Face Width	$F = F_i + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(F_i \tan \psi \cos \phi + r_i \sin \phi + \overline{yb} - \overline{cd})P}{\pi \cos \phi}$ LIMIT ($m > 1.0$)		
Lead of Helix	$L = \frac{\pi D}{\tan \psi}$		L
Direction of Helix	Opposite Hand of Gear		H

FIGURE 12. Form E Design calculation sheets for helical master gear (Int. gear - Ext. mate) - Continued.

FORM E	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (INT GEAR – EXT MATE)		Sheet 4 of 4
CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER			
Secant of Maximum Pressure Angle with Meas Wires	$\sec \phi_{wx} = \frac{M_w + .005 - d_w}{D_b}$		
Inv ϕ_{wx}	From suitable Table		
Involute of Operating Pressure Angle (Min)	$\text{inv}\,\phi_{rN} = \frac{P\left(\frac{d_w}{\cot \phi \sin \phi_n} - D\,\text{inv}\,\phi_{wx} - t_i\right) + N_i\,\text{inv}\,\phi}{N_i - N}$		
Tan ϕ_{rN}	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(\overline{cd} - C\cos\phi \tan\phi_{rN})^2 + D_b^2}$		D_{ox}

FIGURE 12. Form E Design calculation sheets for helical master gear (Int. gear - Ext. mate) - Continued.

FORM F	Sheet 1 of 4	DWO #
DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – INT MATE)		JOB #
		GEAR # & LEC
		MATE #
		MASTER #

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		NORM. DIAMETRAL PITCH	P_n	
OUTSIDE RAD (MAX)	r_{o1}		NORM. PRESSURE ANGLE	$\phi_n°$	
CONTACT RAD[1]	r_{c1}		OPERATING CTR DIST (MAX)[1]	C_r	
WHOLE DEPTH (MIN)	h_{t1}		MATE[1]		
NORM. ARC TOOTH THK (MAX)	t_{n1}		NUMBER OF TEETH	N_2	
NORM. ARC TOOTH THK TOL	Δt_{n1}		INSIDE RAD (MIN)	r_{i2}	
HELIX ANGLE AND HAND	ψ		$\sec\psi$ =	$\tan\phi_n$ =	
FACE WIDTH	F_1		$\tan\psi$ =	$\sin\phi_n$ =	

CONVERSION TO TRANSVERSE PLANE

Diametral Pitch	$P = \frac{P_n}{\sec\psi}$		
Pressure Angle	$\phi° = \arctan(\tan\phi_n \sec\psi)$		
Sin ϕ	From suitable Table		
Cos ϕ	From suitable Table		
Cot ϕ	From suitable Table		
Inv ϕ	From suitable Table		
Arc Tooth Thickness (Max)	$t_1 = t_{n1}\sec\psi$		
Arc Tooth Thickness Tol	$\Delta t_1 = \Delta t_{n1}\sec\psi$		

CALCULATED GEAR DATA

Pitch Radius		$r_1 = \frac{N_1}{2P}$		
Base Radius		$r_{b1} = r_1 \cos\phi$		
Line Segment $\overline{yb}$		$\overline{yb} = \sqrt{r_{o1}^2 - r_{b1}^2} - r_1 \sin\phi$		
Standard Center Distance[1]		$C_1 = \frac{N_2 - N_1}{2P}$		
Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 – When r_{c1} is Known	$\overline{cd} = \sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 – When $C_r \equiv C_1$	$\overline{cd} = \sqrt{r_{i2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2} - C_1 \sin\phi$		
	Method #3 – When $C_r \neq C_1$	$\sec\phi_r = \frac{C_r}{C_1 \cos\phi}$ $\overline{cd} = \sqrt{r_{i2}^2 - \left(\frac{N_2 \cos\phi}{2P}\right)^2} - C_r \sin\phi_r$		

[1] When r_{c1} is Known, the Mating Gear Data, C_r and C_1 are Not Required, and Conversely

FIGURE 13. Form F Design calculation sheets for helical master gear (Ext. gear - Int. mate).

FORM F	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – INT MATE)		Sheet 2 of 4
CALCULATED MASTER DATA			
Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\phi° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin \phi \cos \phi + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin \phi + .005 \cos \phi}{\sin^2 \phi}$		
	$R^* = 1.375 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \phi$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \phi - \overline{cd})^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \frac{\Delta t_1 \cot \phi}{2}$		
Root Clearance	$c_R = C + h_{t1} - R_o - r_{o1} - \Delta C$ LIMIT ($c_R \geqq c_R$ min, Table 1)		
ALTERNATE MASTER DATA (When $c_R < c_R$ min, Substitute Alternate Master Data)			
Root Clearance (Min)	Obtain c_R min from Table 1		
Factor J	$J = (r_1 \sin \phi - \overline{cd}) \cos \phi$		
Factor H	$H = r_1 \cos^2 \phi + \overline{cd} \sin \phi + h_{t1} - r_{o1} - \Delta C - c_R \text{ min}$		
Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \phi$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \phi - \overline{cd})^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM			
Secant of Pressure Angle At Outside Radius	$\sec \phi_o = \frac{R_o}{R_b}$		
Inv ϕ_o	From suitable Table		

FIGURE 13. Form F Design calculation sheets for helical master gear (Ext. gear - Int. mate) - Continued.

FORM F — DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – INT MATE) — Sheet 3 of 4

Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\emptyset - \text{inv}\,\emptyset_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D \cos \emptyset$		D_b
Form Diameter	$D_F = \sqrt{(D \sin \emptyset - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F
Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		
Angle β (Degrees)	$\beta° = \left(\frac{\pi - Pt}{N \sec^2 \phi}\right)\frac{180°}{\pi}$		
Sin β	From suitable Table		
Cos $(\emptyset_n° + \beta°)$	From suitable Table		
Meas Wire Dia - Theoretical	$d_w^* \approx \frac{D \sin \beta \sec \phi}{\cos(\emptyset°_n + \beta°)}$		
Meas Wire Dia - Actual	Select d_w as nearest standard to d_w^*		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv}\,\emptyset_w = \frac{d_w \sin \emptyset}{D_b \sin \emptyset_n} + \frac{t}{D} + \text{inv}\,\emptyset - \frac{\pi}{N}$		
Sec $\emptyset_w$	From suitable Table		
Tan $\emptyset_w$	From suitable Table		
Meas over Wires	$M_w = D_b \sec \emptyset_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{arc} \tan\left(\tan \emptyset_w - \frac{d_w \sin \emptyset_n}{D_b \sin \emptyset}\right)\right]$ LIMIT ($D_F < D_{cw} < D_o$)		
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$ $D_R = D_o - 2h_{t1}$		D_R
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(F_1 \tan \phi \cos \emptyset + r_1 \sin \emptyset + \overline{yb} - \overline{cd})P}{\pi \cos \emptyset}$ LIMIT ($m > 1.0$)		
Lead of Helix	$L = \frac{\pi D}{\tan \phi}$		L
Direction of Helix	Opposite Hand of Gear		H

FIGURE 13. Form F Design calculation sheets for helical master gear (Ext. gear - Int. mate) - Continued.

FORM F

DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR - INT MATE)

Sheet 4 of 4

CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER

Secant of Maximum Pressure Angle with Meas Wires	$\sec \emptyset_{wx} = \dfrac{M_w + .005 - d_w}{D_b}$		
Inv $\emptyset_{wx}$	From suitable Table		
Involute of Operating Pressure Angle (Max)	$\text{inv}\,\emptyset_{rx} = \dfrac{P\left(t_1 + D\,\text{inv}\emptyset_{wx} - \dfrac{d_w}{\cot\emptyset \sin\emptyset_n}\right) + N_1\,\text{inv}\emptyset}{N + N_1}$		
Tan $\emptyset_{rx}$	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(C\cos\emptyset \tan\emptyset_{rx} - \overline{cd})^2 + D_b^2}$		D_{ox}

FIGURE 13. Form F Design calculation sheets for helical master gear (Ext. gear - Int. mate) - Continued.

FORM G DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE WITH ASKEW AXES)	Sheet 1 of 4	
		DWO #
		JOB #
		GEAR # & LEC
		MATE #
		MASTER #

KNOWN DATA

GEAR			GEAR AND MATE		
NUMBER OF TEETH	N_1		NORM. DIAMETRAL PITCH	P_n	
OUTSIDE RAD (MAX)	r_{o1}		NORM. PRESSURE ANGLE	$\phi_n°$	
CONTACT RAD[1]	r_{c1}		SHAFT ANGLE	$\Sigma°$	
WHOLE DEPTH (MIN)	h_{t1}		OPERATING CTR DIST (MIN)[1]	C_r	
NORM. ARC TOOTH THK (MAX)	t_{n1}		MATE[1]		
NORM. ARC TOOTH THK TOL	Δt_{n1}		NUMBER OF TEETH	N_2	
HELIX ANGLE AND HAND	ψ_1		OUTSIDE RAD (MAX)	r_{o2}	
FACE WIDTH	F_1		HELIX ANGLE AND HAND	ψ_2	

Sec ψ_1	Tan ϕ_n	Sec ψ_2
Tan ψ_1	Sin ϕ_n	

CONVERSION TO TRANSVERSE PLANES

Diametral Pitch	$P = \dfrac{P_n}{\sec \psi_1}$		
Pressure Angle (Gear)	$\phi° = \arctan(\tan \phi_n \sec \psi_1)$		
Sin ϕ	From suitable Table		
Cos ϕ	From suitable Table		
Cot ϕ	From suitable Table		
Inv ϕ	From suitable Table		
Arc Tooth Thickness (Max)	$t_1 = t_{n1} \sec \psi_1$		
Arc Tooth Thickness Tol	$\Delta t_1 = \Delta t_{n1} \sec \psi_1$		
Pressure Angle (Mate)	$\phi_2° = \arctan(\tan \phi_n \sec \psi_2)$		
Sin ϕ_2	From suitable Table		
Cos ϕ_2	From suitable Table		

CALCULATED GEAR DATA

Pitch Radius (Gear)	$r_1 = \dfrac{N_1}{2P}$		
Base Radius (Gear)	$r_{b1} = r_1 \cos \phi$		
Pitch Radius (Mate)	$r_2 = \dfrac{N_2 \sec \psi_2}{2P_n}$		
Base Radius (Mate)	$r_{b2} = r_2 \cos \phi_2$		
Line Segment $\overline{yb}$	$\overline{yb} = \sqrt{r_{o1}^2 - r_{b1}^2} - r_1 \sin \phi$		
Standard Center Distance[1]	$C_1 = r_1 + r_2$		

[1] When r_{c1} is Known the Mating Gear Data C_r and C_1 are Not Required, & Conversely

FIGURE 14. Form G Design calculation sheets for helical master gear (Ext. gear - Ext. mate with askew axes).

FORM G

Sheet 2 of 4

DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE)

Line Segment $\overline{cd}$		As Applicable Below		
Choose One Method As Applicable	Method #1 – When r_{c1} is Known	$\overline{cd} = -\sqrt{r_{c1}^2 - r_{b1}^2}$		
	Method #2 – When $C_r \equiv C_1$	$\overline{yc} = \frac{\left(\sqrt{r_{o2}^2 - r_{b2}^2} - r_2 \sin \emptyset_2\right) \sin \emptyset_2}{\sin \emptyset}$ $\overline{cd} = r_1 \sin \emptyset - \overline{yc}$		
	Method #3 – When $C_r \neq C_1$	See para 3.12		

CALCULATED MASTER DATA

Trial Pitch Radius (Select the Largest Value of R* Obtained by the Three Given Expressions)	$R^* = \frac{\overline{yb} \cos 5°}{\sin(\emptyset° - 5°)}$		
	$K = \sqrt{\overline{yb} \sin \emptyset \cos \emptyset + .0025}$ $R^* = \frac{.1K + \overline{yb} \sin \emptyset + .005 \cos \emptyset}{\sin^2 \emptyset}$		
	$R^* = 1.375 + \frac{1.157}{P}$		
Number of Teeth (Round to Next Larger Even Integer)	$N = 2PR^*$		N
Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = -\sqrt{(C \sin \emptyset - \overline{cd})^2 + R_b^2}$		
Center Distance Variation	$\Delta C = \frac{\Delta i_1 \cot \emptyset}{2}$		
Root Clearance	$c_R = C + h_{t1} - R_o - r_{o1} - \Delta C$ LIMIT ($c_R \geq c_R$ min, Table 1)		

ALTERNATE MASTER DATA

When $c_R < c_R$ min, Substitute Alternate Master Data	Root Clearance (Min)	Obtain c_R min from Table 1		
	Factor J	$J = (r_1 \sin \emptyset - \overline{cd}) \cos \emptyset$		
	Factor H	$H = r_1 \cos^2 \emptyset + \overline{cd} \sin \emptyset + h_{t1} - r_{o1} - \Delta C - c_R \min$		
	Trial Outside Radius	$R_o^* = \frac{H^2 + J^2}{2H}$		
	Number of Teeth (Round to Next Larger Even Integer)	$N = 2(PR_o^* - 1)$		N

FIGURE 14. Form G Design calculation sheets for helical master gear (Ext. gear - Ext. mate with askew axes) - Continued.

FORM G — DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE) — Sheet 3 of 4

When $cR < cR$ min, Substitute Alternate Master Data: Pitch Radius	$R = \frac{N}{2P}$ LIMIT ($R \leqq 4.0$, See para 3.7)		
Base Radius	$R_b = R \cos \emptyset$		
Center Distance	$C = \frac{N + N_1}{2P}$		
Outside Radius	$R_o = \sqrt{(C \sin \emptyset - \overline{cd})^2 + R_b^2}$		
TEST FOR POINTED ADDENDUM			
Secant of Pressure Angle At Outside Radius	$\sec \emptyset_o = \frac{R_o}{R_b}$		
Inv $\emptyset_o$	From suitable Table		
Arc Tooth Thickness At Outside Radius	$t_o = 2R_o\left(\frac{\pi - Pt_1}{N} + \text{inv}\,\emptyset - \text{inv}\,\emptyset_o\right)$ LIMIT ($t_o \geqq .005$, See para 3.9)		
FINAL MASTER DATA			
Outside Diameter	$D_o = 2R_o$		D_o
Pitch Diameter	$D = \frac{N}{P}$		D
Base Diameter	$D_b = D \cos \emptyset$		D_b
Form Diameter	$D_F = \sqrt{(D \sin \emptyset - 2\overline{yb})^2 + D_b^2} - \frac{.050}{P}$ LIMIT ($D_F \geqq D_b$)		D_F
Arc Tooth Thickness	$t = \frac{\pi}{P} - t_1$		t
Angle β (Degrees)	$\beta° = \left(\frac{\pi - Pt}{N \sec^2 \phi}\right)\frac{180°}{\pi}$		
Sin β	From suitable Table		
Cos ($\emptyset_n° + \beta°$)	From suitable Table		
Meas Wire Dia - Theoretical	$d_w^* \approx \frac{D \sin \beta \sec \psi}{\cos(\emptyset°_n + \beta°)}$		
Meas Wire Dia - Actual	Select d_w as nearest standard to d_w^*		d_w
Involute of Pressure Angle with Actual Meas Wires	$\text{inv}\,\emptyset_w = \frac{d_w \sin \emptyset}{D_b \sin \emptyset_n} + \frac{t}{D} + \text{inv}\,\emptyset - \frac{\pi}{N}$		
Sec $\emptyset_w$	From suitable Table		
Tan $\emptyset_w$	From suitable Table		
Meas over Wires	$M_w = D_b \sec \emptyset_w + d_w$		M_w
Contact Diameter with Measuring Wire	$D_{cw} = D_b\left[\sec \text{arc} \tan\left(\tan \emptyset_w - \frac{d_w \sin \emptyset_n}{D_b \sin \emptyset}\right)\right]$ LIMIT ($D_F < D_{cw} < D_o$)		

FIGURE 14. Form G Design calculation sheets for helical master gear (Ext. gear - Ext. mate with askew axes) - Continued.

FORM G	DESIGN CALCULATIONS FOR HELICAL MASTER GEAR (EXT GEAR – EXT MATE)		Sheet 4 of 4
Root Diameter (Select the Smallest Value Obtained by the Two Given Expressions)	$D_R = M_w - 2d_w - .010$		D_R
	$D_R = D_o - 2h_{t1}$		
Minimum Face Width	$F = F_1 + \frac{1}{4}$ (See para 3.10)		F
Contact Ratio	$m = \frac{(F_1 \tan \phi \cos \varnothing + r_1 \sin \varnothing + \overline{yb} - \overline{cd})P}{\pi \cos \varnothing}$ LIMIT ($m > 1.0$)		
Lead of Helix	$L = \frac{\pi D}{\tan \phi}$		L
Direction of Helix	Opposite Hand of Gear		H
CALCULATIONS FOR ADJUSTED OUTSIDE DIAMETER			
Secant of Maximum Pressure Angle with Meas Wires	$\sec \varnothing_{wx} = \frac{M_w + .005 - d_w}{D_b}$		
Inv $\varnothing_{wx}$	From suitable Table		
Involute of Operating Pressure Angle (Max)	$\text{inv}\varnothing_{rx} = \frac{P\left(t_1 + D \text{ inv}\varnothing_{wx} - \frac{d_w}{\cot \varnothing \sin \varnothing_n}\right) + N_1 \text{inv}\varnothing}{N + N_1}$		
Tan $\varnothing_{rx}$	From suitable Table		
Adjusted Outside Dia (Max)	$D_{ox} = \sqrt{4(C \cos \varnothing \tan \varnothing_{rx} - \overline{cd})^2 + D_b^2}$		D_{ox}

FIGURE 14. Form G Design calculation sheets for helical master gear (Ext. gear - Ext. mate with askew axes) - Continued.

APPENDIX

PARTIAL LISTING OF
GEAR INSPECTION EQUIPMENT SUPPLIERS

10. Scope. This appendix is not a mandatory part of this handbook. The information contained herein is intended for guidance only.

10.1 Disclaimer. This appendix is a partial listing of suppliers of gear inspection equipment or instruments and it is not to be interpreted as a recommendation or endorsement by the Government nor construed as a listing of the only qualified sources, but rather as a guide in the planning for an adequate gear inspection facility.

10.2 Spur and Helical gears. For spur and Helical gears, coarse and fine pitch, either external or internal.

10.2.1 Fellows Gear Shaper Company - Springfield, Vermont

a. Gear rolling instruments or red liners both hand roll and motor driven of variable center distance type of various sizes and capacities up to 20" pitch diameters, with or without electrical charting recorders, adjustable pressures and backlash measuring devices.

b. Involute measuring machines, two sizes for checking gears up to 24" pitch diameter from master cam without special base rolls. Supplied with or without electrical charting recorders with dual ratio device for representing 1/2° or 3° of involute roll increments on chart.

c. Lead measuring instruments for checking leads of gears up to 24" pitch diameter - sine bar or continuous-originating type, not a comparator. Furnished with electrical recorder only.

d. Cone, spacing, taper, and composite error inspection instrument for checking gears up to 12". Furnished with .0001 indicator.

e. Single convolution worm attachment for use on #4 Red Liner for checking tooth thickness, space width and runout of gears.

f. Variable center distance checking devices for use in automated lines.

10.2.2 Illinois Tool Works

a. Hand rolling checkers - coarse and fine pitch, stud and column types, in all sizes, with chart recorder attachment.

b. Machine type rolling checkers with chart recorders.

c. Involute checkers - coarse and fine pitch - with chart recorder.

d. Analytical checkers, lead, involute spacing - with chart recorder coarse and fine pitch. Automatic tooth space comparator - with chart recorder coarse and fine pitch.

e. Helical lead measuring machines in several sizes for coarse pitch with chart recorder.

f. Base pitch and space measuring machine to check spacing - both circular and base pitch on spur and helical gears.

10.2.3 Michigan Tool Company

a. Gear rolling fixtures - bench type.

b. Gear speeders - four sizes - machine type (coarse pitch) to check rolling action under power and high speed and to determine tooth contact pattern under load.

c. Involute checkers (sine bar principle) coarse or fine pitch - two models 14 sizes 0 to 36" diameter gears.

d. Lead checker - spur - helical and worm gears one model in four sizes 24" to 66" between centers - with chart recorder.

e. Angular index table - two models 12" diameter plate - ultra precise for checking teeth spacing.

10.2.4 National Broach & Machine Company

a. Analytical gear checkers, general purpose for spur and helical gears. Adjustable internal and external models. Check pitch diameter, size, tooth spacing, eccentricity and lead of helical gears; pitch diameter, size, tooth spacing, eccentricity and parallelism of spur gears. Electronic recorder optional.

b. Gear rolling fixtures, general purpose with parallel axes. Adjustable internal and external models. Electronic recorder optional.

c. Gear rolling fixtures, general purpose with crossed axes (zero to 90 degrees). Adjustable internal and external models. Electronic recorder optional.

d. Gear speeders, general purpose. Adjustable internal and external models. Used to check gears operating at any of four standard test speeds under modified load conditions.

e. Automatic gear gages, single purpose for high-production application. Check pitch diameter, size and lead of helical gears; pitch diameter, size and parallelism of spur gears. Electronic recorder optional.

f. Automatic gear speeders, single purpose for high-production application. Uses electronic sound discrimination principle. Used to check gears operating at prescribed test speed under modified load conditions. Electronic recorder optional.

10.2.5 Vinco Corporation

a. Optical Master Inspection Dividing Head for precision inspection of tooth spacing, increment inspection and verification of helix and gear tooth profiles, calibration of master involutes. Direct readings in degrees, minutes and seconds. Available in 12", 24" and 30" sizes - standard and heavy duty, either vertical or horizontal axes. Accuracy: two seconds of arc, total variation (± one second), spindle runout .000025 TIR max; No recorder.

b. Precisiondex - Mechanical Index Table and Dividing Head, recommended for inspection of tooth spacing, increment inspection of helix and involutes, direct readings in degrees, minutes and seconds, table release for rapid rotation. Accuracy: Standard Model - 15 seconds total variation; special model - 8 seconds; No recorder.

c. Involute checker - Sine bar principle, for coarse and fine pitch gears, capacity 24" diameter 55° and 110° roll angle. Accuracy: .0001, furnished with calibrated master involute; Recorder optional.

d. Gear rolling fixtures - All types and sizes, parallel and crossed axes, Recorder and Power Drive optional.

10.3 Bevel gears.

10.3.1 Fellows Gear Shaper Company, Springfield, Vermont

a. Red liners are used for checking conjugate action on any shaft angle by the use of special attachments.

10.3.2 Gleason Works, Rochester, New York

a. Universal Gear Testing Machines.

b. Hypoid Gear Testers (90° shaft angle).

c. Angular Hypoid Gear Testers (coarse pitch).

d. Angular Hypoid Gear Testers (fine pitch).

e. Tooth Spacing Tester.

f. Concentricity Tester.

10.3.3 Illinois Tool Works

a. A Bevel Gear Rolling Machine to check conjugate action of bevel gears, any shaft angle, coarse and fine pitch.

b. Bevel and Hypoid Gear Checker with chart recorder (for coarse and medium fine pitch bevel gears).

10.4 Worms and Worm Wheels.

10.4.1 Fellows Gear Shaper Company, Springfield, Vermont

a. Red liners are used for checking center distance variations by the use of special attachments.

b. Fellows 3-M hourglass red liner charts amount of clearance between worm and mating sector or roller (for hourglass worms).

10.4.2 Illinois Tool Works

a. Worm lead measuring machine in several sizes to check lead index for single and multiple thread worms, with chart recorder (coarse and fine pitch).

b. Worm and gear rolling fixtures and machines in several sizes, manual or motor drive.

10.4.3 Michigan Tool Company

a. Worm lead checker (sine bar principle) one size - 8" diameter work - 14" between centers. Will check up to 7.2 inch lead also checks index (with chart recorder).

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