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Department OF DEFENSE

TEST & EVALUATION of SYSTEM RELIABILITY AVAILABILITY and MAINTAINABILITY

A Primer

DIRECTOR TEST AND EVALUATION

Office of the Under Secretary of Defense
for Research and Engineering

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TEST AND EVALUATION

OF

SYSTEM

RELIABILITY AVAILABILITY MAINTAINABILITY

- A PRIMER -

Dr. John C. Conlon
Army Materiel Systems Analysis Activity

Mr. Walter A. Lilius
Army Management Engineering Training Activity

Lt. Col. Frank H. Tubbesing, Jr., USAF
Office of the Director Defense Test and Evaluation

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RESEARCH AND
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FOREWORD

This handbook is issued under the authority of DoD Instruction 3235.1, "Test and Evaluation of System Reliability, Availability, and Maintainability," February 1, 1982. Its purpose is to provide instruction in the analytical assessment of System Reliability, Availability, and Maintainability (RAM) performance.

The provisions of this handbook apply to the Office of the Secretary of Defense, the Military Departments, the Organization of the Joint Chiefs of Staff, and the Defense Agencies (hereafter referred to as "DoD Components").

This handbook is effective immediately and may be used on an optional basis by DoD Components engaged in system RAM.

Send recommended changes to the handbook to:

Office of the Director
Defense Test and Evaluation OUSDRE/DDTE
Washington, D.C. 20301

DoD Components may obtain copies of this handbook through their own publication channels- Other federal agencies and the public may obtain copies from the Naval Publication and Forma Center, 5801 Tabor Avenue, Philadelphia,Pa. 19120.

ISHAM LINDER
Director Defense
Test and Evaluation

OFFICE OF THE UNDER SECRETARY OF DEFENSE

WASHINGTON. D.C. 20301

RESEARCH AND ENGINEERING

MEMORANDUM FOR TEXT USERS

SUBJECT : The Application of Statistical Concepts to Test and Evaluation

Test and Evaluation of military systems and equipment is conducted by DoD to support the assessment of system performance characteristics. These assessments are an integral part of the decision process inherent in the acquisition cycle.

In many hardware and software development programs, testing has become a controversial issue. Questions which often arise are: How much testing is enough? Is the hardware/software ready for testing? Are hardware require - ments, assessment parameters and critical issues adequately defined? Does the test effort represent the minimum time and resource program consistent with meaningful results? Have the development and operational testing cycles been integrated so as to form an efficient evaluation program? And so on.

This text presents concepts and techniques for designing test plans which can verify that previously established system suitability requirements have been achieved. We realize, of course, that test resource availability ❑ay be adversely affected by cost, schedule and operational urgency constraints. In such cases, alternate test plans which represent the most meaningful, timely and cost effective approach, consistent with these constraints, must be developed. In any event, it is essential that all participants understand the critical issues being addressed and the acquisition risks inherent in conducting a limited test program.

The design and execution of sound test programs is NO accident. It requires numerous hours of research and planning and a thorough understanding of testing techniques, the test system and its operating scenario. Further, the test results must support the development of realistic performance estimates for the entire production run, after having tested relatively few systems. Herein lies the usefulness of the statistical concepts contained in this text.

The topics addressed in this text will familiarize the reader with the statistical concepts relevant to test design and performance assessment. In short, these topics, when combined with common sense and technical expertise formulate the basis of all sound test programs.

Isham Linder
Director Defense
Test and Evaluation

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CHAPTER 1

INTRODUCTION

One step in the acquisition of military weapon systems and equipment is the verification that the candidate systems do, in fact, perform in accordance with previously specified operational requirements. The verification process involves the design of test programs which provide an adequate data base to support realistic assessments of hardware characteristics. This text outlines the concepts and techniques to be used in structuring such test programs and analyzing the resulting data.

Since verifying the performance of every hardware item to be procured is neither practical nor possible, we base our projection of the entire population's performance on the assessment of an available sample. This sample may consist of the first 10 preproduction aircraft of a projected buy of 725 or 50 percent of a lot of high-reliability pyrotechnic devices. In either case, we are utilizing statistical techniques to project or estimate the true value of some population's characteristic, such as reliability, maintainability, weight, size, etc.

The material contained in the following chapters is designed to familiarize the reader with the various statistical concepts and techniques required to thoroughly understand the relationships among test design, assessment and projection of population characteristics.

The beginning chapters present background material on the three basic quantitative system parameters: reliability, availability and maintainability. The various definitions of these basic terms are discussed, as are the relationships among these parameters. The statistical concepts of confidence and producer's and consumer's risk are next introduced, with the majority of the text devoted to the practical application and significance of these concepts. The chapters utilize a combination of narrative and case studies to introduce and illustrate the usefulness of the concepts. It will prove quite useful to refer to the case studies while reading through the chapters. This study technique will prove especially helpful in Chapters 7 and 8, which present information on analyzing test data and structuring statistically adequate test programs. Chapter 9 contains an introductory discussion of the reliability growth concept. Chapter 10 presents qualitative aspects of test planning along with a description of data collection requirements.

It should be noted that there is no intent here to indicate that all DoD test programs must produce statistically significant test results. Obviously some will, but it is essential to understand the risk associated with a proposed test program and the confidence associated with specific results before recommending a course of action.

A first glance at the middle of one of the more "intense" chapters will quickly bring the reader to the conclusion that the real title of the text should have been "Everything You Never Knew about Statistics and Never Will"--SLAM!! In fact, the text could be entitled "What To Do Until the Statistician Arrives." Anyone able to work through the entire text without any questions needs no statistician--for most tasks. The text, however, makes no attempt to eliminate the need for expert advice but rather attempts to aid the reader in recognizing the simplicity of some concepts, the complexity of others, the assumptions and limitations associated with all of them, and the importance of the topic to test and evaluation in general.

CHAPTER 2

RELIABILITY

Reliability is a term used to describe quantitatively how failure-free a system is likely to be during a given period of operation. The ability to express reliability numerically is crucial, because it enables us to concretely identify the user's needs, contractual specifications, test guidelines and performance assessment.

DEFINITION OF TERMS AND CONCEPTS

Reliability

Reliability is defined as the probability that an item will perform its intended function for a specified interval under stated conditions . This definition does not specifically consider the effect of the age of the system.

The following adaptation is useful for systems that are repairable. Reliability, for repairable systems, is the probability that an item will perform its intended function for a specified interval, under stated conditions, at a given age, if both corrective and preventive maintenance are performed in a specified manner.

If a system is capable of performing multiple missions, or if it can perform one or more of its missions while operating in a degraded condition or if the mission test profiles represent only typical usage, then, the concept of a unique mission reliability becomes difficult to define. In such cases , 't is preferable to use a reliability measure that is not based solely on the length of a specified time interval but rather on the definition of a specific mission profile or set of profiles. This concept is illustrated in Case Study 2-7.

The meaning of the terms "stated conditions' and "specified interval" are important to the understanding of reliability. The term "stated conditions" refers to the complete definition of the scenario in which the system will operate. For a ground combat vehicle, these conditions include climatic conditions, road surface, and loads that would be experienced during a selected mission profile. These conditions should reflect operational usage. The term "specified interval" refers to the length of the mission described in a mission profile. This interval may include ❑ultiple factors. For example, an air defense system mission profile will define an interval containing X rounds fired, Y hours of electronics on-time and Z miles of travel. For a simpler system, say an air-burst artillery round, the interval may include a single event--round detonation.

Mean Time Between Failures

Mean time between failures (MTBF) is defined as the total functioning life of a population of an item during a specific measurement interval, divided by the total number of failures within the population during that interval. MTBF can be interpreted as the expected length of time a system will be operational between failures. The definition is true for time, cycles, miles, events, or other measure-of-life units. These various measure-of-life units permit the MTBF term to be tailored to the reliability requirements of a specific system. Some examples of this tailoring are:

- Mean rounds between failure (MRBF)
- Mean miles between operational mission failure (MMBOMF)
- Mean time between unscheduled maintenance actions (MTBUMA)
- Mean rounds between any maintenance actions (MRBAMA)

Failure Rate

Failure rate is defined as the number of failures of an item per measure-of-life unit (e.g., cycles, time, miles or events as applicable). This measure is more difficult to visualize from an operational 'standpoint than the MTBF measure, but is a useful mathematical term which frequently appears in many engineering and statistical calculations. As we will see in later chapters the failure rate is the reciprocal of the MTBF measure, or

$$\text{Failure Rate} = \frac{1}{\text{MTBF}}$$

SYSTEM RELIABILITY DESIGN OBJECTIVES

There are two very different system reliability design objectives. One is to enhance system effectiveness; the other is to minimize the burden of owning and operating the system. The first objective is addressed by means of mission reliability, the second by means of logistics-related reliability. Measures of mission reliability address only those incidents that affect mission accomplishment. Measures of logistics-related reliability address all incidents that require a response from the logistics system.

Mission Reliability

Mission reliability is the probability that a system will perform mission essential functions for a period of time under the conditions stated in the mission profile. Mission reliability for a single shot type of system, i.e., a pyrotechnic device, would not include a time period constraint. A system with a high mission reliability has a high probability of successfully completing the defined mission.

Measures of mission reliability address only those incidents that affect mission accomplishment. A mission reliability analysis must, therefore, include the definition of mission essential functions. For example, the mission essential functions for a tank might be to move, shoot and communicate. More specific requirements could specify minimum speed, shooting accuracy and communication range'.

See Case Study 2-7.

Logistics (Maintenance/Supply) Related Reliability

Logistics related reliability measures, as indicated above, must be selected so that they account for or address all incidents that require a response from the logistics system.

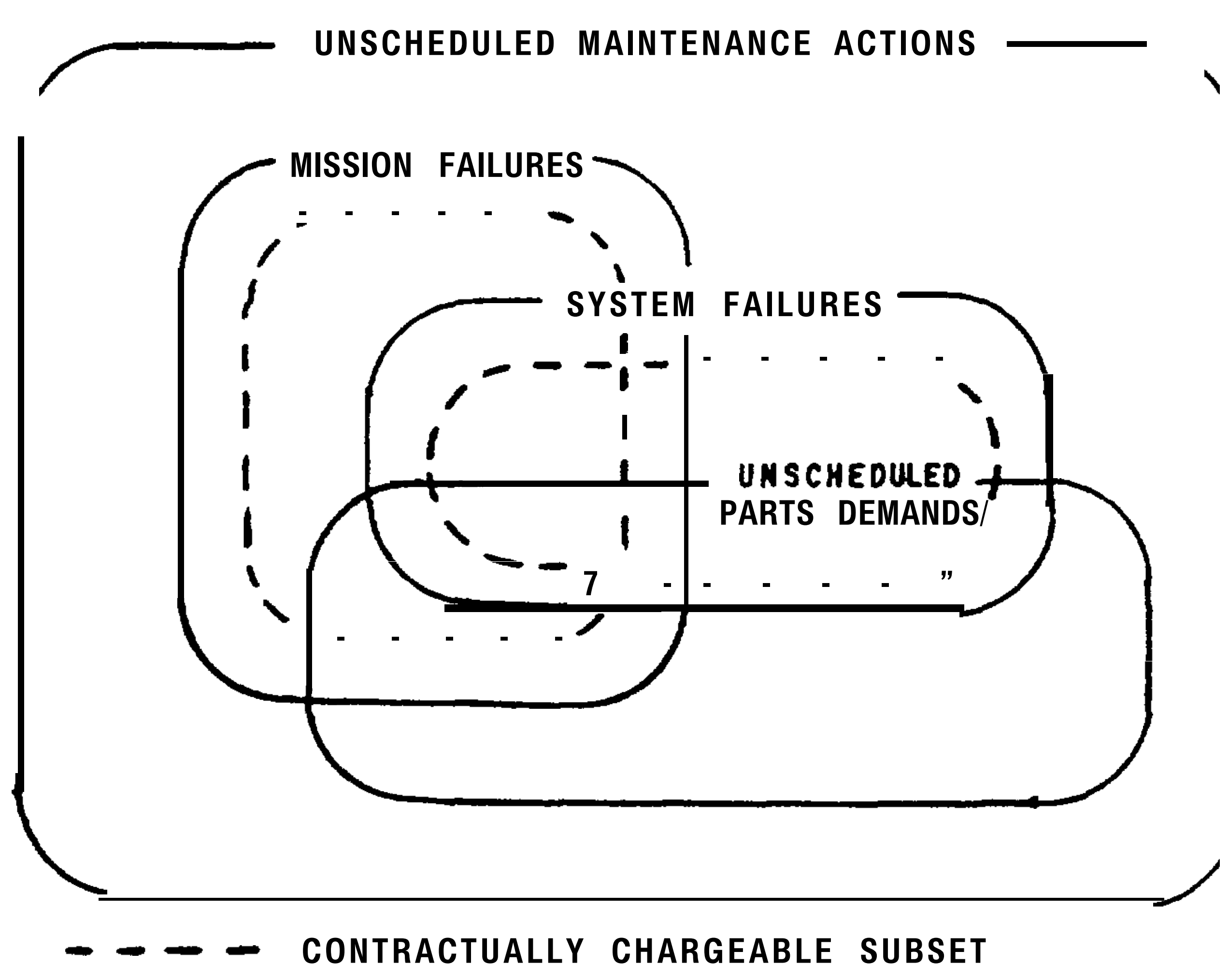
Logistics related reliability may be further subdivided into maintenance related reliability and supply related reliability. These parameters respectively represent the probability that no corrective maintenance or the probability that no unscheduled supply demand will occur following the completion of a specific mission profile.

The mathematical models used to evaluate mission and logistics reliability for the same system may be entirely different. This is illustrated in Case Study 2-3.

RELIABILITY INCIDENT CLASSIFICATION

An understanding of the relationships existing between the above reliability measures and other terms is essential to the knowledgeable application of these parameters. Figure 2-1 illustrates the effects of these relationships not their causes. For example, system failures may be caused by the hardware itself, by the operator, or by inadequate/faulty maintenance.

FIGURE 2-1 RELIABILITY INCIDENT CLASSIFICATION



Mission Failures

Mission failures are the loss of any of the mission's essential functions. Along with system hardware failures, operator errors and errors in publications that cause such a loss are included in this region. Mission failures are related to mission reliability measures because they prevent complete mission accomplishment.

System Failures

System failures are hardware malfunctions: they may or may not affect the mission's essential functions, and they may or may not require spares for correction. A system failure generally requires unscheduled maintenance so system failures heavily influence maintenance-related reliability.

Unscheduled Spares Demands

Unscheduled spares demands are used to evaluate supply-related reliability. All unscheduled spares demands require a response from the supply system, so they form the basis for evaluating supply-related reliability.

System/Mission Failures Requiring Spares

System/mission failures that require spares for correction are the most critical. Mission, maintenance and supply reliabilities are affected, and the system runs the risk of being held in a non-mission-ready status for an extended period of time by logistics delays.

Contractually Chargeable Failures

Contract requirements are often established for the subset of mission failures and/or system failures for which the contractor can be held accountable. Normally excluded from contractual chargeability are such failure categories as: operator or maintenance errors; item abuse; secondary failures caused by another (primary) failure; and failures for which a "fix" has been identified (but not incorporated in the test article that failed). It should be noted that, in operation, all failures (in fact, all unscheduled maintenance actions) are relevant regardless of contractual chargeability, and should therefore be included in operational evaluations.

SYSTEM RELIABILITY MODELS

System reliability models are utilized to describe visually and mathematically the relationship between system components and their effect on the resulting system reliability. A reliability block diagram or structural model provides the visual representation while the mathematical or "math" model provides the analytical tool to calculate quantitative reliability values.

The following notation is used in the discussion of reliability models:

R_s = reliability of the system

R_i = reliability of the i^{th} subsystem

$Q_s = 1 - R_s$ = unreliability of the system

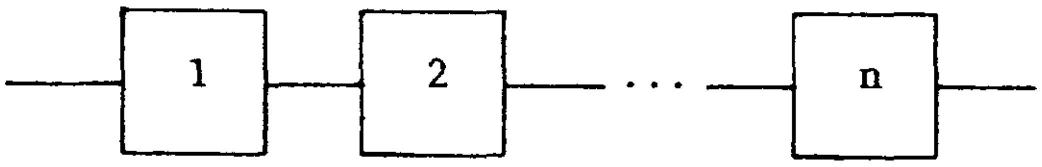
$Q_i = 1 - R_i$ = unreliability of the i^{th} subsystem

Π = product of (Note: This operator is used in the same fashion as Σ for summation, but it indicates multiplication rather than addition.)

Note: In the following discussion it is assumed that all subsystems function independently of one another, that is, failures of different subsystems are statistically independent of each other. For many systems this represents a realistic assumption. The reliability analysis for dependent subsystems is significantly more complex. Independent operation, practically speaking, means that a failure of one system will not cause a change in the failure characteristics of one or more other subsystems. Therefore, replacement of the single failed subsystem should permit continued operation of the entire system, because other subsystems were not affected.

Series Model

When a group of components or subsystems is such that all must function properly for the system to succeed, they are said to be in series. A system consisting of a series arrangement of n subsystems is illustrated in the following block diagram:



The mathematical model is

$$R_s = R_1R_2 \ldots R_n = \prod_{i=1}^{n} R_i. \tag{2.1}$$

See Case Studies 2-1, 2-2, 2-3, 2-5, and 2-6 for examples of reliability series models.

Redundant Models

The mission reliability of a system containing independent subsystems can usually be increased by using subsystems in a redundant fashion, that is, providing more subsystems than are absolutely necessary for satisfactory performance. The incorporation of redundancy into a system design and the subsequent analysis and assessment of that design is a complex task and will not be addressed here in detail. We will define the elements of redundancy and present several simplified examples.

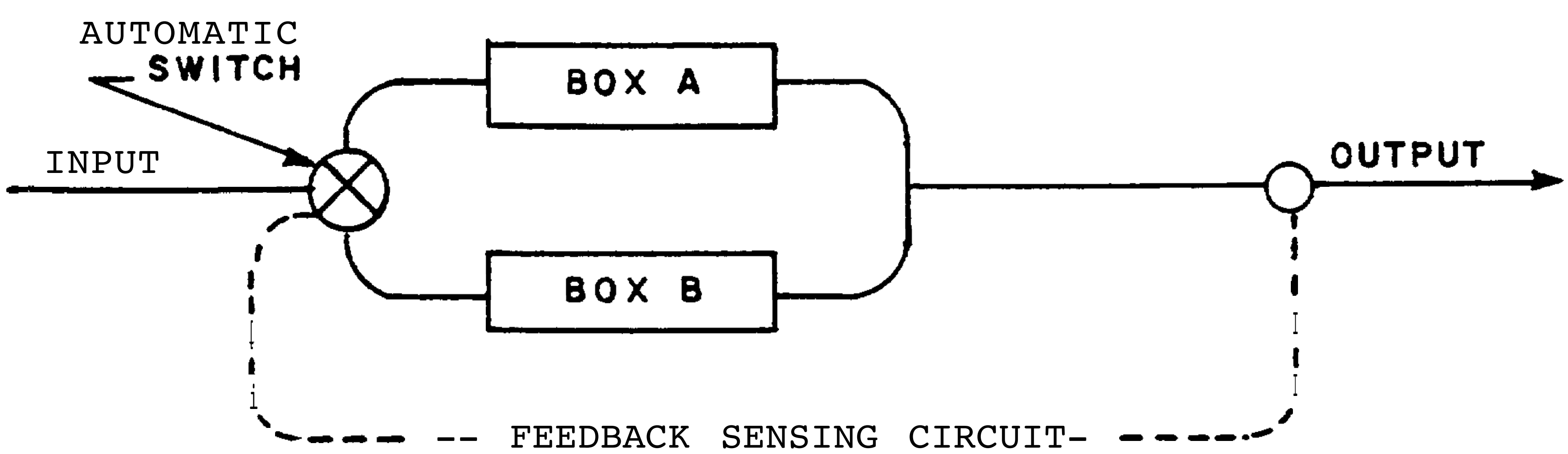
Redundance Characteristics. Redundance can be defined by three basic characteristics.

- First, the level at which redundancy is applied. For example, we could have redundant pieceparts, redundant black boxes, or complete redundant systems.

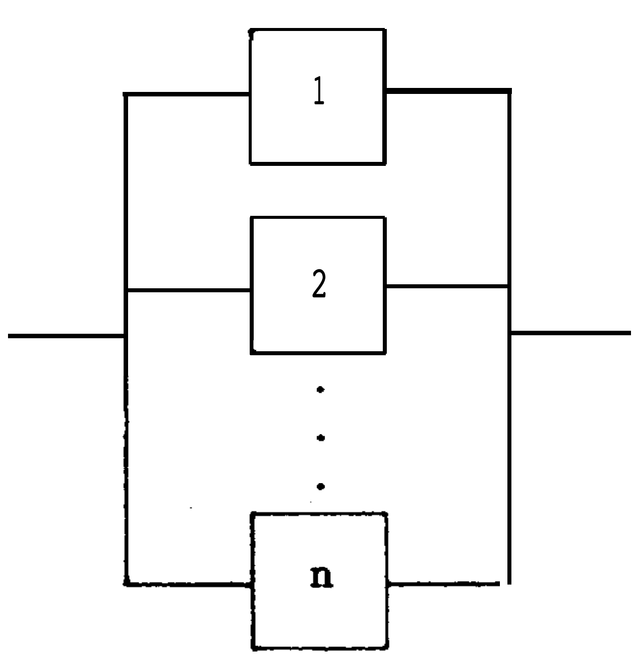
- Second, the operating state of the redundant element. The redundant part, subsystem, etc. , may exist in the circuit as an active functioning element or as a passive, power off, element. For example, an airport that maintains two separate operating ground control approach radars at all times has active redundancy for that capability. Carrying a spare tire in your trunk is an example of passive redundancy.

- Third, the method used to activate the redundant element. Consider the passive redundancy case of the spare tire. The vehicle driver represents the switching device that decides to activate the spare. Obviously mission time is lost in installing the spare. The opposite case is represented by the use of an electronic switching network that senses the failure of Box A and automatically switches to Box B without lost time or mission interruption.

FIGURE 2-2 PASSIVE REDUNDANCY WITH AUTOMATIC SWITCHING



Our examples will consider only simple active redundancy. In this type of redundancy, all the operable subsystems are operating, but only **one** is needed for satisfactory performance. There are no standby subsystems, and no repair is permitted during the mission. Such a system of n subsystems is illustrated in block diagram form as:



Note: Simple active redundancy requires that only one of the n subsystems be operating for mission success.

The mathematical model is

$$Q_s = Q_1 Q_2 \ldots Q_n = \prod_{i=1}^{n} Q_i = \prod_{i=1}^{n} (1-R_i).$$

$$R_s = 1-Q_s = 1- \prod_{i=1}^{n} (1-R_i). \tag{2.2}$$

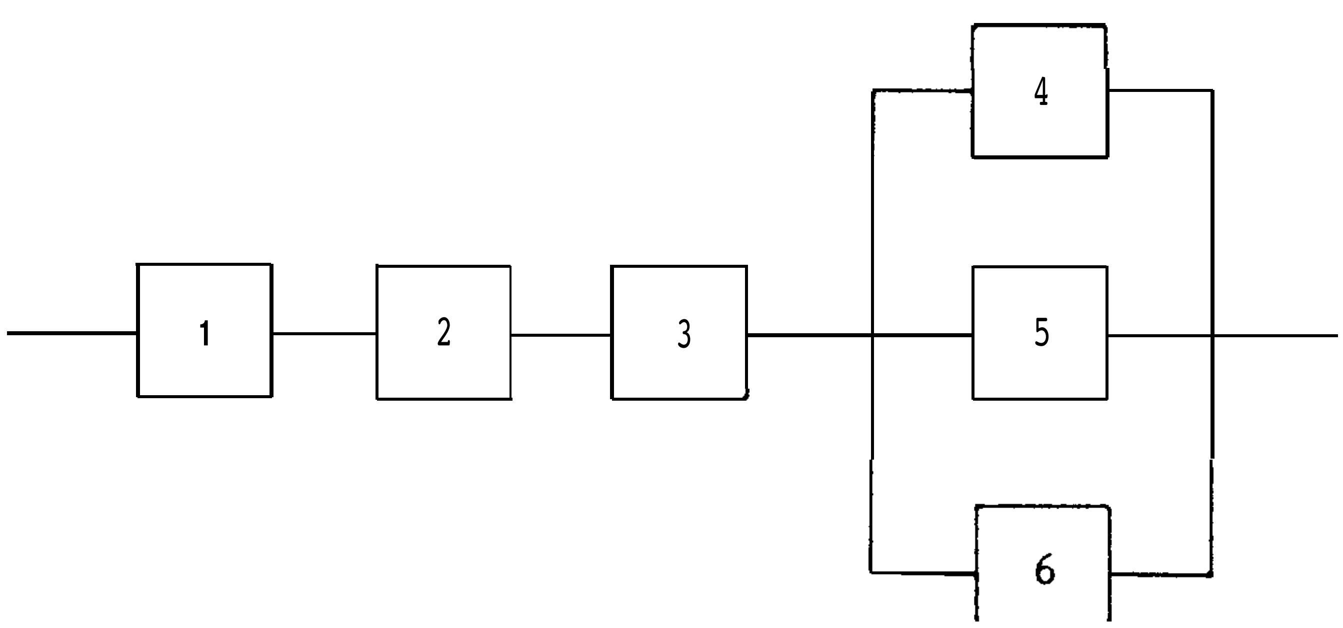
This model again assumes that there is statistical independence among failures of the subsystems. This assumption is important because dependence between subsystems can have a significant effect on system reliability. Calculations based on an assumption of independence can be erroneous and misleading. In fact, erroneously assuming failure independence will often result in overestimating system reliability for an active redundant system and underestimating reliability for a series system.

Implications of Redundant Design. While redundant design does improve mission reliability, its use must' be weighed against the inherent disadvantages. These disadvantages include greater initial cost, increased system size and weight, increased maintenance burden and higher spares demand rates. These factors must be considered by using and procuring agencies and by testing organizations when assessing the true mission capability and support requirements .

Although there are some possible exceptions, redundancy generally improves mission reliability and degrades logistics reliability. Case Study 2-3 gives a numerical comparison between ❑ission- and maintenance-related reliability.

Mixed Models

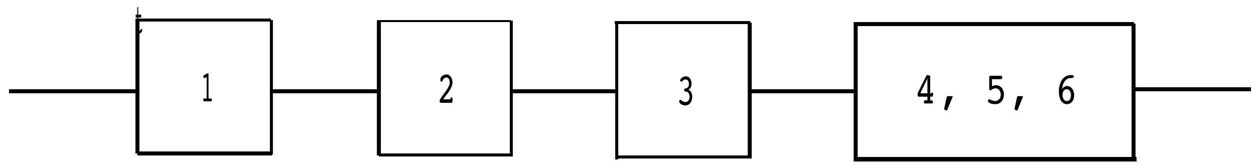
One system configuration that is often encountered is one in which subsystems are in series, but redundancy (active) is applied to a certain critical subsystem(s). A typical block diagram follows:



This type of model (or any mixed model, for that matter) is characterized by working from low to high levels of assembly. In this case, assuming independence and active redundancy, we can apply equation 2.2.

$$R_{4,5,6} = 1-(1-R_4)(1-R_5)(1-R_6) \tag{2.3}$$

We can now represent the redundant configuration of 4, 5, and 6 by a single block on the diagram.



We can now apply equation 2.1:

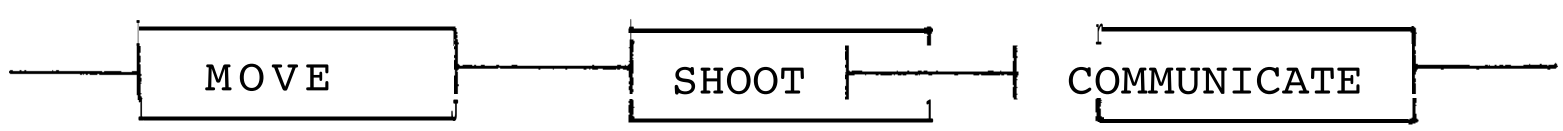
$$R_S = R_1 R_2 R_3 R_{4,5,6}$$

$$R_s = R_1R_2R_3[1-(1-R_4)(1-R_5)(1-R_6)] \tag{2.4}$$

See Case Study 2-4 for the numerical analysis of a mixed model.

Functional Models

The series, redundant and mixed models mentioned above, are hardware-oriented in that they display hardware capabilities. In some cases, it is desirable to model a system from a functional standpoint. As an example, the functional reliability block diagram for a tank is shown below:



The functions may be defined as:

- MOVE. The mobility system must be capable of effectively maneuvering such that the system can maintain its assigned position within a tactical scenario. Specific requirements are determined for speed, turning, terrain, etc.

- SHOOT. The main gun must be capable of delivering effective fire at the rate of X rounds per minute.

- COMMUNICATE. The communication system must be capable of providing two-way communication with other vehicles and with fixed stations within specific ranges and terrain confines.

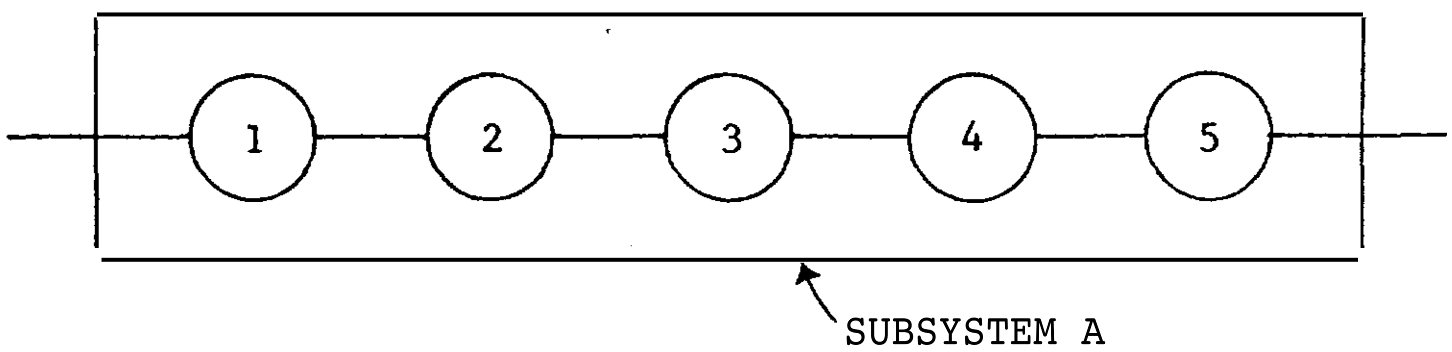
Note that this concept addresses mission-essential functions, but in no way implies how these functions will be accomplished. Generally the functional model approach is helpful in the program formulation stages of a program when specific hardware information is not necessary and frequently not desired. This type of model can provide a useful transition from operational requirement to engineering specifications.

Case Study 2-7 utilizes this concept to evaluate the multi-mission, multi-mode capabilities of a system.

BELIABILITY ALLOCATION

The previous section presented the topic of functional reliability models and indicated that these models provided a useful means of transitioning from operational requirements to engineering specifications. The process of transitioning from operational requirements to engineering specifications is known as reliability allocation. The reliability allocation process "allocates" the reliability "budget" for a given system or subsystem to the individual components of that system or subsystem. An example will prove helpful.

Suppose we have previously determined that the reliability of an electronic subsystem A, must equal or exceed 0.90, and that this subsystem has been designed with 5 parts all functioning in series. For this example, we will assume Parts 1, 2 and 3 are the same and the best available piece part for Part 4 has a reliability of 0.990. How can we allocate the reliability budget for this subsystem to its individual parts?



Using equation 2.1 we have

$$R_{Total} = R_1R_2R_3R_4R_5$$

$$= R_1R_2R_3(0.990)R_5$$

Solving for $R_1R_2R_3R_5$ we have

$$R_1R_2R_3R_5 = \frac{0.900}{0.990} = 0.909$$

If we assume $R_1 = R_2 = R_3 = R_5$ then,

$$R_1 = R_2 = R_3 = R_5 = \sqrt[4]{0.909} = 0.976$$

If we can locate piece parts for Part 5 with $R_5 = 0.985$, then

$$R_1R_2R_3 = \frac{0.909}{R_5} = \frac{0.909}{0.985} = 0.923. \text{ So,}$$

$$R_1 = R_2 = R_3 = \sqrt[3]{0.923} = 0.973.$$

Summarv of Allocation

Case I	Case II
$R_1 = R_2 = R_3 = R_5 = 0.976$	$R_1 = R_2 = R_3 = 0.973$
$R_4 = 0.990$	$R_4 = 0.990$
	$R_5 = 0.985$

Another, and somewhat more frequently used approach to reliability allocation is one in which reliability is allocated on the basis of allowable failures or failure rates.

The understanding of reliability allocation is important to those individuals who must be concerned with hardware operating characteristics below the system level. This is especially true to development and testing organizations who are frequently faced with predicting system performance early in development, when no full-up system exists but when subsystem or component test data may be available.

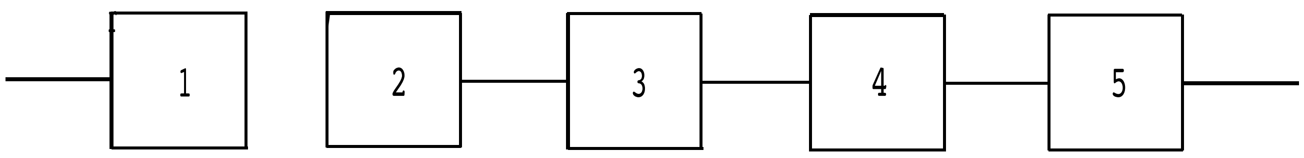
See Case Study 2-5 for another example of reliability allocation.

CASE STUDY NO. 2-1

Background

A system is composed of 5 subsystems, each of which must succeed for system success. Past records indicate the subsystem reliabilities to be as shown on the diagram.

$R_1 = 0.9 \quad R_2 = 0.95 \quad R_3 = 0.99 \quad R_4 = 0.99 \quad R_5 = 0.9$



Determine

System reliability.

Solution

Applying equation 2.1:

$$R_S = R_1R_2R_3R_4R_5 = (0.9)\ (0.95)(0.99)(0.99)(0.9) = 0.75$$

Commentary

Note that the system reliability is lower than that of the worst subsystem. This is generally the case for a series structured system.

CASE STUDY NO. 2-2

Background

An electronic system has 1000 components in reliability series. The reliability of each component is 0.999.

Determine

System reliability.

Solution

Applying equation 2.1:

$$R_s = \prod_{i=1}^{1000} 0.999 = (0.999)^{1000} \cong 0.368$$

Commentary

1. Even though a component reliability of 0.999 sounds good, the sheer number of these components causes a low system reliability.

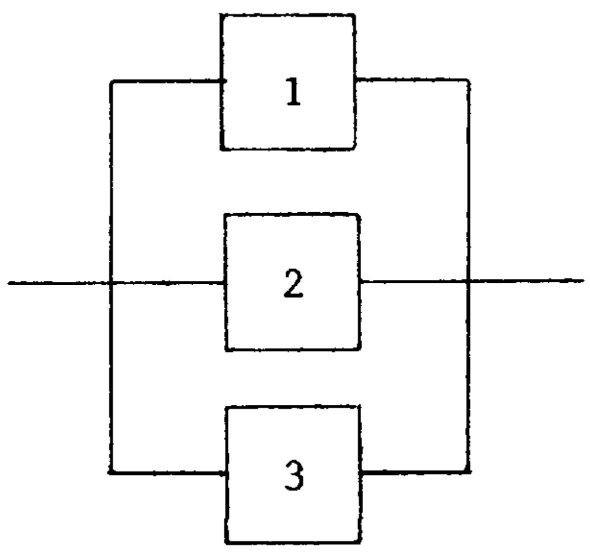
2. Even though $0.999 \cong 1.0$, the difference is crucial. For high reliability values, the probability of failure often gives a clearer picture. For example, increasing the component reliability from 0.999 to 0.9999 requires a ten-fold reduction of the failure probability.

3. Problems such as this, involving large powers, are solved effortlessly with an electronic calculator with a power capability. The more traditional approach is, of course, the use of logarithms and anti-logarithm tables.

CASE STUDY NO. 2-3

Background

The mission reliability of the system is described by the following block diagram. All subsystems are identical and each has a reliability of R = 0.90. No repairs are possible during a mission, but will be made following missions in which failures occur. Failures occur independently. For this case, we assume that any mission failure will require an unscheduled maintenance action.



Determine

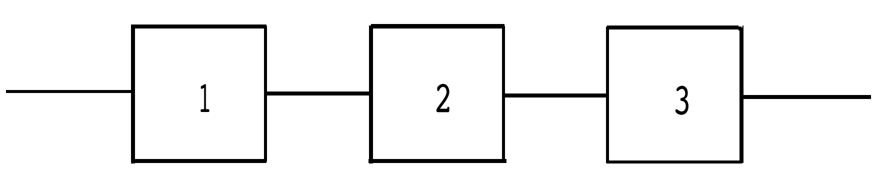
System mission reliability and maintenance reliability (probability of no corrective maintenance following a mission).

Solution

System mission reliability: Applying equation 2.2:

$$R_s = 1-(1-R)^n = 1-(1-0.9)^3 = 1-(0.1)^3 = 1-0.001 = 0.999$$

Maintenance reliability: An unscheduled maintenance action required by any subsystem is chargeable to the maintenance burden of the entire system, i.e. , a failure, defined in this case to be a requirement for corrective maintenance of one subsystem, is charged as a system failure. As a consequence, we model maintenance reliability for this system using a series structure.



Applying equation 2.1:

$$R_S = R^n = (0.9)^3 = 0.729$$

Commentary

1. Note that we evaluated system mission reliability; that is, the reliability of the hardware alone.

2. Based on the given information, it is apparent that a system consisting of only one of the above subsystems will have a probability of mission failure equal to 0.1 and a probability of corrective maintenance action also equal to 0.1. The system with triple active redundancy has a mission reliability of 0.999, which corresponds to a probability of mission failure equal to 0.001 (a 100-fold reduction). The same system has a maintenance reliability of 0.729 which corresponds to a probability of corrective maintenance action equal to 0.271 (approximately a 3-fold increase). The procuring and using agencies must decide whether to contract for redundancy and how much to require based on consideration of these parameters.

3. Note that with active redundancy the system reliability is generally greater than the reliability of the best subsystem.

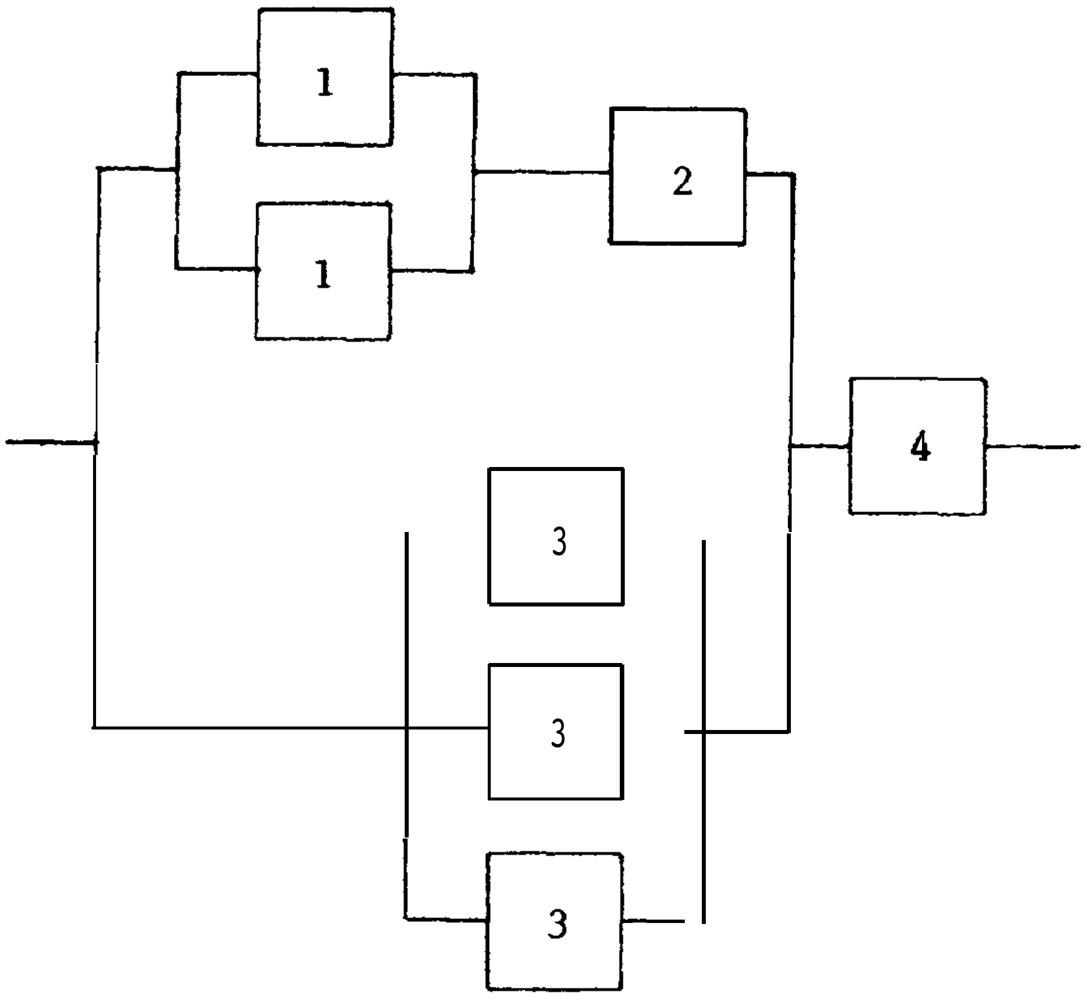
4. For this example, we stipulated that any mission failure would require an unscheduled maintenance action. The reader should note that this is not always the case.

5. It is possible, for example, to experience the failure of one of two redundant mission critical subsystems and still successfully complete the mission. After successful mission completion, an unscheduled maintenance action would be necessary to repair/replace the defective critical redundant subsystem.

CASE STUDY NO. 2-4

Background

Consider the following block diagram. Components with the same number are identical and consequently have the same reliability.



$R_1 = 0.80$

$R_2 = 0.95$

$R_3 = 0.70$

$R_4 = 0.90$

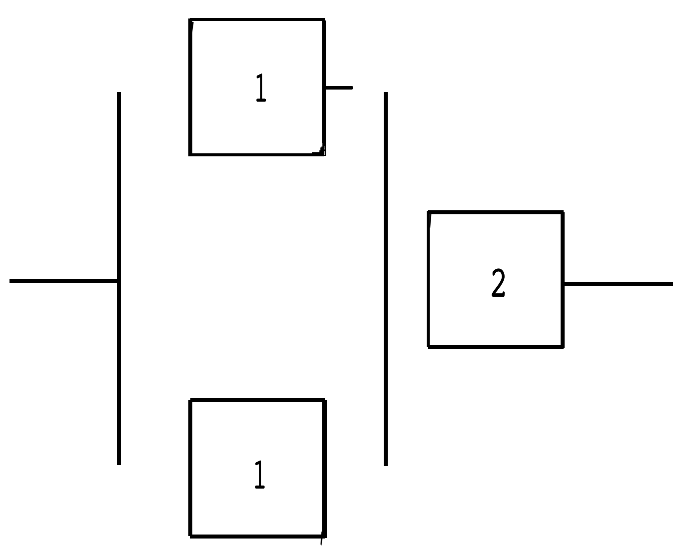
Determine

System reliability assuming independence and active redundancy.

Solution

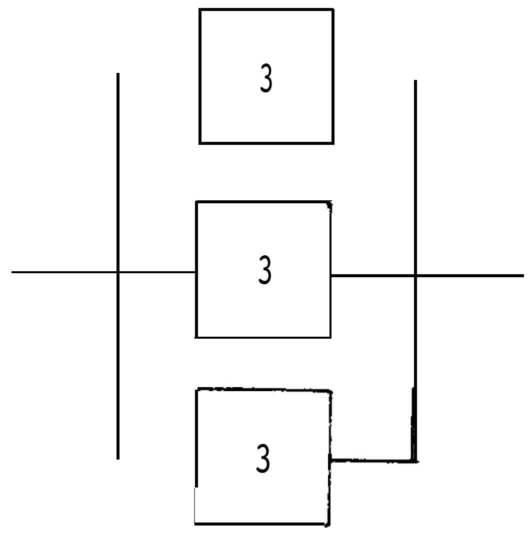
Break the system into subsystems and find the reliability for each using equations 2.1 and 2.2 and then combine into a system model. Define:

Subsystem I as



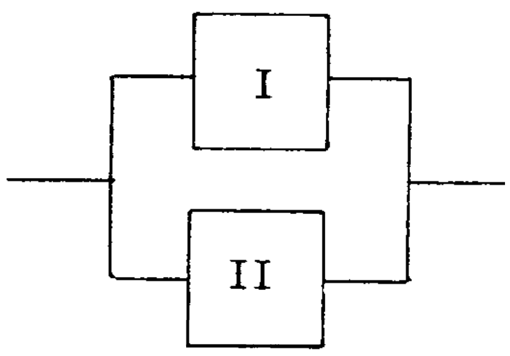
$$R_I = [1-(1-R_1)^2]R_2$$

Subsystem II as



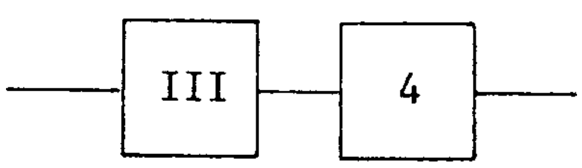
$$R_{II} = 1-(1-R_3)^3$$

Subsystem III as



$$R_{III} = 1-(1-R_I)(1-R_{II}) = 1-\{1-[1-(1-R_1)^2]R_2\}(1-R_3)^3$$

Then the system becomes



$$R_s = R_4 \cdot R_{III} = R_4\left\{1-\{1-[1-(1-R_1)^2]R_2\}(1-R_3)^3\right\}$$

$$= 0.90\{1-\{1-[1-(1-0.80)^2]\ (0.95)\ \}(1-0.70)^3\} = 0.879$$

Commentary

The general mathematical model is often of more use than the numerical solution, since it permits evaluating a variety of alternatives.

CASE STUDY NO. 2-5

Background

An air defense system comprises a weapon subsystem, a vehicle subsystem and a fire control subsystem. One of the mission profiles for the total system requires firing 250 rounds, traveling 10 miles, and operating the fire control system for 15 hours. The respective subsystem reliabilities for this mission profile are:

$$R_{WE} = 0.95$$

$$R_{VE} = 0.99$$

$$R_{FC} = 0.90$$

Determine

The system is to have its reliability improved by redesign of the fire control subsystem. What reliability is required of this subsystem to attain a system reliability of 0.90?

Solution

This is a series system, so equation 2.1 is applied:

$$R_S = R_{WE}R_{VE}R_{FC}$$

Using stars to represent requirements:

$$R^*_{FC} = \frac{R^*_S}{R_{WE}R_{VE}} = \frac{0.90}{(0.95)(0.99)} = 0.957$$

Commentary

This represents a very simple form of reliability apportionment; that is, allocating a system-level requirement among lower levels of assembly.

Determine

What reliability is required of the fire control system to attain a system reliability of 0.95?

$$R^*_{FC} = \frac{R^*_S}{R_{WE}R_{VE}} = \frac{0.95}{(0.95)(0.99)} = 1.01$$

Since reliability camot exceed one, the requirement cannot be met. At the very best, $R_{FC} = 1.0$, which would yield:

$$R_S = R_{WE}R_{VE}R_{FC} = (0.95)(0.99)(1.0) = 0.9405$$

Commentary

The "you can' t get there from here" solution points out that more than just the fire control system ❑ust be improved if a reliability of 0.95 is to be attained.

CASE STUDY NO. 2-6

Background

An electronic system currently in use has approximately 1000 series components. The reliability for a 24-hour mission is 0.95. A proposed system will utilize approximately 2000 similar components in series. Assume a similarity in design practices, quality of manufacture, and use conditions.

Determine

1. What is the average part reliability for the current system?

2. What reliability should be expected of the proposed system for a 24-hour mission?

Solution 1

The "average" component reliability can be found by applying equation 2.1 to the old system:

$$R_S = (R_i)^{1000} = 0.95$$

$$R_i = (0.95)^{0.001} = 0.9999487$$

For the new system:

$$R_S = (0.9999487)^{2000} = 0.9025$$

Solution 2

The new system is the reliability equivalent of two of the old systems used in reliability series. Applying equation 2.1:

$$R_S = (0.95)(0.95) = (0.95)^2 = 0.9025$$

Commentary

This type of prediction based on parts count is particularly useful for evaluating feasibility.

CASE STUDY NO. 2-7

Background

A design for a new long-range ocean reconnaissance/weapons control system has been proposed. The system will be used aboard an aircraft whose reliability is not considered in this evaluation. The system has been designed to accomplish six specific missions.

These missions are defined in the table below. Due to size, weight and power limitations, the hardware elements peculiar to each mission must be combined with hardware elements peculiar to other missions in order to form a complete mission hardware set.

For example, as depicted in the table below, the mission hardware set to accomplish Mission E is a combination of hardware elements 3, 4, and 5.

Mission	Mission Description	Mission Hardware Set
A	Long-Range A/C Surveillance	1
B	Long/Short-Range Surface Ship Detection	1,2
c	Area Sea State Information	1,3
D	Subsurface Surveillance	1,3,4
E	Long-Range Terminal Guidance of Ship Launched Missiles	3,4,5
F	Wide Area Weather Data	1,2,3,6

Mission-Peculiar Subsystem Reliabilities

Hardware Element:	1	2	3	4	5	6
Hardware Element Reliability:	95%	93%	99%	91%	90%	95%

All missions are three hours in length and require the operation of all elements in the hardware set for the full three hours.

The mission-peculiar hardware can support several missions simultaneously.

Determine

1. What is the probability of successfully completing each of the six missions?

2. What is the probability of successfully completing all six ❑issions during a three-hour period?

Solution

1. Since the elements function in a series relationship, the individual reliabilities are multiplied. Hence,

$$R_A = R_1 = 0.95$$

$$R_B = R_1 \times R_2 = 0.95 \times 0.93 = 0.88$$

$$R_C = R_1 \times R_3 = 0.95 \times 0.99 = 0.94$$

$$R_D = R_1 \times R_3 \times R_4 = 0.95 \times 0.99 \times 0.91 = 0.85$$

$$R_E = R_3 \times R_4 \times R_5 = 0.99 \times 0.91 \times 0.90 = 0.81$$

$$R_F = R_1 \times R_2 \times R_3 \times R_6 = 0.95 \times 0.93 \times 0.99 \times 0.95 = 0.83$$

2. The probability of successfully completing all six missions during a single three-hour period is determined by multiplying together the individual hardware element reliabilities. This is done because all individual hardware elements must function throughout the three-hour period to enable all missions to be completed successfully.

Note that the probability of completing all six missions successfully is not correctly calculated by multiplying together the individual mission reliabilities R_A through R_F. This approach would erroneously take individual hardware element reliability into account more than once.

$P_{\substack{Total\\Mission}}$ = Probability of successfully completing six missions during a three-hour period

$$P_{\substack{Total\\Mission}} = R_1 \times R_2 \times R_3 \times R_4 \times R_5 \times R_6$$

$$= 0.95 \times 0.93 \times 0.99 \times 0.91 \times 0.90 \times 0.95$$

$$= 0.68$$

Commentary

The significant point illustrated by this problem is that the reliability of multi-mission/multi-mode systems should be presented in terms of their individual mission reliabilities. This is a useful technique because it permits us to evaluate a system's development progress relative to its individual capabilities rather than its total mission reliability which may prove less meaningful. For example, if we assume that Missions A and B are the primary missions, we see that the system has an 88% chance of successfully completing both functions during a three-hour period. However, if we evaluate Missions A and B along with the remaining four lower-priority missions, we fund that our analysis of the total system capability is far different, i.e., 68% chance of success. Consequently, for this case, approval to proceed with system development would likely be given based on the criticality of Missions A and B and the relatively good probability of successfully completing Missions A and B.

In summary, for multi-mission/multi-mode systems, the presentation of individual *mission* relabilities provides a more meaningful picture of a system's development status and its current and projected combat capabilities as these relate to primary mission achievement.

NOTE : If the individual mission completion times had not been equally constrained to the three-hour time period, we would have been required to use the more sophisticated techniques presented in Chapter 5.

CHAPTER 3

MAINTAINABILITY

INTRODUCTION

Maintainability and reliability are the two major system characteristics that combine to form the commonly used effectiveness index--availability. While maintainability is important as a factor of availability, it also merits substantial consideration as an individual system characteristic. The importance of this parameter in the national defense posture becomes even more obvious when we consider that at least one branch of the armed services spends one-third of its budget on system maintenance activities.

Several aspects of system maintainability must be addressed before an accurate assessment can be undertaken. First, the difference between maintainability and maintenance must be understood. Maintainability is a design consideration, whereas maintenance is the consequence of design. The maintenance activity must live with whatever maintainability is inherent in the design, that is, it must preserve the existing level of maintainability and can do nothing to improve that level. Maintenance is therefore defined as "all actions necessary for retaining a hardware item in or restoring it to an optimal design condition." The second consideration is that maintainability requirements can be specified, measured and demonstrated. Unlike reliability, detailed and quantitative study of maintainability was not initiated until the early 1950s. Until recently, maintainability often was viewed as a "common sense" ingredient of design. It is now seen as a factor of the design process and an inherent design characteristic that is quantitative in nature and therefore lends itself to specification, demonstration, and trade-off analysis with such characteristics as reliability and logistics support.

DEFINITION OF TERMS AND CONCEPTS

Maintainability

Maintainability is defined as a characteristic of design and installation. This characteristic is expressed as the probability that an item will be retained in, or restored to, a specified condition within a given period if prescribed procedures and resources are used.

A commonly used working definition states that Maintainability is a design consideration. It is the inherent characteristic of a finished design that determines the type and amount of maintenance required to retain that design in, or restore it to, a specified condition.

Maintenance

This term is defined as all actions required to retain an item in, or restore it to, a specified condition. This includes diagnosis, repair and inspection.

Preventive Maintenance

This term is defined as systematic inspection, detection and correction of incipient failures either before they occur or before they develop into major defects. Adjustment, lubrication and scheduled checks are included in the definition of preventive maintenance.

Corrective Maintenance

This term is defined as that maintenance performed on a non-scheduled basis to restore equipment to satisfactory condition by correcting a malfunction.

CONSIDERATEONS IN PLANNING MAINTAINABILITY ASSESSMENT

An understanding of the principal elements of maintainability is essential to the assessment planning process. Certain elements of a design basically define a system's inherent maintainability and thus determine the related maintenance burden and affect system availability.

It is apparent, from the definition of maintainability, that the ability and need to perform maintenance actions is the underlying consideration when assessing maintainability. The factors which affect the frequency with which maintenance is needed are reliability and the preventive maintenance schedule. Those which affect the ability to perform ❑aintenance on a given weapon system may be broken down into three categories: the physical design of the system, the technical personnel performing the maintenance and the support facilities required.

The consideration of maintenance when designing a system is not new. There have been very successful efforts in the development of automatic check out and design for accessibility, etc. What is new is the emphasis on quantitative treatment and assessment which results in a complete change in design philosophy, design approach and design management. In the past, design for "maximum" or "optimum" reliability and maintainability was emphasized. This resulted in "unknown" reliability and maintainability. New techniques permit us to bring qualitative design judgments into an area of quantitative measurement. We can thus establish quantitative design goals and orient the design to specific mission thresholds, not to "optimum" or "maximum" goals. Maintainability design considerations and testing intended to assess system maintainability characteristics must be based on established quantitative requirements (thresholds and goals). In addition to verifying these values, the maintainability test and evaluation program also should address the impact of physical design features and maintenance action frequency on system maintenance.

Some physical design features affect the speed and ease with which maintenance can be performed. These features and pertinent questions are:

- Accessibility: Can the item to be repaired or adjusted be reached easily?

- Visibility: Can the item being worked on be seen?

- Testability: Can system faults be detected and isolated to the faulty replaceable assembly level?

- Complexity: How many subsystems are in the system? How many parts are used? Are the parts standard or special-purpose?

- Interchangeability: Can the failed or malfunctioning unit be "swapped around" or readily replaced with an identical unit with no need for recalibration?

In addition to the listed physical design factors, the frequency with which each maintenance action must be performed is a major factor in both corrective and scheduled or preventive maintenance. Thus, reliability could have a significant impact on corrective maintenance, and such design features as self-check-out, reduced lubrication requirements and self-adjustment would affect the need for preventive maintenance.

Personnel and human factor considerations are of prime importance. These considerations include the experience of the technician, training, skill level, supervision required, supervision available, techniques used, physical coordination and strength and number of technicians and teamwork requirements.

Support considerations cover the logistics system and maintenance organization required to support the weapon system. They include availability of supplies, spare parts, technical data (TOS and ❑anuals), built-in test equipment, external test equipment and required tools (standard or special) and servicing equipment.

While some elements of maintainability can be assessed individually, it should be obvious that a true assessment of system maintainability generally must be developed at the system level under operating conditions and using production configuration hardware.

QUANTITATIVE MAINTAINABILITY INDICES

The following paragraphs describe the various mathematical indices used to quantify maintainability. It is important to remember, however, that these relationships merely categorize data derived from planned testing. For maintainability, the test planning phase is equal in importance to the assessment phase. Testing that does not adequately demonstrate the effect of the above physical design features and personnel and support aspects provides data that effectively conceal the impact of these critical elements.

Indices used to support maintainability analysis must be composed of measurable quantities, must provide effectiveness-oriented data and must be readily obtainable from operational and applicable development testing. If they are, system designers, users and testers can evaluate candidate system characteristics and logistics and maintenance practices more precisely.

Mean-Time-to-Repair (MTTR) or Met

MTTR is the total corrective maintenance down time accumulated during a specific period divided by the total number of corrective maintenance actions completed during the same period. MTTR commonly is used as an on-equipment measure but can be applied to each maintenance level individually. The MTTR considers active corrective maintenance time only. Because the frequency of corrective maintenance actions and the number of man-hours expended are not considered (clock hours are used) , this index does not provide a good measure of the maintenance burden.

Maximum-Time-to-Repair (MaxTTR) or MmaxC

MmaxC is the maximum corrective maintenance down time within which either 90 or 95 percent (as specified) of all corrective maintenance actions can be accomplished. An MmaxC requirement is useful in those special cases in which there is a tolerable down time for the system. Ideally, we would like to be able to state an absolute maximum, but this is impractical because there will inevitably be failures that require exceptionally long repair times. A 95th percentile MmaxC specification requires that no more than 5 percent of all corrective maintenance actions take longer than MmaxC.

Maintenance Ratio (MR)

MR is the cumulative number of man-hours of maintenance expended in direct labor during a given period of time, divided by the cumulative number of end-item operating hours (or rounds or miles) during the same time. The MR is expressed at each level of maintenance and summarized for all levels of maintenance combined. Both corrective and preventive maintenance are included. Man-hours for off-system repair of replaced components and man-hours for daily operational checks are included for some classes of systems.

Particular care must be taken that the operating hour base be clearly defined. For example, in the case of combat vehicles, either system operating hours or engine hours could be used.

The MR is a useful measure of the relative maintenance burden associated with a system. It provides a means of comparing systems and is useful in determining the compatibility of a system with the size of the maintenance organization.

For fielded systems, the MR is useful in maintenance scheduling. Some care must be exercised in relating the MR to maintenance costs, because an in-house maintenance organization will have a fixed labor cost, independent of the amount of actual use of the system, but principally fixed by the size of the maintenance staff.

Mean-Time-Between-Maintenance-Actions (MTBMA)

MTBMA is the mean of the distribution of the time intervals between either corrective maintenance actions, preventive maintenance actions or all maintenance actions. This index is frequently used in availability calculations and in statistically-oriented maintenance analyses.

Average Number of Technicians Required

The average number of technicians required at each maintenance level provides a quantitative means of expressing the personnel aspects of the overall maintenance concept. This index also provides a conversion factor from active down time to labor hours.

Off-System Maintainability Indices

The indices MTTR, MmaxC and MR all specifically exclude off-system maintenance actions. Off-system measures are particularly important if a system's maintenance concept involves extensive use of modular removal and replacement, since this type of concept transfers the maintenance burden to off-system maintenance. As an assessment tool, off-system maintainability measures are essential. Without them, it is not possible to assess the ability of combat environment off-system repair and logistics capability to maintain the system. Because of the system-peculiar nature of these parameters, none are specified here. Suffice it to say that a complete set of on- and off-system indices is required to adequately assess system maintainability and total maintenance burden.

Annual Support Cost (ASC)

This is the direct, annual cost of maintenance personnel, repair, parts and transportation for all corrective (either on-system, off-system or both) and preventive maintenance actions when the system operates X hours per year during the Nth year of M years service life, where the system is defined as Y units of item A, Z units of item B, etc.

The ASC provides another means of quantifying the ❑aintenance burden of a system. The unique feature of the ASC measure is the recognition that maintenance requirements may not be uniform over the life of a system. For example, a combat vehicle will experience a high-cost year when its engine requires replacement or overhaul. This measure provides a means of interrelating durability requirements and policies for scheduled maintenance.

Case Study No. 3-1 illustrates the use of several maintainability indices.

DIAGNOSTIC SYSTEMS

Introduction

One aspect of maintainability that has received significant attention in recent system designs is the use of automatic diagnostic systems. These systems include both internal or integrated diagnostic systems, referred to as built-in-test (BIT) or built-in-test-equipment (BITE), and external diagnostic systems, referred to as automatic test equipment (ATE), test sets or off-line test equipment. The following discussion will focus on BIT but most of the key points apply equally to other diagnostic systems.

Need for Automatic Diagnostic Systems - BIT

As technology advances continue to increase the capability and complexity of modern weapon systems, we are relying more on the use of automatic diagnostics, i.e., BIT, as a means of attaining the required level of failure detection capability. Our need for BIT is driven by operational availability requirements which do not permit the lengthy MTTRs associated with detecting and isolating failure modes in microcircuit technology equipment. We also find that because BIT operates within the basic system and at the same functioning speed, it therefore affords us the capability to detect and isolate failures which conventional test equipment and techniques could not provide. Finally, a well designed BIT system can substantially reduce the need for highly trained field level maintenance personnel by permitting less skilled personnel to locate failures and channel suspect hardware to centralized intermediate repair facilities which are equipped to diagnose and/or repair defective hardware.

As we shall discuss, BIT is not a comprehensive solution to all system maintenance problems but rather a necessary tool required to deal with the complexity of modern electronic systems.

Specifying BIT Performance

One of the more complex tasks inherent in the acquisition of modern systems is the development of realistic and ❑eaningful operational requirements and their subsequent conversion into understandable and achievable contractual specifications. This situation is equally applicable to BIT. Before discussing this topic in more detail, we will present typical performance measures or figures of merit which are used to specify BIT performance.

Percent Detection. The percent of all faults or failures that the BIT system detects.

Percent Isolation. The percent of detected faults or failures that the system will isolate to a specified level of assembly. For example, the BIT might isolate to one box or to three or less printed circuit boards in a box.

Automatic Fault Isolation Capability (AFIC). The AFIC is the product of percent isolation times percent detection.

AFIC = % detection x % isolation

Percent of False Alarms. The percent of the BIT indicated faults where, in fact, no failure is found to exist.

Percent of False Removals. The percentage of units removed because of BIT indications which are subsequently found to test "good" at the next higher maintenance station.

For each of the above parameters, there is a considerable span of interpretation. For example, does the percent detection refer to failure modes or the percentage of all failures that could potentially occur? Does the detection capability apply across the failure spectrum, i.e. , ❑echanical systems, instrumentation, connections and so ftare, or is its diagnostic capability applicable only to electronic hardware systems?

A major contractual issue relates to the definition of failure. Should BIT performance be viewed in terms of "BIT addressable" failures , which normally exclude cable/connector, etc. , problems as not contractually chargeable, or in terms of all operationally relevant maintenance actions?

An important consideration relates to exactly what failures BIT can detect. Our BIT system will operate ineffectively if the 80% of detectable failures occur infrequently while the remaining 20% occur with predictable regularity. It, therefore, becomes important to specify BIT performance measures in relation to overall mission availability requirements.

Relative to isolation characteristics , will the BIT isolate failures while the basic system is in an operational mode, or must the basic system be "shut down" to permit the isolation software package to be run? How does this characteristic impact mission requirements? Also, to what "level" will the BIT system isolate failures? Early BIT systems were frequently designed to fault isolate to the module level. This resulted in BIT systems as complex as, and frequently less reliable than, the basic system. The current trend is to isolate to the subsystem or box level based on BIT's ability to detect abnormal output signal patterns. Intermediate and depot level maintenance facilities will frequently use BIT or external diagnosic equipment to isolate to the board or piece-part level.

The percent of false alarms is a difficult parameter to measure accurately because an initial fault detection followed by an analysis indicating that no fault exists can signify several different occurrences, such as:

- The BIT system erroneously detected a fault.

- An intermittent out-of-tolerance condition exists--somewhere.

- A failure exists but cannot be readily reproduced in a maintenance environment.

The percent of false removals can be a more difficult problem to address. False removals may be caused by:

- Incorrect BIT logic.

- Wiring or connection problems which manifest themselves as faulty equipment.

- Improper match of tolerances between the BIT and test equipment at the next higher maintenance level.

The resolution of each type of false alarm and false removal requires a substantially different response. From a logistic viewpoint, false alarms often lead to false removals creating unnecessary demands on supply and maintenance systems. Of potentially more concern is the fact that false alarms and removals create a lack of confidence in the BIT system to the point where maintenance or operations personnel may ignore certain fault detection indications. Under these conditions, the BIT system in particular and the maintenance concept in general cannot mature nor provide the support required to meet mission requirements.

The specification of BIT performance must be tailored to the specific system under consideration as well as the available funds and, most importantly, the overall mission requirements. This tailoring activity must include a comprehensive definition of BIT capability based upon the figures of merit presented above.

Characteristics External to BIT

There are two important considerations, external to BIT, which must be addressed in any discussion of BIT and diagnostics. First, reliable performance of the weapon system determines, to a large extent, the criticality of BIT performance. If the basic system is very reliable, more than expected, a shortfall in the BIT performance may have very limited impact on the system's operational utility. Second, it must be remembered that generally all system faults that are correctable by maintenance action must eventually be detected and isolated. Therefore, the techniques, tools, manuals, test equipment and personnel required to detect and isolate non-BIT detectable faults can be a major maintenance consideration.

The following example illustrates the impact of BIT on the overall maintenance effort. It further attempts to illustrate the effect of external factors on BIT performance.

Case Description. An attack aircraft radar is composed of five line replaceable units (LRUs) with the following BIT and system performance characteristics.

System:	Five (5) LRUS
MTTR (w/BIT):	2 hours (includes failures which have been both detected and isolated)
MTTR (no/BIT):	5 hours (includes failures which have not been isolated but ❑ay have been detected)
MFHBF :	50 flying hours
Time Period of Interest:	2500 flying hours
BIT Specified:	Percent detection = 90'% Percent isolation = 90% (to the LRU level) False alarm rate = 5% (of all BIT indications)

NOTE : Operating time is assumed to be flight time for this example.

Before beginning the analysis of this system, note that we have specified a relatively high capability BIT system. An off-handed examination would likely conclude that with this extensive BIT coverage there is minimal additional maintenance action required.

- How many total failures will be experienced (on the average) during the 2500 flying hours?

  2500 total hours ÷ 50 mean hours between failures = 50 failures

- How many of these failures (on the average) will BIT detect?

  50 failures x 90% = 45 BIT detected failures

- How many detected failures (on the average) will be isolated to an LRU?

  45 detected failures x 90% isolation ≅ 40 failures

- What is the Automatic Fault Isolation Capability (AFIC)?

  AFIC = % detection x % isolation (LRU level)

  $= 0.9 \times 0.9 = 0.81 = 81\%$

- How many false alarm indications are expected to occur during the 2500 flight hours?

  Total BIT indications = true failure detections + false alarms

  = x

  and,

  x = (BIT detection rate)(total failures) +(false alarm rate)(total BIT indications)

  $x = (0.90)(50) + (0.05)(X)$

  $(1-0.05)x = 45$

  $X = 47.36$

  Therefore,

  False Alarms = total BIT indications - true indications

  $= 47.36 - 45$

  $= 2.36$

  $\cong 2$

- What is the total corrective maintenance time (on the average) required to repair the 40 detected/isolated failures?

  Time = 40 failures x 2 hours (MTTR w/BIT) = 80 hours

- What is the total corrective maintenance time (on the average) required to repair the remaining 10 no/BIT detected/isolated failures?

  Time = 10 failures x 5 hours (MTTR no/BIT) = 50 hours

- If we assume that manual or no/BIT maintenance time is required to resolve the false alarm indications, what total no/BIT corrective maintenance time is required for the 2500 flying hour period?

  Total (no/BIT) time = no/BIT failure repair time

  + false alarm maintenance time

  = (10)(5) + (2)(5) = 60 hours

- What is the total corrective maintenance time M_t anticipated during the 2500 hours?

  M_t = BIT maintenance + no/BIT maintenance = 80 + 60 = 140 hours

- Note that even with a relatively high AFIC of 81% the no/BIT oriented corrective maintenance represents 43% of the total anticipated corrective maintenance hours.

- Note that we have not considered the impact of any scheduled/ preventive maintenance for our system. This additional maintenance is generally not associated with BIT.

The information presented in this example is greatly simplified in that we have ignored many of the pitfalls and controversial areas discussed in the previous sections. Also note that we are basically dealing with planning type information in that we are assuming that the BIT AFIC will be 81%. If, in fact, the AFIC is 81% then 57% of the ❑aintenance effort will be oriented toward BIT detected/isolated failures. If the true AFIC is found to be lower, it will be necessary to reevaluate the overall effectiveness of the entire maintenance and logistics programs as well as total mission effectiveness. Our next section discusses some of the difficulties involved in the design and evaluation of a BIT system which ❑ust perform in accordance with established specifications.

Basic System/BIT Development and Evaluation Considerations

The development and evaluation of BIT and diagnostics has traditionally been an activity that has chronologically followed basic system efforts. The argument usually presented is that "the basic system has to be designed and evaluated before we know what the BIT is suppose to test." This argument has some basis in fact, but there are significant drawbacks associated with lengthy schedule differentials between basic system and BIT design and testing. For example, design considerations relating to the basic system such as partitioning and subsystem layout determine to a large extent the required BIT design. The BIT design is also driven by the prediction and occurrence of basic system failures modes which BIT is expected to address. Consequently, the two design efforts cannot be conducted in isolation from one another.

From an evaluation viewpoint, conducting the BIT evaluation after the basic system tests are completed may preclude BIT improvement options from being incorporated because the already finalized basic system design would be substantially impacted. Likewise, an inadequate evaluation of BIT which leads to an erroneous confirmation of its capabilities (AFIC) will result in a substantial impact to system operational effectiveness.

Determination of Basic System Failure Modes and Frequency of Occurrence

The design of BIT is based upon two assumptions regarding the reliability of the basic weapon system: accurate identification of failure modes and correct estimation of the frequency of occurrence of the failure modes. If either of these assumptions is proven incorrect by test or operational experience, the resultant BIT performance is likely to be inadequate or at least less effective than anticipated. The following two situations, based on the previous example, will illustrate the impact of these two assumptions.

- Situation 1. An unforeseen failure mode is observed in the radar system every 250 flying hours. What impact does this have on the no/BIT maintenance?

  New failures = 2500 flying hours x 1 failure per 250 hours

  = 10 failures (new).

  Maintenance time associated with no/BIT detected failures

  = 10 x 5 hours/failure

  = 50 hours.

  Total Maintenance hours = 80 + 60 + 50 = 190 hours.

  Total no/BIT maintenance = 60 + 50 = 110 hours.

  This represents 58% of total maintenance.

  BIT (detected/isolated) maintenance = 80 hours = 42% of total.

  This represents 42% of total maintenance.

  Note that the discovery of one unforeseen, no/BIT detectable failure has a relatively significant impact on the comparable magnitude of the two maintenance percentages.

TABLE 3-1. PERCENT OF TOTAL MAINTENANCE HOURS

	BIT	No/BIT
Previous Estimate	57%	43%
Current Estimate (including new failure)	42%	58%

- Situation 2. One of the original BIT detectable failures was predieted to have a very low frequency of occurrence. BIT detection for this failure was considered unnecessary and was, therefore, not included in the original BIT design to detect 90% of the failures. It is now found that the failure occurs five times as often as expected. This is a realistic situation and one which again directly impacts the no/BIT maintenance man-hours.

Test and Evaluation of BIT Systems. The test and evaluation of BIT systems and the reliable prediction of BIT performance are areas of controversy. The following paragraphs present some of the factors supporting this statement.

BIT systems are hardware/software logic networks designed to detect the presence of an unwanted signal or the absence of a desired signal, each representing a failure mode. Each failure mode is detected by a specific logic network tailored to detect a specific failure. While the same network may be designed to detect a specific failure in several components, there is no assurance that the logic is correct until verified by test. It is possible to project, using statistical techniques, BIT performance assuming we have a large enough representative sample of failures.

Unlike reliability testing which has matured over the past 40 years, BIT testing and BIT system design represent less mature technologies and are just now beginning to receive increased management emphasis. This lack of maturity and attention has resulted in circumstances which are not conducive to gathering an adequate size, representative data base needed to support accurate and defendable estimates of BIT performance at decision milestones. A lack of confidence in BIT performance assessments has resulted because of these circumstances.

Since we are not content nor have the time to just wait for the basic system to experience random failures, we decide to "cause" failures using "synthetic fault insertion." These faults are generally selected from a contractor-provided list of possible faults--all of which are thought to be detectable. We insert these faults and BIT detects and isolates 93% of those inserted. This does not mean that we have a 93% AFIC BIT system. Why? Because the data is not a representative random sample of the entire failure population and, therefore, cannot be used to make statistically valid predictions of future performance.

While synthetic fault insertion has recognized limitations in predicting future operational BIT performance, it is a valuable and necessary tool during the engineering development phase. Also, fault insertion can be used to simulate random failures which we know will occur but as yet have not been detected during DT or OT testing. These include problems such as faulty connector and circuit board plug-in points as well as the effects of poor maintenance or rough handling.

Because of the lack of system maturity (especially software) and the required use of fault insertion, we find that there is normally insufficient good data available to support early, accurate and defendable estimates of BIT performance. It has generally required a few years of operational exposure to develop an adequate data base to support a BIT performance analysis.

Current trends support early reliability testing during development to facilitate identification of failure modes and timely incorporation of design improvements. These tests provide a data base to support preliminary estimates of system reliability. What is most frequently overlooked is that this data, after minimal screening, could also be used to monitor, verify, and upgrade BIT performance, assuming, of course, that the BIT system is functional at this stage in development. This action requires a disciplined approach toward the use of BIT in failure detection early in the development cycle which has not been prevalent in previous programs.

In summary, there is, and will remain, a requirement to assess BIT performance during the system development and Initial Operational Test and Evaluation (IOT&E) phase. The developing and testing organizations must support this assessment using all available data. This includes combining random failure detection data with data from contractor demonstrations and fault insertion trials . Early emphasis on BIT design will generally result in accelerated BIT system maturity and more accurate early projections of BIT performance. BIT assessment should be actively pursued throughout the deployment phase to assure that required software and hardware changes are incorporated.

CASE STUDY NO. 3-1

Background

A system has a mean time to repair (MTTR) of 30 minutes, and a mean time between unscheduled maintenance actions (MTBUMA) of 50 hours. The intended utilization (actual hours of operation) of the system is 5,000 hours per year.

Determine

1. How many unscheduled maintenance actions are to be expected each year?

2. How many clock-hours of unscheduled maintenance are to be expected each year?

'3. If an average of 1.5 technicians is required to perform unscheduled maintenance, how many' corrective maintenance man-hours are required each year?

4. Ten hours of scheduled maintenance are required every 1,000 hours of operation. Scheduled maintenance requires only one technician. What is the maintenance ratio (MR) for this system?

Solution

MTTR = 30 minutes

MTBUMA = 50 hours

5,000 hours/year

1. Unscheduled maintenance actions $= \frac{5000}{50} = 100\text{/year}$

2. 30 ❑inutes x 100 = 3000 ❑inutes = 50 "hours
   50 hours mean repair time/year

3. 1.5 x 50 = 75 man-hours/year

4. Scheduled maintenance = 10 x 5 = 50 man-hours/year

$$MR = \frac{\text{maintenance man-hours}}{\text{operating hours}}$$

$$MR = \frac{50 + 75}{5000}$$

$$MR = 0.025$$

CHAPTER 4

AVAILABILITY

INTRODUCTION

Availability is the parameter that translates system reliability and maintainability characteristics into an index of effectiveness. It is based on the question, "Is the equipment available in a working condition when it is needed?" The ability to answer this question for a specific system represents a powerful concept in itself, and there are additional side benefits that result. An important benefit is the ability to use the availability analysis as a platform to support both the establishment of reliability and maintainability parameters and trade-offs between these parameters. As part of our review of availability, we will separate maintainability into its components (preventive and corrective maintenance and administrative and logistics delay times) to determine the impact of these individual elements on overall system availability.

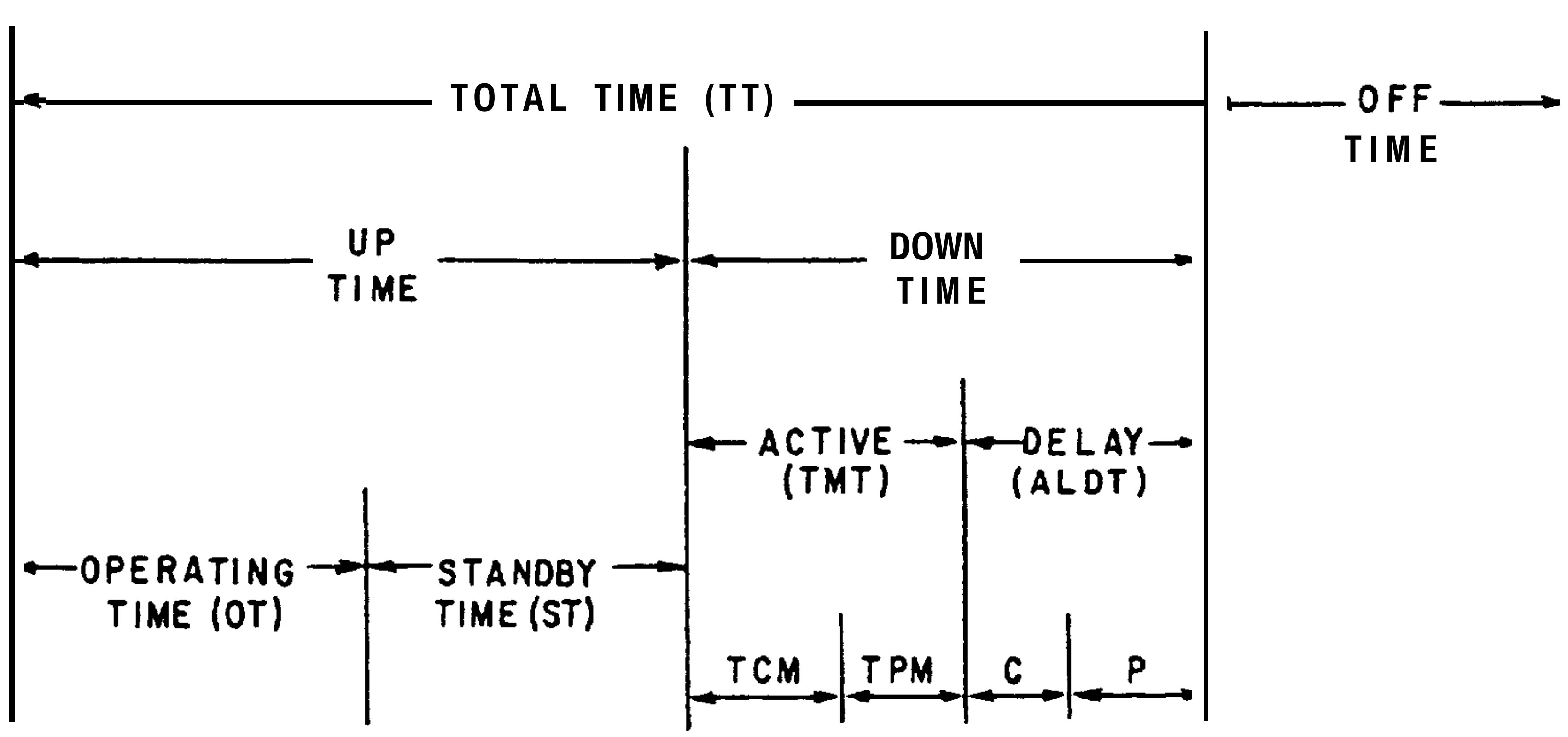
DEFINITION OF AVAILABILITY

Availability is defined as a measure of the degree to which an item is in an operable and committable state at the start of a mission when the mission is called for at a random point in time.

ELEMENTS OF AVAILABILITY

As is evident by its very nature, approaches to availability are time-related. Figure 4-1 illustrates the breakdown of total equipment time into those time-based elements on which the analysis of availability is based. Note that the time designated as "off time" does not apply to availability analyses because during this time system operation is not required. Storage and transportation periods are examples of "off time".

FIGURE 4-I. BREAKDOWN OF TOTAL EQUIPMENT TIME



The letters "C" and "P" represent those periods of time attributed to corrective or preventive maintenance, respectively, which are expended in active repair of hardware or in waiting (delay) for resources to effect needed repairs. This waiting or delay period can further be subdivided into administrative and logistics delay periods.

DEFINITION OF TERMS

Definitions of commonly used availability elements are given below. Several are displayed pictorially in Figure 4-1.

TT = Total intended utilization period, total time.

TCM = Total corrective (unscheduled) maintenance time per specified period.

TPM = Total preventive (scheduled) maintenance time per specified period.

ALDT = Administrative and logistics down time spent waiting for parts, administrative processing, maintenance personnel, or transportation per specified period. See Figure 4-1, Delay-Down Time (no maintenance time).

TMT = Total maintenance time = TCM + TPM. See Figure 4-1, Active-Down Time.

TDT = Total down time = TMT + ALDT.

OT = Operating time (equipment in use). See Figure 4-1.

ST = Standby time (not operating but assumed operable) in a specified period. See Figure 4-1.

MTBF = Mean time between failures.

MTBM = Mean time between maintenance actions.

MTBUMA = Mean time between unscheduled maintenance actions (unscheduled means corrective).

MDT = Mean down time.

MTTR = Mean time to repair.

MATHEMATICAL EXPRESSIONS OF AVAILABILITY

The basic mathematical definition of availability is

$$\text{Availability} = A = \frac{\text{Up Time}}{\text{Total Time}} = \frac{\text{Up Time}}{\text{Up Time + Down Time}} \quad (4.1)$$

Actual assessment of availability is accomplished by substituting the time-based elements defined above into various forms of this basic' equation. Different combinations of elements combine to formulate different definitions of availability.

Operational availability is the most desirable form of availability to be used in assessing a system's combat potential. Achieved, and to a lesser degree inherent availability are primarily the concern of the developing agency in its interface with the contractor and other co-developing agencies.

Ao is an important measure of system effectiveness because it relates system hardware, support and environment characteristics into one meaningful parameter-- a figure of merit depicting the equipment state at the start of a mission. Because it is an effectiveness-related index, availability is used as a starting point for nearly all effectiveness and force sizing analyses.

Inherent Availability (Ai)

Under certain conditions, it is necessary to define system availability with respect only to operating time and corrective maintenance. Availability defined in this manner is called inherent availability (Ai).

$$A_i = \frac{MTBF}{MTBF + MTTR} \tag{4.2}$$

Under these idealized conditions, we choose to ignore standby and delay times associated with scheduled or preventive maintenance, as well as administrative and logistics down time. Because only corrective maintenance is considered in the calculation, the MTBF becomes MTBUMA, and, likewise, MTTR is calculated using only times associated with corrective maintenance.

Inherent availability is useful in determining basic system operational characteristics under conditions which might include testing in a contractor's facility or other controlled test environment. Likewise, inherent availability becomes a useful term to describe combined reliability and maintainability characteristics or to define one in terms of the other during early conceptual phases of a program when, generally, these terms cannot be defined individually. Since this definition of availability is easily measured, it is frequently used as a contract-specified requirement.

As is obvious from this definition, inherent availability provides a very poor estimate of true combat potential for most systems, because it provides no indication of the time required to obtain required field support. This term should normally not be used to support an operational assessment.

Case Study No. 4-1 displays the usefulness of inherent availability.

Operational Availability (Ao)

Operational availability, unlike inherent availability, covers all segments of time that the equipment is intended to be operational (total time in Figure 4-1). The same up-down time relationship exists but has been expanded. Up time now includes operating time plus nonoperating (stand-by) time (when the equipment is assumed to be operable). Down time has been expanded to include preventive and corrective maintenance and associated administrative and logistics lead time. All are measured in clock time.

$$\text{Operational Availability} = A_o = \frac{OT + ST}{OT + ST + TPM + TCM + ALDT}. \tag{4.3}$$

This relationship is intended to provide a realistic measure of equipment availability when the equipment is deployed and functioning in a combat environment. Operational availability is used to support operational testing assessment, life cycle costing, and force development exercises.

One significant problem associated with determining Ao is that it becomes costly and time-consuming to define the various parameters. Defining ALDT and TPM under combat conditions is not feasible in most instances. Nevertheless, the operational availability expression does provide an accepted technique of relating standard reliability and maintainability elements into an effectiveness-oriented parameter. As such, it is a useful assessment tool.

Case Study 4-4 illustrates how this relationship can be used to define and analyze the various elements of reliability and maintainability. Case Study 4-2 illustrates the calculation of Ao.

One important aspect to take note of when assessing Ao is that it is affected by utilization rate. The less a system is operated in a given period, the higher Ao will be. It is important therefore when defining the "total time" period to exclude lengthy periods during which little or no system usage is anticipated. Case Study 4-3 attempts to display this characteristic of Ao.

One other frequently encountered expression for operational availability is

$$A_o = \frac{MTBM}{MTBM + MDT}. \tag{4.4}$$

where

MTBM = mean time between maintenance actions and MDT = mean down time.

While maintenance-oriented, this form of Ao retains consideration of the same basic elements. The MDT interval includes corrective and preventive maintenance and administrative and logistics down time. This form of the Ao relationship would generally prove more useful in support of early maintenance parameter sensitivity and definition analysis. Note that the above definition assumes that standby time is zero.

Achieved Availability (As)

This definition of availability is mathematically expressed as

$$A_a = \frac{OT}{OT + TCM + TPM} \tag{4.5}$$

Aa is frequently used during development testing and initial production testing when the system is not operating in its intended support environment Excluded are operator before-and-after maintenance checks and standby, supply, and administrative waiting periods. Aa is much more a system hardware-oriented measure than is operational availability, which considers operating environment factors. It is, however, dependent on the preventive maintenance policy, which is greatly influenced by non-hardware considerations.

A GENERAL APPROACH FOR EVALUATING AVAILABILITY

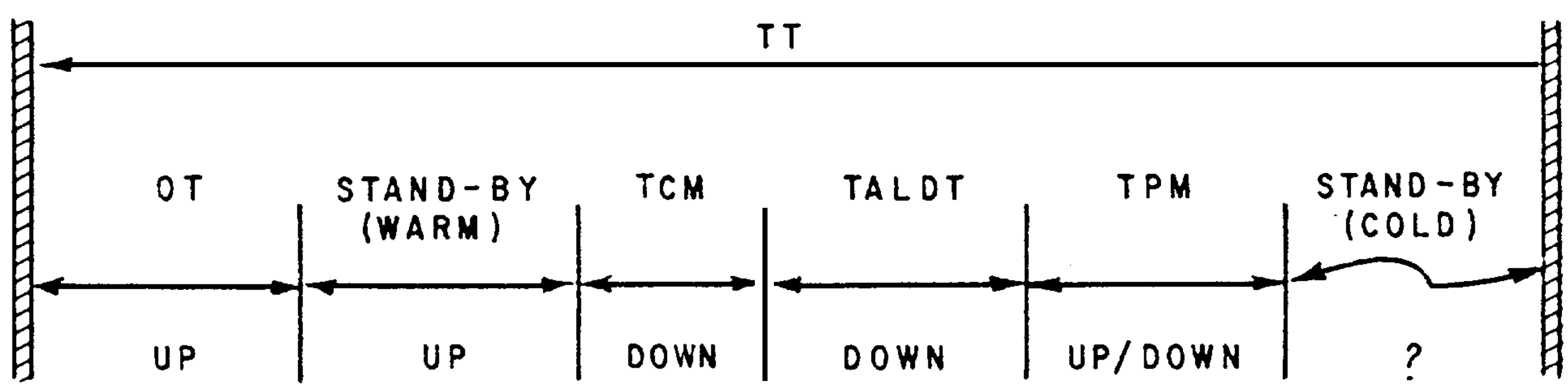
The following paragraphs present a generalized approach for evaluating system availability. It is important to note that for such an analysis to be meaningful to an equipment user or developer it must reflect the peculiarities of the system being considered.

General Procedure

1. The operational and maintenance concepts associated with system utilization must be defined in detail using terminology compatible with the user, developer and contractor.

2. Using the above definitions, construct a time line availability model (see Figure 4-2) which reflects the mission availability parameters.

FIGURE 4-2 MISSION AVAILABILITY TIME LINE MODEL GENERALIZED FORMAT



NOTE : Figure 4-2 displays elements of availability frequently included in a quantitative assessment of availability. The up or down status of a specific system during preventive maintenance must be closely examined. Generally, a portion of the preventive maintenance period may be considered as uptime. Cold standby time must also be examined closely before determining system up or down status during this period.

3. With the aid of the time line model, determine which time elements represent "uptime" and "downtime." Don't be mislead by the apparent simplicity of this task. For example, consider that the maintenance concept may be defined so that the equipment must be maintained in a committable state during the performance of preventive maintenance.

Additionally, for multi-mission and/or multi-mode systems, it will be necessary to determine up and down times as a function of each mission/mode. This generally will require the use of a separate time line model for each identifiable mission/mode.

Likewise, separate time line models are generally required to support the availability analyses of systems which experience significantly different peacetime, sustained combat and surge utilization rates.

4. Determine quantitative values for the individual time elements of the 'time line models. Coordinate these values with the user, developer and contractor.

5. Compute and track availability using the definitions of availability appropriate for the current stage of system development.

6. Continue to check availability model status and update the model as required. Special attention should be given to updating the model as the operational , maintenance, and logistics support concepts mature.

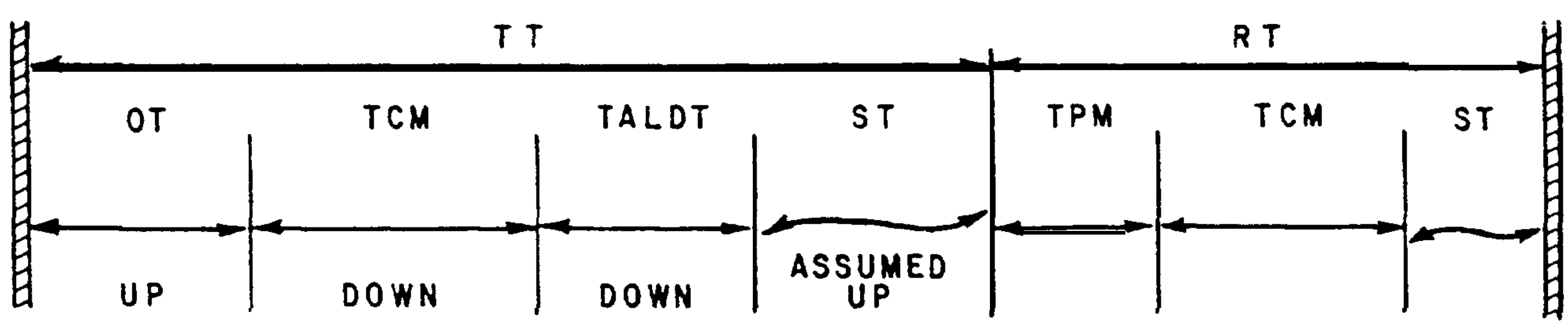
System Availability Assessment Considerations

As indicated in the above paragraphs, the quantitative evaluation of availability must be carefully and accurately tailored to each system. For this reason, no detailed examples are presented in this text. However, the following paragraphs do present concepts which will apply to various classes of systems.

Recovery Time. Normally, availability measures imply that every hour has equal value from the standpoint of operations and the performance of maintenance and logistics activities. Normally, the operational concept requires the system to function only for selected periods. The remaining time is traditionally referred to as "off-time," during which no activity is conducted.

An alternative to the "off-time" or "cold. standby" concepts is the use of the term "recovery time" (RT).

FIGURE 4-3 MISSION AVAILABILITY TIME LINE MODEL RECOVERY TIME FORMAT



Recovery time represents an interval of time during which the system may be up or down. Recovery time does not appear in the availability calculation which is based only on the TT time period. Take special note of the fact that corrective maintenance time (TCM) is found in both TT and RT time intervals. Corrective maintenance performed during the TT period is maintenance required to keep the system in a mission ready or available status. Corrective maintenance performed during the RT period generally addresses hardware malfunctions which do not result in a non mission-ready status.

The principal advantage of using the "recovery time" analysis is that it can provide a more meaningful availability assessment for systems whose periods of required availability are predictable and whose preventive maintenance constitutes a significant but delayable portion of the maintenance burden.

The recovery time calculation technique concentrates the availability calculation during the operational time period, thereby focusing attention on critical up and down time elements.

The above discussion presents an alternate technique of computing system availability, i.e., the use of the recovery time concept. Whatever technique is selected for computing availability, it must be carefully tailored to *the* system undergoing assessment

Definition of the terms used in availability analysis must be stressed. For example, what total time (TT) period has been chosen for an analysis base? Assume for a moment that we are assessing the Ao of an operational squadron and that we have chosen a 7-day TT period. If the aircraft normally are not flown on weekends or are left in an up condition on Friday night it is obvious that Ao will be higher than if a 5-day total time were selected. Reference the discussion of recovery and standby time. See Case Study 4-3.

Other definitions associated with Ao are not quite so obvious and must be included in pretest definition. For example, are "before and after" operational checks conducted in conjunction with preventive maintenance excluded from down time because the equipment is assumed operable? Similarly, are corrective maintenance diagnostic procedures logged against down time? What if the hardware is not found defective? How is ALDT arrived at? Is it assumed, calculated or observed? What is the operational status of a system during the warm standby period?

HARDWARE REQUIREMENT ESTABLISHMENT AND TRADE-OFFS

The expression for availability frequently provides the vehicle needed to analyze other system requirements both directly and by way of trade-offs. Case Studies 4-4 and 4-5 provide examples of this application.

AVAILABILITY FOR MULTI-MISSION SYSTEMS

For many modern weapon systems, availability is not simply an "up" or "down" condition. Systems such as AEGIS and HAWK have multi-mission/mode capabilities and thus require detailed techniques to characterize the associated availability states. While these multi-mission/mode characterizations may appear different, they are indeed based on the expressions presented previously. The definition of terms, modes and states is equally important in the analysis of these complex systems. The reliability of a multi-mission system is examined in Case Study 2-7.

SIMULATION MODELS

There are a number of computer simulation models available which are well suited for evaluating interactions between system design characteristics, logistic support, and relevant operational output measures such as operational availability or sortie rate. Examples of such models include LCOM (aircraft), CASEE and PRISM (carrier-based aircraft), ARMS (Army aircraft), TIGER (Ship systems), RETCOM (combat vehicles), etc. These models provide visibility of manpower and test equipment, queueing effects, and the impact of spares stockage levels on operational availability, which generally cannot be evaluated with simple analytical formulas. Simulation models are particularly useful for using test results to project operational availability under conditions different from the test environment (e.g., to project availability under wartime surge conditions). One drawback to simulation models is that they are usually more cumbersome to use than straightforward analytical techniques.

CASE STUDY NO. 4-1

Background

Early in the development phase of a new avionics system, it is determined that an inherent availability of 0.92 is required. The reliability and maintenance engineering personnel in the program office desire to analyze only what effect this requirement has on the relationship between their disciplines, which is appropriate in a first-look consideration.

Determine

How can this analysis be accomplished?

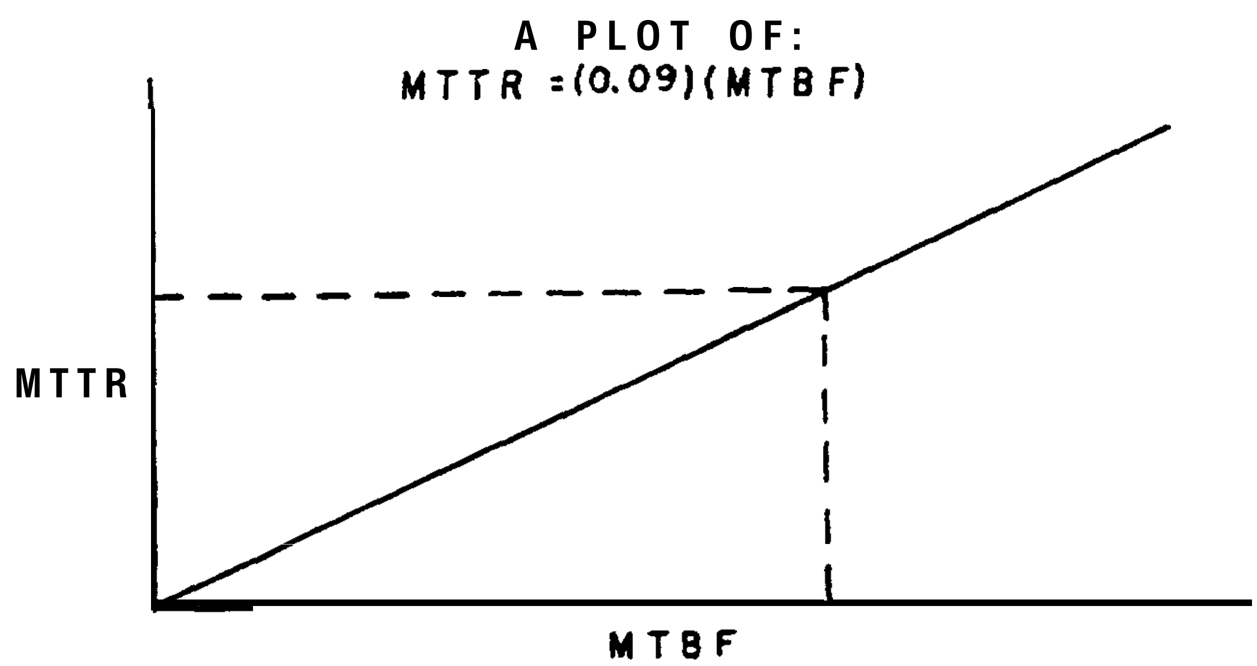
Solution

A straightforward analysis can be conducted by using the definition of Ai. Remember Ai does not consider delay times nor preventive maintenance. Should the engineers so desire and if it is considered important for this system, they could redefine MTTR to include all maintenance.

$$Ai = \frac{MTBF}{MTBF + MTTR} = 0.92$$

$$MTBF = (0.92)(MTBF + MTTR)$$
$$MTBF = (11.5)(MTTR) \text{ or}$$
$$MTTR = (0.09)(MTBF)$$

The function MTTR = (0.09)(MTBF), may be used directly, or it maybe plotted as shown below. The graph is a straight line, passing through the origin, with a slope of 0.09. For the same form of equation, the general solution is MTTR = [(1-A)/A] (MTBF), where A is inherent availability.



CASE STUDY NO. 4-2

Background

A system has an MTTR of 30 minutes and an MTBUMA of 50 hours. The intended utilization of the system is 5,000 hours per year. Ten hours of preventive maintenance are required for each 1,000 hours of operation. A mean administrative and logistic delay time of approximately 20 hours is associated with each unscheduled maintenance action.

Determine

For a one-year period, determine OT, TCM, TPM, ALDT, ST, Ao, Aa, and Ai for a utilization of 5,000 hours per year. Determine Ao if MTTR were reduced to zero. Determine the maximum number of operation hours in a year. Compare Ao and Ai.

Solution

$$TT = (365)(24) = 8{,}760 \text{ hours}$$

$$OT = 5{,}000 \text{ hours}$$

$$TCM = \frac{5{,}000}{50}(0.5) = 50 \text{ hours}$$

$$TPM = \frac{10}{1{,}000}(5{,}000) = 50 \text{ hours}$$

$$ALDT = \frac{5{,}000}{50}(20) = 2{,}000 \text{ hours}$$

$$ST = 8{,}760 - (5{,}000+50+50+2{,}000) = 8{,}760 - 7{,}100 = 1{,}660$$

$$Ao = \frac{5{,}000 + 1{,}660}{8{,}760} = 0.76$$

$$Aa = \frac{5{,}000}{5{,}000 + 50 + 50} = 0.98$$

$$Ai = \frac{5{,}000}{5{,}000 + 50} = 0.99$$

If MTTR (for corrective maintenance only) were reduced to essentially zero

$$Ao = \frac{5{,}000 + (1{,}660+50)}{5{,}000 + (1{,}660+50) + 50 + 0 + 2{,}000} = \frac{6{,}710}{8{,}760}$$

$$Ao = 0.77$$

NOTE : 50 hours added to numerator represents additional available standby time. This time had been spent on repair when MTTR was non-zero.

Assuming ST = O, the maximum possible operating hours in a year can be determined as follows:

$$Ao = \frac{5,000}{5,000 + 50 + 50 + 2,000} = 0.704$$

$$(Ao)(\text{hours/year}) = (0.704)(8,760) = 6,153 \text{ hours maximum.}$$

An alternative method for determining maximum possible operating hours assuming ST = O is to solve the following equation for x.

$$8760 - \frac{0.5}{50}x + \frac{10}{1000}x + \frac{20}{50} \quad X = X,$$

where

$$\frac{0.5}{50}x = TCM$$

$$\& \quad x = TPM$$

$$\frac{20}{50}x = ALDT.$$

The solution is x = 6,169. The difference in the two values occurs as a consequence of rounding Ao (0.704) to three significant digits.

CASE STUDY NO. 4-3

Background

Test planning requires that an assessment be made of some basic program requirements . During this assessment, you observe that some of the assumptions concerning availability assessment are questionable. Consider the case where system availability is being computed, and let us assume that we have the option of using either a 5-day test period or a 7-day test period. Note that neither system utilization nor maintenance occurs on 2 of the 7 days. A close review of these conditions is warranted, particularly one which permits the utilization of a 7-day week for total time when in fact additional system usage does not occur during 2 days of this period.

Determine

What is the impact of the utilization period choice on Ao?.

For purposes of this review, we will utilize the following parameters:

OT = 10 hours
TPM = 5 hours
TCM = 60 hours
ALDT = 22 hours

$$A_o = \frac{OT + ST}{OT + ST + TPM + TCM + ALDT}$$

Solution

For:

7 Days

OT = 10 hours
ST = 158 hours

$$Ao = \frac{168}{168 + 87}$$

$$Ao = 0.66$$

5 Days

OT = 10 hours
ST = 110 hours

$$Ao = \frac{120}{120 + 87}$$

$$Ao = 0.58$$

Commentary

Note the higher value obtained by including the two additional non-usage days.

CASE STUDY NO. 4-4

Background

Because operational availability is a composite of system, support, and environmental factors, it is a useful tool in conducting analyses of the various parameters. The following is an example of this analysis.

Determine

Develop an expression which defines MTBF as a function of OT, TT, availability, and logistics down time.

Solution

We start with the expression for operational availability:

$$A_o = \frac{OT + ST}{OT + ST + TPM + TCM + ALDT} . \tag{1}$$

Since TPM + TCM + ALDT = TDT, total down time,

$$A_o = \frac{OT + ST}{OT + ST + TDT} . \tag{2}$$

The denominator of this equation is total time,

$$OT + ST + TDT = TT,$$

and the numerator equals total time less TDT, thus

$$A_o = \frac{TT - TDT}{TT}$$

$$A_o = 1 - \frac{TDT}{TT} . \tag{3}$$

Define DTF as the down time per failure. It is necessary to base the Ao value on MTBF so that the MTBF may be isolated and computed.

The number of failures, r, is equal to OT/MTBF. Total down time is then'

$$TDT = (DTF)(OT/MTBF) + TPM + (ALDT)_p . \text{ Assume } (ADLT)_p \cong 0.$$

Substituting this expression in the last equation of step 3, we obtain

$$A_o = 1 - \frac{(DTF)\,(OT)}{(TT)(MTBF)} - \frac{TPM}{TT} , \tag{4}$$

Solving for the MTBF, we obtain

$$\text{MTBF} = \frac{(\text{DTF})(\text{OT})}{(1-\text{Ao})(\text{TT}) - \text{TPM}} \qquad (5)$$

Using the following values

TT = 90 days x 24 hrs/day = 2,160 hours

OT = 23 missions x 40 hours per mission = 920 hours

DTF = 24 hours per failure

TPM = 100 hours

and substituting into (5), we obtain

$$\text{MTBF} = \frac{(24)(920)}{(1-0.8)(2,160) - 100} = 50.0 \text{ hours.}$$

NOTE : When using this definition of MTBF, it is important to verify that the standby time is not forced below a zero value by erroneously defining OT, TDT, and TT.

CASE STUDY NO. 4-5

Background

A system currently in late development has exhibited an operational availability of 0.66. The user has stipulated that an operational availability of a 0.75 minimum is required. In order to improve system operational availability, it is decided to change the maintenance concept on several low-reliability units. The current circuit card remove/replace concept will be changed to a black box remove/replace. The following tabulations list the characteristics of the existing equipment and those desired after the system maintenance concept has been revised in an attempt to improve operational availability.

Existing Elements	Desired Elements
TT = 168 hours	TT = 168 hours
TPM = 5 hours	TPM = 5 hours
TCM = 60 hours	TCM = to be determined
ALDT = 22 hours	ALDT = 22 hours
Ao = 0.66	Ao = 0.75

Determine

New required value of TCM which must be realized if the desired Ao increase is to be achieved.

Solution

$$\text{Ao} = \frac{\text{OT} + \text{ST}}{\text{OT} + \text{ST} + \text{TCM} + \text{TPM} + \text{ALDT}} = \frac{\text{TT}}{\text{TT} + \text{TCM} + \text{TPM} + \text{ALDT}}$$

$$\text{Ao} = \frac{168}{168 + 5 + \text{TCM} + 22} = \frac{168}{\text{TCM} + 195}$$

Since Ao = 0.75,

TCM = 27.4 hours

Commentary

Of course, the reasonableness or attainability of such a reduction must be considered. Increased operational availability also can be obtained by decreasing TPM or ALDT.

CHAPTER 5

MATHEMATICAL MODELS OF RELIABILITY

INTRODUCTION

In a general sense, we can say that the science of statistics attempts to quantify our uncertainty about the outcome of an experiment or test. Before we conduct an experiment, we are aware of' the possible outcomes, but we are not sure which of the outcomes will actually result. The mathematical model, a logical extension of our assumptions about the experiment, quantifies our uncertainty and provides us with a tool for analysis of the outcome.

Suppose that a system is to be put into operation. We know that after a certain amount of operating time it will experience a failure. We would like to know when the system will fail. Generally, any prediction about the actual time of failure will not be accurate enough to be worthwhile. However, we can address more confidently questions such as: "Will the system operate free of failure for at least 500 hours?" A model that describes the experiment in question will help us answer these questions with a certain amount of assurance.

In this chapter, we consider three models that are pertinent to RAM considerations: the binomial, the Poisson, and the exponential. The binomial and Poisson are discrete models in that they essentially count numbers of failures. The exponential model is continuous in that it describes times between failures. Although the Poisson model addresses discrete events, it is a continuous time model. It counts failures as they occur over a period of time, i.e., the possible outcomes of the experiment, conducted on a continuous time basis, are enumerated as numbers of failures. This distinction will become clearer as we contrast the poisson model and the binomial model.

In its most basic form, a mathematical model of a statistical experiment is a mathematical expression (function) that defines the probability associated with each of the outcomes of the experiment. For our purposes, we will discuss the two basic types of models: discrete and continuous. The type of model--discrete or continuous--is defined by the type of outcome that the experiment provides.

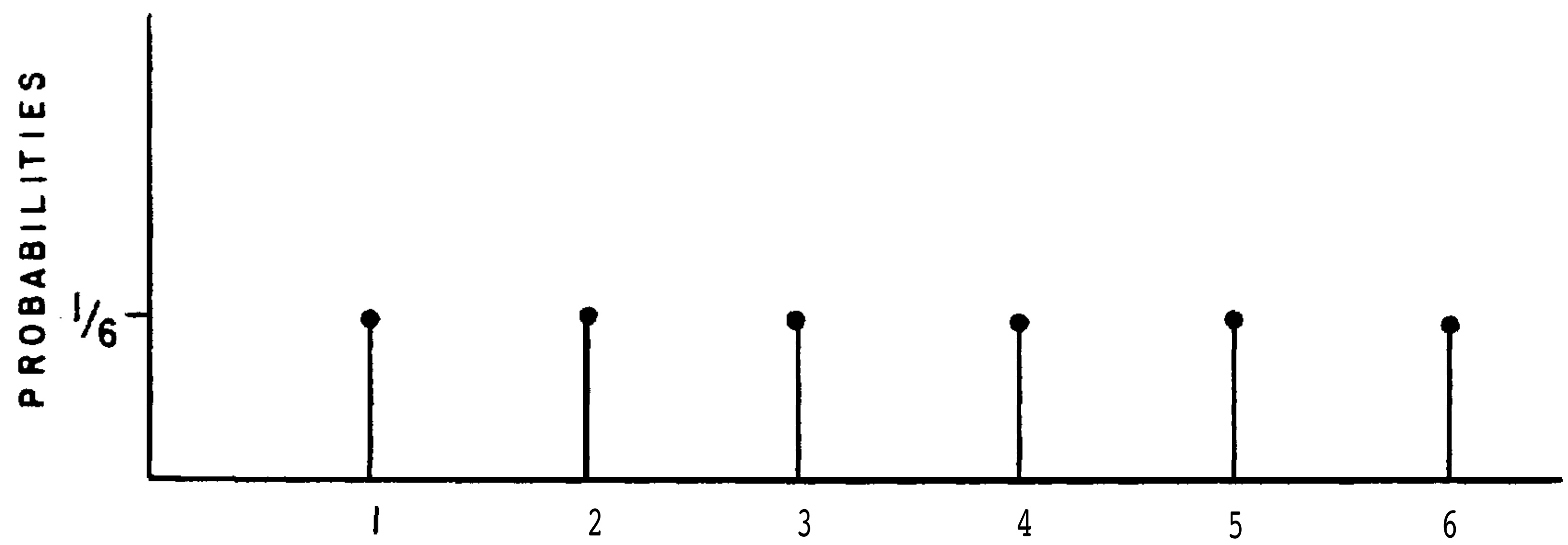
DISCRETE MODELS

A discrete model is appropriate when the possible outcomes of an experiment can be enumerated or counted. In its most basic form, the discrete model is a mathematical expression (function) that defines the probability of each individual outcome. A simple example is the following. Suppose a die is to be tossed once, and the outcome of interest is the number of dots facing up when the die comes to rest. The outcomes are {1, 2, 3, 4, 5, 6}. If we assume that the die is "fair," the probabilities can be expressed as follows:

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.$$

Graphically, we display the probabilities in Figure 5-1.

FIGURE 5-1



Suppose our experiment is to toss two dice, and the outcome of interest is the sum of the dots facing up. The set of all possible outcomes is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. The probabilities can be expressed as follows:

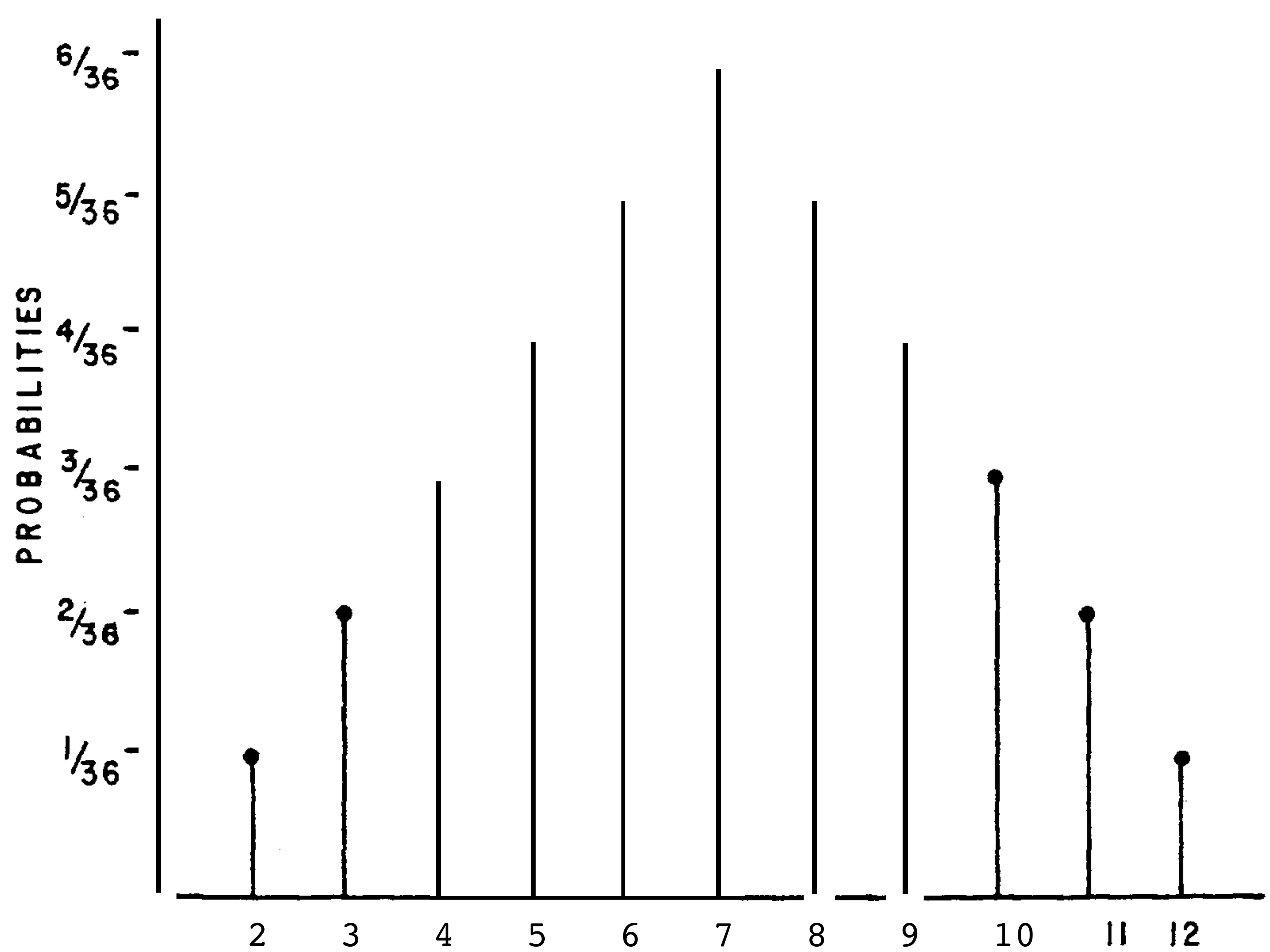
$p(2) = p(12) = 1/36$ $\qquad$ $p(5) = p(9) = 4/36$

$p(3) = p(11) = 2/36$ $\qquad$ $p(6) = p(8) = 5/36$

$p(4) = p(10) = 3/36$ $\qquad$ $p(7) = 6/36.$

Graphically, we display the probabilities in Figure 5-2.

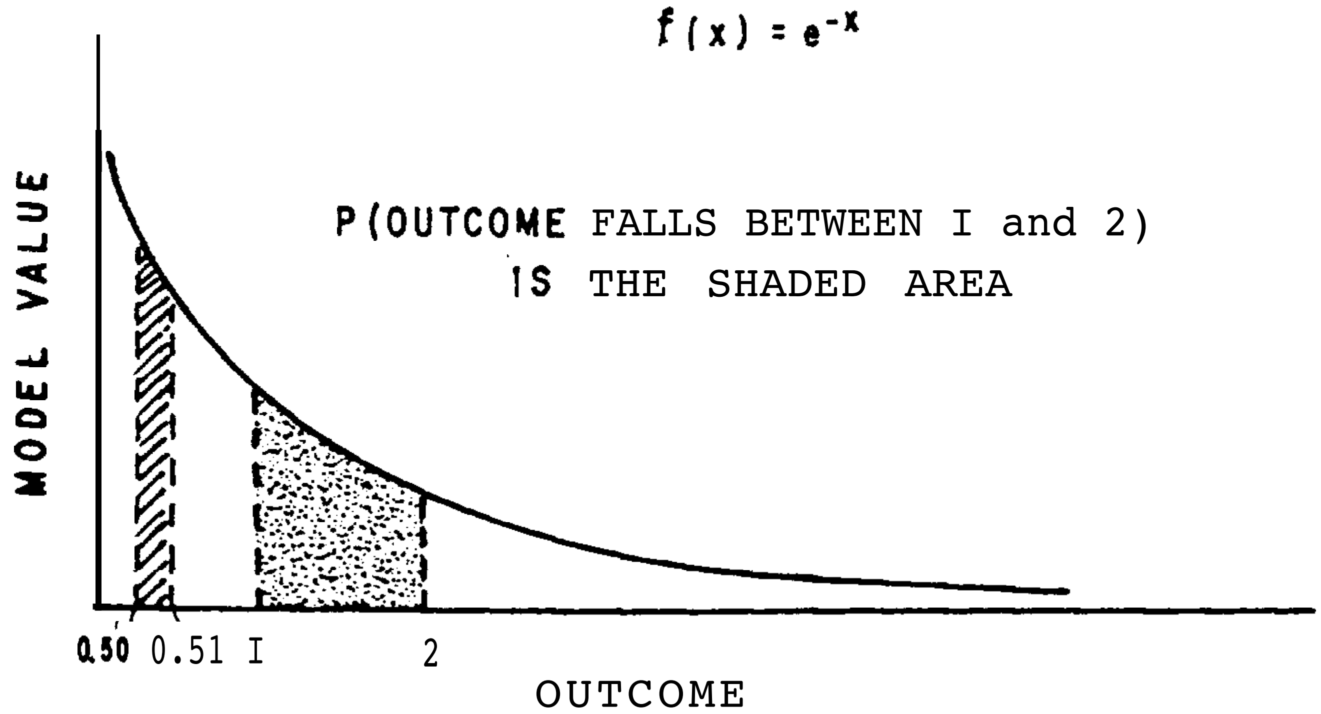
FIGURE 5-2



CONTINUOUS MODELS

A statistical experiment often results in an outcome that is measured on a continuous scale. Time and distance are perhaps the most common continuous variables. In its most basic form, the continuous model is a mathematical expression (function) useful in computing probabilities of certain outcomes. It differs from probabilities for discrete models in that it does not define probabilities directly. Generally, for a continuous model, it only makes sense to consider the probability of an outcome within a range of values or a certain interval--between 100 and 150 miles, more than 10 hours. The probability of an outcome falling within a given range is the area which lies beneath the continuous model curve over that range. Consider the examples below.

FIGURE 5-3 GRAPH OF THE EXPONENTIAL MODEL



The probability that an outcome is between 1 and 2 is defined by the area under the curve between the values 1 and 2. Therefore,

P(outcome falls between 1 and 2)

$$= \int_1^2 e^{-x} dx = -e^{-x} \Big|_1^2 = e^{-1} - e^{-2} = 0.233.$$

The probability that an outcome is between 0.50 and 0.51 is

$$\int_{0.50}^{0.51} e^{-x} dx$$

which is $e^{-0.50} - e^{-0.51}$ or 0.006.

FIGURE 5-4 GRAPH OF UNIFORM MODEL

f(x) = 0.1 FOR 10 < x < 20

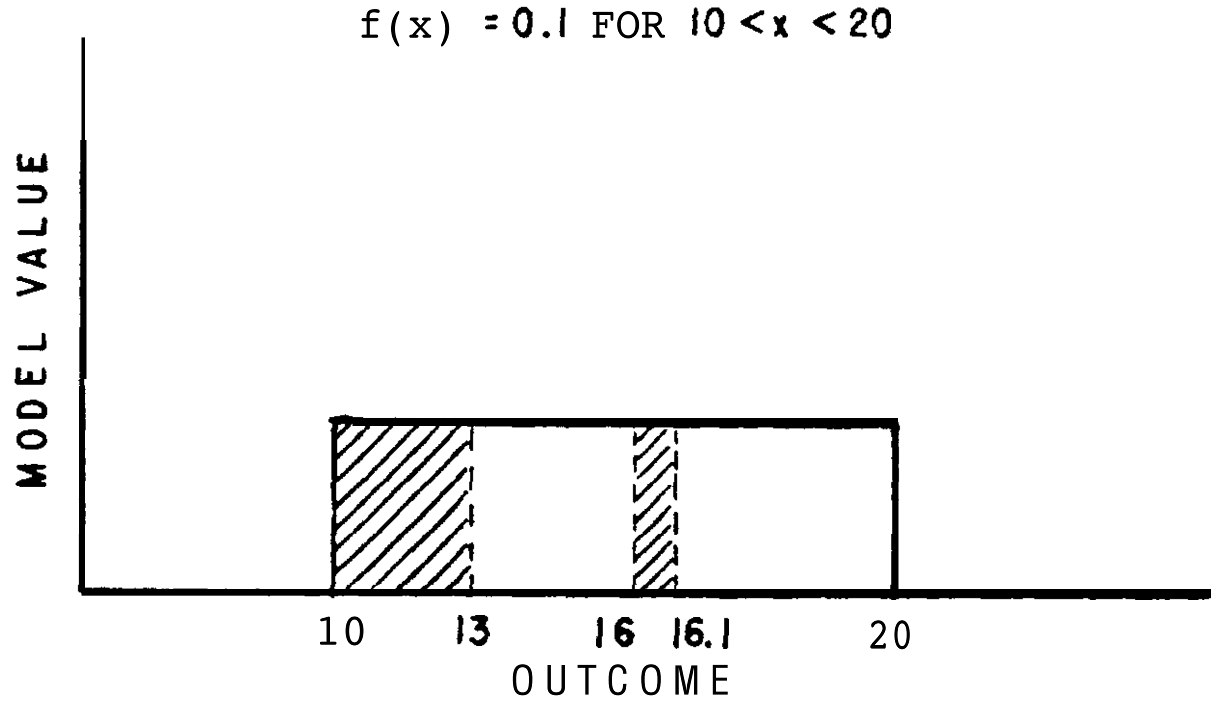


Figure 5-4 illustrates another possible continuous model with the function being f (x) = 0. 1 defined on the interval 10 < x < 20.

The probability that an outcome is less than 13 is

$$\int_{10}^{13} 0.1\, dx,$$

which is 1.3-1.0, or 0.3. The probability that an outcome is between 16 and 16.1 is

$$\int_{16}^{16.1} 0.1\, dx,$$

which is 1.61-1.60, or 0.01.

BINOMIAL MODEL

The model that is used most commonly to describe the outcomes of success/fail test programs is the binomial model. In order for a testing program to be a binomial experiment, four conditions are required. They are:

- The test period consists of a certain number (n) of identical trials.
- At any individual time unit (trial) , the test results in a success or failure.
- The outcome at any individual time unit is independent of the outcomes of all other time units.
- The reliability (probability of success) of the system remains unchanged for each trial.

In a lot acceptance sampling test program, the third and fourth conditions are not precisely satisfied. In such a case, the hypergeometric distribution provides the exact analysis and must be used for small lot sizes. However, if the sample size (n) is small compared to the lot size (N), say $n/N < 0.05$, then the binomial model provides a reasonable basis for analysis. #e do not present a discussion of the hypergeometric distribution in this text. The reader can refer to any number of moderate level statistics textbooks for information on the use of hypergeometric distribution.

Examples of binomial experiments are:

- firing a missile (is it launched successfully?);

- firing a missile (does it hit the intended target?);

operating any system over time (does it achieve its durable life?).

For the remainder of this section on binomial models, the following notation is used.

n: the number of trials or test units.

p: the probability of failure for any trial. (We use p here as the probability of failure instead of the more classical probability of success because of the failure-oriented approach used by the Poisson model) .

$\binom{n}{k}$: binomial coefficient, which by definition is equal to $n!/[k!(n-k)!]$, where k must be some integer between 0 and n inclusive. By definition, $n!=n(n-1)(n-2)\ldots1$ and $0!=1$.

$b_{n,p}(k)$: the probability of k failures out of n trials with p the probability of failure on any one trial.

$B_{n,p}(k)$: the probability of k or fewer failures out of n trials with p the probability of failure on any one trial.

Any binomial experiment (test program) is completely characterized by the number of trials and the probability of failure for any given trial. The probability of failure (p) is generally an unknown value about which testing requirements are stated.

In Chapter 7, the binomial is used in this reversed role: the test results will have been observed and we will make inferences about the probability of success based on the binomial model.

For the binomial model an exact mathematical formula is used to compute probabilities. We denote the probability of exactly k failures out of n trials for a fixed probability of failure, p, by $b_{n,p}(k)$, and

$$b_{n,p}(k) = \binom{n}{k} p^k (1-p)^{n-k}. \tag{5.1}$$

The probability of k or fewer failures, termed the binomial cumulative distribution function, is $B_{n,p}(k)$, where

$$B_{n,p}(k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \sum_{i=0}^{k} b_{n,p}(i). \tag{5.2}$$

Figure 5-5 presents graphs of three binomial models. The graphs chosen portray models for which the values of p are equal to 0.2, 0.5, and 0.7 with n, the number of trials, equal to 10 in each case.

For the case $p = 0.2$, the graph shows that a large number of failures--more than 5--is unlikely. Note that the most likely number of failures is 2, which corresponds to a percentage of failures equal to 0.2. For the case $p = 0.7$, the graph shows that a small number of failures--fewer than 5--is unlikely. Once again, the most likely number of failures, 7, corresponds to a percentage of failures equal to 0.7. For the case $p = 0.5$, the graph shows that a moderate number of failures--between 3 and 7--is likely, with 5 failures (half of the trials) being most likely.

Computation of binomial probabilities using $b_{n,p}$ or $B_{n,p}$ is cumbersome when n is large. Three alternative methods for determining binomial probabilities are:

use of a statistics package for a calculator,

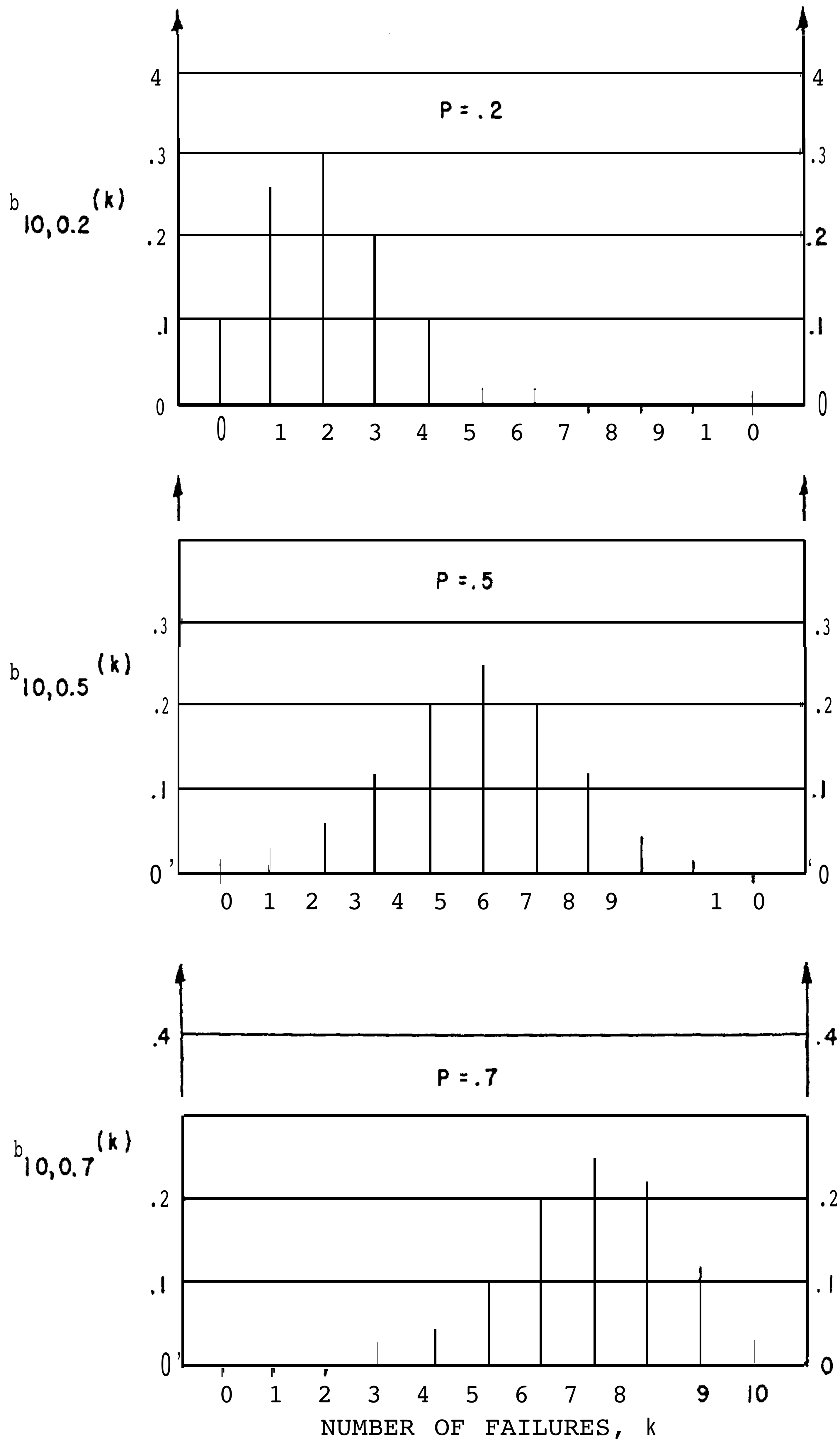
use of binomial tables, and

use of an approximating distribution (Poisson or normal).

Tables of $b_{n,p}$ or $B_{n,p}$ for n less than or equal to 20 are generally published in elementary statistics textbooks. More extensive tables are available but are not as easy to locate. A table of cumulative binomial probabilities for selected values of n, k, and p is given in Appendix B, Table 1. When n is larger than 20, either the Poisson distribution ($p > 0.8$ or $p < 0.2$) or the normal distribution ($0.2 < p < 0.8$) provides reasonable approximations to binomial probabilities. (See Appendix A for details on these procedures.)

In Case Studies 5-1, 5-2 and 5-3, we demonstrate the application of the binomial model and compute probabilities associated with the model.

FIGURE 5-5 BINOMIAL PROBABILITIES



POISSON MODEL

The most commonly used model for describing the outcomes of a continuous time testing program is the Poisson model. In order for a testing program to be a Poisson experiment, it is required that no more than one event (failure) can occur at the same time and that the number of events (failures) is related directly to the amount of test time. Examples of Poisson experiments are:

> number of failures of a system during a test cycle;
>
> number of unscheduled maintenance actions required during a given time period; and,
>
> number of misfires of an automatic weapon firing over a given time period.

For the remainder of the section on Poisson models and the succeeding section on exponential models, the following notation is used.

- λ: Failure rate or average number of failures per unit time.
- T: The length of the interval (hours, ❑iles, etc.) of :interest (e.g., mission duration, test exposure).
- $g_{\lambda,T}(k)$: The probability of exactly k failures during a test period of length T when the failure rate is A.
- $G_{\lambda,T}(k)$: The probability of k or fewer failures during a test period of length T when the failure rate is A.

Any Poisson experiment (test program) is completely characterized by the length of time on test and the mean value function. The mean value function for a specific test length is the average number of failures to occur during the specified length. When the system has a constant failure rate for the entire interval T, this function is simply AT. A discussion of constant failure rate assumptions can be found in Chapter 7. The value A is the failure rate of the system on test, A more familiar parameter is the mean time between failures (MTBF) which is the reciprocal of A. System requirements are generally stated in terms of the mean time between failures.

As with the binomial model, Poisson probabilities can be computed using an exact mathematical formula. We denote the probability of exactly k failures during a test period of length T where the failure rate is A, by $g_{\lambda,T}(k)$, and

$$g_{\lambda,T}(k) = \frac{(\lambda T)^k e^{-\lambda T}}{k!} \tag{5.3}$$

The number of failures may be any integer value including 0 and 0! = 1. The probability of k or fewer failures, termed the Poisson cumulative distribution function, is $G_{\lambda,T}(k)$, where

$$G_{\lambda,T}(k) = \sum_{i=0}^{k} \frac{(\lambda T)^i e^{-\lambda T}}{i!}$$

$$= \sum_{i=0}^{k} g_{\lambda,T}(i). \qquad (5.4)$$

Figure 5-6 presents graphs of three Poisson models. Values for λT of 2, 5, and 7 were chosen to demonstrate the effect of time T on the numbers of failures likely to be seen. When λ, the failure rate, is fixed, the number of failures will, in all likelihood, increase with the amount of operating time T.

Alternative methods for computing Poisson probabilities include:

use of a statistics package for a calculator,

use of Poisson tables or charts, and

use of an approximating function.

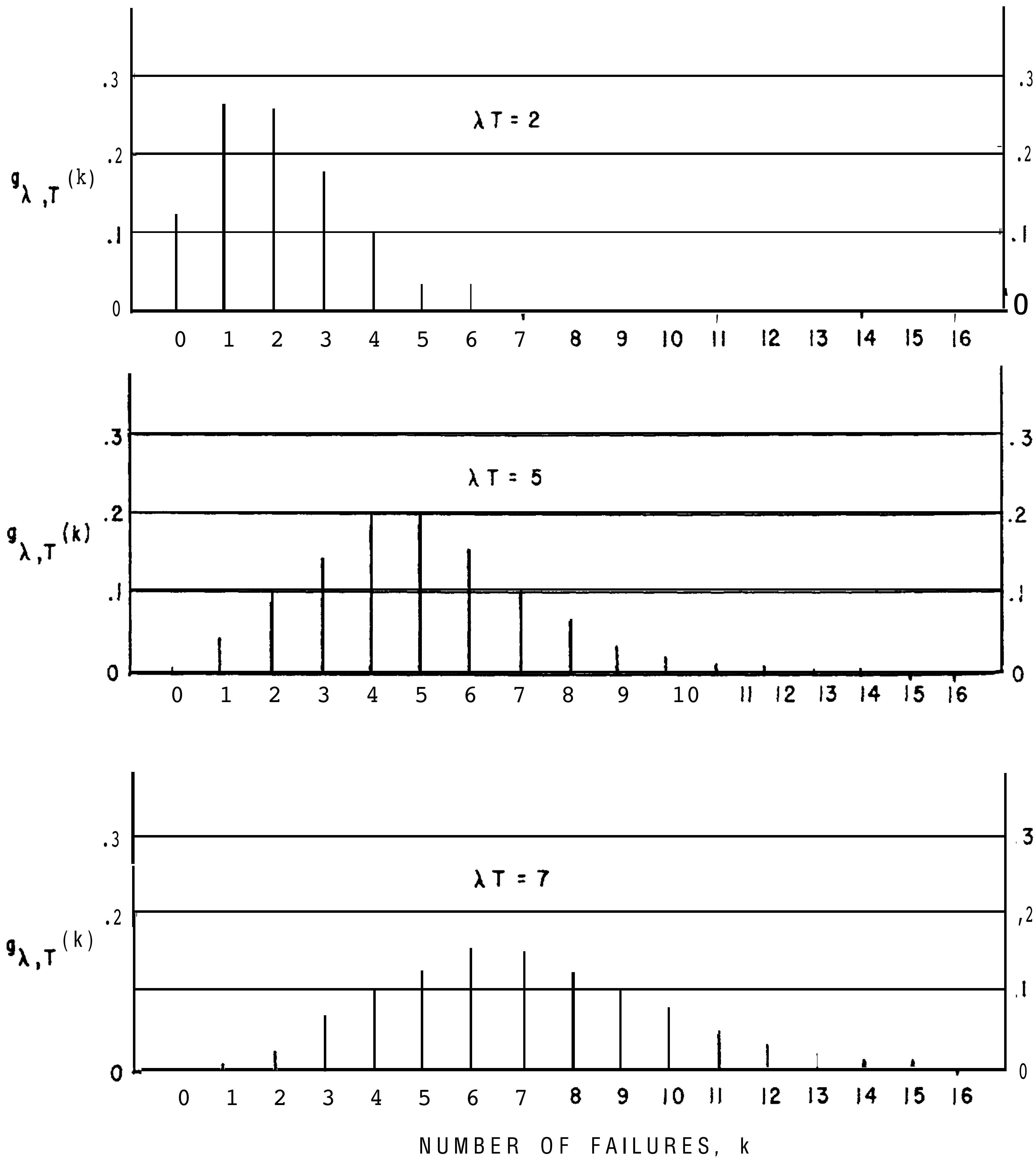
In Case Studies 5-5 and 5-6 we demonstrate the application of the Poisson model and compute probabilities associated with the model. Tables of $g_{\lambda,T}(k)$ or $G_{\lambda,T}(k)$ are available in many textbooks. A table of cumulative Poisson probabilities is given in Appendix B, Table 3. Appendix B, Chart 1 is also useful in determining cumulative Poisson probabilities. When the product λT is greater than 5, the normal distribution provides reasonable approximations to Poisson probabilities. (See Appendix A for details on this procedure.)

EXPONENTIAL MODEL

Generally, it is more informative to study times between failures, rather than numbers of failures, for a continuous time testing program. The most commonly used model for describing the times between failures for a continuous time testing program is the exponential model. In order for a testing program to qualify as an exponential experiment, the following conditions are required: (1) the system is as good as new after each repair, and (2) the probability of failure in any given interval of time is the same no matter how old a system is and no matter how many failures it has experienced. The second condition is an intuitive description of the so-called memoryless property. The system cannot "remember" how old it is, nor can it "remember" how often it has failed. The examples listed for Poisson experiments can serve also as examples of exponential experiments.

For the exponential model, there is an exact mathematical formula used for computing probabilities that a certain amount of time will pass before the

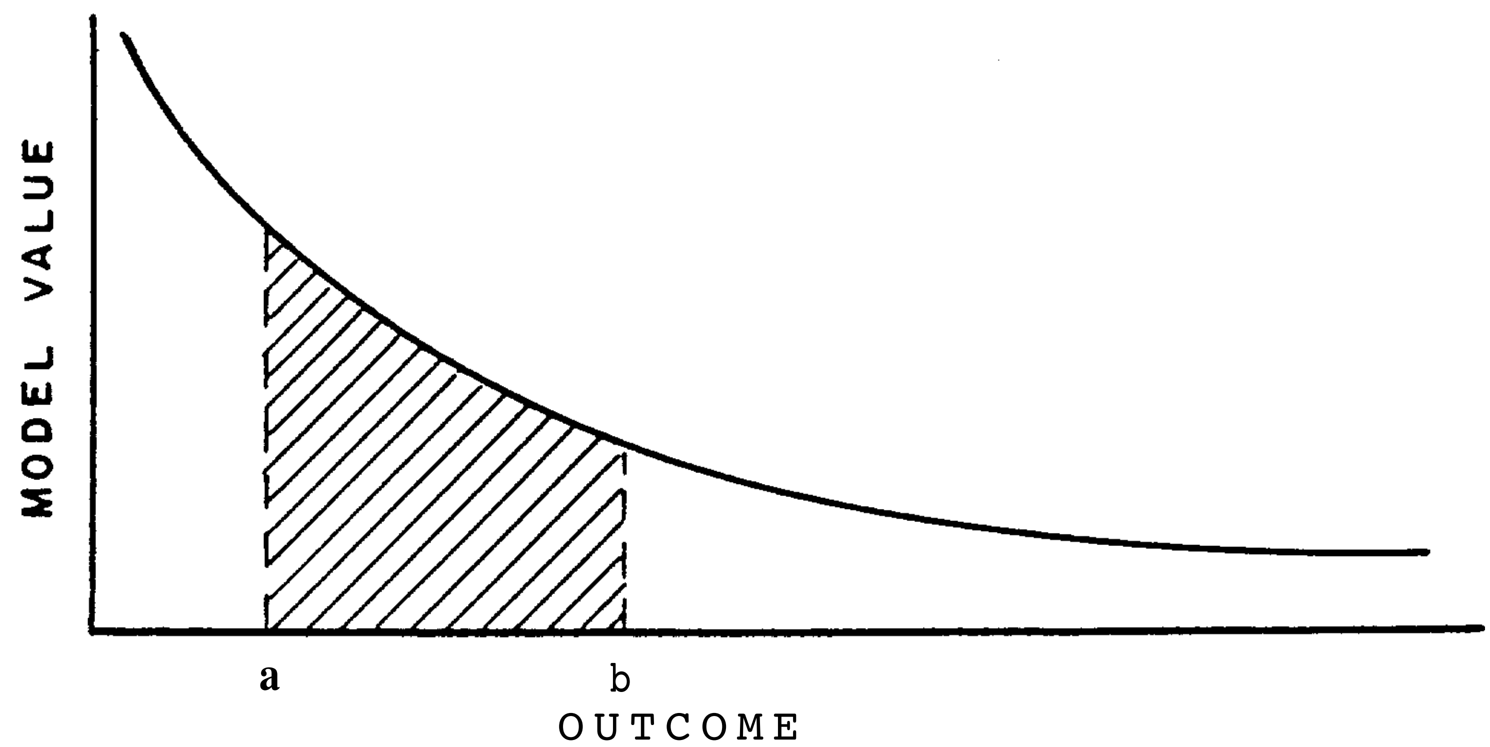
FIGURE 5-6 POISSON PROBABILITIES



next failure. The probability that a failure will occur in some future interval of time (a, b) for a system with failure rate A is

$$\int_a^b \lambda e^{-\lambda x} dx. \tag{5.5}$$

FIGURE 5-7 GENERAL EXPONENTIAL **MODEL: $f(x) = \lambda e^{-\lambda x}$**



The exponential cumulative distribution function, $F_\lambda(t)$, defines the probability that the system will fail before time t. By definition,

$$F_\lambda(t) = \int_0^t \lambda e^{-\lambda x} dx = 1 - e{-}At. \tag{5.6}$$

A function of more interest to us is **the** reliability function, R(t), which defines the probability that the system will **operate** for t units of time without failure. By definition,

$$R_\lambda(t) = \int_t^\infty \lambda e^{-\lambda x} dx = e^{-At}. \tag{5.7}$$

The reliability function, R(t), can also be expressed as $e^{-(t/\theta)}$, where θ, the reciprocal of A, is the mean time between failures (MTBF). The reliability function R(t) translates the effectiveness parameters λ, the failure rate, or θ, the MTBF, into reliability. Reliability is the probability of failure-free operation for a specified length of time, t.

We referred to the variable t as a time variable in the above discussion. Measure of life units which can be appropriate are hours, miles, **cycles and** rounds.

As an example, suppose that the mission profile for a system requires a mission duration (MD) of 40 hours and the system has a mean time between operational mission failure (MTBOMF) of 400 hours. Then the probability that the system successfully completes a mission can be evaluated using the reliability function, R(t). Now

$$R(t) = e^{-At} = e^{-t/\theta} = e^{-(MD/MTBOMF)}$$

Since MD = 40 hours and MTBOMF = 400 hours, the mission reliability is $e^{-40/400}$, which reduces to 0.905. In other words, the system has a 90.5% chance of completing the 40-hour mission.

NOTE : A useful approximation for the reliability function is :

$$e^{-t/\theta} \cong 1 - t/e \text{ for } t/\theta \leq 0.1.$$

For the above example, t/El = O. 1 so that the approximation yields 0.90.

In Case Study 5-4, we demonstrate the application of the exponential model with computations based on both models. See Case Studies 5-4, 5-5, and 5-6, for more illustrations of this computation and other computations associated with the Poisson/exponential model.

The reliability function e^{-At} may also be symbolized as $\exp(-\lambda t)$ or in'¹(-At) . That is, it is the exponential function evaluated at the point, -At, or it is the inverse of the natural logarithm function evaluated at that point. Some calculators evaluate the exponential function as the inverse natural logarithm function.

CASE STUDY NO. 5-1

Background

A sensor device has an operational mission reliability of 0.90 for a 4 hour mission. At least 3 sensors are required to locate targets. Failed sensors will not be repaired during a mission.

Determine

1. If a system employs 3 sensors, what is the probability of successfully completing the mission?

2. If a system employs 4 sensors, what is the probability of successfully completing the mission?

Solution

1. We use the mathematical formula for the binomial model given in equation 5.1, with

$$R = 1 - p = 0.9$$

$$p = 1 - 0.9 = 0.1$$

$$n = 3, \text{ and}$$

$$k = 0$$

Applying equation 5.1, the probability of 0 failures is:

$$b_{3,0.1}(0) = \binom{3}{0}(0.1)^0(0.9)^3$$

The binomial coefficient is:

$$\binom{3}{0} = \frac{3!}{0!3!} = 1$$

The probability of 0 failures is:

$$(1)(1)(0.9)^3 = 0.729.$$

2. We use the binomial cumulative distribution function given in equation 5.2, with

$$R = 1 - p = 0.9$$

$$p = 1 - 0.9 = 0.1$$

$n = 4$, and

$$k = 1$$

Applying equation 5.2, the probability of 1 or fewer failures is:

$$B_{4,0.1}(1) = \sum_{k=0}^{1} \binom{4}{k} (0.1)^k (0.9)^{(4-k)}$$

$$= \binom{4}{0}(0.1)^0(0.9)^4 + \binom{4}{1}(0.1)^1(0.9)^3$$

The binomial coefficients are:

$$\binom{4}{0} = \frac{4!}{0!4!} = 1, \text{ and } \binom{4}{1} = \frac{4!}{1!3!} = 4.$$

The probability of 1 or fewer failures is:

$$(1)(1)(0.9)^4 + (4)(0.1)^1(0.9)^3$$

$$= 0.656 + 0.292 = 0.948.$$

Commentary

For the second problem, we use the binomial cumulative distribution function since we are required to compute the probability of 0 failures or 1 failure, i.e., the cumulative probabilities of both outcomes.

CASE STUDY NO. 5-2

Background

A lot of 500,000 rounds of ammunition is available for sale. The buyer will purchase the lot if he has reasonable assurance that the lot is no more than 15% defective.

Determine

1. If the true proportion of defects is 0.15, what is the probability a sample of size 10 will yield fewer than 2 defects? More than 3 defects?

2. If the true proportion of defects is 0.05, what is the probability a sample of size 20 will yield fewer than 3 defects? More than 1 defect?

3. If the true proportion of defects is 0.02, what is the probability a sample of size 100 will yield fewer than 5 defects?

4. If the true proportion of defects is 0.15, what is the probability a sample of size 50 will yield more than 10 defects? Between 5 and 10 defects, inclusive?

Solutions

This is a lot acceptance sampling problem. Although the binomial model is not technically correct, it will provide very good approximations in this case because the ratio of sample size n to lot size N(N = 500,000) is not more than 0.0002. A further explanation of the use of the binomial model for lot acceptance sampling problems is found on page 5-5.

1. The probability of failure, p, is 0.15. For fewer than 2 defects, we look in the tables for n = 10, the column p = 0.15, and the row c = 1. The probability is 0.5443.

The probability of more than 3 defects is the difference between 1 and the probability of fewer than 4 defects. The probability of fewer than 4 defects is 0.9500, so the probability of more than 3 defects is 0.05.

2. The probability of fewer than 3 defects out of 20 is obtained directly in the tables for n = 20, the column p = 0.05, and the row c = 2. The probability is 0.9245.

The probability of more than 1 defect is the difference between 1 and the probability of fewer than 2 defects. The probability of fewer than 2 defects is 0.7358, so the probability of more than 1 defect is 0.2642.

3. A binomial table for n = 100 is not given in the Appendix and would be difficult to locate in other sources. However, because the sample size, n, is large, we may use an approximation fairly confidently. Recall that there are two approximations (Poisson and normal) to the binomial presented in the text of Chapter 5. The procedures for using these approximations are detailed in

Appendices A-1 and A-2. Using the normal approximation (Appendix A-1), the probability of 4 or fewer defects out of 100 trials is

$$1 - P(z \geq (4 + 0.5 - (100)(0.02))/\sqrt{100(0.02)(0.98)}) ,$$

which reduces to

$$1 - P(Z \geq 1.79) = 0.9633 .$$

Using the Poisson approximation (Appendix A-2), we set m = 2, c = 4, and obtain the probability directly from Appendix B, Table 3 as 0.947.

The exact value for the probability of fewer than 5 defects, obtained from the formula, is 0.9491, Note that, although each approximation is reasonably close to the exact value, the Poisson has provided the better approximation. As noted on page 5-6, the Poisson is more appropriate when p is very large or very small (in our case, p = 0.02).

4. The probability of more than 10 defects is the difference between 1 and the probability of fewer than 11 defects. The probability of fewer than 11 defects using the binomial tables for n = 50, the column p = 0.15, and the row c = 10, is 0.8801. The probability of more than 10 defects is thus 0.1199. The probability of between 5 and 10 defects inclusive is the probability of fewer than 11 defects less the probability of fewer than 5 defects. These two numbers are 0.8801 and 0.1121, and the difference is 0.7680. The normal distribution is appropriate for approximating this probability. Using this approximation, we find that the probability of more than 10 (11 or more) defects is

$$P(Z \geq (11 - 0.5 - (50)(0.15))/4\overline{50(0.15)(0.85)}) ,$$

which reduces to

$$P(z \geq 1.19) = 0.1170 .$$

The approximate probability of between 5 and 10 defects inclusive is

$$P(z \geq (5 - 0.5 - 7.5)/\sqrt{6.38}) - P(z \geq (lo + 0.5 - 7.5)/\sqrt{6.38})$$

$$= P(Z \geq -1.19) - P(Z \geq 1.19) = 0.7660 .$$

Commentary

We have calculated probabilities for certain outcomes which could result from an inspection of a sample from the lot of 500,000. To calculate these values it is necessary to assume that we know the true proportion of defects in the entire lot. Of couse, we do not, in fact, know this true proportion of defects, but we perform this exercise in order to develop a rational plan for sampling from the lot in order to determine whether we should accept or reject the lot. Consider, for example, the solution to the second part of question 1. Namely, the probability of 4 or more defects out of 10 trials when the lot contains 15% defects is 0.05. Consequently, if the sample of 10 should yield 4 or more defects, the buyer has reasonable assurance that the lot contains more than 15% defects and should be rejected. Consider now part 2 of question 4. In a preliminary step to the solution, we determined that the probability of 4 or fewer failures when the lot contains 15% defects is 0.1121. Consequently, if a sample of 50 yields 4 or fewer defects, the buyer has reasonable assurance that the lot contains fewer than 15% defects and should be accepted. We discuss in Chapter 8 methods for preparing test plans which provide reasonable assurance to both producer and consumer. Our purpose in presenting the above discussion is to introduce the reader to the probabilistic analysis that takes place in the preparation of test plans or lot acceptance sampling plans .

CASE STUDY NO. 5-3

Background

A mechanized infantry battalion has 36 armored personnel carriers (APC), 20 weapon stations (WS), and 5 command posts (CP). The mission reliability of an APC is 0.70, of a WS is 0.85, and of a CP is 0.95. The entire battalion can perform satisfactorily if at least 24 APCS, 17 WSS, and 5 CPS are operable throughout the miss ion.

Determine

1. What is the probability each type of system has enough units operating to complete the mission?

2. What is the probability the mission will be successful?

3. How small do we need to make p, the probability of failure, in order to ensure that each set of systems has a probability 0.90 of performing satisfactorily?

4. If each of the probabilities of failure is fixed, then how many more of each type of system is required to achieve the goal mentioned in number 3?

Solutions

The probability that a sufficient number of APCS operate throughout the mission is the probability of 12 or fewer failures out of 36 trials. Note that p, the probability of failure, is equal to 0.30. The probability that a sufficient number of WSS operate throughout the mission is the probability that 2 or fewer failures occur out of 20 trials. The probability that all CPS operate throughout the mission is the probability of no failures.

la. The probability of 12 or fewer failures out of 36 trials where p = 0.30.

i. Use Appendix B, Table 1. On page B-20 for n = 36, we look in the column labeled 0.300 and the row r = 12. The value is 0.7365.

ii. Use Normal Approximation. (See Appendix A-1.) Since np = (36)(0.3) = 10.8 and np(1-p) = (36)(0.3)(0.7) = 7.56, the approximate probability is

$$1 - P(Z \geq (12.5 - np)/\sqrt{np(1-p)})$$

$$= 1 - P(Z \geq (12.5 - 10.8)/\sqrt{7.56})$$

$$= 1 - P(Z \geq 0.62)$$

$$= 0.7324$$

We obtain this value using Appendix B, Table 2, page B-42 in the row marked $z_\alpha = 0.62$ and the column labeled $P(Z \geq z_\alpha)$.

iii. Use Poisson Approximation. (See Appendix A-2.) We make the identification $m = np = (36)(0.3) = 10.8$ and use Appendix B, Table 3. On page B-47 we obtain the value from the column labeled 12 and interpolate between rows labeled 10.5 and 11.0. The value is 0.718.

Note that since $0.2 < p < 0.8$, the normal yields a better approximation than the Poisson.

lb. The probability of 3 or fewer failures out of 20 trials where $p = 0.15$.

i. Use Appendix B, Table 1. On page B-7 for $n = 20$, we look in the column labeled 0.150 and the row $r = 3$. The value is 0.6477.

ii. Use Normal Approximation. (See Appendix A-1.) Since $np = (20)(0.25) = 3$ and $np(1-p) = (20)(0.15)(0.85) = 2.55$, the approximate probability is

$$1 - P(Z \geq (3.5-np)/\sqrt{np(1-p)})$$

$$= 1 - P(z \geq (3.5-3)/4-")$$

$$= 1 - P(Z \geq 0.32) = 1 - 0.3745$$

$$= 0.6255$$

We obtain this value using Appendix B, Table 2, page B-42 in the row marked $z_\alpha = 0.32$ and the column labeled $P(Z \geq z_\alpha)$.

iii. Use Poisson Approximation. (See Appendix A-2.) We make the identification $m = np = (20)(0.15) = 3$ and use Appendix B, Table 3. On page B-46 we obtain the value from the column labeled 3 and the row labeled 3.00. The value is 0.647.

Note that since $p < 0.2$, the Poisson yields a better approximation than the normal.

l.c. The probability of O failures out of 5 trials where $p = 0.05$. This probability is given by equation 5.1.

$$b_{n,p}(k) = \binom{n}{k} p^k(1-p)^{n-k}$$

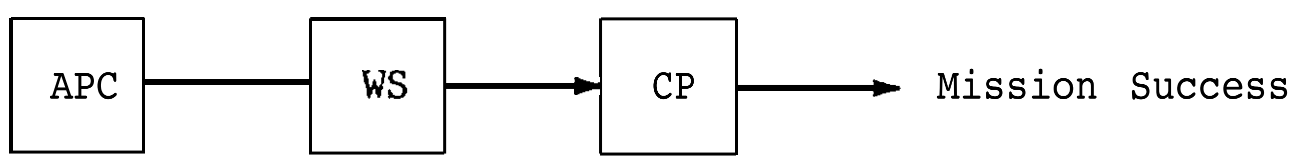
$$\binom{5}{0}(0.05)^0(0.95)^5 = (0.95)^5 = 0.774$$

Note the value of n is much too small to use an approximation. See discussion of this concept on page 5-6.

System	Author-ized	Minimum Required	Max. # of Allowable Failures	System Mission Reliability	Probability of Mission Success Binomial	Normal	“Poisson
APC	36	24	12	0.70	0.7365	0.7324	0.718
WS	20	17	3	0.85	0.6477	0.6255	0.647
CP	5	5	0	0.95	0.774	-	

The term System Mission Reliability is the probability that an individual APC, WS , or CP will successfully complete a mission. The term Probability of Mission Success is the probability that for the individual systems at least the required minimum number of vehicles will successfully complete the mission.

2.



We use a series model to evaluate the probability of mission success. This probability is the product of the probabilities of success for APC's (0.737), WS’S (0.648), and CP’S (0.774). The product is 0.37.

(See Chapter 2’s section entitled “Series and Redundancy Models.”)

3. In part 1 above , we determined that the mission success probabilities for APC’S, WS’S, and CP's, are approximately 0.73, 0.65, and 0.77, respectively. If we assume that the authorized number of units of each type system remains fixed, then we must improve the mission reliability of each type system to achieve a success probability of 0.90 for each type system. Generally, statistics books do not address procedures for solving this type problem but the solutions are straightforward and logical.

a. For APC’S, the probability of 12 or fewer failures must be at least 0.90.

i. Use Appendix B, Table 1. Note that for $n = 36$ and $c = 12$, the values in the body of the table increase as p, the probability of failure decreases. This fact is actually true for any values of n and c. As we move in the body of the table from right to left, we find that the first time the probability exceeds 0.90 is for a p of 0.25. Consequently a p of 0.25 or less will achieve the goal.

ii. The normal approximation provides a method for determining the desired probability, p. The approximate probability of 12 or fewer failures out of 36 trails for **any** p is

$$1 - P(Z \geq (12.5 - 36p)/\sqrt{36p(1-p)}) .$$

The procedure is to set the above quantity equal to 0.90 and solve for p. To accomplish this, note that we can reduce the equation to

$$P(Z \geq (12.5 - 36p)/\sqrt{36p(1-p)}) = 0.10 .$$

Since $P(Z \geq 1.28) = 0.10$, the above equality occurs when

$$(12.5 - 36p)/\sqrt{36p(1-p)} = 1.28 .$$

To solve the equation for p, we multiply both sides by

$$\sqrt{36p(1-p)}$$

and square both sides to obtain the quadratic equation

$$156.25 - 900p + 1296p^2 = 58.98p(1-p), \text{ or}$$

$$1354.98p^2 - 958.98p + 156.25 = 0 .$$

We used the quadratic formula to find p = 0.254.

iii. The Poisson approximation provides a method for determining the desired probability, p. Note that for c = 12 in the Poisson tables, the probabilities increase as the value of m decreases. Searching the body of the table in the column labeled c = 12, we find that the probability exceeds 0.90 for the first time at M = 8.6. Recall that m = np for the Poisson approximation. Now n = 36 and m= 8.6, so p = 8.6/36 = 0.24.

b. For WS's, the probability of 3 or fewer failures must be at least 0.90. The procedures for determining the probability of faiure, p, are identical to those in part a above.

i. Appendix B, Table 1. For c = 3, as p decreases, the value in the table exceeds 0.90 for the first time at p = 0.09.

ii. Normal Approximation. The equation to solve is

$$(3.5 - 20p)/\sqrt{20p(1-p)} = 1.28 ,$$

and the value of p that solves the equation is p = 0.092.

iii. Poisson Approximation. For c = 3, as m decreases, the value in the table exceeds 0.90 for the first time at m = 1.7. Consequently, p = 1.7/20 = 0.085.

c. For CP's, the probability of 0 failures must be at least 0.90. Using a direct computation, we solve the following equation for p:

$$\binom{5}{0}p^0(1-p)^5 = 0.90.$$

Equivalently,

$$(1-p)^5 = 0.90 \text{ or } 1-p = (0.90)^{0.2} .$$

The solution to the above equation is a p of 0.02.

4. If the mission reliability of the different systems cannot be improved, then in order to achieve a success probability of 0.90 for each system, the number of authorized units must be increased- This allows for more units to fail during the mission, while maintaining the required strength to achieve mission success.

a. For APC'S, the probability of at least 24 units performing successfully must be at least 0.90.

i. Appendix B, Table 1.

n = 36	P(I2 or fewer failures) = 0.7365
n = 37	P(13 or fewer failures) = 0.8071
n = 38	P(14 or fewer failures) = 0.8631
n = 39	P(15 or fewer failures) = 0.9056

The requirement is satisfied for an n of 39.

ii. Normal Approximation. Since the number of failures allowed increases by one for every increase in allocation while the number of successes remains constant at 34, we can formulate the solution to this question more easily in terms of successes . The approximate probability of 24 or more successes out of n trials, for any n, when p = 0.7 is

$$P(Z \geq (23.5 - 0.7n)/\sqrt{0.21n}) .$$

We set the above term equal to 0.90 and solve for n. Since $P(Z \geq -1.28) = 0.90$, the value of n required is the one which solves the equation

$$(23.5 - 0.7n)/\sqrt{0.21n} = -1.28 .$$

We perform the same manipulations as in part 3 above to obtain a quadratic equation in n, one of whose solutions is n = 38.8.

iii. Poisson Approximation. Let us use Chart 6 to approximate the solution. The abscissa of the graph is labeled 0/T which is the reciprocal of m = np. Consider the points where the curves cross the 0.90 ordinate line.

c	θ/T	n = (0.36/2′)$^{-1}$	# Successes
12	0.12	27.8 ≅ (28)	16 or more
13	0.108	30.9 ≅ (31)	18 or more
14	0.097	34.4 ≅ (35)	21 or more
15	0.09	37.0 ≅ (38)	23 or more
16	0.083	40.2 ≅ (41)	25 or more

This method shows that 38 units are not enough and that 41 units are too many. Either 39 or 40 units are appropriate.

b. For WS'S, the probability of at least 17 units performing successfully must be at least 0.90.

i. Appendix B, Table 1.

n = 20 P(3 or fewer failures) = 0.6477
n = 21 P(4 or fewer failures) = 0.8025
n = 22 P(5 or fewer failures) = 0.9001

The requirement is satisfied for an n of 22.

ii. Normal Approximation. The equation to solve is

$$(16.5 - 0.85n)/\sqrt{0.1275n} = -1.28 \ .$$

The solution is n = 21.9.

iii. Poisson Approximation. Use Chart 6.

c	θ/T	n = (0.15θ/T)$^{-1}$	# Successes
4	0.42	15.9 (16)	12
5	0.33	20.2 (21)	16
6	0.26	25.6 (26)	20

Clearly, this method indicates that either 22 or 23 should be sufficient.

c. For CP's, the probability of at least 5 units performing satisfactorily must be at least 0.90. The number of units is so small that approximations are not appropriate. We can solve the problem very easily using the formula for computation of binomial probabilities.

$$b_{n,p}(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \frac{n!}{[k!(n-k)!]} p^k (1-p)^{n-k}$$

n	Probability of 5 or More Successes
5	$\binom{5}{0}(0.05)^{0}(0.95)^{5} = 0.774$
6	$\binom{6}{1}(0.05)^{1}(0.95)^{5} + \binom{6}{0}(0.05)^{0}(0.95)^{6} = 0.96$

An n of 6 is sufficient.

CASE STUDY NO. 5-4

Background

A new combat assault vehicle is being proposed. The mean miles between mission failure (MMBMF) is to be 320 miles. The mean miles between durability failures (MMBDF) is to be 5771 miles. Historically, both mission failures and durability failures have occurred at a fairly constant rate.

Determine

1. What is the mission reliability for a 50-mile mission?

2. What is the probability of operating for 4000 miles without experiencing a durability failure?

3. What MMBDF is required to have an 80% probability of operating for 4000 miles without a failure?

Solutions

1. We use the reliability function,

$$R(t) = e^{-t/\theta},$$

where t is mission duration and θ is mission MMBF. Since t = 50 miles and θ = 320 miles, the mission reliability is

$$e^{-50/320} = 0.855 .$$

2. Durability is defined as the probability of completing 4000 miles of operation without suffering a power train durability failure. Again we use the reliability function,

$$R(t) = e^{-t/o},$$

where t is the durability requirement and θ is the power train MMBDF. Since t = 4000 miles and θ = 5771 miles, the mission reliability is

$$e^{-4000/5771} = 0.50 .$$

3. Once again we use the reliability function,

$$R(t) = e^{-t/\theta},$$

but, in this case, t = 4000 miles and R(4000) = 0.80. By solving the above equation for θ, we have

$$\theta = \frac{t}{-\log_e R(t)}$$

Consequently,

$$\theta = \frac{4000}{-\log_e 0.80}$$

$$= 17{,}926 \text{ miles} .$$

Commentary

Usually durability requirements are imposed on specific subsystems such as the power train. When individual subsystems are considered, durability failures are likely to occur with an increasing failure rate due to physical wearout of components. In such a case, the use of the exponential model would not be appropriate. Other distributional models of life length (such as the Weibull distribution) would be appropriate for use in analyzing durability.

CASE STUDY NO. 5-5

Background

A heavy truck uses 10 road wheels. Each tire' s mean miles between failures (MMBF) is 10,000 miles. Assume a series model for the 10 tire system.

Determine

1. What is the failure rate of each tire.

2. What is the probability that a single tire will experience no failure during a 1000 mile mission?

3. Assuming the tires operate independently, what is the probability of no tire failures in a 500 mile mission?

4. What is the MMBF of the entire system of 10 tires.

5. How many spares should be kept so that a 500 mile mission can be completed at least 90% of the time? 99% of the time?

Solutions

1. The failure rate (λ) is the reciprocal of the mean miles between failures (MMBF). Consequently,

$$\lambda = \frac{1}{MMBF} = \frac{1}{10,000} = 0.0001 .$$

2. Using the formula for computing Poisson probabilities (equation 5.3) with the following parameters

Mission Length (T) = 1000 miles
Failure Rate (A) = 0.0001,

we have

$$g_{\lambda,T}(k) \quad \frac{(\lambda T)^k e^{-\lambda T}}{k!}$$

$$P(\text{no failures}) = \frac{(\lambda T)^0 e^{-\lambda T}}{0!} = e^{-\lambda T} = e^{-(0.0001)(1000)}$$

$$= e^{-0.1} = 0.905 .$$

This value could have been obtained using Appendix B, Table 3, using m = AT = 0.1 and c = O. In addition, we could have used Appendix B, Chart 1 using 0/T = 1/AT = 10 and c = O.

3. For a single tire the probability of no failures in a 500 mile mission is

$$g_{\lambda,T}(k) = \frac{(\lambda T)^k e^{-\lambda T}}{k!}$$

$$= \frac{[(0.0001)(500)]^0 e^{-(0.0001)(500)}}{0!}$$

$$= 0.951 \ .$$

Since all tires are in series, have the same failure rate and operate independently, the probability of no failures among all 10 tires is

$$(0.951)^{10} = 0.605 \ .$$

4. In order to determine the MMBF of the system of 10 tires, we use the reliability function

$$R(T) = e^{-T/\theta},$$

where T is mission length and θ is MMBF. Solving this equation for θ, we obtain

$$\theta = \frac{T}{-\log_e R(T)} \ .$$

In problem 3, we determined that R(500) = 0.605 for the 10-tire system. Thus,

$$\theta = \frac{500}{-\log_e 0.605}$$

$$= 1000 \text{ miles.}$$

5. Mission Length (T) = 500 miles.
System Failure Rate (λ) = 0.001.
(Note: MMBF = 1000 from question 4)

Using Appendix B, Table 3 with m = AT = 0.5, we have

P(no failures) = 0.607
P(less than or equal to 1 failures) = 0.910
P(less than or equal to 2 failures) = 0.986
P(less than or equal to 3 failures) = 0.998

For a 90% chance of mission completion, one spare will suffice. For a 99% chance of mission completion, 3 spares are required. However, the improvement in reliability from ().986 (2 spares) to 0.998 (3 spares) does not appear to warrant the extra spare.

CASE STUDY NO. 5-6

Background

A new vehicle is under development which is required to have a MTBF of at least 200 hours (λ no more than 0.005) and a mean time to repair (MTTR) of no more than 40 man-hours. Assume that the specifications (200 hours MTBF and 40 hours MTTR) have been met by contractor A.

Determine

1. What is the probability that a development test of 1000 hours will show at least 10 failures?

2. What is the probability of no failures during the first 400 hours of the test?

3. How many man-hours of repair time are expected during the 1000-hour test?

4. How much calendar time in days--operating and maintenance--based upon an 8-hour day should be programmed so that at least 10 total repairs will be performed by a team of 4 mechanics working on 10 identical systems? We desire that there be at least 10 failures with probability 0.90. We desire to take only a 10% risk that repair time allotted will be insufficient. Assume that each repair takes 40 hours, the mean repair time. Rework the problem assuming each repair is completed within 60 hours.

Solutions

1. The "background to this study indicates that this is a continuous time testing situation. In addition, we are interested in numbers of failures and not in times between failures, so the Poisson model is used for this analysis. Test time (T) is 1000 hours and MTBF is 200 hours (λ = 0.005). The unknown value is the probability of 10 or more failures.

Use Appendix B, Table 3, with AT = (0.005)(100.0) = 5.0. The probability of 10 or more failures = 1 - P(9 or fewer failures).

For AT = m = 5.0, we see in the column labeled c = 9 that the probability of 9 or fewer failures is 0.968. Consequently, the probability of 10 or more failures is 0.032.

2. Test time (T) is 400 hours and MTBF is 200 hours (λ = 0.005). The unknown value is the probability of 0 failures.

Use Appendix B, Table 3, with λT = (0.005)(400) = 2.0.

For λT = m = 2.0 we see in the column labeled c = 0 that the probability of zero failures is 0.135.

3. Test time (T) is 1000 hours and MTBF is 200 hours (λ = 0.005). MTTR is 40 hours (λ = 0.025). Each repair takes 40 hours on the average. There are

1000(0.005) or 5 failures expected. The expected repair time is 5(40) or 200 man-hours.

4. Let To be operating time needed and T_r be repair time needed. We desire that To be large enough that the probability of seeing 10 or more failures be at least 0.9. However we must allow repair time for that number of failures (n_{10}) which represents all but the upper 10% of the distribution of numbers of failures, i.e., the probability that more than n_{10} failures occurring is 0.10.

a. Determine T_o: Probability of at least 10 failures must be at least 0.90.

Note: The probability of 10 or more failures = 1 - the probability of 9 or fewer failures.

The probability of nine or fewer failures must be no more than 0.10. Under the column labeled c = 9.0 of Table 3, we find by interpolation that a 0.10 probability of 9 or fewer failures is achieved when m = 14.2.

Since m = AT and m = 14.2

T = To and

A= 0.005

We solve the following equation for T_0:

To = m/A = 14.2/0.005 = 2,840 hours.

b. Determine n_{10}: Allow for 19 failures since the probability of 20 or more failures is just under 0.10.

c. Determine T_r: (19)(40) = 760 man-hours

T_r = 760/4 = 190 clock hours.

d. Determine number of days needed:

Operating hours: 2,840
Ten systems operating hours: 284 for each system
Maintenance hours: 190

Total hours: 474
Days Required: 59.25

e. If each repair takes 60 man-hours. then 1140 man-hours are required. This corresponds to 285 clock hours. Consequently, the total number of hours is 569 which represents 71.1 working days.

Commentary

For number 4, we have allowed 40 man-hours for each repair. If the average repair time is indeed 40 man-hours, then 760 total man-hours should be a reasonably close estimate. Computation of the risk involved with this approach is complicated and beyond the scope of this work. One procedure to adopt is to increase the allotted time per repair as we have noted in 4e.

CHAPTER 6

STATISTICAL CONCEPTS

INTRODUCTION

As we mentioned in Chapter 5, our assumptions about a given testing situation lead us to the choice of a mathematical model to characterize the reliability of a system. However, we cannot determine the actual reliability of the system using the model until the parameters of the model, p for the binomial model and A (or θ) for the Poisson or exponential model, have been specified. The values of the parameters are never known with absolute certainty. As a consequence, some form of sampling or testing is required to obtain estimates for these parameters. The quality of the estimates is, of course, directly related to the quality and size of the sample.

POINT ESTIMATES

point estimates represent a single "best guess" about model parameters, based on the sample data. A distinguishing symbol commonly is used to designate the estimate of a parameter. Most commonly, a caret or "hat" is used to designate point estimates (e.g., $\hat{\theta}$, $\hat{R}(x)$, $\hat{\lambda}$). Quite often, and for our purposes, the caret further indicates that the estimator is a maximum likelihood estimator; that is, it is the most likely value of the parameter of the model which is presumed to have generated the actual data.

There are criteria other than maximum likelihood used for a single "best guess." One other is unbiasedness. For an estimator to be unbiased, we mean that, in the long run, it will have no tendency toward estimating either too high or too low. The point estimates which we propose for p in the binomial model and for A in the Poisson and exponential models are both maximum likelihood and unbiased.

CONFIDENCE STATEMENTS

Point estimates represent a single "best guess" about parameters, based on a single sample. The actual computed values could greatly overestimate or underestimate the true reliability parameters, particularly if they are based on a small amount of data. As an example, suppose that 20 rounds of ammunition were tested and 18 fired successfully.

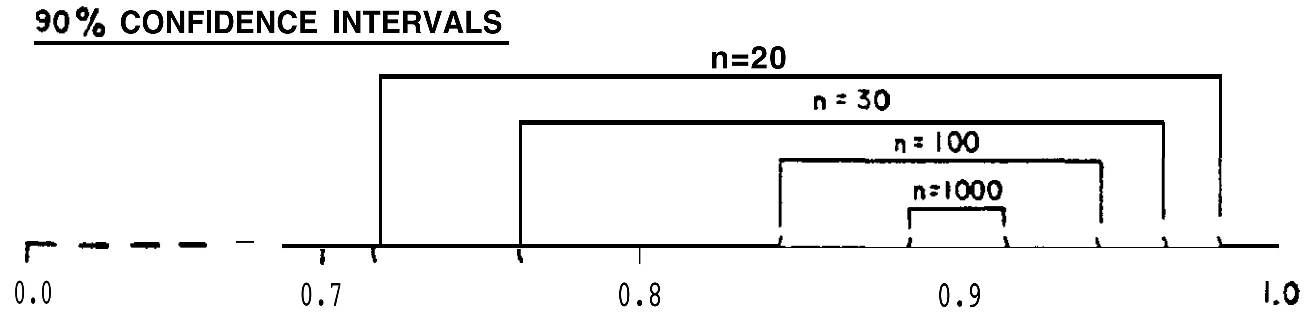
The maximum likelihood and unbiased estimate of reliability is $\hat{R} = 18/20 = 0.9$. In other words, the system most likely to have generated 18 successes is one whose reliability is 0.9. Note that 0.9 is the percentage of successes actually observed in the sample. However, a system whose true reliability is somewhat less than or somewhat more than 0.9 could reasonably have generated this particular data set.

We use confidence limits to address how high or low the value of a parameter could reasonably be. A 90% confidence interval for reliability is: $0.717 < R < 0.982$. In other words, if being reasonable signifies being 90% confident of being right, then it is unreasonable to consider that a system whose reliability is actually less than 0.717 or one whose reliability is actually more than 0.982 generated the 18 successful rounds . When we desire to be more confident, say 95% confident, that our interval contains the true system reliability, we widen our interval, i.e. , we expand the group of systems considered to have reasonably generated the data. A 95% confidence interval for the reliability of our example system is: $0.683 < R < 0.988$. Since we are now allowing for the possibility that the system reliability could be a little lower than 0.717 -- namely, as low as 0.683 -- or a little higher than 0.982 -- namely, as high as 0.988 -- we can now afford to be more confident that our interval indeed contains the *true* value. For a fixed amount of testing, we can only increase our confidence by widening the interval of reasonable values.

Suppose that we desire to reduce the size of the interval while maintaining the same level of confidence or to increase the level of confidence while maintaining approximately the same size interval. Either of these objectives is accomplished through increased testing, i.e., taking a larger sample. If the system test had resulted in 27 successful firings out of 30 attempts (vice 18 out of 20) , the point estimate is still 0.9. However, the 90% confidence interval for system reliability is: $0.761 < R < 0.972$. The length of this interval represents a 20% reduction in the length of the 90% confidence interval resulting from our test of 20 units. The 95% confidence interval for system reliability is: $0.734 < R < 0.979$. This interval represents an 8% reduction in size, but our confidence has increased to 95%. Figure 6-1 graphically portrays the effect on interval length induced by changing confidence levels or increasing sample size.

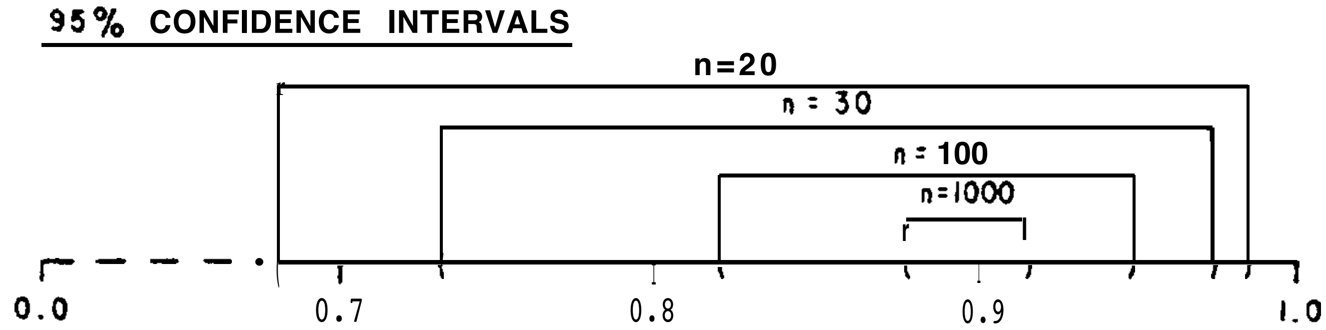
FIGURE 6-1 CONFIDENCE INTERVALS

90% CONFIDENCE INTERVALS



IF n = 20 THEN (0.717, 0.982)
n = 30 (0.761, 0.972)
n = 100 (0.842, 0.943)
n = 1000 (0.886, 0.914)

95% CONFIDENCE INTERVALS

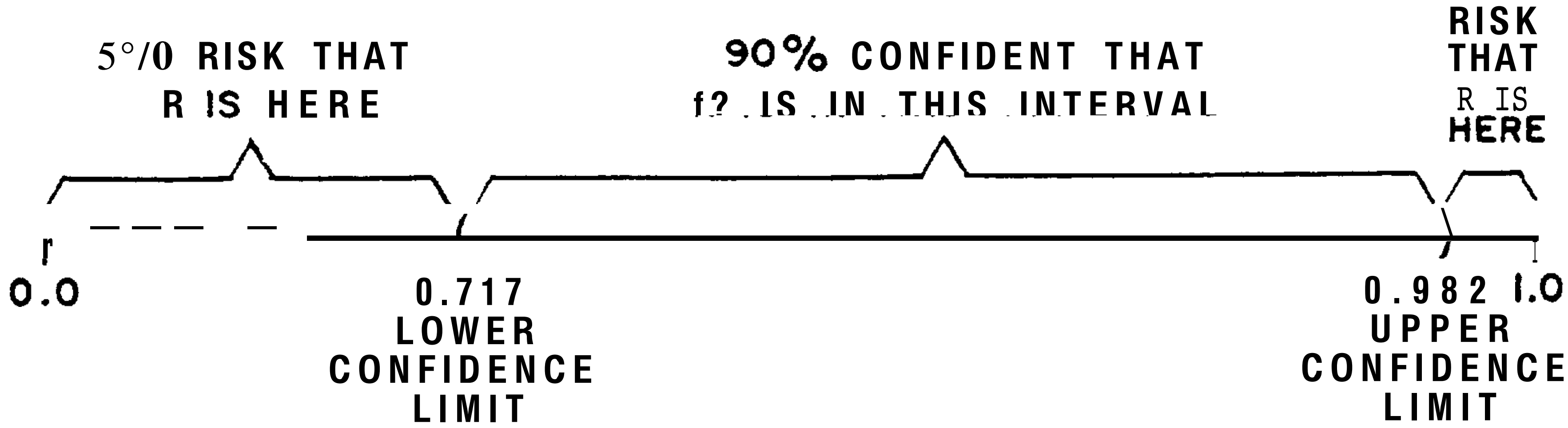


IF n = 20 THEN (0. 683, 0.988)
n = 30 (0. 734, 0.979)
n = 100 (0. .922, 0.951)
n = 1000 (0.882, 0.918)

A cautious, conservative person who buys safe investments, wears a belt and suspenders, and qualifies his statements carefully is operating on a high-confidence level. He is certain he won't be wrong very often. If he is wrong once in 100 times, he is operating on a 99% confidence level. A less conservative person who takes more chances will be wrong more often, and hence he operates on a lower confidence level. If he is wrong once in 20 times, he is operating on a 95% confidence level. The confidence level, therefore, merely specifies the percentage of the statements that a person expects to be correct. If the experimenter selects a confidence level that is too high, the test program will be prohibitively expensive before any very precise conclusions are reached. If the confidence level is too low, precise conclusions will be reached easily, but these conclusions will be wrong too frequently, and, in turn, too expensive if a large quantity of the item is made on the basis of erroneous conclusions. There is no ready answer to this dilemma.

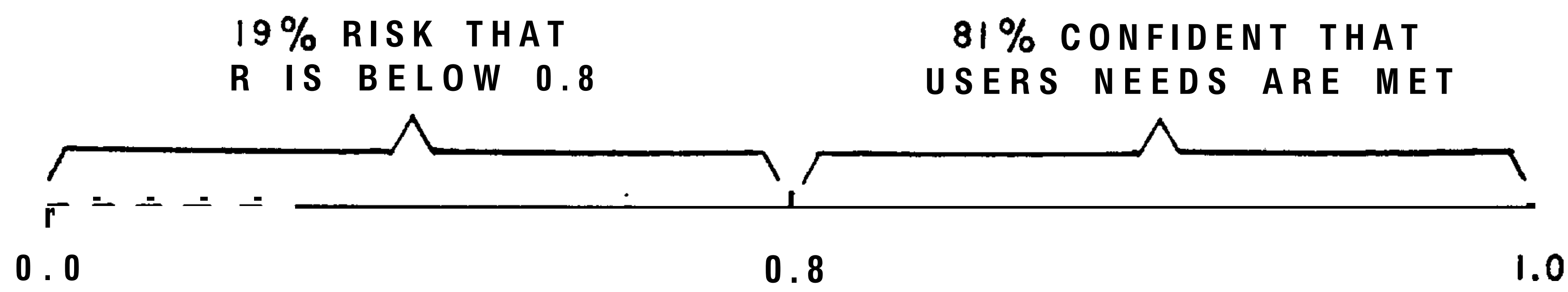
We can interpret confidence statements using the concept of risk. With a 90% confidence statement, there is a 10% risk; with a 99% confidence statement, there is a 1% risk. Confidence intervals generally are constructed so that half of the total risk is associated with each limit or extreme of the interval. Using this approach with a 90% interval for reliability, there is a 5% risk that the true reliability is below the lower limit and also a 5% risk that the true reliability is above the upper limit. We can therefore state for the example system with 18 of 20 successes that we are 95% confident that: $R > 0.717$. This is a lower confidence limit statement. We are also 95% confident that: $R < 0.982$. This is an upper confidence limit statement. See Figure 6-2.

FIGURE 6-2



The classical textbook approach to confidence intervals has been to specify the desired confidence level and determine the limit associated with this confidence level. This approach creates a twofold problem. First, the desired confidence level has to be determined. Second, the limits that are generated are generally not, in themselves, values of direct interest. A very practical modification is to determine the level of confidence associated with a predetermined limit value. For example, the minimum value of a reliability measure that is acceptable to the user is a logical lower limit. The confidence in this value can then be interpreted as the assurance that the user's needs are met. See Figure 6-3.

FIGURE 6-3 CONFIDENCE INTERVALS- ACCEPTABLE LOWER LIMITS



The confidence level for a lower limit of 0.8 is 81%. A system reliability of 0.8 is the user's minimum acceptable value (MAV).

HYPOTHESIS TESTING

While confidence limits are generally used to define the uncertainty of a parameter value, an alternative approach is hypothesis testing. Both approaches essentially give the same information. Hypothesis testing can be used to distinguish between two values or two sets of values for the proportion of failures in a binomial experiment, or for the failure rate in a Poisson/exponential experiment. Let us examine hypothesis testing using a binomial example. Typically, for a binomial experiment, it is hypothesized that the probability of failure, p, is a specified value. While there is seldom any belief that p is actually equal to that value, there are values of p which would be considered unacceptable in a development program. These unacceptable values are specified in an alternative hypothesis. Consider the following examples.

(1) One-Sided Tests

H_0: $p = 0.3$ (Null Hypothesis)

H_1: $p > 0.3$ (Alternative Hypothesis)

In Case (1), the evaluator hopes that p is no more than 0.3. He considers a p of more than 0.3 to be unacceptable. This is a classical one-sided test. Another type of one-sided test has the alternative hypothesis $p < 0.3$.

(2) Two-Sided Tests

H_0: $p = 0.3$

H_1: $p \neq 0.3$

In Case (2), the evaluator hopes that p is approximately 0.3. Values of p much larger than or much smaller than 0.3 are unacceptable. This is a classical two-sided test.

(3) Simple vs. Simple Tests

H_0: p = 0.3

H_1: p = 0.5

In Case 3, the evaluator hopes that p is no more than 0.3. He considers a p of more than 0.5 to be unacceptable. The region between 0.3 and 0.5 is an indifference region in that it represents acceptable but not hoped for values. This is actually a classical simple versus simple test. This type of test is treated extensively and exclusively in Chapter 8.

In order to conduct a statistical test of hypothesis, the following steps are employed:

1. The hypothesis, null and alternative, are specified. For our purposes, the null hypothesis is the contractually specified value (SV) and the alternative hypothesis is the minimum acceptable value (MAV).

2. A sample size, n, is determined. This value must be large enough to allow us to distinguish between the SV and MAV. Chapter 8 is devoted to procedures for determining a sufficiently large value of n.

3. An accept/reject criterion is established. For our purposes, this criterion is established by specifying a value c, which is the maximum number of failures permitted before a system will be rejected.

4. The sample is taken and the hypothesis is chosen based upon the accept/reject criterion. If c or fewer failures occur, we accept the system. If more than c failures occur, we reject the system.

PRODUCER'S AND CONSUMER'S RISKS

There are two possible errors in making a hypothesis-testing decision. We can choose the alternative hypothesis, thereby rejecting the null hypothesis, when, in fact, the null hypothesis is true. The chance or probability of this occurring is called the producer's risk, α. On the other hand, we can choose the null hypothesis, i.e., accept it as reasonable, when in fact the alternative hypothesis is true. The chance or probability of this occurring is termed the consumer's risk, β. See Chapter 8 for an additional discussion of this topic.

Consider the following: A system is under development. It is desired that it have a 300-hour MTBF. However, an MTBF of less than 150 hours is unacceptable, i.e., the MAV is 150 hours. How would we set up a hypothesis test to determine the acceptability of this new system? Our null hypothesis (desired value) is that the MTBF is 300 hours. Our alternative hypothesis (values of interest) is that the MTBF has a value which is less than 150 hours. To decide which hypothesis we will choose, we determine a test exposure and a decision criterion. The α risk (producer's risk) is the probability that the decision criterion will lead to a rejection decision when in fact the system meets the specification of 300 hours MTBF. The β risk (consumer's risk) is the probability that the decision criterion will lead to an acceptance decision when in fact the system falls short of the 150 hours MTBF.

For a given test, the decision criteria can be altered to change the α and β risks. Unfortunately, a decision criterion which decreases one automatically increases the other. The only way to decrease both risks is to increase the test exposure, that is, the number of test hours. We address this area below in Chapter 8, "Reliability Test Planning."

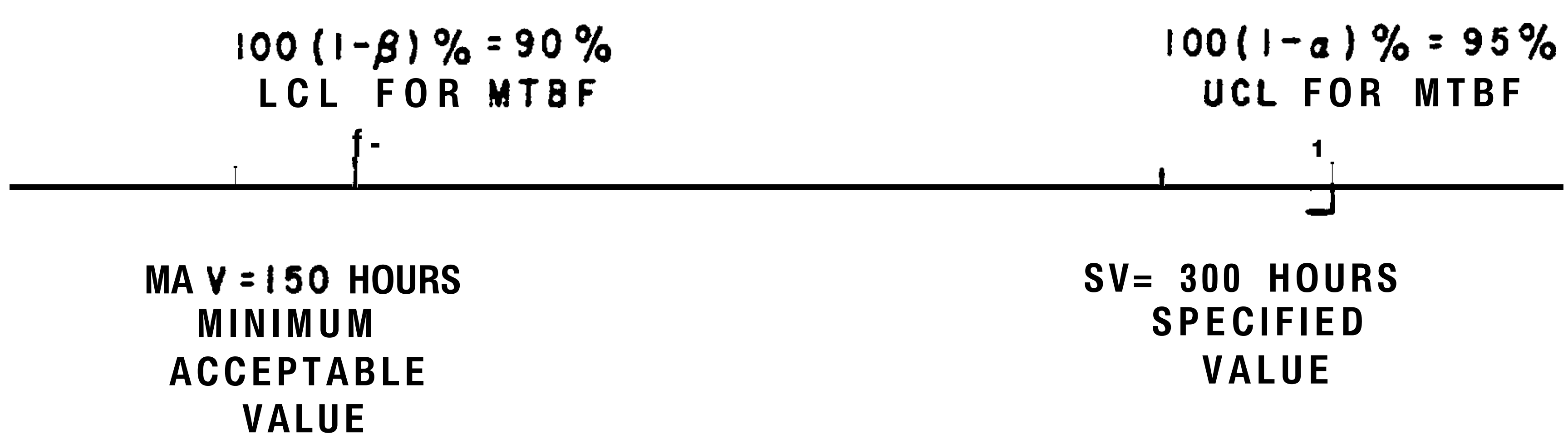
INTERFACE BETWEEN HYPOTHESIS TESTING AND CONFIDENCE STATEMENTS

In both test planning and data analysis situations, either hypothesis testing or confidence statements provide an avenue of approach. The interface between the two approaches can be best understood through the following example.

Suppose α is the desired producer's risk ($\alpha = 0.05$) for the specified MTBF of 300 hours. Suppose further that β is the desired consumer's risk ($\beta = 0.1$) for the minimum acceptable MTBF of 150 hours. The hypothesis testing approach determines a required sample size and a specified accept/reject criterion. We show how the same information can be obtained through confidence statements in the following two cases. The abbreviations LCL and UCL represent Lower Confidence Limit and Upper Confidence Limit, respectively.

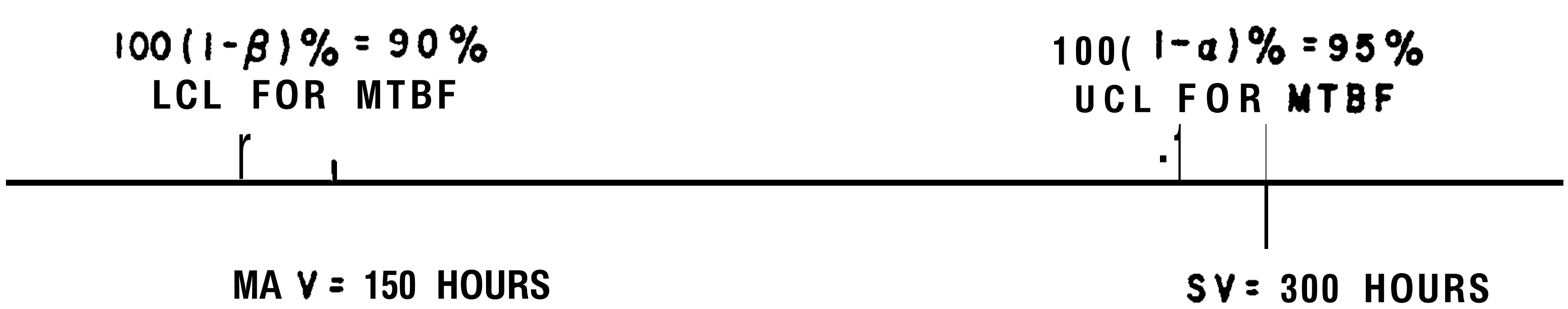
Note that the distance between the upper and lower limits is the same as the distance between the SV and the MAV. When this is the case we shall always be able to make a clear-cut decision and the risks associated with the decision will be as specified at the outset of testing.

FIGURE 6-4 ACCEPTANCE DECISION



Note that in Figure 6-4 the $100(1-\beta)\% = 90\%$ lower limit exceeds the MAV of 150 hours. In addition, the $100(1-\alpha)\% = 95\%$ upper limit exceeds the specified value of 300 hours. The consumer is 90% confident that the 150-hour MAV has been met or exceeded and the producer has demonstrated that the system could reasonably have a 300-hour MTBF. Consequently, we would make the decision to accept the system.

FIGURE 6-5 REJECTION DECISION



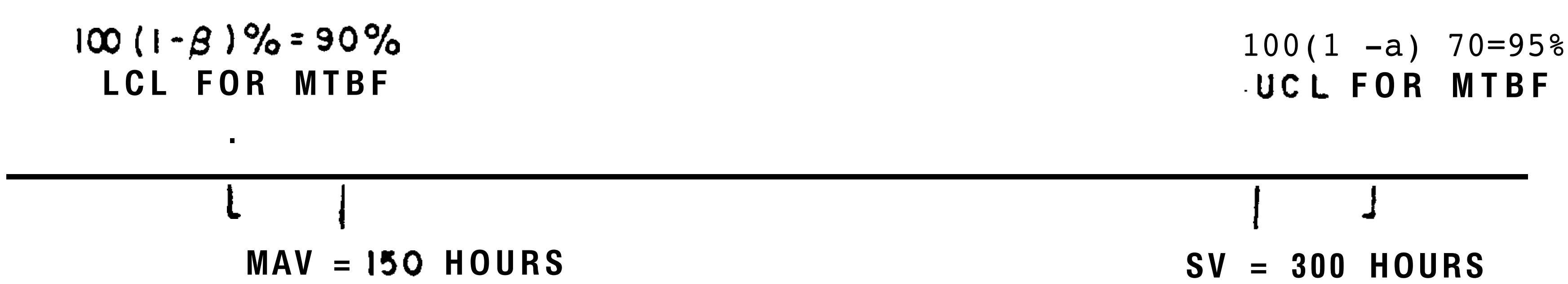
Note that in Figure 6-5 the $100(1-\beta)\% = 90\%$ lower limit falls below the MAV of 150 hours. In addition, the 100(1-cY)% = 95% upper limit falls below the SV of 300 hours. Therefore, the true MTBF could reasonably be below 150 hours and the producer has not demonstrated that an MTBF of 300 hours is reasonable. Consequently, we make the decision to reject the system.

TEST EXPOSURE

Perhaps one of the most important subjects to be considered in the evaluation of RAM characteristics is the subject of test exposure. The term "test exposure" refers to the amount (quantity and quality) of testing performed on a system or systems in an effort to evaluate performance factors. In Chapter 10, we discuss the qualitative aspects of test exposure which should be considered by the test designer. The primary purpose of Chapter 8, "Reliability Test Planning," is to document procedures which ensure that the quantitative aspects of test planning are adequate.

Recall the comment we made in the previous section to the effect that the difference in the distance between the upper and lower confidence <limits was equal to the difference in the distance between the SV and the NAV. When this condition is achieved, we have obtained the most efficient test exposure for the stated requirements and risks. Examples of situations where test exposure is inadequate or excessive are given below. See Case Study 6-2 for an illustration of the evaluation of a proposed test exposure.

FIGURE 6-6 INADEQUATE TEST DURATION



Note that in Figure 6-6 the $100(1-\beta)\% = 90\%$ lower limit falls below the MAV of 150 hours. The 100(1-cY)% = 95% upper limit exceeds the SV of 300 hours. The true MTBF could reasonably be below 150 hours or above 300 hours. Test exposure is insufficient to discriminate between the MAV of 150 hours and the SV of 300 hours with the required risk levels of 10% and 5%. If we reject the system, the producer can legitimately claim that an MTBF of 300 hours is reasonable for his system. On the other hand, if we accept the system, we❑ay be fielding an inadequate system.

Note that in Figure 6-7 the $100(1-\beta)\% = 90\%$ lower limit exceeds the MAV of 150 hours. The 100(1-cY)% = 95% upper limit falls below the SV of 300 hours. The consumer has 90% confidence that the 150-hour MAV has been met or exceeded. However, the producer has not demonstrated the specified 300-hour MTBF. The test exposure is more than required to obtain the risks of 10% and 5% for the stated values of MAV and SV. Since the MAV has been met or exceeded, we will probably accept the system. We may have paid a premium to obtain information that allowed us to construct a confidence interval more narrow than required.

FIGURE 6-7 EXCESSIVE TEST DURATION

CASE STUDY NO. 6-1

Background

A contract for a new electronic system specifies an MTBF of 1000 hours. The minimum acceptable value is 500 hours MTBF. A design qualification test is to be conducted prior to production. The test risks are to be 20% for consumer and 10% for producer.

Determine

Describe the events which lead to acceptance or rejection of the system-

Solution

In accordance with procedures defined in Chapter 7, "Reliability Data Analysis," the appropriate hypothesis test is set up, the sample is taken, and the data are analyzed.

The Positive Chain of Events

1. The contractor has met (or exceeded) an MTBF of 1000 hours.

2. There is (at least) a 0.90 probability of "passing" the test.

3. "Passing" the test will give the user (at least) 80% confidence that the MAV of 500 hours MTBF has been exceeded.

4. The user is assured that his needs have been met.

The Negative Chain of Events

1. The contractor has met an MTBF of 500 hours (or less).

2. There is (at least) a 0.80 probability of "failing" the test.

3. "Failing" the test gives the procuring activity (at least) 90% confidence that the contractually obligated SV of 1000 hours MTBF has not been met.

4. The procuring activity is assured that the contractual obligations have not been met.

CASE STUDY NO. 6-2

Background

The specified MTBF of a targeting system is 500 hours and the minimum acceptable MTBF is 400 hours. The contractor has proposed a development test consisting of 6000 hours on the initial prototype system and 2000 hours on a second prototype system which will contain some minor engineering advances.

The proposed test plan of 8000 hours can distinguish between the SV of 600 hours and the MAV of 400 hours for consumer's and producer's risks of slightly over 20%. If the producer is willing to accept a 30% producer's risk, the proposed plan will yield a 12% consumer's risk.

Determine

Comment on the adequacy of the proposed test.

Solution

These risks seem to be larger than should be considered for an important system. The test exposure seems to be inadequate for the following reasons:

- Test time is not of sufficient length.
- Prototypes are not identical. Test time on the second prototype may not be long enough to determine if the design improvements increase reliability.
- Only two systems on test may be insufficient. Ideally, more systems should be used for shorter periods of time.

A test plan having four systems accumulating about 4000 hours each will yield producer and consumer risks of just over 10%. A further benefit is that using four systems and operating them for a period of time about 10 times the minimum MTBF should paint a pretty clear picture of the system capability throughout a significant part of its expected age.

Note : Chapter 8 will present the analytical tools required to evaluate the above test plan. Our objective here is to qualitatively review the various aspects of a statistically relevant test program.

CHAPTER 7

RELIABILITY DATA ANALYSIS

INTRODUCTION

It is important to understand that for any realistic situation the true reliability characteristics of a system, or fleet of systems, are never known with complete certainty. This is true, of course, because we have not, in fact, actually tested every system in the total population and, practically speaking, never could. To compensate for this lack of total information, some form of sampling is used to obtain information about the reliability characteristics inherent in a system and to quantify the level of uncertainty about them. Of course, uncertainty continues to exist, and, as a consequence, the reliability parameters can only be estimated. This chapter presents procedures which can be used to determine estimates for the various reliability parameters and to quantify the uncertainty inherent in these estimates.

These procedures support the analysis of data gathered in previously conducted tests. Planning these tests to assure that adequate sample sizes are obtained is the topic of Chapter 8. The objective of the data analysis effort is to determine "best estimates" of system performance parameters, such as reliability, and to estimate the uncertainty associated with these "best estimate" values.

As in previous chapters, the case studies illustrate the application and manipulation of the mathematical concepts presented in the chapter text. Note that in the typical Chapter 7 case study, you are provided the results of a hypothetical test program and requested to develop a best estimate and confidence interval for a reliability parameter.

TYPES OF RELIABILITY TESTS

Fixed Configuration and Growth Tests

There are basically two types of reliability tests. One is a test of fixed configuration. The other is the growth, or developmental, test, which centers on reliability improvement seen as the configuration changes during the test. There is not, however, a clean line between these two types. For the truly fixed configuration test of continuously operated systems, any changes in reliability are due to the inherent characteristics of the hardware and how it is maintained. The analysis is done as a function of system age. If there are design changes, they have to be considered on a separate basis, perhaps by a data set for each configuration. See Chapter 10 for more details on this procedure.

For the growth type of test, the statistical models currently available assume that all changes in reliability are attributable to the design changes. In other words, they assume that the inherent reliability of the hardware is constant. The basic analysis for the growth type of test is done as a function of test exposure, rather than age, since it is test exposure that provides information for design improvements. The effects of system age can be dealt with separately, primarily by considering the failure modes that are observed. Chapter 9 summarizes the topic of reliability growth and illustrates the associated analysis techniques.

Discrete and Continuous Tests

The most elementary consideration in beginning a data analysis is to determine whether test time is measured continuously or discretely. Usually, this distinction is quite obvious. An example of a test which can be analyzed either way is the following. Suppose that a system has a durability requirement of 5000 hours and ten systems are available for testing. Each system is tested until it either experiences a durability failure or successfully completes the 5000 hour test period. We can let each system be a test unit and count as a failure any system which fails before 5000 hours. This is a discrete time approach. Alternatively, we could let hours be our test units, with the total operating hours of the 10 -systems as the test exposure. This is a continuous time approach. Another example is the firing of an automatic weapon, where many rounds are fired. This is a one-shot, discrete time test if we are analyzing the ammunition, but could be considered a continuous time test if we are analyzing the gun or any of its components. Generally, when either appreach is appropriate, more information is obtained from the continuous time approach.

DISCRETE TIME TESTING

Suppose that the systems under test are single-shot systems. Each test unit results in a distinguishable success or failure. As discussed in Chapter 5, the binomial model will be used to represent or model system reliability when discrete time or success/fail operations are of interest. It is assumed throughout this discussion on discrete time testing that the conditions of a binomial model are reasonably satisfied. (See Chapter 5.) We present data analysis for success/fail (discrete) tests in the form of point estimates, confidence intervals, and tests of hypotheses.

Binomial Model: Point Estimate of Failure Probability

Once the number of trials has been specified (see Chapter 8), all the information contained in a binomial experiment rests in the number of failures that occur. We use this information to make an assessment or an estimate of the true probability of failure, p. Thus, our best estimate of the value of p is the ratio of the number of failures to the number of trials . This ratio is called the sample proportion of failures and is designated by the symbol $\hat{p}$, called p-hat. We use this sample proportion of failures, $\hat{p}$, to construct confidence intervals for p and in testing hypotheses about p. By definition, then

$$\hat{p} = \frac{\text{number of failures}}{\text{number of trials}} = \text{sample proportion of failures}$$

$$\hat{p} = \text{best estimate for } p$$

$$p = \text{true proportion of failures}$$

Note that true system reliability is the probability of successful operation, therefore

$R = 1 - p$, where R is true system reliability, and

$\hat{R} = 1 - \hat{p}$ = best estimate of system reliability.

It is important that the test designer and/or evaluator understand that a point estimate for p represents a small portion of the information contained in the data generated by a binomial experiment. Other useful information includes upper and lower confidence limits for the unknown parameter, p.

Binomial Model: Confidence Limits for Failure Probability

Confidence limits and their interpretation should play a vital role in designing and evaluating a binomial experiment. Not only does the actual interval relay a significant amount of information about the data, but also the method of interval construction can aid the test designer in determining adequate test exposure to meet his needs. An extensive discussion on the interpretation of confidence intervals is given in Chapter 6.

Suppose that we observe "s" failures out of "n" trials in a binomial experiment. This translates to a sample proportion of failures equal to s/n and a sample proportion of successes equal to (n-s) /n. Tables of exact confidence limits for the true proportion of failures for values of n less than or equal to 30 are given in Appendix B, Table 4. As an example, suppose that n = 25 trials and s = 4 failures. A 90% upper confidence limit for p is 0.294. We obtain this value using Appendix B, Table 4 with n = 25 in the column labeled 90% upper limit and the row labeled s = 4. For the same data, a 98% confidence interval is

$$0.034 \leq p \leq 0.398.$$

In this case, the values are found in the columns labeled 98% interval and the row labeled s = 4. More examples using Table 4 are given in Case Study 7-3.

Binomial Model: Confidence Levels for Pre-Established Reliability Limits

If, after conducting a test in which we observed s failures (c = n-s successes) out of n trials, we wish to determine how confident we are that a pre-established level of reliability (such as the MAV) has been met or exceeded, we may use equation 7.1 below.

Let R_L designate the desired pre-established level of reliability. To find the confidence that R_L has been met or exceeded, we evaluate the expression:

$$B_{n,R_L}(c-1) = \sum_{k=0}^{c-1} \binom{n}{k} R_L^{\,k}(1-R_L)^{n-k} \qquad (7.1)$$

If we denote the value of this expression as $1 - \alpha$, then we are $100(1-\alpha)\%$ confident that $R \geq R_L$.

If, on the other hand, we wish to determine how confident we are that a pre-established level of reliability (such as the SV) has not been attained, we may use equation 7.2.

Let R_U designate the desired pre-established level of reliability. To find the confidence that R_U has not been attained, we evaluate the expression:

$$B_{n,R_U}(c) = \sum_{k=0}^{c} \binom{n}{k} R_U{}^k (1-R_U)^{n-k} \tag{7.2}$$

If we denote the value of this expression as α, then we are $100(1 - \alpha)\%$ confident that $R \leq R_U$.

See Case Study 7-1 for an example of this technique.

The Greek letter α is used numerous times throughout this chapter to represent a generalized value or designation of "RISK." In this chapter, α is not necessarily to be interpreted as producer' s risk as in Chapters 6 and 8.

Approximate Binomial Confidence Limits (Normal Approximation)

If the number of failures and the number of successes both are greater than or equal to 5, we can obtain approximate confidence limits using the normal distribution. The approximate 100(1-cY)% lower limit for p, the true proportion of failures, is

$$p \geq p_L$$

$$p \geq \hat{p} - z_\alpha \sqrt{\hat{p}(1-\hat{p})/n} \, , \tag{7.3}$$

where $\hat{p} = s/n$. The approximate 100(1-u)% upper confidence limit for p is

$$p \leq p_U$$

$$p \leq \hat{p} + z_\alpha \sqrt{\hat{p}(1-\hat{p})/n} \, . \tag{7.4}$$

The two-sided 100(1-u)% confidence limits for p are

$$p_L \leq p \leq p_U$$

$$\hat{p} - z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} \, . \tag{7.5}$$

Values for z_α and $z_{\alpha/2}$ are obtained from Appendix B, Table 2.

As an example, suppose that 5 failures in 30 trials occurred during a test. An approximate 90% ((Y = 0.10) upper limit for the true proportion of failures is

$$\hat{p} + z_{0.10}\sqrt{\hat{p}(1-\hat{p})/n} \ .$$

Substituting n = 30, and p = 5/30 = 0.166, we obtain

$$0.166 + 2_{0.10}\sqrt{(0.166)(0.834)/30} \ .$$

'he value 'f '$_{0.10}$ is determined using Appendix B, Table 2. Under the column labeled $P(Z \geq z_\alpha)$ we search the values until we find the number closest to 0.10, the value of α. The number in the column labeled z_α is the desired value. In this case, for $\alpha = 0.10$, $z_\alpha = 1.28$. The upper limit is then

$$0.166 + 1.28\sqrt{(0.166)(0.834)/30} \ ,$$

which reduces to 0.253. We are thus 90% confident that the true proportion of failures is 0.253 or smaller.

See Case Study 7-3 for construction of confidence limits using normal approximation.

<u>Approximate Binomial Confidence Limits (Poisson/Exponential Approximation)</u>

When the sample proportion of failures is small, and the number of trials is reasonably large--at least 30--we can obtain approximate confidence limits using techniques described in the section on Exponential Model: Confidence Intervals and Limits for MTBF. This is an especially useful technique for situations involving very few failures in fairly large samples. We use the procedure for <u>failure terminated testing</u> with the identifications: T = n (the number of trials) and r = s (the number of failures). We obtain approximate confidence limits for p, the probability of failure, by constructing confidence limits for 6, the system MTBF. Because p and A are failure-oriented parameters and θ is a success-oriented parameter (remember that by definition $\theta = 1/A$), an approximate confidence limit for p is the reciprocal of the confidence limit for θ. An important consequence of the reciprocity mentioned above is that an upper confidence limit for θ yields a lower confidence limit for p and vice versa.

Consider the situation described in Chapter 6, where 3 failures out of 30 trials of a binomial experiment were observed. To construct an approximate 90% confidence interval for the true proportion of failures, we let T be 30 and r be 3. The 95% confidence interval for θ is

$$\frac{2T}{\chi^2_{\alpha/2,2r}} < \theta \leq \frac{2T}{\chi^2_{1-\alpha/2,2r}}$$

since T = n= 30, r = s = 3, and $\alpha = 0.05$, we have

$$\frac{2(30)}{\chi^2_{0.025,6}} \leq \theta \leq \frac{2(30)}{\chi^2_{0.975,6}}$$

Values for $\chi^2_{0.025,6}$ and $\chi^2_{0.975,6}$ are obtained from Appendix B, Table 5. The explanation of how to extract these values is presented below in the section entitled "Exponential Model: Confidence Intervals and Limits for MTBF." The values are $\chi^2_{0.025,6} = 14.46$ and $\chi^2_{0.975,6} = 1.24$. Thus the interval for θ is

$$\frac{2(30)}{14.46} \leq \theta \leq \frac{2(30)}{1.24} ,$$

which, upon simplification, becomes

$$4.15 \leq \theta \leq 48.39 .$$

Taking the reciprocals of the limits for θ, we have that the approximate 95% confidence interval for the true proportion of failures is

$$0.021 \leq p \leq 0.241 .$$

Since reliability is 1 - p, the approximate 95% confidence interval for system reliability is

$$0.759 \leq R \leq 0.979 .$$

This statement can also be interpreted as follows: We are 95% confident that the true system reliability is between 0.759 and 0.979. This interval is based on our test results where 3 out of 30 trials ended in failure.

See Case Study 7-2 for another example of this procedure.

Point Estimates and Confidence Limits for the Difference/Ratio of Proportions

Suppose that tests have been conducted on two different types of systems resulting in sample proportions of failures of $\hat{p}_1$ and $\hat{p}_2$ with sample sizes of n_1 and n_2, respectively. The point estimates for the difference $(p_1 - p_2)$ and ratio (p_1/p_2) of proportions are the difference and ratio of the sample proportions, i.e., $\hat{p}_1 - \hat{p}_2$ and $\hat{p}_1/\hat{p}_2$, respectively. We present the procedures for determining confidence limits for the difference and for the ratio of the two population proportions (p_1 and p_2) using the normal distribution. The approximate $100(1 - \alpha)\%$ lower confidence limit for the true difference in proportions is

$$\begin{aligned} p_1 - p_2 &\geq (p_1 - p_2)_L \\ &\geq \hat{p}_1 - \hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2} . \end{aligned}$$

The approximate 100 (1 - α)% upper confidence limit for the true difference in proportions is

$$p_1 - p_2 \le (p_1 - p_2)_U$$

$$\le \hat{p}_1 - \hat{p}_2 + z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2} \; .$$

The approximate 100(1 - α)% confidence interval for the true difference in proportions is

$$(p_1 - p_2)_L \le p_1 - p_2 \le (p_1 - p_2)_U$$

$$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2} \le p_1 - p_2$$

$$\le \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2} \; .$$

With high reliability systems, it is sometimes more informative for comparing two systems to look at the ratio of proportions of failures. As an example, suppose that the true proportions of failures for two systems are 0.01 and 0.001. We can say that one system is ten times better than the other even though the difference is a mere 0.009. An approximate 100(1 - α)% lower confidence limit for the true ratio of proportions is

$$p_1/p_2 \ge (p_1/p_2)_L$$

$$p_1/p_2 \ge \hat{p}_1/\hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} \; .$$

The approximate 100 (1 - α)% upper confidence limit for the true ratio of proportions is

$$p_1/p_2 \le (p_1/p_2)_U$$

$$p_1/p_2 \le \hat{p}_1/\hat{p}_2 + z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} \; .$$

The approximate 100 (1 - α)% confidence interval for the true ratio of proportions is

$$(p_1/p_2)_L \le p_1/p_2 \le (p_1/p_2)_U$$

$$\hat{p}_1/\hat{p}_2 - z_{\alpha/2}\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} \le p_1/p_2$$

$$\le \hat{p}_1/\hat{p}_2 + z_{\alpha/2}\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} \; .$$

In Case Study 7-4, we construct confidence limits for the difference and ratio of population proportions.

CONTINUOUS TIME TESTING

Suppose the systems under test operate as a function of hours, kilometers, or other continuous measure. In such a case, the data are not solely success/ failure oriented. Generally, the times at which failures occur and the time in operation without failures must also be considered. These types of tests are analyzed by using a Poisson model. When the failure rate remains constant throughout the test, the exponential distribution describes the times between failures and provides all the information needed for the data analysis. For the analysis presented in subsequent sections of this chapter, we will assume that the failure rate is constant. We present below a graphical procedure to determine if that assumption is reasonable.

Continuous Time Testing: Failure Pattern Identification

When confronted with data from a continuous time test the analyzer should first construct an average failure rate plot. The purpose of constructing an average failure rate plot is to help the analyst determine whether the failure rate is increasing, decreasing, or is constant. The type of failure rate plot that will be described considers hardware that did not have significant design changes made, so that changes in the failure rate are due primarily to the age of the equipment. (When substantial design changes are made, there may be reliability growth. In that case, a different type of average failure rate plot is used, which is based on cumulative test exposure rather than the age of the equipment.)

The average failure rate plot is constructed as follows:

1. Determine the lowest and highest equipment ages which the test experience covers. These need not be ages at which failures occurred. This establishes the lower and upper limits of the plot. For convenience, working limits may be set at "round" numbers above and below the lower and upper limits , respectively.

2. Divide the interval encompassed by the working limits into subintervals. The subintervals need not be of equal size.

3. Count the number of failures in each subinterval. (A minimum of 5 failures per subinterval is desirable, though not absolutely necessary.)

4. Add up the hours (or miles, rounds, etc.) of operation within each subinterval.

5. Compute the average failure rate for each subinterval by dividing the number of failures in the subinterval by the hours (or miles, rounds, etc.) of operation in the subinterval.

6. Construct a graph, with the system age (in hours, miles, rounds, etc.) on the horizontal scale, and failure rate on the vertical scale. The average failure rates computed for each subinterval are shown as horizontal lines over the length of each subinterval.

7. If the average failure rate plot has too much fluctuation to show any kind of trend, reduce the number of subintervals and repeat steps 3

through 6. For very small amounts of data, it may be necessary to use only two subintervals.

8. From the final version of the average failure rate plot, judge whether the failure rate trend remains constant, increases , or decreases as the equipment ages. For small amounts of data it may be difficult to make this judgment. In any case, statistical tests for trend may be used.

9. If the data are judged to have no trend, analyses based on the exponential distribution may generally be used with validity.

10. If the failure rate is judged to be increasing or decreasing, as a minimum, a note to this effect should accompany any analyses based on the assumption of exponential times between failures. To analyze data that appear to have a trend more explicitly, a non-homogeneous Poisson process may be fitted to the data. We do not present any analysis using a non-homogeneous Poisson process in this chapter. If the average failure rate plot indicates that a constant failure rate assumption is unwarranted, the data analyst may refer to a statistics text which covers the topic of stochastic processes in depth to aid in his analysis.

11. See Case Studies 7-5 and 7-6 for examples of average failure rate plots.

<u>Exponential Model: Point Estimate of MTBF</u>

When data are judged to show a constant failure rate, the exponential distribution may be used for data analysis. Exponential analysis does not require the use of actual failure times.

<u>Notation</u>

T = total test exposure, the total hours, miles, etc. , accumulated among all the items included in the sample

r = number of failures observed

$\hat{\theta}$ = point estimate of MTBF

$\hat{R}(x)$ = point estimate of reliability for a specified exposure, x

$\hat{\lambda}$ = the point estimate of the failure rate

<u>Formulas</u>

$$\hat{\theta} = \frac{T}{r} \tag{7.6}$$

<u>Exponential Model: Point Estimates of Reliability and Failure Rate</u>

Point estimates of reliability and failure rate may be developed from point estimates of MTBF as follows:

$$i(x) = e^{-x/ii} \tag{7.7}$$

$$\hat{\lambda} = 1/\hat{\theta} \tag{7.8}$$

See Case Studies 7-7, 7-8, and 7-9 for illustrations of computing point estimates.

<u>Exponential Model: Confidence Intervals and Limits for MTBF</u>

<u>Notation</u>

T = total test exposure, the total hours, miles, etc., accumulated among all the items included in the sample

r = the number of failures observed

$X^2_{\_\_,\_\_}$ = a chi-square value, identified by two subscripts. To determine a chi-square value using Appendix B, Table 5, we use the first subscript, a function of the risk (α), to indicate the column, and the second subscript, a function of the number of failures (r), to indicate the row.

α = the risk that a confidence statement is in error. Note: The symbol a used here does not necessarily represent the producer's risk as discussed in Chapter 6.

6 = MTBF R(x) = reliability for a period x λ = failure rate	Used in conjunction with	no subscript = true but unknown value L subscript = lower limit U subscript = upper limit

Formulas (All the formulas listed below will yield statements at the 100(1-a%) level of confidence.)

Confidence Limits

Time Terminated

When the test exposure ends at a time other than a failure occurrence, use Appendix B, Table 8a multipliers or the following formulas.

Interval for specified confidence level

$$\theta_L \le \theta \le \theta_U$$

$$\frac{2T}{\chi^2_{\alpha/2,2r+2}} \le \theta \le \frac{2T}{\chi^2_{1-\alpha/2,2r}} \qquad (7.9a)$$

See Case Studies 7-7 and 7-8.

Lower limit for specified confidence level

$$\theta \ge \theta_L$$

$$\theta \ge \frac{2T}{\chi^2_{\alpha,2r+2}} \qquad (7.10a)$$

Upper limit for specified confidence level

$$\theta \le \theta_U$$

$$\theta \le \frac{2T}{\chi^2_{1-\alpha,2r}} \qquad (7.11a)$$

See Case Study 7-7,

Failure Terminated

When the test exposure ends at a failure occurrence, use Appendix B, Table 8b multipliers or the following formulas.

Interval for specified confidence level

$$\theta_L \le \theta \le \theta_U$$

$$\frac{2T}{\chi^2_{\alpha/2,2r}} \le \theta \le \frac{2T}{\chi^2_{1-\alpha/2,2r}} \qquad (7.9b)$$

See Case Study 7-9.

Lower limit for specified confidence level

$$\theta \ge \theta_L$$

$$\theta \ge \frac{2T}{\chi^2_{\alpha,2r}} \qquad (7.10b)$$

Upper limit for specified confidence level

$$\theta \le \theta_U$$

$$\theta \le \frac{2T}{\chi^2_{1-\alpha,2r}} \qquad (7.11b)$$

See Case Study 7-9.

Confidence That A Specific Lower Limit Has Been Attained

Time Terminated

Confidence that a specific lower limit, θ_L, has been attained

$$\chi^2_{\alpha,2r+2} = \frac{2T}{\theta_L} \tag{7.12a}$$

Search χ^2 tables in row labeled 2r + 2 for the numerical value, $2T/\theta_L$, and find the associated value for α.

Confidence that

$$\theta \geq \theta_L$$

is 100(1-a)%.

The value, α, may also be determined in closed form as follows:

$$\alpha = \sum_{k=0}^{r} \frac{(T/\theta_L)^k \, e^{-(T/\theta_L)}}{k!} \tag{7. 13a}$$

(Use Appendix B, Table 3 or Chart 1 to evaluate this expression.)

Confidence that

$$\theta \geq \theta_L$$

is 100(1-a)%.

See Case Studies 7-7 and 7-8.

Failure Terminated

Confidence that a specific lower limit, θ_L, has been attained

$$\chi^2_{\alpha,2r} = \frac{2T}{\theta_L} \tag{7.12b}$$

Search χ^2 tables in row labeled 2r for the numerical value, $2T/\theta_L$, and find the associated value for α.

Confidence that

$$\theta \geq \theta_L$$

is 100(1-CY)%.

The value, α, may also be determined in closed form as follows:

$$\alpha = \sum_{k=0}^{r-1} \frac{(T/\theta_L)^k \, e^{-(T/\theta_L)}}{k!} \tag{7.13b}$$

(Use Appendix B, Table 3 or Chart 1 to evaluate this expression.)

Confidence that

$$\theta \geq \theta_L$$

is 100(1-a)%.

See Case Study 7-9.

Confidence That A Specific Upper Limit Has Not Been Attained

Time Terminated

Confidence that a specified upper limit, θ_U, has not been attained

$$\chi^2_{1-\alpha,2r} = \frac{2T}{\theta_U} \qquad (7.14a)$$

Search χ^2 tables in the row labeled 2r for the numerical value, $2T/\theta_U$, and find the associated value for $1-\alpha$.

Confidence that

$$\theta \leq \theta_U$$

is 100(1-CY)%.

The value α may also be determined in closed form using the following equation:

$$1-\alpha = \sum_{k=0}^{r=1} \frac{(T/\theta_U)^k \, e^{-(T/\theta_U)}}{k!} \qquad (7.15a)$$

(Use Appendix B, Table 3 or Chart 1 to evaluate this expression.)

Confidence that

$$\theta \leq \theta_U$$

is $100(1-\alpha)\%$.

See Case Study 7-7.

Failure Terminated

Confidence that a specified upper limit, θ_U, has not been attained

$$\chi^2_{1-\alpha,2r} = \frac{2T}{\theta_U} \qquad (7.14b)$$

Search X^2 tables in the row labeled 2r for the numerical value, $2T/\theta_U$, and find the associated value for $1-\alpha$.

Confidence that

$$\theta \leq \theta_U$$

is 100(1-a)%.

The value α may also be determined in closed form using the following equation:

$$1-\alpha = \sum_{k=0}^{r-1} \frac{(T/\theta_U)^k \, e^{-(T/OU)}}{k!} \qquad (7.15b)$$

(Use Appendix B, Table 3 or Chart 1 to evaluate this expression.)

Confidence that

$$\theta \leq \theta_U$$

is 100(1-Q)%.

See Case Study 7-9.

Exponential Model: Confidence Intervals and Limits for Reliability and Failure Rate

Intervals for reliability and failure rate with 100(1-a)% confidence are

$$R_L(x) \le R(x) \le R_U(x)$$

$$e^{-(x/\theta_L)} \le R(x) \le e^{-(x/\theta_U)} \tag{7.16}$$

and

$$\lambda_L \le \lambda \le \lambda_U$$

$$1/\theta_U \le A \le I/e_L \tag{7.17}$$

where θ_L and 0_u are the lower and upper limits of the 100(1-a)% confidence interval for θ (MTBF).

Lower limit for reliability and upper limit for failure rate with 100(1-α)% confidence are

$$R(x) \ge R_L(x)$$

$$R(x) \ge e^{-(x/eL)} \tag{7.18}$$

and

$$\lambda \le \lambda_U$$

$$\lambda \le 1/\theta_L \tag{7.19}$$

where θ_L is the 100(1-α)% lower confidence limit for θ (MTBF).

Upper limit for reliability and lower limit for failure rate with 100(1-α)% confidence are

$$R(x) \le R_U(x)$$

$$R(x) \le e^{-(x/eu)} \tag{7.20}$$

and

$$\lambda \ge \lambda_L$$

$$\lambda \ge 1/\theta_U \tag{7.21}$$

Where θ_U is the 100(1-a)% upper confidence limit for θ (MTBF).

CASE STUDY NO. 7-1

Background

The engine for a light armored vehicle must have a 0.90 probability of completing 100,000 miles without an operational durability failure. In order to evaluate durability, four vehicles are tested. Each vehicle is operated until a durability failure occurs or until it successfully completes 100,000 miles of operation without experiencing an operational durability failure.

Determine

1. If no failures occur, what confidence do we have that the requirement has been met or exceeded?

2. If 1 failure occurs, what confidence do we have that the probability is at least 0.75?

3. If 2 failures occur, what confidence do we have that the probability is at least 0.50?

Solution

1. Since no failures have occurred, the number of successes is 4. We use equation 7.1 with

$n = 4$

$S = 4$

$R_L = 0.90.$

The confidence is:

$$\sum_{k=0}^{3}\binom{4}{k}(0.9)^k(0.1)^{4-k} = \binom{4}{0}(0.9)^0(0.1)^4 + \binom{4}{1}(0.9)^1(0.1)^3$$

$$+ \binom{4}{2}(0.9)^2(0.1)^2 + \binom{4}{3}(0.9)^3(0.1)^1$$

$$= (1)(0.0001) + (4)(0.0009) + (6)(0.0081) + (4)(0.0729)$$

$$= 0.0001 + 0.0036 + 0.0486 + 0.2916 = 0.3439.$$

We are 34% confident that the reliability meets or exceeds 0.90.

2. The number of successes is 3. We use equation 7.1 with

$n = 4$

$S = 3$

$R_L = 0.75.$

The confidence is:

$$\sum_{k=0}^{2} \binom{4}{k} (0.75)^k (0.25)^{4-k} = 0.2617.$$

We are 26% confident that the reliability meets or exceeds 0.75.

3. The number of successes is 2. We use equation 7.1 with

$n = 4$

$S = 2$

$R_L = 0.5.$

The confidence is:

$$\sum_{k=0}^{1} \binom{4}{k} (0.5)^k (0.5)^{4-k} = 0.3125.$$

We are 31% confident that the reliability meets or exceeds 0.50.

Commentary

It is interesting to note that with the small sample size, we can only reach 34% confidence that the requirement has been met or exceeded, even though we encountered zero failures. In many cases, durability requirements are impossible to demonstrate at high confidence levels because sample sizes are almost always constrained to be small.

CASE STUDY NO. 7-2

Background

A launcher for a medium range anti-tank missile has been tested. of 100 missiles, 95 were launched successfully.

Determine

1. Point estimate of reliability.

2. Construct a 90% upper limit on the true proportion of failures using the Poisson/exponential approximation.

3. Construct an 80% confidence interval on the true reliability using the Poisson/exponential approximation.

Solution

1. Point estimate of p, the true proportion of failures is 5/100 = 0.05. Consequently, the point estimate for the reliability, R, is

$$\hat{R} = 1 - \hat{p} = 1 - 0.05 = 0.95.$$

2. We set T=n=1OO, r= s = 5, and α = 0.10. The approximate 90% upper limit for p, the true proportion of failures, is obtained by first determining a 90% lower limit for 0. The 90% lower limit for θ is

$$\theta \geq \theta_L$$

$$\geq \frac{2T}{\chi^2_{\alpha,2r}}$$

$$\geq \frac{2(100)}{15.99}$$

$$\geq 12.51.$$

Consequently, the 90% upper limit for p is

$$p \leq p_U$$

$$\leq \frac{1}{\theta_L}$$

$$\leq \frac{1}{12.51}$$

$$\leq 0.08.$$

Thus, we are 90% confident that the true proportion of failures does not exceed 0.08.

3. Weset T=n=lOO, r=s =5, and α = 0.20. The 80% interval for R, the launcher reliability, is obtained by first determining an 80% interval for θ. The 80% interval for θ is

$$\theta_L \le \theta \le \theta_U$$

$$\frac{2T}{\chi^2_{\alpha/2,2r}} \le \theta \le \frac{2T}{\chi^2_{1-\alpha/2,2r}}$$

$$\frac{2(100)}{18.31} \le \theta \le \frac{2(100)}{3.94}$$

$$10.92 \le \theta \le 50.76.$$

Consequently, an 80% interval for p, the true proportion of failures is

$$p_L \le p \le p_U$$

$$\frac{1}{\theta_U} \le p \le \frac{1}{\theta_L}$$

$$\frac{1}{50.76} \le p \le \frac{1}{10.92}$$

$$0.02 \le p \le 0.09.$$

The 80% interval for the reliability, R, is

$$R_L \le R \le R_U$$

$$1 - p_U \le R \le 1 - p_L$$

$$1 - 0.09 \le R \le 1 - 0.01$$

$$0.91 \le R \le 0.98.$$

We are 80% confident that the true launcher reliability is between 0.91 and 0.98.

CASE STUDY NO. 7-3

Background

A new missile system has been under development and is ready for production. The contract specifies that the producer must demonstrate a proportion of successes at least equal to 0.85 (SV). The user will accept as a minimum a demonstration of at least 0.70 (MAV). An initial production test of 30 firings was conducted for the missile system, and 6 missiles fired improperly.

Determine

1. What is our best single value estimate for the true proportion of failures?

2. Construct exact 90%, 95%, and 99.5% lower confidence limits for the true proportion of failures.

3. Construct exact 90%, 95%, and 99.5% upper confidence limits for the true proportion of failures.

4. Construct approximate 60%, 70%, 80%, and 90% two-sided confidence limits for the true proportion of failures, using the normal approximation to the binomial.

5. Provide an accept/reject criterion which permits the greatest number of acceptable failures which still meets a consumer's risk of no more than 10%. What is the producer's risk for this criterion? Is the system acceptable under this criterion?

6. Increase the sample size to 40 and 50. Provide an accept/reject criterion to meet a producer's risk of 15%. What is the consumer's risk for each criterion?

Solutions

1. Point estimate: 6/30 = 0.20. This corresponds to an 80% reliability.

2. Lower confidence limits: Use Appendix B, Table 4.

   a. 90% Lower limit, $n = 30$, $s = 6$.
   Lower limit = 0.109.

   b. 95% Lower limit, $n = 30$, $s = 6$.
   Lower limit = 0.091.

   c. 99.5% Lower limit, $n = 30$, $s = 6$.
   Lower limit = 0.054.

Note that the three solutions above are lower confidence limits on the true proportion of failures, i.e., lower limits on unreliability. If we subtract any of the lower limits from 1, we obtain an upper limit on reliability. To convert the 90% lower limit on unreliability (0.109) to an upper limit on reliability, we subtract it from 1, i.e., 1 - 0.109 = 0.891. This means that we are 90% confident that the true reliability does not exceed 0.891.

3. Upper confidence limits: Use Appendix B, Table 4.

a. 90% Upper limit, n = 30, s = 6.
Upper limit = 0.325.

b. 95% Upper limit, n = 30, s = 6.
Upper limit = 0.357.

c. 99.5% Upper limit, n = 30, s = 6.
Upper limit = 0.443.

Note that the three solutions above are upper confidence limits on the true proportion of failures, i.e., upper limits on unreliability. To obtain a lower limit on reliability, we subtract the corresponding upper limit on unreliability from 1. The 90% lower limit on reliability is thus: 1 - 0.325 = 0.675. This means that we are 90% confident that the true reliability exceeds 0.675.

4. Approximate two-sided limits (normal), for $\hat{p} = s/n = 6/30 = 0.2$:

$$\text{Lower limit} = \hat{p} - z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$$

$$\text{Upper limit} = \hat{p} + z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$$

Note that the values for $z_{a/2}$ can be found in Appendix B, Table 2, To use the table for two-sided limits, we convert the confidence percentage (say 60%) to a value for $\alpha(0.40)$, divide that value by $2(\alpha/2 = 0.20)$, and locate the value for $z_{a/2}$ ($z_{0.20} = 0.84$).

a. 60% $\alpha = 0.40$ $z_{\alpha/2} = z_{0.20} = 0.84$

Lower limit = 0.139
Upper limit = 0.261

b. 70% $\alpha = 0.30$ $z_{\alpha/2} = z_{0.15} = 1.04$

Lower limit = 0.124
Upper limit = 0.276

c. 80% $\alpha = 0.20$ $z_{a/2} = z_{0.10} = 1.28$

Lower limit = 0.107
Upper limit = 0.293

d. 90% $\alpha = 0.10$ $z_{\alpha/2} = z_{0.05} = 1.645$

Lower limit = 0.080
Upper limit = 0.320

5. a. Use Appendix B, Table 1, n = 30. The probability of 5 or fewer failures when p is 0.3 is 0.0766. (Recall that p = 0.3 corresponds to a reliability of 0.7.) The probability of 6 or fewer failures when p is 0.3 is 0.1595. Because the consumer' s risk is not to exceed 10%, we must make our decision criterion to accept with 5 or fewer failures and reject with more than 5 failures. The decision criterion to accept with 6 or fewer failures results in a consumer's risk of 15.95%, which exceeds the requirement of a 10% consumer's risk. Note that the actual consumer' s risk for the criterion to accept with 5 or fewer failures is 7.66%.

b. Use Appendix B, Table 1, n = 30. The producer' s risk is the probability of rejecting the system when it has met the specification of 0.15 proportion of failures (i.e. , a reliability of 0.85) . We reject the system if 6 or more failures occur. The probability of 6 or more failures is the difference between 1 and the probability of 5 or fewer failures. The probability of 5 or fewer failures when p is 0.15 is 0.7106. Consequently, the producer's risk is 1 - 0.7106 or 0.2894 (28.94%) .

c. The system is not acceptable because in fact more than 5 failures occurred.

6. a. Appendix B, Table 1, n = 40, p = 0.15. Producer' s risk must not exceed 0. 15.

r	P(r or fewer failures)	P(r+1 or more failures)
7	0.7559	0.2441
8	0.8646	0.1354

The criterion is to reject if 9 or more failures occur; otherwise, accept.

The consumer' s risk, the probability of accepting the system when, in fact, it has fallen below the MAV of 0.7, is the probability that 8 or fewer failures occur when the true proportion of failures, p, is 0.3. This value is 0.1110. Thus , there is an 11. 1% chance of accepting a bad system with this plan.

b. Appendix B, Table 1, n = 50, p = 0.15. Producer's risk must not exceed 0. 15.

r	P(r or fewer failures)	P(r+1 or more failures)
9	0.7911	0.2089
10	0.8801	0.1199

The criterion is to reject if 11 or more failures occur; otherwise, accept.

The consumer's risk (note above definition) is the probability that 10 or fewer failures occur when p is 0.3. This value is 0.0789. Thus, there is a 7.89% chance of accepting a bad system with this plan.

Note that for a fixed producer's risk (approximately 13%), the consumer's risk decreases as the sample size increases. An increased sample size will also result in a decreased producer's risk when the consumer's risk is held approximately constant.

CASE STUDY NO. 7-4

Background

Two contractors are competing for a contract to produce an electronic guidance system. Twenty-five units from Contractor I and thirty units from Contractor 2 have been tested. The results of the test are: Contractor 1 had 2 failures, Contractor 2 had 7 failures.

Determine

1. What is the point estimate of the difference in the true proportions of failures for the two contractors?

2. What is the point estimate of the ratio of the true proportions of failures for the two contractors?

3. Construct an approximate 90% lower confidence limit for the difference in the true proportions.

4. Construct an approximate 90% lower confidence limit for the ratio of the true proportions.

5. What confidence do we have that Contractor 1's system is at least twice as bad as Contractor 2's? At least 50% worse than Contractor 2's?

Solutions

1. $\hat{p}_1 - \hat{p}_2 = 7/30 - 2/25 = 0.233 - 0.080 = 0.153.$

2. $\hat{p}_1/\hat{p}_2 = 7/30 \div 2/25 = (7)(25)/2(30) = 2.91.$

3. Lower limit $= \hat{p}_1 - \hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}$

$$= 0.153 - z_\alpha\sqrt{(0.233)(0.767)/30 + (0.08)(0.92)/25}$$

For a 90% lower limit, $\alpha = 0.10$ and $z_\alpha = 1.28$. (See Appendix B, Table 2.) The lower limit for the difference in true proportions is 0.031. This means that we are 90% confident that the difference in the true proportions of failures is at least 0.031.

4. Lower limit $= \hat{p}_1/\hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2}$

$$= 2.91 - z_\alpha\sqrt{(0.233)\ (0.92)/25(0.08)2}\ .$$

For a 90% lower limit, $\alpha = 0.10$ and $z_\alpha = 1.28$. The lower limit is thus 1.43. This means that we are 90% confident that Contractor 1's system is 1.43 times worse than Contractor 2's system.

5. To find the confidence that Contractor 1's system is at least twice as bad as Contractor 2's, we must find the confidence associated with a lower limit of 2 for the ratio. Since, by definition, the lower limit is

$$\hat{p}_1/\hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} .$$

We set this expression equal to 2, and solve for z_α to obtain

$$z_\alpha = (\hat{p}_1/\hat{p}_2 - 2)/\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} .$$

Substituting 0.233 for $\hat{p}_1$, 0.08 for $\hat{p}_2$, and 25 for n_2, we have

$$z_\alpha = 0.788 .$$

We look in Appendix B, Table 2 to find the value of α which corresponds to z_α = 0.788. Since, by definition, $P(Z \geq z_\alpha) = \alpha$, the desired value of α is located under the column labeled $P(Z \geq z_\alpha)$. Thus the value of α is 0.215. This represents a 100(1-(Y)% = 78.5% lower confidence limit, so we are 78.5% confident that Contractor 1's system is at least twice as bad as Contractor 2's.

To find the confidence that Contractor 1's system is at least 50% worse than Contractor 2's, we solve the following equation for z_α,

$$\hat{p}_1/\hat{p}_2 - z_\alpha\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} = 1.5 .$$

The solution is:

$$z_\alpha = (\hat{p}_1/\hat{p}_2 - 1.5)/\sqrt{\hat{p}_1(1-\hat{p}_2)/n_2\hat{p}_2^2} .$$

Substituting 0.233 for $\hat{p}_1$, 0.08 for $\hat{p}_2$, and 25 for n_2, we have

$$z_\alpha = 1.22 .$$

This corresponds to an α of 0.1112, so we are 88.88% confident that Contractor 1's system is at least 50% worse than Contractor 2's.

CASE STUDY NO. 7-5

Background

Six electronic systems were tested. All systems had the same configuration, and no design changes were introduced during the test. The test experience is tabulated below.

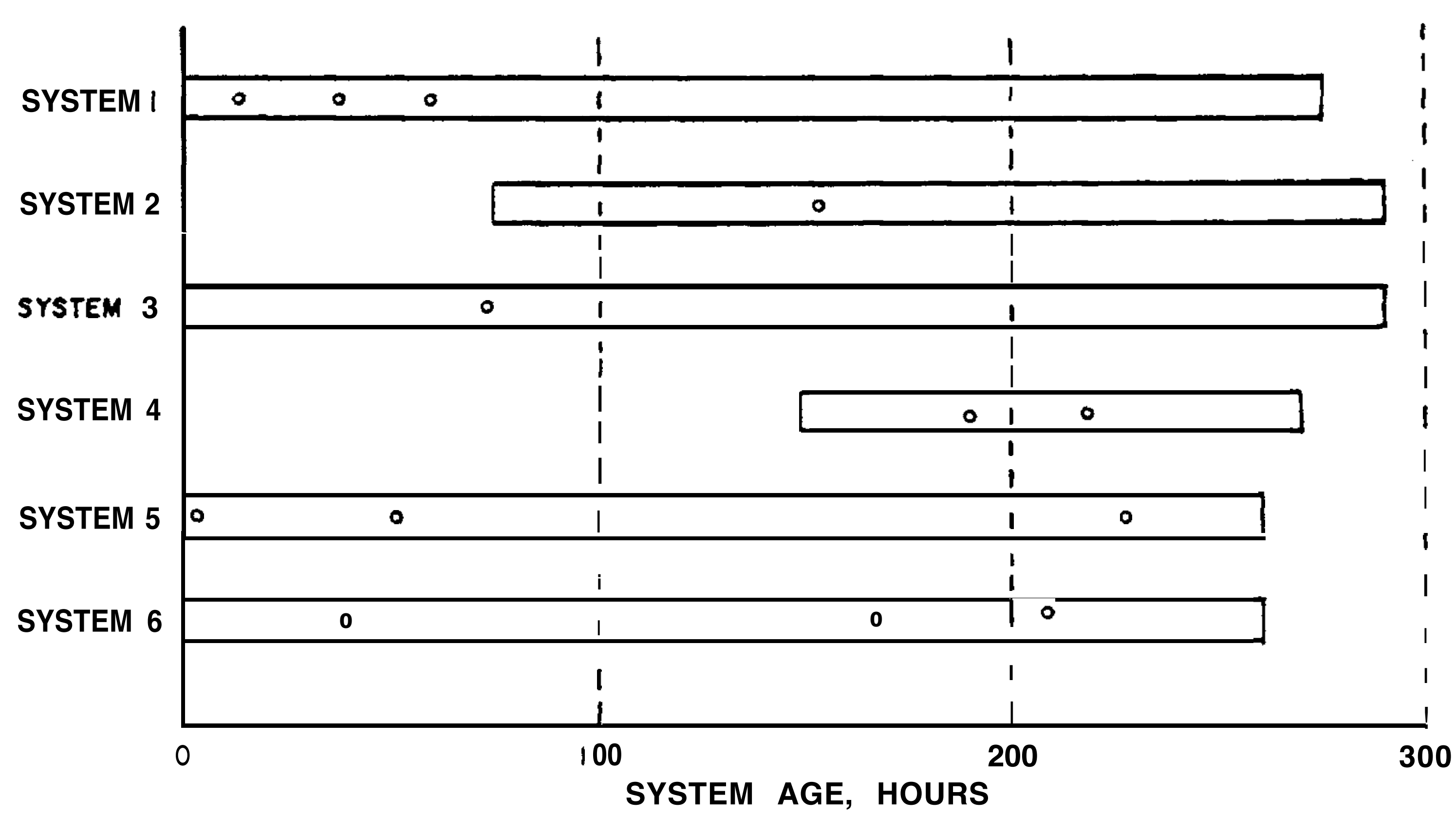
System	System Age at Start of Test, Hrs.	System Age at Failure(s) Hrs.	System Age at End of Test, Hrs.
1	0	13, 37, 60	275
2	75	154	290
3	0	73	290
4	150	190, 218	270
5	0	3, 52, 227	260
6	0	39, 166, 167, 209	260

Determine

Is exponential data analysis appropriate for this data set?

Solution

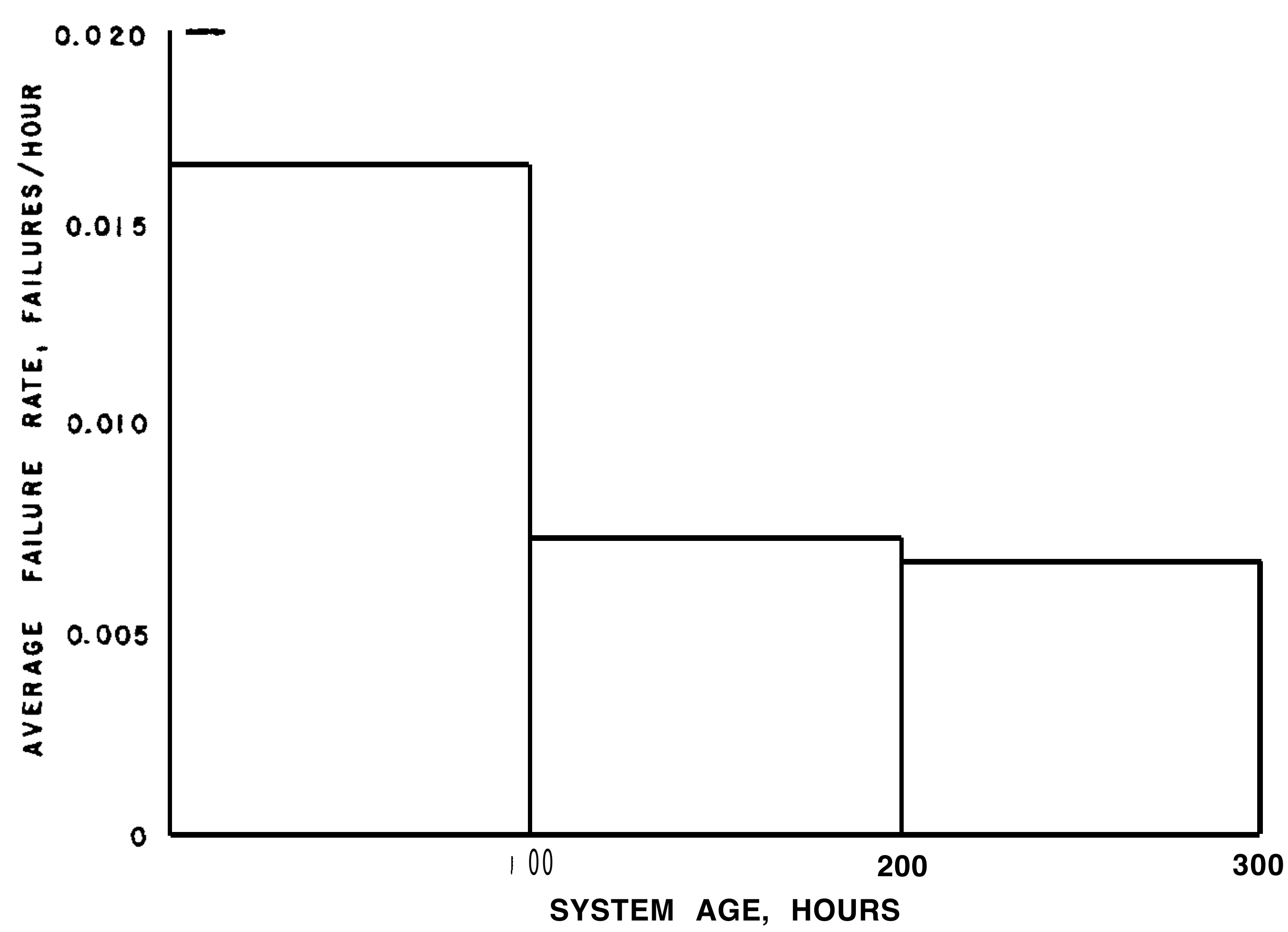
An average failure rate plot will be used to determine if there is a trend to the data. The following graph, although not a necessary part of the analysis, is included to aid visualization of the data.



The data will be broken down into three equal intervals. The steps involved in arriving at the average failure rate for each interval are contained in the following table.

Interval	Failures	Operating Hours	Average Failure Rate
o-1oo	7	425	7/425 = 0.0165
100-200	4	550	4/550 = 0.0073
200-300	3	445	3/445 = 0.0067

These average failure rates are plotted on the following graph.



The average failure rate plot suggests very strongly that there is a decreasing failure rate as the system ages, and exponential data analysis should not be used unless, at a minimum, a caveat about the decreasing failure rate is included.

Commentary

1. Although this is a fictional data set, the pattern to the data is frequently observed in real data sets.

2. For a data set of this type, it is generally useful to consider the actual failure types and corrective actions encountered. This tends to clarify how permanent the high initial failure rate might be.

CASE STUDY NO. 7-6

Background

Three developmental vehicles were operated under test conditions that closely matched the operational mode summary and mission profile for the system. All vehicles had the same physical configuration. Only one relatively minor design change was introduced to the test vehicles during the test. Scoring of test incidents determined that there were 7 operational-mission failures. 'The following table displays the operational mission failure data.

Vehicle Number	Odometer at Start(km)	Odometer at Failure	Odometer at End (km)
1	0	None	6,147
2	0	3,721; 6,121; 6,175 9,002	11,000
3	0	216; 561; 2,804	5,012

Determine

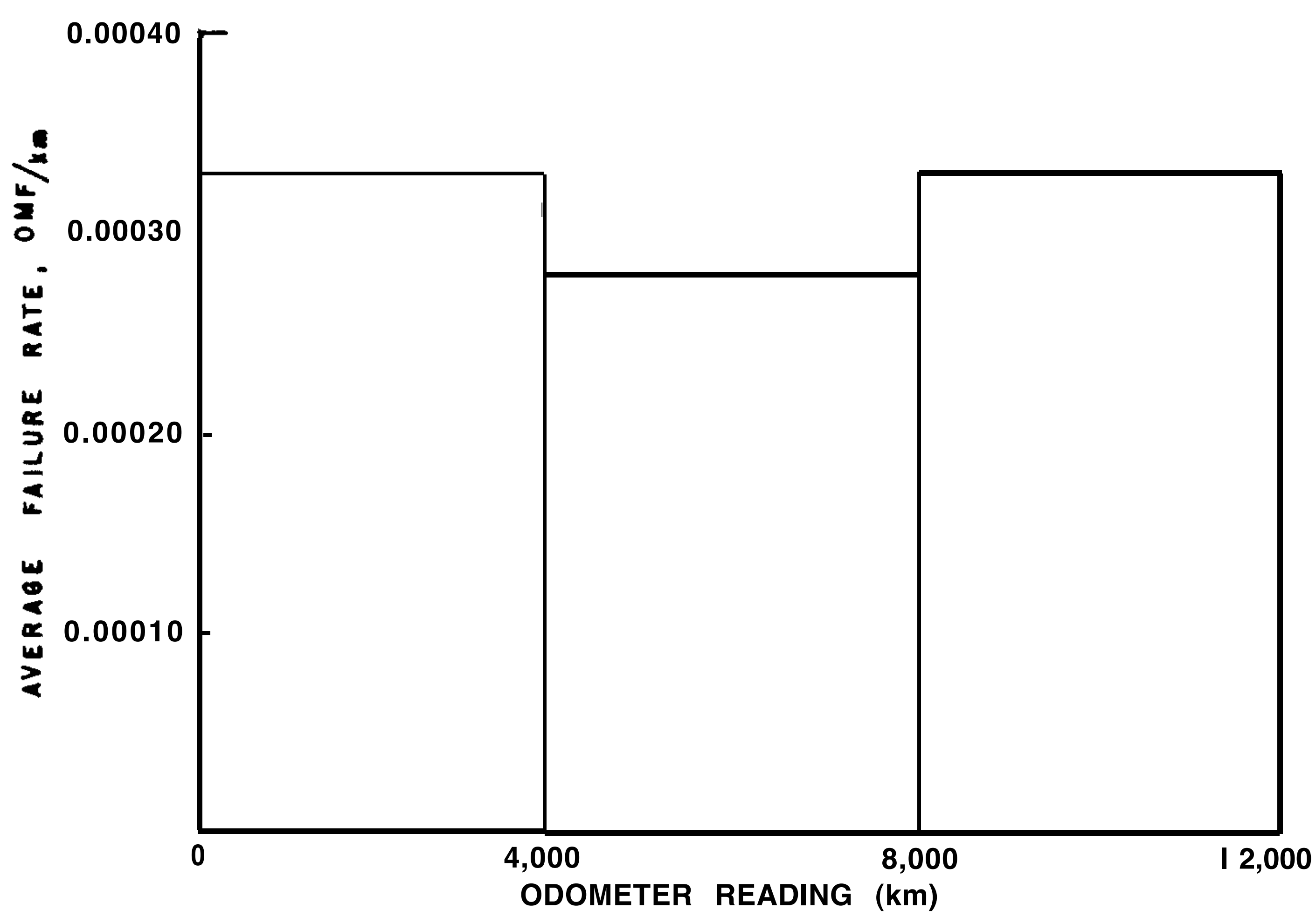
Is exponential data analysis appropriate for this data set?

Solution

An average failure rate plot will be used to determine if there is a trend to the data. Three equal intervals will (arbitrarily) be used.

Interval	Failures	Kilometers	Average Failure Rate
0-4,000	4	12,000	4/12,000 = 0.00033
4,000-8,000	2	7,159	2/7,159 = 0.00028
8,000-12,000	1	3,000	1/3,000 = 0.00033

These average failure rates are plotted on the following graph.



Since the average failure rate plot is essentially horizontal, there is virtually no evidence of trend in the data, and exponential data analysis procedures may be used.

Commentary

For large data sets , the average failure rate plot gives a very precise picture of the actual failure rate pattern. For small data sets, such as this one, chance plays a very heavy role. For example, in this case we observed one failure in the last interval. Just one more, or one less failure in this interval would make a drastic difference in the observed average failure rate. More formal trend tests address whether such variations could reasonably be due to chance alone.

CASE STUDY NO. 7-7

Background

The vehicle system discussed in Case Study 7-6 has a mission duration of 100 kilometers. The user has stated a minimum acceptable value (MAV) of 2,000 mean kilometers between operational mission failure (MKBOMF) and the contractual reliability requirement is equivalent to a 4,000 MKBOMF specified value (SV). The analysis in Case Study 7-6 showed no trend to the data. The test gave 22,159 km of exposure, and 7 operational mission failures were observed.

Determine

1. Point estimate of MKBOMF, mission reliability and failure rate.
2. 80% confidence interval for MKBOMF and mission reliability.
3. 80% lower confidence limit for MKBOMF.
4. 80% upper confidence limit for MKBOMF.
5. What confidence do we have that the MAV has been met or exceeded?
6. What confidence do we have that the SV has not been obtained?
7. Does the statistical evidence suggest that the reliability is satisfactory, or not?

Solutions

Because Case Study 7-6 gave no evidence of trend, exponential data analysis procedures will be used. Note that they are all based on test exposure, T = 22,159 kilometers, and number of failures, r = 7. Actual odometer readings at failure need not be considered, except to note that the test exposure is "time" terminated.

1. Point estimates of θ, R(100), and λ.

   a. Apply equation 7.6

$$\hat{\theta} = \frac{T}{r} = \frac{22,159}{7} = 3165.6 \text{ MKBOMF}$$

   Convert to mission reliability using equation 7.7:

$$\hat{R}(x) = e^{-x/\hat{\theta}}$$

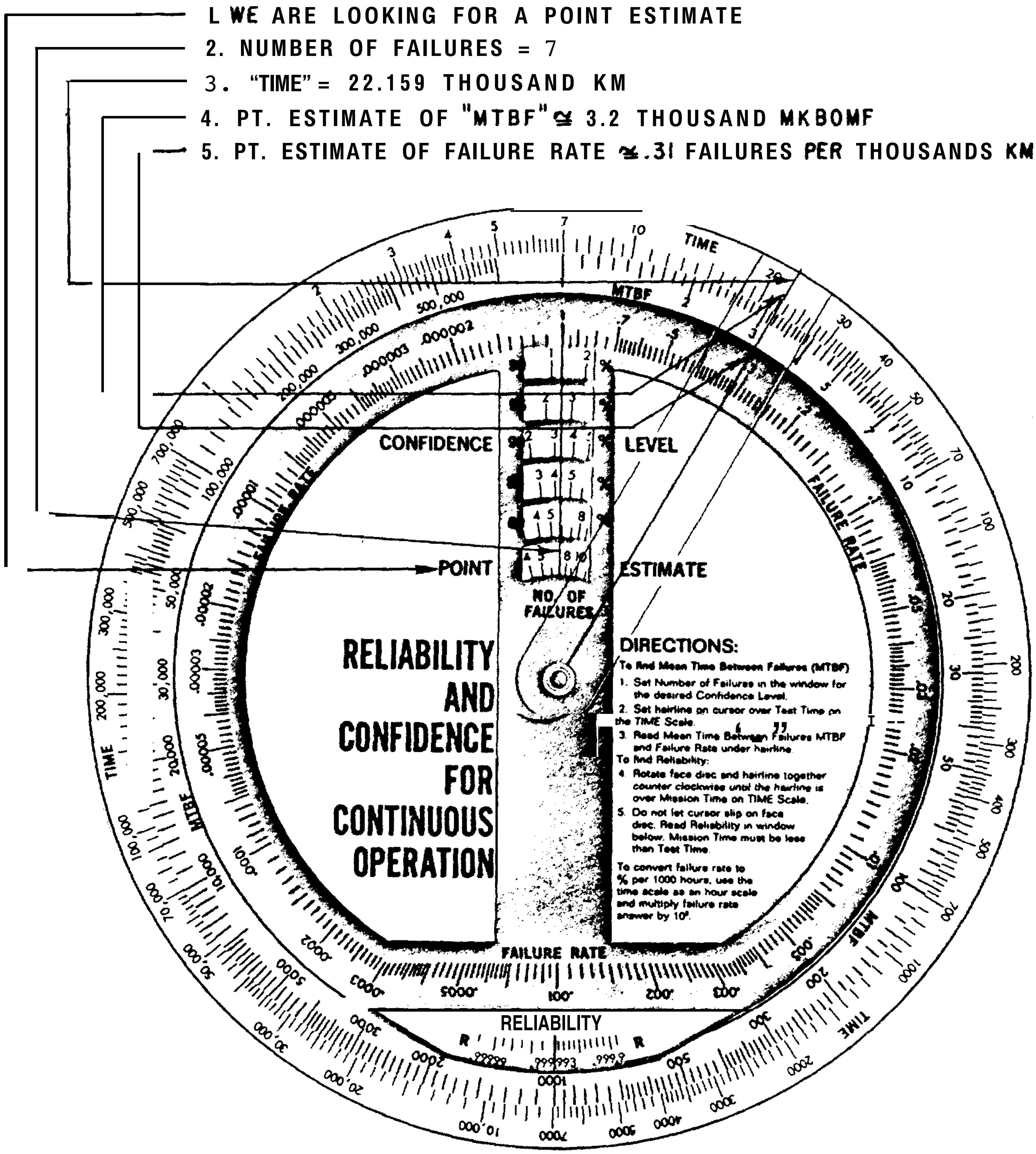
$$\hat{R}(100) = e^{-100/3165.6} = e^{-0.0316} = 0.969.$$

   Convert to failure rate using equation 7.8:

$$\hat{\lambda} = \frac{1}{\hat{\theta}} = \frac{1}{3165.6} = 0.000316 \text{ failures per km.}$$

   b. Use a reliability computer.

The next two figures illustrate the two-step solution procedure for part 1 using the reliability computer.

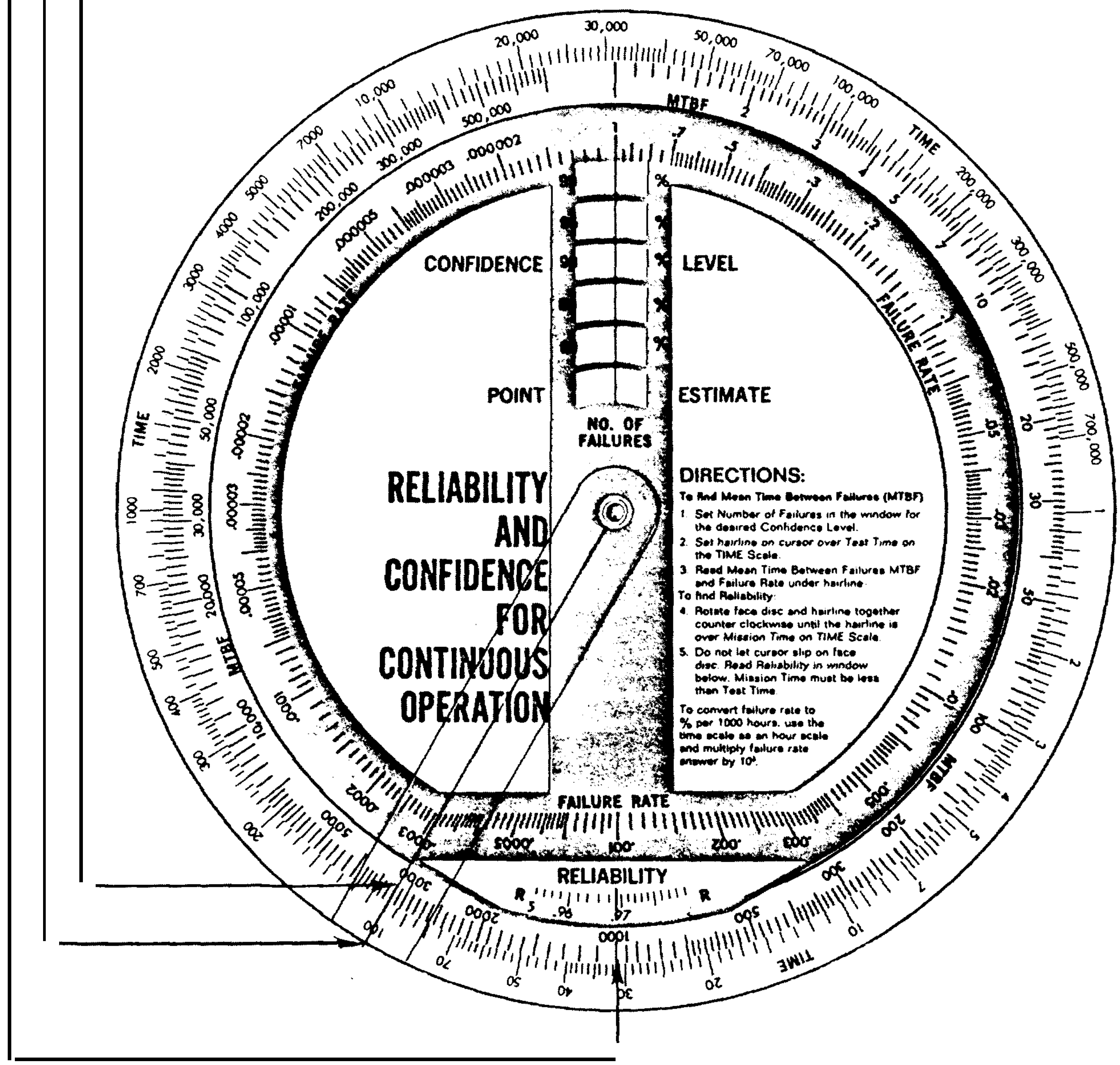


NOTE: THE "RELIABILITY COMPUTER" SHOWN IN THIS ILLUSTRATION CAN BE PURCHASED FROM TECHNICAL AND ENGINEERING AIDS FOR MANAGEMENT, BOX 25 TAMWORTH, N. H., 03886

6. PT. ESTIMATE OF "MTBF" ≅ 3,200 MKBOMF

7. "TIME" = MISSION DURATION = 100 KM

8. PT. ESTIMATE OF RELIABILITY ≅ .969



2. An 80% confidence interval for θ and reliability for a 100-kilometer mission R(100).

a. Using Table 8, Appendix B we obtain the confidence limit multipliers for the case of 7 failures and 80% confidence interval, i.e., 90% upper and 90% lower confidence limits. These multipliers are 0.665 and 2.797 for the 90% lower and upper confidence limits, respectively. Note we use Table 8a because this is a kilometers (i.e., time) terminated test.

$$\theta_L = \text{multiplier } (\hat{\theta}) = (0.595)(3165.6) = 1883.5 \text{ MKBOMF}$$

$${}'_u = \text{multiplier } (\hat{\theta}) = (1.797)(3165.6) = 5688.6 \text{ MKBOMF}$$

We are therefore 80% confident that

$$1883.5 \leq \theta \leq 5688.6 \text{ MKBOMF}$$

b. Using inequality 7.9a, we find, for $\alpha = 0.20$,

$$\frac{2T}{\chi^2_{\alpha/2,2r+2}} \leq \theta \leq \frac{2T}{\chi^2_{1-\alpha/2,2r}}$$

$$\frac{2(22,159)}{\chi^2_{0.10,16}} \leq \theta \leq \frac{2(22,159)}{{}^*{}^2_{0.90,14}}$$

Using Appendix B, Table 5 for the appropriate χ^2 values, we have

$$\frac{44,318}{23.55} \leq {}_0 \leq \frac{44,318}{7.79}$$

We are 80% confident that

$$1881.9 \leq \theta \leq 5689.0 \text{ MKBOMF}$$

In other words, we are reasonably sure that the MKBOMF is not less than 1881.9, nor greater than 5689.0.

c. Converting to mission reliability using inequality 7.16, we find

$$e^{-x/\theta_L} \leq R(x) \leq e^{-x/\theta_U}$$

$$e^{-100/1881.9} \leq R(100) \leq e^{-100/5689.0}$$

We are 80% confident that

$$0.948 \leq R(100) \leq 0.983$$

Commentary

The reliability computer cannot be used for confidence intervals since it does not have a capability for upper limits.

3. An 80% lower confidence limit for θ.

a. Use Table 8a, Appendix B to find the multiplier for an 80% lower confidence limit with 7 failures.

$$\theta_L = \text{multiplier}\ (\hat{\theta}) = (0.684)(3165.6) = 2165.3 \text{ MKBOMF}$$

Therefore, we are 80% confident that

$$\theta \geq 2165.3 \text{ MKBOMF}$$

b. Using inequality 7.10a, we find

$$\theta \geq \frac{2T}{\chi^2_{\alpha,\ 2r+2}}$$

$$\theta \geq \frac{2(22,159)}{\chi^2_{0.20,16}}$$

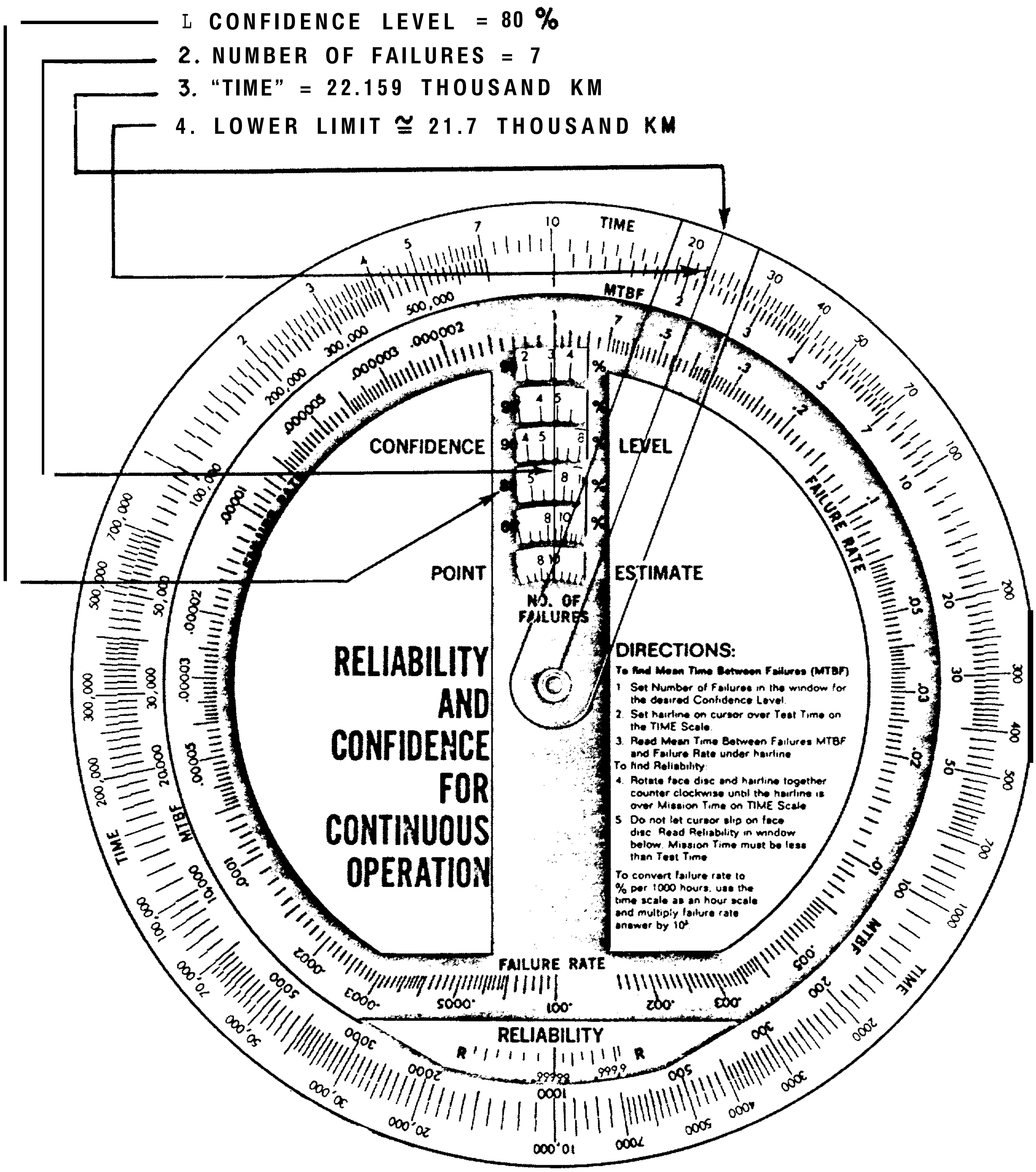
Using Appendix B, Table 5 for the χ^2 value, we have

$$\theta \geq \frac{44,318}{20.47}$$

We are 80% confident that

$$\theta \geq 2165.0 \text{ MKBOMF}$$

c. Using a reliability computer, we find



L CONFIDENCE LEVEL = 80 %
2. NUMBER OF FAILURES = 7
3. "TIME" = 22.159 THOUSAND KM
4. LOWER LIMIT ≅ 21.7 THOUSAND KM
TIME
MTBF
CONFIDENCE
LEVEL
POINT
ESTIMATE
NO. OF FAILURES
FAILURE RATE
RELIABILITY
RELIABILITY AND CONFIDENCE FOR CONTINUOUS OPERATION
DIRECTIONS:
To find Mean Time Between Failures (MTBF)
1. Set Number of Failures in the window for the desired Confidence Level.
2. Set hairline on cursor over Test Time on the TIME Scale.
3. Read Mean Time Between Failures MTBF and Failure Rate under hairline
To find Reliability:
4. Rotate face disc and hairline together counter clockwise until the hairline is over Mission Time on TIME Scale
5. Do not let cursor slip on face disc. Read Reliability in window below. Mission Time must be less than Test Time
To convert failure rate to % per 1000 hours, use the time scale as an hour scale and multiply failure rate answer by 10⁵

4. An 80% upper confidence limit for 0.

a. Use Table 8a, Appendix B to find the multiplier for an 80% upper confidence limit with 7 failures.

$$\theta_U = \text{multiplier } (\hat{\theta}) = (1.479)(3165.6) = 4681.9 \text{ MKBOMF}$$

Therefore, we are 80% confident that

$$\theta \leq 4681.9 \text{ MKBOMF}$$

b. Using inequality 7.11, we find

$$\theta \leq \frac{2T}{\chi^2_{1-\alpha,2r}}$$

$$\theta \leq \frac{2(22,159)}{\chi^2_{0.80,14}}$$

Using Appendix B, Table 5 for the χ^2 value, we find

$$\theta \leq \frac{44,318}{9.47}$$

We are 80% confident that

$$\theta \leq 4679.8 \text{ MKBOMF}$$

Commentary

The reliability computer does not have a capability for upper limits.

5. What confidence do we have that $\theta \geq 2,000$?

a. Using equation 7.12a, we find

$$\chi^2_{\alpha,2r+2} = \frac{2T}{\theta_L}$$

$$\chi^2_{\alpha,16} = \frac{2(22,159)}{2000} = 22\ 159$$

Searching Appendix B, Table 5 in the row labeled 16 for a value of 22.159, we find values of 20.47 and 23.55. Interpolating, we obtain $\alpha \cong 0.14$. Confidence is $100(1-\alpha)\% \cong 100(1-0.14)\%$. We are approximately 86% confident that

$$\theta \geq 2000 \text{ MKBOMF}$$

b. Using equation 7.13a, we find

$$\alpha = \sum_{k=0}^{r} \frac{(T/\theta_L)^k e^{-(T/\theta_L)}}{k!}$$

$$T/\theta_L = \frac{22,159}{2000} = 11.0795$$

$$r = 7$$

$$\alpha = \frac{(11.0795)^0 e^{-11.0795}}{1} + \frac{(11.0795)^1 e^{-11.0795}}{1}$$

$$+ \frac{(11.0795)^2 e^{-11.0795}}{2} + \frac{(11.0795)^3 e^{-11.0795}}{6}$$

$$+ \frac{(11.0795)^4 e^{-11.0795}}{2\quad 4} + \frac{(11.0795)^5 e^{-11.0795}}{120}$$

$$+ \frac{(11.0795)^6 e^{-11.0795}}{720} + \frac{(11.0795)^7 e^{-11.0795}}{5040}$$

$$= 0.0000 + 0.0002 + 0.0009 + 0.0035 + 0.0097$$

$$+ 0.0215 + 0.0396 + 0.0627$$

$$= \mathbf{0.1381}$$

We are 86.2% confident that

$$\theta \geq 2000 \text{ MKBOMF}$$

6. What confidence do we have that 6 < 4,000?

The confidence that $\theta < 4,000$ is the same as the confidence that $\theta < 4,000$. The former statement is easier to interpret, although the latter is the usual expression.

a. Using equation 7.14a, we find

$$\chi^2_{1-\alpha,2r} = \frac{2T}{\theta_U}$$

$$\chi^2_{1-\alpha,\ 14} = \frac{2(22,159)}{4000} = 11.08$$

Searching Appendix B, Table 5 in the row labeled 14 for a value of 11.08, we find values of 10.16 and 13.34. Interpolating, we obtain $1-\alpha \cong 0.68$. Confidence is $100(1-\alpha)\% \cong 100(0.68)\%$. We are approximately 68% confident that

$$\theta < 4000 \text{ MKBOMF}$$

b. Using equation 7.15, we obtain

$$1-\alpha = \sum_{k=0}^{r-1} \frac{(T/\theta_U)^k \, e^{-(T/\theta_U)}}{k!}$$

$$T/\theta_U = \frac{22,159}{4000} = 5.53975$$

$$r-1 = 6$$

$$1-\alpha = \frac{(5.53975)^0 e^{-5.53975}}{1} + \frac{(5.53975)^1 e^{-5.53975}}{1}$$

$$+ \frac{(5.53975)^2 e^{-5.53975}}{2} + \frac{(5.53975)^3 e^{-5.53975}}{6}$$

$$+ \frac{(5.53975)^4 e^{-5.53975}}{24} + \frac{(5.53975)^5 e^{-5.53975}}{120}$$

$$+ \frac{(5.53975)^6 e^{-5.53975}}{720}$$

$$= 0.0039 + 0.0218 + 0.0603 + 0.1113 + 0.1541 + 0.1708$$

$$+ 0.1577$$

$$= 0.6798$$

We are 68% confident that

$$\theta < 4000 \text{ MKBOMF}$$

7. Does the reliability appear satisfactory?

We are 86% confident that the user's needs have been met, but only 68% confident that contractual obligations were not met. There is stronger evidence that the reliability is satisfactory than not. If many more failures were experienced, we would have low confidence that the user's needs were met, and we would also have higher confidence that the contractual obligations were not met, suggesting that the reliability is not satisfactory from both standpoints.

CASE STUDY NO. 7-8

Background

An avionics system has the following reliability requirements: the minimum acceptable value (MAV) = 150 hrs. MTBF, and the specified value (SV) = 450 hrs. MTBF. Three of these systems were tested for 100 hours (each) under test conditions that closely duplicated the expected operational environment. No failures were observed during this test.

Determine

An 80% lower confidence limit for MTBF, and the confidence that the MAV has been attained.

Commentary

The case of a test with zero failures has some interesting features. With no failures, there is no way to determine the type of failure pattern. If we have some assurance that the equipment will not degrade as it ages, we can make a constant failure rate assumption, which, in turn, permits an exponential data analysis.

If we attempt to obtain a point estimate of 9, we get:

$$\hat{\theta} = \frac{T}{r} = \frac{T}{0} = \text{indeterminate}$$

Similarly, the upper limit is indeterminate. We can, however, obtain lower confidence limits.

Solutions

1. 80% lower confidence limit for 9.

   a. Note that the technique of using the multipliers from Table 8, Appendix B, cannot be used for the case of zero failures.

   b. Using inequality **7.10a**, **we** find

   $$\theta \geq \frac{2T}{\chi^2_{a,\ 2r+2}}$$

   We have in this case, T = 300, α = 0.2 and r = 0, so

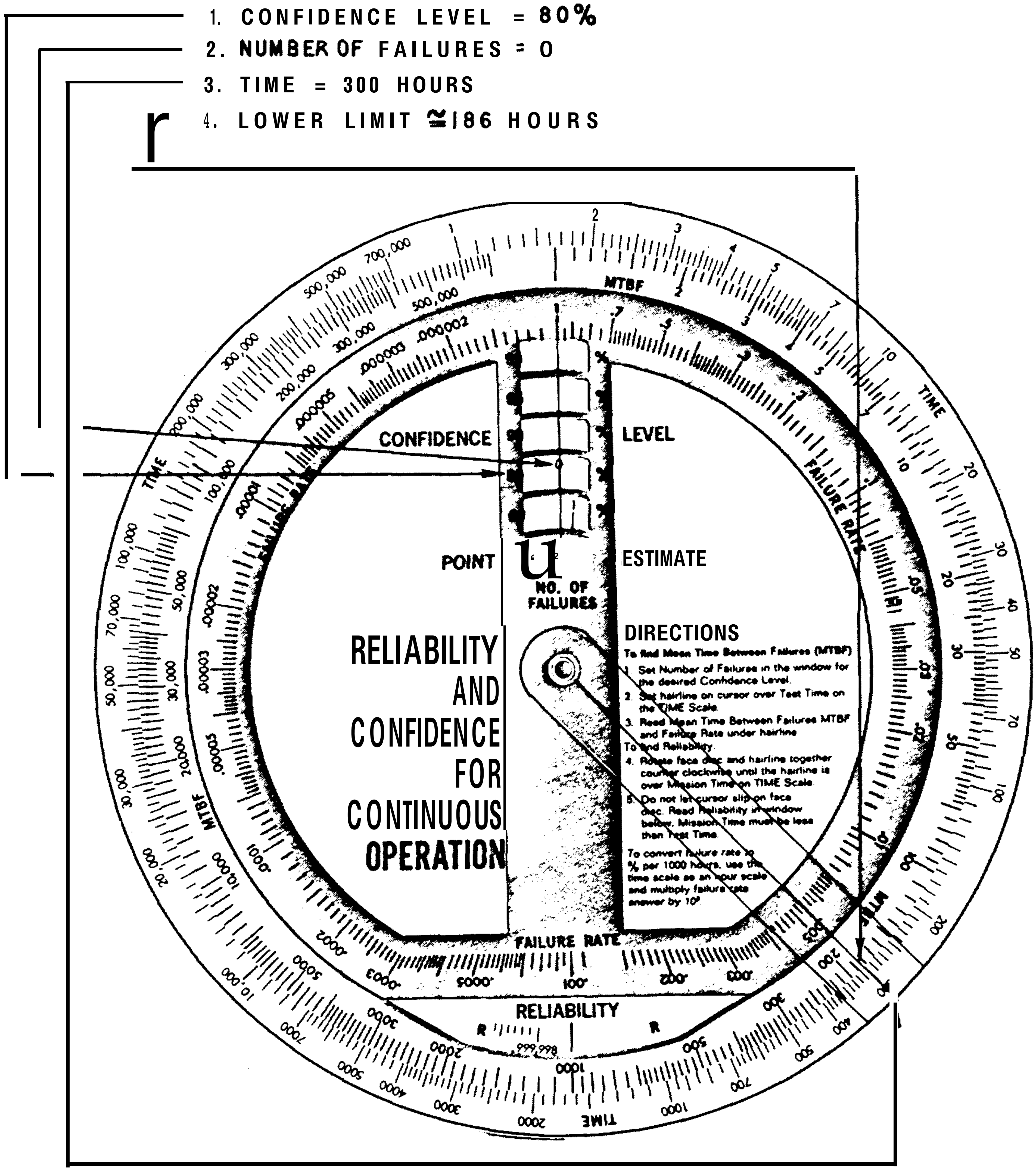
   $$\theta \geq \frac{2(300)}{\chi^2_{0.2,2}}$$

Using Appendix B, Table 5 for the χ^2 value, we find

$$\theta \geq \frac{600}{3.22}$$

We are 80% confident that $\theta \geq 186.3$ hrs MTBF

b. Using a reliability computer, we find



2. Confidence that $\theta \geq 150$

a. As noted in part 1 of this problem, Table 8 cannot be used for the case of zero failures.

b. Using equation 7.12a, we find

$$\chi^2_{\alpha,2r+2} \cdot \frac{2T}{\theta_L}$$

$$\chi^2_{\alpha,2} = \frac{2(300)}{150} = 4.0$$

Searching Appendix B, Table 5 in the row labeled 2 for a value of 4.0, we find values of 3.22 and 4.6o. Interpolating, we obtain $\alpha \cong .13$. Confidence is $100(1-\alpha)\% = 100(1-0.13)\%$. We are approximately 87% confident that

$\theta \geq 150$ hrs MTBF

c. Using equation 7.13a, we find

$$\alpha = \sum_{k=0}^{r} \frac{(T/\theta_L)^k \; e^{-(T/\theta_L)}}{k!}$$

For r =0, this simplifies to

$$\alpha = e^{-(T/\theta_L)}$$

In this case,

$$\alpha = e^{-(300/150)} = e^{-2} = \mathbf{0.135}$$

We are 86.5% confident that

$\theta \geq 150$ hrs MTBF

CASE STUDY NO. 7-9

Background

A system is being tested using test plan XIIC from Appendix B, Table 6. The upper test value (SV) is 100 hours MTBF, and the lower test value (MAV) is 50 hours MTBF. The required test duration is 940 hours, and 14 failures are rejectable. The source of data for this test plan is not relevant for this case study, but is presented here for future reference. Chapter 8 contains detailed discussions on the formation and use of this and other test plans.

The seventh failure has just occurred after only 57 hours of test exposure. Because of the excessive number of failures , an evaluation is to be done at this point in the test. Preliminary analysis of the data showed no evidence of trend, i.e., failure rate appeared constant,

Determine

1. Point estimate of MTBF.

2. 80% confidence interval for MTBF.

3. 80% lower confidence limit for MTBF.

4. 80% upper confidence limit for MTBF.

5. What confidence do we have that the lower test value has been met or exceeded?

6. What confidence do we have that the upper test value has not been attained?

7. Does the statistical evidence suggest that the reliability is satisfactory or not?

Commentary

Because an evaluation is being made at this point based on what was observed, we do not have a legitimate random sample. The true risks in making decisions based on such an analysis are difficult to determine. They are, in fact, substantially higher than the ones associated with the original plan. Consequently, the following analyses are all somewhat pessimistically biased.

Solutions

Since the seventh failure has just occurred, this is failure terminated data.

1. Point estimate of θ.

Applying equation 7.6, we obtain

$$\theta = \frac{T}{r} = \frac{57}{7} = 8.14 \text{ hrs MTBF}$$

2. An 80% confidence interval for θ.

a. Use Table 8b, Appendix B to obtain the confidence limit multiplier for the case of 7 failures and 80% confidence interval, i.e., 90% upper and lower confidence limits. Note we are using Table 8b because the test is <u>failure</u> terminated.

$$\theta_U = \text{multiplier } (\hat{\theta}) = (2.797)(8.14) = 14.63 \text{ hrs. MTBF}$$

$$\theta_L = \text{multiplier } (\hat{\theta}) = (0.665)(8.14) = 5.41 \text{ hrs. MTBF}$$

We are therefore 80% confident that

$$5.41 \leq \theta < 14.63 \text{ hrs. MTBF}$$

b. Using inequality 7.9b, we find, for $\alpha = 0.20$,

$$\frac{2T}{\chi^2_{\alpha/2,2r}} \leq \theta \leq \frac{2T}{\chi^2_{1-\alpha/2,2r}}$$

Using Appendix B, Table 5 for X^2 values:

$$\frac{114}{21.07} \leq \theta \leq \frac{114}{7.79}$$

We are 80% confident that

$$5.41 \leq \theta \leq 14.63 \text{ hrs MTBF}$$

3. An 80% *lower* confidence limit for θ.

a. Use Table 8b, Appendix B to find the multiplier for an 80% lower confidence limit with 7 failures.

$$'_L = \text{multiplier } (\hat{\theta}) = (0.771)(8.14) = 6.28 \text{ hrs. MTBF}$$

Therefore, we are 80% confident that

$$\theta \geq 6.28 \text{ hrs. MTBF}$$

b. Using inequality 7.10b, we find

$$\theta \geq \frac{2T}{\chi^2_{\alpha,2r}}$$

Using Appendix B, Table 5 for the X^2 value, we find

$$\theta \geq \frac{114}{18.15}$$

We are 80% confident that

$$\theta \geq 6.28 \text{ hrs MTBF}$$

4. An 80% upper confidence limit for θ.

a. Use Table 8b, Appendix B to find the multiplier for an 80% upper confidence limit with 7 failures.

$$\theta_U = \text{multiplier } (\hat{\theta}) = (1.479)(8.14) = 12.04 \text{ hrs. MTBF}$$

Therefore, we are 80% confident that

$$\theta \leq 12.04 \text{ hrs. MTBF}$$

b. Using inequality 7.11, we find

$$\theta \leq \frac{2T}{\chi^2_{1-\alpha,2r}}$$

Using Appendix B, Table 5 for the X^2 value, we obtain

$$\theta \leq \frac{114}{9.47}$$

We are 80% confident that

$$\theta \leq 12.04 \text{ hrs MTBF}$$

5. What confidence do we have that $\theta \geq 50$?

a. Using equation 7.12b, we find

$$\chi^2_{\alpha,2r} = 2T/\theta_L$$

$$\chi^2_{\alpha,14} = 2(57)/50 = 2.28$$

Searching Appendix B, Table 5 in the row labeled 14 for a value of 2.28, we find that we are beyond the end of the table, and $\alpha > 0.995$. The confidence is $100(1-\alpha)\%$, $100(1-0.995)\%$, or less than 0.5%.

We are less than 0.5% confident that

$$\theta \geq 50 \text{ hrs MTBF}$$

b. Using equation 7.13b, we find

$$\alpha = \sum_{k=0}^{r-1} \frac{(T/\theta_L)^k e^{-(T/\theta_L)}}{k!}$$

where

$T/\theta_L = 57/50 = 1.14$ and $r - 1 = 7 - 1 = 6$.

Solving equation 7.13b, we find $\alpha = 0.9998$.

We are 0.02% confident that $\theta \geq 50$ hours MTBF.

6. **What** confidence do we have that $\theta \leq 100$?

a. Using equation 7.14, we find

$$\chi^2_{1-\alpha,14} = \frac{2(57)}{100} = 1.14$$

Searching Appendix B, Table 5 in the row labeled 14 for a value of 1.14, we find that we are well beyond the end of the table, and $1-\alpha \cong 1.0$. The confidence is 100(1-o)%, 100(1.0)%, or essentially 100%.

We are essentially 100% confident that

$\theta \leq 100$ hrs MTBF

b. Using equation 7.15, we find

$$1-\alpha = \sum_{k=0}^{r-1} \frac{(T/\theta_U)^k e^{-(T/\theta_U)}}{k!}$$

where

$T/\theta_U = 0.57$ and $r - 1 = 7 - 1 = 6$

Solving equation 7.15, we find $1-\alpha = 0.99999$.

We are essentially 100% confident that

$\theta \leq 100$ hrs MTBF

7. **Does** the reliability appear satisfactory?

Since we have essentially 0% confidence that the lower test value is met or exceeded, and since we have essentially 100% confidence that the upper test value is not met, there is overwhelming evidence that the reliability is not satisfactory, even taking into consideration the fact that the analysis may be somewhat pessimistically biased.

In this case, the evidence is so strong that we can even state that we are 99. 98% confident that $\theta \leq 50$ hrs MTBF, though, ordinarily, upper limit statements are associated with the upper test value, and lower limit statements are associated with the lower test value.

Commentary

Test plan XIIC from Appendix B, Table 6 required a test duration of 940 hours to achieve true producer's and consumer's risks of 0.096 and 0.106, respectively. Since the system appears to be "failing miserably," the user has chosen to stop testing after 57 hours. No doubt this is a wise decision from an economic standpoint. However, the user should be fully cognizant that the risks associated with his abnormally terminated test are not 0.096 and o.1o6, nor are they the ones advertised in the preceding computations. The calculation of the true risks is well beyond the scope of this work.

CHAPTER 8

RELIABILITY TEST PLANNING

INTRODUCTION

This chapter presents the techniques for determining the amount of test exposure required to satisfy previously established program reliability requirements. The reader will note that Chapter 7 addresses the topic of reliability data analysis. There, we assumed that the test data had already been gathered. We then used the available data to determine point estimates for reliability parameters and to stipulate the uncertainty associated with these estimates.

Chapter 8 presents techniques for designing test plans which can verify that previously specified reliability requirements have been achieved. We realize, of course, that the required test exposure and/or sample size may exceed the available resources. In such cases, alternative test plans, consistent with program constraints, must be developed. In this chapter, we also present methods which make it possible to clearly identify the inherent risks associated with a limited test program.

PRIMARY TEST DESIGN PARAMETERS

Upper and Lower Test Values

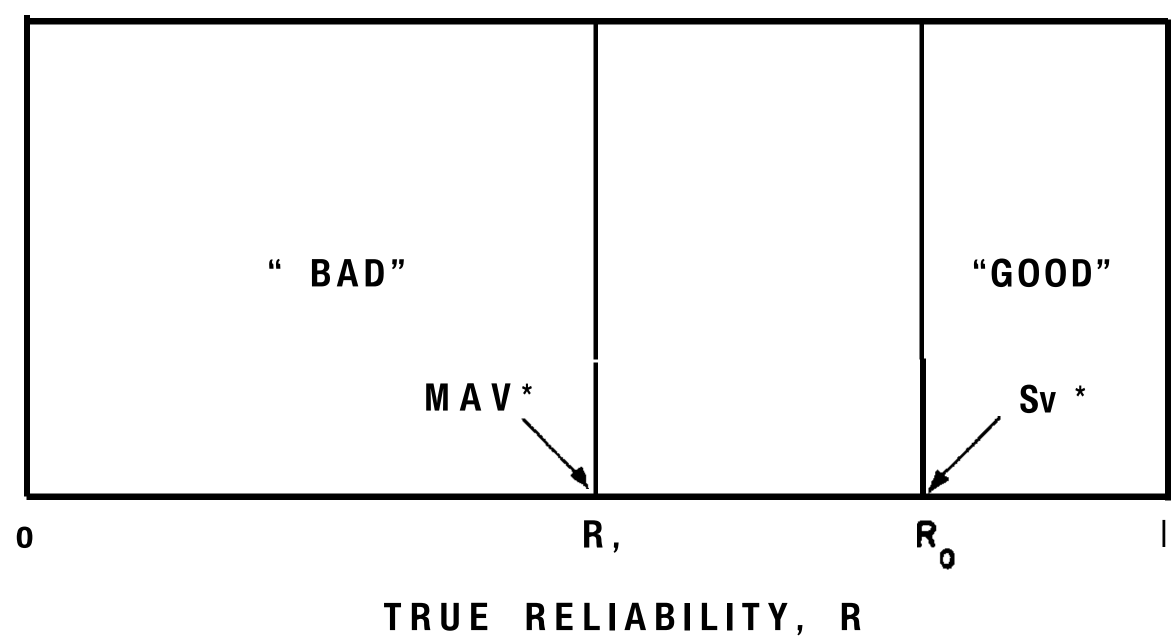
Two values of system reliability are of particular importance in the design of a reliability test plan. These are referred to as the upper test and lower test values. In some cases, only a single value is initially apparent, the second value being only implied. These two values and the risks associated with them determine the type and magnitude of testing required.

The upper test value is the hoped for value of the reliability measure. An upper test MTBF is symbolized as θ_0, and an upper test reliability is symbolized as R_0. A test plan is designed so that test systems whose true reliability parameters exceed θ_0 and R_0 will, with high probability, perform during the test in such a way as to be "accepted."

The lower test value is commonly interpreted in two different ways that may initially appear contradictory. One interpretation is that this lower value of the reliability measure represents a rejection limit. The other interpretation is that this value is minimally acceptable. The apparent conflict is resolved by viewing the lower test value as the fine line between the best rejectable value and the worst acceptable value. A lower test MTBF is symbolized as θ_1, and a lower test reliability is symbolized as R_1. Systems whose true reliability parameters having values less than θ_1 and R_1 will, with high probability, perform in such a way as to be "rejected."

The upper and lower test values serve to divide the reliability, or MTBF, scale into three distinct regions as shown in Figure 8-1. Note that the region between R_1 and R_0 is neither bad enough to demand rejection nor is it good enough to demand acceptance. This region is necessary since we will never precisely know the true reliability of the system.

FIGURE 8-1 REGIONS DEFINED BY R_0 AND R_1



*MAY BE DIFFERENT DEPENDING ON ACTUAL MATURITY. SEE FOLLOWING PARAGRAPH.

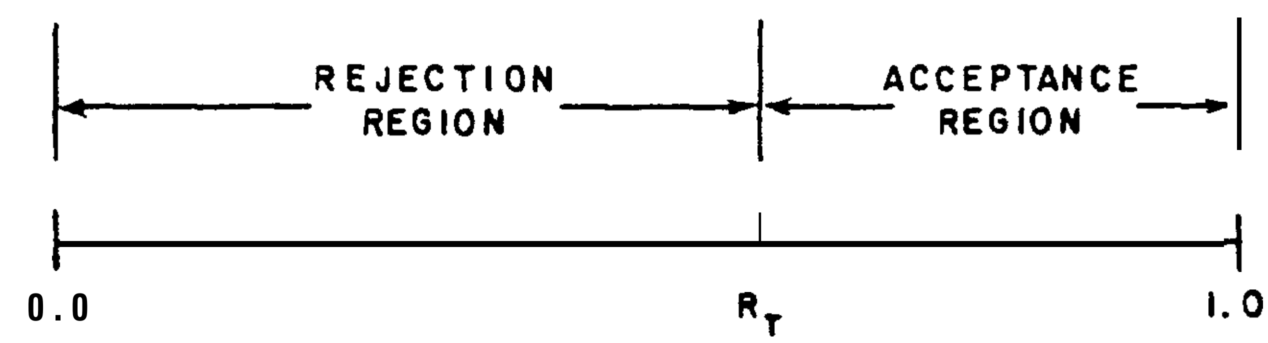
The user's reliability requirement should be stated as a minimum acceptable value (MAV); that is, the worst level of reliability that the user can tolerate and accept. The contractually specified value (SV) is a value somewhat higher than the MAV. For reliability qualification tests prior to production, the lower test value is the MAV, and the upper test value is the SV. Earlier in the development process, fixed configuration tests may be conducted to demonstrate the attainment of lower levels of reliability at specified milestones. In such cases, upper test and lower test values should be consistent with the stage of the development process.

In the above paragraphs, we have been discussing population parameter values only. These values are never known with absolute certainty, so we are forced to base an evaluation of system performance characteristics on sample data. Let us conclude this section with a discussion of sample reliability values and how we can interpret them to aid us in making our system reliability assessment.

One objective of this chapter is the determination of an accept/reject criterion for a test to be conducted. As an example, consider the value R_T in Figure 8-2 below. The term R_T is that value of the sample reliability which corresponds to the maximum allowable number of failures that can occur during testing and still result in acceptance of the system.

If we test our determined number of articles and find that R_{sample} is larger than R_T, then we accept the system because there is high probability that the

FIGURE 8-2 SAMPLE RELIABILITY RANGE



sample system(s) come from a population of systems whose true reliability R exceeds R_1, the minimum acceptable value (MAV) (see Figure 8-1) for this test. Note that when R_{sample} is larger than R_T, we have confidence that the true reliability exceeds the MAV. We should not interpret this result as an indication that the contractor has met the SV. Further, if R_{sample} is smaller than R_T, we will reject the system because there is high probability that the sample system(s) come from a population whose true reliability R is lower than 'o ' the SV for this test. Note that when R_{sample} is smaller than R_T, we have confidence that the true reliability falls short of the SV. We should not interpret this result as an indication that the MAV has not been met, but rather that the MAV has not been demonstrated at a sufficiently high level of confidence.

<u>Consumer' s Risk (β) and Producer' s Risk (α)</u>

The consumer's risk (β) is the probability of accepting the system if the true value of the system reliability measure is less than the lower test value. It can be interpreted in the following ways:

1. β represents the maximum risk that the true value of the reliability measure is, in fact, less than the lower test value.

2. From an alternative viewpoint, if the acceptance criterion is met, there will be at least 100(1-β)% confidence that the true value of the reliability measure equals or exceeds the lower test value.

The producer' s risk (α) is the probability of rejection if the true value of the reliability measure is greater than the upper test value. It can be interpreted in the following ways:

1. The probability of acceptance will be at least (1-α) if the upper test value is, in fact, met or exceeded.

2. From an alternative viewpoint, if there is a rejection decision, there will be at least 100(1-cY)% confidence that the true value of the reliability measure is less than the upper test value.

Case study 8-1 illustrates the relationship between α and β.

Pre- and Post-Test Risk Considerations

Before proceeding on with the application of the consumer's and producer's risk concept, it is important to understand the contrast that exists between pre-and post-test risks.

The α and β risks represent uncertainties that exist in the test planning or pre-data environment discussed in this chapter. Once data has been gathered and we have accepted or rejected the system, we find that the risk environment is altered. For example, we take a sample and decide to accept the system. At this point the producer's risk is eliminated; the consumer's risk remains but is less than the maximum that would exist had the sample reliability, R_{sample}, been exactly equal to R_T.

If, on the other hand, R_{sample} is less than R_T, i.e., we reject the system, we find that the consumer's risk is eliminated since there is no risk of accepting a bad system. Likewise, the producer's risk is less than the maximum that would exist had the sample reliability R_{sample} been exactly equal to R_T.

In this chapter, we are concerned with pre-test risks. We determine the maximum α and β risks and then calculate the required test exposure and acceptable number of failures which will limit our risk to the maximum levels.

TEST DESIGN FOR DISCRETE TIME TESTING: BINOMIAL MODEL

Four values specify the plan for a binomial test. They are:

- the specified or desired proportion of failures (p_0),
- the maximum acceptable proportion of failures (p_1),
- the consumer's risk (β),
- the producer's risk (α).

The test plan itself consists of a sample size (n) and an acceptance criterion (c). The value c represents the maximum number of failures which still results in acceptance of the system. It is usually not possible to construct a plan which attains the exact values of α and β. There are however plans which attain risks which do not exceed α and β. We shall present methods for determining these types of plans, though in a real world situation, the user and producer may trade off some protection to achieve other goals.

The following paragraphs present exact and approximate procedures to be used in planning a Discrete Time-Binomial Model test program. The "exact procedure" presents the equations used to determine the two values required to specify a binomial test plan. These equations are presented here for the sake of completeness. The "approximate solution" procedure, which makes use of the binomial tables to simplify the procedure, is intended for use by our readers.

Exact Solution Procedure

The exact procedure for determining test plans for the four values listed above is to solve the following two inequalities simultaneously for c and n.

$$\sum_{k=0}^{c} \binom{n}{k} p_1^k (1-p_1)^{n-k} \leq \beta \tag{8.1}$$

$$\sum_{k=c+1}^{n} \binom{n}{k} p_0^k (1-p_0)^{n-k} \leq \alpha \tag{8.2}$$

There are an infinite number of solutions to this pair of inequalities. The plans of interest are, of course, those which minimize the sample size (n) required. Solving inequalities 8.1 and 8.2 directly is next to impossible without the aid of a computer. MIL-STD-105D contains numerous binomial test plans which may be used for reliability applications. We should point out that the user unfamiliar with this document will find it difficult to interpret, thus we present the following procedures.

Approximate Solution Procedures

The following so-called approximate procedures utilize the normal and Poisson distributions to obtain approximate solutions to equations 8.1 and 8.2 and thereby estimate values of the sample size (n) and the acceptance criterion (c). After approximate values for these parameters have been obtained, we may then use the values in conjunction with the binomial tables (Appendix B) and the previously selected and fixed values of α and β to "fine tune" the approximate values of n and c.

Test Planning Using Normal Approximation. The normal distribution provides good approximations for solving inequalities 8.1 and 8.2, especially for moderate values of p ($0.1 \leq p \leq 0.9$). Using this information, we obtain the approximate solutions for n and c as follows.

$$n = \frac{z_\alpha^2(p_0-p_0^2) + z_\beta^2(p_1-p_1^2) + 2z_\alpha z \sqrt{p_0p_1(1-p_0)(1-p_1)}}{(p_1-p_0)^2}, \tag{8.3}$$

$$c = z_\alpha \sqrt{np_0(1-p_0)} + np_0 - 0.5 . \tag{8.4}$$

Generally, the values computed using equations 8.3 and 8.4 are good approximations for the test plainer. When p. and p_1 are very small (less than 0.05), the procedure is not recommended. Fine-tuning of the test plan may still require solving the original inequalities or some bargaining with user and/or producer.

As an example, suppose that the minimum acceptable reliability of a system is 0.85 (p_1 = 0.15), while the contractually specified reliability is 0.95 (P. =

0.05). Consumer and producer risks of 0.11 are required, i.e., $\alpha = \beta = 0.11$. For $\alpha = 0.11$, $z_\alpha = 1.225$ and for $\beta = 0.11$, $z_\beta = 1.225$. (These values of z_α and z_β are obtained from Appendix B, Table 2.) Using the normal approximation, we have

$$n = \{(1.225)^2(0.05-0.0025) + (1.225)^2(0.15-0.0225)$$

$$+ 2(1.225)^2\sqrt{(0.05)(0.15)(0.95)(0.85)}\,]/(0.15-0.05)^2$$

$$= 49.6$$

and

$$c = 1.225\sqrt{(49.6)(0.05)(0.95)} + (49.6)(0.05) - 0.5$$

$$= 3.9\ .$$

The values of n = 49.6 and c = 3.9 are initial approximations. In order to fine tune the test plan, we round these values to n = 50 and c = 4 and use the binomial tables (Appendix B, Table 1). For an n of 50 and a c of 4, the probability of c or fewer failures when $p = p_1 = 0.15$ is 0.1121. In addition, the probability of c or fewer failures when $p = p_0 = 0.05$ is 0.8964. Thus, for the test using a sample size of 50 with a maximum acceptable number of failures of 4, the producer's risk $\alpha = 1 - 0.8964 = 0.1036$, and the consumer's risk $\beta = 0.1121$. Note that these values were obtained directly from Appendix B, Table 1. It would, however, have been difficult at best to decide where to begin looking in the binomial tables without having first used the normal approximation for guidance.

Test Planning Using Poisson Approximation. The Poisson distribution also provides reasonable approximations to inequalities 8.1 and 8.2. All this amounts to is substituting np for λt or t/θ in the Poisson distribution equation. Consequently, approximate values for n and c are obtained by solving the following inequalities.

$$\sum_{k=0}^{c} \frac{(np_1)^k e^{-np_1}}{k!} \le \beta. \tag{8.5}$$

$$\sum_{k=0}^{c} \frac{(np_0)^k e^{-np_0}}{k!} \ge 1 - \alpha. \tag{8.6}$$

Standard test plans and procedures for the Poisson (exponential) are readily available and may be used in lieu of solving inequalities 8.5 and 8.6. This subject is discussed in the "Sources of Exponential Test Plans" section of this chapter. To use these plans in this context, we let $\theta_0 = 1/p_0$, $\theta_1 = 1/p_1$, n = T, and use the acceptable number of failures as given.

As an example, suppose that the minimum acceptable reliability of a system is 0.9 (p_1 = 0.1) and the contractually specified reliability is 0.95 (p_0 = 0.05). Consumer and producer risks are to be 20%, i.e., $\alpha = \beta = 0.20$. To use the Poisson approximation, we define $\theta_0 = 1/p_0 = 1/0.05 = 20$ and $\theta_1 = 1/p_1 = 1/0.1 = 10$. The discrimination ratio, θ_0/θ_1, is 2. Note that test plan XIVC in Appendix B, Table 6, has α and β risks of 19.9% and 21.0%, respectively. This plan requires a test duration T, corresponding to n for this example, of $(7.8)(\theta_1)$ or 78, with five or fewer failures being acceptable.

The term "discrimination ratio" and the use of Appendix B, Table 6, test plans are discussed in detail in the following section.

Case Studies 8-1, 8-2, and 8-3 demonstrate the development of binomial test plans for a variety of α and β values.

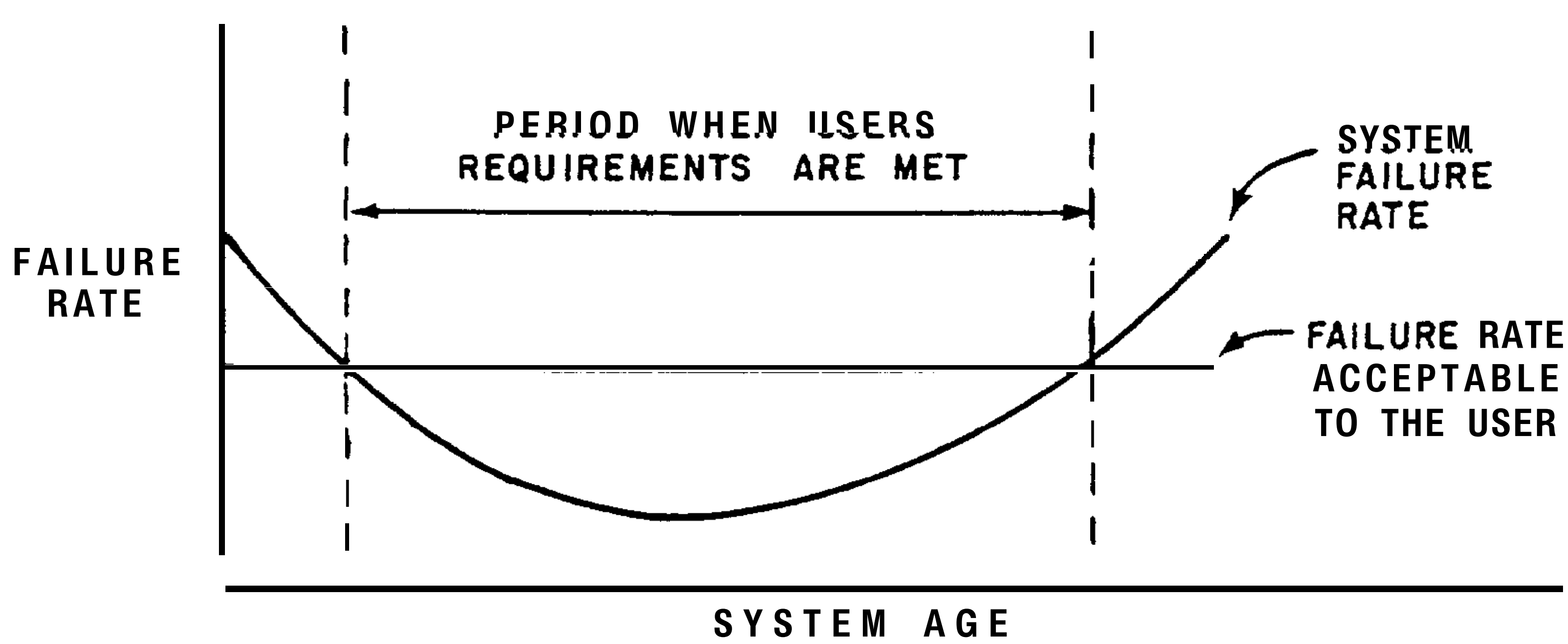
TEST DESIGN FOR CONTINUOUS TIME TESTING: EXPONENTIAL MODEL

The main feature of test planning for continuously operating systems based on the exponential distribution is the assumption that the systems have a constant failure rate.

Requirement Interpretation

When the user's requirement is stated in terms of an MTBF, there is an implication of a constant failure rate. This does not mean that the system must have a constant failure rate. It means, instead, that the need remains constant. Figure 8-3 illustrates that the user's needs may be met during only a portion of the time during the life of a system.

FIGURE 8-3 USER REQUIREMENTS vs SYSTEM PERFORMANCE



Constant System Failure Rate Assumption

The assumption that the system to be tested has a constant failure rate may not be a good one, but it is a practical necessity for determining the amount of testing required. In theory, with the constant failure rate assumption only the total test exposure is important. That is (in theory), one system could be tested for the required test exposure, or many systems could be tested for a short time.

In practice, a test should be planned with a moderate number of systems on test for a moderate period of time. This makes the test relatively insensitive to the constant failure rate assumption. For example, one organization recommends that at least three systems be tested for at least three times the MAV (each) . These are constraints imbedded in the required total test exposure.

Discrimination Ratio

The discrimination ratio, $d = \theta_0/\theta_1$, is a parameter useful in test planning for the exponential model. (For the binomial model, it is necessary to consider the upper and lower test values, p_0 and p_1, explicitly along with the α and β risks.) An interesting feature of the exponential model is that only the ratio of the upper and lower test values, $d = \theta_0/\theta_1$, along with the α and β risks need to be considered. As a consequence, test plans for the exponential models address explicitly the discrimination ratio as a planning parameter.

Sources of Exponential Test Plans

There are numerous methods and references available for developing exponential test plans. Three such approaches are:

1. MIL-STD 105D and MIL-HBK 108.

2. MIL-STD 781C Test Plans.

3. Poisson Distribution Equations .

Reference to MIL-STD 105D and MIL-HBK 108 is included here solely for the sake of completeness. It is our intention that the reader become familiar with methods of exponential test planning using MIL-STD 781C and the Poisson distribution equations. These methods are described below. All the necessary excerps from MIL-STD 781C are provided in Appendix B, Tables 6 and 7.

MIL-STD 105D and MIL-HBK 108. MIL-STD 105D is a document devoted primarily to binomial and Poisson sampling plans, and as such, is mentioned in the previous section. The Poisson sampling plans may be used for continuous time reliability tests. MIL-HBK 108 is devoted to reliability testing based on the exponential distribution. However, it is limited in use for our purposes because it describes test plans for the situation when the test time per unit on test is preset and the number of units is determined. We iterate here that these documents are difficult to interpret, and as such, should only be used by a person familiar with their content.

MIL-STD 781C. The required excerpts from MIL-STD 781C are provided in Appendix B, Tables 6 and 7. Both tables provide information which enable the reader to design a test program which addresses established requirements. The following paragraphs detail the use of both tables.

Appendix B, Table 6: Exponential Test Plans for Standard Discrimination Ratios. This table presents information which supports the de-velopment of a test plan based on discrimination ratios of 1.5, 2.0, and 3.0. For each of these discrimination ratios, four test plans are provided which attain approximate α and β risks of 10% and 10%, 10% and 20%, 20% and 20%, and 30% and 30%. Figure 8-4, in conjunction with the following example problem, illustrates the use of Appendix B, Table 6.

FIGURE 8-4. HOW TO USE APPENDIX B, TABLE 6

1. Identify rows corresponding to the specified "d". (If not in Appendix B, Table 6, use Table 7 plans.)
2. Identify desired α and β risks.
3. Identify test plan number (for reference).
4. Identify test duration multiplier.
5. Determine total test time as θ_1 times multiplier.
6. Identify accept/reject criteria.

Test Plan	True Decision Risks α	True Decision Risks β	Discrimination Ratio θ_0/θ_1	Test Duration Multiplier (M) $T = M\theta_1$	Accept-Reject Failures: Reject (Equal or More)	Accept-Reject Failures: Accept (Equal or Less)
IXC*	12.0%	9.9%	1.5	45.0	37	36
XC	10.9%	21.4%	1.5	29.9	26	25
XIC	17.8%	22.4%	1.5	21.1	18	17
XIIC	9.6%	10.6%	2.0	18.8	14	13
XIVC	19.9%	21.0%	2.0	7.8	6	5
XVC	9.4%	9.9%	3.0	9.3	6	5
XVIC	10.9%	21.3%	3.0	5.4	4	3
XVIIC	17.5%	19.7%	3.0	4.3	3	2

*NOTE : C refers to Revision C of MIL-STD-781.

How To Use Appendix B, Table 6. As an example, suppose that the upper test MTBF is 900 hours and the lower test MTBF is 300, so that the discrimination ratio (d) is 900/300 = 3. Consumer and producer risks of approximately 10% for each are required. Now, as shown in Figure 8-4, test plan XVC has α and β risks of 9.4% and 9.9%, respectively, and the discrimination ratio is 3. Test plan XVC requires a test length (T) of 9.3 times the lower MTBF of 300, so T = (9.3)(300) = 2790 hours. The acceptance criterion is to accept with 5 or fewer failures and reject with 6 or more failures. Note that if the upper test MTBF had been 90 and the lower test MTBF had been 30, the same test plan is appropriate. However, in this situation the test duration (T) is (9.3)(30) or 279 hours, whereas the accept/reject criterion remains the same.

Case Study 8-4 is another example illustrating the use of this table.

Appendix B, Table 7: Supplemental Exponential Test Plans. This table presents information which supports the development of a test plan based on combinations of α and β risks of 10%, 20%, and 30%. Figure 8-5, in conjunction with the following example problem, illustrates the use of Appendix B, Table 7.

How to Use Appendix B, Table 7. Concerning Figure 8-5 and Appendix B, Table 7, note the following:

If the discrimination ratio, d, is not given exactly in the tables, going to the next lower value will give a conservative (i.e., longer) test time requirement.

Consider once again, the example where $\theta_0 = 900$, $\theta_1 = 300$, and the desired α and β risks are 10% each. Recall that the discrimination ratio was 3. To select a test plan from Figure 8-5, we search the column labeled as a 10% producer's risk to find the number closest to 3. In this case, test plan 10-6 has a discrimination ratio of 2.94. The test duration is (9.27)(300) or 2781 hours with 5 being the maximum acceptable number of failures. Note how this plan compares with test plan XVC which has the same acceptance criterion, requires 2790 hours of test time, and has a discrimination ratio of 3. Case Study 8-5 further illustrates the use of this table.

Graphical Representation of Test Planning Parameters. Figure 8-6 graphically illustrates the interaction between α and β risks and test length for three-commonly used discrimination ratios. The graphs do not provide complete test planning information since no acceptance criterion is specified. These curves are useful tools for conducting tradeoff analyses between risk levels and test length. Note that some of the specific test plans presented in Appendix B, Tables 6 and 7 are displayed on the curves, i.e., 30-7, 10-19, 20-7, etc.

To illustrate how the graphs may be used, consider that $\theta_1 = 100$ hours, $\theta_0 =$ 150 hours, and a test duration of 2500 hours is affordable. To enter the

FIGURE 8-5. HOW TO USE APPENDIX B, TABLE 7

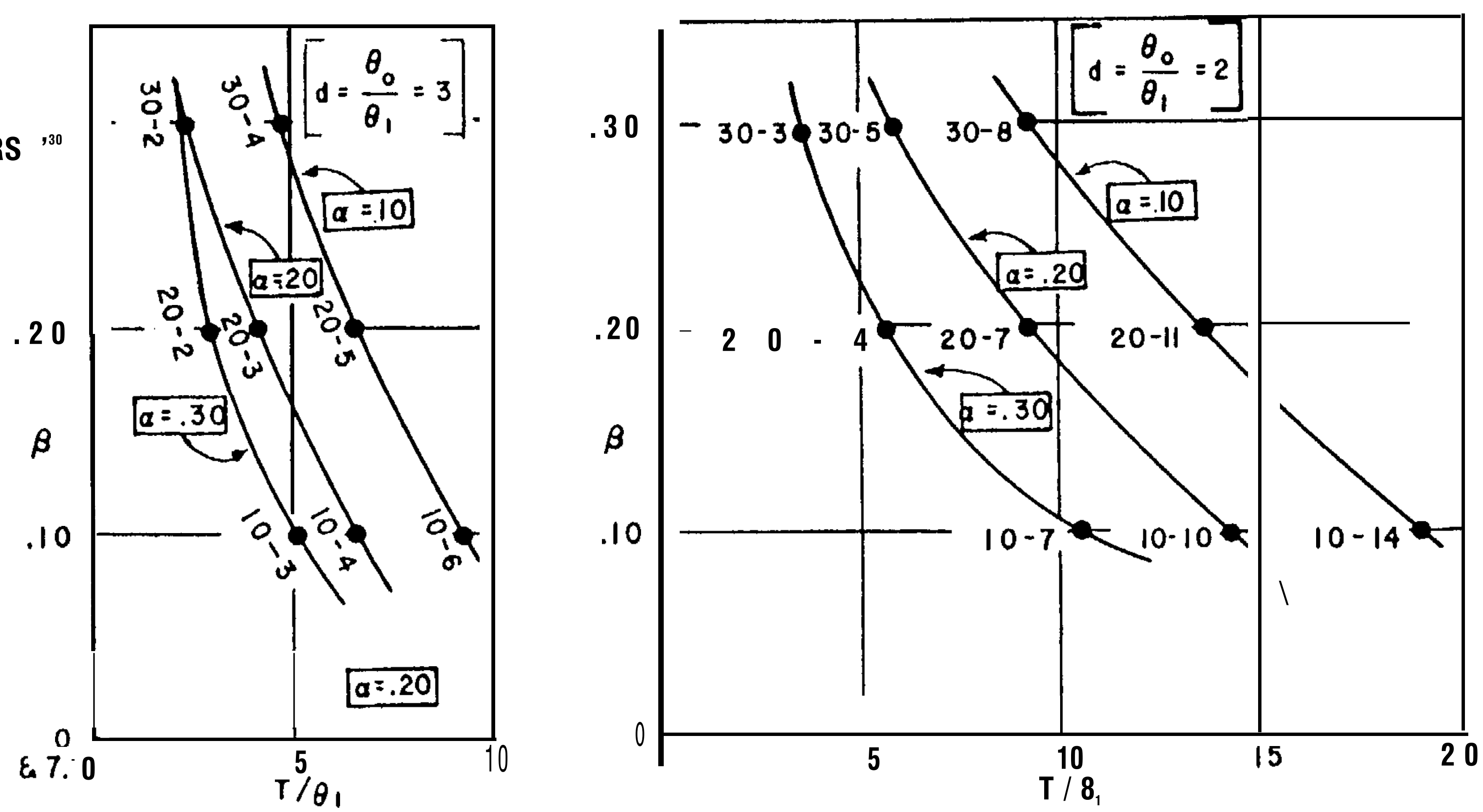
1. Identify desired β risk (10% β risk table shown below).
2. Identify column for desired α risk.
3. Identify row for d.
4. Identify test plan number (for reference).
5. Identify total test time multiplier.
6. Determine total test time as θ_1 times multiplier.
7. Identify accept/reject criteria.

Test Plan No's	No. Failures Acc.	No. Failures Rej.	Test Duration Multiplier (M) $T = M\theta_1$	Discrimination Ratio θ_0/θ_1 For Producer's Risk 30%	20%	10%
10-1	0	1	2.30	6.46	10.32	21.85
10-2	1	2	3.89	3.54	4.72	7.32
10-3	2	3	5.32	2.78	3.47	4.83
10-4	3	4	6.68	2.42	2.91	3.83
10-5	4	5	7.99	2.20	2.59	3.29
10-6	5	6	9.27	2.05	2.38	2.94
10-7	6	7	10.53	1.95	2.22	2.70
10-8	7	8	11.77	1.86	2.11	2.53
10-9	8	9	12.99	1.80	2.02	2.39
10-10	9	10	14.21	1.75	1.95	2.28
10-11	10	11	15.41	1.70	1.89	2.19
10-12	11	12	16.60	1.66	1.84	2.12
10-13	12	13	17.78	1.63	1.79	2.06
10-14	13	14	18.96	1.60	1.75	2.00
10-15	14	15	20.13	1.58	1.72	1.95
10-16	15	16	21.29	1.56	1.69	1.91
10-17	16	17	22.45	1.54	1.67	1.87
10-18	17	18	23.61	1.52	1.62	1.84
10-19	18	19	24.75	1.50	1.62	1.81
10-20	19	20	25.90	1.48	1.60	1.78

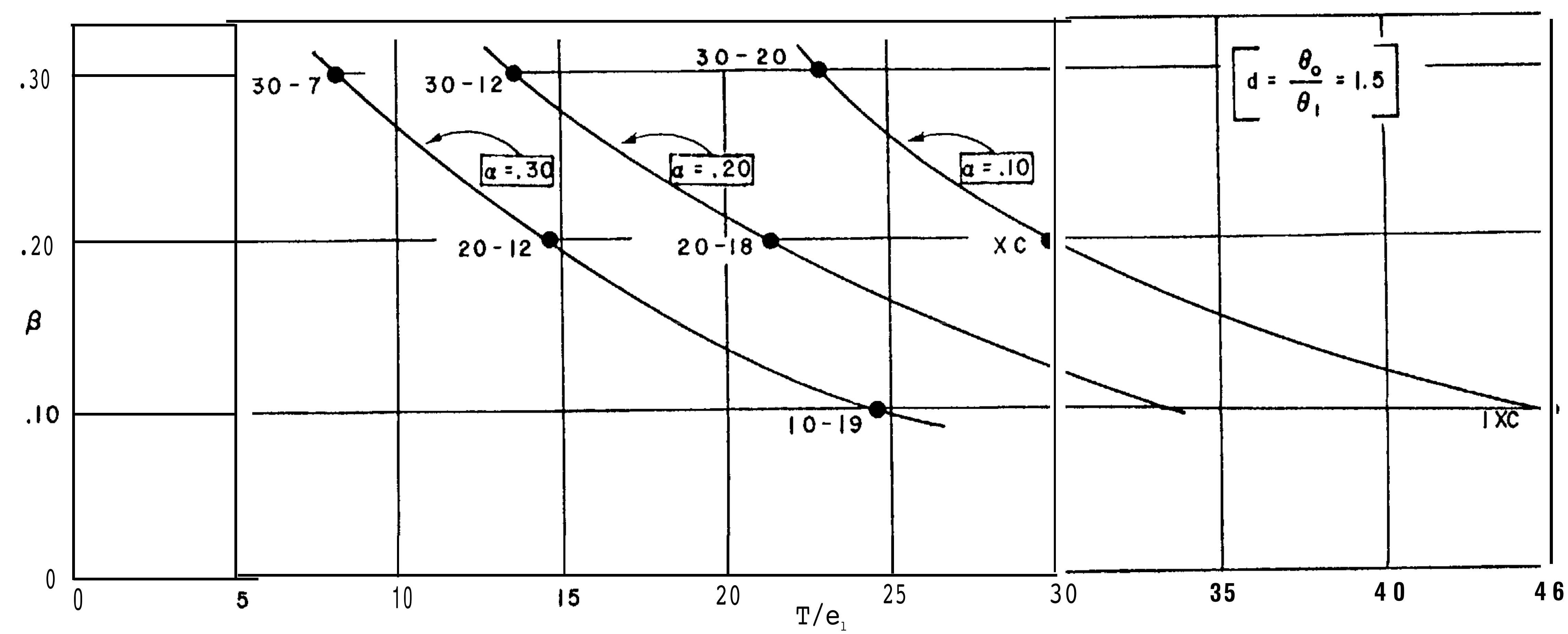
FIGURE 8-6

GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS

T = TOTAL TEST EXPOSURE



NOTE:
PLOTTING POINTS ARE CODED TO INDICATE THE MIL-STD 781C TEST PLAN THESE PLANS ARE CONTAINED IN APPENDIX B, TABLES 6 & 7.



graphs, use $d = \theta_0/\theta_1 = 150/100 = 1.5$ and $T/\theta_1 = 2500/100 = 25$. Reading up through the three curves , we find the following risk combinations:

$\alpha = 0.30 \qquad \beta = 0.10$

$\alpha = 0.20 \qquad \beta = 0.16$

$\alpha = 0.10 \qquad \beta = 0.26$

If one of these combinations is tolerable, the test length is adequate. To reduce one or both risks, the test duration must be increased. Tolerating greater risks permits reduction of the test duration. Case Study 8-6 further illustrates the use of these graphs.

Figure 8-7 is a graphical portrayal of the interaction between test length and risk when α and β risks are equal. Curves for each of the three values (1.5, 2.0, 3.0) of the discrimination ratio appear on the same graph. Case Study 8-6 illustrates the use of Figure 8-7.

Poisson Distribution Equations. When a standard test plan for a specific combination of θ_0, θ_1, α, and β is not available, the test designer may use the Poisson equations to develop a test plan.

The following notation is used in the discussion of the Poisson equation technique.

T = Total test exposure

θ = True MTBF

c = Maximum acceptable number of failures

θ_0 = Upper test MTBF

θ_1 = Lower test MTBF

α = Producer's risk

β = Consumer's risk

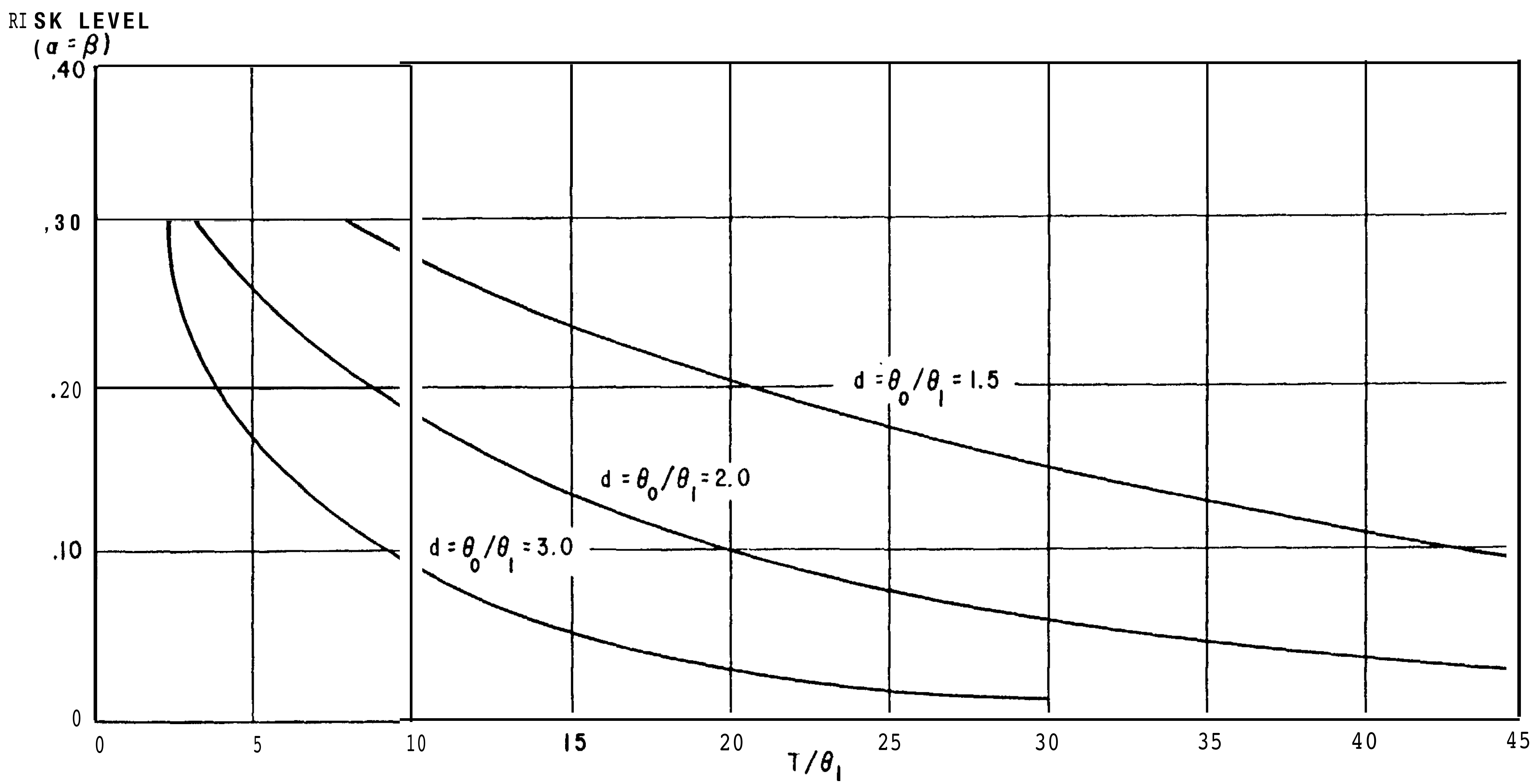
$P(ac|\theta)$ = Probability of accepting the system assuming the true MTBF is θ.

$P(rej|\theta)$ = Probability of rejecting the system assuming the true MTBF is θ.

The probability of acceptance is the probability that no more than a certain (acceptable) number of failures will occur. This probability can be computed using the equation:

$$P(ac|\theta) = \sum_{k=0}^{c} \frac{(T/\theta)^k e^{-(T/\theta)}}{k!} \qquad (8.7)$$

FIGURE 8-7 GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS FOR $\alpha = \beta$



This is the Poisson Distribution Equation. This distribution and assumptions regarding its applications are discussed in Chapter 5.

The consumer's risk (β) is the probability that during the test no more than the acceptable number of failures will occur when the true MTBF is θ_1. Consequently,

$$\beta = P(ac \quad \theta=\theta_1)$$

$$= P(c \text{ or fewer failures } \theta=\theta_1) \text{ ,}$$

where c is the maximum acceptable number of failures. Thus ,

$$\beta = \sum_{k=0}^{c} \frac{(T/\theta_1)^k \; e^{-(T/\theta_1)}}{k!} \quad . \tag{8.8}$$

The producer's risk (α) is the probability that during the test more than the acceptable number of failures will occur when the true MTBF is θ_0. Consequently,

$$\alpha = P(rej \quad \theta=\theta_0)$$

$$= P(c + 1 \text{ or more failures } \theta=\theta_0) \text{ .}$$

Since

$$P(c + 1 \text{ or more failures } \theta=\theta_0) = I - P(c \text{ or fewer failures } \theta=\theta_0) \text{ ,}$$

we have that

$$\alpha = 1 - \sum_{k=0}^{c} \frac{(T/\theta_0)^k \; e^{-(T/\theta_0)}}{k!} \text{ ,}$$

or equivalently,

$$1 - \alpha = \sum_{k-0}^{c} \frac{(T/\theta_0)^k \; e^{-(T/\theta_0)}}{k!} \tag{8.9}$$

In order to determine the complete test plan, we must solve equations 8.8 and 8.9 simultaneously for T and c.

Solving these equations directly without the aid of a computer is too tedious and time consuming to be considered practical. We therefore present the following graphical solution procedure which utilizes the Poisson Chart, Chart No. 1 in Appendix B.

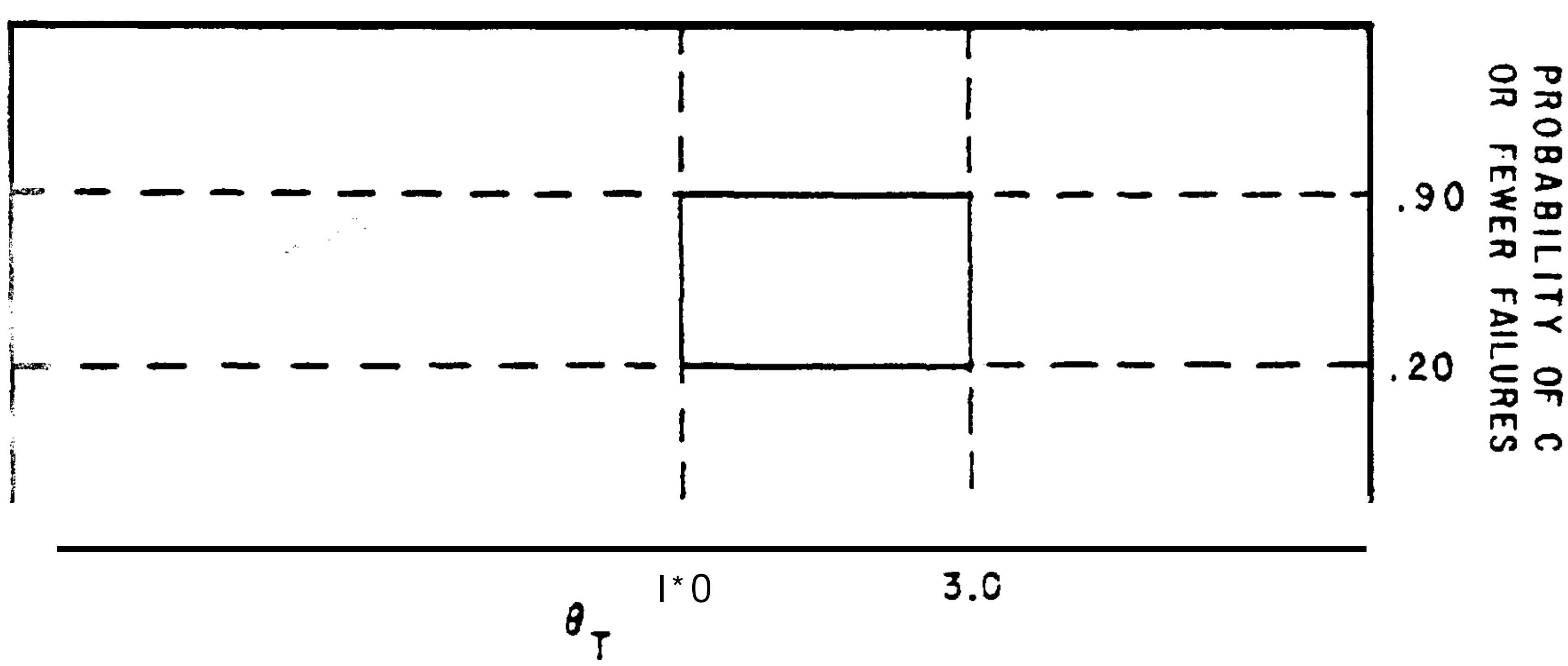
Graphical Poisson Solution Procedure. We wish to find a test exposure, T, and an acceptable number of failures, c, such that the probability of acceptance is β when $\theta = \theta_1$ and $1 - \alpha$ when $\theta = \theta_0$. This is done graphically with the use of a transparent overlay.

On an overlay sheet, draw vertical lines at $\theta/T = 1$ and $\theta/T = \theta_0/\theta_1$. Draw horizontal lines at probabilities β and $1 - \alpha$, forming a rectangle. Slide the overlay rectangle horizontally until a curve for a single value of c passes through the lower left and upper right corners. (It may not be possible to hit the corners exactly. Conservative values of c will have curves that pass through the horizontal lines of the rectangle.) This value of c is the acceptable number of failures. Read the value of θ/T corresponding to the left side of the rectangle. Divide θ_1 by this value to find T, the required test exposure. The following numerical example illustrates the use of the graphical Poisson Solution Procedure.

We wish to find the required test exposure, T, and acceptable number of failures c; such that when the MTBF, $\theta = \theta_1 = 100$ hours, the probability of acceptance, β, will be 0.20 and then $\theta = \theta_0 = 300$ hours the probability of acceptance, $1 - \alpha$, will be 0.90.

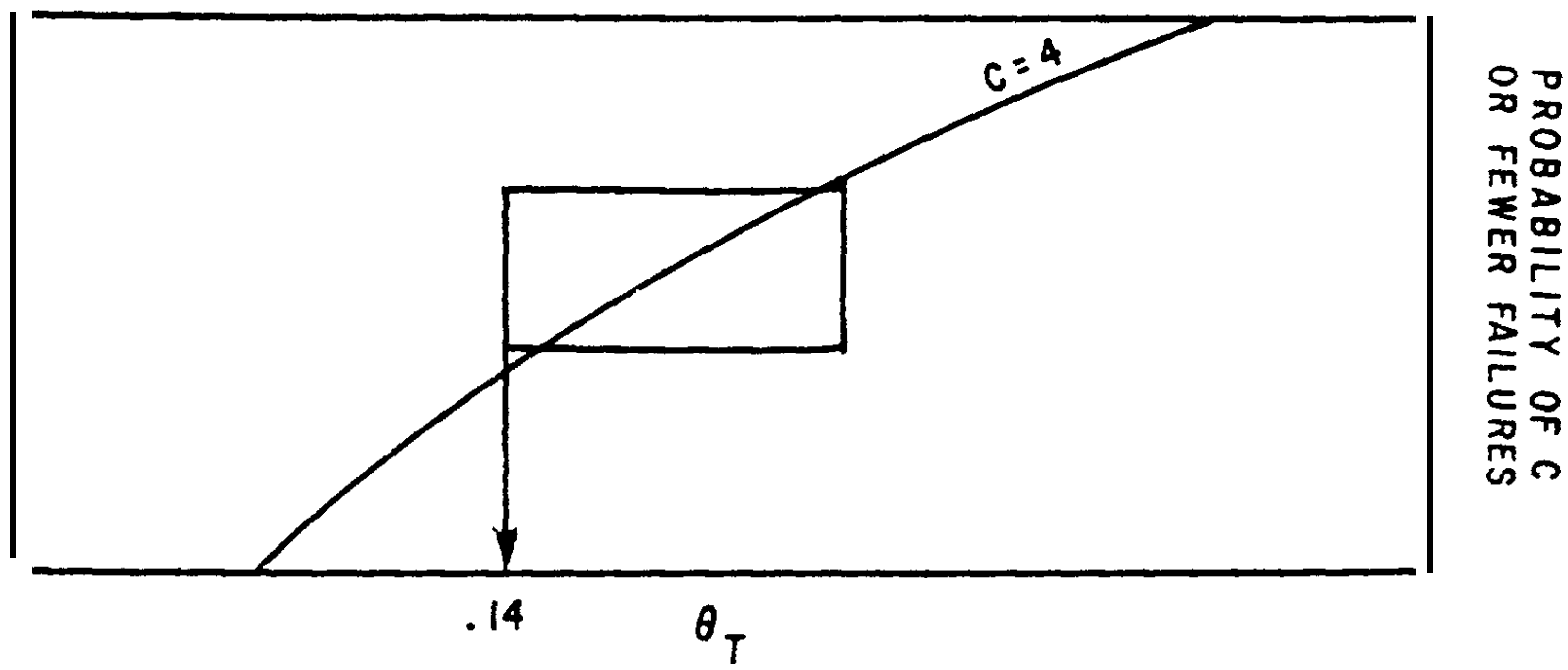
An overlay rectangle is constructed as shown.

FIGURE 8-3 OVERLAY CONSTRUCTION TECHNIQUE



Sliding the rectangle to the left, we find that when c = 3 the fit is close, but slightly higher risks must be tolerated. Going to c = 4, the curve passes through the horizontal lines of the rectangle. At the left of the rectangle, $\theta/T = 0.14$, so the required test exposure is approximately 100/0.14 = 714 hours and the acceptance criterion is 4 or fewer failures.

FIGURE 8-9 OVERLAY CURVE MATCHING PROCEDURE



OPERATING CHARACTERISTIC (OC) CURVES

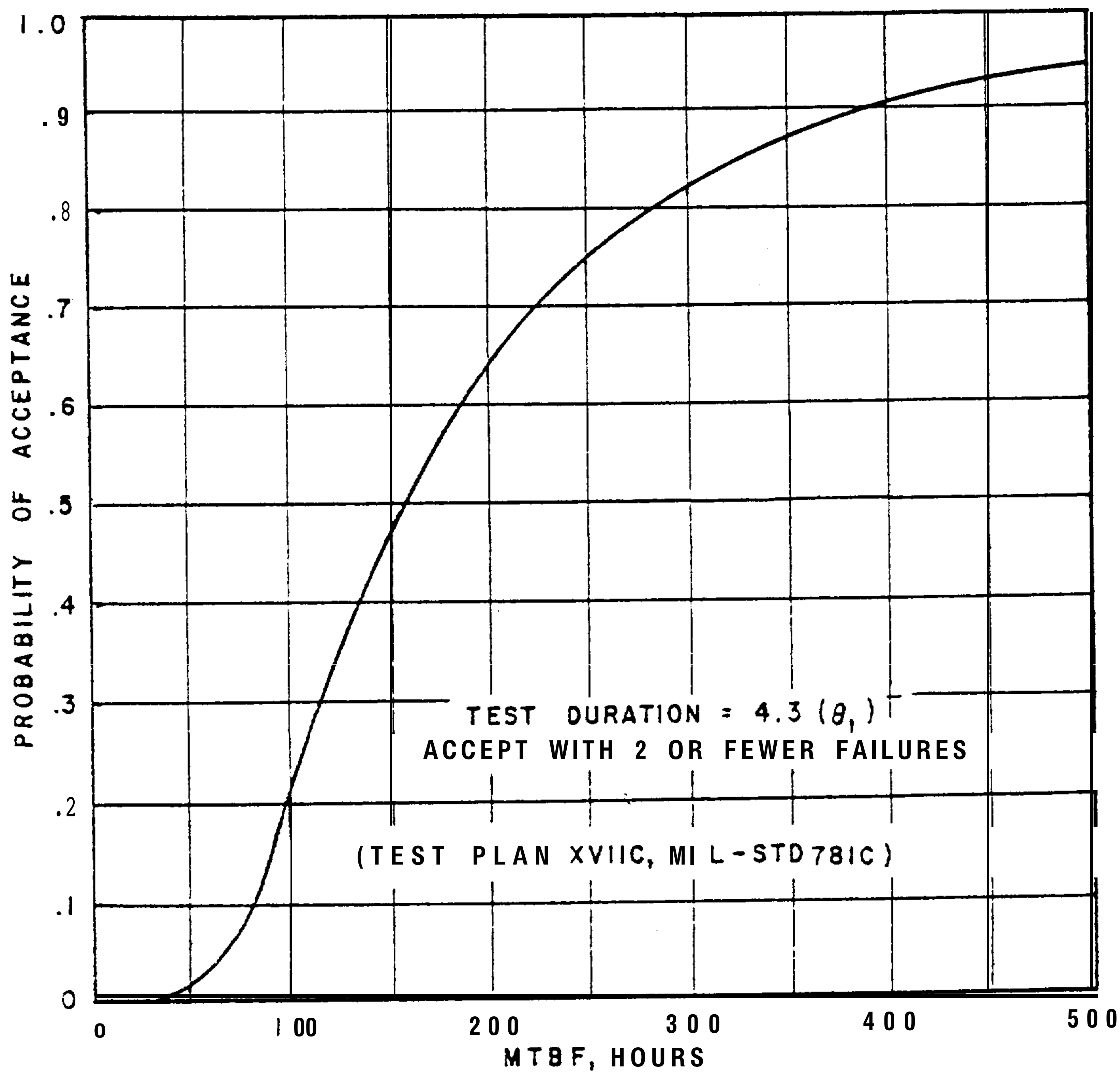
Introduction

In the previous sections of this chapter, we have discussed methods for developing test plans which achieve required α and β risks. The test plan itself is specified by the test exposure and the maximum acceptable number of failures. For a test plan developed using the methods in this chapter, we know that the producer's risk (the probability of rejecting a good system) for the specified value (SV) is α and the consumer's risk (the probability of acceptance) for the minimum acceptable value (MAV) is β. In addition, to assess a test plan proposed by another party, we have shown methods for computing the producer's risk and the consumer's risk for the SV and MAV, respectively. A graphical tool which provides more complete information about a specific test plan is the operating characteristic (OC) curve. The OC curve displays both acceptance and rejection risks associated with all possible values of the reliability parameter and not merely the SV and MAV. By definition, an OC curve is a plot of the probability of acceptance (the ordinate) versus the reliability parameter value (the abscissa) .

Figure 8-10 contains the operating OC curve for test plan XVICC from MIL-STD 781C, with θ_1, the lower test MTBF, assumed to be 100 hours.

Consider a single point on the curve, say an MTBF of 200 hours and a probability of acceptance of 0.63. This means that for test plan XVIIC (test duration of 430 hours, accept with 2 or fewer failures) , a system which has a true MTBF of 200 hours has a 63% chance of passing this test, i.e. , being accepted. A system requires an MTBF of around 400 hours in order for the producer to be at least 90% confident that the system will be accepted. A system whose true MTBF is about 80 hours has only a 10% chance of being accepted.

FIGURE 8-10 OPERATING CHARACTERISTIC (OC) CURVE



FOR THIS EXAMPLE θ_1 IS ASSUMED TO EQUAL 100 HOURS

Operating characteristic curves for all the test plans in Appendix B, Table 6 of this text can be found in Appendix C of MIL-STD 781C. However, OC curves for the test plans in Appendix B, Table 7, of this text are not available in MIL-STD 781C.

OC Curve Construction

The OC curve shown in Figure 8-10 is a representation of the mathematical model used to compute the reliability for a system. We have discussed two basic models in previous sections. The Poisson/exponential model is used for systems undergoing continuous time testing and the binomial model is used for discrete time tests.

The OC curve specifically displays the relationship between the probability of acceptance and MTBF.

For the Poisson/exponential model, we indicated in equation 8.7 that

$$P(ac|\,0) = \sum_{k=0}^{c} \frac{(T/\theta)^k\, e^{-(T/\theta)}}{k!} \qquad (8.10)$$

where

k = Number of failures

T = Total test time

θ = Values of MTBF

c = Maximum acceptable number of failures

Referring to Figure 8-10, let us assume that θ_1 = 100 hours and that we have selected test plan XVIIC from Appendix B, Table 6 which permits a maximum of two failures. Using θ_1 = 100 hours, which corresponds to a test duration of 430 hours for test plan XVIIC (T = 4.301), and c = 2, we can determine points on the curve by calculating P(ac θ) for different values of θ. As an example, for θ= 215 hours

$$P(ac|\theta{=}215) = \frac{\left(\frac{430}{215}\right)^0 e^{-\left(\frac{430}{215}\right)}}{0!} + \frac{\left(\frac{430}{215}\right)^1 e^{-\left(\frac{430}{215}\right)}}{1!} + \frac{\left(\frac{430}{215}\right)^2 e^{-\left(\frac{430}{215}\right)}}{2!}$$

$$= \frac{2^0 e^{-2}}{0} + \frac{2^1 e^{-2}}{1} + \frac{22e-2}{2}$$

$$= 0.135 + 2(0.135) + 2(0.135)$$

$$= 0.676\ .$$

By choosing a sufficient number of values for θ between 0 and 500 and computing the probability of acceptance for each, we can construct a smooth curve.

For the binomial model, the probability of acceptance is expressed by the equation

$$P(ac|\,p) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=0}^{c} \frac{n!}{[k!\ (n-k)!]}\, p^k\, (1-p)^{n-k}$$

where

n = Number of trials

c = Maximum acceptable number of failures

p = Probability of failure on any trial.

By inserting specific values for n, k and by varying the probability of failure on any trial, p, we can compute values of the probability of acceptance which permit us to construct an OC curve.

For example, by letting n = 5 and c = 2, calculate the probability of acceptance for p = 0.1.

$$P(ac \quad p=0.1) = \frac{5!}{0!(5-0)!}(0.1)^0(1-0.1)^{5-0} + \frac{5!}{1!(5-1)!}(0.1)^1(1-0.1)^{5-1}$$

$$+ \frac{5!}{2!(5-2)!}(0.1)^2(1-0.1)^{5-2}$$

$$= \frac{(120)(1)(0.9)^5}{120} + \frac{(120)(0.1)(0.9)^4}{24} + \frac{(120)(0.1)^2(0.9)^3}{(2)(6)}$$

$$= (0.9)^5 + (0.5)(0.9)^4 + (0.1)(0.9)^3$$

$$= 0.59 + 0.33 + 0.07 = 0.99.$$

Thus ,

$$P(ac \quad p=0.1) = 0.99 .$$

As expected, the probability of acceptance is very high since we have designed a relative easy test to pass.

CASE STUDY NO. 8-1

Background

A mechanical system which controls the aiming point of a large-caliber gun is under development. The specified and minimum acceptable values for the probability of aiming correctly are 0.85 and 0.70 respectively. Testing requires that expensive projectiles be fired for each trial, and only 20 rounds are allotted for testing.

Determine

1. Propose a test plan which equalizes the consumer's and producer's risks. What are the risks?

2. The user can tolerate a risk of no worse than 5%. What test plan gives the best (smallest) producer's risk?

Solutions

1. As mentioned in Chapter 6, "Statistical Concepts", consumer' s risk increases when producer's risk decreases, and vice versa, when the sample size is fixed. Theoretically, there is a point where they are equal or almost equal.

It is also important to understand that the analytical interpretation of producer' s and consumer' s risk when determining the solution to question no. 1. The producer' s risk is the probability of rejecting a system which meets the SV of 0.15 proportion of failures (reliability of 0.85). For a given accept/ reject criterion (determined by a value c which represents the maximum number of failures which results in acceptance of the system) , the producer' s risk, α, is the probability that c + 1 or more failures occur. The consumer's riks is the probability of accepting a system which exceeds the MAV of 0.30 proportion of failures (reliability of 0.70) . For the same accept/reject criterion, the consumer' s risk, β, is the probability that c or fewer failures occur. Below is a section of binomial tables for n = 20, extracted from Appendix B, Table 1.

c	β = P(c or fewer failures) $p_1 = 0.30$	$1-\alpha$ = P(c or fewer failures) $p_0 = 0.15$	α = P(c+1 or more failures) $p_0 = 0.15$
0	0.00	0.04	0.96
1	0.00	0.18	0.82
2	0.03	0.40	0.60
3	0.11	0.65	0.35
4	0.24	0.83	0.17
5	0.42	0.93	0.07
6	0.61	0.98	0.02
7	0.77	0.99	0.01

The proposed test plan is to accept with 4 or fewer failures and reject with 5 or more failures. The consumer's and producer's risks are 0.24 and 0.17, respectively.

2. From the above table, we see that test plans which have the maximum acceptable number of failures (c) of O, 1, and 2, and satisfy the consumer's risk of no more than 5%. The best (smallest) producer' s risk occurs when c = 2, the risk being 0.60.

CASE STUDY NO. 8-2

Background

A new, highly reliable missile system is under development. The specified reliability (SV) is 0.98, and the minimum acceptable reliability (MAV) is 0.85.

Determine

1. Design test plans for producer's risks of 5%, 10%, and 20%, with a consumer's risk of 5%.

2. Design test plans for a producer's risk of 10% and for a consumer's risk of 10%.

3. Redo number 2 if the MAV is 0.92 instead of 0.85.

Solutions

Note that a reliability of 0.98 corresponds to a proportion of failures, or unreliability, of 0.02, and a reliability of 0.85 corresponds to a proportion of failures, or unreliability, of 0.15. Thus, we list our test planning parameters p_0 and p_1 as 0.02 and 0.15, respectively.

1a. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.05$ $\beta = 0.05$

i. Normal Approximation. In order to determine a starting point for our analysis, we calculate approximate values of n and c using equations 8.3 and 8.4. For values of z_α and z_β, use Appendix B, Table 2.

$$n = \{(1.645)^2(0.02-0.0004) + (1.645)^2(0.15-0.0225)$$

$$+ 2(1.645)^2 \sqrt{(0.02)(0.15)(0.98)(0.85)}\}/(0.15-0.02)^2$$

$$= 39.6$$

$$c = (1.645)\sqrt{(39.6)(0.02)(0.98)} + (39.6)(0.02) - 0.5$$

$$= 1.7$$

ii. Poisson Approximation (Appendix B, Chart 1, with np = T/El, the reciprocal of θ/T)

c	np_1	n	np_0	β	1-α	α
0	3.0	20.0	0.40	0.05	0.69	0.32
1	4.7	34.3	0.63	0.06	0.88	0.12
2	6.3	42.0	0.84	0.05	0.94	0.06
3	7.8	52.0	1.04	0.05	0.97	0.03

NOTE :
α = P(c + 1 or more failures)
β = P(c or fewer failures)
$1-\alpha$ = P(c or fewer failures)

iii. Proposed Test Plans. It appears from i and ii above that a good starting point for fine tuning is an n of 40 and c of 2. Using Appendix B, Table 1 to fine tune, we propose the following test plans.

n	c	α	β
40	2	0.04	0.05
39	2	0.04	0.05
38	2	0.04	0.06
37	2	0.04	0.07
* 36	2	0.03	0.08

*The protection afforded by this plan seems to be adequate though the consumer's risk is 8% (slightly above the required 5%).

1b. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.10$ $\beta = 0.05$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.645)^2(0.15-0,0225)$$

$$+ 2(1.28)(1.645)\sqrt{(0.02)\ (0.15)(0.98)(085)}\ ]/(0.15-0.02)^2$$

$$= 34.8$$

$$c = (1.28)\sqrt{(34.8)\ (0.02)(0.98)} + (34.8)(0.02) - 0.5$$

$$= 1.3$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_o	β	$1-\alpha$	α
0	3.0	20.0	0.40	0.05	0.69	0.31
1	4+7	34.3	0.63	0.05	0.88	0.12
2	6.3	42.0	0.84	0.05	0.94	0.06
3	7.8	52.0	1.04	0.05	0.97	0.03

iii. It appears that a good starting point for fine tuning is an n of 35 and c of 1. The following test plans are proposed.

	n	c	α	β
	35	1	0.15	0.03
	34	1	0.15	0.03
	33	1	0.14	0.03
	32	1	0.13	0.04
	31	1	0.13	0.04
	30	1	0.12	0.05
	29	1	0.11	0.05
*	28	1	0.11	0.06

'The actual risks exceed the required risks of 10% and 5% but not to any significant extent.

1.c. $p_0 = 0.02 \quad p_1 = 0.15 \quad \alpha = 0.20 \quad \beta = 0.05$

i. Normal Approximation

$$n = \{(0.84)^2(0.02-0.0004) + (1.645)^2(0.15-0.0225)$$

$$+ 2(0.84)(1.645)\sqrt{(0.02)(0.15)(0.98)(0.85)}\,\}/(0.15-0.02)^2$$

$$= 29.4$$

$$c = (0.84)\sqrt{(29.4)(0.02)(0.98)} + (29.4)(0.02) - 0.5$$

$$= 0.72$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
0	3.0	2.0	0.40	0.05	0.69	0.31
1	4.7	34.3	0.63	0.05	0.88	0.12

iii. It appears that a good starting point for fine tuning is an n of 25 and c of 1. The following programs are proposed.

	n	c	α	β
*	25	1	0.09	0.09
	25	**0**	**0.40**	**0.02**

$'Generally, there is no reasonable test plan for the input values given. A very large sample size is required to achieve an α of 0.20. (For n = 40, α = 0.19, and β = 0.01, with a c of O.) This sample size seems unwarranted. Our recommendation is to use the n of 25 and the c of 1.

2. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.10$ $\beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.28)^2(0.15-0.0225)$$

$$+ 2(1.28)^2\sqrt{(0.02)(0.15)(0.98)(0.85)}\}/(0.15-0.02)^2$$

$$= 24.2$$

$$c = (\ .28)\sqrt{(24.2)(0.02)(0.98)} + (24.2)(0.02) - 0.5$$

$$= 0\ 86$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
o	2.3	15.3	0.31	0.10	0.74	0.26
1	3.9	26.0	0.52	0.10	0.90"	0.10
2	5.2	35.0	0.70	0.10	0.96	0.04

iii. It appears that a good starting point for fine tuning is an n of 25 and c of 1. The following programs are proposed.

	n	c	α	β
*	25	1	0.09	0.09
	24	1	0.08	0.11

*The test plans with a sample size of 25 fits well. The sample size can be reduced by 1 to 24 if the consumer allows his risk to be 11%.

3. $p_0 = 0.02$ $p_1 = 0.08$ $\alpha = 0.10$ $\beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.28)^2(0.08-0.0064)$$

$$+ 2(1.28)^2 \sqrt{(0.02)(0.08)(0.98)(0.92)}\}/(0.08-0.02)^2$$

$$= 77.0$$

$$c = (1.28)\sqrt{(77.0)(0.02)(0.98)} + (77.0)(0.02) - 0.5$$

$$= 2.6$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
1	3.9	48.8	0.87	0.10	0.75	0.25
2	5.3	66.3	1.32	0.10	0.88	0.12
3	6.7	83.8	1.67	0.10	0.91	0.09
4	8.0	100.0	2.00	0.10	0.94	0.06

iii. It appears that a good starting point for fine tuning is an n of 75 and c of 3. The following programs are proposed.

n	c	α	β
75	3	0.06	0.14
76	3	0.07	0.13
77	3	0.07	0.13
78	3	0.07	0.13
79	3	0.07.	0.11
80	3	0.08	0.10

CASE STUDY NO. 8-3

Background

An operational test is being considered for a disposable survival ratio which must work for at least three hours. The ratio has a specified mission reliability of 0.85 and a minimum acceptable mission reliability of 0.7. A number of radios will be put on test for-three hours each and the-number of failures recorded.

Determine

1. Propose some test plans for a producer's risk of about 10% and consumer's risks of about 10%, 20%, and 30%.

2. Propose a test plan for a minimum acceptable reliability of 0.5 with a user risk of 2% and a producer risk of 20%.

Solutions

Note that a reliability of 0.85 corresponds to a proportion of failures or "unreliability" p. of 0.15.

1a. $p_0 = 0.15 \quad p_1 = 0.3 \quad \alpha = 0.10 \quad \beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (1.28)^2(0.3-0.09)$$

$$+ 2(1.28)^2 \sqrt{\{(0.15)(0.3)(0.85)(0.7)]}/(0.3-0.15)^2$$

$$= 48.4$$

$$c = 1.28\sqrt{(48.4)(0.15)(0.85)} + (48.4)(0.15) - 0.5$$

$$= 9.9$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
7	12.0	40.0	6.0	0.10	0.75	0.25
8	13.0	43.3	6.5	0.10	0.80	0.20
9	14.0	46.7	7.0	0.10	0.83	0.17
10	**15.5**	**51.7**	**7.7**	**0.10**	**0.85**	**0.15**
11	16.5	55.0	8.2	0.10	0.88	0.12
12	18.0	60.0	9.0	0.10	0.89	0.11
13	19.0	63.0	9.5	0.10	0.90	0.10

iii. It appears that a good starting point for fine tuning is an n of 50 and c of 10. The following programs are proposed.

n	c	α	β
50	10	0.12	0.08
49	10	0.11	0.09
48	10	0.10	0.10

1b. $p_0 = 0.15$ $p_1 = 0.3$ $\alpha = 0.10$ $\beta = 0.20$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (0.84)^2(0.3-0.09)$$

$$+ 2(0.84)(1.28)\sqrt{(0.15)(0.3)(0.85)(0.7)}\}/(0.3-0.15)^2$$

$$= 31.5$$

$$c = (1.28)\sqrt{(31.5)(0.15)(0.85)} + (31.3)(0.15) - 0.5$$

$$= 6.78$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
6	9.1	30.7	4.6	0.20	0.82	0.18
7	10.3	34.3	5.1	0.20	0.85	0.15
8	11.5	38.3	5.7	0.20	0.88	0.12
9	12.5	41.7	6.3	0.20	0.90	0.10
10	13.8	46.0	6.9	0.95	0.20	0.05

iii. It appears that a good starting point for fine tuning is an n of 35 and c of 7. The following programs are proposed.

n	c	α	β
35	7	0.14	0.13
34	7	0.12	0.16
33	7	0.11	0.19
32	7	0.10	0.21

1.c. $p_0 = 0.15$ $p_1 = 0.30$ $\alpha = 0.10$ $\beta = 0.30$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (0.526)^2(0.3-0.09)$$
$$+ 2(0.526)(1.28)\sqrt{(0.15)(0.3)(0.85)(0.7)}\,]/(0.3-0.15)^2$$
$$= 21.7$$

$$C = (1.28)\sqrt{(21.7)(0.15)(0.85)} + (21.7)(0.15) - 0.5$$
$$= 4.9$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
4	5.9	19.7	2.9	0.30	0.82	0.18
5	7.0	23.3	3.5	0.30	0.87	0.13
6	8.4	28.0	4.2	0.30	0.89	0.11

iii. It appears that a good starting point for fine tuning is an n of 22 and c of 5. The following programs are proposed.

n	c	α	β
22	5	0.10	0.31
23	5	0.12	0.27

2. $p_0 = 0.15$ $p_1 = 0.5$ $\alpha = 0.20$ $\beta = 0.02$

i. Normal Approximation

$$n = \{(0.84)^2(0.15-0.0225) + (2.06)^2(0.5-0-25)$$
$$+ 2(0.84)(2.06)\sqrt{(0.15)(0.5)(0.85)(0.5)}\}/(0.5-0.15)^2$$
$$= 14.4$$

$$C = (0.84)\sqrt{(14.4)(0.15)(0.85)} + (14.4)(0.15) - 0.5$$
$$= 2.8$$

ii. Poisson Approximation (Appendix B, Chart 6)

c	np_1	n	np_o	β	1-α	α
2	7.5	15.0	2.25	0.02	0.63	0.37
3	9.1	18.2	2.70	0.02	0.73	0.27
4	10.6	21.2	3.20	0.02	0.80	0.20

iii. It appears that a good starting point for fine tuning is an n of 15 and c of 3. The following programs are proposed.

n	c	α	β
15	3	0.18	0.02
14	3	0.16	0.03

CASE STUDY NO. 8-4

Background

A communication system has minimum acceptable value (MAV) of 100 hrs MTBF, and a specified value (SV) of 150 hrs MTBF.

Determine

How many hours of test are required for design qualification prior to a production decision if we desire α and β risks of 10% each?

Solution

$$\theta_0 = 150,\ \theta_1 = 100,\ d = \frac{150}{100} = 1.5,\ \alpha \cong \beta \cong 0.10$$

In this case, a "standard" test plan may be selected from Appendix B, Table 6. Test plan IXC satisfies these inputs. The required test duration is

$$T = (45.0)(\theta_1) = (45.0)(100) = 4500 \text{ hrs.}$$

The accept/reject criterion for this test plan is to accept if we encounter 36 or fewer failures. Bear in mind though, that the acceptance criteria may require violation, depending on the nature of the failures and the verification of corrective action.

Commentary

1. In order to make the test less sensitive to the constant failure rate assumption, it would be desirable to have at least 3 systems tested for at least 3(100) = 300 hours each. The remainder of the 4500 hours may be satisfied with these or other systems. See section entitled "Constant Failure Rate Assumption" for a discussion of this topic.

2. The test duration of 4500 hours is very long! (The equivalent of 187.5 24-hour days). Putting more systems on test will reduce the calendar time requirement, but 4500 hours of test exposure are still required. The required test exposure is high because of the low discrimination ratio, d, and the relatively low α and β risks. Plans with higher risks may be worth consideration to see how much the amount of testing may be reduced.

CASE STUDY NO. 8-5

Background

An air defense system has a minimum acceptable value (MAV) of 80 hours MTBF and a specified value (SV) of 220.

Determine

How many hours of testing are required to give the user 80% assurance that his needs have been met? The producer requires 90% assurance that his product will be accepted if it meets the SV.

Solution

The 80% user assurance is equivalent to a consumer's risk of $\beta = 0.20$, and the 90% contractor assurance is equivalent to a producer's risk of $\alpha = 0.10$.

$$\theta_0 = 220, \ \theta_1 = 80, \ d = \frac{220}{80} = 2.75.$$

Because the discrimination ratio is 2.75, the "standard" test plans from Appendix B, Table 6, cannot be used. Appendix B, Table 7, will be considered.

For the 20% β risk, and entering the 10% α risk column, we find a discrimination ratio of 2.76 available, which is very close to 2.75. This is test plan number 20-5. The required test duration is

$$T = (6.72)(\theta_1) = (6.72)(80) = 537.6 \text{ hrs.}$$

The accept/reject criterion is to accept with 4 or fewer failures.

CASE STUDY NO. 8-6

Background

A radar system is under development. A test is to be run which will be essentially a fixed configuration test. At this stage of development, an MTBF of 200 hours is planned, but assurance is desired that the MTBF is not lower than 100 hours. For cost and schedule reasons, a test exposure of 300 hours has been proposed.

Determine

Is this amount of test exposure adequate? If not, what is an adequate amount of testing?

Solution

The upper test value, θ_0, is 200, and the lower test value, θ_1, is 100. Test exposure, T, is 300 hrs.

For a quick and simple look at the adequacy of the proposed test, we will use Figure 8-7. (For the convenience of the reader, the graphs in Figures 8-6 and 8-7 have been reproduced and annotated below.) Entering Figure 8.7 with $T/\theta_1 = 300/100 = 3$ and $d = \theta_0/\theta_1 = 200/100 = 2$, we find that the proposed test exposure results in risks slightly above 30%. The amount of testing proposed is minimally adequate for this stage in the program.

We may use Figure 8-7 to determine an adequate test duration. At about 580 hours, $\alpha = \beta = 0.25$. At about 900 hours, $\alpha = \beta = 0.20$. At about 2000 hours, $\alpha = \beta = 0.10$. At 580 hours, the risks are fairly high. A β risk of 25% is perhaps tolerable, but an α risk of 25% means a 25% chance of an erroneous "back to the drawing board" decision.

A test duration of about 900 hours looks reasonable, particularly if we reduce α by letting β increase. To investigate this possibility, we may use Figure 8-6. From the graph for d = 2, we find that test plan 30-8 with $\alpha = 0.10$, $\beta = 0.30$ and T = 981 looks reasonable. (Test plan 30-5 with $\alpha = 0.20$, $\beta = 0.30$ and T = 589 is another attractive possibility) . A test duration of about 900 hours is recommended.

Commentary

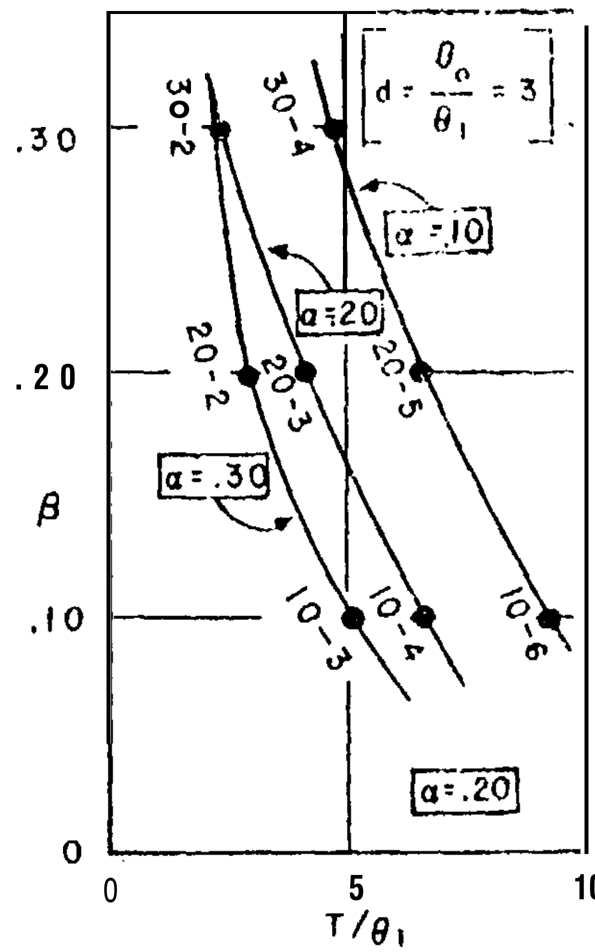
The process of trading test length for testing risks is inherently somewhat subjective. Actual problems of the type illustrated in this case should, of course, explicity address time, cost, and other penalties associated with the test.

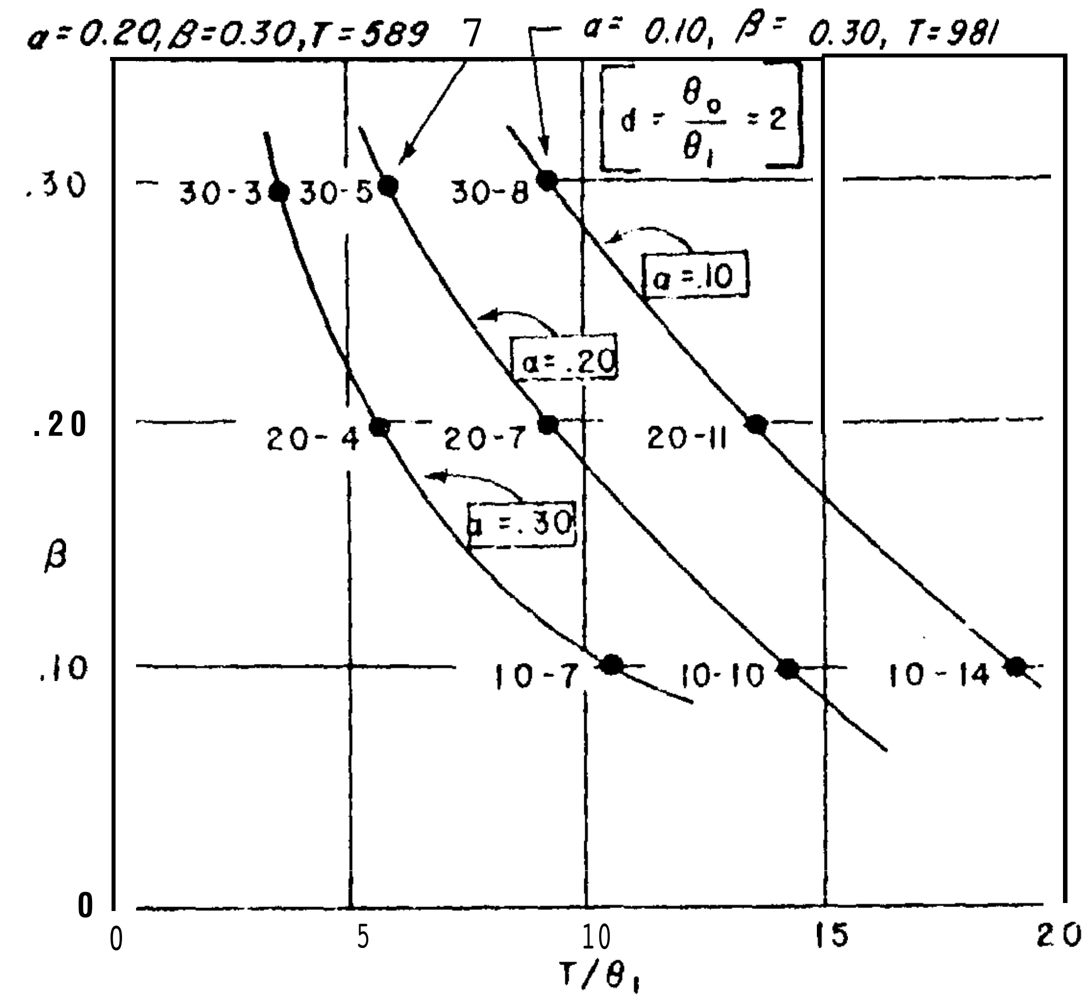
FIGURE 8-6

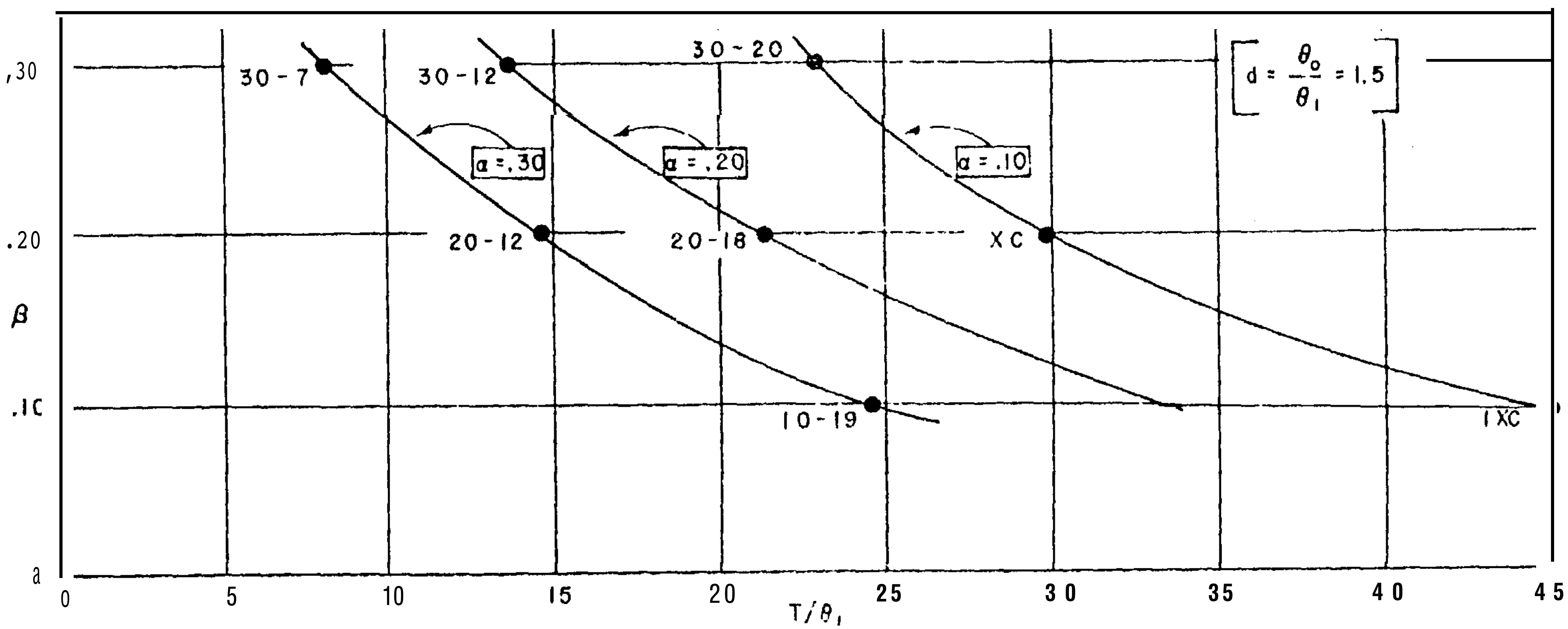
GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS

T = TOTAL TEST EXPOSURE

NOTE.
PLOTTING POINTS ARE CODED TO INDICATE THE MIL-STD 781C TEST PLAN. THESE PLANS ARE CONTAINED IN APPENDIX B, TABLE. S 6L7.

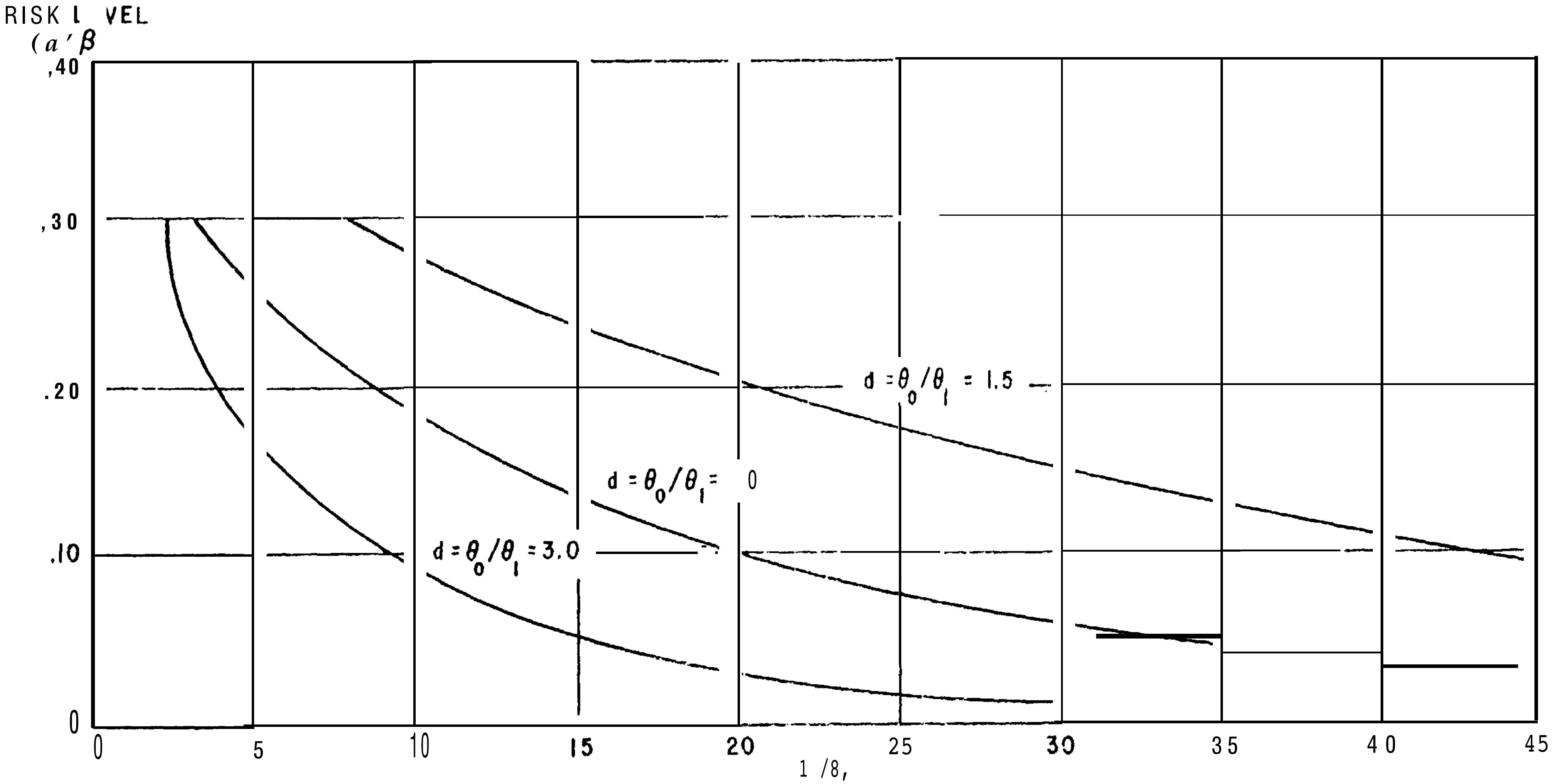






8-35

FIGURE 8-7 GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS FOR $\alpha = \beta$



CASE STUDY NO. 8-7

Background

A program manager desires to demonstrate an MTBF of at least 200 at a 90% confidence level.

Determine

The minimum test length required, and an evaluation of the proposed test plan.

Commentary

The correct interpretation of the background statement is that the MAV is 200 hours and the consumer's risk is 10%.

Solution

The absolute minimum test length is that which permits making the desired confidence statement with zero failures. Applying inequality 7. 10a we have:

$$\theta \geq \frac{2T}{\chi^2_{\alpha,2r+2}}$$

$$200 = \frac{2T}{\chi^2_{0.10,2}}$$

$$T = \frac{200(4.60)}{2} = 460.0 \text{ hours}$$

of test exposure.

NOTE : Values of $\chi^2_{\alpha,2r+2}$ are found in Appendix B, Table 5.

An MTBF of 200 can be demonstrated at a 90% confidence level by completing 46o.o hours of test exposure with zero failures.

To evaluate this proposed test plan, we will use an OC curve. To construct the OC curve, we use equation 8.10.

$P(ac|\theta)$ = Probability of acceptance for a given value of θ

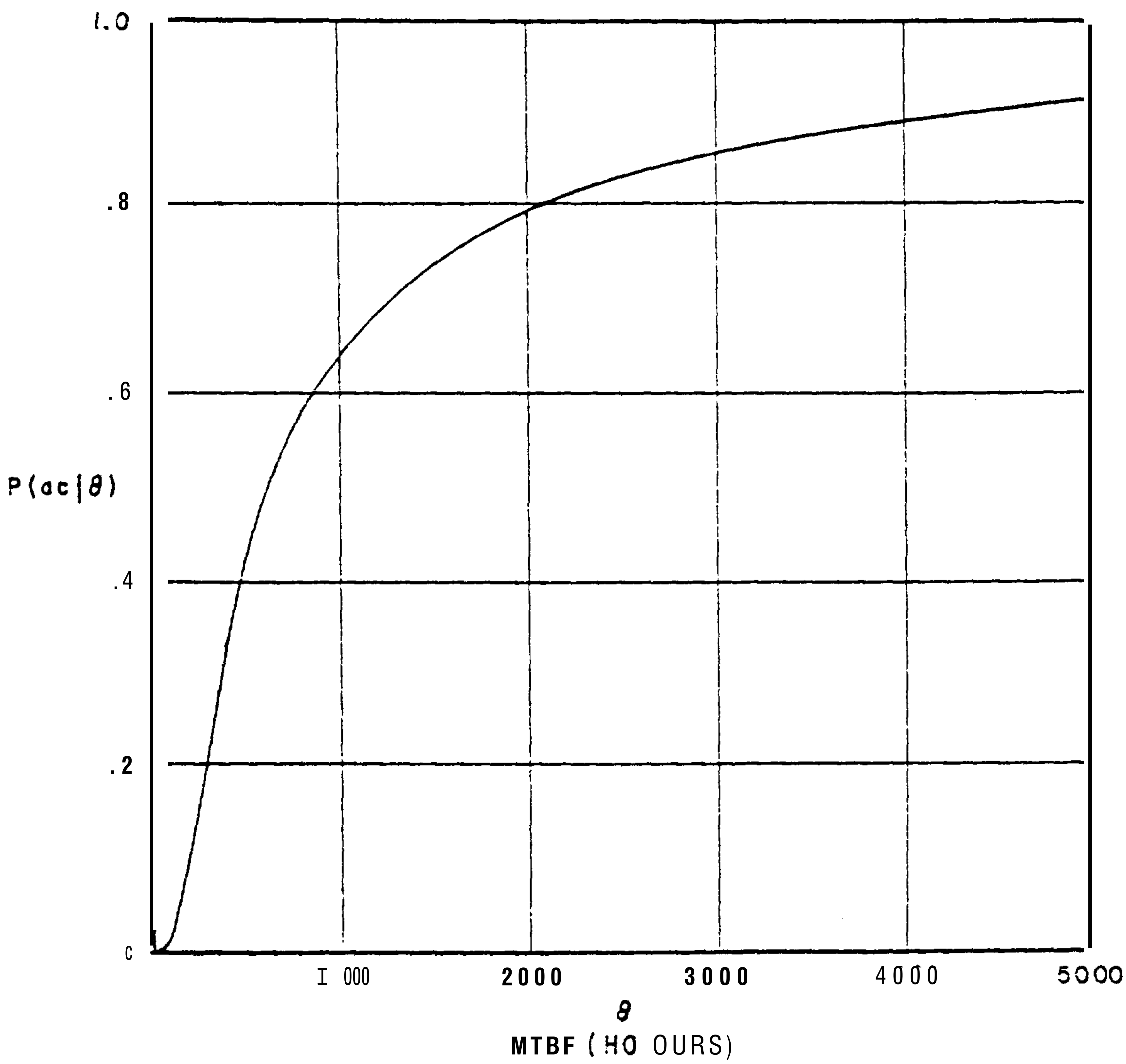
$$P(ac|\theta) = \sum_{k=0}^{c} \frac{(T/\theta)^k e^{-(T/\theta)}}{k!} .$$

For T = 460.0 and C = O

$$P(ac\ \theta) = e^{-(460.0/0)}$$

A few of the points for plotting the OC curve are tabulated below.

θ	W
100	0.010
200	0.100
500	0.398
1000	0.631
2000	0.794
3000	0.858
4000	0.891



Commentary

The curve shows that the proposed test plan does , in fact, achieve a consumer's risk of 10% for the MAV of 200 hours. Let us now examine the OC curve for this test plan through the eyes of the contractor.

The curve shows that a system whose true MTBF is 650 hours has only a 50% chance of passing the test, i.e., being accepted. In addition, for a system to have a 90% chance of being accepted, it must have a true MTBF of 4,400 hours. In other words, for the contractor to obtain a producer' s risk of 10%, he needs to manufacture a system whose true MTBF is 4,400 hours. To have a 50/50 chance of passing the test, he needs to manufacture a system whose true MTBF is 650 hours. The lesson to be learned here is that consideration must be given to both the upper test value (SV) , the lower test value (MAV) , and the risks associated with them in designing or evaluating a test plan- The test planner or evaluator should be concerned with obtaining the minimal test exposure plan which protects both the consumer and producer. To ignore either aspect can be a dangerous policy.

CHAPTER 9

RELIABILITY GROWTH

INTRODUCTION

Initial prototype models of complex weapon systems will invariably have inherent reliability and performance deficiencies that generally could not have been foreseen and eliminated in early design stages. To uncover and eliminate these deficiencies, we subject these early prototypes and later more mature models to a series of development and operational tests. These tests have been specifically planned to stress the system components to predetermined realistic levels at which inadequate design features will surface as system failures. These failures are analyzed, design modifications incorporated, and then the modified system is tested to verify the validity of the design change.

This testing philosophy utilizes the test-analyze-fix-test (TAFT) procedure as the basic catalyst in achieving system reliability growth. The ultimate goal of a reliability growth program, and, ideed, the entire test program, is to increase system reliability to stated requirement levels by eliminating a sufficient number of inherent system failure modes.

A successful system reliability growth program is dependent on several factors. First, an accurate determination must be made of the current system reliability status. Second, a test program must be planned which subjects the system to test exposure and stress levels adequate to uncover inherent failure models and to verify design modifications. Third, the program manager must address the availability of test schedule and resource required to support the "TAFT" procedure.

To adequately control the above and other factors inherent in the reliability growth process, it is important to track reliability growth throughout the testing program. This is accomplished by periodically assessing system reliability (e.g., at the end of every test phase) and comparing the current reliability to the planned level of achievement for that point in time. These assessments provide the necessary data and visibility to support necessary corrective management initiatives.

The following paragraphs present the analytical tools required to plan a reliability growth program and those useful in tracking the actual growth of a system during consecutive test phases.

RELIABILITY GROWTH CONCEPTS

Idealized Growth

For a system under development, reliability generally increases rapidly early on and at a much slower rate towards the end of development. It is useful at the beginning of a development program to depict the growth in reliability as a smooth curve which rises at slower and slower rates as time progresses. This curve, known as the idealized growth curve, does not necessarily convey precisely how the reliability will actually grow during development. Its purpose is to present a preliminary view as to how a program should be progressing in order for the final reliability requirements to be realized. The model for the idealized curve is the Duane Growth Model, the primary feature of which is the every decreasing rate of growth as testing progresses.

The development testing program will usually consist of several major test phases. Within each test phase, the testing may be conducted according to a program which incorporates fixes or design changes while testing is in process, at the end of the test phase, or both. If we divide the development testing program into its major phases and join by a smooth curve the proposed reliability values for the system at the end of these test phases , the resulting curve represents the overall pattern for reliability growth. This is called the idealized reliability growth curve. The idealized curve is very useful in quantifying the overall development effort and serves as a significant tool in the planning of reliability growth.

Planned Growth

The planning of reliability growth is accomplished early in the development program, before hard reliability data are obtained, and is typically a joint effort between the program manager and the contractor. Its purpose is to give a realistic and detailed indication of how system reliability enhancement is planned to grow during development. Reliability growth planning addresses program schedules, testing resources and the test exposure levels. The objective of growth planning is to determine the number and length of distinct test phases, whether design modifications will be incorporated during or between distinct test phases and the increases in reliability to ensure that the achieved reliability remains within sight of the idealized growth values.

Growth Tracking

The primary objective in tracking reliability growth is to obtain demonstrated reliability values at the end of each test phase. The demonstrated reliability is usually determined by one of two methods. The first and preferred method is reliability growth analysis. However, should the data not lend themselves to this type of analysis, then the second method, an engineering analysis, should be used. Reliability growth analysis is useful for combining test data to obtain a demonstrated estimate in the presence of changing configurations within a given test phase. Engineering analysis is employed when the reliability growth analysis procedure is inappropriate. We do not address engineering analysis in this text.

IDEALIZED GROWTH CURVE DEVELOPMENT

The first step in planning reliability growth is the development of an idealized growth curve. The development of this curve is based on the following three parameters :

t_1 = length of initial test phase.

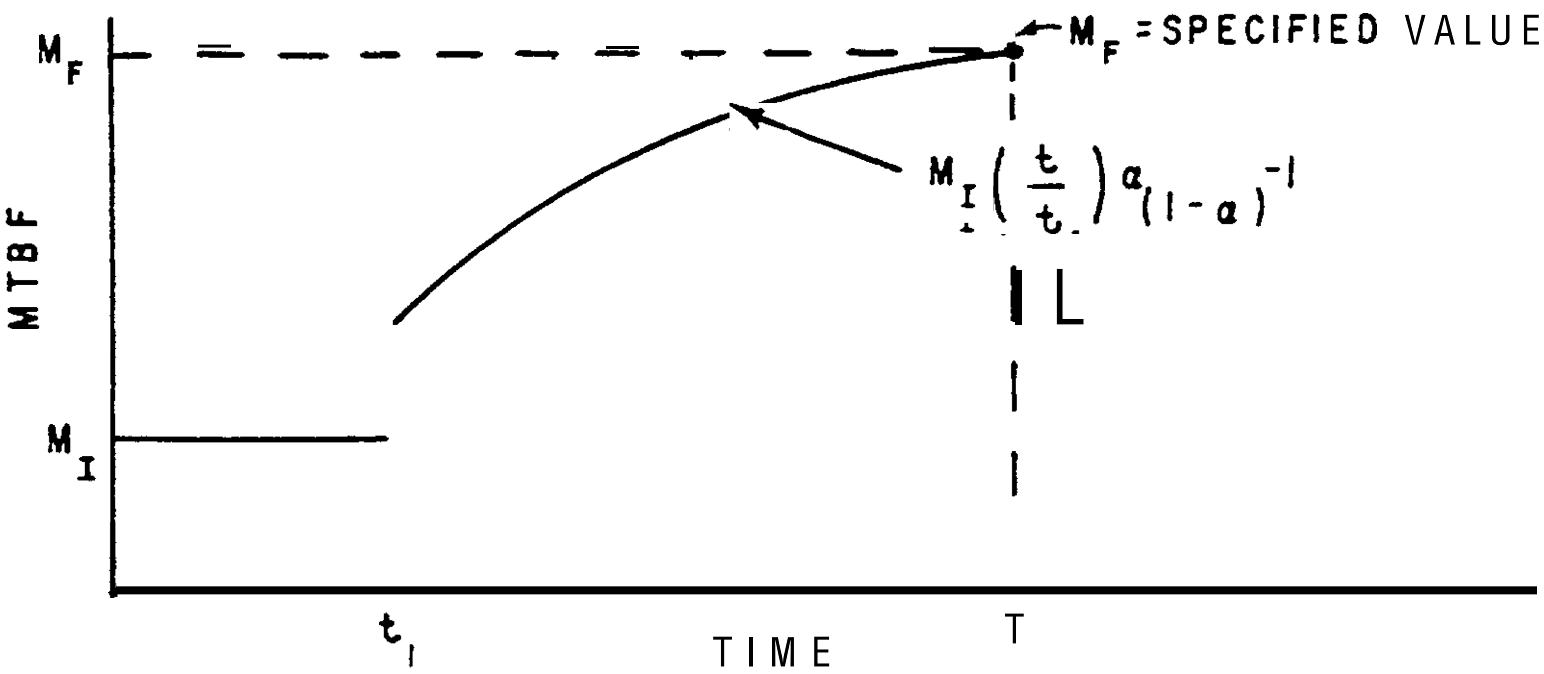
M_I = average MTBF over the first test phase, t_1.

α = a parameter which addresses the rate of growth.

The idealized curve, illustrated in Figure 9-1, is a graph of the function M(t) where:

$$M(t) = \begin{cases} M_I & \text{in the interval } 0<t\leq t_1 \\ & \text{and} \\ M_I(t/t_1)^{\alpha}(1-\alpha)^{-1} & \text{in the interval } t>t_1 . \end{cases} \tag{9.1}$$

FIGURE 9-1 IDEALIZED GROWTH CURVE



The idealized growth curve development procedure starts with the determination of the initial test phase length (t_1) and the average MTBF over the initial test phase (MI) . There is no exact procedure for determining values of these parameters. The initial test phase length (t_1) may be determined through a joint effort of both the contractor and the program manager. Perhaps an initial test has already been performed, in which case both t_1 and M_I are known. If this is not the case, then the determination of a value for M_I would in all likelihood require the expertise of individuals familiar with present day capabilities of the actual system in question or other similar systems. The parameter, M_I , should be a realistic estimate of what the system's average MTBF will be during the initial test phase, i.e. , before any significant design weaknesses can be detected and modifications developed, implemented and tested.

The parameter α represents the rate of growth necessary to achieve an MTBF of M_F (the contractually specified value) after a total of T hours of testing. The specified value M_F represents the user's desired capability and is determined by means of extensive battlefield as well as logistics analyses. The total amount of testing T is a value which is determined through a joint contractor and program manager effort and is based upon considerations of calendar time and number of prototypes available in addition to cost constraints. For fixed values of t_1, $_I$, T, and M_F, the value for α is calculated algebraically by solving the equation

$$M_F = M_I(T/t_1)^{\alpha}(1-\alpha)^{-1} . \tag{9.2}$$

There is no closed form solution for α in equation 9.2. However, an approximation for α is given below.

$$\alpha = \log_e(t_1/T)-1 + \{(\log_e(T/t_1)+1)^2 + 2\log_e(M_F/M_I)\}^{1/2} . \tag{9.3}$$

This is a reasonably good approximation when α is smaller than 0.4. The approximation will always be on the high side but within two decimal places for values of α less than 0.3. Programs which require a growth rate (α) greater than 0.3 should be viewed somewhat skeptically and those which require an α greater than 0.4 are far too ambitious to be realistic.

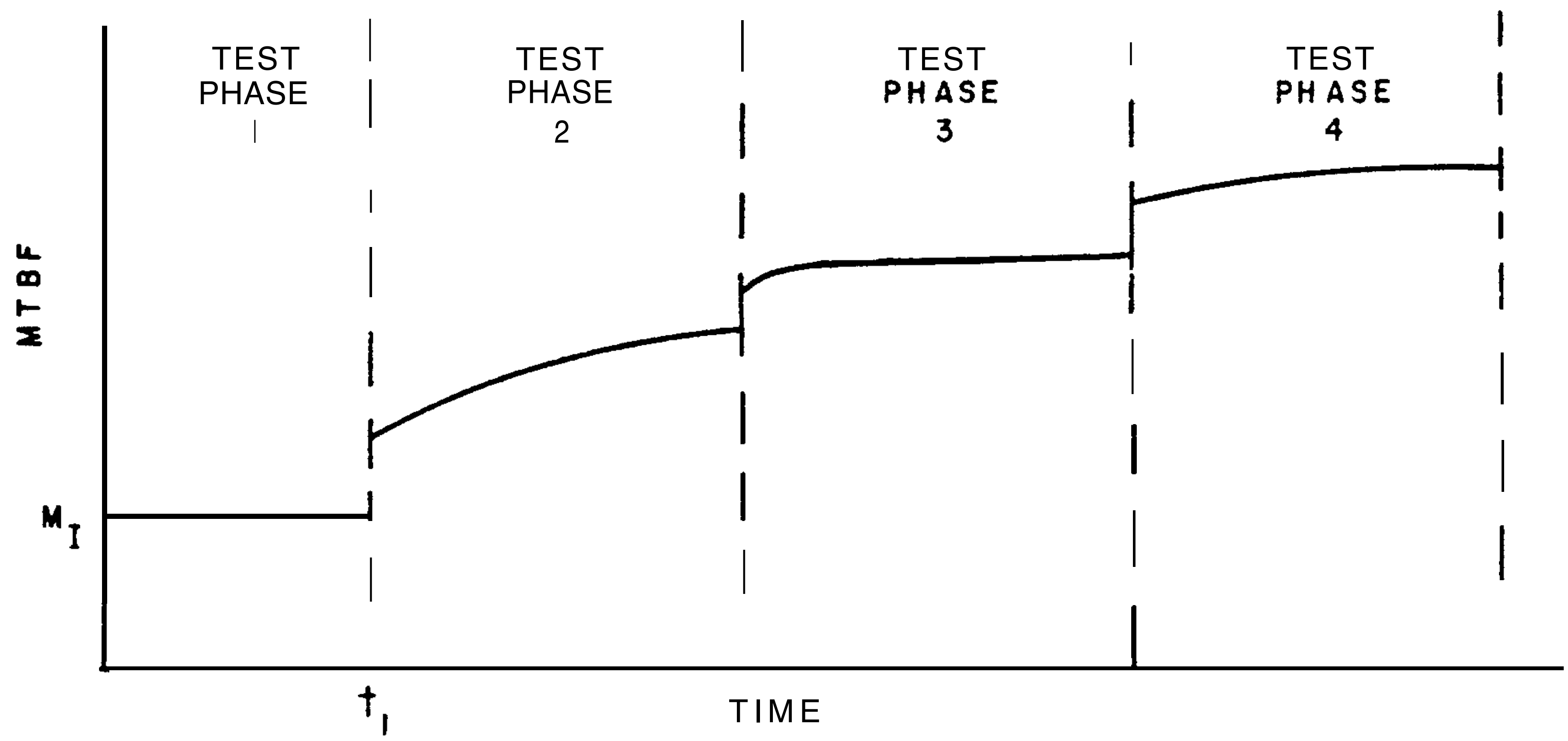
PLANNED GROWTH CURVE DEVELOPMENT

Once the idealized curve has been constructed, it is used as a basis for developing a planned growth curve. The planned growth curve displays, in graphic terms, how the producer plans by stages to achieve the required final MTBF . The curve is divided into portions which represent the different test phases. The entire curve indicates graphically where in the development program reliability is expected to grow, and where it is expected to remain constant. The curve depicts increases in reliability resulting from design improvements. At any given time during development testing, the planned growth curve value can be higher than, lower than, or equal to the idealized growth curve value. The idealized curve serves as a guide for the preparation of the planned curve. At no time in the planning of reliability growth should the separation between values on the curve be large. If this is the case, then unquestionably the re is some point during development where an unrealistic jump in reliability is expected to occur.

As we mentioned earlier, the planned growth curve should graphically display how reliability is expected to grow. Growth, of course, will generally occur as a result of incorporating design modifications. These modifications may be incorporated during the test phase, resulting in a smooth gradual improvement in reliability, or at the end of the test phase, resulting in a jump in reliability from the end of one test phase to the beginning of the subsequent test phase. In Figure 9-2 , we present a planned growth curve which illustrates the effect on reliability of design improvements incorporated during, and at the completion of, the various test phases. Note that the rate of growth is gradually decreasing as the system matures.

The portion of the planned growth curve between time zero and t_1 is identical to the idealized growth curve.

FIGURE 9-2 PLANNED GROWTH CURVE

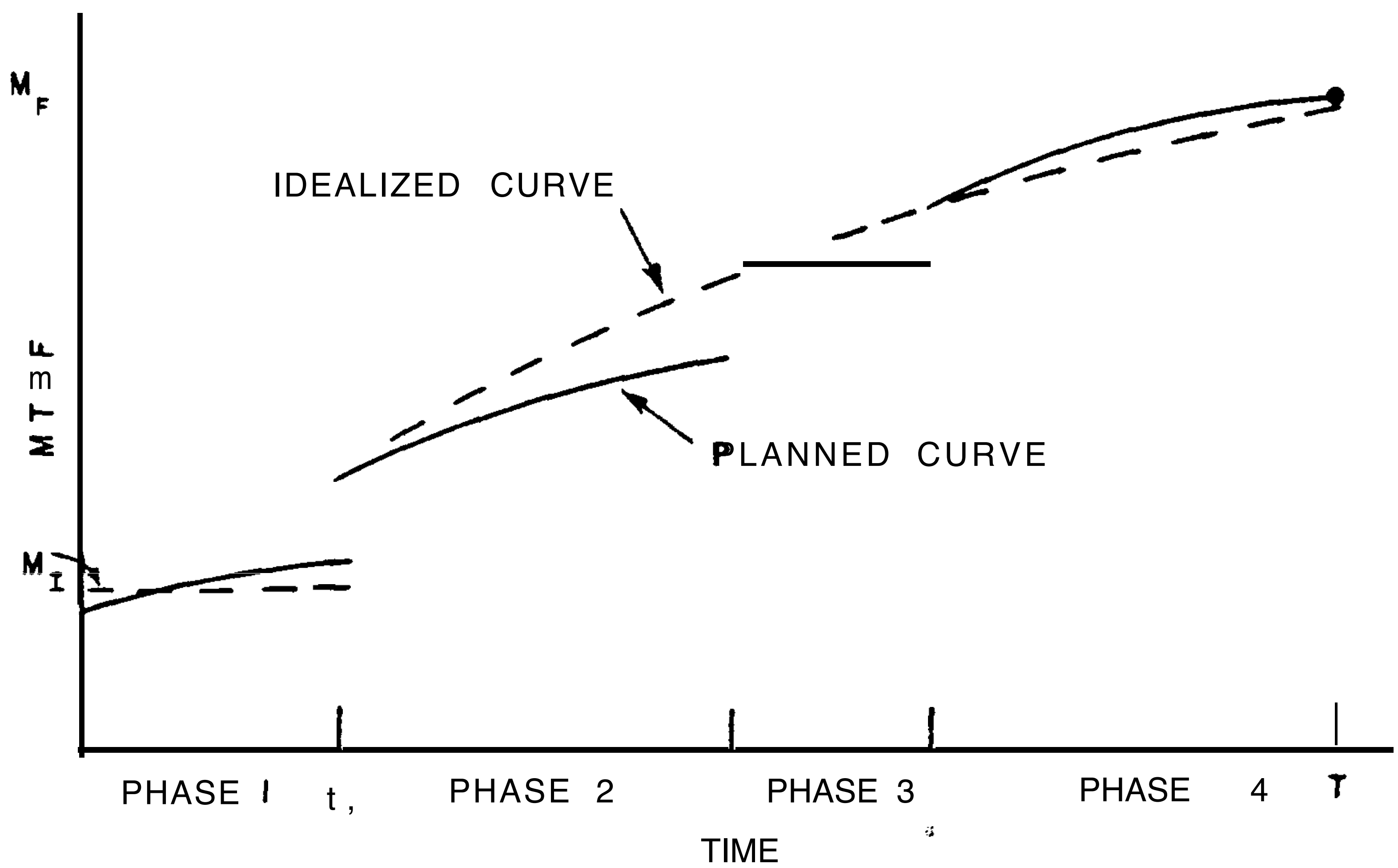


Delayed fixes are incorporated after each of the first three test phases. During all of test phase 2 and early in test phase 3, fixes are incorporated. Fixes are incorporated during the final test phase, and the MTBF grows to the required specified value. It is not a good practice to allow for a jump in reliability at the end of the final test phase even though fixes may be incorporated. The reason is that there is no test time available to determine the impact of these fixes.

The planned growth curve is an indication of how the required MTBF might be achieved and is developed by using the idealized curve as a guide.

Figure 9-3 illustrates the graphical relationship between the planned growth curve and the corresponding idealized curve. A point on the planned curve at any given time in the program represents the level of reliability to be achieved at that time.

FIGURE 9-3 EXAMPLE OF A PLANNED GROWTH CURVE AND CORRESPONDING IDEALIZED CURVE



RELIABILITY GROWTH TRACKING

The objectives of growth tracking include:

- Determining if growth is occurring and to what degree,
- Estimating the present reliability, and
- Formulating a projection of the reliability expected at some future time.

The methods discussed in this section are directed toward reliability growth tracking using a mathematical model. Parameters of the model are estimated using data which have been accumulated during a given test phase. Using this model and the parameter estimates, we can determine present and projected reliability values. The present value represents the reliability inherent in the existing configuration. A projected value represents the reliability of the system expected at some future time. Projected values take into account the effect of design improvements intended to correct observed failure modes or failure modes which further testing will surface. Generally growth tracking analysis is performed at the end of a major test phase.

The mathematical model we shall use for growth tracking describes the failure rate as a function of time. The value r(t) denotes the failure rate of the system after t units of testing, and

$$r(t) = \lambda\beta t^{\beta-1}, \tag{9.4}$$

where A and β are parameters of the model which determine the scale and the shape of the curve. The reciprocal of r(t) is the MTBF of the system after t units of testing. We fit the model to the actual test data using maximum likelihood estimates for A and β. (See Chapter 6 for a discussion of maximum likelihood estimates.)

When actual failure times $(t_1, t_2, \ldots t_N)$ are known and the test phase is <u>time truncated,</u> i.e. , at time T, the estimate for β is

$$\hat{\beta} = \frac{N}{N\log_e T - \sum_{i=1}^{N} \log_e t_i} \tag{9.5}$$

The estimate for A is

$$\hat{\lambda} = N/T^{\hat{\beta}}. \tag{9.6}$$

When the test phase is <u>failure truncated,</u> i.e. , at time t_N, the estimates are

$$\hat{\beta} = \frac{N}{(N-1)\log_e t_N - \sum_{i=1}^{N-1} \log_e t_i}, \tag{9.7}$$

and

$$\hat{\lambda} = N/T^{\hat{\beta}}. \tag{9.8}$$

In either case, the estimate of r(t) is

$$\hat{r}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}. \tag{9.9}$$

The reciprocal of $\hat{r}(t)$ is the estimate of the MTBF of the system after a test period of length t, that is

$$\hat{M}(t) = \frac{1}{\hat{r}(t)}.$$

Confidence limits for MTBF may be determined by multiplying point estimates of MTBF by the multipliers found in Table 9 of Appendix B.

When actual failure times are not known, the calculation of maximum likelihood estimates requires a complicated iterative procedure which can only be achieved using a computer algorithm. In addition, the estimates are not as accurate as they would be if actual failure times are known and used. It is important then to collect the actual failure times (in total test time) during development testing. See Chapter 10 for more information on this topic.

In Case Studies 9-1 and 9-2, we demonstrate the procedures for preparing idealized and planned growth curves and for tracking reliability growth.

CASE STUDY NO. 9-1

Background

A new helicopter system has been proposed. It is required to have a Mean Time between Mission Failure (MTBMF) of 50 hours. Past experience has shown that an average MTBMF of 20 hours can be expected during the initial test phase. Four test phases are planned, and the manufacturer intends to use test-analyze-fix-test (TAFT) during all but the final test phases. Delayed fixes will be incorporated at the end of all but the final test phase.

Determine

1. Construct the idealized curve for the program when the initial test phase is 100, 200, 300 hours, and the total test time is 1000 hours.

2. Construct an idealized curve and a planned growth curve when the total test time is 2000 hours, and the four test phases are of equal length.

Solutions

1a. $t_1 = 100$ $\quad T = 1{,}000$

$M_I = 20$ $\quad M_F = 50$

i. Solve for α in the model, using the approximation 9.3

$$\alpha = \log_e(100/1000)-1 + \{(\log_e(1000/100)+1)^2 + 2\log_e(50/20)\}^{1/2}$$

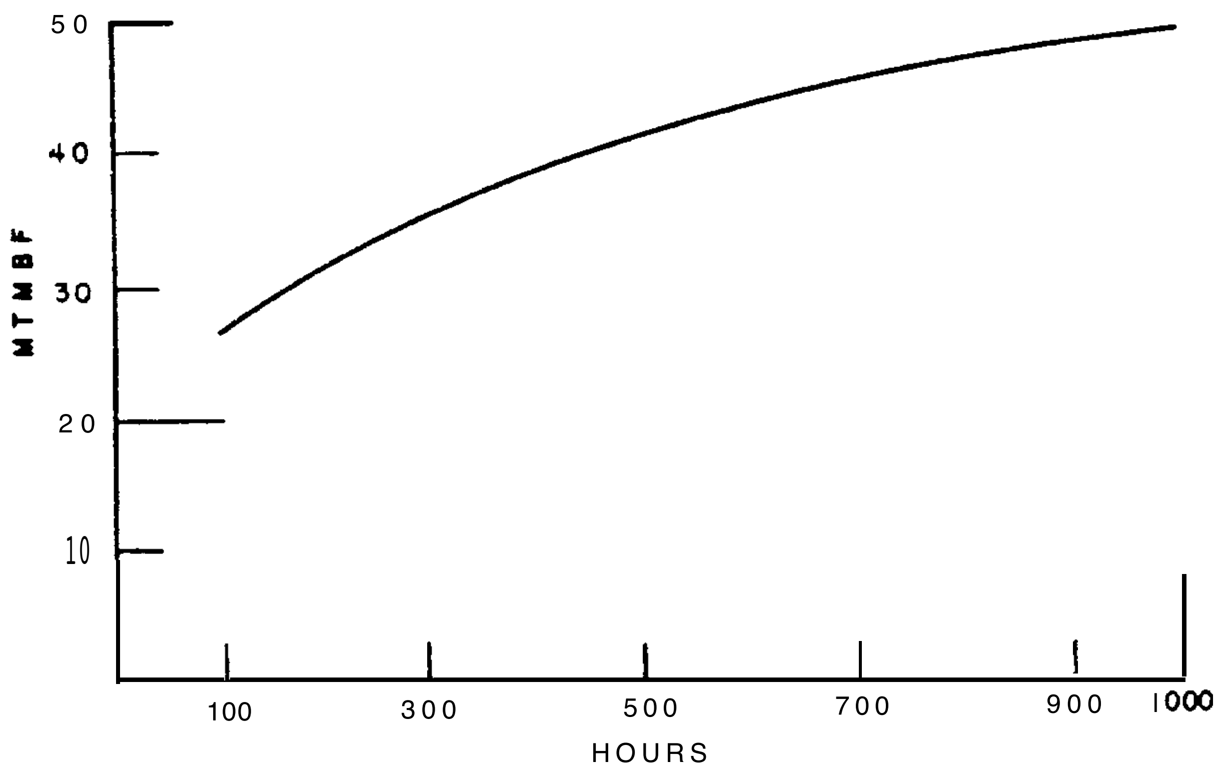
$$= 0.267$$

ii. Determine points on the curve using equation 9.1

$$M(t) = M_I(t/t_1)^{\alpha}(1-\alpha)^{-1}$$

t	M(t)
<100	20
100	27
300	36
500	42
700	46
900	49
1000	50

iii. Sketch the curve.



1b. $t_1 = 200$ $T = 1000$

$M_I = 20$ $M_F = 50$

i. Solve for α

$$\alpha = \log_e(200/1000)-1 + \{(\log_e(1000/200)+1)^2$$

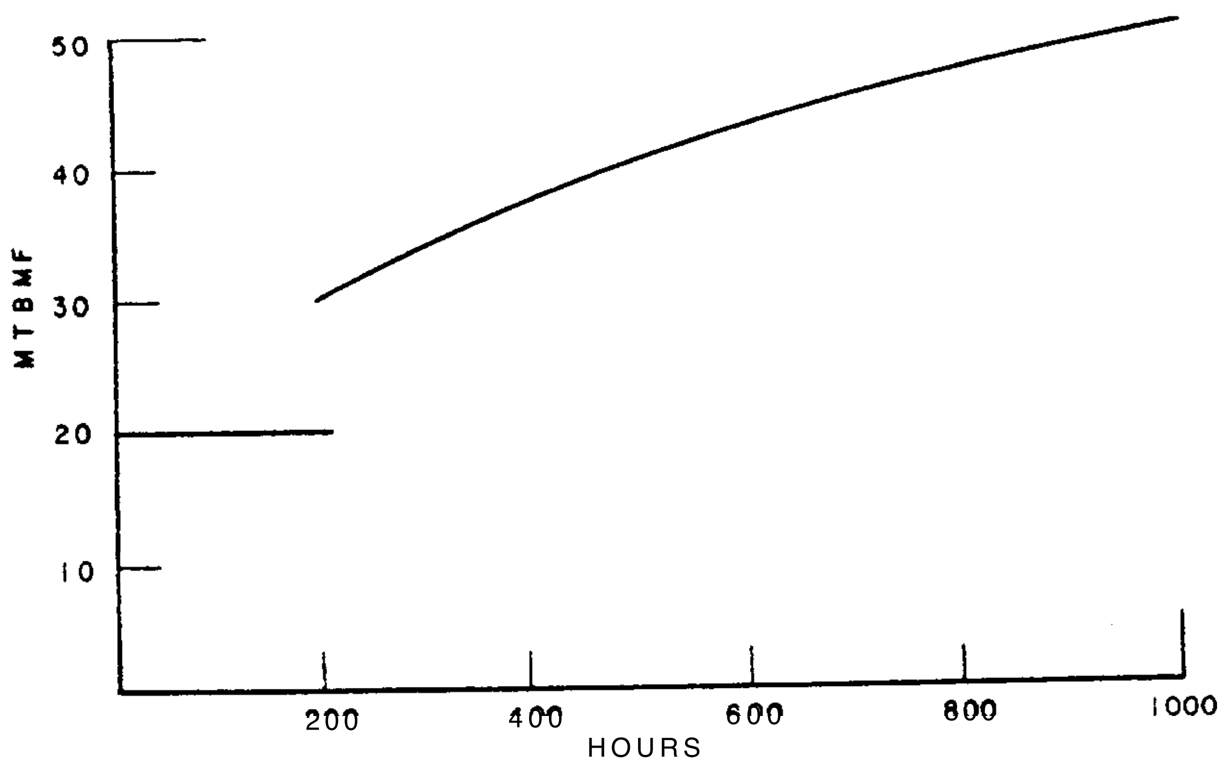
$$+ 2\log_e(50/20)\}^{1/2}$$

$$= 0.33$$

ii. Determine points on the curve, using equation 9.1

t	M(t)
<200	20
200	30
400	37
600	43
800	47
1000	50

iii. Sketch the curve.



1.c. $t_1 = 300$ $T = 1000$

$M_I = 20$ $M_F = 50$

i. Solve for α

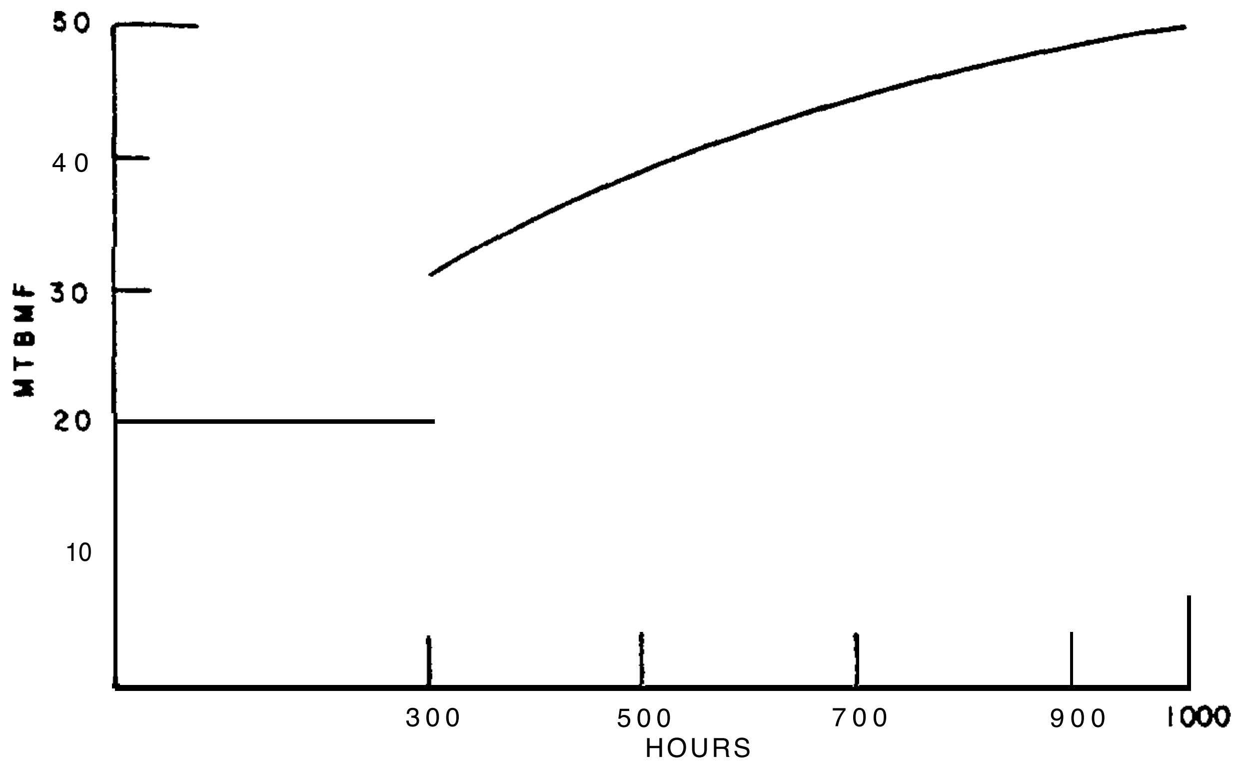
$$\alpha = \log_e(300/1000) - 1 + \{(\log_e(1000/300)+1)^2 + 2\log_e(50/20)\}^{1/2}$$

$$= 0.38$$

ii. Determine points on the curve, using equation 9.1

$\underline{t}$	M(t)
<300	20
300	32
500	39
700	44
900	49
1000	50

iii. Sketch the curve



2. $t_1 = 500$ $\quad T = 2000$

$M_I = 20$ $\quad M_F = 50$

i . Solve for α

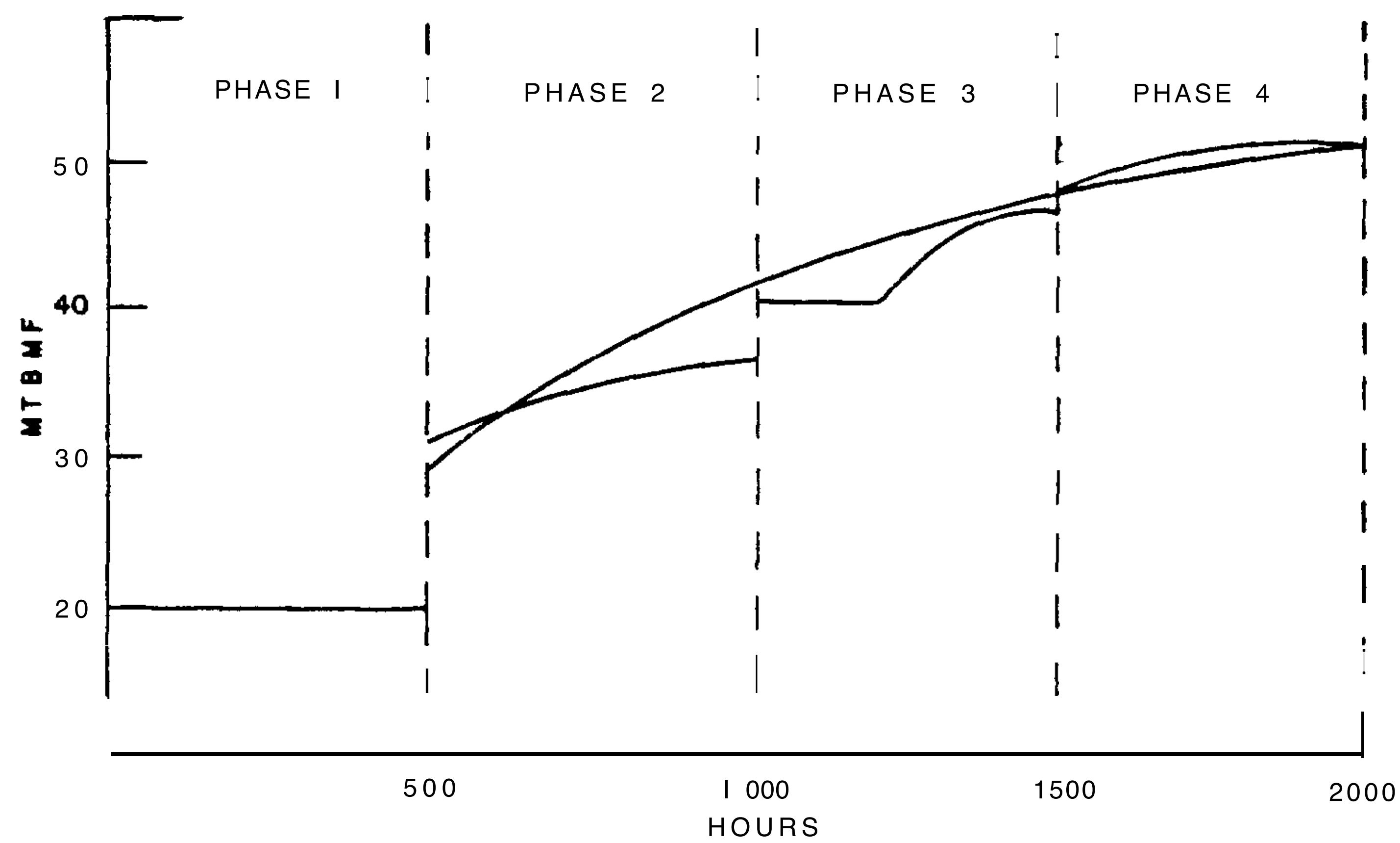
$$\alpha = \log_e(500/2000) - 1 + \{(\log_e(2000/500)+1)^2 + 2\log_e(50/20)\}^{1/2}$$

$$= 0.356$$

ii. Determine points on the idealized curve, using equation 9.1

<u>t</u>	M(t)
<500	20
500	31
1000	40
1500	46
2000	50

iii. Sketch the idealized curve and superimpose a planned growth curve.



Commentary

1. Note, for the solution to question 1, how the length of the initial test phase (t_1) affects the growth parameter α. The α of 0.38 in part c. may even be too ambitious. The initial test phase **length** (t_1) should, however, be long enough so that the average MTBF of M_I is achievable.

2. Note that, at various times during the test, the planned growth curve either exceeds or falls below the idealized curve. The relatively low values of the planned curve toward, and at the end of, the second test phase may be cause for some concern. Some fairly substantial increases in reliability are required during the third test phase to get the program back on track.

CASE STUDY NO. 9-2

Background

For the system proposed in Case Study No. 9-1, the data from the final test phase have been collected. The failure times as measured in total test hours are {12, 70, 105, 141, 172, 191, 245, 300, 340, 410, 490}.

Determine

1. Calculate the MTBMF of the system at the end of the final test phase.

2. Calculate a 90% lower limit on the system MTBMF.

Solution

1a. If we assume no growth during this test phase, the estimated MTBMF is the ratio of the total time to the number of failures . This value is 500/11 or 45.4 hours.

1b. If fixes are being incorporated during this test phase as suggested in the background for Case Study No. 9-1, then a reliability growth analysis is more appropriate than one based upon the assumption of no growth, as in 1a.

i. Maximum likelihood estimate for β is

$$\hat{\beta} = 11/\{((11)\ \log_e 500) - (\log_e 12+\log_e 70+\log_e 105+\log_e 141+ \log_e 172+\log_e 191+\log_e 245+\log_e 300+\log_e 340+\log_e 410+\log_e 490)\}$$

$$\hat{\beta} = 0.89$$

ii. Maximum likelihood estimate for λ is

$$\hat{\lambda} = \frac{11}{(500)^{0.89}} = 0.044$$

iii. Estimated MTBMF after final test phase

$\hat{r}(500) = 0.0198$, and the estimated MTBMF is the reciprocal of $\hat{r}(500)$, which is 50.6 hours.

2. Using Appendix B, Table 9,. for a time terminated test, we find the lower 90% confidence limit multiplier for 11 failures to be 0.565. The lower limit is

$$\theta \geq \theta_L$$

$$\geq (0.565)\hat{\theta}$$

$$\geq (0.565)(50.6)$$

$\geq 28.6.$

We are 90% confident that the true MTBMF is at least 28.6 hours.

Commentary

The estimated MTBMF assuming no growth is 45.4 hours. (See la. above.) Note that this estimate was computed using the number of failures and does not take into account the actual failure times. It cannot show that the times between successive failures seem to be increasing. If fixes are being incorporated during the test, then the reliability growth analysis is more appropriate- With this analysis, the estimated MTBMF is 50.6 hours. The type of analysis used, however, should not be determined by the data but rather by a realistic assessment of the test program.

APPENDIX A

A-1. NORMAL APPROXIMATION TO BINOMIAL

When p, the probability of failure on a given trial, is moderate ($0.2 \leq p \leq 0.8$) and n the number of trials is large ($n \geq 30$), the normal distribution provides reasonable approximations to binomial probabilities. This approximation is detailed below. Note that Z is the notation for the standard normal variable. (See Appendix B, Table 2.)

The probability of k or fewer failures out of n trials is approximately equal to

$$1 - P(Z \geq (k+0.5-np)/\sqrt{np(1-p)}) .$$

The probability of at least k failures out of n trials is approximately equal to

$$P(Z \geq (k-0.5-np)/\sqrt{np(1-p)}) .$$

The probability of between k_1 and k_2 failures out of n trials inclusive is approximately equal to

$$P(Z \geq (k_1-0.5-np)/\sqrt{np(1-p)}) - P(Z \geq (k_2+0.5-np)/\sqrt{np(1-p)})$$

We have listed the approximations in the form

$$P(Z \geq a)$$

so that the use of Appendix B, Table 2 is direct.

As an example, suppose that n = 40 and p = 0.3. The probability of between 10 and 20 failures inclusive is

$$P(z \geq (10-0.5-(40)(0.3)/\sqrt{(40)(0.3)(0.7)})$$

$$- P(Z \geq (20+0.5-(40)(0.3)/\sqrt{(40)(0.3)\ (0.7)}) .$$

Simplifying we obtain

$$P(z \geq -0.86) - P(Z \geq 2.93).$$

Now from Appendix B, Table 2, we find that $P(Z \geq -0.86) = 0.8051$ and $P(Z \geq 2.93) = 0.0017$. Consequently, the probability that between 10 and 20 failures inclusive occur is approximately 0.8034.

The value using a set of binomial tables is 0.8017.

A-2 . POISSON APPROXIMATION TO BINOMIAL

When p, the probability of failure on a given trial, is extreme ($p \leq 0.2$ or $p \geq 0.8$) and n, the number of trails, is large ($n \geq 30$), the Poisson distribution provides reasonable approximations to binomial probabilities. We make the identification $m = np$ and use Poisson tables to determine the probabilities of events in a binomial experiment.

As an example, suppose that $n = 40$ and $p = 0.05$, so that $m = 40(0.05) = 2$. The probability of between 5 and 10 failures is the difference between the probability of 10 or fewer failures (1 .000) and the probability of 4 or fewer failures (0.947) . (See Appendix B, Table 3.) The difference is 0.053. Using a set of binomial tables we obtain 0.0480.

A-3. NORMAL APPROXIMATION TO POISSON

When the product AT is greater than 5, the normal distribution provides reasonable approximations to Poisson probabilities. The approximation is detailed below. Note that Z is the notation for the standard normal variable. (See Appendix B, Table 2)

The probability of k or fewer failures during time T is approximately

$$1 - P(Z \geq (k+0.5-\lambda T)/\sqrt{\lambda T}) .$$

The probability of at least k failures during time T is approximately

$$P(Z \geq (k-0.5-\lambda T)/\sqrt{\lambda T}) .$$

The probability of between k_1 and k_2 failures inclusive during time T is approximately

$$P(Z \geq (k_1-0.5-\lambda T)/\sqrt{\lambda T}) - P(Z \geq (k_2+0.5-\lambda T)/\sqrt{\lambda T}) .$$

We have listed the approximations in the form

$$P(Z \geq a)$$

so that the use of Appendix B, Table 2 is direct.

As an example, suppose that the failure rate A is 0.01 and the test time T is 1000 hours. The probability of between 8 and 15 failures inclusive is

$$P(Z \geq (8-0.5-(0.01)(1000)/\sqrt{(0.01)(1000)}\ )$$

$$- P(2 \geq (15+0.5-(0.01)(1000)/\sqrt{(0\ .01)(1000))}) .$$

The above expression reduces to

$$P(Z \geq -0.79) - P(2 \geq 1.74).$$

Now P(Z > -0.79) = 0.7852 and P(Z $\geq$ 1.74) = 0.049, so the probability that between 8–and 15 failures inclusive occur is approximately 0.7443.

Using the Poisson tables (Appendix B, Table 3), we obtain the probability more precisely as 0.731.

APPENDIX B

*Extracted from MIL-STD 781C, Tables II and III. Operating characteristic (OC) curves for the test plans shown in Table 6 can be found in MIL-STD 781C, Appendix C.

TABLE 1

BINOMIAL TABLES

Table gives probability of c or fewer failures,

NOTE: Only excerpts of binomial tables required
to work case studies are provided in Table 1.

p = P(failure)

	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 10	0	.8171	.5987	.4344	.3894	.2318	.1969	.0563	.0335	.0282	.0010
	1	.9838	.9139	.8121	.7746	.5967	.5443	.2440	.1689	.1493	.0107
	2	.9991	.9885	.9599	.9460	.8552	.8202	.5256	.4154	.3828	.0547
	3	1.0000	.9990	.9942	.9912	.9637	.9500	.7759	.6813	.6496	.1719
	4	1.0000	.9999	.9994	.9990	.9935	.9901	.9219	.8695	.8497	.3770
	5	1.0000	1.0000	1.0000	.9999	.9992	.9986	.9803	.9609	.9527	.6230
	6	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9965	.9917	.9894	.8281
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9988	.9984	.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000
n = 11	0	.8007	.5688	.3996	.3544	.2003	.1673	.0422	.0238	.0198	.0005
	1	.9805	.8981	.7819	.7399	.5471	.4922	.1971	.1299	.1130	.0059
	2	.9988	.9848	.9481	.9305	.8200	.7788	.4552	.3444	.3127	.0327
	3	1.0000	.9984	.9915	.9871	.9489	.9306	.7133	.6047	.5696	.1133
	4	1.0000	.9999	.9990	.9983	.9895	.9841	.8854	.8153	.7897	.2744
	5	1.0000	1.0000	.9999	.9998	.9984	.9973	.9657	.9346	.9218	.5000
	6	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.9924	.9828	.9784	.7256
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988	.9968	.9957	.8867
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9994	.9673
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9941
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000
n = 12	0	.7847	.5404	.3677	.3225	.1730	.1422	.0317	.0170	.0138	.0002
	1	.9769	.8816	.7513	.7052	.4999	.4435	.1584	.0994	.0850	.0032
	2	.9985	.9804	.9348	.9134	.7829	.7358	.3907	.2826	.2528	.0193
	3	.9999	.9978	.9880	.9820	.9314	.9078	.6488	.5298	.4925	.0730
	4	1.0000	.9998	.9984	.9973	.9840	.9761	.8424	.7547	.7237	.1938
	5	1.0000	1.0000	.9998	.9997	.9972	.9954	.9456	.9002	.8822	.3872
	6	1.0000	1.0000	1.0000	1.0000	.9996	.9993	.9857	.9689	.9614	.6128
	7	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9972	.9928	.9905	.8062
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9988	.9983	.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 13	0	.7690	.5133	.3383	.2935	.1495	.1209	.0238	.0121	.0097	.0001
	1	.9730	.8646	.7206	.6707	.4555	.3983	.1267	.0756	.0637	.0017
	2	.9980	.9755	.9201	.8946	.7444	.6920	.3326	.2249	.2025	.0112
	3	.9999	.9969	.9837	.9758	.9112	.8820	.5843	.4586	.4206	.0461
	4	1.0000	.9997	.9976	.9959	.9768	.9658	.7940	.6899	.6543	.1334
	5	1.0000	1.0000	.9997	.9995	.9954	.9925	.9198	.8583	.8346	.2905
	6	1.0000	1.0000	1.0000	.9999	.9993	.9987	.9757	.9492	.9376	.5000
	7	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9944	.9859	.9818	.7095
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990	.9970	.9960	.8666
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9993	.9539
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9888
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9983
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000
n = 14	0	.7536	.4877	.3112	.2670	.1292	.1028	.0178	.0086	.0068	.0001
	1	.9690	.8470	.6900	.6368	.4139	.3567	.1010	.0573	.0475	.0009
	2	.9975	.9699	.9042	.8745	.7051	.6479	.2811	.1854	.1608	.0065
	3	.9999	.9958	.9786	.9685	.8885	.8535	.5213	.3927	.3552	.0287
	4	1.0000	.9996	.9965	.9941	.9679	.9533	.7415	.6233	.5842	.0898
	5	1.0000	1.0000	.9996	.9992	.9929	.9885	.8883	.8098	.7805	.2120
	6	1.0000	1.0000	1.0000	.9999	.9988	.9978	.9617	.9230	.9067	.3953
	7	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.9897	.9753	.9685	.6047
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9978	.9938	.9917	.7880
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9988	.9983	.9102
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9998	.9713
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9935
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000
n = 15	0	.7386	.4633	.2863	.2430	.1116	.0874	.0134	.0061	.0047	.0000
	1	.9647	.8290	.6597	.6035	.3751	.3186	.0802	.0433	.0353	.0005
	2	.9970	.9638	.8870	.8531	.6655	.6042	.2361	.1485	.1268	.0037
	3	.9998	.9945	.9727	.9601	.8636	.8227	.4613	.3330	.2969	.0176
	4	1.0000	.9994	.9950	.9918	.9571	.9383	.6865	.5569	.5155	.0592
	5	1.0000	.9999	.9993	.9987	.9895	.9832	.8516	.7561	.7216	.1509
	6	1.0000	1.0000	.9999	.9998	.9980	.9964	.9434	.8904	.8689	.3036
	7	1.0000	1.0000	1.0000	1.0000	.9997	.9994	.9827	.9602	.9500	.5000
	8	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9958	.9885	.9848	.6964
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992	.9974	.9963	.8491
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9993	.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 16	0	.7238	.4401	.2634	.2211	.0964	.0743	.0100	.0044	.0033	.0000
	1	.9601	.8108	.6299	.5711	.3393	.2839	.0635	.0326	.0261	.0003
	2	.9963	.9571	.8689	.8306	.6260	.5614	.1971	.1182	.0994	.0021
	3	.9998	.9930	.9658	.9504	.8366	.7899	.4050	.2790	.2459	.0106
	4	1.0000	.9991	.9932	.9889	.9444	.9209	.6302	.4924	.4499	.0384
	5	1.0000	.9999	.9990	.9981	.9851	.9765	.8103	.6987	.6598	.1051
	6	1.0000	1.0000	.9999	.9997	.9968	.9944	.9204	.8517	.8247	.2272
	7	1.0000	1.0000	1.0000	1.0000	.9995	.9989	.9729	.9401	.9256	.4018
	8	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9925	.9804	.9743	.5982
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9984	.9948	.9929	.7728
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9989	.9984	.8949
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.9616
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9894
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9979
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
n = 17	0	.7093	.4181	.2423	.2012	.0833	.0631	.0075	.0031	.0023	.0000
	1	.9554	.7922	.6005	.5396	.3063	.2525	.0501	.0245	.0193	.0001
	2	.9956	.9497	.8497	.8073	.5870	.5198	.1637	.0936	.0774	.0012
	3	.9997	.9912	.9581	.9397	.8080	.7556	.3530	.2333	.2019	.0064
	4	1.0000	.9988	.9911	.9855	.9297	.9013	.5739	.4312	.3887	.0245
	5	1.0000	.9999	.9985	.9973	.9795	.9681	.7653	.6393	.5968	.0717
	6	1.0000	1.0000	.9998	.9996	.9952	.9917	.8929	.8077	.7752	.1662
	7	1.0000	1.0000	1.0000	1.0000	.9991	.9983	.9598	.9147	.8954	.3145
	8	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9876	.9688	.9597	.5000
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9969	.9907	.9873	.6855
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994	.9977	.9968	.8338
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9993	.9283
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9755
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9936
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 18

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6951	.3972	.2229	.1831	.0720	.0536	.0056	.0022	.0016	.0000
1	.9505	.7735	.5719	.5091	.2759	.2241	.0395	.0163	.0142	.0001
2	.9948	.9419	.8298	.7832	.5488	.4797	.1353	.0737	.0600	.0007
3	.9996	.9891	.9494	.9277	.7779	.7202	.3057	.1931	.1646	.0038
4	1.0000	.9985	.9884	.9814	.9132	.8794	.5187	.3742	.3327	.0154
5	1.0000	.9998	.9979	.9962	.9728	.9581	.7175	.5794	.5344	.0481
6	1.0000	1.0000	.9997	.9994	.9931	.9882	.8610	.7592	.7217	.1189
7	1.0000	1.0000	1.0000	.9999	.9986	.9973	.9431	.8839	.8593	.2403
8	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.9807	.9532	.9404	.4073
9	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9946	.9844	.9790	.5927
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988	.9957	.9939	.7597
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9990	.9986	.8811
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.9519
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9846
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9962
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 19

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6812	.3774	.2051	.1666	.0622	.0456	.0042	.0016	.0011	.0000
1	.9454	.7547	.5440	.4798	.2482	.1985	.0310	.0137	.0104	.0000
2	.9939	.9335	.8092	.7585	.5117	.4413	.1113	.0577	.0462	.0004
3	.9995	.9868	.9398	.9147	.7468	.6841	.2631	.1587	.1332	.0022
4	1.0000	.9980	.9853	.9765	.8948	.8556	.4654	.3221	.2822	.0096
5	1.0000	.9998	.9971	.9949	.9647	.9463	.6678	.5203	.4739	.0318
6	1.0000	1.0000	.9996	.9991	.9903	.9837	.8251	.7074	.6655	.0835
7	1.0000	1.0000	.9999	.9999	.9978	.9959	.9225	.8479	.8180	.1796
8	1.0000	1.0000	1.0000	1.0000	.9996	.9992	.9713	.9332	.9161	.3238
9	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9911	.9754	.9674	.5000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9977	.9924	.9895	.6762
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9981	.9972	.8204
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9994	.9165
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9682
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9904
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9978
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 20

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6676	.3585	.1887	.1516	.0537	.0388	.0032	.0011	.0008	.0000
1	.9401	.7358	.5169	.4516	.2229	.1756	.0243	.0102	.0076	.0000
2	.9929	.9245	.7879	.7334	.4759	.4049	.0913	.0450	.0355	.0002
3	.9994	.9841	.9294	.9007	.7148	.6477	.2252	.1296	.1071	.0013
4	1.0000	.9974	.9817	.9710	.8746	.8298	.4148	.2750	.2375	.0059
5	1.0000	.9997	.9962	.9932	.9551	.9327	.6172	.4632	.4164	.0207
6	1.0000	1.0000	.9994	.9987	.9868	.9781	.7858	.6535	.6080	.0577
7	1.0000	1.0000	.9999	.9998	.9968	.9941	.8982	.8075	.7723	.1316
8	1.0000	1.0000	1.0000	1.0000	.9994	.9987	.9591	.9087	.8867	.2517
9	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9861	.9632	.9520	.4119
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9961	.9875	.9829	.5881
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991	.9965	.9949	.7483
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9992	.9987	.8684
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.9423
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9793
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9941
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9987
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 21

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6543	.3406	.1736	.1380	.0464	.0329	.0024	.0008	.0006	.0000
1	.9347	.7170	.4906	.4246	.1999	.1550	.0190	.0076	.0056	.0000
2	.9919	.9151	.7663	.7081	.4415	.3705	.0745	.0350	.0271	.0001
3	.9993	.9811	.9181	.8856	.6823	.6113	.1917	.1053	.0856	.0007
4	1.0000	.9968	.9775	.9646	.8529	.8025	.3674	.2331	.1984	.0036
5	1.0000	.9996	.9950	.9912	.9442	.9173	.5666	.4090	.3627	.0133
6	1.0000	1.0000	.9991	.9982	.9825	.9713	.7436	.5987	.5505	.0392
7	1.0000	1.0000	.9999	.9997	.9954	.9917	.8701	.7631	.7230	.0946
8	1.0000	1.0000	1.0000	1.0000	.9990	.9980	.9439	.8795	.8523	.1917
9	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.9794	.9475	.9324	.3318
10	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9936	.9805	.9736	.5000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9983	.9939	.9913	.6682
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9984	.9976	.8083
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9994	.9054
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9608
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9867
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9964
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 22

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6412	.3235	.1597	.1256	.0401	.0280	.0018	.0006	.0004	.0000
1	.9290	.6982	.4652	.3988	.1790	.1367	.0149	.0056	.0041	.0000
2	.9907	.9052	.7442	.6826	.4086	.3382	.0606	.0271	.0207	.0001
3	.9991	.9778	.9059	.8696	.6496	.5752	.1624	.0850	.0681	.0004
4	.9999	.9960	.9727	.9575	.8297	.7738	.3235	.1963	.1645	.0022
5	1.0000	.9994	.9936	.9888	.9318	.9001	.5168	.3584	.3134	.0085
6	1.0000	.9999	.9988	.9976	.9773	.9632	.6994	.5441	.4942	.0262
7	1.0000	1.0000	.9998	.9996	.9937	.9886	.8385	.7158	.6713	.0669
8	1.0000	1.0000	1.0000	.9999	.9985	.9970	.9254	.8460	.8135	.1431
9	1.0000	1.0000	1.0000	1.0000	.9997	.9993	.9705	.9279	.9084	.2617
10	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9900	.9710	.9613	.4159
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9971	.9900	.9860	.5841
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993	.9971	.9957	.7383
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9989	.8569
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9331
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9738
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9915
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9978
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 23

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.6283	.3074	.1469	.1143	.0347	.0238	.0013	.0004	.0003	.0000
1	.9233	.6794	.4408	.3742	.1601	.1204	.0116	.0042	.0030	.0000
2	.9806	.8948	.7219	.6570	.3774	.3080	.0492	.0209	.0157	.0000
3	.9990	.9742	.8930	.8528	.6168	.5396	.1370	.0683	.0538	.0002
4	.9999	.9951	.9674	.9496	.8052	.7440	.2832	.1643	.1356	.0013
5	1.0000	.9992	.9920	.9860	.9179	.8811	.4685	.3117	.2688	.0053
6	1.0000	.9999	.9984	.9968	.9711	.9537	.6537	.4906	.4399	.0173
7	1.0000	1.0000	.9997	.9994	.9915	.9848	.8037	.6663	.6181	.0466
8	1.0000	1.0000	1.0000	.9999	.9979	.9958	.9037	.8085	.7709	.1050
9	1.0000	1.0000	1.0000	1.0000	.9995	.9990	.9592	.9043	.8799	.2024
10	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9851	.9586	.9454	.3388
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9954	.9846	.9786	.5000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988	.9951	.9928	.6612
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9986	.9979	.7976
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9995	.8950
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9534
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9827
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9947
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9987
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 24	0	.6158	.2920	.1352	.1040	.0299	.0202	.0010	.0003	.0002	.0000
	1	.9174	.6608	.4173	.3508	.1431	.1059	.0090	.0031	.0022	.0000
	2	.9882	.8841	.6994	.6316	.3479	.2798	.0398	.0161	.0119	.0000
	3	.9988	.9702	.8793	.8352	.5842	.5049	.1150	.0547	.0424	.0001
	4	.9999	.9940	.9614	.9409	.7796	.7134	.2466	.1366	.1111	.0008
	5	1.0000	.9990	.9900	.9827	.9026	.8606	.4222	.2692	.2288	.0033
	6	1.0000	.9999	.9979	.9958	.9639	.9428	.6074	.4391	.3886	.0113
	7	1.0000	1.0000	.9996	.9992	.9887	.9801	.7662	.6157	.5647	.0320
	8	1.0000	1.0000	.9999	.9999	.9970	.9941	.8787	.7675	.7250	.0758
	9	1.0000	1.0000	1.0000	1.0000	.9993	.9985	.9453	.8767	.8472	.1537
	10	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9787	.9430	.9258	.2706
	11	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9928	.9771	.9686	.4194
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9979	.9920	.9885	.5806
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9976	.9964	.7294
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9990	.8463
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9242
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9680
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9887
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9967
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
n = 25	0	.6035	.2774	.1244	.0946	.0259	.0172	.0008	.0002	.0001	.0000
	1	.9114	.6424	.3947	.3286	.1277	.0931	.0070	.0023	.0016	.0000
	2	.9868	.8729	.6768	.6063	.3200	.2537	.0321	.0123	.0090	.0000
	3	.9986	.9659	.8649	.8169	.5521	.4711	.0962	.0436	.0332	.0001
	4	.9999	.9928	.9549	.9314	.7530	.6821	.2137	.1130	.0905	.0005
	5	1.0000	.9988	.9877	.9790	.8858	.8385	.3783	.2310	.1935	.0020
	6	1.0000	.9998	.9972	.9946	.9555	.9305	.5611	.3901	.3407	.0073
	7	1.0000	1.0000	.9995	.9989	.9853	.9745	.7265	.5648	.5118	.0216
	8	1.0000	1.0000	.9999	.9998	.9959	.9920	.8506	.7238	.6769	.0539
	9	1.0000	1.0000	1.0000	1.0000	.9990	.9979	.9287	.8453	.8106	.1148
	10	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.9703	.9239	.9022	.2122
	11	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9893	.9673	.9558	.3450
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9966	.9877	.9825	.5000
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991	.9960	.9940	.6550
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9989	.9982	.7878
	15	1.0000	1.0009	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9995	.8852
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9461
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9784
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9927
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9980
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 26

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5914	.2635	.1144	.0861	.0224	.0146	.0006	.0001	.0001	.0000
1	.9052	.6241	.3731	.3076	.1138	.0817	.0055	.0017	.0011	.0000
2	.9852	.8614	.6543	.5813	.2938	.2296	.0258	.0094	.0067	.0000
3	.9983	.9613	.8499	.7979	.5205	.4385	.0802	.0346	.0260	.0000
4	.9999	.9915	.9477	.9211	.7257	.6505	.1844	.0930	.0733	.0003
5	1.0000	.9985	.9851	.9747	.8678	.8150	.3371	.1971	.1626	.0012
6	1.0000	.9998	.9965	.9932	.9461	.9167	.5154	.3443	.2965	.0047
7	1.0000	1.0000	.9993	.9985	.9813	.9679	.6852	.5145	.4605	.0145
8	1.0000	1.0000	.9999	.9997	.9944	.9904	.8195	.6780	.6274	.0378
9	1.0000	1.0000	1.0000	1.0000	.9986	.9970	.9091	.8103	.7705	.0843
10	1.0000	1.0000	1.0000	1.0000	.9997	.9993	.9599	.9013	.8747	.1635
11	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9845	.9548	.9397	.2786
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9948	.9818	.9745	.4225
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9985	.9936	.9906	.5775
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9981	.9970	.7214
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9991	.8365
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9157
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9622
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9855
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 27

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5796	.2503	.1053	.0784	.0193	.0174	.0004	.0001	.0001	.0000
1	.8990	.6061	.3524	.2876	.1014	.0716	.0042	.0012	.0008	.0000
2	.9836	.8495	.6318	.5567	.2694	.2074	.0207	.0072	.0051	.0000
3	.9981	.9563	.8347	.7784	.4897	.4072	.0666	.0273	.0202	.0000
4	.9998	.9900	.9398	.9100	.6978	.6187	.1583	.0762	.0591	.0002
5	1.0000	.9981	.9821	.9699	.8485	.7903	.2989	.1671	.1358	.0008
6	1.0000	.9997	.9956	.9916	.9354	.9014	.4708	.3019	.2563	.0030
7	1.0000	1.0000	.9991	.9980	.9765	.9602	.6427	.4655	.4113	.0096
8	1.0000	1.0000	9998	.9996	.9926	.9862	.7859	.6309	.5773	.0261
9	1.0000	1.0000	1.0000	.9999	.9980	.9958	.8867	.7722	.7276	.0610
10	1.0000	1.0000	1.0000	1.0000	.9995	.9989	.9472	.8751	.8434	.1239
11	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9784	.9394	.9202	.2210
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9922	.9740	.9641	.3506
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9976	.9902	.9857	.5000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993	.9968	.9950	.6494
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9991	.9985	.7790
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.8761
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.9390
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9739
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9904
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9970
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 28

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5680	.2378	.0968	.0713	.0167	.0106	.0003	.0001	.0000	.0000
1	.8925	.5883	.3326	.2688	.0902	.0627	.0033	.0009	.0006	.0000
2	.9820	.8373	.6094	.5325	.2465	.1871	.0166	.0055	.0038	.0000
3	.9978	.9509	.8180	.7585	.4597	.3772	.0551	.0215	.0157	.0000
4	.9998	.9843	.9314	.8982	.6695	.5869	.1354	.0621	.0474	.0001
5	1.0000	.9977	.9787	.9645	.8280	.7646	.2638	.1409	.1128	.0005
6	1.0000	.9996	.9945	.9896	.9236	.8868	.4279	.2631	.2202	.0019
7	1.0000	1.0000	.9988	.9974	.9709	.9514	.5997	.4184	.3648	.0063
8	1.0000	1.0000	.9998	.9995	.9904	.9823	.7501	.5833	.5275	.0178
9	1.0000	1.0000	1.0000	.9999	.9973	.9944	.8615	.7315	.6825	.0436
10	1.0000	1.0000	1.0000	1.0000	.9993	.9985	.9321	.8454	.8087	.0925
11	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9706	.9208	.8972	.1725
12	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9888	.9641	.9509	.2858
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9962	.9856	.9792	.4253
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9989	.9949	.9923	.5747
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9984	.9975	.7142
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9993	.8275
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9075
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9564
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9822
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9937
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9981
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 29

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5566	.2259	.0891	.0649	.0144	.0090	.0002	.0001	.0000	.0000
1	.8860	.5708	.3138	.2510	.0802	.0549	.0025	.0007	.0004	.0000
2	.9802	.8249	.5873	.5087	.2253	.1684	.0133	.0042	.0028	.0000
3	.9975	.9452	.8013	.7381	.4307	.3487	.0455	.0169	.0121	.0000
4	.9997	.9864	.9223	.8856	.6410	.5555	.1153	.0504	.0379	.0001
5	1.0000	.9973	.9740	.9585	.8064	.7379	.2317	.1182	.0932	.0003
6	1.0000	.9995	.9932	.9873	.9106	.8667	.3868	.2274	.1880	.0012
7	1.0000	.9999	.9984	.9967	.9645	.9414	.5568	.3737	.3214	.0041
8	1.0000	1.0000	.9997	.9993	.9878	.9777	.7125	.5358	.4787	.0121
9	1.0000	1.0000	.9999	.9999	.9964	.9926	.8337	.6888	.6360	.0307
10	1.0000	1.0000	1.0000	1.0000	.9991	.9978	.9145	.8126	.7708	.0680
11	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.9610	.8991	.8706	.1325
12	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9842	.9516	.9348	.2291
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9944	.9794	.9707	.3555
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9982	.9922	.9883	.5000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9974	.9959	.6445
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9992	.9987	.7709
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.8675
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9320
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9693
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9879
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9959
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

= P(failure)

n = 30

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5455	.2146	.0820	.0591	.0125	.0076	.0002	.0000	.0000	.0000
1	.8795	.5535	.2958	.2343	.0713	.0480	.0020	.0005	.0003	.0000
2	.9783	.8122	.5654	.4855	.2055	.1514	.0106	.0032	.0021	.0000
3	.9971	.9392	.7842	.7175	.4028	.3217	.0374	.0132	.0093	.0000
4	.9997	.9844	.9126	.8723	.6124	.5245	.0979	.0408	.0302	.0000
5	1.0000	.9967	.9707	.9519	.7839	.7106	.2026	.0987	.0766	.0002
6	1.0000	.9994	.9918	.9848	.8964	.8474	.3481	.1963	.1595	.0007
7	1.0000	.9999	.9980	.9959	.9571	.9302	.5143	.3317	.2814	.0026
8	1.0000	1.0000	.9996	.9990	.9846	.9722	.6736	.4891	.4315	.0081
9	1.0000	1.0000	.9999	.9998	.9952	.9903	.8034	.6448	.5888	.0214
10	1.0000	1.0000	1.0000	1.0000	.9987	.9971	.8943	.7770	.7304	.0494
11	1.0000	1.0000	1.0000	1.0000	.9997	.9992	.9493	.8742	.8407	.1002
12	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9784	.9365	.9155	.1808
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9918	.9714	.9599	.2923
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9973	.9885	.9831	.4278
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992	.9959	.9936	.5722
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9987	.9979	.7077
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9994	.8192
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.8998
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9506
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9786
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9919
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9974
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

n = 31

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5346	.2039	.0754	.0537	.0108	.0065	.0001	.0000	.0000	.0000
1	.8728	.5366	.2787	.2185	.0633	.0420	.0015	.0004	.0002	.0000
2	.9763	.7992	.5438	.4629	.1873	.1359	.0084	.0024	.0016	.0000
3	.9967	.9329	.7667	.6966	.3760	.2961	.0307	.0103	.0072	.0000
4	.9996	.9821	.9024	.8584	.5839	.4940	.0828	.0329	.0239	.0000
5	1.0000	.9961	.9661	.9448	.7606	.6827	.1764	.0820	.0627	.0001
6	1.0000	.9993	.9901	.9818	.8811	.8269	.3117	.1682	.1346	.0004
7	1.0000	.9999	.9975	.9949	.9489	.9178	.4727	.2927	.2448	.0017
8	1.0000	1.0000	.9995	.9988	.9809	.9659	.6338	.4438	.3865	.0053
9	1.0000	1.0000	.9999	.9997	.9938	.9876	.7710	.5999	.5416	.0147
10	1.0000	1.0000	1.0000	1.0000	.9982	.9961	.8716	.7389	.6879	.0354
11	1.0000	1.0000	1.0000	1.0000	.9995	.9989	.9356	.8462	.8076	.0748
12	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9711	.9186	.8931	.1405
13	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9885	.9613	.9466	.2366
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9959	.9836	.9761	.3601
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9987	.9938	.9905	.5000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9979	.9966	.6399
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9989	.7634
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.8595
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9252
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9646
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9853
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9947
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9983
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5239	.1937	.0694	.0489	.0093	.0055	.0001	.0000	.0000	.0000
1	.8660	.5200	.2624	.2037	.0561	.0366	.0012	.0003	.0002	.0000
2	.9742	.7861	.5226	.4409	.1704	.1218	.0067	.0018	.0012	.0000
3	.9963	.9262	.7489	.6756	.3503	.2721	.0252	.0081	.0055	.0000
4	.9996	.9796	.8915	.8438	.5556	.4644	.0698	.0264	.0189	.0000
5	1.0000	.9954	.9610	.9370	.7365	.6544	.1530	.0679	.0510	.0001
6	1.0000	.9991	.9881	.9785	.8647	.8053	.2779	.1434	.1131	.0003
7	1.0000	.9999	.9969	.9937	.9397	.9042	.4325	.2568	.2118	.0011
8	1.0000	1.0000	.9993	.9984	.9765	.9587	.5935	.4003	.3440	.0035
9	1.0000	1.0000	.9999	.9997	.9920	.9844	.7367	.5550	.4961	.0100
10	1.0000	1.0000	1.0000	.9999	.9976	.9948	.8464	.6989	.6440	.0251
11	1.0000	1.0000	1.0000	1.0000	.9994	.9985	.9196	.8153	.7717	.0551
12	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9622	.8977	.8674	.1077
13	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9841	.9490	.9306	.1885
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9940	.9772	.9673	.2983
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9980	.9908	.9862	.4300
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994	.9967	.9948	.5700
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9989	.9982	.7017
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9995	.8115
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8923
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9449
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9749
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9900
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9965
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9989
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 32

p = ∞(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5134	.1840	.0638	.0445	.0080	.0047	.0001	.0000	.0000	.0000
1	.8592	.5036	.2470	.1897	.0498	.0320	.0009	.0002	.0001	.0000
2	.9721	.7728	.5018	.4196	.1549	.1090	.0053	.0014	.0009	.0000
3	.9959	.9192	.7308	.6544	.3258	.2495	.0205	.0063	.0042	.0000
4	.9995	.9770	.8801	.8287	.5277	.4355	.0586	.0211	.0149	.0000
5	1.0000	.9946	.9554	.9286	.7119	.6259	.1322	.0559	.0414	.0000
6	1.0000	.9989	.9860	.9747	.8473	.7826	.2467	.1216	.0944	.0002
7	1.0000	.9998	.9962	.9923	.9295	.8803	.3938	.2242	.1822	.0007
8	1.0000	1.0000	.9991	.9980	.9715	.9505	.5533	.3590	.3043	.0023
9	1.0000	1.0000	.9998	.9995	.9899	.9805	.7009	.5104	.4497	.0068
10	1.0000	1.0000	1.0000	.9999	.9968	.9932	.8190	.6574	.5993	.0175
11	1.0000	1.0000	1.0000	1.0000	.9991	.9979	.9013	.7818	.7334	.0401
12	1.0000	1.0000	1.0000	1.0000	.9998	.9994	.9516	.8740	.8387	.0814
13	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9787	.9342	.9116	.1481
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9916	.9691	.9563	.2434
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9970	.9869	.9805	.3642
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990	.9950	.9922	.5000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9983	.9972	.6358
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9991	.7566
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.8519
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9186
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9599
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9825
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9932
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9977
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 33

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.5031	.1748	.0587	.0405	.0069	.0040	.0001	.0000	.0000	.0000
1	.8523	.4877	.2323	.1767	.0441	.0270	.0007	.0001	.0001	.0000
2	.9698	.7593	.4814	.3989	.1406	.0975	.0042	.0010	.0006	.0000
3	.9954	.9119	.7125	.6333	.3026	.2285	.0167	.0048	.0032	.0000
4	.9995	.9741	.8682	.8130	.5002	.4076	.0491	.0168	.0117	.0000
5	.9999	.9937	.9494	.9196	.6869	.5973	.1138	.0459	.0334	.0000
6	1.0000	.9987	.9835	.9706	.8284	.7591	.2180	.1027	.0785	.0001
7	1.0000	.9998	.9954	.9907	.9183	.8733	.3570	.1946	.1558	.0004
8	1.0000	1.0000	.9989	.9975	.9658	.9413	.5134	.3201	.2677	.0015
9	1.0000	1.0000	.9998	.9994	.9874	.9760	.6640	.4668	.4061	.0045
10	1.0000	1.0000	1.0000	.9999	.9959	.9913	.7894	.6151	.5545	.0122
11	1.0000	1.0000	1.0000	1.0000	.9988	.9972	.8807	.7460	.6932	.0288
12	1.0000	1.0000	1.0000	1.0000	.9997	.9992	.9390	.8474	.8071	.0607
13	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9719	.9169	.8897	.1147
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9883	.9590	.9429	.1958
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9956	.9818	.9732	.3038
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9985	.9927	.9887	.4321
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9974	.9957	.5679
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9991	.9985	.6962
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.8042
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8853
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9393
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9712
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9878
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9955
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9985
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 34

p = P(failure)

n = 35

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4931	.1661	.0540	.0369	.0060	.0034	.0000	.0000	.0000	.0000
1	.8453	.4720	.2184	.1644	.0390	.0243	.0005	.0001	.0001	.0000
2	.9675	.7458	.4615	.3789	.1275	.0870	.0033	.0008	.0005	.0000
3	.9940	.9042	.6940	.6122	.2806	.2088	.0136	.0037	.0024	.0000
4	.9994	.9710	.8557	.7968	.4733	.3807	.0410	.0134	.0091	.0000
5	.9999	.9927	.9429	.9100	.6615	.5689	.0976	.0375	.0269	.0000
6	1.0000	.9985	.9808	.9660	.8096	.7348	.1920	.0863	.0650	.0001
7	1.0000	.9997	.9945	.9889	.9061	.8562	.3223	.1682	.1326	.0003
8	1.0000	1.0000	.9986	.9969	.9593	.9311	.4743	.2840	.2341	.0009
9	1.0000	1.0000	.9997	.9992	.9845	.9708	.6263	.4245	.3646	.0030
10	1.0000	1.0000	.9999	.9998	.9947	.9890	.7581	.5724	.5100	.0083
11	1.0000	1.0000	1.0000	1.0000	.9984	.9963	.8579	.7083	.6516	.0205
12	1.0000	1.0000	1.0000	1.0000	.9996	.9989	.9244	.8182	.7729	.0448
13	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9637	.8969	.8650	.0877
14	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9842	.9469	.9269	.1553
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9938	.9752	.9641	.2498
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9978	.9895	.9840	.3679
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993	.9960	.9936	.5000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9986	.9977	.6321
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9992	.7502
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.8447
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9123
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9552
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9795
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9917
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9970
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4832	.1578	.0497	.0335	.0052	.0029	.0000	.0000	.0000	.0000
1	.8302	.4567	.2053	.1529	.0345	.0212	.0004	.0001	.0000	.0000
2	.9650	.7321	.4420	.3596	.1154	.0776	.0026	.0006	.0004	.0000
3	.9943	.8963	.6754	.5912	.2597	.1906	.0110	.0029	.0018	.0000
4	.9993	.9676	.8428	.7802	.4471	.3550	.0342	.0106	.0071	.0000
5	.9999	.9917	.9359	.8998	.6359	.5406	.0835	.0306	.0216	.0000
6	1.0000	.9982	.9778	.9610	.7894	.7099	.1684	.0723	.0536	.0000
7	1.0000	.9997	.9934	.9869	.8930	.8380	.2897	.1446	.1124	.0002
8	1.0000	.9999	.9983	.9962	.9521	.9199	.4363	.2506	.2037	.0006
9	1.0000	1.0000	.9996	.9990	.9810	.9649	.5883	.3841	.3254	.0020
10	1.0000	1.0000	.9999	.9998	.9933	.9863	.7251	.5298	.4663	.0057
11	1.0000	1.0000	1.0000	1.0000	.9979	.9952	.8329	.6691	.6091	.0144
12	1.0000	1.0000	1.0000	1.0000	.9994	.9985	.9078	.7865	.7365	.0326
13	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9539	.8742	.8373	.0662
14	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9791	.9325	.9083	.1215
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9914	.9671	.9530	.2025
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9968	.9854	.9781	.3089
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9989	.9941	.9907	.4340
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9979	.9964	.5660
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9988	.6911
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.7975
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8785
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9338
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9674
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9856
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9943
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9980
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 36

n = 37

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.280	.300	.500
0	.4735	.1499	.0457	.0305	.0045	.0024	.0000	.0000	.0000	.0000
1	.8311	.4418	.1928	.1422	.0306	.0184	.0003	.0001	.0000	.0000
2	.9625	.7183	.4231	.3410	.1044	.0692	.0021	.0004	.0003	.0000
3	.9938	.8881	.6567	.5704	.2401	.1736	.0089	.0022	.0014	.0000
4	.9992	.9641	.8294	.7632	.4216	.3303	.0284	● fIfIP6	.0055	.0000
5	.9999	.9905	.9285	.8891	.6102	.5128	.0711	.0248	.0172	.0000
6	1.0000	.9979	.9744	.9555	.7686	.6845	.1472	.0603	.0440	.0000
7	1.0000	.9996	.9921	.9845	.8789	.8188	.2594	.1238	.0947	.0001
8	1.0000	.9999	.9979	.9953	.9441	.9076	.3996	.2201	.1763	.0004
9	1.0000	1.0000	.9995	.9988	.9771	.9581	.5503	.3456	.2889	.0013
10	1.0000	1.0000	.9999	.9997	.9917	.9831	.6909	.4878	.4241	.0038
11	1.0000	1.0000	1.0000	.9999	.9973	.9939	.8060	.6290	.5663	.0100
12	1.0000	1.0000	1.0000	1.0000	.9992	.9980	.8891	.7527	.6983	.0235
13	1.0000	1.0000	1.0000	1.0000	.9998	●9994	.9423	.8490	.8071	.0494
14	1.0000	1.0000	1.0000	I.norm	1.0000	.9998	.9728	.9157	.8870	.0939
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9883	.9571	.9306	.1620
16	1.0000	1.0000	1.0000	1.0000	I.norm	1.0000	.9955	.9801	.9705	.2557
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9984	.9916	.9869	.3714
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9968	.9947	.5000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9989	.9981	.6286
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9994	.7443
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.8380
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9061
23	1.0000	1.0000	1.0000	I.norm	1.0000	1.0000	1.0000	1.0000	1.0000	.9506
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9765
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	I.norm	1.0000	1.0000	.9900
26	.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	].norm	1.0000	.9962
27	.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9987
28	.0000	1.0000	1.0000	I.norm	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
29	.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
30	.0000	1.0000	1.0000	I.norm	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	.0000	1.0000	1.0000	1.0000	1.0000	I.norm	1.0000	1.0000	I.norm	I.norm
32	1.0000	1.0000	1.0000	1.0000	1.0000	I.norm	1.0000	1.0000	I.norm	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	I.norm	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	I.norm

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4641	.1424	.0421	.0278	.0039	.0021	.0000	.0000	.0000	.0000
1	.8240	.4272	.1811	.1321	.0270	.0160	.0002	.0000	.0000	.0000
2	.9599	.7045	.4047	.3231	.0944	.0615	.0016	.0003	.0002	.0000
3	.9931	.8796	.6380	.5497	.2217	.1579	.0072	.0017	.0011	.0000
4	.9991	.9603	.8156	.7458	.3970	.3068	.0235	.0066	.0043	.0000
5	.9999	.9891	.9205	.8777	.5846	.4854	.0604	.0201	.0137	.0000
6	1.0000	.9975	.9707	.9495	.7470	.6588	.1282	.0501	.0359	.0000
7	1.0000	.9995	.9907	.9819	.8639	.7986	.2313	.1055	.0795	.0001
8	1.0000	.9999	.9974	.9943	.9352	.8943	.3646	.1923	.1518	.0002
9	1.0000	1.0000	.9994	.9984	.9726	.9505	.5126	.3095	.2551	.0008
10	1.0000	1.0000	.9999	.9996	.9897	.9793	.6558	.4469	.3835	.0025
11	1.0000	1.0000	1.0000	.9999	.9965	.9923	.7772	.5883	.5236	.0069
12	1.0000	1.0000	1.0000	1.0000	.9990	.9974	.8693	.7171	.6587	.0168
13	1.0000	1.0000	1.0000	1.0000	.9997	.9992	.9290	.8213	.7745	.0365
14	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9652	.8965	.8631	.0717
15	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9844	.9452	.9238	.1279
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9937	.9735	.9612	.2088
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9977	.9863	.9820	.3136
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992	.9953	.9924	.4357
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9983	.9971	.5643
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9990	.6864
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9997	.7912
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8721
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9283
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9635
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9832
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9931
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9975
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 38

p = P(failure)

c	.020	.050	.080	.090	. 36	. 50	.250	.288	.300	.500
0	.4548	.1353	.0387	.0253	.0033	.0018	.0000	.0000	.0000	.0000
1	.8168	.4129	.1699	.1227	.0239	.0139	.0002	.0000	.0000	.0000
2	.9571	.6906	.3868	.3059	.0852	.0547	.0013	.0002	.0001	.0000
3	.9925	.8709	.6194	.5293	.2043	.1435	.0058	.0013	.0008	.0000
4	.9990	.9562	.8014	.7282	.3731	.2845	.0194	.0052	.0033	.0000
5	.9999	.9877	.9121	.8659	.5591	.4586	.0512	.0162	.0109	.0000
6	1.0000	.9971	.9667	.9430	.7249	.6328	.1112	.0414	.0293	.0000
7	1.0000	.9994	.9891	.9790	.8480	.7777	.2055	.0895	.0664	.0000
8	1.0000	.9999	.9969	.9932	.9255	.8799	.3313	.1673	.1301	.0001
9	1.0000	1.0000	.9992	.9981	.9675	.9421	.4756	.2757	.2241	.0005
10	1.0000	1.0000	.9998	.9995	.9874	.9750	.6200	.4073	.3450	.0017
11	1.0000	1.0000	1.0000	.9999	.9956	.9903	.7469	.5476	.4816	.0047
12	1.0000	1.0000	1.0000	1.0000	.9986	.9966	.8455	.6800	.6182	.0119
13	1.0000	1.0000	1.0000	1.0000	.9996	.9989	.9138	.7913	.7397	.0266
14	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9561	.8748	.8365	.0541
15	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9796	.9312	.9056	.0998
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9914	.9653	.9500	.1684
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9967	.9840	.9758	.2612
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9988	.9933	.9803	.3746
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9974	.9957	.5000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9991	.9984	.6254
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9995	.7388
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.8316
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9002
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9459
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9734
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9881
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9983
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 39

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4457	.1285	.0356	.0230	.0029	.0015	.0000	.0000	.0000	.0000
1	.8095	.3991	.1594	.1140	.0211	.0121	.0001	.0000	.0000	.0000
2	.9543	.6767	.3694	.2894	.0769	.0486	.0010	.0002	.0001	.0000
3	.9918	.8619	.6007	.5092	.1881	.1302	.0047	.0010	.0006	.0000
4	.9988	.9520	.7868	.7103	.3502	.2633	.0160	.0041	.0026	.0000
5	.9999	.9861	.9033	.8535	.5338	.4325	.0433	.0130	.0086	.0000
6	1.0000	.9966	.9624	.9361	.7024	.6067	.0962	.0347	.0238	.0000
7	1.0000	.9993	.9873	.9758	.8313	.7559	.1820	.0757	.0553	.0000
8	1.0000	.9999	.9963	.9919	.9150	.8646	.2998	.1449	.1110	.0001
9	1.0000	1.0000	.9990	.9976	.9618	.9328	.4395	.2445	.1959	.0003
10	1.0000	1.0000	.9998	.9994	.9847	.9701	.5839	.3694	.3087	.0011
11	1.0000	1.0000	1.0000	.9999	.9945	.9880	.7151	.5072	.4406	.0032
12	1.0000	1.0000	1.0000	1.0000	.9982	.9957	.8209	.6419	.5772	.0083
13	1.0000	1.0000	1.0000	1.0000	.9995	.9986	.8968	.7542	.7032	.0192
14	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9456	.8508	.8074	.0403
15	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9738	.9149	.8849	.0769
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9884	.9555	.9367	.1341
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953	.9787	.9680	.2148
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9983	.9906	.9852	.3179
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994	.9962	.9937	.4373
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9986	.9976	.5627
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9991	.6821
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.7852
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8659
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9231
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9597
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9808
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9917
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9968
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9989
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 40

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4368	.1221	.0328	.0209	.0025	.0013	.0000	.0000	.0000	.0000
1	.8023	.3855	.1495	.1058	.0186	.0105	.0001	.0000	.0000	.0000
2	.9514	.6629	.3526	.2736	.0693	.0431	.0008	.0001	.0001	.0000
3	.9910	.8526	.5822	.4894	.1730	.1179	.0038	.0008	.0004	.0000
4	.9987	.9475	.7719	.6922	.3281	.2433	.0132	.0032	.0020	.0000
5	.9998	.9844	.8940	.8406	.5088	.4071	.0365	.0105	.0068	.0000
6	1.0000	.9961	.9576	.9286	.6794	.5805	.0839	.0281	.0192	.0000
7	1.0000	.9992	.9853	.9722	.8137	.7335	.1605	.0637	.0458	.0000
8	1.0000	.9998	.9955	.9905	.9036	.8483	.2704	.1250	.0943	.0001
9	1.0000	1.0000	.9988	.9971	.9554	.9226	.4046	.2158	.1705	.0002
10	1.0000	1.0000	.9997	.9992	.9816	.9645	.5478	.3334	.2749	.0007
11	1.0000	1.0000	.9999	.9998	.9931	.9853	.6823	.4675	.4010	.0022
12	1.0000	1.0000	1.0000	1.0000	.9977	.9945	.7944	.6031	.5362	.0058
13	1.0000	1.0000	1.0000	1.0000	.9993	.9982	.8778	.7254	.6654	.0138
14	1.0000	1.0000	1.0000	1.0000	.9998	.9994	.9334	.8244	.7762	.0298
15	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9667	.8965	.8616	.0586
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9848	.9438	.9211	.1055
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9936	.9720	.9586	.1744
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9976	.9872	.9801	.2664
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991	.9946	.9912	.3776
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9979	.9964	.5000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9987	.6224
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.7336
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8256
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.8945
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9414
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9702
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9862
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9942
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9978
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 41

θ = P(failure)

n = 42

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.4281	.1160	.0301	.0190	.0022	.0011	.0000	.0000	.0000	.0000
1	.7950	.3724	.1402	.0981	.0164	.0091	.0001	.0000	.0000	.0000
2	.9485	.6490	.3364	.2585	.0624	.0382	.0006	.0001	.0001	.0000
3	.9902	.8431	.5639	.4700	.1589	.1067	.0030	.0006	.0003	.0000
4	.9985	.9427	.7567	.6739	.3070	.2245	.0108	.0025	.0015	.0000
5	.9998	.9826	.8842	.8272	.4842	.3826	.0307	.0084	.0054	.0000
6	1.0000	.9955	.9525	.9207	.6562	.5545	.0714	.0230	.0155	.0000
7	1.0000	.9990	.9831	.9683	.7955	.7106	.1411	.0534	.0378	.0000
8	1.0000	.9998	.9947	.9888	.8914	.8311	.2429	.1073	.0798	.0000
9	1.0000	1.0000	.9985	.9965	.9484	.9114	.3711	.1897	.1476	.0001
10	1.0000	1.0000	.9996	.9990	.9780	.9582	.5120	.2996	.2436	.0005
11	1.0000	1.0000	.9999	.9998	.9916	.9822	.6487	.4289	.3632	.0014
12	1.0000	1.0000	1.0000	.9999	.9971	.9932	.7664	.5640	.4957	.0040
13	1.0000	1.0000	1.0000	1.0000	.9991	.9976	.8570	.6902	.6267	.0098
14	1.0000	1.0000	1.0000	1.0000	.9997	.9992	.9195	.7959	.7430	.0218
15	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9584	.8757	.8360	.0442
16	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9803	.9302	.9033	.0821
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9914	.9639	.9474	.1400
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9966	.9828	.9736	.2204
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9987	.9925	.9879	.3220
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9970	.9949	.4388
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9989	.9980	.5612
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9993	.6780
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.7796
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8600
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9179
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9558
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9782
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9902
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9960
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9986
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.280	.300	.500
0	.4195	.1102	.0277	.0173	.0019	.0009	.0000	.0000	.0000	.0000
1	.7876	.3595	.1314	.0910	.0145	.0079	.0001	.0000	.0000	.0000
2	.9454	.6352	.3207	.2441	.0561	.0339	.0005	.0001	.0000	.0000
3	.9894	.8334	.5457	.4510	.1458	.0964	.0024	.0004	.0003	.0000
4	.9986	.9377	.7413	.6556	.2869	.2069	.0089	.0019	.0012	.0000
5	.9998	.9806	.8740	.8134	.4601	.3589	.0257	.0067	.0042	.0000
6	1.0000	.9949	.9471	.9123	.6328	.5287	.0612	.0188	.0125	.0000
7	1.0000	.9988	.9807	.9640	.7765	.6872	.1237	.0447	.0311	.0000
8	1.0000	.9998	.9938	.9870	.8783	.8130	.2175	.0918	.0672	.0000
9	1.0000	1.0000	.9982	.9958	.9406	.8994	.3390	.1660	.1272	.0001
10	1.0000	1.0000	.9996	.9988	.9740	.9512	.4768	.2679	.2148	.0003
11	1.0000	1.0000	.9999	.9997	.9897	.9786	.6145	.3917	.3273	.0010
12	1.0000	1.0000	1.0000	.9999	.9963	.9915	.7370	.5251	.4559	.0027
13	1.0000	1.0000	1.0000	1.0000	.9988	.9969	.8343	.6539	.5874	.0069
14	1.0000	1.0000	1.0000	1.0000	.9997	.9990	.9038	.7655	.7081	.0158
15	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9486	.8527	.8081	.0330
16	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9748	.9145	.8831	.0631
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9886	.9542	.9342	.1110
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953	.9774	.9658	.1802
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9982	.9897	.9836	.2712
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994	.9957	.9928	.3804
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9983	.9971	.5000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9989	.6196
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.7288
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8198
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.8890
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9369
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9670
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9842
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9931
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9973
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 43

p = P failure

c	.020	.050	.080	.090	.136	.150	.250	.278	.300	.500
0	.4111	.1047	.0255	.0158	.0016	.0008	.0000	.0000	.0000	.0000
1	.7803	.3471	.1231	.0844	.0128	.0069	.0000	.0000	.0000	.0000
2	.9422	.6214	.3056	.2303	.0505	.0300	.0004	.0001	.0000	.0000
3	.9885	.8235	.5277	.4324	.1336	.0871	.0019	.0003	.0002	.0000
4	.9982	.9325	.7257	.6372	.2677	.1903	.0073	.0015	.0009	.0000
5	.9998	.9784	.8634	.7992	.4366	.3361	.0215	.0053	.0033	.0000
6	1.0000	.9941	.9412	.9034	.6094	.5032	.0523	.0153	.0100	.0000
7	1.0000	.9986	.9780	.9593	.7570	.6634	.1081	.0372	.0255	.0000
8	1.0000	.9997	.9927	.9849	.8645	.7941	.1940	.0782	.0564	.0000
9	1.0000	.9999	.9979	.9950	.9322	.8864	.3086	.1446	.1092	.0001
10	1.0000	1.0000	.9995	.9985	.9694	.9434	.4423	.2386	.1885	.0002
11	1.0000	1.0000	.9999	.9996	.9876	.9745	.5801	.3560	.2936	.0006
12	1.0000	1.0000	1.0000	.9999	.9954	.9896	.7064	.4867	.4173	.0018
13	1.0000	1.0000	1.0000	1.0000	.9985	.9961	.8100	.6168	.5479	.0048
14	1.0000	1.0000	1.0000	1.0000	.9995	.9987	.8865	.7333	.6719	.0113
15	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9374	.8276	.7781	.0244
16	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9682	.8967	.8606	.0481
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9852	.9427	.9188	.0871
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9936	.9707	.9563	.1456
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9975	.9861	.9782	.2257
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9991	.9940	.9900	.3258
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9976	.9958	.4402
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9991	.9983	.5598
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9994	.6742
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.7743
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8544
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9129
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9519
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9756
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9887
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9952
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9982
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 44

p = P(failure)

c	.020	.050	.080	.090	.136	.[illegible]5[illegible]	.250	.288	.300	.500
0	.4029	.0994	.0235	.0144	.0014	.0007	.0000	.0000	.0000	.0000
1	.7729	.3350	.1153	.0782	.0117	.0060	.0000	.0000	.0000	.0000
2	.9390	.6077	.2910	.2172	.0453	.0265	.0003	.0000	.0000	.0000
3	.9876	.8134	.5099	.4142	.1223	.0785	.0016	.0003	.0001	.0000
4	.9980	.9271	.7098	.6187	.2425	.1748	.0049	.0012	.0007	.0000
5	.9997	.9761	.8524	.7846	.4196	.3142	.0149	.0042	.0026	.0000
6	1.0000	.9934	.9350	.8940	.5839	.4782	.0496	.0124	.0080	.0000
7	1.0000	.9984	.9750	.9543	.7359	.6394	.0971	.0309	.0209	.0000
8	1.0000	.9997	.9916	.9826	.8469	.7745	.1825	.0664	.0471	.0000
9	1.0000	.9999	.9975	.9941	.9294	.8726	.2800	.1255	.0934	.0000
10	1.0000	1.0000	.9993	.9982	.9631	.9349	.4489	.2115	.1647	.0001
11	1.0000	1.0000	.9998	.9995	.9844	.9608	.5757	.3222	.2620	.0004
12	1.0000	1.0000	1.0000	.9999	.9941	.9873	.6848	.4491	.3802	.0012
13	1.0000	1.0000	1.0000	1.0000	.9984	.9952	.7641	.5793	.5088	.0033
14	1.0000	1.0000	1.0000	1.0000	.9998	.9983	.8273	.6998	.6347	.0080
15	1.0000	1.0000	1.0000	1.0000	.9990	.9995	.9647	.8004	.7462	.0178
16	1.0000	1.0000	1.0000	1.0000	.0000	.9008	.9805	.8768	.8358	.0362
17	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9909	.9295	.9014	.0676
18	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9915	.9626	.9451	.1163
19	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9965	.9817	.9717	.1856
20	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9987	.9917	.9865	.2757
21	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9995	.9965	.9940	.3830
22	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.9099	.9987	.9976	.5000
23	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	.9995	.9991	.6170
24	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	.9998	.9997	.7243
25	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	.9999	.8144
26	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.8837
27	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9324
28	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9638
29	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9822
30	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9920
31	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9967
32	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9988
33	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9996
34	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	.9999
35	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	.0000	1.0000	1.0000	1.0000

n = 45

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.3948	.0945	.0216	.0131	.0012	.0006	.0000	.0000	.0000	.0000
1	.7655	.3232	.1079	.0725	.0099	.0052	.0000	.0000	.0000	.0000
2	.9357	.5940	.2769	.2047	.0407	.0234	.0002	.0000	.0000	.0000
3	.9866	.8031	.4924	.3964	.1118	.0707	.0012	.0002	.0001	.0000
4	.9978	.9214	.6938	.6003	.2322	.1604	.0048	.0009	.0005	.0000
5	.9997	.9737	.8410	.7697	.3913	.2933	.0149	.0033	.0020	.0000
6	1.0000	.9925	.9284	.8842	.5624	.4536	.0379	.0101	.0063	.0000
7	1.0000	.9982	.9718	.9489	.7164	.6152	.0817	.0256	.0170	.0000
8	1.0000	.9996	.9902	.9801	.8345	.7543	.1529	.0562	.0392	.0000
9	1.0000	.9999	.9970	.9931	.9130	.8579	.2531	.1085	.0795	.0000
10	1.0000	1.0000	.9992	.9979	.9587	.9255	.3767	.1867	.1433	.0001
11	1.0000	1.0000	.9998	.9994	.9823	.9646	.5115	.2903	.2328	.0003
12	1.0000	1.0000	.0000	.9999	.9931	.9847	.6425	.4125	.3448	.0008
13	1.0000	1.0000	.0000	1.0000	.9976	.9940	.7568	.5418	.4702	.0023
14	1.0000	1.0000	.0000	1.0000	.9992	.9978	.8465	.6651	.5969	.0057
15	1.0000	1.0000	.0000	1.0000	.9998	.9993	.9104	.7714	.7128	.0129
16	1.0000	1.0000	.0000	1.0000	.9999	.9998	.9516	.8548	.8690	.0270
17	1.0000	1.0000	.0000	1.0000	1.0000	.9999	.9758	.9143	.8817	.0519
18	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9889	.9531	.9319	.0920
19	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9953	.9762	.9637	.1510
20	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9981	.9888	.9820	.2307
21	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9993	.9951	.9918	.3294
22	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9998	.9981	.9965	.4415
23	1.0000	1.0000	0000	1.0000	1.0000	1.0000	.9999	.9993	.9986	.5585
24	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.6706
25	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.7693
26	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8490
27	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9080
28	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9481
29	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9730
30	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9871
31	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9943
32	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9977
33	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9992
34	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
35	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
36	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
46	1.0000	1.0000	1 0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 46

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.268	.300	.500
0	.3869	.0897	.0199	.0119	.0010	.0005	.0000	.0000	.0000	.0000
1	.7581	.3117	.1010	.0671	.0087	.0045	.0000	.0000	.0000	.0000
2	.9323	.5805	.2634	.1928	.0365	.0207	.0002	.0000	.0000	.0000
3	.9856	.7926	.4752	.3792	.1022	.0636	.0010	.0001	.0001	.0000
4	.9976	.9155	.6777	.5820	.2158	.1469	.0039	.0007	.0004	.0000
5	.9997	.9711	.8292	.7545	.3696	.2733	.0124	.0026	.0016	.0000
6	1.0000	.9916	.9214	.8739	.5392	.4295	.0322	.0081	.0050	.0000
7	1.0000	.9979	.9684	.9431	.6954	.5910	.0708	.0211	.0138	.0000
8	1.0000	.9995	.9888	.9773	.8184	.7334	.1351	.0474	.0326	.0000
9	1.0000	.9999	.9965	.9919	.9023	.8423	.2281	.0934	.0674	.0000
10	1.0000	1.0000	.9990	.9974	.9525	.9154	.3458	.1642	.1242	.0000
11	1.0000	1.0000	.9997	.9993	.9791	.9587	.4778	.2605	.2060	.0002
12	1.0000	1.0000	.9999	.9998	.9916	.9817	.6098	.3773	.3112	.0005
13	1.0000	1.0000	1.0000	1.0000	.9970	.9926	.7282	.5046	.4326	.0015
14	1.0000	1.0000	1.0000	1.0000	.9990	.9973	.8241	.6296	.5589	.0040
15	1.0000	1.0000	1.0000	1.0000	.9997	.9991	.8944	.7408	.6780	.0093
16	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9413	.8308	.7801	.0200
17	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9698	.8972	.8599	.0395
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9856	.9410	.9169	.0719
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9937	.9695	.9547	.1215
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9974	.9852	.9765	.1908
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990	.9933	.9888	.2800
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9972	.9951	.3854
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9989	.9980	.5000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996	.9992	.6146
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.7200
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8092
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.8785
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9281
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9605
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9800
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9907
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9960
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9985
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
46	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
47	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 47

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.3742	.0853	.0183	.0108	.0009	.0004	.0000	.0000	.0000	.0000
1	.7506	.3006	.0945	.0622	.0077	.0039	.0000	.0000	.0000	.0000
2	.9288	.5670	.2504	.1815	.0327	.0183	.0001	.0000	.0000	.0000
3	.9845	.7820	.4582	.3624	.0932	.0572	.0008	.0001	.0001	.0000
4	.9973	.9093	.6615	.5637	.2003	.1344	.0032	.0005	.0003	.0000
5	.9996	.9683	.8171	.7389	.3487	.2544	.0103	.0021	.0012	.0000
6	1.0000	.9905	.9140	.8631	.5161	.4061	.0272	.0065	.0040	.0000
7	1.0000	.9976	.9646	.9368	.6742	.5667	.0611	.0174	.0112	.0000
8	1.0000	.9994	.9871	.9742	.8017	.7120	.1190	.0398	.0269	.0000
9	1.0000	.9999	.9958	.9906	.8909	.8260	.2048	.0802	.0570	.0000
10	1.0000	1.0000	.9988	.9969	.9457	.9044	.3164	.1438	.1071	.0001
11	1.0000	1.0000	.9997	.9991	.9755	.9522	.4448	.2327	.1814	.0004
12	1.0000	1.0000	.9999	.9998	.9899	.9782	.5768	.3437	.2796	.0010
13	1.0000	1.0000	1.0000	.9999	.9962	.9909	.6986	.4679	.3961	.0028
14	1.0000	1.0000	1.0000	1.0000	.9987	.9966	.8001	.5936	.5210	.0066
15	1.0000	1.0000	1.0000	1.0000	.9996	.9988	.8768	.7088	.6423	.0147
16	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9296	.8049	.7405	.0297
17	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9626	.8781	.8359	.0557
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9816	.9290	.8998	.0967
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9916	.9616	.9430	.1562
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9965	.9807	.9698	.2354
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9986	.9910	.9852	.3327
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9995	.9961	.9932	.4427
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9984	.9971	.5573
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9989	.6673
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9996	.7646
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9999	.8438
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9033
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9443
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9703
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9853
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9934
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9972
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9996
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
46	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
47	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
48	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 48

p = P(failure)

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.3716	.0810	.0166	.0098	.0008	.0003	.0000	.0000	.0000	.0000
1	.7432	.2899	.0884	.0575	.0068	.0034	.0000	.0000	.0000	.0000
2	.9252	.5537	.2379	.1707	.0293	.0161	.0001	.0000	.0000	.0000
3	.9834	.7713	.4416	.3461	.0850	.0513	.0006	.0001	.0000	.0000
4	.9971	.9030	.6452	.5456	.1858	.1228	.0026	.0004	.0002	.0000
5	.9996	.9653	.8046	.7232	.3285	.2364	.0085	.0016	.0009	.0000
6	.9999	.9894	.9063	.8520	.4933	.3833	.0230	.0053	.0032	.0000
7	1.0000	.9972	.9605	.9302	.6527	.5426	.0527	.0143	.0090	.0000
8	1.0000	.9994	.9853	.9708	.7844	.6902	.1046	.0333	.0222	.0000
9	1.0000	.9999	.9952	.9891	.8788	.8089	.1834	.0685	.0480	.0000
10	1.0000	1.0000	.9986	.9964	.9382	.8926	.2885	.1255	.0921	.0000
11	1.0000	1.0000	.9996	.9989	.9714	.9450	.4127	.2071	.1502	.0001
12	1.0000	1.0000	.9999	.9997	.9880	.9743	.5438	.3117	.2502	.0002
13	1.0000	1.0000	1.0000	.9999	.9954	.9890	.6681	.4321	.3612	.0007
14	1.0000	1.0000	1.0000	1.0000	.9984	.9957	.7747	.5574	.4835	.0019
15	1.0000	1.0000	1.0000	1.0000	.9995	.9985	.8576	.6756	.6059	.0047
16	1.0000	1.0000	1.0000	1.0000	.9998	.9995	.9164	.7772	.7173	.0106
17	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9544	.8570	.8100	.0222
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9769	.9143	.8806	.0427
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9891	.9522	.9300	.0762
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953	.9752	.9618	.1264
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9981	.9880	.9805	.1958
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993	.9946	.9908	.2841
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9978	.9960	.3877
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9991	.9983	.5000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9994	.6123
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.7159
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.8042
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.8736
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9238
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9573
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9779
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9894
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9953
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9981
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9993
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
46	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
47	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
48	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
49	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 49

p = P(failure)

n = 50

c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
0	.3642	.0769	.0155	.0090	.0007	.0003	.0000	.0000	.0000	.0000
1	.7358	.2794	.0827	.0532	.0059	.0029	.0000	.0000	.0000	.0000
2	.9216	.5405	.2260	.1605	.0263	.0142	.0001	.0000	.0000	.0000
3	.9822	.7604	.4253	,3303	.0 774	.0460	.0005	.0001	.0000	.0000
4	.9968	.8964	.6290	.5277	.1721	.1121	.0021	.0003	.0002	.0000
5	.9995	.9622	.7419	.7072	.3091	.2194	.0070	.0013	.0007	.0000
6	● L)C)QV	.9882	.8981	.8404	.4709	.3613	.0194	.0042	.0025	.0000
7	1.0000	.9968	.9562	.9232	.6310	.5188	.0453	.0117	.0073	.0000
8	1.0000	.9992	.9833	.9672	.7665	.6681	.0916	.0278	.0183	.0000
9	1.0000	.9998	.9944	.9875	.8660	.7911	.1637	.0584	.0402	.0000
10	1.0000	1.0000	.9983	.9957	.9302	.8801	.2622	.1041	.0789	.0000
11	1.0000	1.0000	.9995	.9987	.9669	.9372	.38 1(!	.1836	.1390	.0000
12	1.0000	1.0000	.9999	.9996	.9857	.9699	.5110	.2816	.2229	.0002
13	1.0000	1.0000	1.0000	.9999	.9944	.9868	.6370	.3975	.3279	.0005
14	1.0000	1.0000	1.0000	1.0000	.9980	.9947	.7481	.5213	.4468	.0013
15	1.0000	l.norm	1.0000	1.0000	.9993	.9981	.8369	.6415	.5692	.0033
16	1.0000	1.0000	1.0000	1.0000	.9998	.9993	.9017	.7479	.6839	.0077
17	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9449	.8340	.7822	.0164
18	1.0000	1.0000	1.0000	l.norm	1.0000	.9999	.9713	.8978	.8594	.0325
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9861	.9413	.9152	.0595
20	l.norm	1.0000	1.0000	1.0000	1.0000	1.0000	.9937	.9686	.9522	.1013
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9974	.9843	.9749	.1611
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9990	.9927	.9877	.2399
23	1.0000	1.0000	1.0000	l.norm	1.0000	1.0000	.9996	.9969	.9944	.3359
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9987	.9976	.4439
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	noon	.9995	.9991	.5561
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	.9998	.9997	.6641
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	.9999	.9999	.7601
28	1.0000	1.0000	1.0000	.0000	l.norm	1.0000	.0000	1.0000	1.0000	.8389
29	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	.8987
30	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	.9405
31	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	.9675
3?	1.0000	1.0000	1.0000	.0000	l.norm	1.0000	.0000	1.0000	1.0000	.9836
33	].norm	1.0000	l.norm	.0000	1.0000	1.0000	.0000	1.0000	1.0000	.9923
34	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	.9967
35	1.0000	l.norm	l.norm	.0000	l.norm	1.0000	.0000	1.0000	1.0000	.9987
36	l.norm	1.0000	].norm	,0000	1.0000	1.0000	.0000	1.0000	1.0000	.9995
37	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.n00[1	1.0000	1.0000	.9998
38	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
39	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
40	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
41	l.norm	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
42	1.0000	l.norm	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
43	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
44	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
46	1.0000	1.0000	1.0000	1.0000	l.norm	1.0000	,(100(1	1.0000	1.0000	1.0000
47	1.0000	l.norm	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
48	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
49	1.0000	1.0000	l.norm	1.0000	1.0000	1.0000	.0000	1.0000	1.0000	1.0000
50	1.0000	1.0000	1.0000	1.0000	1.1)(10(1	1.0000	.0000	1.0000	1.0000	1.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.280	.300	.500
n = 73	0	.2288	.0236	.0023	.0010	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5697	.1145	.0167	.0084	.0003	.0001	.0000	.0000	.0000	.0000
	2	.8202	.2867	.0619	.0347	.0018	.0007	.0000	.0000	.0000	.0000
	3	.9412	.5011	.1548	.0963	.0074	.0031	.0000	.0000	.0000	.0000
	4	.9844	.6986	.2962	.2029	.0229	.0105	.0000	.0000	.0000	.0000
	5	.9965	.8421	.4659	.3483	.0566	.0286	.0001	.0000	.0000	.0000
	6	.9993	.9277	.6332	.5113	.1167	.0648	.0002	.0000	.0000	.0000
	7	.9999	.9708	.7724	.6657	.2072	.1259	.0008	.0001	.0000	.0000
	8	1.0000	.9895	.8722	.7916	.3247	.2149	.0024	.0002	.0001	.0000
	9	1.0000	.9966	.9349	.8815	.4583	.3204	.0061	.0007	.0003	.0000
	10	1.0000	.9990	.9698	.9385	.5930	.4565	.0141	.0019	.0010	.0000
n = 74	0	.2242	.0225	.0021	.0009	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5629	.1100	.0155	.0077	.0003	.0001	.0000	.0000	.0000	.0000
	2	.8152	.2781	.0582	.0323	.0016	.0006	.0000	.0000	.0000	.0000
	3	.9387	.4904	.1474	.0907	.0067	.0027	.0000	.0000	.0000	.0000
	4	.9835	.6888	.2849	.1933	.0208	.0094	.0000	.0000	.0000	.0000
	5	.9963	.8349	.4523	.3352	.0520	.0259	.0000	.0000	.0000	.0000
	6	.9993	.9234	.6198	.4967	.1085	.0594	.0002	.0000	.0000	.0000
	7	.9999	.9686	.7612	.6518	.1949	.1168	.0007	.0001	.0000	.0000
	8	1.0000	.9886	.8642	.7803	.3087	.2016	.0020	.0002	.0001	.0000
	9	1.0000	.9963	.9299	.8734	.4402	.3114	.0052	.0006	.0003	.0000
	10	1.0000	.9989	.9671	.9333	.5746	.4373	.0121	.0016	.0008	.0000
n = 75	0	.2198	.0213	.0019	.0008	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5561	.1056	.0145	.0071	.0002	.0001	.0000	.0000	.0000	.0000
	2	.8101	.2697	.0548	.0301	.0014	.0005	.0000	.0000	.0000	.0000
	3	.9363	.4798	.1402	.0855	.0060	.0024	.0000	.0000	.0000	.0000
	4	.9826	.6789	.2739	.1840	.0189	.0084	.0000	.0000	.0000	.0000
	5	.9960	.8276	.4390	.3224	.0478	.0234	.0000	.0000	.0000	.0000
	6	.9992	.9190	.6064	.4821	.1008	.0544	.0002	.0000	.0000	.0000
	7	.9999	.9664	.7499	.6378	.1831	.1062	.0005	.0000	.0000	.0000
	8	1.0000	.9876	.8560	.7687	.2933	.1889	.0016	.0001	.0001	.0000
	9	1.0000	.9959	.9247	.8650	.4223	.2949	.0044	.0005	.0002	.0000
	10	1.0000	.9988	.9641	.9279	.5564	.4184	.0103	.0013	.0006	.0000
n = 76	0	.2154	.0203	.0018	.0008	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5494	.1014	.0135	.0066	.0002	.0001	.0000	.0000	.0000	.0000
	2	.8051	.2615	.0516	.0281	.0013	.0004	.0000	.0000	.0000	.0000
	3	.9337	.4693	.1334	.0805	.0054	.0021	.0000	.0000	.0000	.0000
	4	.9817	.6689	.2632	.1752	.0171	.0075	.0000	.0000	.0000	.0000
	5	.9958	.8202	.4257	.3100	.0439	.0212	.0000	.0000	.0000	.0000
	6	.9992	.9144	.5930	.4678	.0936	.0497	.0001	.0000	.0000	.0000
	7	.9999	.9640	.7384	.6238	.1719	.1001	.0004	.0000	.0000	.0000
	8	1.0000	.9865	.8475	.7569	.2783	.1768	.0014	.0001	.0000	.0000
	9	1.0000	.9955	.9192	.8564	.4047	.2790	.0037	.0004	.0002	.0000
	10	1.0000	.9986	.9609	.9223	.5381	.3999	.0088	.0011	.0005	.0000

p = P(failure)

	c	.020	.050	.080	.090	.136	.150	.250	.280	.300	.500
n = 77	0	.2111	.0193	.0016	.0007	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5427	.0973	.0125	.0060	.0002	.0001	.0000	.0000	.0000	.0000
	2	.7999	.2535	.0485	.0261	.0011	.0004	.0000	.0000	.0000	.0000
	3	.9312	.4589	.1269	.0758	.0048	.0019	.0000	.0000	.0000	.0000
	4	.9807	.6589	.2528	.1667	.0155	.0067	.0000	.0000	.0000	.0000
	5	.9955	.8126	.4127	.2974	.0402	.0191	.0000	.0000	.0000	.0000
	6	.9991	.9047	.5796	.4536	.0868	.0454	.0001	.0000	.0000	.0000
	7	.9998	.9615	.7268	.6098	.1613	.0925	.0004	.0000	.0000	.0000
	8	1.0000	.9854	.8388	.7449	.2638	.1653	.0011	.0001	.0000	.0000
	9	1.0000	.9950	.9134	.8474	.3875	.2631	.0031	.0003	.0001	.0000
	10	1.0000	.9985	.9576	.9164	.5200	.1817	.0076	.0009	.0004	.0000
n = 78	0	.2068	.0183	.0015	.0006	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5361	.0934	.0117	.0056	.0001	.0000	.0000	.0000	.0000	.0000
	2	.7948	.2457	.0457	.0243	.0010	.0003	.0000	.0000	.0000	.0000
	3	.9285	.4486	.1206	.0713	.0043	.0016	.0000	.0000	.0000	.0000
	4	.9797	.6489	.2427	.1585	.0141	.0060	.0000	.0000	.0000	.0000
	5	.9952	.8049	.3999	.2860	.0369	.0173	.0000	.0000	.0000	.0000
	6	.9990	.9048	.5663	.4395	.0805	.0415	.0001	.0000	.0000	.0000
	7	.9998	.9589	.7150	.5957	.1512	.0855	.0003	.0000	.0000	.0000
	8	1.0000	.9842	.8298	.7328	.2499	.1544	.0009	.0001	.0000	.0000
	9	1.0000	.9945	.9075	.8382	.3707	.2489	.0026	.0002	.0001	.0000
	10	1.0000	.9983	.9541	.9101	.5020	.3640	.0064	.0007	.0003	.0000
n = 79	0	.2027	.0174	.0014	.0006	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5295	.0897	.0108	.0051	.0001	.0000	.0000	.0000	.0000	.0000
	2	.7896	.2380	.0429	.0226	.0009	.0003	.0000	.0000	.0000	.0000
	3	.9259	.4385	.1146	.0671	.0038	.0014	.0000	.0000	.0000	.0000
	4	.9787	.6389	.2330	.1506	.0127	.0053	.0000	.0000	.0000	.0000
	5	.9949	.7971	.3874	.2746	.0338	.0156	.0000	.0000	.0000	.0000
	6	.9990	.8999	.5530	.4257	.0746	.0378	.0001	.0000	.0000	.0000
	7	.9998	.9562	.7031	.5816	.1416	.0789	.0002	.0000	.0000	.0000
	8	1.0000	.9829	.8206	.7204	.2364	.1440	.0008	.0001	.0000	.0000
	9	1.0000	.9940	.9013	.8287	.3543	.2347	.0022	.0002	.0001	.0000
	10	1.0000	.9981	.9503	.9037	.4841	.3468	.0055	.0006	.0003	.0000
n = 80	0	.1986	.0165	.0013	.0005	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5230	.0861	.0101	.0047	.0001	.0000	.0000	.0000	.0000	.0000
	2	.7844	.2306	.0404	.0211	.0008	.0003	.0000	.0000	.0000	.0000
	3	.9231	.4284	.1089	.0631	.0034	.0013	.0000	.0000	.0000	.0000
	4	.9776	.6289	.2235	.1431	.0115	.0047	.0000	.0000	.0000	.0000
	5	.9946	.7892	.3750	.2634	.0309	.0140	.0000	.0000	.0000	.0000
	6	.9989	.8947	.5397	.4121	.0690	.0345	.0001	.0000	.0000	.0000
	7	.9998	.9534	.6911	.5676	.1324	.0727	.0002	.0000	.0000	.0000
	8	1.0000	.9816	.8112	.7079	.2235	.1342	.0006	.0000	.0000	.0000
	9	1.0000	.9935	.8948	.8190	.3383	.2211	.0018	.0002	.0001	.0000
	10	1.0000	.9979	.9464	.8969	.4665	.3300	.0047	.0005	.0002	.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.278	.300	.500
n = 81	0	.1947	.0157	.0012	.0005	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5165	.0826	.0094	.0043	.0001	[illegible]	.0000	.0000	.0000	.0000
	2	.7792	.2234	.0380	.0196	.0007	.0002	.0000	.0000	.0000	.0000
	3	.9204	.4186	.1034	.0593	.0031	[illegible]	.0000	.0000	.0000	.0000
	4	.9766	.6189	.2143	.1359	.0104	[illegible]	.0000	.0000	.0000	.0000
	5	.9942	.7812	.3629	.2526	.0283	[illegible]	.0000	.0000	.0000	.0000
	6	.9988	.8894	.5265	.3987	.0638	[illegible]	.0000	.0000	.0000	.0000
	7	.9998	.9505	.6790	.5536	.1238	[illegible]	.0002	.0000	.0000	.0000
	8	1.0000	.9802	.8016	.6953	.2112	.1250	.0005	.0000	.0000	.0000
	9	1.0000	.9929	.8881	.8090	.3227	[illegible]	[illegible]	.0001	.0000	.0000
	10	1.0000	.9977	.9423	.8894	.4490	[illegible]	[illegible]	.0004	.0002	.0000
n = 82	0	.1908	.0149	.0011	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	1	.5101	.0792	.0087	.0040	.0001	[illegible]	.0000	.0000	.0000	.0000
	2	.7730	.2164	.0357	.0182	.0006	[illegible]	.0000	.0000	.0000	.0000
	3	.9175	.4088	.0981	.0557	.0027	[illegible]	.0000	.0000	.0000	.0000
	4	.9754	.6088	.2055	.1290	.0094	[illegible]	.0000	.0000	.0000	.0000
	5	.9939	.7731	.3510	.2421	.0258	[illegible]	.0000	.0000	.0000	.0000
	6	.9987	.8840	.5134	.3856	.0590	[illegible]	.0000	.0000	.0000	.0000
	7	.9998	.9474	.6668	.5397	.1157	[illegible]	.0001	.0000	.0000	.0000
	8	1.0000	.9787	.7918	.6826	.1993	[illegible]	.0004	.0000	.0000	.0000
	9	1.0000	.9922	.8812	.7987	.3075	.1956	[illegible]	.0001	.0000	.0000
	10	1.0000	.9974	.9380	.8826	.4318	.2978	.0034	.0003	.0001	.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 106	0	.1175	.0044	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3716	.0286	.0015	.0005	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6430	.0957	.0076	.0030	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8366	.2181	.0260	.0115	.0002	.0000	.0000	.0000	.0000	.0000
	4	.9378	.3840	.0672	.0331	.0007	.0002	.0000	.0000	.0000	.0000
	5	.9799	.5622	.1403	.0768	.0026	.0008	.0000	.0000	.0000	.0000
	6	.9944	.7200	.2472	.1495	.0074	.0025	.0000	.0000	.0000	.0000
	7	.9986	.8386	.3801	.2522	.0183	.0068	.0000	.0000	.0000	.0000
	8	.9997	.9159	.5231	.3760	.0395	.0161	.0000	.0000	.0000	.0000
	9	.9999	.9602	.6585	.5133	.0758	.0341	.0000	.0000	.0000	.0000
	10	1.0000	.9828	.7727	.6432	.1312	.0649	.0000	.0000	.0000	.0000
n = 107	0	.1151	.0041	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3665	.0274	.0014	.0005	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6355	.0924	.0071	.0028	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8327	.2120	.0245	.0107	.0002	.0000	.0000	.0000	.0000	.0000
	4	.9358	.3757	.0639	.0312	.0007	.0002	.0000	.0000	.0000	.0000
	5	.9791	.5532	.1344	.0724	.0023	.0007	.0000	.0000	.0000	.0000
	6	.9941	.7121	.2387	.1430	.0067	.0022	.0000	.0000	.0000	.0000
	7	.9986	.8327	.3695	.2430	.0168	.0061	.0000	.0000	.0000	.0000
	8	.9997	.9120	.5117	.3666	.0366	.0147	.0000	.0000	.0000	.0000
	9	.9999	.9580	.6477	.5012	.0708	.0314	.0000	.0000	.0000	.0000
	10	1.0000	.9817	.7636	.6315	.1237	.0603	.0000	.0000	.0000	.0000
n = 108	0	.1128	.0039	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3615	.0263	.0013	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6330	.0891	.0066	.0026	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8288	.2060	.0231	.0100	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9337	.3676	.0607	.0294	.0006	.0002	.0000	.0000	.0000	.0000
	5	.9782	.5444	.1286	.0691	.0021	.0006	.0000	.0000	.0000	.0000
	6	.9938	.7041	.2303	.1367	.0061	.0020	.0000	.0000	.0000	.0000
	7	.9985	.8267	.3590	.2340	.0154	.0055	.0000	.0000	.0000	.0000
	8	.9997	.9081	.5003	.3555	.0339	.0134	.0000	.0000	.0000	.0000
	9	.9999	.9557	.6368	.4891	.0662	.0289	.0000	.0000	.0000	.0000
	10	1.0000	.9805	.7543	.6198	.1165	.0559	.0000	.0000	.0000	.0000
n = 109	0	.1106	.0037	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3565	.0251	.0012	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6276	.0860	.0062	.0024	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8249	.2002	.0218	.0093	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9316	.3595	.0577	.0276	.0005	.0001	.0000	.0000	.0000	.0000
	5	.9773	.5355	.1233	.0656	.0019	.0005	.0000	.0000	.0000	.0000
	6	.9935	.6961	.2222	.1306	.0056	.0018	.0000	.0000	.0000	.0000
	7	.9984	.8205	.3487	.2252	.0142	.0050	.0000	.0000	.0000	.0000
	8	.9996	.9040	.4890	.3446	.0314	.0122	.0000	.0000	.0000	.0000
	9	.9999	.9533	.6259	.4770	.0618	.0266	.0000	.0000	.0000	.0000
	10	1.0000	.9792	.7449	.6080	.1097	.0519	.0000	.0000	.0000	.0000

p = P(failure)

n	c	.020	.050	.080	.090	.136	.150	.250	.288	.300	.500
n = 110	0	.1086	.0035	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3516	.0241	.0011	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6222	.0829	.0058	.0022	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8210	.1945	.0206	.0087	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9295	.3515	.0549	.0260	.0005	.0001	.0000	.0000	.0000	.0000
	5	.9764	.5267	.1181	.0621	.0017	.0005	.0000	.0000	.0000	.0000
	6	.9932	.6881	.2143	.1247	.0051	.0016	.0000	.0000	.0000	.0000
	7	.9983	.8143	.3386	.2167	.0130	.0045	.0000	.0000	.0000	.0000
	8	.9996	.8998	.4778	.3338	.0290	.0112	.0000	.0000	.0000	.0000
	9	.9999	.9508	.6149	.4651	.0577	.0244	.0000	.0000	.0000	.0000
	10	1.0000	.9779	.7354	.5962	.1031	.0481	.0000	.0000	.0000	.0000
n = 111	0	.1062	.0034	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3468	.0230	.0010	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6168	.0800	.0054	.0020	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8170	.1889	.0194	.0081	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9273	.3437	.0521	.0244	.0004	.0001	.0000	.0000	.0000	.0000
	5	.9755	.5180	.1130	.0589	.0015	.0004	.0000	.0000	.0000	.0000
	6	.9929	.6800	.2066	.1191	.0046	.0014	.0000	.0000	.0000	.0000
	7	.9982	.8080	.3287	.2084	.0119	.0041	.0000	.0000	.0000	.0000
	8	.9996	.8955	.4666	.3233	.0269	.0102	.0000	.0000	.0000	.0000
	9	.9999	.9483	.6040	.4533	.0538	.0224	.0000	.0000	.0000	.0000
	10	1.0000	.9766	.7258	.5844	.0970	.0445	.0000	.0000	.0000	.0000
n = 112	0	.1041	.0032	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3419	.0221	.0009	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6114	.0771	.0051	.0019	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8130	.1835	.0183	.0076	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9251	.3359	.0495	.0230	.0004	.0001	.0000	.0000	.0000	.0000
	5	.9745	.5093	.1082	.0558	.0014	.0004	.0000	.0000	.0000	.0000
	6	.9925	.6719	.1991	.1137	.0042	.0013	.0000	.0000	.0000	.0000
	7	.9981	.8016	.3189	.2004	.0109	.0037	.0000	.0000	.0000	.0000
	8	.9996	.8912	.4556	.3130	.0248	.0092	.0000	.0000	.0000	.0000
	9	.9999	.9456	.5930	.4416	.0501	.0206	.0000	.0000	.0000	.0000
	10	1.0000	.9752	.7160	.5726	.0911	.0412	.0000	.0000	.0000	.0000
n = 113	0	.1020	.0030	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3372	.0211	.0009	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6060	.0744	.0047	.0017	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8089	.1781	.0172	.0071	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9220	.3283	.0470	.0216	.0003	.0001	.0000	.0000	.0000	.0000
	5	.9735	.5006	.1035	.0528	.0012	.0003	.0000	.0000	.0000	.0000
	6	.9922	.6638	.1918	.1085	.0038	.0011	.0000	.0000	.0000	.0000
	7	.9980	.7951	.3093	.1926	.0100	.0033	.0000	.0000	.0000	.0000
	8	.9995	.8867	.4447	.3028	.0229	.0084	.0000	.0000	.0000	.0000
	9	.9999	.9429	.5820	.4300	.0467	.0189	.0000	.0000	.0000	.0000
	10	1.0000	.9737	.7062	.5608	.0855	.0381	.0000	.0000	.0000	.0000

p = P(failure)

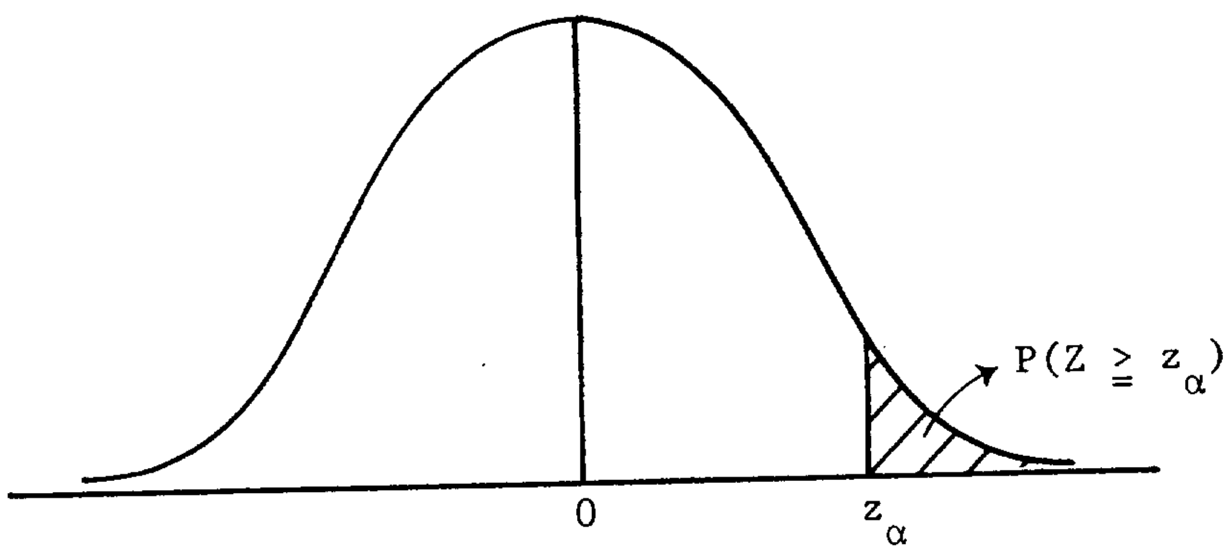
n	c	.020	.050	.080	.090	.136	.150	.250	.280	.300	.500
n = 114	0	.0999	.0029	.0001	.0000	.0000	.0000	.0000	.0000	.0000	noon
	1	.3325	.0202	.0008	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	2	.6006	.0717	.0044	.0016	.0000	.0000	.0000	.0000	.0000	.0000
	3	.8049	.1729	.0162	.0066	.0001	.0000	● oon	.0000	.0000	.0000
	4	.9206	.3208	.0446	.0203	.0003	.0001	.0000	.()(1 (10	.0000	.0000
	5	.9725	.4920	.0989	.0500	.0011	.0003	.0000	.0000	.0000	.0000
	6	.9918	.6556	.1848	.1035	.0035	.0010	.0000	.0000	.0000	.0000
	7	.9978	.7886	.2999	.1850	.0092	.0030	.0000	.0000	.0000	.0000
	8	.9995	.8821	.4338	.2929	.0212	.0076	.0000	.0000	.0000	.0000
	9	.9999	.9401	.5710	.4186	.0434	.0173	.0000	.0000	.0000	.0000
	10	1.0000	.9722	.6962	.5491	.0802	.0352	.0000	.0000	.0000	.0000
n = 115	0	.0979	.0027	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.3270	.0193	.0008	.0002	.0000	.0000	.0000	.0000	.0000	.0000
	2	.5952	.0692	.0041	.0015	.0000	.0000	.0000	.0000	noon	.0000
	3	.8008	.1679	.0153	.0061	.0001	.0000	.0000	.0000	.0000	.0000
	4	.9183	.3134	.0423	.0190	.0003	.0001	.0000	.0000	.0000	.0000
	5	.9715	.4834	.0946	.0473	.0010	.0003	.0000	.0000	.0000	.0000
	6	.9914	.6475	.1779	.0987	.0032	.0009	.0000	.0000	.0000	.0000
	7	.9977	.7819	.2907	.1777	.0084	.0027	.0000	.0000	.0000	.0000
	8	.9995	● p774	.4231	.2832	.0195	.0069	.0000	.0000	.0000	.0000
	9	.9999	.9372	.5600	.4073	.0404	.0159	.0000	.0000	.0000	.0000
	10	1.0000	.9706	.6862	.5373	.0752	.0326	. norm	● oon	noon	.0000

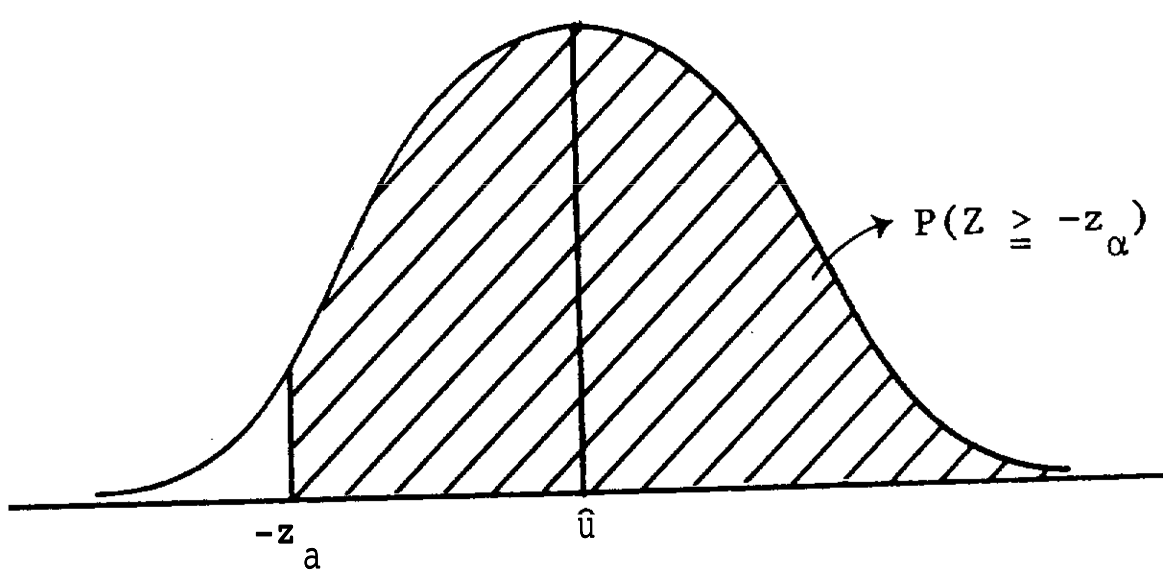
TABLE 2

NORMAL TABLES

The letter Z represents the standard normal variable. The notation z_α is defined by the relationship

$$P(Z \geqq z_\alpha) = \alpha.$$





Note that

$$P(Z \leqq z_\alpha) = P(Z \geqq -z_\alpha)$$

TABLE 2. NORMAL TABLES

z_α	$P(Z\geq -z_\alpha)$	$P(Z\geq z_\alpha)=\alpha$
.00	.5000	.5000
.01	.5040	.4960
.02	.5080	.4920
.03	.5120	.4880
.04	.5160	.4840
.05	.5199	.4801
.06	.6239	.4761
-07	.5279	.4721
.08	.5319	.4681
.09	.5389	.4641
.10	.5398	.4602
.11	.5438	.4562
-12	.5478	.4522
.13	.5517	.4483
.14	.5557	.4443
.15	.5696	.4-404
.16	.5636	.4364
.17"	.5675	.4325
.18	.5714	.4286
.19	. 57s3	.4247
-20	.5793	.4207
.21	.5832	.4168
.22	.5871	.4129
.23	.5910	.4090
.24	.6948	.4052
.25	.6987	.4013
.26	.6026	.3974
.27	.6064	.3936
.28	.6103	.3897
.29	.6141	.3859
.30	.6179	.3821
.31	.6217	.3783
.32	.6256	.3745
.33	.6293	.3707
.34	.6331	.3660
.35	.6368	.3632
.36	.6406	.3594
.37	.6443	.3557
.38	.6480	.3520
.39	.6517	.3483
.40	.6554	.3446
.41	.6591	.3409
.42	.6628	.3372
.43	.6864	.3336
.44	.6700	.3300
.45	.6736	.3264
.46	.6772	.3228
.47	.6808	.3192
.48	.6844	.3156
.49	.6879	.3121
.60	.6915	.3085

z_α	$P(Z\geq -z_\alpha)$	$P(Z\geq z_\alpha)=\alpha$
.50	.6915	.3085
.51	.6950	.3050
.52	.6985	. 301s
.53	.7019	.2981
.54	.7054	.2946
.56	.7088	.2912
.56	.7123	.2877
.67	.7157	.2843
.58	.7190	.2810
.59	.7224	. 2776
.60	.7257	.2743
.61	.7281	.2709
.62	.7324	.2676
.63	.7357	.2643
.64	.7389	.2611
.65	.7422	.2578
.66	.7454	.2546
.67	.7486	.2514
.68	.7517	.2483
.69	.7549	.2451
.70	.7580	.2420
.71	.7611	.2389
.72	.7642	.2358
73	.7673	.2327
:74	.7704	.2296
.75	.7734	.2266
.76	.7764	.2236
.77	.7794	.2206
.78	.7823	.2177
.79	.7852	.2148
.80	.7881	.2119
.81	.7910	.2090
.82	.7939	.2061
.83	.7967	.2033
.84	.7995	.2005
.85	.8023	.1977
.88	.8051	.1949
.87	.8079	.1921
.88	.8106	.1894
.89	.8133	.1867
.90	.8159	.1841
.91	.8186	.1814
.92	.8212	.1788
.93	.8238	.1762
.94	-8264	.1736
.95	. 8289	.1711
.96	.8315	.1685
.97	.8340	.1660
,98	-8365	.1635
.99	.8389	.1611
1.00	.8413	.1587

z_α	$P(Z\geq -z_\alpha)$	$P(Z\geq z_\alpha)=\alpha$
1.00	.8413	.1587
1.01	.8438	.1562
1.02	.8461	.1539
1.03	.8485	.1515
1.04	.8508	.1402
1.05	.8531	.1469
1.06	.8554	.1448
1.07	.8577	.1423
1.08	.8599	.1401
1.09	.8621	.1379
1.10	.6643	.1357
1.11	.8665	.1335
1.12	.8686	.1314
1.13	.8708	.1292
1.14	.8729	.1271
1.15	.8749	.1251
1.16	.8770	.1230
1.17	.8790	.1210
1.18	.8810	.1190
1-.19	.8830	.1170
1.20	.8849	.1151
1.21	.8869	.1131
1.22	.8888	.1112
1.23	.8907	.1093
1.24	.8925	.1075
1.25	.8944	1056
1.26	.8962	:1038
1.27	.8980	.1020
1.28	.8997	.1003
1.29	.9015	.0085
1.30	.9032	.0968
1.31	.9049	.09s1
1.32	.9066	.0934
x.33	.9082	.0918
1.34	.9099	.0901
1.35	.9115	.0885
1.36	.9131	.0869
1.37	.9147	.0853
1.38	.9162	.0838
1.39	.9177	.0823
1.40	.9192	.0808
1.41	.9207	.0793
1.42	.9222	.0778
1.43	.9236	.0764
1.44	.9251	.0749
1.45	.9265	.0735
1.46	.9279	.0721
1.47	.9292	.0708
1.48	.9306	.0694
1.49	.9319	.0681
1.50	.9332	.0668

TABLE 2. NORMAL TABLES (CONT.)

z_α	$P(Z \geq -z_\alpha)$	$P(Z \geq z_\alpha)=\alpha$
1.50	.9332	.0668
1.s1	.9345	.0655
1.52	.0357	.0643
1.53	.9370	.0630
1.64	.9382	.0618
1.s5	.9394	-0606
1.56	.9406	.0594
1.57	.9418	.0582
1.58	.9429	.0571
1.69	.9441	.0559
1.60	.9452	.0548
1.61	.9483	.0537
1.62	.9474	.0526
1.63	.9484	.0516
1.64	.9495	.0505
1.6S	.9505	.0495
1.66	.9515	.0485
1.67	. 9S25	.0475
1.68	.9535	.0465
1.69	.9545	.0455
1.70	.9554	.0446
1.71	.95$4	.0436
1.72	.9573	.0427
1.73	.9582	.0418
1.74	.9591	.0409
1.75	.9599	.0401
1.76	. 9608	.0392
1.77	.9616	.0384
1.7s	.9625	.0375
1.79	.9633	.0367
1.80	.9641	.0359
1.81	. 9649	.0351
1.82	.96.56	.0344
1.83	.9664	.0336
1.84	.9671	.0329
1.85	.9678	. 0322
1.86	.9686	.0314
1.87	.9693	.0307
1.86	.9699	.0301
1.89	.9706	.0294
1.90	.9713	.0287
1.91	.9719	.0281
1.92	.9726	.0274
1.93	.9732	.0268
1.94	.9738	.0262
1.95	.974	.0256
1.96	.9750	.0250
1.97	.9756	.0244
1.98	.9761	.0230
1.99	.9767	.0233
2.00	.9773	.0227

z_α	$P(Z \geq -z_\alpha)$	$P(Z \geq z_\alpha)=\alpha$
2.00	.9773	.0227
2.01	.9778	.0222
2.02	.9783	.0217
2.03	.9786	.0212
2.04	.9793	.0207
2.06	.9708	.0202
2.06	.9803	.0197
2.07	.9808	.0192
2.08	.9812	.0188
2.09	.9817	.0183
2.10	.9821	.0179
2.11	. 9826	.0174
2.12	.9830	.0170
2.13	.9834	.0166
2.14	.9838	.0162
2.1s	.9842	.0158
2.16	.9846	.0154
2.17	.9850	.0150
2.18	.9854	.0146
2.19	.9857	.0143
2.20	.9861	.0139
2.21	.9864	.0136
2.22	.9868	.0132
2.23	.9871	.0129
2.24	.9875	.0125
2.25	.9878	.0122
2.26	.9881	.0119
2.27	.9884	.0116
2.28	.9687	.0113
2.29	.9890	.0110
2.30	.9893	.0107
2.31	.9806	.0104
2.32	. 9898	.0102
2.33	.9901	.0099
2.34	.9 *	.0096
2.35	.9906	.0094
2.36	.9909	.0091
2.37	.9911	.0089
2.38	.9913	.0087
2.39	.9916	.0064
2.40	.9918	.0082
2.41	.9920	.0080
2.42	.9922	.0078
2.43	.9925	.0075
2.44	.9927	.0073
2.45	.9929	.0071
2.46	.9931	.0069
2.47	.9932	.0068
2.48	.9934	.0066
2.49	.9936	.0064
2.50	.9038	.0062

z_α	$P(Z \geq -z_\alpha)$	$P(Z \geq z_\alpha)=\alpha$
2.50	.9938	.0062
2.61	.9940	.0060
2.62	.9941	.0059
2.53	.9943	.0057
2.64	.9945	.0055
2.65	.9946	.0054
2.56	.9948	.0052
2.57	. 9949	.0051
2.58	.9951	.0049
2.59	.9952	.0048
2.60	.9953	. 0047
2.61	.9955	.0045
2.62	.9956	.0044
2.63	.9957	.0043
2.64	.9959	.0041
2.65	.9960	.0040
2.66	.9961	.0039
2.67	.9962	.0038
2.68	.9963	.0037
2.69	.9964	.0036
2.70	.9965	.0035
2.71	.9966	.0034
2.72	.9967	.0033
2.73	.9968	.0032
2.74	.9969	.0031
2.75	.9970	.0030
2.76	.9971	.0029
2.77	.0972	. 0028
2.78	.9973	.0027
2.79	.9974	.0026
2.80	.9974	.0026
2.81	.9975	.0025
2.82	.9976	. 0024
2.83	.9977	.0023
2.84	.9977	.0023
2.85	.9978	. 0022
2.86	.9979	.0021
2.87	.9$79	.(X)21
2.88	.9980	.0020
2.89	.9981	.0019
2.90	.9981	.0019
2.91	.9982	.0018
2.92	.9983	.0017
2.03	.9983	.0017
2.04	.9984	.0016
2.9s	.9984	.0016
2.96	. 9985	.(X)15
2.97	.9985	.0015
2.98	.9986	.0014
2.99	.9986	.0014
3.00	.9987	.0013

TABLE 2. Now TABLES (CONT.)

a	?(Z≥-z_α)	P(Z≥z_α)=α
3.00	.9987	.0013
3.01	.0087	. 0013
3.02	. 9987	. 0013
3.03	.9988	.0012
3.04	.9988	. 0012
3.03	.9989	.0011
3.06	.9989	.0011
3.07	. 9989	.0011
3.08	. 9990	.0010
3.09	.9990	.0010
3.10	.9990	.0010
3.11	.9891	.0009
3.12	.9991	.0009
3.13	.9991	.0009
3.14	.9992	.0008
3.16	.9992	.0006
3.16	.9992	.0008
3.17	.9992	.0008
3.18	. 9993	.0007
3.19	.0993	.0007
3.20	.9993	.0007
3.21	.999a	.0007
3.22	.9994	.0006
3.23	.9994	.0006
3.24	.9994	.0006
3.25	.9994	.0006
3.26	.9994	.0006
3.27	.9995	. 0005
3.28	.0996	. 0005
3.29	.9996	.0005
3.30	.9996	.0005
3.31	.9995	.0005
3.32	.9996	.0004
3.23	.9996	.0004
3.34	.9996	.0004
3.35	.9996	.0004
3.36	. 9996	.0004
3.37	.9996	.0004
3.33	.9996	.0004
3.30	.9997	.0003
3.40	.9997	.0003
3.41	.9997	.0003
3.42	.9997	.0003
3.43	.9997	.0003
3.44	.9997	.0003
3.45	.9997	.0003
3.46	.9997	.0003
a.47	.9997	.0003
3.48	.9997	.0003
3.49	.9998	.0002
3.50	. 9998	.0002

TABLE 3

CUMULATIVE POISSON PROBABILITIES

Description and Instructions

This table lists values of

$$G_{\lambda,T}(c) = \sum_{i=0}^{c} \frac{m^i e^{-m}}{i!}, \quad m = AT.$$

The table is indexed horizontally by c and vertically by m, and the tabled probabilities are rounded to the nearest 0.001. See Chapter 5, section ● ntitled "Poisson Model" for a discussion of the Poisson distribution function, G*, T(C).

Example 1. Assume a Poisson model where the failure rate (λ) is 0.016 and the time (T) is 200 hours. Thus, m = AT = (0.016)(200) = 3.2. To find the probability of 5 or fewer failures occurring, we let m = 3.2, c = 5 and read the value directly from the table as 0.895.

Example 2. The probability of ● xactly c failures occurring is

P(c or fewer failures)

- P(c-1 or fewer failures).

For m = 3.2 and c = 5, this probability is

0.895 - 0.781 = 0.114.

Note that this example shows how to obtain values of the function $g_{\lambda,T}(c)$ as defined in Chapter 5.

Example 3. The probability of mnre than c failures occurring is

1 - P(c or fewer failures).

For m = 3.2 and c = 5, this probability is

1 - 0.895 = 0.105.

m \ c	0	1	2	3	4	5
0.01	.990	L. 000				
0.02	.980	L. 000				
0.03	.970	l.000				
0.04	.961	.999	..000			
0.05	.951	.999	l. 000			
0.06	.942	.998	L 000			
0.0'7	.932	.998	l. 000			
0.08	.923	.997	l.000			
0.09	.914	.996	l. 000			
0.10	.905	.995	l. 000			
0.12	.887	.993	l. 000			
0.14	.869	.991	L 000			
0.16	.852	.988	.999	l. 000		
0.18	.635	.986	.999	L 00C		
0.20	.819	.982	.999	L 000		
0.22	.803	.979	.998	l. 00C		
0.24	.787	.975	.998	l. 00C		
0.26	.771	.972	.998	l. 000		
0.28	.756	.967	.997	l. 000		
0.30	.741	.963	.996	l. 00C		
0.32	.726	.959	.996	l.000		
0.34	. ?12	.954	.995	l. 000		
0.36	.698	.949	.994	.999	L 00C	
0.38	.664	.944	.993	.999	l. 00C	
0.40	.670	.938	.992	.999	L 00C	
0.42	.657	.933	.991	.99:	L 00(	
0.44	.644	.927	.990	.99$	L 00(	
0.46	.631	.922	.988	.99$	1. 00(	
0.48	.619	.916	.987	. 99ε	L. 00(	
0.50	.607	.910	.986	. 99ε	L. 00(	
0.52	.595	.904	.984	. 99ε	L. 00(	
0.54	.583	.897	.982	.99ε	l.00(	
0.56	.571	.891	.981	. 997	L. 00(	
0.58	.560	.885	.979	.997	l. 00(	
0.60	.549	.878	.97'7	.997	L. 00(	
0.62	.538	.871	.975	. 996	L. 00(	
0. 64	.527	.865	.973	. 996	· 99:	
0.66	.517	.858	.971	.99:	.999	:. 000
0.68	.507	.851	.968	.99:	.999	..000
0.70	.497	.844	.966	.994	.999	..000
0.72	.487	.627	.963	.99	.999	l. 00C
0.74	.477	.830	.961	.99:	.999	1.000
0.76	.468	.823	.958	.99:	.999	l. 000
0.'78	.458	.816	.955	.99:	.899	L 00C
0.60	.449	.809	.953	.99	.999	L 00C

TABLE 3. CUMULATIVE POISSON PROBABILITIES (CONT.)

m \ c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0.82	.440	.802	.950	.990	.998	1.000									
0.84	.432	.794	.947	.989	.998	1.000									
0.86	.423	.787	.844	.988	.998	1.000									
0.88	.415	.780	.940	.988	.998	1.000									
0.90	.407	.772	.937	.987	.998	1.000									
0.92	.399	.765	.934	.986	.997	1.000									
0.94	.391	.758	.930	.984	.997	1.000									
0.96	.383	.750	.927	.983	.997	1.000									
0.98	.375	.743	.923	.982	.997	.999	1.000								
1.00	.368	.736	.920	.981	.996	.999	1.000								
1.10	.333	.699	.900	.974	.995	.999	1.000								
1.20	.301	.663	.879	.966	.992	.998	1.000								
1.30	.273	.627	.857	.957	.989	.998	1.000								
1.40	.247	.592	.833	.846	.986	.997	.999	1.000							
1.50	.223	.558	.809	.934	.981	.996	.999	1.000							
1.60	.202	.525	.783	.921	.976	.984	.999	1.000							
1.70	.183	.493	.757	.907	.970	.992	.998	1.000							
1.80	.165	.463	.731	.891	.964	.990	.997	.999	1.000						
1.90	.150	.434	.704	.875	.956	.987	.997	.999	1.000						
2.00	.135	.406	.677	.857	.947	.983	.995	.999	1.000						
2.20	.111	.355	.623	.819	.928	.975	.993	.998	1.000						
2.40	.091	.308	.570	.779	.904	.964	.988	.997	.999	1.000					
2.60	.074	.267	.518	.736	.877	.951	.983	.995	.999	1.000					
2.80	.061	.231	.469	.692	.848	.935	.976	.992	.998	.999	1.000				
3.00	.050	.199	.423	.647	.815	.916	.966	.988	.996	.999	1.000				
3.20	.041	.171	.380	.603	.781	.895	.955	.983	.984	.998	1.000				
3.40	.033	.147	.340	.558	.744	.871	.942	.977	.992	.997	.999	1.000			
3.60	.027	.126	.303	.515	.706	.844	.927	.969	.988	.996	.999	1.000			
3.80	.022	.107	.269	.473	.668	.816	.909	.960	.984	.991	.998	.999	1.000		
4.00	.018	.092	.238	.433	.629	.785	.889	.949	.979	.992	.997	.999	1.000		
4.20	.015	.078	.210	.395	.590	.753	.867	.936	.972	.989	.996	.999	1.000		
4.40	.012	.066	.185	.359	.551	.720	.844	.921	.964	.985	.994	.998	.999	1.000	
4.60	.010	.056	.163	.326	.513	.686	.818	.905	.955	.980	.992	.397	.999	1.000	
4.80	.006	.048	.143	.284	.476	.651	.791	.887	.944	.975	.990	.996	.999	1.000	
5.00	.007	.040	.125	.265	.440	.616	.762	.867	.932	.968	.986	.995	.398	.999	1.000
5.20	.006	.034	.109	.238	.406	.581	.732	.845	.918	.960	.982	.993	.997	.999	1.000
5.40	.005	.029	.095	.213	.373	.546	.702	.822	.903	.951	.977	.990	.996	.999	1.000
5.60	.004	.024	.082	.191	.342	.512	.670	.797	.886	.941	.972	.988	.995	.998	.999
5.80	.003	.021	.072	.170	.313	.478	.638	.771	.867	.929	.965	.984	.993	.997	.999
6.00	.002	.017	.062	.151	.285	.446	.606	.744	.847	.916	.957	.980	.391	.996	.999
6.20	.002	.015	.054	.134	.259	.414	.574	.716	.826	.902	.949	.975	.989	.395	.998
6.40	.002	.012	.046	.119	.235	.384	.542	.687	.803	.886	.939	.969	.986	.994	.997
6.60	.001	.010	.040	.105	.213	.355	.511	.658	.780	.869	.927	.963	.982	.992	.997
6.80	.001	.009	.034	.093	.192	.327	.480	.628	.755	.850	.915	.255	.973	.990	.996
7.00	.001	.007	.030	.082	.173	.301	.450	.599	.729	.830	.901	.947	.973	.987	.994
7.20	.001	.006	.025	.072	.156	.276	.420	.569	.703	.810	.887	.937	.967	.984	.993
7.40	.001	.005	.022	.063	.140	.253	.392	.539	.676	.788	.871	.926	.961	.980	.991
7.60	.001	.004	.019	.055	.125	.231	.365	.510	.648	.765	.854	.915	.954	.975	.989
7.80	.000	.004	.016	.048	.112	.210	.338	.481	.620	.741	.335	.902	.945	.971	.986
8.00	.000	.003	.014	.042	.100	.191	.313	.453	.593	.717	.816	.888	.936	.966	.983

m \ c	15	16	17	18	19
5.60	1.000				
5.80	1.000				
6.00	.999	1.000			
6.20	.999	1.000			
6.40	.999	1.000			
6.60	.999	.999	1.000		
6.80	.998	.999	1.000		
7.00	.998	.999	1.000		
7.20	.997	.999	1.000		
7.40	.996	.998	.599	1.000	
7.60	.995	.998	.999	1.000	
7.30	.993	.997	.999	1.000	
8.00	.992	.996	.998	.999	1.000

TABLE 3. CUMULATIVE POISSON PROBABILITIES (CONT.)

m \ c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
8.2	.000	.003	.012	.037	.089	.174	.290	.425	.565	.692	.796	.873	.926	.960	.979
8.4	.000	.002	.010	.032	.079	.157	.267	.399	.537	.666	,774	.857	.915	.952	.975
8.6	.000	.002	.009	.028	.070	.142	.246	.373	.509	.640	.752	.840	.903	.945	.970
8.8	.000	.001	.007	.024	.062	.128	.226	.348	.482	.614	.729	.822	.890	.936	.965
9.0	.000	.001	.006	.021	.055	.116	.207	.324	.456	.587	.706	.803	.876	.926	.959
9.2	.000	.001	.005	.018	.049	.104	.189	.301	.430	.561	.682	.783	.861	.916	.952
9.4	.000	.001	.005	.016	.043	.093	.173	.279	.404	.535	.658	.763	.845	.904	.944
9.6	.000	.001	.004	.014	.038	.084	.157	.258	.380	.509	.633	.741	.828	.892	.936
9.8	.000	.001	.003	.012	.033	.075	.143	.239	.356	.483	.608	.719	.810	.879	.927
0.0		.000	.003	.010	.029	.067	.130	.220	.333	.458	.583	.697	.792	.864	.917
0.5		.000	.002	.007	.021	.050	.102	.179	.279	.397	.521	.639	.742	.825	.888
1.0		.000	.001	.005	.015	.038	.079	.143	.232	.341	.460	.579	.689	.781	.854
1.5		.000	.001	.003	.011	.028	.060	.114	.191	.289	.402	.520	.633	.733	.815
2.0		.000	.001	.002	.008	.020	.046	.090	.155	.242	.347	.462	.576	.682	.772
2.5			.000	.002	.005	.015	.035	.070	.125	.201	.297	.406	.519	.628	.725
3.0			.000	.001	.004	.011	.026	.054	.100	.166	.252	.353	.463	.573	.675
3.5			.000	.001	.003	.008	.019	.041	.079	.135	.211	.304	.409	.518	.623
4.0				.000	.002	.006	.014	.032	.062	.109	.176	.260	.358	.464	.570
4.5				.000	.001	.004	.010	.024	.048	.088	.145	.220	.311	.413	.518
5.0				.000	.001	.003	.008	.018	.037	.070	.118	.185	.268	.363	.466
6.0					.000	.001	.004	.010	.022	.043	.077	.127	.193	.275	.368
7.0					.000	.001	.002	.005	.013	.026	.049	.085	.135	.201	.281
8.0						.000	.001	.003	.007	.015	.030	.055	.092	.143	.208
9.0						.000	.001	.002	.004	.009	.018	.035	.061	.098	.150

m \ c	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
8.2	.990	.995	.998	.999	.000										
8.4	,987	.994	.997	.999	.000										
8.6	,985	.993	.997	.999	.999	.000									
8.8	.982	.991	.996	.998	.999	.000									
9.0	,978	.989	.995	.998	.999	.000									
9.2	,974	.987	.993	.997	.999	.999	.000								
9.4	,969	.984	.992	.996	.998	.999	.000								
9.6	,864	.981	.990	.995	.998	.999	.000								
9.8	,958	.977	.980	.994	.997	.999	.999	1.000							
10.0	.951	.973	.986	.993	.997	.998	.999	1.000							
10.5	.932	.960	.978	.988	.994	.997	.999	.999	.000						
11.0	.907	.944	.968	.982	.991	.995	.998	.999	.000						
11.5	.878	.924	.954	.974	.986	.992	.996	.998	.999	1.000					
12.0	.844	.899	.937	.963	.979	.988	.994	.997	.999	.999	1.000				
12.5	.806	.869	.916	.948	.969	.983	.991	.995	.998	.999	.999	1.000			
13.0	.764	.835	.890	.930	.957	.975	.986	.992	.996	.998	.999	1.000			
13.5	.718	.798	.861	.908	.942	.965	.980	.989	.994	.997	.998	.999	1.000		
14.0	.669	.756	.827	.883	.923	.952	.971	.983	.991	.995	.997	.999	.999	1.000	
14.5	.619	.711	.790	.853	.901	.936	.960	.976	.986	.992	.996	.998	.999	.999	1.000
15.0	.568	.664	.749	.819	.875	.917	.947	.967	.981	.989	.994	.997	.998	.999	1.000
16.0	.467	.566	.659	.742	.812	.868	.911	.942	.963	.978	.987	.993	.996	.998	.999
17.0	.371	.468	.564	.655	.736	.805	.861	.905	.937	.959	.975	.985	.991	.995	.997
18.0	.287	.375	.469	.562	.651	.731	.799	.855	.899	.932	.955	.972	.983	.990	.994
19.0	.215	.292	.378	.469	.561	.647	.725	.793	.849	.893	.927	.951	.989	.980	[illegible]

m \ c	30	31	32	33	34	35
16.0	.999	1.000				
17.0	.999	.999	1.000			
18.0	.997	.998	.999	1.000		
19.0	.993	.996	.998	.999	.999	1.000

TABLE 3. CUMULATIVE POISSON PROBABILITIES (CONT.)

m \ c	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20.0	.000	.001	.002	.005	.011	.021	.039	.066	.105	.157	.221	.297	.381	.470	.559
21.0		.000	.001	.003	.006	.013	.025	.043	.072	.111	.163	.227	.302	.364	.471
22.0		.000	.001	.002	.004	.008	.015	.028	.048	.077	.117	.169	.232	.306	.387
23.0			.000	.001	.002	.004	.009	.017	.031	.052	.082	.123	.175	.238	.310
24.0				.000	.001	.003	.005	.011	.020	.034	.056	.087	.128	.180	.243
25.0				.000	.001	.001	.003	.006	.012	.022	.038	.060	.092	.134	.185
26.0					.000	.001	.002	.004	.008	.014	.025	.041	.065	.097	.139
27.0						.000	.001	.002	.005	.009	.016	.027	.044	.069	.101
28.0						.000	.001	.001	.003	.005	.010	.018	.030	.048	.073
29.0							.000	.001	.002	.003	.006	.012	.020	.033	.051
30.0								.000	.001	.002	.004	.007	.013	.022	.035
31.0								.000	.001	.001	.002	.005	.008	.014	.024
32.0									.000	.001	.001	.003	.005	.009	.016
33.0										.000	.001	.002	.003	.006	.010
34.0											.000	.001	.002	.004	.007

m \ c	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
20.0	.644	.721	.787	.843	.888	.922	.948	.966	.978	.987	.992	.995	.997	.999	.999
21.0	.558	.640	.716	.782	.838	.883	.917	.944	.963	.976	.985	.991	.994	.997	.998
22.0	.472	.556	.637	.712	.777	.832	.877	.913	.940	.959	.973	.983	.989	.994	.996
23.0	.389	.472	.555	.635	.708	.772	.827	.873	.908	.936	.956	.971	.981	.988	.993
24.0	.314	.392	.473	.554	.632	.704	.768	.823	.868	.904	.932	.953	.969	.979	.987
25.0	.247	.313	.394	.473	.553	.629	.700	.763	.818	.863	.900	.929	.950	.966	.978
26.0	.190	.252	.321	.396	.474	.552	.627	.697	.759	.813	.859	.896	.925	.947	.964
27.0	.144	.195	.256	.324	.398	.474	.551	.625	.693	.755	.809	.855	.992	.921	.944
28.0	.106	.148	.200	.260	.327	.400	.475	.550	.623	.690	.752	.805	.850	.888	.918
29.0	.077	.110	.153	.204	.264	.330	.401	.475	.549	.621	.687	.748	.801	.846	.884
30.0	.054	.081	.115	.157	.208	.267	.333	.403	.476	.548	.619	.685	.744	.797	.843
31.0	.038	.058	.084	.119	.161	.212	.271	.335	.405	.476	.548	.617	.682	.741	.794
32.0	.026	.041	.061	.088	.123	.166	.216	.274	.338	.406	.476	.547	.615	.679	.738
33.0	.018	.028	.043	.064	.092	.127	.170	.220	.277	.340	.408	.477	.546	.613	.677
34.0	.012	.019	.030	.046	.067	.095	.131	.173	.224	.280	.343	.409	.477	.545	.612

m \ c	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
20.0	.000														
21.0	.999	.999	1.000												
22.0	.998	.999	.999	1.000											
23.0	.996	.997	.999	.999	1.000										
24.0	.992	.995	.997	.998	.999	.999	1.000								
25.0	.985	.991	.994	.997	.998	.999	.999	1.000							
26.0	.976	.984	.990	.994	.996	.998	.999	.999	1.000						
27.0	.961	.974	.983	.989	.993	.996	.997	.998	.999	.999	1.000				
28.0	.841	.959	.972	.981	.988	.992	.995	.997	.998	.999	.999	1.000			
29.0	.914	.938	.956	.970	.979	.986	.991	.994	.996	.998	.999	.999	1.000		
30.0	.880	.911	.935	.954	.968	.978	.965	.990	.994	.996	.998	.999	.999	.999	1.000
31.0	.839	.877	.908	.932	.951	.966	.976	.984	.989	.993	.996	.997	.998	.999	.999
32.0	.790	.835	.873	.904	.929	.949	.964	.975	.983	.988	.992	.995	.997	.998	.999
33.0	.735	.787	.832	.870	.901	.926	.946	.962	.973	.981	.987	.992	.995	.997	.998
34.0	.674	.732	.783	.828	.866	.898	.924	.944	.960	.971	.980	.966	.991	.994	.996

m \ c	51	52	53	54	55
31.0	1.000				
32.0	.999	1.000			
33.0	.999	.999	1.000		
34.0	.998	.998	.999	.999	1.000

TABLE 3. CUMULATIVE POISSON PROBABILITIES (CONT.)

m \ c	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
35.0	.000	.001	.001	.002	.004	.008	.013	.021	.032	.049	.070	.099	.134	.177	.227
36.0		.000	.001	.001	.003	.005	.008	.014	.022	.035	.051	.074	.102	.138	.181
37.0			.000	.001	.002	.003	.006	.009	.015	.024	.037	.054	.077	.106	.141
38.0			.000	.001	.001	.002	:004	.006	.010	.017	.026	.039	.057	.080	.109
39.0				.000	.001	.001	.002	.004	.007	.011	.018	.028	.041	.059	.083
40.0					. 0OC	.001	.001	.003	.004	.008	.012	.019	.029	.043	.062
41.0						.000	. 00?.	.002	.003	.005	.008	.013	.021	.031	.045
42.0						.000	.001	.001	.002	.003	.006	.009	.014	.022	.033
43.0							.000	.001	.001	.002	.004	.006	.010	.016	.024
44.0								.000	.001	.001	.002	.004	.007	.011	.017
45.0									.000	.001	.002	.003	.004	.007	. 012
46.0									.000	.001	.001	.002	.003	.005	.008
47.0										.000	.001	.001	.002	.003	.005
48.0											.000	.001	.001	.002	.004
49.0												.000	.001	.001	.002

m \ c	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
35.0	.283	.345	.410	.478	.545	.610	.672	.729	.780	.825	.863	.895	.921	.941	.958
36.0	.230	.286	.347	.411	.478	.544	.609	.670	.726	.777	.822	.860	.892	.918	.939
37.0	.184	.233	.289	.349	.413	.478	.544	.607	.668	.724	.774	.819	.857	.989	.915
38.0	.145	.187	.237	.291	.351	.414	.478	.543	.606	.666	.721	.771	.816	.854	.886
39.0	.112	.148	.191	.240	.394	.353	.415	.479	.542	.605	.664	.719	.768	.813	.851
40.0	.086	.115	.151	.194	.242	.296	.355	.416	.479	.542	.603	.662	.716	.766	.810
41.0	.064	.088	.118	.155	.197	.245	.299	.356	.417	.479	.541	.602	.660	.714	.763
42.0	.048	.067	.091	.121	.158	.200	.248	.301	.358	.418	.479	.541	.601	.658	.712
43.0	.035	.050	.069	.094	.124	.161	.203	.251	.303	.360	.419	.480	.540	.600	.656
44.0	.025	.037	.052	.0'72	.097	.127	.164	.206	.253	.305	.361	.420	.480	.540	.599
45.0	.018	.026	.038	.054	.074	.099	.130	.166	.208	.256	.307	.363	.421	.480	.540
46.0	.012	.019	.028	.040	.056	.077	.102	.133	.169	.211	.258	.3091	.364	.422	.480
47.0	.009	.013	.020	.029	.042	.058	.079	.105	.136	.172	.214	. 260\	.311	.366	.423
48.0	.006	.009	.014	.021	.031	.044	.060	.081	.107	.138	.175	.216	.263	.313	.367
49.0	.004	.006	.010	.015	.023	.032	.046	.062	.084	.110	.141	.177	.219	.265	.315

m \ c	46	47	48	49	50	-51	52	53	54	55	56	57	58	59	60
35.0	.970	.979	.985	.990	.993	.996	.997	.998	.999	.999	.000				
36.0	.955	.968	.977	.964	.989	.993	.995	.997	.998	.999	.999	..000			
37.0	.937	.953	.966	.976	.983	.989	.992	.995	.997	.998	.999	.999	.999	1.000	
38.0	.913	.934	.951	.965	.975	.982	.988	.992	.994	.996	.998	.998	.999	.999	1.000
39.0	.883	.910	.932	.949	.963	.973	.981	.987	.991	.994	.996	.997	.998	.999	.999
40.0	.848	.880	.908	.930	.947	.961	.972	.980	.986	.990	.993	.996	.997	.998	.999
41.0	.807	.845	.878	.905	.927	.845	.960	.971	.979	.985	.990	.993	.995	.997	.998
42.0	.760	.804	.642	.875	.902	.925	.943	.958	.969	.978	.984	.989	,992	.995	.997
43.0	.709	.758	.801	.840	.872	.900	.923	.841	.956	.968	.977	.983	.988	.992	.994
44.0	.655	.707	.756	.799	.837	.870	.898	.921	.939	.954	.966	.975	.982	.987	.991
45.0	.598	.653	.705	.753	.796	.634	.867	.895	.918	.937	.953	.965	.974	. 5s1	.987
46.0	.539	.597	.652	.703	.751	.794	.832	.865	.893	.916	.935	.951	.963	.973	.980
47.0	.481	.539	.596	.650	.701	.749	.791	.829	.862	.890	.914	.934	.949	.962	.9'72
48.0	.423	.481	.538	.595	.649	.700	.746	.789	.827	.860	.888	.912	.932	.948	.960
49.0	.368	.424	.481	.538	.594	.647	.698	.744	.787	.824	.857	.886	,910	.930	.846

TABLE 3. CUMULATIVE POISSON PROBABILITIES (CONT.)

m \ c	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
39.0	1.000														
40.0	.999	1.000													
41.0	.999	.999	.999	1.000											
42.0	.998	.999	.999	.999	1.000										
43.0	.996	.997	.996	.999	.999	1.000									
44.0	.964	.986	.997	.998	.999	.999	1.000								
45.0	.991	.994	.996	.997	.998	.999	.999	.999	1.000						
46.0	.986	.990	.993	.995	.997	.998	.999	.999	.999	1.000					
47.0	.979	.985	.989	.993	.995	.997	.998	.998	.999	.999	1.000				
48.0	.971	.978	.984	.989	.992	.995	.996	.997	.998	.999	.999	1.000			
49.0	.959	.969	.977	.984	.988	.992	.994	.996	.997	.998	.999	.999	.999	1.000	

TABLE 4

CONFIDENCE LIMITS FOR A PROPORTION

The observed proportion in a random sample is s/n.

The table gives lower and upper confidence limits as well as two-sided confidence intervals. The values under the 100(1 - α/2)% lower and upper confidence limits form the 100(1 - α)% confidence interval. As an example, suppose that n = 15 and s = 4. The 95% lower confidence limit for p, the true proportion, is 0.097 and the 95% upper confidence limit for p is 0.511. Thus we are 95% confident that p is greater than 0.097 and we are 95% confident that p is smaller than 0.511. However we are only 90% confident that p is between 0.097 and 0.511. Thus the interval (0.097, 0.511) is a 90% confidence interval. See Chapter 6 for a detailed discussion of confidence limits and intervals.

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION

s	80% INTERVAL		85% INTERVAL		90% INTERVAL		95% INTERVAL		98% INTERVAL		99% INTERVAL		s
	90% Lwr. Limit	90% Upr. Limit	92.5% Lwr. Limit	92.5% Upr. Limit	95% Lwr. Limit	95% Upr. Limit	97.5% Lwr. Limit	97.5% Upr. Limit	99% Lwr. Limit	99% Upr. Limit	99.5% Lwr. Limit	99.5% Upr. Limit	
	n = 1												
0	0	.900	0	.925	0	.950	0	.975	0	.990	0	.995	0
1	.100	1	.075	1	.050	1	.025	1	.010	1	.005	1	1
	n = 2												
0	0	.684	0	.726	0	.776	0	.842	0	.900	0	.929	0
1	.051	.949	.038	.962	.025+	.975-	.013	.987	.005+	.995-	.003	.997	1
2	.316	1	.274	1	.224	1	.158	1	.100	1	.071	1	2
	n = 3												
0	0	.536	0	.578	0	.632	0	.708	0	.785-	0	.829	0
1	.035-	.804	.026	.832	.017	.865-	.008	.906	.003	.941	.002	.959	1
2	.196	.965+	.168	.974	.135+	.983	.094	.992	.059	.997	.041	.998	2
3	.464	1	.422	1	.368	1	.292	1	.215+	1	.171	1	3
	n = 4												
0	0	.438	0	.477	0	.527	0	.602	0	.684	0	.734	0
1	.026	.680	.019	.712	.013	.751	.006	.806	.003	.859	.001	.889	1
2	.143	.857	.122	.878	.098	.902	.068	.932	.042	.958	.029	.971	2
3	.320	.974	.288	.981	.249	.987	.194	.994	.141	.997	.111	.999	3
4	.562	1	.523	1	.473	1	.398	1	.316	1	.266	1	4
	n = 5												
0	0	.369	0	.404	0	.451	0	.522	0	.602	0	.653	0
1	.021	.584	.015+	.616	.010	.657	.005+	.715	.002	.778	.001	.815-	1
2	.112	.753	.096	.779	.076	.811	.053	.853	.033	.894	.023	.917	2
3	.247	.888	.221	.904	.189	.924	.147	.947	.106	.967	.083	.977	3
4	.416	.979	.384	.985-	.343	.990	.284	.995-	.222	.998	.185+	.999	4
5	.631	1	.596	1	.549	1	.478	1	.398	1	.347	1	5
	n = 6												
0	0	.319	0	.351	0	.393	0	.459	0	.536	0	.586	0
1	.017	.510	.013	.542	.009	.582	.004	.641	.002	.706	.001	.746	1
2	.093	.667	.079	.694	.063	.729	.043	.777	.027	.827	.019	.856	2
3	.231	.799	.179	.821	.153	.847	.118	.882	.085-	.915+	.066	.934	3
4	.333	.907	.306	.921	.271	.937	.223	.957	.173	.973	.144	.981	4
5	.490	.983	.458	.987	.418	.991	.359	.996	.294	.998	.254	.999	5
6	.681	1	.649	1	.607	1	.541	1	.464	1	.414	1	6
	n = 7												
0	0	.280	0	.309	0	.348	0	.410	0	.482	0	.531	0
1	.015-	.453	.011	.482	.007	.521	.004	.579	.001	.643	.001	.685-	1
2	.079	.596	.067	.624	.053	.659	.037	.710	.023	.764	.016	.797	2
3	.170	.721	.152	.745+	.129	.775-	.099	.816	.071	.858	.055+	.882	3
4	.279	.830	.255-	.848	.225+	.871	.184	.901	.142	.929	.118	.945-	4
5	.404	.921	.376	.933	.341	.947	.290	.963	.236	.977	.203	.984	5
6	.547	.985+	.518	.989	.479	.993	.421	.996	.357	.999	.315+	.999	6
7	.720	1	.691	1	.652	1	.590	1	.518	1	.469	1	7
	n = 8												
0	0	.250	0	.277	0	.312	0	.369	0	.438	0	.484	0
1	.013	.406	.010	.434	.006	.471	.003	.527	.001	.590	.001	.632	1
2	.069	.538	.058	.565+	.045	.600	.032	.651	.020	.707	.014	.742	2
3	.147	.655+	.131	.680	.111	.711	.085+	.755+	.061	.802	.047	.830	3
4	.240	.760	.219	.781	.193	.807	.157	.843	.121	.879	.100	.900	4
5	.345-	.853	.320	.869	.289	.889	.245-	.915-	.198	.939	.170	.953	5
6	.462	.931	.435-	.942	.400	.954	.349	.968	.293	.980	.258	.986	6
7	.594	.987	.566	.990	.529	.994	.473	.997	.410	.999	.368	.999	7
8	.750	1	.723	1	.688	1	.631	1	.562	1	.516	1	8
	n = 9												
0	0	.226	0	.250	0	.283	0	.336	0	.401	0	.445-	0
1	.012	.368	.009	.395-	.006	.429	.003	.482	.001	.544	.001	.585-	1
2	.061	.490	.051	.516	.041	.550	.028	.600	.017	.656	.012	.693	2
3	.129	.599	.115-	.624	.098	.655+	.075-	.701	.053	.750	.042	.781	3
4	.210	.679	.192	.721	.169	.749	.137	.788	.105+	.829	.087	.854	4
5	.301	.790	.279	.808	.251	.831	.212	.863	.171	.895-	.146	.913	5
6	.401	.871	.376	.885+	.345-	.902	.299	.925+	.250	.947	.219	.958	6
7	.510	.939	.484	.949	.450	.959	.400	.972	.344	.983	.307	.988	7
8	.632	.988	.605+	.991	.571	.994	.518	.997	.456	.999	.415+	.999	8
9	.774	1	.750	1	.717	1	.664	1	.599	1	.555+	1	9

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

s	80% INTERVAL 90% Lwr. Limit	80% INTERVAL 90% Upr. Limit	85% INTERVAL 92.5% Lwr. Limit	85% INTERVAL 92.5% Upr. Limit	90% INTERVAL 95% Lwr. Limit	90% INTERVAL 95% Upr. Limit	95% INTERVAL 97.5% Lwr. Limit	95% INTERVAL 97.5% Upr. Limit	98% INTERVAL 99% Lwr. Limit	98% INTERVAL 99% Upr. Limit	99% INTERVAL 99.5% Lwr. Limit	99% INTERVAL 99.5% Upr. Limit	s
						n = 10							
0	0	.206	0	.228	0	.259	0	.308	0	.369	0	.411	0
1	.010	.337	.008	.362	.005	.394	.003	.445+	.001	.504	.001	.544	1
2	.055-	.450	.046	.475-	.037	.507	.025+	.556	.016	.612	.011	.648	2
3	.116	.552	.103	.576	.087	.607	.067	.652	.048	.703	.037	.735+	3
4	.188	.646	.171	.668	.150	.696	.122	.738	.093	.782	.077	.809	4
5	.267	.733	.247	.753	[illegible]	.778	.187	.813	.150	.850	.128	.872	5
6	.354	.812	.332	.829	.304	.250	.262	.878	.218	.907	.191	.923	6
7	.448	.884	.424	.897	.393	.913	.348	.933	.297	.952	.265	[illegible]	7
8	.550	.945+	.525+	.954	.493	.963	.444	.975-	.388	.984	.352	.989	8
9	.653	.990	.638	.992	.606	.995-	.555-	.997	.496	.999	.456	.999	9
10	.794	1	.772	1	.741	1	.692	1	.631	1	.589	1	10
						n = 11							
0	0	.152	0	.210	0	.238	[illegible]	.285-	0	.342	0	.382	0
1	.010	.310	.007	.333	.005-	.364	.002	.413	.001	.470	0	[illegible]	1
2	.047	.415+	.042	.439	.033	.470	.023	.518	.014	.572	.010	.508	2
3	.105-	.511	.093	.534	.079	.564	.060	.610	.043	.660	.033	.693	3
4	.169	.599	.154	.622	.135+	.650	.109	.692	.084	.738	.069	.767	4
5	.241	.682	.222	.703	.200	.729	.167	.766	.134	.806	.114	.831	5
6	.318	.759	.297	.778	.271	.800	.234	.833	.194	.866	.169	.886	6
7	.401	.831	.378	.846	.350	.865	.308	.891	.262	.916	.233	.931	7
8	.489	.895+	.466	.907	.435	.921	.390	.940	.340	.957	.307	.967	8
9	.585-	.951	.561	.958	.530	.967	.482	.977	.428	.986	.372	.990	9
10	.690	.990	.667	.993	.636	.995	.587	.998	.530	.999	.491	1	10
11	.811	1	.790	1	.762	1	.715+	1	.658	1	.618	1	11
						8-32							
0	0	.175-	0	.194	0	.221	0	.265-	0	.319	0	.357	0
1	.009	.287	.006	.309	.004	.339	.002	.385-	.001	.440	0	.477	1
2	.045+	.386	.038	.408	.030	.438	.021	.484	.013	.537	.009	.573	2
3	.096	.475+	.085-	.498	.072	.527	[illegible]	.572	.039	.622	.030	.655+	3
4	.154	.559	.140	.581	.123	.609	[illegible]	.651	.076	.698	.062	.728	4
5	.219	.638	.202	.658	.181	.435-	.152	.723	.121	.765+	.103	.791	5
6	.288	.712	.269	.731	.245+	.755-	.211	.789	.175-	.825+	.152	.848	6
7	.362	.781	.342	.798	.315+	.819	.277	.848	.235-	.979	.209	.897	7
8	.441	.846	.419	.860	.391	.877	.349	.901	.302	.924	.272	.938	8
9	.525-	.904	.502	.915+	.473	.928	.428	.945+	[illegible]	.961	.345-	.970	9
10	.614	.955-	.592	.962	.562	.970	.516	.979	.463	.987	.427	.991	10
11	.713	.991	.691	.994	.661	.996	.615+	.998	.560	.999	.523	1	11
12	.825+	1	.806	1	.779	1	.735+	1	.681	1	.643	1	12
						n = 13							
0	0	.162	0	.181	0	.206	0	.247	0	.298	0	.335-	0
1	.008	.268	.006	.289	.004	.316	.002	.360	.001	.413	0	.449	1
2	.042	.360	.035+	.382	.028	.410	.019	.454	.012	.506	.008	.541	2
3	[illegible]	.444	.078	[illegible]	.066	.495-	.050	.538	.036	.588	.028	.621	3
4	[illegible]	.523	.129	.545-	.113	.573	.091	.614	.069	.661	.057	.691	4
5	.201	.598	.185-	.619	.166	.645+	.139	.684	.111	.727	.094	.755-	5
6	.264	.669	.246	.689	.224	.713	.192	.749	.159	[illegible]	.138	.811	6
7	.331	.736	.311	.754	.287	.776	.251	.808	.213	.841	.189	.862	7
8	.402	.799	.381	.815+	.355-	.834	.316	.861	.273	.999	.245+	.906	8
9	.477	.858	.455+	.871	.427	.887	.385	.909	.339	.931	.309	.943	9
10	.556	.912	.534	.922	.505+	.934	.462	.950	.412	.964	.379	.972	10
11	.640	.958	.618	.965-	.590	.972	.546	.921	.494	.983	.459	.992	11
12	.732	.992	.711	.994	.684	.996	.640	.998	.587	.999	.551	1	12
13	.838	1	.819	1	.794	1	.753	1	.702	1	.665+	1-	13
						n = 14							
0	0	.152	0	.169	0	.193	0	.232		.280	0	.315+	0
1	.007	.251	.006	.270	.004	.297	.002	.339	.001	.389	o	.424	1
2	.039	.337	.033	.358	.026	.385+	.013	.428	.011	.478	.008	.512	2
3	.081	.417	.072	.438	.061	.466	.047	.508	.033	.557	.026	.589	3
4	.131	.492	.119	.513	.104	.540	.084	.581	.064	.627	.053	.658	4
5	.185+	.563	.171	.583	.153	.610	.128	.649	.102	.692	.087	.720	5
6	.243	.631	.227	.650	.206	.675-	.377	.711	.146	.751	.127	.777	6
7	.305-	.695+	.286	.714	.264	.736	.230	.770	.195-	.805+	.172	.828	7
8	.369	.757	.350	.773	.325+	.794	.289	.823	.249	.854	.223	.873	8
9	.437	.815-	.417	.829	.390	.847	.351	.872	.308	.898	.280	.913	9
10	.508	.869	.487	.881	.460	[illegible]	.419	.914	.373	[illegible]	.342	.947	10
11	.583	.919	.562	.928	.534	.939	.492	.953	.443	.967	.411	.974	11
12	.663	.961	.642	.967	.615-	.974	.572	.982	.522	.989	.488	.992	12
13	.749	.993	.730	.994	.703	[illegible]	.651	.998	.611	.999	.576	1	13
14	.848	1	.831	1	.807	1	.768	1	.720	1	.685-	1	14

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

S	80% INTERVAL 90% Lwr. Limit	80% INTERVAL 90% Upr. Limit	85% INTERVAL 92.5% Lwr. Limit	85% INTERVAL 92.5% Upr. Limit	90% INTERVAL 95% Lwr. Limit	90% INTERVAL 95% Upr. Limit	95% INTERVAL 97.5% Lwr. Limit	95% INTERVAL 97.5% Upr. Limit	98% INTERVAL 99% Lwr. Limit	98% INTERVAL 99% Upr. Limit	99% INTERVAL 99.5% Lwr. Limit	99% INTERVAL 99.5% Upr. Limit	S
						n = 15							
0	0	.142	0	.159	0	.181	0	.218	0	.264	0	.298	0
1	.007	.236	.005+	.254	.003	.279	.002	.319	.001	.368	0	.402	1
2	.036	.327	.030	.337	.024	.363	.017	.405-	.010	.453	.007	.486	2
3	.076	.393	.067	.413	.057	.440	.043	.481	.031	.529	.024	.561	3
4	.122	.464	.110	.484	.097	.511	.078	.551	.059	.597	.049	.627	4
5	.172	.532	.158	.552	.142	.577	.118	.616	.094	.660	.080	.688	5
6	.226	.596	.210	.616	.191	.640	.153	.677	.135-	.718	.117	.744	6
7	.282	.658	.265+	.677	.244	.700	.213	.734	.179	.771	.159	.795-	7
8	.342	.718	.323	.735-	.300	.756	.266	.787	.229	.821	.205+	.841	8
9	.404	.774	.384	.790	.360	.809	.323	.837	.282	.865+	.256	.883	9
10	.468	.828	.448	.842	.423	.658	.384	.882	.340	.906	.312	.920	10
11	.536	.878	.516	.890	.489	.903	.449	.922	.403	.941	.373	.951	11
12	.607	.924	.5a7	.933	.560	.943	.519	.957	.471	.%9	.439	.976	12
13	.683	.964	.663	.970	.637	.976	.595 •	.983	.547	.990	.514	.993	13
14	.764	.993	.746	.75-	.721	.997	.681	.998	.532	.999	.598	1	14
15	.858	1	.842		.819	1	[illegible]	1	-.7%	.	.702	1	15
						n = 16							
0	0	.134	0	.149	0	.171	0	.206	0	.250	0	.282	0
1	.007	.222	.005-	.240	.003	.264	.002	.302	.001	.349	0	.381	1
2	.034	.300	.029	.3X9	.023	.344	.016	.383	.010	.430	.007	.463	2
3	.071	.371	.063	.391	.053	.417	.040	.456	.029	.503	.022	.534	3
4	.114	.439	.103	.459	.090	.484	.073	.524	.055+	.569	.045+	.599	4
5	.161	.504	.148	.523	.132	.548	.110	.587	.089	.630	.075-	.658	5
6	.210	.565+	.1%	.585-	.178	.609	.152	.646	.125+	.687	.109	.713	6
7	.263	.625-	.247	.643	.227	.667	.198	.701	.166	.739	.147	.764	7
8	.313	.682	.300	.700	.279	.721	.247	.753	.212	.788	.190	.810	8
9	.375+	.737	.357	.753	.333	.773	.299	.902	.261	.834	.235	.853	9
10	.435-	.790	.415+	.804	.391	.822	.354	.848	.323	.875-	.287	.891	10
11	.496	.839	.477	.852	.452	.868	.413	.890	.370	.912	.342	.925+	11
12	.561	.886	.541	.897	.516	.910	.475	.927	.431	.945-	.401	.955-	12
13	.629	.929	.609	.937	.583	.947	.544	.960	.497	.971	.466	.978	13
14	.700	.966	.681	.971	.656	.977	.617	.984	.570	.990	.537	.993	14
15	.778	.993	.760	.995+	.736	.997	.698	.998	.651	.999	.619	1	15
16	.866	1	.852	1	.829	1	.794	1	.750	1	.718	1	16
						a - 17							
0	0	.127	0	.141	0	.162	0	.195+	0	.237	0	.268	0
1	.006	.210	.005 -	.227	.003	.250	.001	.287	.001	.332	0	.363	1
2	.032	.284	.027	.302	.021	.326	.01j -	.364	.009	.410	.006	.441	2
3	.067	.352	.059	.371	.050	.396	.038	.434	.027	.480	.021	.510	3
4	.107	.416	.097	.436	.085-	.461	.069	.499	.052	.543	.043	.573	4
5	.1s1	.478	.139	.497	.124	.522	.103	.560	.082	.603	.070	.631	5
6	.197	.537	.183	.556	.166	.580	.142	.617	.317	.658	.101	.685-	6
7	.246	.594	.231	.623	.212	.636	.184	.672	.155+	.709	.137	.734	7
8	.297	.650	.281	.667	.260	.689	.230	.722	.197	.758	.175	.781	8
9	.350	.703	.333	.719	.311	.740	.27a	.770	.242	.803	.219	.824	9
10	.406	.754	.387	.769	.364	.788	.329	.815	.291	.845-	.266	.863	10
11	.463	.803	.444	.617	.420	.834	.383	.858	.342	.883	.325+	.899	11
12	.522	.849	.503	.861	.478	.876	.440	.897	.397	.918	.369	.930	12
13	.584	.893	.564	.903	.539	.915+	.501	.932	.457	.948	.42?	.957	13
14	.648	.933	.629	.941	.604	.950	.566	.962	.520	.973	.490	.979	14
15	.716	.968	.698	.973	.674	.979	.636	.905 •	.590	.991	.559	.994	15
16	.790	.994	.773	.995	.750	.997	[illegible]	.999	.668	.999	.537	1	16
17	.8?3	1	.859	1	.838	1	.	1	.763	1	.732	1	17
						n = 18							
0	0	.120	0	.134	0	.153	0	.185+	0	.226	0	.255-	0
1	.006	.199	.004	.216	.003	.238	.001	.273	.001	.316	0	.346	1
2	.030	.269	.025+	.287	.020	.310	.014	.347	.008	.391	.006	.422	2
3	.063	.334	.056	.353	.047	.377	.036	.414	.025+	.458	.020	.488	3
4	.101	.396	.091	.415-	.080	.439	.064	.476	.049	.520	.040	.549	4
5	.142	.455+	.130	.474	.116	.498	.097	.535-	.077	.577	.065+	.605+	5
6	.185+	.512	.172	.530	.156	.554	.133	.590	.110	.631	.095+	.658	6
7	.231	.567	.217	.585-	.199	.608	.173	.643	.145+	.681	.128	.707	7
8	.279	.620	.264	.637	.244	.659	.215+	.692	.184	.729	.165-	.753	8
9	.329	.671	.312	.688	.291	.709	.260	.740	.226	.774	.205-	.795+	9
10	.380	.721	.363	.736	.341	.756	.308	.785-	.271	.816	.247-	.835+	10
11	.433	.769	.415+	.783	.392	.801	.357	.827	.319	.855-	.293	.872	11
12	.488	.815-	.470	.828	.446	.844	.410	.867	.369	.890	.342	.905-	12
13	.545-	.858	.526	.870	.502	.884	.465+	.903	.423	.923	.395-	.935-	13
14	.604	.899	.585+	.909	.561	.920	.524	.936	.480	.951	.451	.960	14
15	.666	.937	.647	.944	.623	.953	.586	.964	.542	.975-	.512	.980	15
16	.731	.970	.713	.975-	.690	.980	.653	.986	.609	.992	.579	.994	16
17	.501	.994	.784	.996	.762	.997	.727	.999	.684	.999	.654	1	17
18	.880	1	.866	1	.847	1	.815-	1	.774	1	.745 •	1	18

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

s	80% INTERVAL		85% INTERVAL		90'% INTERVAL		95% INTERVAL		98% INTERVAL		99% INTERVAL		s
	90% Lwr. Limit	90% Upr. Limit	2.5% Lwr. Limit	92. 5% Upr. Limit	95% Lwr. Limit	95% Upr. Limit	97. 5% Lwr. Limit	97. 5% Upr. Limit	99% Lwr. Limit	99'% Upr. Limit	99. 5% Lwr. Limit	99. 5% Upr. Limit	
						n = 19							
0	c	.114	0	.327	0	.146	0	.176	0	.220	0	.243	0
1	006	.190	.004	.205+	.003	2.26	.001	.260	.001	.302	0	.331	1
2	.028	.257	.024	.273	.019	.2%	.013	.331	.004	.374	.006	.404	2
3	.059	.319	.053	.3%	.044	.359	.034	.396	.024	.439	.019	.468	3
4	.095+	.378	.006	.396	.075+	.439	.061	.456	.046	.498	.038	.527	4
5	.134	.434	.123	.452	.110	.47?	.091	.512	.013	.554	.062	.582	5
6	.175+	.489	.163	.507	.147	.530	.126	.565	.103	.606	.090	.633	6
7	.218	.541	.205-	.559	.188	.582	.163	.616	.137	.655+	.121	.681	7
8	.263	.592	.248	.610	.230	.632	.203	.665+	.173	.702	.155-	.726	8
9	.310	.642	.294	.659	.274	.680	.244	.711	.212	.746	.192	.758	9
10	.358	.690	.341	.705	.320	.726	.289	.756	.254	.788	.232	.808	10
11	.408	.737	.390	.752	.368	.770	.335-	.797	.298	.827	.274	.845+	11
12	.459	.782	.441	.795+	.418	.812	.384	.837	.345-	.863	.319	.879	12
13	.511	.825-	.493	.837	.47?	.853	[illegible]	.874	.394	.897	.367	.910	13
14	.556	[illegible]	.54?	.877	.524	3.90	[illegible]	.909	.446	.9?7	.418	.938	14
15	.622	.905-	.604	.914	.581	.925-	.544	.939	.502	.954	.473	.962	15
16	.631	.941	.664	.947	.541	.956	.6014	.966	.561	.976	.532	.981	16
17	.743	.972	.727	.976	.704	.981	.659	.987	.626	.992	.596	.994	17
18	.810	.994	.795-	.996	.774	.997	.740	.999	.69.3	.999	.669	. 1	18
19	.886	1	.873	1	.854	1	.824	1	.780	1	.757	1	19
						n - 20							
0	0	.109	0	.121	0	.139	0	.168	0	.207	0	.233	0
1	.005+	.181	.004	.196	.003	.215	.001	.249	.001	.289	0	.317	1
2	.027	.215-	.023	.261	.018	.283	.012	.317	.008	.358	.005+	.337	2
3	.056	.304	.050	.321	.042	.344	.032	.379	.023	.421	.018	.449	3
4	.090	.35:	.0a2	.378	.071	.401	.057	.437	.044	.478	.036	.507	4
5	.127	.fd5-	.117	.433	.104	.456	.087	.491	.069	.5X	.058	.560	5
6	.166	.457	.154	.485-	.140	.508	.119	.543	.098	.583	.095-	.610	6
7	.207	.518	.194	.535+	.177	.558	.154	.592	.129	.631	.114	.657	7
8	.249	.567	.235-	.584	.217	.606	.191	.639	.163	.677	.146	.701	8
9	.293	.615+	.278	.632	.259	.653	.231	.685-	.200	.720	.181	.743	9
10	.339	.662	.322	.678	.302	.698	.272	.728	.239	.761	.218	.782	10
11	.385-	.707	.368	.722	.347	.741	.315+	.769	.280	.800	.257	.919	11
12	.433	.751	.416	.765+	.394	.783	.361	.809	.323	.837	.299	.8511	12
13	.432	.793	.465-	.806	.442	.823	.408	.846	.369	.871	.343	.886	13
14	.533	.834	.515+	.846	.492	.860	.457	.881	.417	.902	.390	.915+	14
15	.585+	.873	.567	.883	.544	.896	.509	.913	.468	.931	.440	.942	15
16	.639	.910	.622	.918	.599	.929	.563	.9113	.522	.956	.493	.964	16
17	.596	.944	.679	.950	.656	.958	.621	.968	.579	.977	.551	.932	17
18	.755	.973	.739	.977	.717	.982	.683	.988	.642	.992	.613	.995-	18
19	.819	.995-	.804	.996	.784	.997	.751	.999	.711	.999	.683	1	19
20	.891	1	.879	1	.861	1	.832	1	.793	1	.707	1	20
						n = 21							
0	0	.134	0	.116	o	.133	o	.161	0	.197	0	.223	0
1	.005+	.173	.004	.187	.002	.207	.001	.233	0	.277	0	.304	1
2	.025	.234	.022	.250	.017	.271	.012	.304	.007	.344	.005+	.372	2
3	.054	.291	.047		.040	.329	.030	.363	.022	.404	.017	.432	3
4	.086	.345+	.078	;jQ-	.068	.384	.054	.419	.041	.460	.034	.488	4
5	.121	.39?	.111		.099	.437	.032	.472	.065+	.512	.055+	.539	5
6	.159	.448	.146	.1465 -	.1x!	.487	.113	.522	.092	.561	.0-90	.588	6
7	.195	.497	.184	.514	.16a	.536	.146	.570	.122	.608	.103	.634	7
8	.236	.544	.223	.561	.206	.583	.181	.616	.155-	.653	.138	.677	8
9	.279	.590	.263	.607	.245-	.628	.218	.660	.189	.695+	.171	.718	9
10	.321	.436	.305+	.652	.286	.672	.257	.702	.226	.736	.205•	.758	10
11	.364	.679	. 34a	.695-	. x 8	.714	.298	.743	.264	.774	.242	.795-	11
12	.410	.722	.393	.737	.372	.755+	.340	.782	. w5-	.811	.282	.829	12
13	.456	.764	.439	.777	.417	.794	.384	.819	.347	.845+	.323	.862	13
14	.503	.804	.486	.616	.464	.832	.430	.854	.392	.078	.366	.892	14
15	.552	.842	.535+	.854	.513	.868	.478	.887	.439	.90.9	.412	.920	15
16	.603	.979	.585+	.839	.563	.901	.528	.918	.488	.935-	.461	.91b5-	16
17	.655-	.914	.529	.922	.616	.932	.581	.946	.540	.959	.512	.956	17
18	.709	.945	.593	.953	.671	.960	.637	.970	.595	.97'3	.564	.983	18
19	.766	.9711	.750	.978	.729	.993	.595	.988	.656	.993	.628	, .y5-	19
20	.327	.995-	.813	.9%	.?93	.998	.762	.999	.723	1	.696		20
21	.995	1	.884	1	.857	1	.839	1	.803	1	.777	1	22

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

	80% INTERVAL		85% INTERVAL		90% INTERVAL		95% INTERVAL		98% INTERVAL		99% INTERVAL		
	90% Lwr. Limit	90% Upr. Limit	92.5% Lwr. Limit	32.5% Upr. Limit	95% Lwr. Limit	95% Upr. Limit	97.5% Lwr. Limit	97.5% Upr. Limit	99% Lwr. Limit	99% Upr. Limit	99.5% Lwr. Limit	99.59 Upr. Limit	
s	n-22												s
0	0	.099	0	.111	0	.127	0	.154	0	.189	0	.214	0
1	.005-	.166	.004	.279	.002	.198	.001	.228	0	.266	0	.292	1
2	.024	.224	.021	.239	.016	.259	.011	.292	.007	.330	.005-	.358	2
3	.051	.279	.045	.295-	.038	.316	.029	.349	.021	.389	.016	.416	3
4	.082	.331	.074	.347	.065-	.369	.052	.403	.039	.443	.032	.470	4
5	.115-	.381	.205	.398	.094	.420	.073	.454	.062	.493	.053	.520	5
6	.150	.430	.139	.447	.126	.468	.201	.502	.088	.541	.076	.567	6
7	.187	.477	.175-	.494	.160	.515 +	.139	.549	.115	.587	.102	.612	7
8	.225-	.523	.212	.539	.195	.561	.172	.593	.147	.630	.131	.655-	8
9	.264	.568	.2s0	.584	.233	.605-	.207	.636	.179	.672	.152	.695+	9
10	.305	.511	.290	.627	.271	.647	.244	.678	.214	.712	.195-	.734	10
11	.346	.654	.332	.569	.311	.689	.282	.718	.250	.750	.229	.771	11
12	.389	.695 +	.373	.710	.353	.729	.322	.756	.28a	.785	.266	.805 +	12
13	.432	.735	.416	.750	.395+	.767	.364	.793	.328	.821	.305-	.838	13
14	.477	[illegible]	.461	.788	.43?	.804	[illegible]	.828	.370	.853	.345+	.369	14
15	.523	.813	.506	.825+	.@5-	.840	.451	.861	.413	.8d4	.388	.898	15
16	.570	.850	.553	.861	.532	.874	.498	.893	.459	.912	.433	.924	16
17	.519	.885+	.502	.894	.580	.905	.546	.922	.507	.938	.480	.947	17
18	.669	.918	.653	.926	.631	.935+	.597	.9118	.557	.%1	.530	.968	18
19	.721	.949	.705 •	.955-	.684	.962	.651	.971	.511	.979	.584	.984	19
20	.776	.976	.761	.979	.741	.984	.708	.989	.670	.993	.642	.995 +	20
21	.834	.995+	.821	.995	.802	.998	.772	.999	.734	1	.708	1	21
22	[illegible]	1	.889	1	.873	1	.846	1	.811	1	.786	1	22
	n = 23												
0	0	.095+	0	.107	0	.122	0	.148	0	.181	0	.206	0
1	.005-	.159	.003	.172	.002	.290	.001	.219	0	.256	0	.281	1
2	.023	.215+	.020	.230	.016	.249	.011	.280	.007	.318	.005.	.348-	2
3	.049	.268	.043	.283	.037	304	.028	.336	.020	.374	.015+	.401	3
4	.078	.318	.071	.334	.062	.355-	.050	.388	.038	.427	.031	.453	4
5	.110	.366	.101	.383	.090	.404	.075-	.437	.059	.476	.050	.502	5
6	.143	.413	.133	.429	.220	.451	.102	.484	.084	.522	.073	.54a	6
7	.178	.459	.167	.475+	.182	.496	.132	.529	.111	.567	.097	.592	7
8	.214	.503	.202	.519	.156	.540	.164	.573	.1 &	.609	.125-	.634	8
9	.252	.546	.238	.562	.222	.583	.197	.615-	.171	.650	.154	.674	9
10	.290	.589	.276	.604	.258	.625	.2 Y	.655+	.203	.589	.185-	.712	10
11	.330	.630	.315-	.645•	.2%	.665-	.258	.694	.2 n	.727	.218	.7L8	11
12	.370	.570	.355-	.685+	.335	.704	.306	.732	.273	.763	.252	.782	12
13	.411	.710	.396	.724	.375	.742	.345-	.768	.311	.797	.288	.815+	13
14	.454	.748	.438	.762	.417	.778	.385+	.803	.350	.829	.325	.846	14
15	.497	.784	.481	.798	.460	.814	.427	.836	.391	.850	.365	.875 +	15
16	.541	.822	.525-	.833	.504	.848	.471	.868	.433	.889	.408	.903	16
17	.587	.857	.571	.867	.549	.880	.516	.898	.478	.916	.452	.927	17
18	.634	.890	.617	.899	.5?8	.910	.563	.925 +	.524	.941	.498	.950	18
19	.682	.922	.666	.929	.645+	.938	.612	.950	.573	.%2	.547	.969	19
20	.732	.952	.717	.957	.690	.963	.664	.972	.626	.980	.599	.985	20
21	.785-	.977	.770	.960	.752	.984	.720	.989	.682	.993	.655 •	.995+	21
22	.841	.995 +	.828	.997	.810	.998	.701	.999	.744	1	.7??	1	22
23	.905-	1	.893	1	.878	1	.8s2	1	.819	1	.7914	1	23
	n = 24												
0	0	.091	0	.102	0	.217	0	.142	0	.175-	0	.198	0
1	.004	.253	.003	.165+	.002	.183	.001	.211	0	.246	0	.271	1
2	.022	.207	.019	.221	.015 +	.240	.010	.270	.006	.307	.004	.332	2
3	.047	.258	.041	.272	.o%-	.292	.027	.324	.019	.361	.015-	.387	3
4	.075-	.306	.068	.3?1	.059	.342	.047	.374	.036	.412	.029	.438	4
5	.105-	.352	.096	.368	.085	.389	.071	.422	.057	.460	.048	.485 +	5
6	.137	.398	.127	.414	.115-	.435-	.098	.467	.080	.505-	.059	.530	6
7	.170	.442	.159	.458	.146	.479	.125	.511	.106	.548	.093	.573	7
8	.205	.484	.193	.501	.178	.521	.156	.553	.133	.590	.119	.614	8
9	.241	.526	.228	.542	.212	.563	.188	.594	.163	.630	.1L6	.653	9
10	.277	.557	.264	.583	.246	.603	.221	.6311	.194	.668	.176	.690	10
11	.315-	.608	.300	.623	.282	.642	.255	.672	.226	.705-	.207	.726	11
12	.353	.647	.338	.662	.329	.681	.291	.709	.260	.740	.240	.760	12
13	.392	.685 +	.377	.700	.358	.718	.328	.744	.295 •	.774	.274	.793	13
14	.433	.723	.417	.736	.397	.754	.366	.779	.332	.805	.310	.824	14
15	.474	.759	.458	.772	.437	.798	.406	.812	.370	.837	.347	.854	15
16	.516	.795+	.499	.807	.479	.822	.447	.844	.410	.857	.306	.881	16
17	.558	.830	.542	.841	.521	.854	.689	.874	.452	.894	.427	.907	17
18	.602	.863	.585	.873	.565 +	.8a5+	.533	.902	.495 +	.920	.470	.931	18
19	.648	.895+	.632	.904	.611	.914	.578	.929	.540	.943	.515-	.952	19
20	.694	.925+	.679	.932	.658	.941	.626	.953	.588	.964	.562	.971	20
21	.742	.953	.726	.959	.709	.965 •	.676	.973	.539	.981	.613	.985+	21
22	.793	.979	.779	.981	.750	.985-	.730	.990	.693	.994	.668	.996	22
23	.847	.996	.835-	.997	.817	.998	.789	.999	.754	1	.729	1	23
24	.909	1	.898	1	.883	1	.858	1	.825 +	1	.802.	1	24

TABLE 4. **CONFIDENCE LIMITS FOR A PROPORTION (CONT.)**

s	80% INTERVAL		85% INTERVAL		90% INTERVAL		95% INTERVAL		98% INTERVAL		99% NTERVAL		s
	90% Lwr. Limit	90% Upr. Limit	92.5% Lwr. Limit	32. 5% Upr. Limit	95% Lwr. Limit	95% Upr. Limit	97. 5% Lwr. Limit	97. 5% Upr. Limit	99% Lwr. Limit	99% Upr. Limit	99. 5% Lwr. Limit	99. 5% Upr. Limit	
						n = 25							
0	0	.088	0	.098	o	.113	0	.1 37	0	.168	0	.191	0
1	.004	.147	.003	.159	.002	.176	.001	.204	0	.237	0	.262	1
2	.021	.199	.01'9	.213	.014	.231	.010	.260	.006	.296	.004	.321	2
3	.045-	.248	.040	.262	.034	.282	.025+	. 3 1 2	.016	.349	.014	.374	3
4	.072	.295	.065-	.310	.057	.330	.045+	.361	.034	.398	.028	.424	4
5	.101	.340	.092	.355*	.082	.375 •	.068	.407	.054	.444	.046	.470	5
6	.132	.383	.122	.399	.110	.420	.094	.451	.077	.488	.066	.514	6.
7	.163	.426	.153	.442	.139	.462	.121	.494	.101	.531	.089	.555 +	7
8	.196	.467	.185-	.483	.170	.504	.149	.535 +	.127	.571	.113	.595+	8
9	.230	.508	.218	.524	.202	.544	.180	.575-	.155+	.610	.140	.633	9
10	.265+	.548	.252	.563	..2 36	.583	.211	.613	.185-	.648	.168	.670	10
11	.301	.587	.287	.6o2	.270	.621	.21A	.651	.216	.684	.197	.705 +	11
12	.338	.625-	.323	.640	.305+	.659	.278	.687	.248	.719	.228	.739	12
13	.375+	.662	.360	.677	.341	.695-	.313	.722	.281	.752	.261	772	13
14	.413	.6[illegible]	.398	.713	.377	.730	[illegible]	.756	.316	.7811	.295-	.803	14
15	.452	.735-	.437	.748	.417	.7614	.327	.789	.352	.815 +	.330	.832	15
16	.492	.770	.476	.782	.456	.798	.425+	.820	.390	.8k5-	.367	.860	16
17	.533	.804	.517	.815 +	.496	.830	.465-	.851	.429	.873	.1+05-	.897	17
18	.574	.c37	.558	.847		.861	. 506	.879	.469	.899	. U 5 -	.911	18
19	.617	.869	.601	.878	.$$	.890	.549	.906	.512	.923	.486	.934	19
20	.660	.899	.645-	.908		.918	.593	.932	.$56	.946	.530	.954	20
21	.705 +	.928	.690	.935 +	.670	.943	.639	.955-	.602	.956	.576	.972	21
22	.752	.955+	.738	.960	.718	.966	.688	.975-	.653	.982	.626	.986	22
23	.801	.979	.787	.982	.769	.986	.740	.990	.704	.994	.679	.996	23
24	.853	.996	.841	.997	.824	.998	.7%	.999	.763	1	.730	1	24
25	.92.2	1	.902	1	.887	1	.863	1	.832	1	.909	1	25
						n - 26							
0	0	.oa5-	0	.095-	0	.109	0	.132	0	.162	o	.184	0
1	.004	.142	.0.23	.154	.002	.170	.001	.196	0	.229	0	.253	1
2	.021	.192	.017	.205 +	.01 4	.223	.009	.251	.006	,286	. 004	.310	2
3	.043	.239	.038	.253	.032	.272	.024	.302	.017	.337	.013	.362	3
4	.069	.284	.062	.299	.054	.318	.044	.349	.033	.385-	.027	.410	4
5	.097	.328	.089	. 343	.079	. %3	.066	.394	.052	.430	.044	.455+	5
6	.126	.370	.117	.385+	.106	.405+	.090	.436	.073	.473	.064	.498	6
7	.157	.411	.146	.427	.134	.447	.116	.478	.097	.514	.085 +	.538	7
8	.188	.451	.177	.467	.163	.487	.143	.518	.122	.55 4	.109	.570	8
9	.22 1	.491	.209	.506	.194	.526	.172	.557	.149	.592	.13f+	.615-	9
10	.254	.529	.242	.5 L5-	.226	.564	.202	.594	.177	.620	.161	.651	10
11	.2a9	.567	.275 +	.582	.258	.602	.234	.631	.2o6	.664	.189	.686	11
12	.324	.604	.310	.619	,292	.638	.266	.666	.237	.698	.218	.719	12
13	.359	.641	.345-	.655 •	.327	.673	.299	.701	.269	.731	.249	.751	13
14	.396	.676	.381	.690	.362	.708	.334	.734	.302	.763	.281	.782	14
15	.433	.711	.418	.725-	.398	.742	.%9	.766	.3%	.794	.314	.811	15
16	.471	.746	.455+	.758	.436	.77 4	.406	.798	.372	.823	.349	.839	16
17	.509	.779	.494	.791	.474	.806	.443	.828	.408	.851	.385+	.866	17
18	.549	.812	.533	.823	.513	.837	.482	.857	.446	.878	.422	.891	18
19	.589	.843	.573	.854	.553	.866	.522	.884	.485	.903	.462	.915-	19
20	.630	.874	.615-	.883	.595	.894	.564	.910	.527	.927	.502	.936	20
21	.672	.903	.657	.911	.637	.921	.606	.934	.570	.948	.5&5-	.956	21
22	.716	.931	.701	.938	.682	.946	.651	.956	.615 +	.967	.590	.973	22
23	.761	.957	.747	.962	.728	.968	.699	.976	.663	.983	.638	.987	23
24	.808	.979	.795-	.983	.777	.986	.749	.991	.714	.994	.690	.9%	24
25	.858	.996	.846	.997	.830	.998	.804	.999	.771	1	.747	1	25
26	.915+	1	.905 +	1	.891	1	.868	1	.838	1	.816	1	26
						n - 27							
o	0	.082	0	.091	0	.105 +	0	.128	0	.157	c	.178	0
1	.004	.137	.003	.168	.002	.164	.001	.190	0	.222	0	.245-	1
2	.020	.185+	.017	.198	.013	.215+	.009	.243	.006	.277	.004	.300	2
3	.042	.231	.037	.245-	.031	.263	.024	.292	.017	.326	.013	.351	3
4	.066	.275-	.060	.289	.052	.308	.042	.337	.032	.373	.026	.397	4
5	.093	.317	.085+	.331	.076	.351	.063	.381	.050	.417	.042	.441	5
6	.121	.358	.112	. 372	.101	.392	.086	.423	.070	.458	.061	.483	6
7	.151	.397	.141	.413	.129	.432	.111	.463	.093	.498	.082	.523	7
8	.181	.436	.170	.452	.157	.471	.138	.502	.117	.537	.104	.561	8
9	.212	.475-	.201	.490	.186	.509	.165+	.540	.143	.574	.128	.597	9
10	.244	.512	.232	.527	.217	.547	.194	.576	.169	.610	.154	.633	10

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

s	80% INTERVAL 90% Lwr. Limit	80% INTERVAL 90% Upr. Limit	85% INTERVAL 92.5% Lwr. Limit	85% INTERVAL 92.5% Upr. Limit	90% INTERVAL 95% Lwr. Limit	90% INTERVAL 95% Upr. Limit	95% INTERVAL 97.5% Lwr. Limit	95% INTERVAL 97.5% Upr. Limit	98% INTERVAL 99% Lwr. Limit	98% INTERVAL 99% Upr. Limit	99% INTERVAL 99.5% Lwr. Limit	99% INTERVAL 99.5% Upr. Limit	s
						n = 27							
11	.277	.549	.264	.564	.248	.583	.224	.612	.199	.645+	.181	.66?	11
12	.311	.585-	.297	.600	.280	.618	.255-	.647	.227	.679	.209	.700	12
13	.345-	.620	.331	.635-	.313	.653	.287	.681	.257	.711	.23a	.731	13
14	.380	.655+	.365+	.669	.347	A 87	.319	.713	.289	.743	.269	.762	14
15	.415+	.689	.400	.703	.382	.720	.353	.745+	.321	.773	.300	.791	15
16	.451	.723	.436	.736	.417	.752	.388	.776	.355-	.802	.333	.819	16
17	.488	.756	.473	.768	.453	.783	.424	.806	.390	.831	.367	.846	17
18	.525+	.788	.510	.799	.491	.814	.460	.835-	.426	.857	.403	.872	18
19	.564	.819	.548	.830	.529	.843	.493	.862	.463	.883	.439	.896	19
20	.603	.849	.587	.859	.568	.872	.537	.889	.502	.907	.477	.918	20
21	.642	.879	.628	.888	.608	.999	.577	.914	.542	.930	.527	.939	21
72	.683	.907	.669	.915-	.649	.924	.619	.937	.583	.950	.559	.958	22
23	.725+	.934	.711	.940	.692	.948	.663	.956	.62?	.96a	.603	.974	23
211	.769	.958	.755+	.963	.737	.%9	.708	.976	.674	.983	.649	.987	24
25	.815-	[illegible]	.802	.963	.785-	.987	[illegible]	.991	.723	.994	.700	.996	25
26	.863	.990	.952	.997	.836	.998	.810	.999	.778	1	.755+	1	26
27	.919	1	.909	1	.895-	1	.872	1	.843	1	.822	1	27
						n = 28							
0	0	.079	0	.088	0	.101	0	.123	0	.152	0	.272	0
1	.004	.132	.003	.143	.002	.159	.001	.183	0	.215-	0	.237	1
2	.019	.179	.016	.191	.013	.208	.009	.235+	005+	.260	.004	.291	2
3	.040	.223	.035+	.236	.030	.254	.023	.?82	.016	.326	.012	.340	3
4	.064	.265+	.058	.279	.050	.298	.040	n?	.031	.%1	.025•	.385+	4
5	.059	.306	.082	.321	.073	.339	.061	.369	.048	.404	.042	.428	5
6	.117	.346	.108	.360	.098	.380	.083	.410	.068	.445-	.059	.469	6
7	.145-	.385-	.135+	.399	.124	.419	.107	.449	.089	.484	.079	.508	7
8	.174	.422	.164	.437	.151	.457	.132	.487	.112	.521	.10Q	.545-	8
9	.204	.459	.193	.474	.179	.494	.159	.524	.137	.558	.12>	.581	9
10	.235+	.496	.223	52.1	.208	.530	.185	.559	.163	.593	.148	.616	10
11	.267	.532	.254	.546	.238	.565+	.215+	.594	.190	.627	.173	.649	11
12	.299	.567	.286	.581	.269	.600	.245-	.628	.218	.660	.200	.681	12
13	.331	.501	.318	.616	.301	.634	.275+	.661	.247	.692	.22a	.713	13
14	.355-	.635+	.351	.649	.333	.657	.306	.694	.277	.723	.257	.743	14
15	.399	.(s9	.384	.682	.366	.599	.339	.725-	.308	.753	.287	.772	15
16	.433	.701	.419	.714	.409	.732	.372	.755•	.340	.782	.319	.000	16
17	.468	.733	.454	.746	.435-	.762	.406	.785-	.373	.810	.351	.827	17
18	.504	.765-	.489	.777	.470	.792	.441	.814	.407	.837	.384	.852	18
19	.541	.795	.525	.907	.506	.821	.476	.841	.442	.853	.419	.877	19
20	.578	.826	.563	.835	.543	.849	.51j	.868	.479	.888	.455+	.900	20
21	.617•	.955+	.501	.865-	.581	.876	.551	.893	.516	.911	.492	.921	21
22	.654	.483	.540	.892	.620	.902	.590	.917	.555+	.932	.531	.941	22
23	.694	.9X1	.679	.918	.661	.927	.531	.9>9	.596	.952	.572	.959	23
24	.735-	.936	.721	.942	.702	.950	.673	.960	.639	.959	.615	.975-	24
25	.777	.960	.764	.965-	.745	.970	.719	.977	.684	.984	.650	.928	25
26	.321	.931	.809	.984	.792	.987	.765-	.991	.732	.9y -	.709	.996	26
27	.853	.996	.857	.997	.841	.99a	.917	.999	.785+		.763	1	27
2a	.921	1	.912	1	.399	1	.877	1	.848	1	.828	1	28
						n = 29							
0	0	.075	0	.085	0	.098	0	.119	0	.147	0	.167	0
1	.004	.128	.003	.139	.002	.153	.001	.178	0	.206	0	.230	1
2	.018	.173	.016	.165+	.012	.202	.008	.228	.005+	.260	.004	.262	2
3	.039	.215	.034	.229	.029	.246	.022	.274	.015+	>07	.012	.330	3
4	.062	.257	.056	.270	.049	.288	.039	.317	.030	.350	.024	.374	4
5	.086	.297	.079	.310	.070	.329	.058	.35a	.046	.392	. 0 3	.416	5
6	.112	.335-	.1OII	.349	.094	.369	.080	.397	.065•	.432	.056	.455+	6
7	.140	.372	.131	.397	.119	.406	.103	.435+	.086	.470	.076	.493	7
8	.158	.409	.15a	.424	.145+	.443	.127	.472	.108	.507	.096	.530	8
9	.197	.445+	.185	.460	.172	.479	.153	.50a	.132	.542	.119	.565+	9
10	.226	.481	.215-	.495+	.200	.514	.1"/9	.543	.157	.577	.142	.599	10
11	.257	.515+	.21bs -	.530	.229	.549	.207	.s77	.182	.610	.167	.6 ?	11
12	.288	.550	.245-	.564	.259	.583	.235+	.611	.209	.643	.192	.654	12
13	.319	.583	.304	.597	.2a9	.616	.264	.643	.237	.671	.219	.695-	13
14	.351	.616	.338	.630	.320	.648	.2911	.675	.266	.705-	.247	.724	14
15	.384	.649	.370	.662	.3S2	.680	.325+	.7c5-	.2%+	.734	.276	.753	15
16	.417	.681	.403	.694	.384	.711	.3s7	.736	.326	.763	.305+	.731	16
17	.450	.712	.436	.725•	.417	.741	.339	.765-	.357	.791	.336	.808	17
18	.485-	.743	.470	.75s•	.451	.771	.423	.793	.390	.818	.368	.833	18
19	.519	.774	.505-	.785•	.486	.900	.457	.821	.423	.843	.401	.858	19
20	.555-	.803	.540	.814	.521	.828	.492	.847	.458	.868	.IA35-	.881	20

TABLE 4. CONFIDENCE LIMITS FOR A PROPORTION (CONT.)

s	80% INTERVAL		85% INTERVAL		90% INTERVAL		95% INTERVAL		98% INTERVAL		99% INTERVAL		s
	90% Lwr. Limit	90% Upr. Limit	92.5% Lwr. Limit	92.5% Upr. Limit	95% Lwr. Limit	95% Upr. Limit	97.5% Lwr. Limit	97.5% Upr. Limit	99% Lwr. Limit	99% Upr. Limit	99.5% Lwr. Limit	99.5% Upr. Limit	
							n = 29						
21	.591	.832	.576	.842	.5s7	.855-	.528	.873	.493	.892	.470	.904	21
22	.628	.860	.613	.059	.594	.881	.565-	.897	.530	.914	.507	.924	22
23	.665+	.888	.651	.896	.632	.906	.603	.920	.568	.935-	.545-	.944	23
24	.703	.914	.690	.921	.671	.930	.642	.942	.608	.954	.584	.961	24
25	.743	.938	.730	.944	.712	.951	.683	.961	.650	.970	.626	.976	25
26	.784	.961	.771	.966	.754	. 9 n	.726	.970	.693	.9e5-	.570	.988	26
27	.827	.902	.815-	.984	.798	.988	.772	.992	.740	.995-	.718	.996	27
28	.872	.996	.861	.997	.847	.998	.822	.999	.792	1	.770	1	28
29	.924	1	.915-	1	.902	1	.861	1	.853	1	.833	1	29
							n = 30						
0	0	.074	0	.083	0	.095+	0	.116	0	.1142	0	.162	0
1	.004	[illegible]	.003	.134	.002	.1149	[illegible]	.172	0	.202	0	.223	1
2	.018	.168	.015+	.160	.012	.195+	[illegible]	.221	.005+	.252	.004	.274	2
3	.037	.209	.033	.222	.028	.239	.021	.265+	.015-	.298	.012	.320	3
4	.059	.249	.054	.262	.047	.280	.038	.307	.028	.340	.023	.363	4
5	.083	.287	.076	.301	.068	.319	.056	.347	.045-	.381	.038	.404	5
6	.109	.325-	.101	.339	.091	.357	.077	.386	.063	.420	.054	.443	6
7	.135-	.361	.126	.375+	.115-	.394	.099	.423	.083	.457	.073	.480	7
8	.162	.397	.152	.411	.140	.430	.123	.459	.104	.493	.093	.516	8
9	.190	.432	.179	.446	.166	.465+	.147	.494	.127	.527	.114	.550	9
10	.228	.466	.207	.481	.193	.499	.173	.528	.151	.551	.137	.533	10
11	.248	.500	.235	.514	.221	.533	.199	.561	.176	.594	[illegible]	.616	11
12	.277	.533	.265+	.548	.250	.566	.227	.594	.201	.626	.185+	.647	12
13	.308	.566	.295-	.580	.279	.598	.2s5-	.626	.228	.657	.211	.677	13
14	.333	.599	.325+	.612	.308	.630	.283	.657	.256	.587	.237	.707	14
15	.370	.630	.356	.644	.339	.661	.313	.687	.284	.716	.265-	.735+	15
16	.401	.662	.388	.675-	.370	.692-	.343	.717	.313	.744	.293	.763	16
17	.434	.692	.420	.705+	.402	.721	.374	.745	.343	.772	.323	.769	17
18	.467	.723	.452	.735-	.434	.750	.406	.773	.374	.799	.353	.a15-	18
19	.500	.752	.486	.764	.467	.779	.439	.801	.406	.824	.384	.340	19
20	.53&	.782	.519	.793	.501	.807	.472	.827	.439	.849	.417	.863	20
21	.568	.8s0	.554	.821	.535-	.834	.506	.853	.473	.873	.450	.886	21
22	.603	.838	.s89	.848	.570	.860	.541	.87?	.507	.896	.484	.907	22
23	.539	.865+	.625-	.874	.606	.885+	.577	.901	.543	.917	.520	.927	23
24	.675+	.891	.661	.899	.643	.909	.614	.923	.580	.937	.557	.946	24
25	.713	.91?	.699	.924	.681	.932	.6.53	.944	.619	.955+	.596	.962	25
25.	.751	.941	.738	.946	.720	.953	.693	.962	.660	.972	.637	.977	26
27	.791	.963	.778	.967	.761	.972	.735-	.979	.702	.985*	.680	.988	27
28	.832	.982	.820	.985-	.805-	.988	.779	.992	.748	. y 5 -	.726	.996	28
29	.876	.996	.865	.997	.851	.998	.828	.999	.793		.777	1	29
30	.926	1	.917	1	.905-	1	.884	1	.858	1	.833	1	30

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TABLE 5

CHI SQUARE TABLE

The body of the table contains values of

$$\chi^2_{\Delta,\nu}.$$

The table is indexed horizontally by Δ **and vertically by even values for v. The parameter v is a function of the number of failures** (r) **and the parameter A is a function of the risk (a) .**

Example. The number of failures (r) is 13 and α **is 0.05. find**

$$\chi^2_{1-\alpha,2r+2} \text{ ''}$$

Since ν = **2r + 2 and r = 13, v = 28. The parameter A** is 1 - α **or 0.95. Using the table directly, we obtain that**

$$\chi^2_{0.95,28} = 16.92 \ .$$

For degrees of freedom greater than 100 use

$$\chi^2_{\nu,\Delta} = 0.5(Z_A + \sqrt{2\nu-1})^2 \ .$$

Values of z_Δ **are given in Table 2.**

TABLE 5. CHI SQUARE TABLE

← Δ →

v	.995	.990	.980	.975	.950	.900	.800	.750	.500	.250	.200	.100	.050	.025	.020	.010	.005
2	.01	.02	[illegible]	.05	.10	.21	,45	,5P	1.39	Z*77	3,2Z	4.60	5.'79	7.38	7,82	9.22	10.59
4	,21	.30	.43	.48	.71	1.06	1.65	1.92	3.3b	5.39	5.99	7.78	9.49	11.15	11.66	13.28	14.82
6	.67	.67	1.13	1.24	1.03	Z.20	3.07	3.45	5.35	7.04	8.56	10,65	12.60	14.46	15003	16.01	18*55
8	1.34	1.64	2.03	2.18	2.73	3.49	4.59	5,07	7.34	10.2Z	11.03	13.36	15.51	17.55	18,17	20.06	21,94
10	2.15	2.55	3.06	3.24	3.94	4.86	6.18	6,74	9.34	12.55	13.44	15.99	18.31	20.50	21.17	23.19	25,15
12	3.06	3.57	4.10	4.40	5.22	6.30	7.81	8.44	11.34	14.85	15,81	18.55	Z1,03	Z3.35	24.06	26.25	28.25
14	4.07	4.65	5.36	5.62	6.57	7.79	9.47	10.16	13,34	17.12	18.15	21.07	23.69	26.13	26.88	29.17	31.38
1b	5.14	5.81	6.61	6.90	7.96	9.31	11.15	11.91	15.34	19.3	20.47	23.55	26.30	28.86	29.64	32.03	34.32
18	6.25	7.00	7.90	8.23	9.39	10.86	12.06	13.68	17,34	21.61	22.76	25.99	28.88	31.54	32.35	34.83	37.21
20	7.42	8.25	9.23	9.59	10.85	12.44	14.58	15,45	19.34	23,83	25.04	28.42	31*4Z	34.18	35.03	37.59	40.05
22	8.62	9.53	10.59	10,98	12.34	14.04	16.31	17.24	21.34	26.04	27.30	30.82	33.93	36.79	37,67	40.31	42.84
24	9.87	10.85	11.99	12.40	13*O4	15.66	18.06	19.04	23.34	28.24	29.56	33*ZO	36.42	39.38	40.28	43,00	45.60
26	11*13	12.19	13,40	13.84	15.38	17.29	19.82	20.84	25.34	30.44	31,80	35.57	38.89	41.94	42.86	45.66	48.33
28	12.44	13.55	14.84	15.30	16.92	18.94	21.59	22.66	27.34	32.6Z	34.03	37*9Z	41.34	44.47	45.43	48.30	51.04
30	13.77	14.94	1b, 30	16.77'	18.49	20.60	23.36	24.48	29,34	34,80	36.25	40.26	43.78	46.99	47.97	50.91	53.71
32	15.10	16.35	17.70	18.29	20.07	22.27	25.15	26, 30	31.34	36.9	38.47	42.59	46,20	49.50	50.49	53,51	56.37
34	16.48	17.78	19.27	19.$0	21.66	23.95	26.94	28.13	33.34	39.14	40.68	44.91	48.61	51.98	53*00	56,08	59.00
36	17.86	19.21	20,78	21.33	23.26	25.64	28.73	29,97	35.34	41*30	42.88	47.22	51,00	54.45	55.50	58.64	61.62
38	19.26	20.68	22,29	22.87	24.88	27.34	30.54	31.81	37*34	43.46	45.08	49.52	53.39	56.91	57.98	61.18	64.22
40	20.67	22.14	23.82	24.42	26,51	29.06	3.?,35	33,67	39*34	45.61	47.26	51.00	55.75	59,34	60,44	63,71	66.80
42	Z2.10	23.63	25.37	25.99	28, 14	30.77	34.16	35. 5Z	41,34	47.76	49.45	54.08	58.12	61.78	62.90	66,23	69.37
44	23.55	25.12	26.93	27.56	29.79	32.49	35,98	37.37	43,34	49,91	51.63	56.36	60.48	64.20	65.34	68.73	71.93
46	25.01	26.63	28.49	29.15	31.44	34.22	37,80	39.23	45.34	52.05	53,81	58.63	62.83	66.62	67.78	71.22	74.47
48	26.48	28.15	30.07	30,75	33.10	35*95	39.63	41.09	47*34	54.19	55.99	60.90	65.17	69.03	70.20	73.70	77.00
50	2?,96	29.68	31.65	3.?,35	34, 76	37.69	41.46	42.95	49*34	56.33	58.16	63.16	67s50	71.42	72.62	76.17	79,52
52	29.45	31.22	33..?4	33.96	3h. 44	39.44	43*Z9	44.s1	51.34	58.46	60.32	65.41	69.83	73,81	75.03	78,63	82.03
54	30.95	32.77	34.05	35.58	38.12	41.19	45.12	46,68	53*34	60,59	62,49	67,67	72.15	76,20	77.43	81.09	84.53
56	32.46	34*33	36.45	37.20	39.80	42.94	46,96	48.55	55,34	62.72	64,65	69.91	74.46	78,57	79.82	83,53	87.03
58	33,98	35.89	38.07	38.84	41.49	44.70	48.80	50. 42	57*34	64.85	66.01	72,15	76.77	80.94	82.21	05.97	89.51
60	35.50	37,46	39.69	40.47	43, 19	46.46	50.65	52,30	59.34	66, 98	68,97	74.39	79.08	83.30	84.59	88.40	91.98
62	37.04	3?.04	41.32	42.12	44.89	48.23	52.49	54.18	61.34	69,10	71,12	76.62	01.38	85,66	86.96	90.02	94.45
64	38.58	40.63	42.95	43.77	46.59	50.00	54,34	56, 06	63.34	71.22	73,27	78.05	83.67	88.01	89.33	93.23	96.91
66	40.13	42.22	44.59	45.42	48.30	51.77	56.19	57.94	65.34	73.34	75.42	81008	85.96	90,35	91.69	95.64	99.36
68	41,68	43.62	46,23	47.08	50.02	53.55	58.05	59. 82	67.34	75,46	77.56	83.30	88.25	92.69	94.04	98.04	101.80
70	43,.?5	45.42	47.80	48.75	51.74	55.33	59.90	61, 70	69.34	77.57	79.71	85.52	90.53	95.03	96.39	100.44	104.24
72	44.81	47.03	49.54	50, 42	53.46	57*1Z	61,76	63,59	71*34	79.69	81.85	87,74	92.81	97,36	98.74	102.83	106.68
74	46.39	48.65	51.20	52.10	55.19	58.90	63.62	(>5.48	73*34	81.80	03.99	89.95	95.08	99.68	101.08	105.22	109.10
76	47.97	50.27	52.86	53.78	56.92	60.69	65.48	67, 37	75,34	83.91	06.13	92.16	97.35	102.00	103.42	107.60	111.52
70	49.55	51.69	54.53	55.46	58.65	62.49	67.35	69.26	77.34	86.02	08.27	94.37	99,61	104.32	105.75	109*97	113.94
80	51.14	53.52	56.20	57.15	60.39	64.28	69.21	71*15	79.34	88.13	90.40	96.57	101.88	106,63	108.07	112.34	116,35
82	52.74	55.16	57.88	58.84	62.13	66.08	71.08	73.04	81.34	90.23	92.53	98,77	104.14	108,94	110.40	114071	118,75
84	54.34	56.80	59.56	60,53	63.88	67.88	72.?5	74.94	83034	92.34	94 .66	100.97	106,39	111,25	112.72	117.07	121.15
86	55.95	58.44	61.25	62.23	65.62	69.68	74.62	76.83	85,34	94,44	96,79	103,17	108.65	113.55	115.03	119.43	123.55
88	57.56	60.69	6? .93	63.93	67.37	71.49	76.69	78,73	87,34	96,54	98.92	105.37	110.90	115.84	117.35	121.78	125.94
90	59*17	61.74	64.63	65.64	69.13	73.29	78.56	00.63	89.33	98.65	101.05	107.56	113.14	118.14	119.65	124.13	128.32
92	60.79	63,39	66.32	67.35	70.88	75.10	90.44	82.53	91.33	100.75	103.17	109*75	115.39	120.43	121,96	126.48	130.71
94	62.41	65.05	68.02	69.06	72.64	76.92	02.31	84.43	93,33	102,85	105.30	111094	117.63	122.72	124.26	128.82	133.00
96	64.04	66.71	69.72	70.70	74.40	70,73	04.1'3	86.33	95.33	104.94	107.42	114.13	119.87	125,00	126.56	131,15	135.46
98	65.67	68.38	71.43	72.50	76, 16	80.54	86.07	88.23	97.33	107.04	109.54	116.31	122.11	127.28	128.85	133.49	137.83
100	67.30	70,05	73.13	74.22	77.93	82.36	87.95	90. 14	9'?,33	109.14	111.66	11P*49	124.34	129.56	131.15	135.62	140.19

TABLE 6

EXPONENTIAL TEST PLANS FOR STANDARD DISCRIMINATION RATIOS

Test Plan	True Decision Risks		Discrimination Ratio θ_0/θ_1	Test Duration Multiplier (M) $T = M\theta_1$	Accept-Reject Failures	
					Reject (Equal or More)	Accept (Equal or Less)
	α	β				
IXC	12.0%	9.9%	1.5	45.0	37	36
XC	10.9%	21.4%	1.5	29.9	26	25
XIC	17.8%	22.1%	1.5	21.1	18	17
XIIC	9.6%	10.6%	2.0	18.8	14	13
XIIIC	9.8%	20.9%	2.0	12.4	10	9
XIVC	19.9%	21.0%	2.0	7.8	6	5
XVC	9.4%	9.9%	3.0	9.3	6	5
XVIC	10.9%	21.3%	3.0	5.4	4	3
XVIIC	17.5%	19.7%	3.0	4.3	3	2
XIXC	28.8%	31.3%	1.5	8.0	7	6
XXC	28.8%	28.5%	2.0	3.7	3	2
XXIC	30.7%	33.3%	3.0	1.1	1	0

TABLE 7

SUPPLEMENTAL EXPONENTIAL TEST PLANS

Test Plan No's	No. Failures Acc.	No. Failures Rej.	Test Duration Multiplier (Ii) $T = M\theta_1$	Discrimination Ratio θ_0/θ_1 For Producer's Risk 30%	20%	10%
			10 Percent Consumer's Risk (β) Test Plans			
10-1	0	1	2.30	6.46	10.32	21.85
10-2	1	2	3.89	3.54	4.72	7.32
10-3	2	3	5.32	2.78	3.47	4.83
10-4	3	4	6.68	2.42	2.91	3.83
10-5	4	5	7.99	2.20	2.59	3.29
10-6	5	6	9.27	2.05	2.38	2.94
10-7	6	7	10.53	1.95	2.22	2.70
10-8	7	8	11.77	1.86	2.11	2.53
10-9	8	9	12.99	1.80	2.02	2.39
10-10	9	10	14.21	1.75	1.95	2.28
10-11	10	11	15.41	1.70	1.89	2.19
10-12	11	12	16,60	1.66	1.84	2.12
10-13	12	13	17.78	1.63	1.79	2.06
10-14	13	14	18.96	1.60	1.75	2.00
10-15	14	15	20.13	1.58	1.72	1.95
10-16	15	16	21.29	1.56	1.69	1.91
10-17	16	17	22.45	1.54	1.67	1.87
10-18	17	18	23.61	1.52	1.62	1.84
10-19	18	19	24.75	1.50	1.62	1.81
10-20	19	20	25.90	1.48	1.60	1.78
			20 Percent Consumer's Risk (β) Test Plans			
20-1	0	1	1.61	4.51	7.22	15.26
20-2	1	2	2.99	2.73	3.63	5.63
20-3	2	3	4.28	2.24	2.79	3.88
20-6	3	4	5.51	1.99	2.40	3.16
20-5	4	5	6.72	1.85	2.17	2.76
20-6	5	6	7.91	1.75	2.03	2.51
20-7	6	?	9.07	1.68	1.92	2.33
20-8	7	8	10.23	1.62	1.83	2.20
20-9	8	9	11.38	1.57	1.77	2.09
20-10	9	10	12,52	1.54	1.72	2.01
20-11	10	11	13.65	1.51	1.67	1.94
20-12	11	12	14.78	1.48	1.64	1.89
20-13	12	13	15.90	1.46	1.60	1.84
20-14	13	14	17.01	1.44	1.58	1.80
20-15	14	15	18.12	1.42	1.55	1.76
20-16	15	16	19.23	1.40	1.53	1.73
20-17	16	17	20.34	1.39	1.51	1.70
20-18	17	18	21.44	1.38	1.49	1.67
20-19	18	19	22.54	1.37	1.48	1.65
20-20	19	20	23.63	1.35	1.46	1.63
			30 Percent Consumer's Risk (β) Test Plans			
30-1	0	1	1.20	3.37	5.39	11.43
30-2	1	2	2.44	2.22	2.96	4.59
30-3	2	3	3.62	1.89	2.35	3.28
30-4	3	4	4.76	1.72	2.07	2.73
30-5	4	5	5.89	1.62	1.91	2.43
30-6	5	6	7.00	1.55	1.79	2.22
30-7	6	7	8.11	1.50	1.71	2.08
30-8	7	8	9.21	1.46	1.65	1.98
30-9	8	9	10.30	1.43	1.60	1.90
30-10	9	10	11.39	1.40	1.56	1.83
30-11	10	11	12.47	1.38	1.53	1.78
30-12	11	12	13.55	1.36	1.50	1.73
30-13	12	13	14.62	1.34	1.48	1.69
30-14	13	14	15.69	1.33	1.45	1.66
30-15	14	15	16.76	1.31	1.43	1.63
30-16	15	16	17.83	1.30	1.42	1.60
30-17	16	17	18.90	1.29	1.40	1.58
30-18	17	18	19.96	1.28	1.39	1.56
30-19	18	19	21.02	1.27	1.38	1.54
30-20	19	20	22.08	1.27	1.36	1.52

NOTE: The "Acceptable Observed MTBF" column found in MIL-STD-781C has been deleted because a small but critical rounding error makes them inconsistent with the accept/ reject criteria in the "No. Failures" column.

TABLE 8

EXPONENTIAL CONFIDENCE LIMIT MULTIPLIERS FOR MTBF

Table 8 contains multipliers used to calculate confidence intervals and limits for MTBF. The table entries are based on an exponential model of system failures. Table 8a applies to time terminated tests and Table 8b applies to failure terminated tests.

Example

Given the following test information:

$T = 1000$ hours (time terminated)

$r = 4$ failures

$$\hat{\theta} = \frac{1000}{4} = 250 \text{ MTBF}$$

Find: 90% upper and lower confidence limits and 80% confidence interval for MTBF.

Note: Test is time terminated; therefore, use Table 8a.

Multiplier for 90% upper limit = 2.293

Multiplier for 90% lower limit = 0.500

Upper Limit = θ_U = (2.293)(250) = 573.25 MTBF

Lower Limit = θ_L = (0.500)(250) = 125.00 MTBF

Consequently, an 80% confidence interval is

$$\theta_L \leq \theta \leq \theta_U$$

$$125.0 \leq \theta \leq 573.25 \text{ MTBF}$$

TABLE 8a

EXPONENTIAL CONFIDENCE LIMIT MULTIPLIERS FOR MTBF

(Time Terminated Testing)

Total No. of Failures	40% Interval		60% Interval		80% Interval	
	70% Lower Limit	70% Upper Limit	80% Lower Limit	80% Upper Limit	90% Lower Limit	90% Upper Limit
1	0.410	2.804	0.334	4.481	0.257	9.491
2	0.553	1.823	0.467	2.426	0.376	3.761
3	0.630	1.568	0.544	1.954	0.449	2.722
4	0.679	1.447	0.595	1.742	0.500	2.293
5	0.714	1.376	0.632	1.618	0.539	2.055
6	0.740	1.328	0.661	1.537	0.570	1.904
7	0.760	1.294	0.684	1.479	0.595	1.797
8	0.777	1.267	0.703	1.435	0.616	1.718
9	0.790	1.247	0.719	1.400	0.634	1.657
10	0.802	1..230	0.733	1.372	0.649	1.607
11	0.812	1.215	0.744	1.349	0.663	1.567
12	0.821	1.203	0.755	1.329	0.675	1.533
13	0.828	1.193	0.764	1.312	0.686	1.504
14	0.835	1.184	0.772	1.297	0.696	1.478
15	0.841	1.176	0.780	1.284	0.705	1.456
16	0.847	1.169	0.787	1.272	0.713	1.437
17	0.852	1.163	0.793	1.262	0.720	1.419
18	0.856	1.157	0.799	1.253	0.727	1.404
19	0.861	1.152	0.804	1.244	0.734	1.390
20	0.864	1.147	0.809	1.237	0.740	1.377
30	0.891	1.115	0.844	1.185	0.783	1.291

TABLE 8b

EXPONENTIAL CONFIDENCE LIMIT MULTIPLIERS FOR MTBF

(Failure Terminated Testing)

Total No. of Failures	40% Interval		60% Interval		80% Interval	
	70% Lower Limit	70'% Upper Limit	80% Lower Limit	80% Upper Limit	90% Lower Limit	90% Upper Limit
1	0.801	2.804	0.621	4.481	0.434	9.491
2	0.820	1.823	0.668	2.426	0.515	3.761
3	0.830	1.568	0.701	1.954	0.564	2.722
4	0.840	1.447	0.725	1.742	0.599	2.293
5	0.849	1.376	0.744	1.618	0.626	2.055
6	0.856	1.328	0.759	1.537	0.647	1.904
7	0.863	1.294	0.771	1.479	0.665	1.797
8	0.869	1.267	0.782	1.435	0.680	1.718
9	0.874	1.247	0.796	1.400	0.693	1.657
10	0.878	1.230	0.799	1.372	0.704	1.607
11	0.882	1.215	0.806	1.349	0.714	1.567
12	0.886	1.203	0.812	1.329	0.723	1.533
13	0.889	1.193	0.818	1.312	0.731	1.504
14	0.892	1.184	0.823	1.297	0.738	1.478
15	0.895	1.176	0.828	1.284	0.745	1.456
16	0.897	1.169	0.832	1.272	0.751	1.437
17	0.900	1.163	0.836	1.262	0.575	1.419
18	0.902	1.157	0.840	1.253	0.763	1.404
19	0.904	1.152	0.843	1.244	0.767	1.390
20	0.906	1.147	0.846	1.237	0.772	1.377
30	0.920	1.115	0.870	1.185	0.806	1.291

b

TABLE 9

CONFIDENCE LIMIT MULTIPLIERS FOR MTBF TIME TERMINATED RELIABILITY GROWTH TESTS*

The table gives lower and upper confidence limit multipliers which can also be used as multipliers for two-sided confidence intervals. The table is indexed horizontally by level of confidence and vertically by number of failures observed. To obtain the desired confidence limit, we take the product of the appropriate multipler and the point estimate of MTBF.

When N, the total number of failures, is large (> 100), we may use a normal approximation to obtain a multiplier. Specifically,

Lower Limit Multiplier for 100(1-a)% Confidence

$$= (1 + z_\alpha/\sqrt{2N})^{-2}.$$

Upper Limit Multiplier for 100(1-cY)% Confidence

$$= (1 - z_\alpha/\sqrt{2N})^{-2}.$$

Lower and upper limit multipliers for a $100(1-\alpha)\%$ confidence interval are, respectively

$$(1 + z_{\alpha/2}/\sqrt{2N})^{-2}$$

and

$$(1 - z_{\alpha/2}/\sqrt{2N})^{-2}$$

Values of 'a and $z_{\alpha/2}$ can be found in Appendix B, Table 2.

*Multipliers for failure terminated tests are not included in this text.

TABLE 9

CONFIDENCE LIMIT MULTIPLIERS FOR MTBF

TIME TERMINATED RELIABILITY GROWTH TEST*

Total No. f Failures	80% Interval		90% Interval		95% Interval		98% Interval	
	90% Lower Limit	90% Upper Limit	95% Lower Limit	95% Upper Limit	97.5% Lower Limit	97.5% Upper Limit	99% Lower Limit	99% Upper Limit
2	0.261	18.660	0.200	38.660	0.159	78.660	0.124	198.700
3	0.333	6.326	0.263	9.736	0.217	14.550	0.174	24.100
4	0.385	4.243	0.312	5.947	0.262	8.093	0.215	11.810
5	0.426	3.386	0.352	4.517	0.300	5.862	0.250	8.043
6	0.459	2.915	0.385	3.764	0.331	4.738	0.280	6.254
7	0.487	2.616	0.412	3.298	0.358	4.061	0.305	5.216
8	0.511	2.407	0.436	2.981	0.382	3.609	0.328	4.539
9	0.531	2.254	0.457	2.750	0.403	3.285	0.349	4.064
10	0.549	2.136	0.476	2.575	0.421	3.042	0.367	3.712
11	0.565	2.041	0.492	2.436	0.438	2.852	0.384	3.441
12	0.579	1.965	0.507	2.324	0.453	2.699	0.399	3.226
13	0.592	1.901	0.521	2.232	0.467	2.574	0.413	3.050
14	0.604	1.846	0.533	2.153	0.480	2.469	0.426	2.904
15	0.614	1.800	0.545	2.087	0.492	2.379	0.438	2.781
16	0.624	1.759	0.556	2.029	0.503	2.302	0.449	2.675
17	0.633	1.723	0.565	1.978	0.513	2.235	0.460	2.584
18	0.642	1.692	0.575	1.933	0.523	2.176	0.470	2.503
19	0.650	1.663	0.583	1.893	0.532	2.123	0.479	2.432
20	0.657	1.638	0.591	1.858	0.540	2.076	0.488	2.369
21	0.664	1.615	0.599	1.825	0.548	2.034	0.496	2.313
22	0.670	1.594	0.606	1.796	0.556	1.996	0.504	2.261
23	0.676	1.574	0.613	1.769	0.563	1.961	0.511	2.215
24	0.682	1.557	0.619	1.745	0.570	1.929	0.518	2.173
25	0.687	1.540	0.625	1.722	0.576	1.900	0.525	2.134
26	0.692	1.525	0.631	1.701	0.582	1.873	0.531	2.098
27	0.697	1.511	0.636	1.682	0.588	1.848	0.537	2.068
28	0.702	1.498	0.641	1.664	0.594	1.825	0.543	2.035
29	0.706	1.486	0.646	1.647	0.599	1.803	0.549	2.006
30	0.711	1.475	0.651	1.631	0.604	1.783	0.554	1.980
35	0.729	1.427	0.672	1.565	0.627	1.699	0.579	1.870
40	0.745	1.390	0.690	1.515	0.646	1.635	0.599	1.788
45	0.758	1.361	0.705	1.476	0.662	1.585	0.617	1.723
50	0.769	1.337	0.718	1.443	0.676	1.544	0.632	1.671
60	0.787	1.300	0.739	1.393	0.700	1.481	0.657	1.591
70	0.801	1.272	0.756	1.356	0.718	1.435	0.678	1.533
80	0.813	1.251	0.769	1.328	0.734	1.399	0.695	1.488
100	0.831	1.219	0.791	1.286	0.758	2.347	0.722	1.423

*Multipliers for failure terminated tests are not included in this text.

CHART 1. CUMULATIVE POISSON PROBABILITIES

USE OF CHART 1

General

The chart of cumulative Poisson probabilities, for given values of θ/T, gives the probability of c or fewer failures. The chart was constructed for convenience of use at the expense of precision. For more precise evaluations, use the Cumulative Poisson Table (Appendix B, Table 3) or equation 8.7 (Poisson Distribution Equation) .

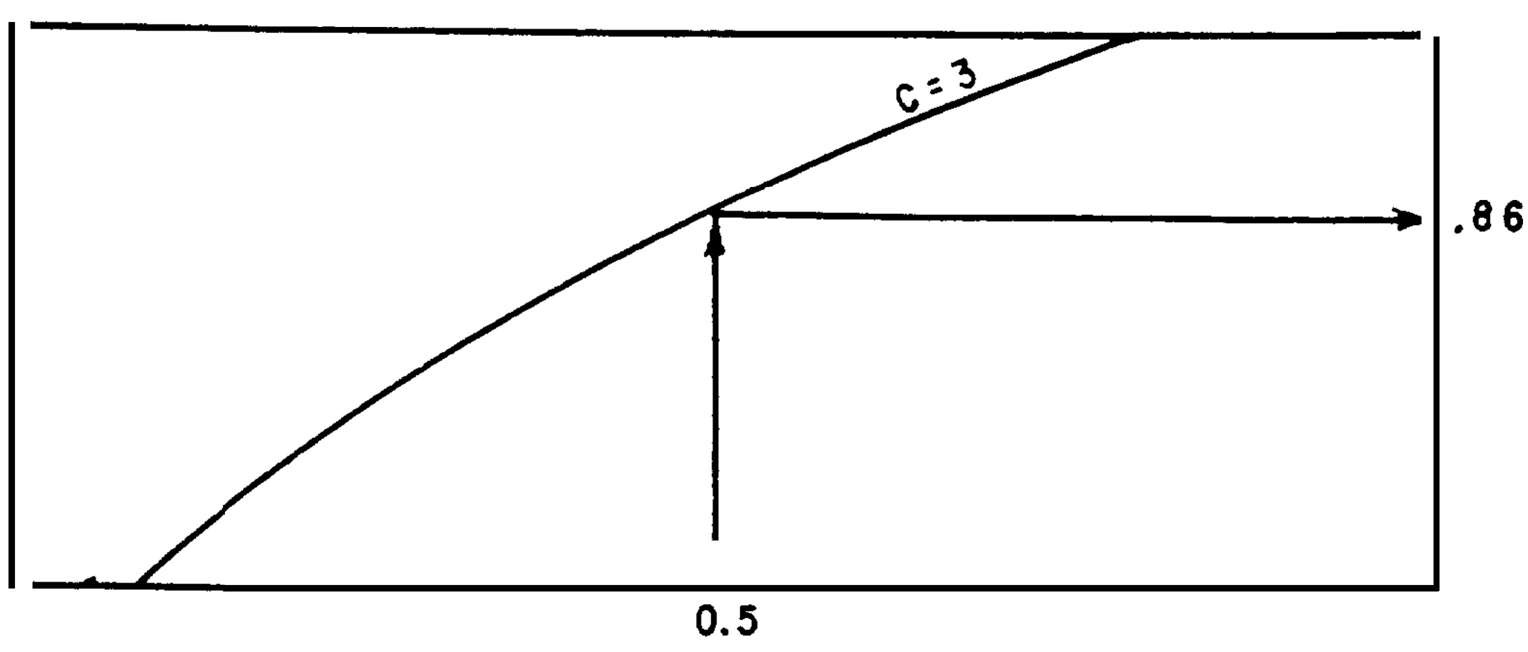
Probability of c or Fewer Failures

To find the probability of c or fewer failures , enter the chart with θ/T on the horizontal scale. Move vertically to the curve for c, then move horizontally to read the probability value on either vertical scale. This is the probability of c or fewer failures .

Example: The MTBF of a system is 100 hours . What is the probability of 3 or fewer failures in 200 hours of use?

$$e = 100$$
$$T = 200$$

$$\theta/T = 100/200 = 0.5$$
$$c = 3$$



The probability of 3 or fewer failures is approximately 0.86.

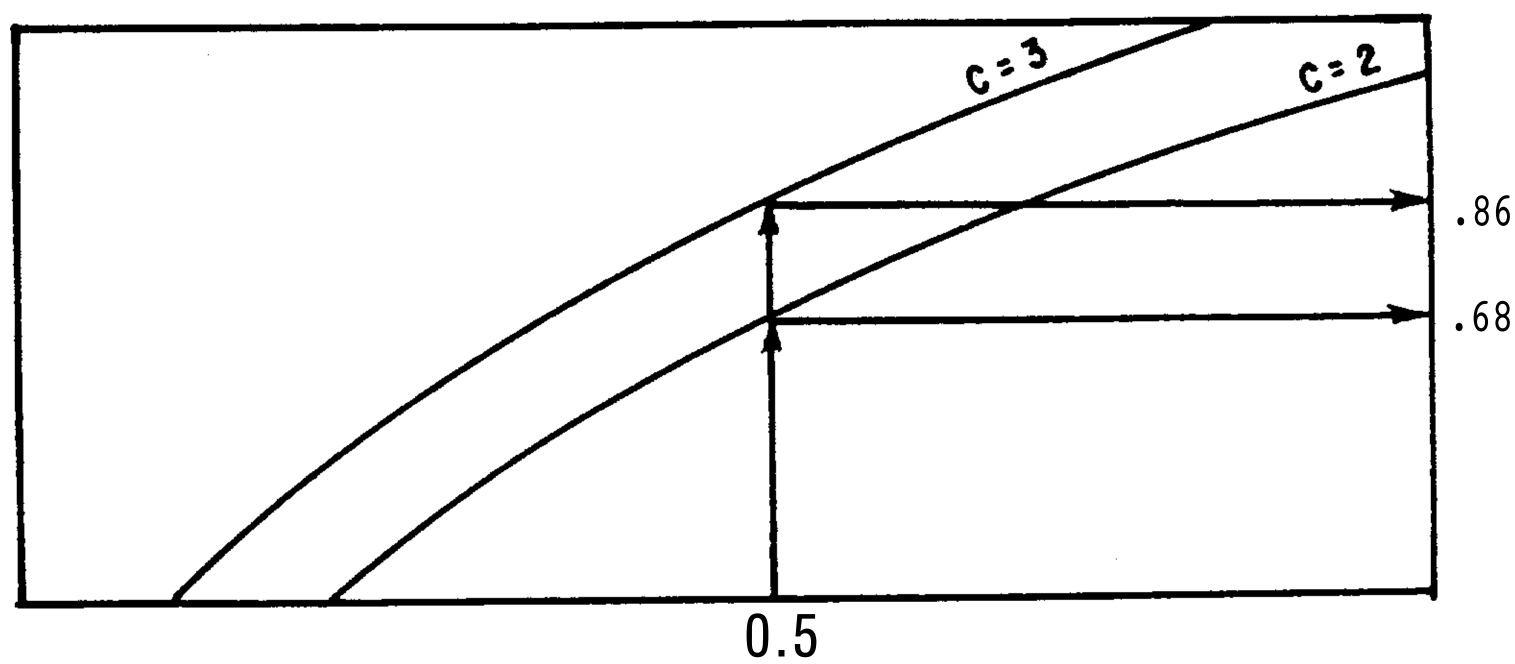
Probability of Exactly c Failures

To find the probability of exactly c failures, find the probability of c or fewer and the probability of c-1 or fewer. The probability of exactly c is the difference between these two probabilities.

Example: The MTBF of a system is 100 hours. What is the probability of exactly 3 failures in 200 hours of use?

$$\theta = 100$$
$$T = 200$$

$$\theta/T = 100/200 = 0.5$$
$$c = 3$$
$$c-1 = 2$$



The probability of exactly 3 failures is approximately 0.86 - 0.68 = 0. 18.

Test Exposure and Acceptance Criterion-Continuous Time Test

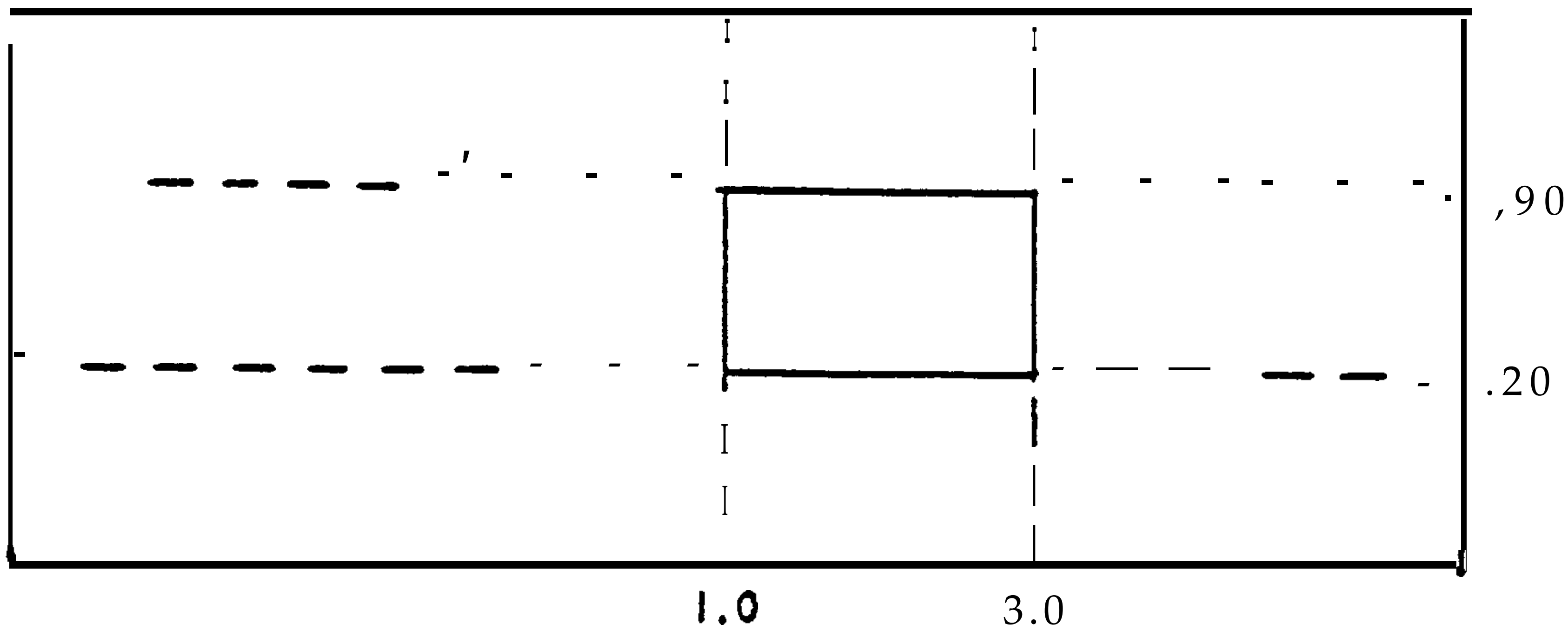
We wish to find a test exposure, T, and an acceptable number of failures, c, such that the probability of acceptance is β when $\theta = \theta_1$ and $1 - \alpha$ when $\theta = \theta_0$. This may be done graphically with the use of an overlay.

On an overlay sheet, draw vertical lines at $\theta/T = 1$ and $\theta/T = \theta_0/\theta_1$. Draw horizontal lines at probabilities β and $1 - \alpha$, forming a rectangle. Slide the overlay rectangle horizontally until a curve for a single value of c passes through the lower left and upper right corners. (It may not be possible to hit the corners exactly. Conservative values of c will have curves that pass through the horizontal lines of the rectangle.) This value of c is the acceptable number of failures. Read the value of θ/T corresponding to the left side of the rectangle. Divide θ_1 by this value to find T, the required test exposure.

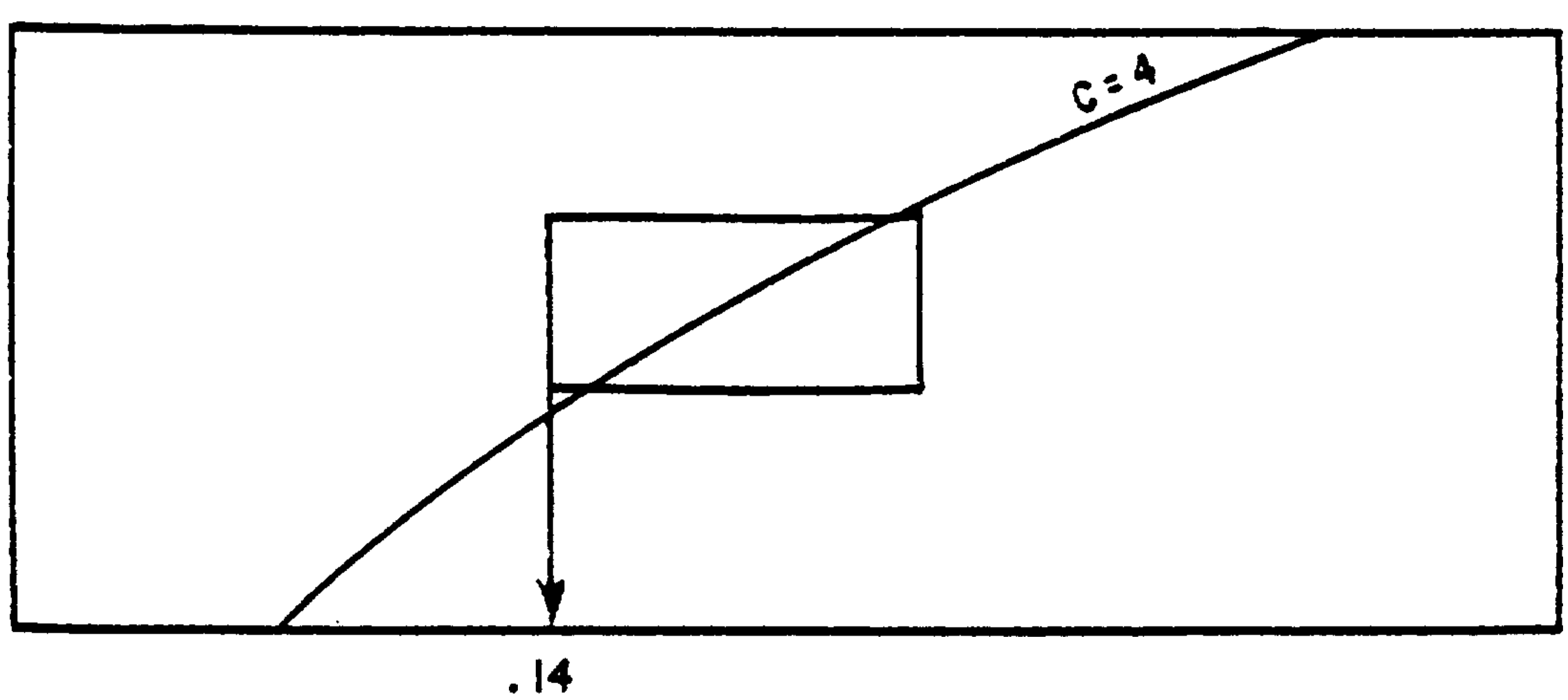
Example

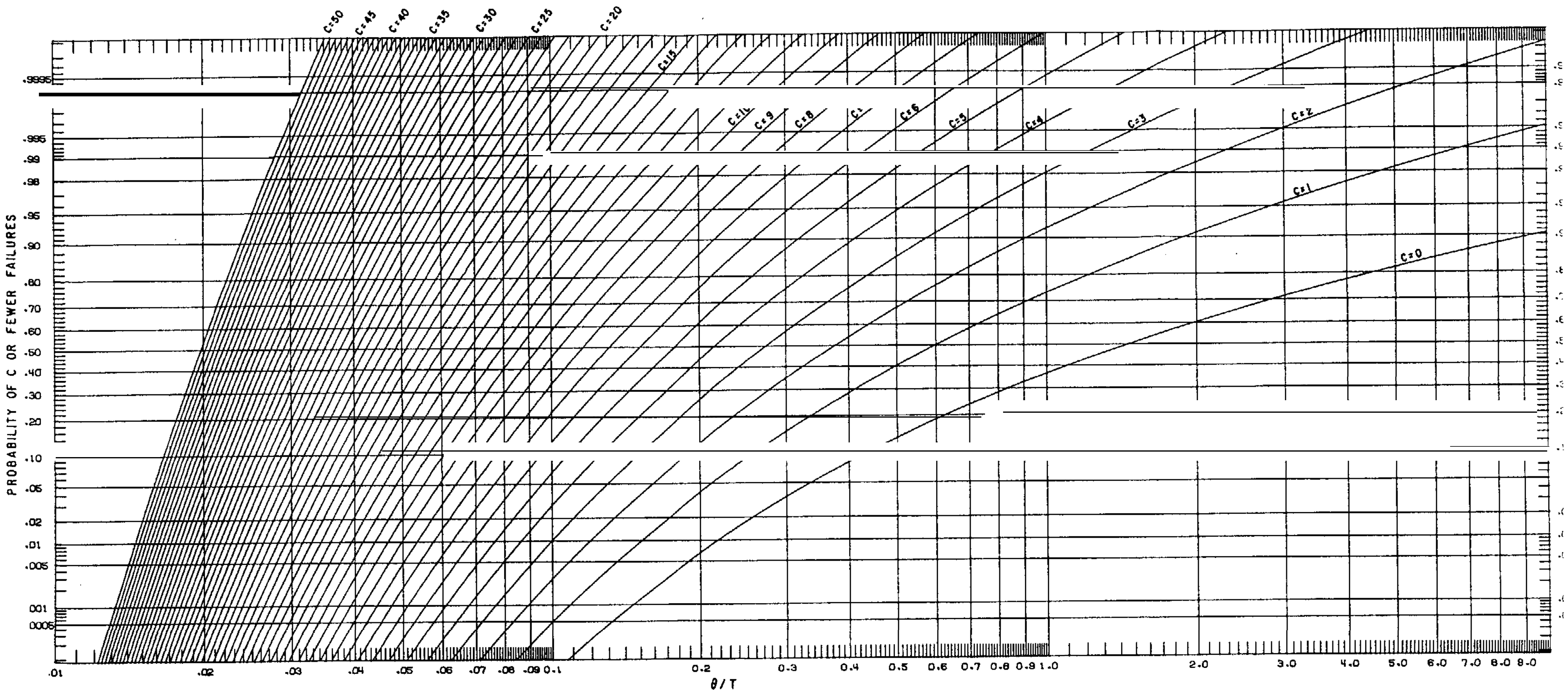
We wish to find the required test exposure, T, and acceptable number of failures c; such that when the MTBF, $\theta = \theta_1 = 100$ hours, the probability of acceptance, β, will be 0.20 and then $\theta = \theta_0 = 300$ hours the probability of acceptance, $1 - \alpha$, will be 0.90.

An overlay rectangle is constructed as shown.



Sliding the rectangle to the left, we find that when c = 3 the fit is close, but slightly higher risks must be tolerated. Going to c = 4, the curve passes through the horizontal lines of the rectangle. At the left of the rectangle, $\theta/T = 0.14$, so the required test exposure is approximately 100/0.14 = 714 hours and the acceptance criterion is 4 or fewer failures.




PROBABILITY OF C OR FEWER FAILURES
.9995
.995
.99
.98
.95
.90
.80
.70
.60
.50
.40
.30
.20
.10
.05
.02
.01
.005
.001
.0005
C=50
C=45
C=40
C=35
C=30
C=25
C=20
C=15
C=10
C=9
C=8
C=6
C=5
C=4
C=3
C=2
C=1
C=0
.01
.02
.03
.04
.05
.06
.07
.08
.09
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
θ/T

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