NOTICE OF CHANGES

INCH-POUND
MIL-HDBK-299(SH)
NOTICE 1
15 OCTOBER 1991

MILITARY HANDBOOK

CABLE COMPARISON HANDBOOK
DATA PERTAINING TO ELECTRICAL SHIPBOARD CABLE

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2. RETAIN THIS NOTICE AND INSERT BEFORE TABLE OF CONTENTS.

3. Holders of MIL-HDBK-299(SH) will verify that page changes and additions indicated above have been entered. This notice page will be retained as a check sheet. This issuance, together with appended pages, is a separate publication. Each notice is to be retained by stocking points until the military handbook is completely revised or canceled.

Preparing activity:
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ELECTRICAL CABLE VOLTAGE DROP CALCULATIONS

10. SCOPE

10.1 Scope. This appendix is intended for use as a guide to determining cable voltage drops for alternating current (ac) and direct current (dc) power, lighting, electronic, interior communication, and weapon control systems.

20. REFERENCED DOCUMENTS

20.1 Government documents.

This paragraph is not applicable to this appendix.

20.2 Nongovernment publications. The following documents form a part of this handbook to the extent specified herein. Unless otherwise specified, the issues of the documents which are DoD adopted are those listed in the issue of the DoDISS cited in the solicitation. Unless otherwise specified, the issues of documents not listed in the DoDISS are the issues of the documents cited in the solicitation.

American Society For Testing And Materials (ASTM)

ASTM B 8 - Standard Specification for Concentric-Lay-Stranded Copper Conductors, Hard, Medium-Hard, or Soft. (DoD adopted)

ASTM B 258 - Standard Specification for Standard Nominal Diameters and Cross-Sectional Areas of AWG Sizes of Solid Round Wire Used as Electrical Conductors.

(Application for copies should be addressed to the American Society for Testing and Materials, 1916 Race Street, Philadelphia PA 19103)

(Nongovernment standards and other publications are normally available from the organizations that prepare or distribute the documents. These documents may be available in or through libraries or other informational services).

30. DEFINITIONS

30.1 Symbols and Abbreviations. The symbols and abbreviations used in this appendix are as follows:

Symbols	Parameter	Units
A	- Cross-sectional area of a conductor.	Cmil
D%	- Cable voltage drop expressed in percent with respect to system or source voltage.	--
DF	- Cable drop factor for equation using current.	$(\text{Amp.ft})^{-1}$
DF′	- Cable drop factor for equation using power.	$(\text{Watt.ft})^{-1}$
d	- Conductor diameter.	Mils
E	- Line-to-neutral rated voltage at the switchboard of a three-phase system or rated voltage of a single single-phase or dc system.	Volt
E_x	- Line-to-line rate voltage of a three-phase system (E_{AB}, E_{BC}, E_{CA}).	Volt
I	- Line current (I_A, I_B or I_C).	Ampere
I_{LO}	- Resultant load currents for each phase (leg) of a three-phase, delta circuit (I_{AB}, I_{BC} or I_{CA}).	Ampere
I_x	- Difference in two line currents of a three-phase system ($I_A - I_B$, $I_B - I_C$, $I_C - I_A$).	Ampere
L	- Cable length.	Feet
P	- Real power for each phase (leg) load of a three-phase delta circuit (P_{AB}, P_{BC}, or P_{CA}).	Watt
P_x	- Net real power in two lines of a three-phase, delta system ($P_A - P_B$, $P_B - P_C$, $P_C - P_A$).	Watt
pf	- Load power factor.	--
Q	- Reactive power for each phase (leg) load of a three-phase, delta circuit (Q_{AB}, Q_{BC} or Q_{CA}).	Var

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Q_x	- Net reactive power in two lines of a three-phase, delta system ($Q_A - Q_B$, $Q_B - Q_C$, $Q_C - Q_A$).	Var
R	- Total cable resistance per phase.	Ohm
R_{dc}	- Conductor dc resistance.	Ohm
S	- Apparent power for each phase (leg) load of a three-phase, delta circuit (S_{AB}, S_{BC} or S_{CA}).	Volt-amp
V	- Terminal voltage or load voltage.	Volt
V_{AN}	- Line-to-neutral terminal load voltage for phase A of a three-phase four-wire system.	Volt
X	- Total cable reactance per phase.	Ohm
Z	- Total cable impedance per phase.	Ohm
z	- Cable impedance per phase per foot.	Ohm/ft
Z_{LO}	- Load impedance in each phase (leg) of a three-phase delta circuit (Z_{AB}, Z_{BC}, Z_{CA}).	Ohm
Z_{AN}	- Load impedance in phase A of a three-phase, four-wire (wye) circuit.	Ohm
α	- Angle between terminal load voltage and cable voltage drop (IZ) .	Degree
α_x	- Angle between terminal load voltage and cable voltage drop (I_xZ).	Degree
β	- Cable impedance angle.	Degree
σ	- Mass density of a selected material.	g/cm^3
Θ	- Load power factor angle or angle between load voltage and line current.	Degree
Θ_x	- Angle between terminal load voltage and current I_x.	Degree
ρ	- Resistivity of a selected material at desired operating temperature.	Ohm.ft
ρ_o	- Resistivity of a selected material at 20°C.	Ohm.ft

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40. GENERAL EQUATIONS FOR CABLE VOLTAGE DROP CALCULATIONS

40.1 Voltage drop calculations for dc systems. The voltage drop for dc circuits is calculated from the following equations as derived in section 80.

40.1.1 Two-wire circuits.

- For all systems except power.

$$D\% = 22.78\left(\frac{IL}{AE}\right) * 100 \qquad (80\text{-}5)$$

- For power systems only.

$$D\% = 24.42\left(\frac{IL}{AE}\right) * 100 \qquad (80\text{-}6)$$

40.2 Voltage drop calculations for ac systems. The voltage drops for ac circuits are calculated from the following equations as derived in appropriate sections 90, 100, 110, and 120:

40.2.1 Single-phase circuits.

- For all systems.

$$D\% = 2IL\left(\frac{z\cos(\alpha)}{E}\right) * 100 \qquad (90\text{-}9)$$

40.2.2 Three-phase circuits.

- Voltage drop in each line for balanced systems such as electronic, interior communication, weapon control systems.

$$D\% = \sqrt{3}I_{LO}L\left(\frac{z\cos(\alpha)}{E}\right) * 100 \qquad (90\text{-}10)$$

- Voltage drop in each line for all systems.

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$$D\% = IL\left(\frac{z\cos(\alpha)}{E}\right) * 100 \qquad (100\text{-}10)$$

- Voltage drop in each phase for all systems.

$$D\% = I_xL\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100 \qquad (100\text{-}17)$$

40.3 Voltage drop calculations using drop factors. The following simplified equations are used for percent drop calculations in lighting and power systems in conjunction with the cable drop factors listed in tables XIII through XVII.

40.3.1 Lighting systems.

40.3.1.1 Single-phase circuit.

$D\% = 2IL(DF)$ Table XVI.

$D\% = 2PL(DF')$ Table XVII.

40.3.1.2 Three-phase circuit.

$D\% = I_xL(DF)$ Table XVI.

$D\% = P_xL(DF')$ Table XVII.

40.3.2 Power Systems.

40.3.2.1 Three-phase circuit. The drop in each line is:

$D\% = IL(DF)$ Tables XIII, XIV, and XV.

Where:

- I, I_{LO}, I_x, Θ, α, Θ_x, and α_x are calculated in section 50.
- P and P_x are calculated in 110.
- DF and DF′ are calculated and tabulated in 70.1.

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50. CURRENT CALCULATIONS FOR AC SYSTEMS

50.1 Single-phase circuits. The line current (I) used in the voltage drop equations is the scalar magnitude of current (I_{LO}) obtained by adding vectorially all load currents in the branch:

$$I = I_{LO}$$

The angles Θ, α, and β are calculated from the following equations as derived in section 90.:

$$\Theta = \cos^{-1}(pf), \quad \beta = \tan^{-1}(X/R)$$

$$\alpha = -\Theta + \beta \tag{90-2}$$

Θ is positive if the load power factor (pf) is lagging.
Θ is negative if the load power factor (pf) is leading.

50.2 Three-phase-delta circuits.

50.2.1 Balanced systems. If the system is balanced, the magnitude of the line current is equal to:

$$I = \sqrt{3}I_{LO} = \sqrt{3}I_{AB} = \sqrt{3}I_{BC} = \sqrt{3}I_{CA}$$

The total load currents $\overline{I}_{AB}$, $\overline{I}_{BC}$, and $\overline{I}_{CA}$ are the vectorial sum of all the currents in their respective phase (leg) loads. If the load power factor angles in legs AB, BC, and CA are Θ_{AB}, Θ_{BC}, and Θ_{CA} respectively, the phase load currents can be written as:

$$\overline{I}_{AB} = I_{AB}\underline{/(0°-\Theta_{AB})}, \quad \overline{I}_{BC} = I_{BC}\underline{/(-120°-\Theta_{BC})}, \quad \overline{I}_{CA} = I_{CA}\underline{/(120°-\Theta_{CA})}$$

Similarly, the phase voltages $\overline{V}_{AB}$, $\overline{V}_{BC}$, and $\overline{V}_{CA}$ are defined as:

$$\overline{V}_{AB} = V_{AB}\underline{/0°}, \quad \overline{V}_{BC} = V_{BC}\underline{/-120°}, \text{ and } \overline{V}_{CA} = V_{CA}\underline{/120°}$$

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50.2.2 Balanced/unbalaced systems. If the system is unbalanced or balanced, the magnitude of line currents I and the associated angles Θ and α are calculated from the following equations as derived in section 100:

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}, \qquad (100\text{-}7)$$

$$\Theta_A = 0° - \underline{/I_A}$$

$$\alpha_A = -\Theta_A + \beta$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}, \qquad (100\text{-}8)$$

$$\Theta_B = -120° - \underline{/I_B}$$

$$\alpha_B = -\Theta_B + \beta$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}, \qquad (100\text{-}9)$$

$$\Theta_C = 120° - \underline{/I_C}$$

$$\alpha_C = -\Theta_C + \beta$$

The currents I_x and the associated angles Θ_x and α_x are calculated as follows:

$$\bar{I}_{x(AB)} = \bar{I}_A - \bar{I}_B, \qquad (100\text{-}12)$$

$$\Theta_{x(AB)} = 0° - \underline{/I_{x(AB)}}$$

$$\alpha_{x(AB)} = -\Theta_{x(AB)} + \beta$$

$$\bar{I}_{x(BC)} = \bar{I}_B - \bar{I}_C \qquad (100\text{-}13)$$

$$\Theta_{x(BC)} = -120° - \underline{/I_{x(BC)}}$$

$$\alpha_{x(BC)} = -\Theta_{x(BC)} + \beta$$

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$$\bar{I}_{x(CA)} = \bar{I}_C - \bar{I}_A, \qquad (100\text{-}14)$$

$$\Theta_{x(CA)} = 120° - \underline{/I}_{x(CA)}$$

$$\alpha_{x(CA)} = -\Theta_{x(CA)} + \beta$$

50.3 Three-phase four-wire circuits. The individual line currents (I) are calculated from the following equation as derived in section 120.:

$$I = I_{LO} = \frac{V_{AN}}{Z_{AN}} \qquad (120\text{-}3)$$

50.4 Example of voltage drop calculations. The following example is a sample calculation of percent voltage drop for a three-phase lighting system (See figures 3 and 4 in section 100).

Step 1.

Determine the load current (I_{LO}) in each phase by adding vectorially all the connected load currents in that phase. Let assume the total load current for each phase as follows:

$$I_{AB} = 6.16A$$

$$I_{BC} = 5.7A$$

$$I_{CA} = 9.5A$$

Also the following parameters are given in Table XVI:

Cable type : LSTSGU-9
Length: $L = 45ft$
Impedance: $z = 1.120(10^{-3})\ \Omega/ft$
Angle: $\beta = 1.82°$

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Step 2.

Calculate phase load currents. Let the terminal load voltages be:

$$\bar{V}_{AB} = V_{AB}\underline{/0°}$$

$$\bar{V}_{BC} = V_{BC}\underline{/-120°}$$

$$\bar{V}_{CA} = V_{CA}\underline{/120°}$$

Assume load power factor for each phase of 0.80 lagging, the phase currents with respect to their respective terminal voltages can be written as follows:

$$\bar{I}_{AB} = I_{AB}\underline{/0° - \Theta_{AB}} = 6.16\underline{/-37°}$$

$$\bar{I}_{BC} = I_{BC}\underline{/-120° - \Theta_{BC}} = 5.7\underline{/-157°}$$

$$\bar{I}_{CA} = I_{CA}\underline{/120° - \Theta_{CA}} = 9.5\underline{/83°}$$

Where $\Theta_{AB} = \Theta_{BC} = \Theta_{CA} = \cos^{-1}(0.80) = 37°$.

Step 3.

Determine line current I, angles Θ and angle α from equations (100-7), (100-8), and (100-9) respectively after converting the phase currents from their polar forms to rectangular forms.

$$\bar{I}_{AB} = 6.16\ \underline{/-37°} = 4.92 - j3.71$$

$$\bar{I}_{BC} = 5.7\underline{/-157°} = -5.25 - j2.23$$

$$\bar{I}_{CA} = 9.5\underline{/83°} = 1.16 + j9.43$$

The line currents are calculated as follows:

$$\bar{I}_{A} = \bar{I}_{AB} - \bar{I}_{CA}, \qquad (100\text{-}7)$$

$$= (4.92 - j3.71) - (-1.16 + j9.43)$$

$$\bar{I}_A = 3.76 - j13.14$$
$$= 13.67\underline{/285.97°}$$

and,

$$\Theta_A = 360° - 285.97°$$
$$= 74.03°$$

$$\alpha_A = -74.03° + 1.82°,$$
$$= -72.21°$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}, \quad (100\text{-}8)$$

$$= (-5.25 - j2.23) - (4.92 - j3.71)$$
$$= -10.17 + j1.48$$
$$= 10.28\underline{/171.72°}$$

and,

$$\Theta_B = -120° - 171.72°$$
$$= -291.72°$$

$$\alpha_B = -(-291.72°) + 1.82°,$$
$$= 293.54°,$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}, \quad (100\text{-}9)$$

$$= (1.16 + j9.43) - (-5.25 - j2.23)$$
$$= 6.41 + j11.66$$
$$= 13.31\underline{/61.20°}$$

and,

$$\Theta_C = 120° - 61.20°$$
$$= 58.8°$$

$$\alpha_C = -58.80° + 1.82°,$$
$$= -56.98°,$$

Step 4.

The drop in each line from the switchboard to the load is calculated as follows:

$$D_A\% = I_A L \left(\frac{z\cos(\alpha_A)}{E}\right) * 100, \quad (100\text{-}10)$$

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$$D_A\% = (13.67)(45)\left(\frac{(1.120 * 10^{-3})\cos(-72.21°)}{(120/\sqrt{3})}\right) * 100$$
$$= 0.30\%$$

$$D_B\% = I_B L\left(\frac{z\cos(\alpha_B)}{E}\right) * 100, \qquad (100\text{-}10)$$

$$= (10.28)(45)\left(\frac{(1.120 * 10^{-3})\cos(293.54°)}{(120/\sqrt{3})}\right) * 100$$
$$= 0.30\%$$

$$D_C\% = I_C L\left(\frac{z\cos(\alpha_C)}{E}\right) * 100, \qquad (100\text{-}10)$$

$$= (13.30)(45)\left(\frac{(1.120 * 10^{-3})\cos(-57.98°)}{(120/\sqrt{3})}\right) * 100$$
$$= 0.53\%$$

The total drop ($D_T\%$) in this portion of cable is the combination of the drops in the lines:

$$D_T\% = D_A\% + D_B\% + D_C\%$$
$$= 0.30\% + 0.30\% + 0.53\%$$
$$= 1.13\%$$

<u>Step 5</u>.

If the voltage drop in each phase is desired, the currents I_x must be determined from the following equations:

The current in loop E_{AB} is:

$$\overline{I}_{x(AB)} = \overline{I}_A - \overline{I}_B, \qquad (100\text{-}12)$$
$$= (3.76 - j13.14) - (-10.17 + j1.48)$$
$$= 13.93 - j14.62$$
$$= 20.19\underline{/313.62°}$$

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$$\overline{I}_{x(AB)} = 13.93 - j14.62$$
$$= 20.19\underline{/313.62°}$$

and,

$$\Theta_{x(AB)} = 360° - 313.62°$$
$$= 46.38°$$

$$\alpha_{x(AB)} = -46.38° + 1.82°$$
$$= -44.54°$$

The current in loop E_{BC} is:

$$\overline{I}_{x(BC)} = \overline{I}_B - \overline{I}_C, \qquad (100\text{-}13)$$

$$= (-10.17 + j1.48) - (6.41 + j11.66)$$
$$= -16.58 - j10.18$$
$$= 19.46\underline{/238.45°}$$

and,

$$\Theta_{x(BC)} = -120° - 238.45°$$
$$= -358.45°$$

$$\alpha_{x(BC)} = -(-358.45°) + 1.82°$$
$$= 360.27°$$

The current in loop E_{CA} is:

$$\overline{I}_{x(CA)} = \overline{I}_C - \overline{I}_A, \qquad (100\text{-}14)$$

$$= (6.41 + j11.66) - (3.76 - j13.14)$$
$$= 2.65 + j24.80$$
$$= 24.94\underline{/83.9°}$$

and,

$$\Theta_{x(CA)} = 120° - 83.90°$$
$$= 36.1°$$

$$\alpha_{x(CA)} = -36.1° + 1.82°$$
$$= -34.28°$$

Step 6.

The percent drop in each phase is calculated as follows:

$$D_{AB}\% = (20.19)(45)\left(\frac{(1.120)(10^{-3})\cos(-44.54°)}{120}\right) * 100$$

$$= 0.60\%$$

$$D_{BC}\% = I_{x(BC)}L\left(\frac{z\cos(\alpha_{x(BC)})}{E_{BC}}\right) * 100, \qquad (100\text{-}17)$$

$$= (19.46)(45)\left(\frac{(1.120)(10^{-3})\cos(360.27°)}{120}\right) * 100$$

$$= 0.82\%$$

$$D_{CA}\% = I_{x(CA)}L\left(\frac{z\cos(\alpha_{x(CA)})}{E_{CA}}\right) * 100, \qquad (100\text{-}17)$$

$$= (24.94)(45)\left(\frac{(1.120)(10^{-3})\cos(-34.28°)}{120}\right) * 100$$

$$= 0.87\%$$

The drop in this portion of the cable is the combination of the results in step 6 as follows:

$$D_T(\%) = 1/2(D_{AB}\% + D_{BC}\% + D_{CA}\%)$$

$$= 1/2(0.60\% + 0.82\% + 0.87\%)$$

$$= 1.14\%$$

60. VOLTAGE DROP CALCULATIONS FOR UNBALANCED SYSTEMS BY SYMMETRICAL COMPONENT METHOD

To calculate cable voltage drop for unbalanced system, the line currents supplying the loads must be calculated. Assume all three phase loads are unbalanced. Therefore, the phase load currents are different and must be calculated individually. From the phase load currents, the individual line currents will be calculated. From the line currents, the voltage drop in each line will be determined. All necessary equations are derived in section 130.

60.1 Calculation Procedure. The step by step procedure for the percent drop determination is as follows:

Step 1.

Locate or determine the following necessary parameters:

- Vectorial sum of all load currents for each phase.
- Real power for each phase load.
- Load power factor for each phase.
- Load impedance for each phase.
- Rated load voltage for each phase.

Step 2.

Calculate the load current for each phase. Two methods will be used. If real power, power factor, and rated voltage of the loads are given, use the following equations:

$$\overline{I}_{AB} = [P_{AB}/V_{AB}\cos(\Theta_{AB})]\underline{/(0 - \Theta_{AB})}^\circ, \quad (130\text{-}31)$$

$$\overline{I}_{BC} = [P_{BC}/V_{BC}\cos(\Theta_{BC})]\underline{/(-120 - \Theta_{BC})}^\circ, \quad (130\text{-}32)$$

$$\overline{I}_{CA} = [P_{CA}/V_{CA}\cos(\Theta_{CA})]\underline{/(120 - \Theta_{CA})}^\circ, \quad (130\text{-}33)$$

If the impedances and rated voltages of the loads are given, use the following equations:

$$\overline{I}_{AB} = \overline{V}_{AB}/\ \overline{Z}_{AB}, \quad (130\text{-}34)$$

$$\overline{I}_{BC} = \overline{V}_{BC}/\ \overline{Z}_{BC}, \quad (130\text{-}35)$$

$$\overline{I}_{CA} = \overline{V}_{CA}/\ \overline{Z}_{CA}, \quad (130\text{-}36)$$

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Step 3.

Calculate the positive and negative sequence currents. Take phase AB as reference, the positive and negative sequence currents are calculated as follows:

$$\bar{I}_{AB1} = (1/3)\ (\bar{I}_{AB} + a\bar{I}_{BC} + a^2\bar{I}_{CA}), \qquad (130\text{-}29)$$

$$\bar{I}_{AB2} = (1/3)\ (\bar{I}_{AB} + a^2\bar{I}_{BC} + a\bar{I}_{CA}), \qquad (130\text{-}30)$$

Where:

$$a = 1\underline{/120°}\ , \qquad a^2 = 1\underline{/240°}$$

Note: In a three-phase and three-wire system, the zero sequence currents are zero as shown in section 130.

Step 4.

Calculate the sequence components of line currents from the following equations:

$$\bar{I}_{A1} = \bar{I}_{AB1}(\sqrt{3}\underline{/-30°}), \qquad (130\text{-}8)$$

$$\bar{I}_{A2} = \bar{I}_{AB2}(\sqrt{3}\underline{/30°}), \qquad (130\text{-}11)$$

$$\bar{I}_{B1} = \bar{I}_{AB1}(\sqrt{3}\underline{/-150°}), \qquad (130\text{-}14)$$

$$\bar{I}_{B2} = \bar{I}_{AB2}(\sqrt{3}\underline{/150°}), \qquad (130\text{-}17)$$

$$\bar{I}_{C1} = \bar{I}_{AB1}(\sqrt{3}\underline{/90°}), \qquad (130\text{-}21)$$

$$\bar{I}_{C2} = \bar{I}_{AB2}(\sqrt{3}\underline{/-90°}), \qquad (130\text{-}25)$$

Step 5.

The line currents flowing into each node of the delta connected loads as shown in figure 7 are calculated as follows:

$$\bar{I}_{A} = \bar{I}_{AB1}(\sqrt{3}\underline{/-30°}) + \bar{I}_{AB2}(\sqrt{3}\underline{/30°}), \qquad (130\text{-}26)$$

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$$\bar{I}_B = \bar{I}_{AB1}(\sqrt{3}\underline{/-150°}) + \bar{I}_{AB2}(\sqrt{3}\underline{/150°}), \quad (130\text{-}27)$$

$$\bar{I}_C = \bar{I}_{AB1}(\sqrt{3}\underline{/90°}) + \bar{I}_{AB2}(\sqrt{3}\underline{/-90°}), \quad (130\text{-}28)$$

The corresponding angles Θ and α are calculated from equations derived in section 100.:

$$\Theta_A = 0° - \underline{/I_{A}}, \qquad \alpha_A = -\Theta_A + \beta, \quad (100\text{-}7)$$

$$\Theta_B = -120° - \underline{/I_{B}}, \qquad \alpha_B = -\Theta_B + \beta, \quad (100\text{-}8)$$

$$\Theta_C = 120° - \underline{/I_{C}}, \qquad \alpha_C = -\Theta_C + \beta, \quad (100\text{-}9)$$

Step 6.

If the cable length and cable impedance are known, the voltage drop in each line is:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E}\right) * 100, \quad (100\text{-}10)$$

60.2 Example of Voltage Drop Calculations.

Step 1.

The following parameters are given:

$$\bar{V}_{AB} = 118\underline{/0°},\ \bar{V}_{BC} = 118\underline{/-120°},\ \bar{V}_{CA} = 118\underline{/120°}$$

$$P_{AB} = 3400W,\ P_{BC} = 2500W,\ P_{CA} = 2900W.$$

$$E_{AB} = E_{BC} = E_{CA} = 120V$$

All load power factors (pf) = 0.80 lagging. From Table XVI, use LSTSGU-50 cable with $z = 0.223(10^{-3})\Omega/ft$, $\beta = 7.8°$

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Step 2.

Calculate the phase currents from the following equations:

$$\begin{aligned}\bar{I}_{AB} &= [P_{AB}/\ V_{AB}\cos(\Theta_{AB})]\underline{/(0 - \Theta_{AB})^\circ}, && (130\text{-}31)\\ &= [3400/(118)(0.8)]\underline{/(0^\circ - 37^\circ)}\\ &= 36\underline{/-37^\circ}\end{aligned}$$

$$\begin{aligned}\bar{I}_{BC} &= [P_{BC}/V_{BC}\cos(\Theta_{BC})]\underline{/(-120 - \Theta_{BC})^\circ}, && (130\text{-}32)\\ &= [2500/(118)(0.8)]\underline{/-120^\circ - 37^\circ}\\ &= 26.5\underline{/-157^\circ}\end{aligned}$$

$$\begin{aligned}\bar{I}_{CA} &= [P_{CA}/V_{CA}\cos(\Theta_{CA})]\underline{/(120 - \Theta_{CA})^\circ}, && (130\text{-}33)\\ &= [2900/(118)(0.8)]\underline{/120^\circ - 37^\circ}\\ &= 30.7\underline{/83^\circ}\end{aligned}$$

Step 3.

Calculate the positive and negative sequences of $\bar{I}_{AB}$ from the following equations:

$$\begin{aligned}\bar{I}_{AB1} &= (1/3)(\bar{I}_{AB} + a\bar{I}_{BC} + a^2\bar{I}_{CA}), && (130\text{-}29)\\ &= (1/3)[36.0\underline{/-37^\circ} + (1\underline{/120^\circ})(26.5\underline{/-157^\circ})\\ &\quad + (1\underline{/240^\circ})(30.7\underline{/83^\circ})]\\ &= 31.0\underline{/-37^\circ}\end{aligned}$$

$$\begin{aligned}\bar{I}_{AB2} &= (1/3)(\bar{I}_{AB} + a^2\bar{I}_{BC} + a\bar{I}_{CA}), && (130\text{-}30)\\ &= (1/3)[36.0\underline{/-37^\circ} + (1\underline{/240^\circ})(26.5\underline{/-157^\circ})\\ &\quad + (1\underline{/120^\circ})(30.7\underline{/83^\circ})\\ &= 2.6\underline{/-62.5^\circ}\end{aligned}$$

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Step 4.

Calculate the sequence components of line currents from the following equations:

$$\bar{I}_{A1} = \bar{I}_{AB1}(\sqrt{3}\underline{/-30°}), \quad (130\text{-}8)$$
$$= (31.0\underline{/-37°})(\sqrt{3}\underline{/-30°})$$
$$= 53.7\underline{/-67°}$$

$$\bar{I}_{A2} = \bar{I}_{AB2}(\sqrt{3}\underline{/30°}), \quad (130\text{-}11)$$
$$= (2.6\underline{/-62.5°})(\sqrt{3}\underline{/30°})$$
$$= 4.5\underline{/-32.5°}$$

$$\bar{I}_{B1} = \bar{I}_{AB1}(\sqrt{3}\underline{/-150°}), \quad (130\text{-}14)$$
$$= (31.0\underline{/-37°})(\sqrt{3}\underline{/-150°})$$
$$= 53.7\underline{/-187°}$$

$$\bar{I}_{B2} = \bar{I}_{AB2}(\sqrt{3}\underline{/150°}), \quad (130\text{-}17)$$
$$= (2.6\underline{/-62.5°})(\sqrt{3}\underline{/150°})$$
$$= 4.5\underline{/87.5°}$$

$$\bar{I}_{C1} = \bar{I}_{AB1}(\sqrt{3}\underline{/90°}), \quad (130\text{-}21)$$
$$= (31.0\underline{/-37°})(\sqrt{3}\underline{/90°})$$
$$= 53.7\underline{/53°}$$

$$\bar{I}_{C2} = \bar{I}_{AB2}(\sqrt{3}\underline{/-90°}), \quad (130\text{-}25)$$
$$= (2.6\underline{/-62.5°})(\sqrt{3}\underline{/-90°})$$
$$= 4.5\underline{/-152.5°}$$

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Step 5.

Calculate the line currents $\overline{I}_A$, $\overline{I}_B$, and $\overline{I}_C$ from the sequence currents as follows:

$$\overline{I}_A = \overline{I}_{A0} + \overline{I}_{A1} + \overline{I}_{A2}, \qquad (130\text{-}26)$$

$$= 0 + 53.7\underline{/-67^\circ} + 4.5\underline{/-32.5^\circ}$$
$$= 24.78 - j51.85$$
$$= 57.47\underline{/-64.46^\circ}$$

Then,

$$\Theta_A = 0^\circ - (-64.46^\circ)$$
$$= 64.46^\circ$$

$$\alpha_A = -\Theta_A + \beta$$

$$= -64.46^\circ + 7.8^\circ$$
$$= -56.66^\circ$$

$$\overline{I}_B = \overline{I}_{B0} + \overline{I}_{B1} + \overline{I}_{B2}, \qquad (130\text{-}27)$$
$$= 0 + 53.7\underline{/-187^\circ} + 4.5\underline{/87.5^\circ}$$
$$= - 53.10 + j11.04$$
$$= 54.24\underline{/168.26^\circ}$$

Then,

$$\Theta_B = -120^\circ - 168.26^\circ$$
$$= -288.26^\circ$$

$$\alpha_B = -\Theta_B + \beta$$

$$= 288.26^\circ + 7.8^\circ$$
$$= 296.06^\circ$$

$$\overline{I}_C = \overline{I}_{C0} + \overline{I}_{C1} + \overline{I}_{C2}, \qquad (130\text{-}28)$$

$$= 0 + 53.7\underline{/53^\circ} + 4.5\underline{/-152.5^\circ}$$
$$= 28.33 + j40.81$$
$$= 49.68\underline{/55.24^\circ}$$

Then,

$$\Theta_C = 120^\circ - 55.24^\circ$$
$$= 64.76^\circ$$

$$\alpha_C = -\Theta_C + \beta$$

$$= -64.76^\circ + 7.8^\circ$$
$$= -56.96^\circ$$

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Step 6.

Determine the percent voltage drops in the lines, and compare the results.

$$D_A\% = I_A L \left(\frac{z\cos(\alpha_A)}{E}\right) * 100, \qquad (100\text{-}10)$$

$$= (57.47)(65)\left(\frac{(0.223)(10^{-3})\cos(-56.66°)}{120/\sqrt{3}}\right) * 100$$

$$= 0.66\%$$

$$D_B\% = I_B L \left(\frac{z\cos(\alpha_B)}{E}\right) * 100, \qquad (100\text{-}10)$$

$$= (54.24)(65)\left(\frac{(0.223)(10^{-3})\cos(296.06°)}{120/\sqrt{3}}\right) * 100$$

$$= 0.50\%$$

$$D_C\% = I_A L \left(\frac{z\cos(\alpha_C)}{E}\right) * 100, \qquad (100\text{-}10)$$

$$= (49.68)(65)\left(\frac{(0.223)(10^{-3})\cos(-56.96°)}{120/\sqrt{3}}\right) * 100$$

$$= 0.57\%$$

The drop in this portion of cable is obtained by combining the drop in the individual lines (phase):

$$D_T\% = D_A\% + D_B\% + D_C\%$$

$$= 0.66\% + 0.50\% + 0.57\%$$

$$= 1.73\%$$

70. CABLE IMPEDANCES AND DROP FACTORS

The drop factors and impedances are calculated based on the

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characteristics of LSTSGU and LS6SGU cables in accordance with MIL-C-24643. The tabulated drop factors for these cables may be used for other types of cables with similar impedance characteristics including lightweight cables in accordance with MIL-C-24640.

70.1 Drop factor calculations. The drop factors in tables XIII through XVII for power and lighting systems are calculated from the following equations derived in sections 90, 100, and 110:

70.1.1 Power systems.

$$DF = \left(\frac{z\cos(\alpha)}{E} + \frac{(2)\,z\tan(\alpha)\sin(\alpha)}{200E} \right) * 100, \qquad (90\text{-}13)$$

70.1.2 Lighting systems.

70.1.2.1 Single-phase Circuits.

- For equations using Current I.

$$DF = \left(\frac{z\cos(\alpha)}{E} \right) * 100, \qquad (100\text{-}20)$$

- For equations using Power P.

$$DF' = \left(\frac{1}{V\cos(\Theta)} \right) \left(\frac{z\cos'(\alpha)}{E} \right) * 100, \qquad (110\text{-}26)$$

70.1.2.2 Three-phase Circuits.

- For equations using Current I_x.

$$DF = \left(\frac{z\cos(\alpha_x)}{E_x} \right) * 100, \qquad (100\text{-}19)$$

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TABLE XI. Impedances for LSTSGU cable, 60Hz electronics and communications.

Size	Cable characteristics				
	A 1/ Cmil	R 2/,3/ Ohms per 100 feet	X 3/	Z	B °C
3	2580	4.503	0.043	4.503	0.55
4	4110	2.825	0.040	2.825	0.81
9	10380	1.119	0.036	1.120	1.82
14	13090	0.887	0.041	0.888	2.63
23	20820	0.558	0.039	0.559	4.02
50	52620	0.221	0.030	0.223	7.81
75	83690	0.138	0.030	0.141	12.07
100	105600	0.110	0.029	0.114	14.92
150	167800	0.069	0.029	0.075	22.58
200	211600	0.055	0.029	0.062	27.80
300	300000	0.040	0.029	0.049	35.75
400	413600	0.029	0.029	0.041	44.80

TABLE XII. Impedances for LSTSGU cable, 400 Hz electronics and communications.

Size	Cable characteristics				
	A 1/ Cmil	R 2/,3/ Ohms per 1000 feet	X 3/	Z	B °C
3	2580	4.503	0.288	4.512	3.66
4	4110	2.825	0.267	2.838	5.40
9	10380	1.119	0.237	1.144	11.98
14	13090	0.887	0.271	0.928	17.01
23	20820	0.558	0.262	0.616	25.12
50	52620	0.221	0.202	0.299	42.44
75	83690	0.143	0.197	0.243	53.98
100	105600	0.116	0.195	0.227	59.29
150	167800	0.077	0.191	0.206	68.10
200	211600	0.065	0.194	0.204	71.44
300	300000	0.051	0.192	0.199	75.14
400	413600	0.042	0.192	0.196	77.65

1/ Conductor cross-sectional areas are based on MIL-C-24643.
2/ Resistances are derived at a temperature of 45°C.
3/ Resistances and reactances per phase are calculated in section 140.

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TABLE XIII. Drop factors for LSTSGU cable, 450V, three-phase, 60Hz power systems. 1/

Size	Cable characteristics					Drop factors at cos (θ) 2/ below (multiply by 10^{-5})														
	A 3/ Cmil	R 4/,5/ Ohms per 1000 feet	X 4/ Ohms per 1000 feet	Z Ohms per 1000 feet	θ °C	1.0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
3	2580	4.827	0.043	4.827	0.51	185.8	177.2	168.4	159.4	150.4	141.5	132.5	123.5	114.8	106.0	97.1	88.2	79.7	71.1	62.0
4	4110	3.028	0.040	3.028	0.76	116.5	111.3	105.8	100.2	94.6	89.1	83.5	77.9	72.4	66.9	61.3	55.7	50.4	45.1	39.9
9	10380	1.199	0.036	1.200	1.70	46.1	44.3	42.2	40.1	37.9	35.8	33.6	31.4	29.3	27.1	24.9	22.7	20.6	18.5	16.4
14	13090	0.951	0.041	0.952	2.45	36.6	35.3	33.7	32.0	30.4	28.7	27.0	25.3	23.6	21.9	20.1	18.4	16.8	15.1	13.4
23	20820	0.598	0.039	0.599	3.75	23.0	22.4	21.4	20.4	19.4	18.4	17.3	16.3	15.2	14.2	13.1	12.0	11.0	9.9	8.9
50	52620	0.237	0.030	0.239	7.29	9.1	9.0	8.7	8.4	8.0	7.6	7.3	6.9	6.5	6.1	5.7	5.3	4.9	4.4	4.0
75	83690	0.148	0.030	0.151	11.27	5.7	5.8	5.6	5.4	5.2	5.0	4.8	4.6	4.4	4.1	3.9	3.6	3.4	3.1	2.9
100	105600	0.118	0.029	0.122	13.94	4.5	4.7	4.6	4.5	4.3	4.2	4.0	3.8	3.7	3.5	3.3	3.1	2.9	2.7	2.9
150	167800	0.074	0.029	0.079	21.20	2.9	3.1	3.0	3.0	2.9	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1	1.9
200	211600	0.059	0.029	0.066	26.18	2.3	2.5	2.5	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.0	1.9	1.9	1.8
300	300000	0.042	0.029	0.051	34.44	1.6	1.9	1.9	2.0	2.0	1.9	1.9	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6
400	413600	0.031	0.029	0.042	42.89	1.2	1.5	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.4

1/ Drop factor equations are derived in section 90.
2/ θ is the load power factor angle or angle between load terminal voltage and line current.
3/ Conductor cross-sectional areas are based on MIL-C-24643.
4/ Resistances and reactances per phase are calculated in section 140.
5/ Resistances are derived at a temperature of 65°C.

TABLE XIV. Drop factors for LSTSGU cable, 450V, three-phase, 400Hz power systems. 1/

Size	Cable characteristics					Drop factors at cos (θ) 2/ below (multiply by 10^{-5})														
	A 3/ Cmil	R 4/,5/ Ohms per 1000 feet	X 4/ Ohms per 1000 feet	Z Ohms per 1000 feet	θ °C	1.0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
3	2580	4.827	0.288	4.836	3.42	185.8	180.1	172.4	164.2	155.9	147.6	139.1	130.5	122.1	113.5	104.9	96.2	87.8	79.3	71.0
4	4110	3.028	0.267	3.040	5.04	116.6	114.0	109.6	104.7	99.8	94.7	89.5	84.3	79.2	73.9	68.5	63.2	58.0	52.7	47.0
9	10380	1.199	0.237	1.222	11.20	46.2	46.7	45.6	44.1	42.5	40.8	39.0	37.2	35.3	33.4	31.4	29.4	27.4	25.4	23.0
14	13090	0.951	0.271	0.989	15.92	36.6	38.0	37.5	36.6	35.6	34.4	33.2	31.9	30.5	29.1	27.6	26.1	24.6	23.0	21.0
23	20820	0.598	0.262	0.653	23.63	23.1	25.0	25.1	24.9	24.5	23.9	23.3	22.7	21.9	21.2	20.3	19.5	18.6	17.7	16.7
50	52620	0.237	0.202	0.311	40.46	9.2	11.1	11.6	11.9	12.0	12.0	11.9	11.8	11.7	11.5	11.3	11.1	10.8	10.5	10.2
75	83690	0.153	0.197	0.249	52.12	6.0	8.0	8.6	9.0	9.3	9.4	9.5	9.6	9.6	9.6	9.5	9.4	9.3	9.2	9.0
100	105600	0.124	0.195	0.231	57.59	4.9	6.9	7.6	8.0	8.3	8.6	8.7	8.8	8.9	8.9	8.9	8.9	8.8	8.7	8.6
150	167800	0.082	0.191	0.208	66.82	3.3	5.4	6.1	6.6	7.0	7.3	7.5	7.7	7.8	7.9	8.0	8.0	8.0	8.0	8.0
200	211600	0.069	0.194	0.206	70.38	2.9	4.9	5.7	6.2	6.6	6.9	7.2	7.4	7.6	7.7	7.8	7.9	7.9	7.9	7.9
300	300000	0.054	0.192	0.200	74.31	2.3	4.4	5.2	5.7	6.1	6.5	6.8	7.0	7.2	7.3	7.5	7.5	7.6	7.7	7.7
400	413600	0.045	0.192	0.197	76.80	2.0	4.1	4.8	5.4	5.9	6.2	6.5	6.8	7.0	7.1	7.3	7.4	7.5	7.5	7.6

1/ Drop factor equations are derived in section 90.
2/ θ is the load power factor angle or angle between load terminal voltage and line current.
3/ Conductor cross-sectional areas are based on MIL-C-24643.
4/ Resistances and reactances per phase are calculated in section 140.
5/ Resistances are derived at a temperature of 65°C.

TABLE XV. Drop factors for LS6SGU cable, 450V, three-phase, 400Hz power systems. 1/

Size	Cable characteristics					Drop factors at cos (θ) 2/ below (multiply by 10^{-5})														
	A 3/ Cmil	R 4/,5/ Ohms per 1000 feet	X 4/	Z	θ °C	1.0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
100	211200	0.062	0.098	0.116	57.60	2.4	3.5	3.8	4.0	4.2	4.3	4.4	4.4	4.4	4.5	4.4	4.4	4.4	4.4	4.3
125	266200	0.049	0.097	0.109	63.29	2.0	3.0	3.3	3.6	3.8	3.9	4.0	4.1	4.1	4.2	4.2	4.2	4.2	4.2	4.1
150	335600	0.041	0.096	0.104	66.83	1.7	2.7	3.0	3.3	3.5	3.6	3.7	3.8	3.9	3.9	4.0	4.0	4.0	4.0	4.0
200	423200	0.035	0.097	0.103	70.12	1.5	2.5	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	3.9	3.9	4.0	4.0	4.0

1/ Drop factor equations are derived in section 90.
2/ θ is the load power factor angle or angle between load terminal voltage and line current.
3/ Conductor cross-sectional areas are based on MIL-C-24643.
4/ Resistances and reactances per phase are calculated in section 140.
5/ Resistances are derived at a temperature of 65°C.

TABLE XVI. Drop factors for LSTSGU cable, 120V, three-phase/single-phase, 60Hz lighting systems using I or I_x. 1/

Size	Cable characteristics					Drop factors at angle θ 2/ for single-phase loads or θ_x 3/ for three-phase loads (multiply by 10^{-5})														
	A 4/ Cmil	R5/,6/ Ohms per 1000 feet	X 5/ Ohms per 1000 feet	Z Ohms per 1000 feet	B °C	-70	-65	-60	-55	-50	-45	-40	-30	-20	-10	0	10	20	30	40
3	2580	4.503	0.043	4.503	0.55	125.0	155.3	184.5	212.3	238.4	262.8	285.1	323.2	351.4	368.9	375.2	370.2	353.9	326.8	289.8
4	4110	2.825	0.040	2.825	0.81	77.4	96.5	114.8	132.3	148.8	164.1	178.2	202.2	220.1	231.3	235.4	232.4	222.4	205.5	182.5
9	10380	1.119	0.036	1.120	1.82	29.1	36.7	44.1	51.1	57.7	63.8	69.5	79.3	86.6	91.3	93.2	92.3	88.6	82.2	73.3
14	13090	0.887	0.041	0.888	2.63	22.1	28.2	34.0	39.6	44.9	49.9	54.4	62.3	68.3	72.2	73.9	73.4	70.6	65.7	58.8
23	20820	0.558	0.039	0.559	4.02	12.8	16.7	20.4	24.0	27.4	30.6	33.5	38.6	42.6	45.2	46.5	46.4	44.8	41.9	37.7
50	52620	0.221	0.030	0.223	7.81	3.9	5.5	7.0	8.5	9.9	11.2	12.5	14.7	16.4	17.7	18.4	18.6	18.2	17.2	15.7
75	83690	0.138	0.030	0.141	12.07	1.6	2.6	3.6	4.6	5.5	6.4	7.2	8.7	10.0	10.9	11.5	11.8	11.6	11.2	10.4
100	105600	0.110	0.029	0.114	14.92	0.8	1.7	2.5	3.3	4.0	4.8	5.5	6.7	7.8	8.6	9.2	9.5	9.4	9.2	8.6
150	167800	0.069	0.029	0.075	22.58	0.3	0.3	0.8	1.3	1.9	2.4	2.9	3.8	4.6	5.2	5.8	6.1	6.2	6.2	5.9
200	211600	0.055	0.029	0.062	27.80	0.7	0.3	0.2	0.6	1.1	1.5	2.0	2.8	3.5	4.1	4.6	4.9	5.1	5.2	5.1
300	300000	0.040	0.029	0.049	35.75	1.1	0.8	0.4	0.1	0.3	0.7	1.0	1.7	2.3	2.9	3.3	3.7	4.0	4.1	4.1
400	413600	0.029	0.029	0.041	44.80	1.4	1.2	0.9	0.6	0.3	0.0	0.3	0.9	1.5	2.0	2.4	2.8	3.1	3.3	3.4

1/ Drop factor equations are derived in section 100.
2/ *θ is the load power factor angle or angle between load terminal voltage and line current I.*
3/ θ_x is the angle between load terminal voltage and current I_x.
4/ Conductor cross-sectional areas are based on MIL-C-24643.
5/ Resistances and reactances per phase are calculated in section 140.
6/ Resistances are derived at a temperature of 45°C.

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TABLE XVII. Drop factors for LSTSGU cable, 120V, three-phase/single-phase, 60Hz lighting systems using P or P_x. 1/

Size	Cable characteristics					Drop factors at angle θ 2/ for single-phase loads or $θ_x$ 3/ for three-phase loads (multiply by 10^{-5})														
	A 4/ Cmil	R 5/,6/ Ohms per 1000 feet	X 5/	Z	θ °C	-70	-65	-60	-55	-50	-45	-40	-30	-20	-10	0	10	20	30	40
3	2580	4.503	0.043	4.503	0.55	317.7	319.6	320.9	321.8	322.6	323.2	323.7	324.5	325.2	325.8	326.3	326.9	327.4	328.1	328.9
4	4110	2.825	0.040	2.825	0.81	196.7	198.5	199.7	200.6	201.2	201.8	202.3	203.0	203.7	204.2	204.7	205.2	205.8	206.4	207.1
9	10380	1.119	0.036	1.120	1.82	74.0	75.6	76.6	77.4	78.0	78.5	78.9	79.6	80.1	80.6	81.1	81.5	82.0	82.6	83.3
14	13090	0.887	0.041	0.888	2.63	56.2	58.0	59.2	60.1	60.8	61.3	61.8	62.6	63.2	63.8	64.3	64.8	65.3	66.0	66.8
23	20820	0.558	0.039	0.559	4.02	32.6	34.3	35.5	36.4	37.0	37.6	38.1	38.8	39.4	39.9	40.4	40.9	41.5	42.1	42.8
50	52620	0.221	0.030	0.223	7.81	10.0	11.3	12.2	12.9	13.4	13.8	14.2	14.7	15.2	15.6	16.0	16.4	16.8	17.3	17.9
75	83690	0.138	0.030	0.141	12.07	4.1	5.4	6.3	6.9	7.5	7.9	8.2	8.8	9.2	9.6	10.0	10.4	10.8	11.2	11.8
100	105600	0.110	0.029	0.114	14.92	2.1	3.4	4.3	4.9	5.4	5.8	6.2	6.7	7.2	7.6	8.0	8.3	8.7	9.2	9.8
150	167800	0.069	0.029	0.075	22.58	0.7	0.5	1.4	2.0	2.5	2.9	3.3	3.8	4.2	4.6	5.0	5.4	5.8	6.2	6.7
200	211600	0.055	0.029	0.062	27.80	1.8	0.5	0.3	1.0	1.5	1.9	2.2	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.7
300	300000	0.040	0.029	0.049	35.75	2.8	1.6	0.7	0.1	0.4	0.8	1.1	1.7	2.1	2.5	2.9	3.3	3.7	4.1	4.6
400	413600	0.029	0.029	0.041	44.80	3.6	2.4	1.5	0.9	0.4	0.0	0.4	0.9	1.3	1.7	2.1	2.5	2.9	3.3	3.9

1/ Drop factor equations are derived in section 110.
2/ θ is the load power factor angle or angle between P and S vectors.
3/ $θ_x$ is the angle between P_x and apparent power S_x vectors.
4/ Conductor cross-sectional areas are based on MIL-C-24643.
5/ Resistances and reactances per phase are calculated in section 140.
6/ Resistances are derived at a temperature of 45°C.

- For equation using Power P_x.

$$DF' = \left(\frac{1}{V\cos(\Theta_x)}\right)\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (110\text{-}23)$$

The drop factors for other cables not listed in the tables can be also calculated from the above equations if the cable characteristic are given.

80. DERIVATION OF VOLTAGE DROP EQUATIONS FOR DC SYSTEMS

80.1 Single-wire circuits. The R_{dc} resistance for a single wire in dc systems is:

$$R_{dc} = \left(\frac{\rho L}{A}\right) \qquad (80\text{-}1)$$

Where:

ρ = Conductor resistivity (copper) of a cable at desired operating temperature (t).
L = Cable length.
A = Conductor cross-sectional area in circular mil(cmil).

The standard nominal cross-sectional area of a conductor in circular mils is calculated in accordance with the following equation:

$$A = d^2, \qquad (80\text{-}2)$$

Where:

d = conductor diameter in mils.

The conductor resistivity (copper) at operating temperature (t) is given by:

$$\rho = \rho_o[1 + 0.00393(t - t_o)], \qquad (80\text{-}3)$$

From section 140, the conductor resistivity (copper) at 20°C is:

$$\rho_o = 10.371\ \Omega.\text{cmil/ft}$$

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At $t = 65°C$,

$$\rho = 10.371[1 + 0.00393(65 - 20)]$$
$$= 12.21\Omega.\text{cmil/ft}$$

At $t = 45°C$,

$$\rho = 10.371[1 + 0.00393(45 - 20)]$$
$$= 11.39\ \Omega.\text{cmil/ft}$$

With respect to the system voltage, the percent voltage drop in a single line is :

$$D\% = \left(\frac{IR_{dc}}{E}\right) * 100 \qquad (80\text{-}4)$$

80.2 Two-wire circuits.

In a two-wire circuit, the percent drop in equation (80-4) must be multiplied by 2 to include the drop in the return path:

- For all systems except power.

$$D\% = 2\left(\frac{IR_{dc}}{E}\right) * 100$$

At 45°C, the resistance R_{dc} is:

$$R_{dc} = 11.39\left(\frac{L}{A}\right)$$

Then,

$$D\% = 22.78\left(\frac{IL}{AE}\right) * 100 \qquad (80\text{-}5)$$

- For power system only.

$$D\% = 2\left(\frac{IR_{dc}}{E}\right) * 100$$

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$$D\% = 2\left(\frac{12.21IL}{AE}\right) * 100$$

$$= 24.42\left(\frac{IL}{AE}\right) * 100 \qquad (80\text{-}6)$$

90. <u>DERIVATION OF VOLTAGE DROP EQUATIONS FOR SINGLE-PHASE/POWER SYSTEMS</u>

The derivation of equations is based on Navy shipboard power systems and following assumptions:

- Cable impedance is purely resistive and inductive.
- The terminal load voltage is used as reference at $V\underline{/0°}$.

Let derive the voltage drop equations based on the following figures:

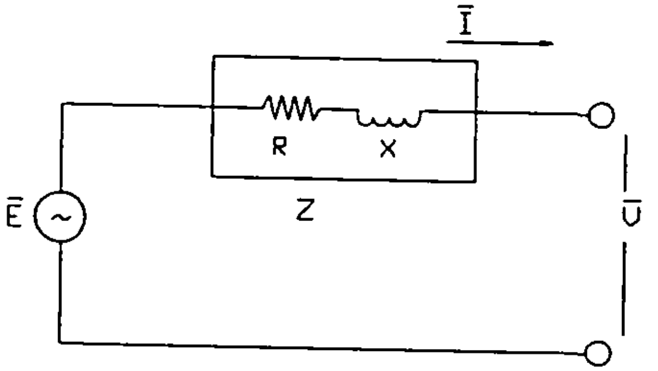

Figure 1. Single-phase circuit representation.

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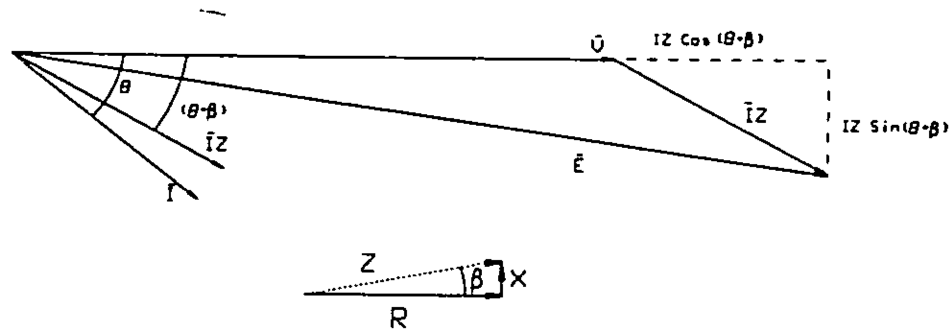

Figure 2. Single-phase Voltage & Current Phasor diagram.

From the above figures, define:

Θ = Load power factor angle or angle between load voltage and line current.
β = Cable impedance angle.

$= \tan^{-1}(X/R)$

The relationship between the sending bus and the terminal load voltage is as follows:

$$\overline{E} = \overline{IZ} + \overline{V}, \qquad (90\text{-}1)$$

$$\overline{E} = (I\underline{/-\Theta})(Z\underline{/\beta}) + V\underline{/0°}$$

$$\overline{E} = IZ\underline{/(-\Theta + \beta)} + V\underline{/0°}$$

Let $\alpha = -\Theta + \beta,$ (90-2)

$$\overline{E} = IZ\cos(\alpha) + jIZ\sin(\alpha) + V$$

$$\overline{E} = (IZ\cos(\alpha) + V) + jIZ\sin(\alpha)$$

$$E = \sqrt{(IZ\cos(\alpha) + V)^2 + I^2Z^2\sin^2(\alpha)}, \qquad (90\text{-}3)$$

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The cable voltage drop in percent is calculated as follows:

$$D\% = \left(\frac{E - V}{E}\right) * 100$$

$$= \left(1 - \frac{V}{E}\right) * 100, \qquad (90\text{-}4)$$

From equation (90-3), the terminal load voltage is:

$$V = \sqrt{E^2 - I^2Z^2\sin^2(\alpha)} - IZ\cos(\alpha)$$

Substitute V into equation (90-4) and simplify:

$$D\% = \left[1 - \frac{\sqrt{E^2 - I^2Z^2\sin^2(\alpha)}}{E} + \frac{IZ\cos(\alpha)}{E}\right] * 100$$

$$D\% = \left[1 - \sqrt{\frac{E^2 - I^2Z^2\sin^2(\alpha)}{E^2}} + \frac{IZ\cos(\alpha)}{E}\right] * 100$$

$$D\% = \left[1 - \sqrt{1 - \frac{I^2Z^2\sin^2(\alpha)}{E^2}} + \frac{IZ\cos(\alpha)}{E}\right] * 100, \qquad (90\text{-}5)$$

Set Z = zL, the exact equation for voltage drop is:

$$D\% = \left[1 - \sqrt{1 - \frac{I^2L^2z^2\sin^2(\alpha)}{E^2}} + \frac{ILz\cos(\alpha)}{E}\right] * 100, \qquad (90\text{-}6)$$

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The rule for approximation by binomial expansion can be applied to simplify equation (90-6):

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!} \pm \frac{n(n-1)(n-2)x^3}{3!} + \ldots, (x^2 < 1)$$

In the above expression, let:

$$x^2 = \left(\frac{I^2L^2z^2\sin^2(\alpha)}{E^2}\right)^2$$ which is approximately equal to zero.

Therefore, the term in equation (90-6) can be written:

$$\sqrt{1 - \frac{I^2L^2z^2\sin^2(\alpha)}{E^2}} = 1 - \frac{1}{2}\left(\frac{I^2L^2z^2\sin^2(\alpha)}{E^2}\right), \qquad (90\text{-}7)$$

Substitute equation (90-7) into equation (90-6):

$$D\% = \left[1 - \left(1 - \frac{1}{2}\left(\frac{I^2L^2z^2\sin^2(\alpha)}{E^2}\right)\right) + \frac{ILz\cos(\alpha)}{E}\right] * 100$$

The voltage drop equation reduces to:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E} + \frac{ILz^2\sin^2(\alpha)}{2E^2}\right) * 100, \qquad (90\text{-}8)$$

Since the quantity $\sin^2(\alpha)/2E^2$ is very small with respect to $\cos(\alpha)/E$, It can be neglected and the acceptable equation for the voltage drop is obtained:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E}\right) * 100, \qquad (90\text{-}9)$$

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Since it is a single-phase circuit, equation (90-9) must be multiplied by 2 to include the drop in the return path.

Equation (90-9) can also be used to determine the percent drop in the individual lines for three-phase balanced systems such as electronic, internal communication, and weapon control systems with the substitution of line current by $\sqrt{3}I_{LO}$:

$$D\% = \sqrt{3}I_{LO}L\left(\frac{z\cos(\alpha)}{E}\right) * 100, \qquad (90\text{-}10)$$

Where I_{LO} is the resultant load current in each phase.

In equation (90-8), since $(z/E)\cos(\alpha) >> (z^2/2E^2)\sin^2(\alpha)$, and by setting D% equal to the assumed drop (AD), an estimate of (IL) may be determined as follows:

$$(AD) = IL[(z/E)\cos(\alpha)] * 100$$

Solve for IL:

$$IL = [(AD)E/100z\cos(\alpha)], \qquad (90\text{-}11)$$

Substitute equation (90-11) into equation (90-8) for IL in the bracket:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E} + \frac{(AD)Ez^2\sin^2(\alpha)}{200zE^2\cos(\alpha)}\right) * 100$$

Simplification of the above equation gives the voltage drop equation which includes the assumed drop (AD) in the systems:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E} + \frac{(AD)z\tan(\alpha)\sin(\alpha)}{200E}\right) * 100, \qquad (90\text{-}12)$$

The assumed drop (AD) for a 60 Hz or 400 Hz power system at normal operation is 2 percent.

In equation (90-12), let define the drop factor as:

$$DF = \left(\frac{z\cos(\alpha)}{E} + \frac{(AD)\, z\tan(\alpha)\sin(\alpha)}{200E} \right) * 100, \qquad (90\text{-}13)$$

Equation (90-12) can be rewritten in the simple form:

$$D\% = IL(DF), \qquad (90\text{-}14)$$

As before, the percent drop for a single-phase circuit in equation (90-14) must be multiplied by 2 to include the drop in the return path:

$$D\% = 2IL(DF), \qquad (90\text{-}15)$$

100. DERIVATION OF VOLTAGE DROP EQUATIONS FOR THREE-PHASE/LIGHTING SYSTEMS

In order to determine the best cable selection for three-phase systems, voltage drop calculations for all phases should be performed. The combination of the individual drops will be the drop of the cable. The following equation derivations are for a three-phase lighting system in delta configuration.

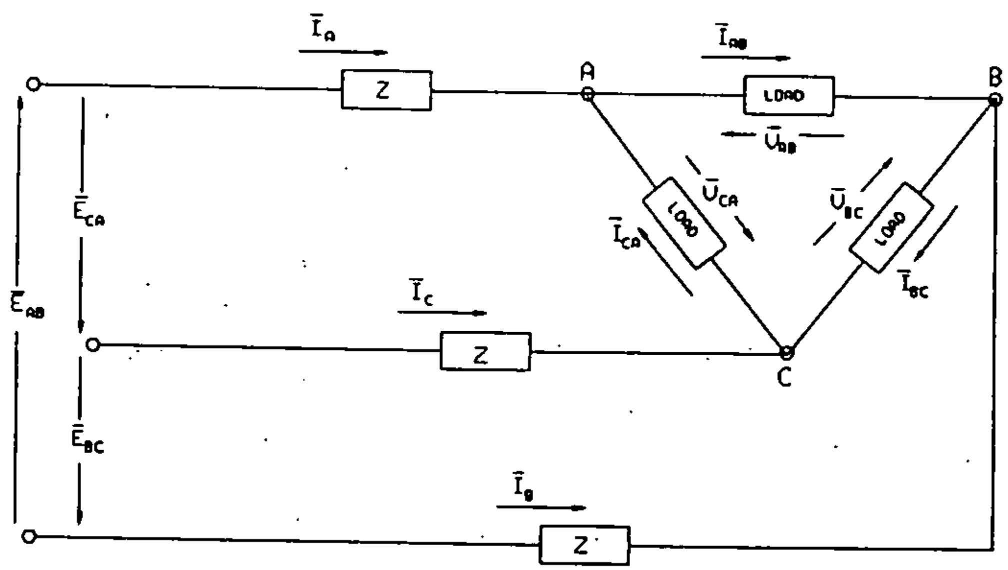

Figure 3. Three-phase circuit representation.

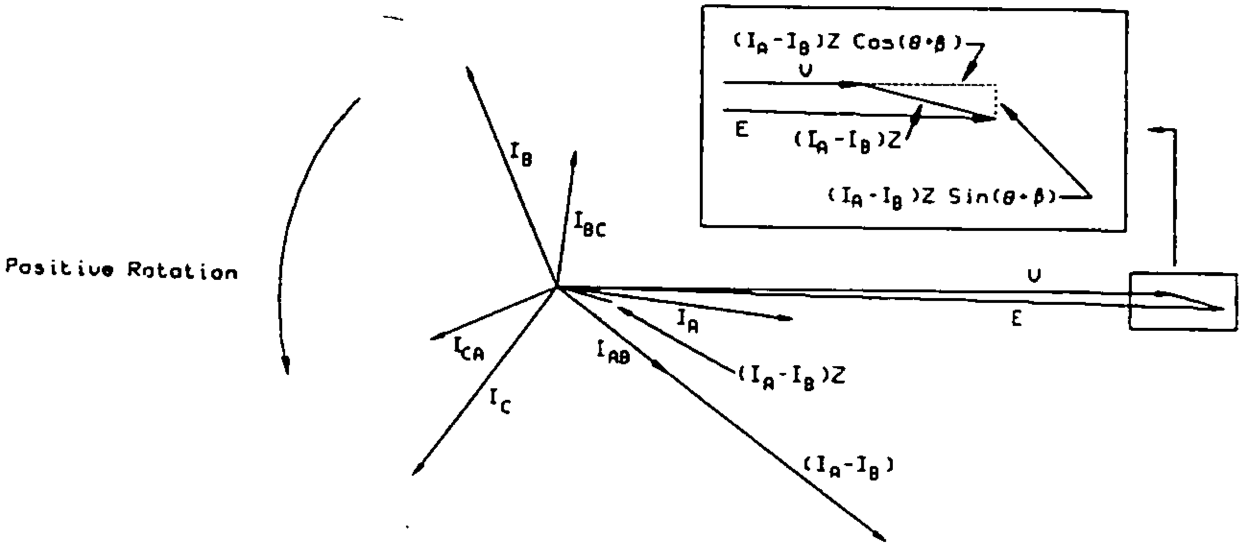

Figure 4. Three-phase voltage and current phasor diagram.

From figure 3., the voltage loop method gives:

$$\overline{E}_{AB} - \overline{I}_A\overline{Z} - \overline{V}_{AB} + \overline{I}_B\overline{Z} = 0, \tag{100-1}$$

$$\overline{E}_{BC} - \overline{I}_B\overline{Z} - \overline{V}_{BC} + \overline{I}_C\overline{Z} = 0, \tag{100-2}$$

$$\overline{E}_{CA} - \overline{I}_C\overline{Z} - \overline{V}_{CA} + \overline{I}_A\overline{Z} = 0, \tag{100-3}$$

Regroup the above equations as follows:

$$\overline{E}_{AB} = \overline{V}_{AB} + \overline{Z}(\overline{I}_A - \overline{I}_B), \tag{100-4}$$

$$\overline{E}_{BC} = \overline{V}_{BC} + \overline{Z}(\overline{I}_B - \overline{I}_C), \tag{100-5}$$

$$\overline{E}_{CA} = \overline{V}_{CA} + \overline{Z}(\overline{I}_C - \overline{I}_A), \tag{100-6}$$

Next, the difference in line current terms in equations (100-4) through (100-6) must be solved. From figures 3 and 4:

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$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}, \qquad (100\text{-}7)$$

$$\Theta_A = 0° - \underline{/I_{A}}$$

$$\alpha_A = -\Theta_A + \beta$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}, \qquad (100\text{-}8)$$

$$\Theta_B = -120° - \underline{/I_{B}}$$

$$\alpha_B = -\Theta_B + \beta$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}, \qquad (100\text{-}9)$$

$$\Theta_C = 120° - \underline{/I_{C}}$$

$$\alpha_C = -\Theta_C + \beta$$

Where:

$\underline{/I}_A$, $\underline{/I}_B$, $\underline{/I}_C$: angle of $\bar{I}_A$, $\bar{I}_B$, and $\bar{I}_C$.

Θ_A : angle between $\bar{V}_{AB}$ and $\bar{I}_A$.

Θ_B : angle between $\bar{V}_{BC}$ and $\bar{I}_B$

Θ_C : angle between $\bar{V}_{CA}$ and $\bar{I}_C$.

The phase load currents are expressed as:

$$\bar{I}_{AB} = I_{AB}\underline{/(0°-\Theta_{AB})}$$

$$\bar{I}_{BC} = I_{BC}\underline{/(-120°-\Theta_{BC})}$$

$$\bar{I}_{CA} = I_{CA}\underline{/(120°-\Theta_{CA})}$$

From equations (100-7), (100-8), and (100-9), the percent voltage drop in each line is computed as follows:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E}\right) * 100, \qquad (100\text{-}10)$$

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Let the drop factor as follow:

$$DF = \left(\frac{z\cos(\alpha)}{E}\right) * 100,$$

Equation (100-10) reduces to:

$$D\% = IL(DF), \tag{100-11}$$

The difference in line currents $\overline{I}_x$ are computed as follows:

$$\begin{aligned} \overline{I}_{x(AB)} &= \overline{I}_A - \overline{I}_B, \\ &= \overline{I}_{AB} - \overline{I}_{BC} - \overline{I}_{CA} + \overline{I}_{AB} \\ &= 2\overline{I}_{AB} - \overline{I}_{BC} - \overline{I}_{CA} \end{aligned} \tag{100-12}$$

and $\Theta_{x(AB)} = 0° - \underline{/I}_{x(AB)}$ –

$$\alpha_{x(AB)} = -\Theta_{x(AB)} + \beta$$

$$\begin{aligned} \overline{I}_{x(BC)} &= \overline{I}_B - \overline{I}_C \\ &= \overline{I}_{BC} - \overline{I}_{CA} - \overline{I}_{AB} + \overline{I}_{BC} \\ &= 2\overline{I}_{BC} - \overline{I}_{CA} - \overline{I}_{AB} \end{aligned} \tag{100-13}$$

and $\Theta_{x(BC)} = -120° - \underline{/I}_{x(BC)}$ –

$$\alpha_{x(BC)} = -\Theta_{x(BC)} + \beta$$

$$\begin{aligned} \overline{I}_{x(CA)} &= \overline{I}_C - \overline{I}_A, \\ &= \overline{I}_{CA} - \overline{I}_{AB} - \overline{I}_{BC} + \overline{I}_{CA} \\ &= 2\overline{I}_{CA} - \overline{I}_{AB} - \overline{I}_{BC} \end{aligned} \tag{100-14}$$

and $\Theta_{x(CA)} = 120° - \underline{/I}_{x(CA)}$ –

$$\alpha_{x(CA)} = -\Theta_{x(CA)} + \beta$$

Where:

$\Theta_{x(AB)}$: Angle between $\overline{V}_{AB}$ and $\overline{I}_{x(AB)}$.

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$\Theta_{x(BC)}$: Angle between $\overline{V}_{BC}$ and $\overline{I}_{x(BC)}$.

$\Theta_{x(CA)}$: Angle between $\overline{V}_{CA}$ and $\overline{I}_{x(CA)}$.

From equations (100-4), (100-5), and (100-6), the general equation for the percent drop in the individual phase is:

$$D\% = \frac{|\overline{E}_x| - |\overline{V}|}{E_x} * 100, \qquad (100\text{-}15)$$

The magnitude of $\overline{V}$ may be approximated as:

$$V = E_x - I_x Z\cos(\alpha_x)$$

Substitution of V in equation (100-15) gives:

$$D\% = I_x Z\left(\frac{\cos(\alpha_x)}{E_x}\right) * 100 \qquad (100\text{-}16)$$

Let Z = zL, equation (100-16) becomes:

$$D\% = I_x L\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100 \qquad (100\text{-}17)$$

For example, the percent drop in phase AB is:

$$D_{AB}\% = I_{x(AB)} L\left(\frac{z\cos(\alpha_{x(AB)})}{E_{AB}}\right) * 100$$

Equation (100-17) can also be written as:

$$D\% = I_x L(DF) \qquad (100\text{-}18)$$

With,

$$DF = \left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (100\text{-}19)$$

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For a single-phase system, equation (100-19) becomes:

$$DF = \left(\frac{z\cos(\alpha)}{E}\right) * 100, \qquad (100\text{-}20)$$

Then, the percent drop equation is as follows:

$$D\% = 2IL(DF) \qquad (100\text{-}21)$$

110. DERIVATION OF VOLTAGE DROP EQUATIONS FOR THREE-PHASE/LIGHTING SYSTEMS USING WATTS AND VARS

In lighting systems, the use of watts (P) and vars (Q) instead of currents (I) in voltage drop equations avoids necessity for calculating all phase and line currents. This is an advantage when real and reactive power or the apparent power (S) and power factor of connected loads are known.

Consider a balanced three-phase voltage and current vector diagram below:

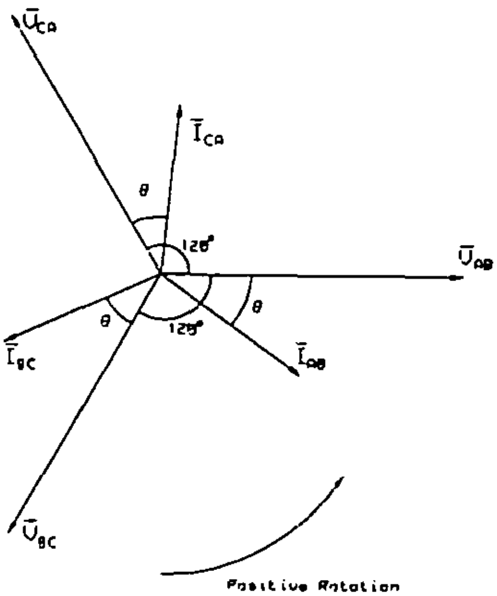

Figure 5. Three-phase voltage and current phasor diagram.

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Assume all phase loads are inductive. The total complex power for each phase can be determined as follows:

$$\bar{S} = \bar{V}\,\bar{I}_L^{\,*}$$

Where $\bar{I}_L^{\,*}$ is the complex conjugate of $\bar{I}_L$.

The apparent power (S) for each phase (leg) is defined as:

$$S = |\,S\,| = \bar{V}I_L$$

For phase AB:

$$S_{AB} = V_{AB}I_{AB}$$

In rectangular form:

$$S_{AB} = V_{AB}(1 + j0) * I_{AB}[\cos(\Theta_{AB}) + j\sin(\Theta_{AB})]$$

$$S_{AB} = V_{AB}I_{AB}\cos(\Theta_{AB}) + jV_{AB}I_{AB}\sin(\Theta_{AB}), \qquad (110\text{-}1)$$

Since,

$$P_{AB} = V_{AB}I_{AB}\cos(\Theta_{AB}), \qquad Q_{AB} = V_{AB}I_{AB}\sin(\Theta_{AB})$$

Equation (110-1) becomes:

$$S_{AB} = P_{AB} + jQ_{AB}, \qquad (110\text{-}2)$$

The apparent powers S_{BC} for phase BC and S_{CA} for phase CA can be referenced to the same axis as S_{AB}. For inductive loads, their horizontal projections are:

$$S_{BC}\cos(-120° - \Theta_{BC}) \quad \text{and} \quad S_{CA}\cos(120° - \Theta_{CA})$$

From the trigonometric identity:

$$\cos(\Theta_1 - \Theta_2) = \cos(\Theta_1)\cos(\Theta_2) + \sin(\Theta_1)\sin(\Theta_2)$$

$$S_{BC}\cos(-120° - \Theta_{BC}) = S_{BC}[\cos(-120°)\cos(\Theta_{BC}) + \sin(-120°)\sin(\Theta_{BC})$$

$$= S_{BC}[-(1/2)\cos(\Theta_{BC}) - (\sqrt{3}/2)\sin(\Theta_{BC})]$$

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For phase CA:

$$S_{CA}\cos(120° - \Theta_{CA}) = S_{CA}[\cos(120°)\cos(\Theta_{CA}) + \sin(120°)\sin(\Theta_{CA})$$
$$= S_{CA}[-(1/2)\cos(\Theta_{CA}) + (\sqrt{3}/2)\sin(\Theta_{CA})]$$

As indicated above, the two quantities can be rewritten:

$$S_{BC}\cos(-120° - \Theta_{BC}) = -(1/2)P_{BC} - (\sqrt{3}/2)Q_{BC}$$
$$S_{CA}\cos(120° - \Theta_{CA}) = -(1/2)P_{CA} + (\sqrt{3}/2)Q_{CA}$$

The apparent power S_A and real power P_A in line A (or at node A) can be calculated as follows:

$$S_A = S_{AB} - S_{CA}, \qquad P_A = P_{AB} - P_{CA}$$

Substitute P_{CA} in the above equation:

$$P_A = P_{AB} - S_{CA}\cos(120° - \Theta_{CA})$$
$$= P_{AB} - (1/2)P_{CA} + (\sqrt{3})Q_{CA})$$
$$= P_{AB} + (1/2)P_{CA} - (\sqrt{3})Q_{CA}, \qquad (110\text{-}3)$$

For lines B and C:

$$P_B = P_{BC} - P_{AB}$$
$$= P_{BC} + (1/2)P_{AB} - (\sqrt{3})Q_{AB}, \qquad (110\text{-}4)$$

$$P_C = P_{CA} - P_{BC}$$
$$= P_{CA} + (1/2)PBC - (\sqrt{3})Q_{BC}, \qquad (110\text{-}5)$$

Similarly,

$$Q_A = Q_{AB} - Q_{CA}$$
$$= Q_{AB} + (1/2)Q_{CA} - (\sqrt{3}/2)P_{CA}, \qquad (110\text{-}6)$$
$$Q_B = Q_{BC} - Q_{AB},$$
$$= Q_{BC} + (1/2)Q_{AB} - (\sqrt{3}/2)P_{AB}, \qquad (110\text{-}7)$$

$$Q_C = Q_{CA} - Q_{BC},$$
$$= Q_{CA} + (1/2)Q_{BC} - (\sqrt{3}/2)P_{BC}, \qquad (110\text{-}8)$$

The net power P_x at two nodes of a three-phase delta connected load is as follows:

Phase AB:

$$P_{x(AB)} = P_A - P_B, \qquad (110\text{-}9)$$

In equation (110-4) for line B, P_B can be expressed as:

$$P_B = P_{BC} - P_{AB}$$
$$= S_{BC}[-(1/2)\cos(\Theta_{BC}) - (\sqrt{3}/2)\sin(\Theta_{BC})] - P_{AB}$$
$$= - P_{AB} - (1/2)P_{BC} - (\sqrt{3}/2)Q_{BC}, \qquad (110\text{-}10)$$

Substitute P_A and P_B in equation (110-9), equation for P_x in phase AB is obtained:

$$P_{x(AB)} = 2P_{AB} + (1/2)(P_{BC} + P_{CA}) + (\sqrt{3}/2)(Q_{BC} - Q_{CA}), \qquad (110\text{-}11)$$

For phases BC and CA:

$$P_{x(BC)} = 2P_{BC} + (1/2)(P_{CA} + P_{AB}) + (\sqrt{3}/2)(Q_{CA} - Q_{AB}), \qquad (110\text{-}12)$$
$$P_{x(CA)} = 2P_{CA} + (1/2)(P_{AB} + P_{BC}) + (\sqrt{3}/2)(Q_{AB} - Q_{BC}), \qquad (110\text{-}13)$$

Similarly, Q_x can be shown as:

$$Q_{x(AB)} = -[2Q_{AB} + (1/2)(Q_{BC} + Q_{CA}) - (\sqrt{3}/2)(P_{BC} - P_{CA})], \qquad (110\text{-}14)$$
$$Q_{x(BC)} = -[2Q_{BC} + (1/2)(Q_{CA} + Q_{AB}) - (\sqrt{3}/2)(P_{CA} - P_{AB})], \qquad (110\text{-}15)$$
$$Q_{x(CA)} = -[2Q_{CA} + (1/2)(Q_{AB} + Q_{BC}) - (\sqrt{3}/2)(P_{AB} - P_{BC})], \qquad (110\text{-}16)$$

Equations for capacitive loads, or mixed inductive and capacitive loads can also be derived by assigning the proper sign to each phase angle (negative for inductive loads and positive for capacitive loads) in determining the cosine and sine of ($\pm$ 120° $\pm$ Θ). Therefore, for capacitive loads, the signs of the last terms in equations (110-11) through (110-16) should be reversed.

It should be noted that if these equations are divided by load voltage V, similar expressions involving resistive and reactive components of current will result. In some cases, the use of these equations may be more convenient than the vector rotation method when currents are used in the calculations.

For checking current carrying capacity of conductors, only the largest phase apparent power (S) or (S_x) needs to be considered. The general equations are:

$$S = \sqrt{P^2 + Q^2} \qquad S_x = \sqrt{P_x^2 + Q_x^2}, \qquad (110\text{-}17)$$

$$I = \frac{S}{V} = \left(\frac{P}{V}\right)\left(\frac{1}{\cos(\Theta)}\right) \qquad I_x = \frac{S_x}{V} = \left(\frac{P_x}{V}\right)\left(\frac{1}{\cos(\Theta_x)}\right), \qquad (110\text{-}18)$$

$$\Theta = \cos^{-1}\left(\frac{P}{S}\right) \qquad \Theta_x = \cos^{-1}\left(\frac{P_x}{S_x}\right), \qquad (110\text{-}19)$$

or

$$\Theta = \tan^{-1}\left(\frac{Q}{P}\right) \qquad \Theta_x = \tan^{-1}\left(\frac{Q_x}{P_x}\right), \qquad (11\text{-}20)$$

Θ_x is positive if Q_x is positive.

Θ_x is greater than 90° if P_x is negative.

Recall the general voltage drop equation for three-phase lighting systems derived in section 100:

$$D\% = I_x L\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (100\text{-}17)$$

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Since,

$$P_x = I_x V\cos(\Theta_x)$$

The current I_x is then:

$$I_x = \left(\frac{P_x}{V\cos(\Theta_x)}\right) \qquad (110\text{-}21)$$

Substitution of I_x in equation (100-17) gives:

$$D\% = P_x L\left(\frac{1}{V\cos(\Theta_x)}\right)\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (110\text{-}22)$$

From equation (110-22), a new drop factor is defined as:

$$DF' = \left(\frac{1}{V\cos(\Theta_x)}\right)\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (110\text{-}23)$$

Finally,

$$D\% = P_x L(DF) \qquad (110\text{-}24)$$

For single-phase circuits, in equation (110-22), the parameters E_x, P_x, α_x, and Θ_x become E, P, α, and Θ respectively. The percent voltage drop for a single-phase circuit is then:

$$D\% = 2PL(DF) \qquad (110\text{-}25)$$

With,

$$DF' = \left(\frac{1}{V\cos(\Theta)}\right)\left(\frac{z\cos(\alpha)}{E}\right) * 100, \qquad (110\text{-}26)$$

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The computed drop factors for LSTSGU cables from equations (110-23) or (110-26) are shown in table XVII. Now, let derive equation for D% in function of watts (P) and vars (Q). Equation (110-18) can be written as:

$$P_x = S_x \cos(\Theta_x), \tag{110-27}$$

Substitution of P_x in equation (110-22) gives:

$$D\% = \left(\frac{S_x L z \cos(\Theta_x)\cos(\alpha_x)}{E_x V \cos(\Theta_x)}\right) * 100, \tag{110-28}$$

$$D\% = \left(\frac{S_x L z \cos(\alpha_x)}{E_x V}\right) * 100, \qquad (110\text{-}28)$$

Rewrite equation (110-28) as:

$$D\% = \left(\frac{S_x L z \cos(\beta \pm \Theta_x)}{E_x V}\right) * 100, \tag{110-29}$$

$$D\% = \left(\frac{S_x L z [\cos(\beta)\cos(\Theta_x) \pm \sin(\beta)\sin(\Theta_x)]}{E_x V}\right) * 100, \tag{110-30}$$

Recall:

$$P_x = S_x \cos(\Theta_x), \qquad Q_x = S_x \sin(\Theta_x),$$

$$R = z\cos(\beta), \qquad X = z\sin(\beta)$$

After the substitution, equation (110-30) becomes:

$$D\% = \left(\frac{L(RP_x \pm XQ_x)}{E_x V}\right) * 100, \tag{110-31}$$

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For lighting systems, let assume:

$$E_x = 120V, \qquad V = 115V$$

Equation (110-31) can be written as:

$$D\% = (7.25)(10^{-5})L(RP_x \pm XQ_x) * 100, \qquad (110\text{-}32)$$

For a single phase system, equation (110-31) becomes:

$$D\% = 2\left(\frac{L(RP \pm XQ)}{EV}\right) * 100, \qquad (110\text{-}33)$$

The signs " + " and " − " are for inductive and capacitive loads respectively. The values of R and X for different types of cables can be found from Tables XI through XVII.

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120. DERIVATION OF VOLTAGE DROP EQUATIONS FOR THREE-PHASE FOUR-WIRE SYSTEMS

Consider a three-phase four-wire (Wye) system as shown below:

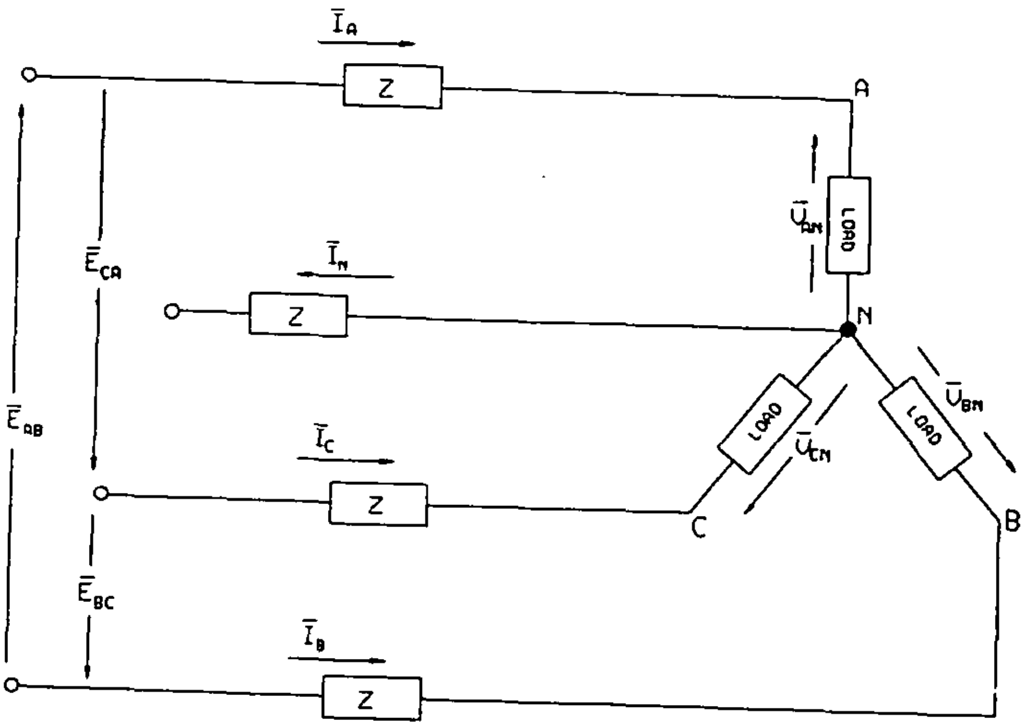

Figure 6. Three-phase Four-wire System.

If the system is balanced, the voltage drops for all lines are the same. For example, in line A, the percent drop is:

$$D_A\% = \left(\frac{E_{AN} - V_{AN}}{E_{AN}}\right) * 100, \qquad (120\text{-}1)$$

Or

$$D_A\% = IL\left(\frac{z\cos(\alpha_A)}{E_{AN}}\right) * 100, \qquad (120\text{-}2)$$

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Where:

$$I = I_A = \frac{V_{AN}}{Z_{AN}}, \qquad (120\text{-}3)$$

$$\alpha = -\theta_A + \beta, \qquad (100\text{-}7)$$

With, $E_{AN} = (E_{AB}/\sqrt{3}) = E$, equation (120-2) becomes:

$$D_A\% = IL\left(\frac{z\cos(\alpha_A)}{E}\right) * 100, \qquad (120\text{-}4)$$

Or

$$D_A\% = IL(DF), \qquad (120\text{-}5)$$

With,

$$DF = \left(\frac{z\cos(\alpha_A)}{E}\right) * 100$$

Now, let derive equations for the drop in each phase (the drop in phase A plus the drop in phase B). From figure 6, the source voltage in line A to line B is:

$$\bar{E}_{AB} = \bar{V}_{AN} - \bar{V}_{BN} + \bar{Z}(\bar{I}_A - \bar{I}_B), \qquad (120\text{-}5)$$

For $\bar{V}_{BN} = \bar{V}_{AN}(1\underline{/-120°})$,

$$\begin{aligned}\bar{E}_{AB} &= \bar{V}_{AN} - \bar{V}_{AN}(1\underline{/-120°}) + \bar{Z}(\bar{I}_A - \bar{I}_B) \\ &= \bar{V}_{AN}(1.5 + j0.866) + \bar{Z}(\bar{I}_A - \bar{I}_B) \\ &= \bar{V}_{AN}(\sqrt{3}\underline{/30°}) + \bar{Z}(\bar{I}_A - \bar{I}_B)\end{aligned}$$

Since $\bar{V}_{AB} = \bar{V}_{AN}(\sqrt{3}\underline{/30°})$, equation (120-5) becomes:

$$\bar{E}_{AB} = \bar{V}_{AB} + \bar{Z}(\bar{I}_A - \bar{I}_B), \qquad (120\text{-}6)$$

Similarly:

$$\bar{E}_{BC} = \bar{V}_{BC} + \bar{Z}(\bar{I}_B - \bar{I}_C), \qquad (120\text{-}7)$$

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and

$$\bar{E}_{CA} = \bar{V}_{CA} + \tilde{Z}(\bar{I}_C - \bar{I}_A), \qquad (120\text{-}8)$$

Note that equations (120-6),(120-7), and (120-8) are similar to equations (100-4), (100-5), and (100-6) in section 100. After the line currents I are computed from equation (120-3), the currents I_x and the associated angles Θ_x and α_x can be determined in accordance with the following equations as derived in section 100 :

$$\bar{I}_{x(AB)} = \bar{I}_A - \bar{I}_B, \qquad (100\text{-}12)$$

$$\Theta_{x(AB)} = 0° - \underline{/I}_{x(AB)}-$$

$$\alpha_{x(AB)} = -\Theta_{x(AB)} + \beta$$

$$\bar{I}_{x(BC)} = \bar{I}_B - \bar{I}_C, \qquad (100\text{-}13)$$

$$\Theta_{x(BC)} = -120° - \underline{/I}_{x(BC)}-$$

$$\alpha_{x(BC)} = -\Theta_{x(BC)} + \beta$$

$$\bar{I}_{x(CA)} = \bar{I}_C - \bar{I}_A, \qquad (100\text{-}14)$$

$$\Theta_{x(CA)} = 120° - \underline{/I}_{x(CA)}-$$

$$\alpha_{x(CA)} = -\Theta_{x(CA)} + \beta$$

The percent drop in each phase is calculated from equation (100-17) as derived in section 100:

$$D\% = I_x L\left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100, \qquad (100\text{-}17)$$

With,

$$DF = \left(\frac{z\cos(\alpha_x)}{E_x}\right) * 100$$

Equation (100-17) reduces to:

$$D\% = I_x L(DF)$$

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130. DERIVATION OF VOLTAGE DROP EQUATIONS FOR UNBALANCED SYSTEMS BY SYMMETRICAL COMPONENT METHOD

For analyzing unbalanced circuit, all loads are assumed unequal. Therefore, the currents in all phase loads are different. The symmetrical component method is preferred here to analyze the network. To use this method, it is convenient to keep the voltages, currents, and impedances in phasor forms. The following figures represent a three-phase unbalanced system and a three-phase unbalanced currents.

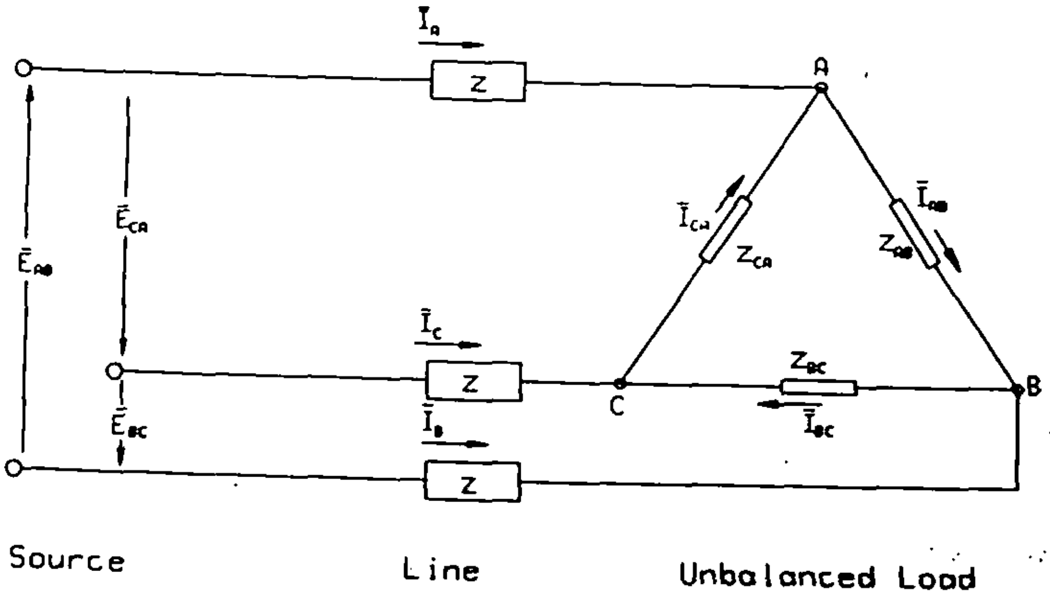

Figure 7. Three-phase unbalanced system.

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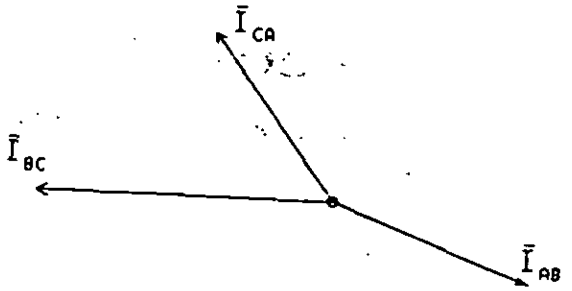

Figure 8. Three-phase unbalanced currents.

Assume all three loads are unbalanced. The load currents are:

$$|\bar{I}_{AB}| \neq |\bar{I}_{BC}| \neq |\bar{I}_{CA}|$$

and

$$\Theta_{AB} \neq \Theta_{BC} \neq \Theta_{CA}$$

The line currents, as functions of load currents at nodes A, B, and C of figure 7, are determined from equations derived in section 100 as follows:

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}, \quad (100\text{-}7)$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}, \quad (100\text{-}8)$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}, \quad (100\text{-}9)$$

Rearranging equation (100-7) as follows:

$$\bar{I}_A + \bar{I}_{CA} - \bar{I}_{AB} = 0, \quad (130\text{-}1)$$

Substitution of equation (100-9) into equation (130-1) yields:

$$\bar{I}_A + \bar{I}_C + \bar{I}_{BC} - \bar{I}_{AB} = 0, \quad (130\text{-}2)$$

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Substitution of equation (100-8) into equation (130-2) yields:

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0, \tag{130-3}$$

Consider the zero sequence component of $\bar{I}_A$:

$$\bar{I}_{A0} = (1/3)(\bar{I}_A + \bar{I}_B + \bar{I}_C), \tag{130-4}$$

Substitution from equation (130-4) gives:

$$\bar{I}_{A0} = (1/3)(0) = 0,$$

By similar derivation:

$$\bar{I}_{A0} = \bar{I}_{B0} = \bar{I}_{C0} = 0, \tag{130-5}$$

Thus, in a three-phase delta system the zero sequence currents are zero.

Now, let calculate the positive and negative sequence components of the line currents in term of current in phase AB.

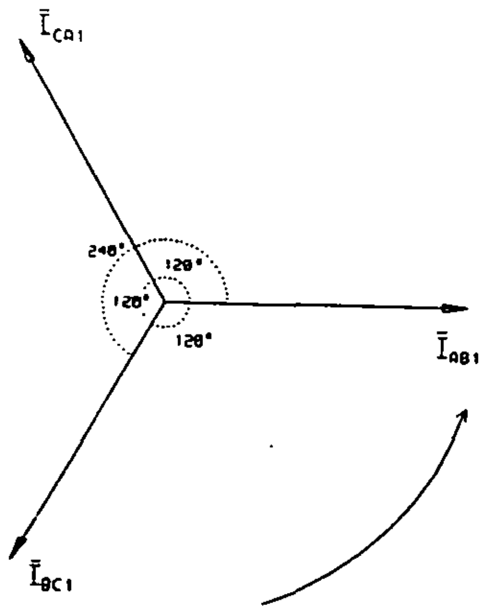

Figure 9. Positive sequence currents.

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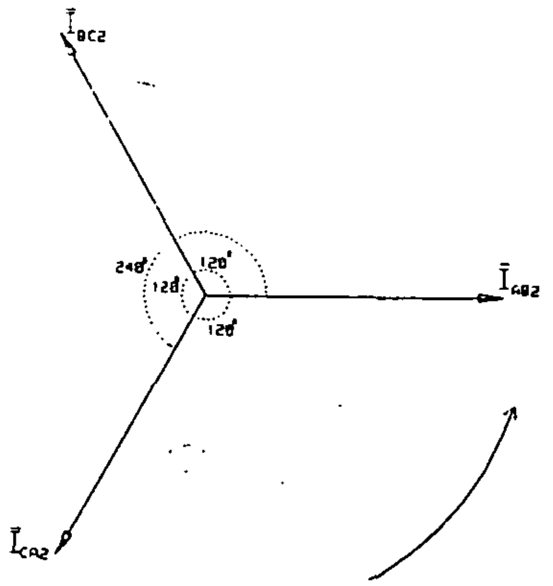

Figure 10. Negative sequence currents.

<u>Components of $\bar{I}_A$</u>

From equation (306), the positive sequence current is:

$$\bar{I}_{A1} = \bar{I}_{AB1} - \bar{I}_{CA1}, \qquad (130\text{-}6)$$

From figure 9:

$$\bar{I}_{CA1} = \bar{I}_{AB1}\underline{/120°}, \qquad (130\text{-}7)$$

Substitution equation (130-7) into equation (130-6) yields:

$$\begin{aligned} \bar{I}_{A1} &= \bar{I}_{AB1} - \bar{I}_{AB1}\underline{/120°}, \\ &= \bar{I}_{AB1}(1\underline{/0°} - 1\underline{/120°}) \\ &= \bar{I}_{AB1}(1 + 0.5 - j0.866) \\ &= \bar{I}_{AB1}(\sqrt{3}\underline{/-30°}), \end{aligned} \qquad (130\text{-}8)$$

From equation (130-7), the negative sequence current is:

$$\bar{I}_{A2} = \bar{I}_{AB2} - \bar{I}_{CA2}, \qquad (130\text{-}9)$$

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From figure 10:

$$\bar{I}_{CA2} = \bar{I}_{AB2}\underline{/240^\circ}, \qquad (130\text{-}10)$$

Substitution of equation (130-10) into equation (130-9) gives:

$$\begin{aligned}\bar{I}_{A2} &= \bar{I}_{AB2} - \bar{I}_{AB2}\underline{/240^\circ} \\ &= \bar{I}_{AB2}(1\underline{/0^\circ} - 1\underline{/240^\circ}) \\ &= \bar{I}_{AB2}(1 + 0.5 + j0.866) \\ &= \bar{I}_{AB2}(\sqrt{3}\underline{/30^\circ}), \qquad (130\text{-}11)\end{aligned}$$

<u>Components of $\bar{I}_B$</u>

From equation (100-8), the positive sequence current is:

$$\bar{I}_{B1} = \bar{I}_{BC1} - \bar{I}_{AB1}, \qquad (130\text{-}12)$$

From figure 9:

$$\bar{I}_{BC1} = \bar{I}_{AB1}\underline{/240^\circ}, \qquad (130\text{-}13)$$

Substitution of equation (13-13) into equation (130-12) yields:

$$\begin{aligned}\bar{I}_{B1} &= \bar{I}_{AB1}/240^\circ - \bar{I}_{AB1} \\ &= \bar{I}_{AB1}(1\underline{/240^\circ} - 1\underline{/0^\circ}) \\ &= \bar{I}_{AB1}(-1 - 0.5 - j.866) \\ &= \bar{I}_{AB1}(\sqrt{3}\underline{/-150^\circ}), \qquad (130\text{-}14)\end{aligned}$$

From equation (100-8), the negative sequence current is:

$$\bar{I}_{B2} = \bar{I}_{BC2} - \bar{I}_{AB2}, \qquad (130\text{-}15)$$

From figure 10:

$$\bar{I}_{BC2} = \bar{I}_{AB2}\underline{/120^\circ}, \qquad (130\text{-}16)$$

Substitution of equation (130-16) into equation (130-15) yields:

$$\bar{I}_{B2} = \bar{I}_{AB2}\underline{/120°} - \bar{I}_{AB2}$$
$$= \bar{I}_{AB2}(1\underline{/120°} - 1\underline{/0°})$$

$$\bar{I}_{B2} = \bar{I}_{AB2}(-1 - 0.5 + j.866)$$
$$= \bar{I}_{AB2}(\sqrt{3}\underline{/150°}), \qquad (130\text{-}17)$$

<u>Components of $\bar{I}_C$</u>

From equation (100-9), the positive sequence current is:

$$\bar{I}_{C1} = \bar{I}_{CA1} - \bar{I}_{BC1}, \qquad (130\text{-}18)$$

From figure 9:

$$\bar{I}_{CA1} = \bar{I}_{AB1}\underline{/120°}, \qquad (130\text{-}19)$$
$$\bar{I}_{BC1} = \bar{I}_{AB1}\underline{/240°}, \qquad (130\text{-}20)$$

Substitution of equations (130-19) and (130-20) into equation (130-18) gives:

$$\bar{I}_{C1} = \bar{I}_{AB1}\underline{/120°} - \bar{I}_{AB1}\underline{/240°}$$
$$= \bar{I}_{AB1}(1\underline{/120°} - 1\underline{/240°})$$
$$= \bar{I}_{AB1}(0 + j1.732)$$
$$= \bar{I}_{AB1}(\sqrt{3}\underline{/90°}), \qquad (130\text{-}21)$$

From equation (100-9), the negative sequence current is:

$$\bar{I}_{C2} = \bar{I}_{CA2} - \bar{I}_{BC2}, \qquad (130\text{-}22)$$

From figure 9:

$$\bar{I}_{CA2} = \bar{I}_{AB2}\underline{/240°}, \qquad (130\text{-}23)$$
$$\bar{I}_{BC2} = \bar{I}_{AB2}\underline{/120°}, \qquad (130\text{-}24)$$

Substitution of equations (130-23) and (130-24) into equation (130-22) gives:

$$\begin{aligned}\bar{I}_{C2} &= \bar{I}_{AB2}\underline{/240^\circ} - \bar{I}_{AB2}\underline{/120^\circ} \\ &= \bar{I}_{AB2}(1\underline{/240^\circ} - 1\underline{/120^\circ}) \\ &= \bar{I}_{AB2}(0 - j1.732) \\ &= \bar{I}_{AB2}(\sqrt{3}\underline{/-90^\circ}), \qquad (130\text{-}25)\end{aligned}$$

Take phase AB as the reference, the line currents can be written in terms of the sequence currents as follows:

$$\begin{aligned}\bar{I}_A &= \bar{I}_{A0} + \bar{I}_{A1} + \bar{I}_{A2} \\ &= \bar{I}_{AB1}(\sqrt{3}\underline{/-30^\circ}) + \bar{I}_{AB2}(\sqrt{3}\underline{/30^\circ}), \qquad (130\text{-}26)\end{aligned}$$

$$\begin{aligned}\bar{I}_B &= \bar{I}_{B0} + \bar{I}_{B1} + \bar{I}_{B2} \\ &= \bar{I}_{AB1}(\sqrt{3}\underline{/-150^\circ}) + \bar{I}_{AB2}(\sqrt{3}\underline{/150^\circ}), \qquad (130\text{-}27)\end{aligned}$$

$$\begin{aligned}\bar{I}_C &= \bar{I}_{C0} + \bar{I}_{C1} + \bar{I}_{C2} \\ &= \bar{I}_{AB1}(\sqrt{3}\underline{/90^\circ}) + \bar{I}_{AB2}(\sqrt{3}\underline{/-90^\circ}), \qquad (130\text{-}28)\end{aligned}$$

To complete the above equations, the positive and negative sequence currents of phase AB must be calculated. From figure 9, it can be shown that:

$$\bar{I}_{AB1} = (1/3)(\bar{I}_{AB} + a\bar{I}_{BC} + a^2\bar{I}_{CA}), \qquad (130\text{-}29)$$

$$\bar{I}_{AB2} = (1/3)(\bar{I}_{AB} + a^2\bar{I}_{BC} + a\bar{I}_{CA}), \qquad (130\text{-}30)$$

Where:

$$a = 1\underline{/120^\circ}, \qquad a^2 = 1\underline{/240^\circ}$$

Finally, to determine $\bar{I}_{AB}$, $\bar{I}_{BC}$, and $\bar{I}_{CA}$, the information supplied by the systems must be used. This information must consist of at least one of the following sets:

Set 1:

- Power (P) in kW of each phase load with pf = cos(Θ).

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- Terminal load voltage (V) of each phase.

Then,

$$\bar{I}_{AB} = [P_{AB}/V_{AB}\cos(\Theta_{AB})]\ \underline{/0° - \Theta_{AB}},\qquad (130\text{-}31)$$

$$\bar{I}_{BC} = [P_{BC}/V_{BC}\cos(\Theta_{BC})]\underline{/-120° - \Theta_{BC}},\qquad (130\text{-}32)$$

$$\bar{I}_{CA} = [P_{CA}/V_{CA}\cos(\Theta_{CA})]\underline{/120° - \Theta_{CA}},\qquad (130\text{-}33)$$

<u>Set 2</u>:

- *Load impedance* (Z_{LO}) *of each phase.*
- Load terminal voltage (V) of each phase.

Then,

$$\bar{I}_{AB} = \bar{V}_{AB}/\ \bar{Z}_{AB},\qquad (130\text{-}34)$$

$$\bar{I}_{BC} = \bar{V}_{BC}/\ \bar{Z}_{BC},\qquad (130\text{-}35)$$

$$\bar{I}_{CA} = \bar{V}_{CA}/\ \bar{Z}_{CA},\qquad (130\text{-}36)$$

Once the line currents $\bar{I}$ have been determined, the percent voltage drop for each line can be calculated from equation (100-10) as derived in section 100:

$$D\% = IL\left(\frac{z\cos(\alpha)}{E}\right) * 100,\qquad (100\text{-}10)$$

Where:

L : Cable length.
z : Conductor impedance per phase per foot.
E : Line-to-neutral source voltage.

$$\alpha = -\Theta + \beta,\qquad (90\text{-}2)$$

In the above equation, Θ is the angle by which the line current I lags the terminal load voltage, and it is calculated from equations (100-7), (100-8), or (100-9) as derived in section 100.

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140. DERIVATION OF CABLE RESISTANCES AND REACTANCES

This section contains the derivation of cable resistances and reactances used in tables XI through XVII. These cables reflect conductor diameters of standard American Wire Gage (AWG) in accordance with MIL-C-24643.

140.1 Calculations of cable resistance. the ideal method of obtaining accurate cable resistance and reactance is by test measurements. In the absence of test data,these parameters can also be determined mathematically.

The method used in this handbook is based upon the one specified in the ASTM B 258 and ASTM B 8. This method may be used to calculate with reasonable accuracy the resistances of any cables of concentric-lay-stranded conductors. The overall approach is as follows:

- Calculate dc resistance at 20° Celsius for solid conductor using ASTM B 258.
- Adjust for concentric-lay-stranded conductors using ASTM B 8.
- Adjust for temperature for which cable service will be designed.
- Adjust for ac resistance by multiplying the dc resistance by the (ac/dc) resistance conversion ratio at any desired frequencies (60/400 Hz).

From the ASTM B 258, dc resistance at 20°C of conductor in ohm per 1000 feet is given by:

$$R_{dc} = 105.35\left(\frac{\rho_o}{\delta d^2}\right) \qquad (140\text{-}1)$$

Where:

ρ_o = resistivity of conductor (copper) at 20°C

= 875.20 Ω lb/mile^2

δ = conductor (copper) density at 20°C

= 8.89g/cm^3

d = conductor diameter in mil.

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Equation (140-1) becomes:

$$R_{dc} = 10371.46\left(\frac{1}{d^2}\right)\ \Omega/1000\text{ft}, \qquad (140\text{-}2)$$

From equation (140-2), R_{dc} resistances for type SG equivalent solid conductors are computed and shown in table XVIII below.

TABLE XVIII. Dc Conductor Resistances at 20°C.

Designation	Conductor Diameter (mil)	R_{dc} ($\Omega/10^3$ft)
SG-3	50.79	4.021
SG-4	64.11	2.523
SG-9	101.88	0.999
SG-14	114.41	0.792
SG-23	144.29	0.498
SG-50	229.39	0.197
SG-75	289.29	0.124
SG-100	324.96	0.098
SG-150	409.63	0.062
SG-200	460.00	0.049
SG-300	547.72	0.035
SG-400	643.12	0.025

The data for these solid conductors must be adjusted for concentric-lay-stranded SG conductors. Due to the lay stranded conductors, the resistance per unit length of a stranded conductor will be slightly greater than that for an equivalent diameter solid conductor. ASTM B 8 provides a mathematical method for deriving the multiplying factor that is used to modify dc resistance of a solid conductor for an equivalent concentric-lay-stranded conductor. A lay factor (m_{ind}) is determined for each wire in a concentric-lay-stranded conductor from:

$$m_{ind} = \sqrt{1 + (9.8696/n^2)}, \qquad (140\text{-}3)$$

Where:

n = length of lay/diameter of wire helical path.

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The lay factor (m) for the complete stranded conductor is the numerical average of the lay factors (m_{ind}) of the individual wires in the conductor. Finally, the increment factor K is calculated from:

$$K = (m-1) * 100, \qquad (140\text{-}4)$$

K is the percentage increase in resistance of a solid conductor for an equivalent concentric-lay-stranded conductor. In lieu of performing many calculations based upon detailed conductor geometry, which would be required by the above method, an increment factor of 2 percent will be used in accordance with table 3 of ASTM B 8. Therefore, the concentric-lay-stranding of a solid conductor results in a nominal increase in electrical resistance of 2 percent. The previous tables for dc resistance of solid conductors can be used to generate a table for concentric-lay-stranded conductors by applying this increment factor.

TABLE XIX. Dc Stranded Conductor Resistances at 20°C

Designation	Number of Strands	R_{dc} ($\Omega/10^3$ ft)
SG-3	7	4.101
SG-4	7	2.573
SG-9	7	1.019
SG-14	7	0.808
SG-23	7	0.508
SG-50	19	0.201
SG-75	37	0.126
SG-100	61	0.100
SG-150	61	0.063
SG-200	61	0.050
SG-300	91	0.036
SG-400	127	0.026

Next, R_{dc} must be adjusted to the designed temperature of cable service. Resistance (R_t) at a selected temperature (t) can be calculated from the following equation:

$$R_t = R_{to}[1 + 0.00393(t - 20)], \qquad (140\text{-}5)$$

Where:

R_{to} = wire resistance at 20°C.

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t = temperature at which resistance is required.

(0.00393) = temperature coefficient of resistance at 20°C.

At t = 65°C:

$$\frac{R_t}{R_{to}} = [1 + 0.00393(65 - 20)]$$
$$= 1.177$$

Therefore, resistances in the preceding table must be multiplied by the above ratio to yield the dc resistances at 65°C. The following results are obtained:

TABLE XX. Dc stranded conductor resistances at 65°C

Designation	R_{dc} ($\Omega/10^3$ft)
SG-3	4.827
SG-4	3.028
SG-9	1.199
SG-14	0.951
SG-23	0.598
SG-50	0.237
SG-75	0.148
SG-100	0.118
SG-150	0.074
SG-200	0.059
SG-300	0.042
SG-400	0.031

Similarly, the (R_t/R_{to}) ratio at 45°C is as follows:

$$\frac{R_t}{R_{to}} = [1 + 0.00393(45 - 20)]$$
$$= 1.098$$

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The dc stranded-conductor resistances at 45°C for various cables are tabulated below:

TABLE XXI. Dc stranded conductor resistances at 45°C

Designation	R_{dc} ($\Omega/10^3$ft)
SG-3	4.503
SG-4	2.825
SG-9	1.119
SG-14	0.887
SG-23	0.558
SG-50	0.221
SG-75	0.138
SG-100	0.110
SG-150	0.069
SG-200	0.055
SG-300	0.040
SG-400	0.029

The dc resistances must be converted to ac resistances. The converting factor known as the skin effect ratio (SER) is determined as follows:

Determine the factor F:

$$F = 0.0635598\sqrt{(f\mu/R)}, \qquad (140\text{-}6)$$

Where,

f = System frequency.
μ = Wire permeability.
R = Dc resistance in Ω/mile.

Equation (140-6) can be rewritten as:

$$F = 0.027677\sqrt{f/R}, \qquad (140\text{-}7)$$

Where the wire permeability u is assumed equal to 1.0000 and the dc resistance R is in ohm per 1000 feet. Then, from the table below, the skin effect ratios are determined.

TABLE XXII. Skin effect ratio

F	SER	F	SER	F	SER
0.0	1.00000	1.3	1.01470	2.6	1.20056
0.1	1.00000	1.4	1.01969	2.7	1.22753
0.2	1.00000	1.5	1.02582	2.8	1.25620
0.3	1.00004	1.6	1.03323	2.9	1.28644
0.4	1.00013	1.7	1.04205	3.0	1.31809
0.5	1.00032	1.8	1.05240	3.1	1.35102
0.6	1.00067	1.9	1.06440	3.2	1.38504
0.7	1.00124	2.0	1.07816	3.3	1.41999
0.8	1.00212	2.1	1.09375	3.4	1.45570
0.9	1.00340	2.2	1.11126	3.5	1.49202
1.0	1.00519	2.3	1.13069	3.6	1.52879
1.1	1.00758	2.4	1.15207	3.7	1.56587
1.2	1.01071	2.5	1.17538	3.8	1.60314

With R_{dc} given in the previous tables, the ac resistance at 60 Hz and 400 Hz can be determined using the ac to dc resistance ratios in the following table:

TABLE XXIII. Dc to ac resistance conversion ratios

Designation	Ac/dc Ratio at 60 Hz	Ac/dc Ratio at 400 Hz
SG-3	1.000	1.000
SG-4	1.000	1.000
SG-9	1.000	1.000
SG-14	1.000	1.000
SG-23	1.000	1.000
SG-50	1.000	1.000
SG-75	1.000	1.033
SG-100	1.000	1.052
SG-150	1.000	1.111
SG-200	1.000	1.175
SG-300	1.007	1.286
SG-400	1.015	1.456

Therefore, ac resistance of conductors at 60/400 Hz and at any temperatures can be calculated by multiplying the appropriate (ac/dc) resistance conversion factors by the R_{dc} of concentric-lay-stranded conductors in the previous tables.

TABLES XXIV. Ac resistance for SG conductors at 65°C

Designation	R_{ac} at 60 Hz ($\Omega/10^3$ft)	R_{ac} at 400 Hz ($\Omega/10^3$ft)
SG-3	4.827	4.827
SG-4	3.028	3.028
SG-9	1.199	1.199
SG-14	0.951	0.951
SG-23	0.598	0.598
SG-50	0.237	0.237
SG-75	0.148	0.153
SG-100	0.118	0.124
SG-150	0.074	0.082
SG-200	0.059	0.069
SG-300	0.042	0.054
SG-400	0.031	0.045

TABLES XXV. Ac resistance for SG conductors at 45°C

Designation	R_{ac} at 60 Hz ($\Omega/10^3$ft)	R_{ac} at 400 Hz ($\Omega/10^3$ft)
SG-3	4.503	4.503
SG-4	2.825	2.825
SG-9	1.199	1.199
SG-14	0.887	0.887
SG-23	0.558	0.558
SG-50	0.221	0.221
SG-75	0.138	0.143
SG-100	0.110	0.116
SG-150	0.069	0.077
SG-200	0.055	0.065
SG-300	0.040	0.051
SG-400	0.029	0.042

These resistance values are used in tables XI through XIV, and XVI through XVII. For type 6SG cables in table XV, two conductors are in parallel for each phase. Therefore, the resistance per phase is half of the ac resistance (400 Hz) calculated earlier at the required designed temperature of 65°C. The resulting resistances are shown in table XXVI.

TABLE XXVI. Ac resistance for 6SG conductors at 65°C

Designation	R_{ac} at 400 Hz ($\Omega/10^3$ft)
6SG-100	0.062
6SG-125	0.049
6SG-150	0.041
6SG-200	0.035

140.2 Calculations of cable reactances. The determination of cable reactances is relatively complicated since the reactances are not only dependent upon the geometry of conductors in the cable, but also on the external environment of the cable, e.g., conducting armor material and closeness to surrounding steel. Below are the step-by-step calculations for cable reactances per phase. These calculations give a reasonable estimate of the total cable reactances. The actual reactances will differ slightly when all magnetic effect are considered, and only for larger cables at high frequencies.

Step 1.

Determine the conductor geometric mean radius (GMR) as follows:

$$GMR = 0.779(d/2), \qquad (140\text{-}8)$$

Where d is the conductor diameter.

Step 2.

Determine the conductor geometric mean distance (GMD) as follows:

$$GDM = \sqrt[3]{D_{AB}D_{BC}D_{CA}} \qquad (140\text{-}9)$$

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Where,

D_{AB} : Distance between centers of conductors A and B.

D_{BC} : Distance between centers of conductors B and C.

D_{CA} : Distance between centers of conductors C and A.

Step 3.

Determine the reactance per phase as follows:

$$X = 0.05292(f/60)\log_{10}(GMD/GMR)\ \Omega/1000ft, \qquad (140\text{-}10)$$

Where f is the frequency at which the reactance is calculated.

From equation (140-10) with conductor characteristics listed in MIL-C-24643 the following tables for reactances can be generated.

TABLE XXVII. Reactances for SG conductors

Designation	GMR	GMD	X at 60 Hz ($\Omega/10^3$ft)	X at 400 Hz ($\Omega/10^3$ft)
SG-3	0.0198	0.1300	0.0432	0.2883
SG-4	0.0250	0.1430	0.0401	0.2672
SG-9	0.0397	0.1870	0.0356	0.2374
SG-14	0.0446	0.2620	0.0407	0.2713
SG-23	0.0562	0.3100	0.0392	0.2616
SG-50	0.0893	0.3340	0.0303	0.2021
SG-75	0.1127	0.4070	0.0295	0.1967
SG-100	0.1266	0.4530	0.0293	0.1953
SG-150	0.1596	0.5570	0.0287	0.1915
SG-200	0.1792	0.6340	0.0290	0.1936
SG-300	0.2133	0.7480	0.0288	0.1922
SG-400	0.2463	0.8620	0.0288	0.1919

Again, the reactances for 6SG conductors are equal to half of the SG conductor reactances as shown in Table XXVIII.

TABLE XXVIII. Reactances for 6SG conductors at 65°C

Designation	X at 400 Hz ($\Omega/10^3$ft)
6SG-100	0.0977
6SG-125	0.0974
6SG-150	0.0958
6SG-200	0.0968