## ReSEARCH AND DEVELOPMENT OF MATERIEL <br> ENGINEERING DESIGN HȦNDBOOK carriages and mounts series CRADLES

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## PREFACE

This handbook on Cradles has been prepared as one of a series on Carriages and Mounts. It presents information on the fundamental operating principles and design of cradles.

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# CARRIAGES AND MOUNTS SERIES CRADLES* 

## I. INTRODUCTION

A. GENERAL

1. This is one of a series of handbooks on Carriages and Mounts. This handbook deals with the design of cradles.

## B. PURPOSE

2. The cradle was first introduced in Ordnance Corps Famphiet ORDP $20.340 \dagger$ where it was discussed as one of the elemenis that make up a carriage or mount. This handbook deals specifically with the cradle. The various types are discussed along with their compossents and pertinent design data.
[^0]
## C. FUNCTIONS

3. The cradle is one of the tipping parts and serves as the supporting structure for all other tipping parts. Its primary function is to suppori the gun tube. It provides the guides or tracks on which the tube slides during recoil and counterrecoil. It anchors the recoil mechanism. It prevents the tube from rotating. It transmits all firing loads, including those due to recoil, tube :rhip, and rifling torque, to the carriage. It provides the base for mounting sighting equipment. Figure 1 shows a typical cradle installation on a weapon.

## II. EQUIPMENT ASSOCIATED WITH CRADLES

## A. RECOIL MECHANISM

4. The fixed part of the recoil mechanism


Figure 1. Weapon Showing Typical Components
is attached to the cradle, and the movable prtions are attached to the recoiling parts. There are, in general, two basic arrangements for the recoil mechanism. Ore has the recoil cylinder and recuperator fixed to the cradle and the piston rod fixed to the gun lug or breechblock. The other arrangement fixes the recoil cylinder and recuperator to the gun and the rod to the cradle. In the former case, the cylinder and recuperator are either integral parts of the cradle or separate parts rigidly attached to it. It is well to have the recoil mechanism installed as near as possible to the tube, not only for compactness but also for lower bending moments on the cradle which are axiomatic to lower stresses and, therefore, lighter structure.

## B. TRUNNIONS

5. The trunnions are considered to be components of the cradle whether the trunnion bearings are located on the side frames or in the cradle itself. The trunnions, through which the firing loads are transmitted, are the main attachment to the top carriage and also serve as the pivot about which the tipping parts rotate during elevation. In the plan view, their axis should lie normal to the direction of recoil. In the side view, the trunnion axis should be located on or near the line parallel to the bore and passing through the center of gravity of the recoiling parts. This reduces tipping moments during firing and relieves the elevating arc of large loads.

## C. Elevating mechanism

6. The elevating mechanism terminates a' the elevating arc which is a gear segment rigidly attached to the cradle. It is here that the torque required to elevate is applied to the tipping parts. The pitch radius of the alevating arc is centered at the trunnions and should be as large as possible and still remain compatible with the size of the rest of the structure. A large radius results in small gear tooth loads and less effort to elevate the gun. Also, if the arc is large, the attachments to the cradle can be located farther apart and, although the torque transmitted to the tipping parts remains unaffected, the corresponding loads at the attachment points are decreased.

## D. EQUILIBRATOR

7. One end of the equilibrator is attached to the top carriage and the other to the cradle. A large turning radius about the trunnion is desirable for the equilibrator as it lowers the forces. Hence, a more efficient design results. The attachment on the cradle may be at any convenient location on the structure or on the elevating arc, provided that clearances and strength requirements are met. Equilibrator design is discussed in Ordnance Corps Pamphlet ORDP 20-345.*

## III. TYPES OF CRADLE

8. There are two basic types of cradle, designated according to the general form of cross section as the U-type and the O-type. Each has its own method of seating the gun tube. The U-type seats the tube on top and retains it by guides. The O-type holds the tube in a hollow cylinder whose inner wall conforms to the mating portion of the tube.

## A. THE U-TYPE CRADLE

9. The degree of resemblance between the U-type cradle and the letter $U$ depends on constructional features. If the recoil cylinder and recuperator are attached to the gun tube so that they become part of the recoiling system (Figure 2), the cradle may be approximately U-shaped, with provisions for accommodating rails and trunnions. If the recoil cylinder and recuperator are integral with the cradle (Figure 3), the resemblance to a Usection is lost. However, the term, U-type, still applies to indicate the general construction. For simplicity, the recuperators are omitted from Figures 2 and 3.
10. If the recoil cylinder and recuperator are attached to and recoil with the gun tube, the structure which supports them is called a sleigh. The sleigh carries the rails and thus supports the tube in the cradle. It may be a forging or a weldment. If a forging, the cylinders of the recoil mechanism are bored directly into it.

[^1]

Figure 2. U-Type Cradle With Attached Recoil Mechanism


Figure 3. U-Type Cradie With Integral Recoil Mechanism
11. The sliding surfaces of the recoiling parts are called rails. Either the rails or their supporting guides may be channel-shaped to prevent them from separating due to the upsetting moments and rifing torques. Either, but not both, may be discontinuous, that is, made of several shorter lengths spaced at convenient distances. Rails may ive attached to the sleigh. Whichever is used, the simnlo rail assembly or the sleigh, although not constituent parts, each should be treated as a component of the cradle.
12. The U-type cradle has several advantages. With a sleigh, the added weight of the recoiling parts reduces either recoil force or length of recoil. Ordinarily, the height of the weapon is decreased (lower silhoustte) by having the recoil mechanism below the gun tube. The design of the gun tube is not influenced to any great extent by the fixtures that hold it in the cradle. Since the gun tube does not form the sliding surfaces for recoil, its contour, and hence wall thickness, need only conform to the gas pressure distribution along its length.
13. There are several disadvantages associated with the U-type cradle. Some are discussed below.
a. Fabrication is difficult. The structure is complex and a high degree of accuracy is required in machining the slides and rails to
the proper alignment and fit Production costs are high.
b. If clearances are not sufficient for an underslung recoil mechanism, the trunnion height must be increased, with an accompanying increase in overturning moment and a higher silhouette.
c. It is difficult to arrange the ideal loading pattern with the resultant of the recoil forces passing through the centerline of the trunnions. This arrangement is always attempted in order to minimize the elevating gear loads during recoil.
d. During extended firing, heat transmitted from gun tubes to rails may cause warpage and eventual binding.
e. Misalignment may occur in discontinuous rails or slides causing them to bind during recoil and counterrecoil. Binding of this nature may prevent the gun tubes from returning to the in-battery position.

## B. THE O-TYPE CRADLE

14. This type has a cylindrical tube for its basic structural element (Figure 4). Each end contains suitable bearings in which the gun

(a) EXTERNAL RECOIL MECHANISM TYPE

(b) CONCENTRIC RECOIL MECHANISM TYPE

Figure 4. O-Type Cradles
tube slides. The outside surface of the tube is cylindrical for a considerable length forward of the breech. : This surface is machined smooth and the tube itself serves as its own slide, the bearings functioning as guides during recoil and counterrecoil. A key transmits the rifling torque to the cradle to prevent rotation of the tube. Brackets or some similar structure are provided on the cylindrical portion of the sradle to attach the recoil mechanism, the trunnions, and the elevating arc.
15. The use of an O-type cradle offers several advantages. It is convenient to locate the trunnions on the line of action of the recoiling parts to relieve the elevating gear of firing loads. The structure is comparatively light which helps to increase mobility. Use of the O-type cradle does not require the sliding surfaces to be attached to the gun tube, thus eliminating this fabrication problem. The cylindrical surfaces reduce machining problems and provide more accurate alignment. A. choice of a favorable location is available for the recoil mechanism. When the recoil mechanism is on the top of the tube, it does not present clearance problems while the tipping parts are being elevated.
16. There are several disadvantages inherent to the O-type cradle. The sliding surface of the gun tube is exposed to the weather, although this can be eliminated by the installation of a shield. It dictates, to some extent, the diameter of the tube forward of the chamber because it cannot be tapered along the sliding surface. If the forward portion of the sliding surface is made smaller in diameter, then two sleeve bearings of different diameters are necessary. The effects of heating the gun tube can be serious if the expansion exceeds the clearances in the bearings. The clearances which must be provided to avoid binding may result in sloppy fits while the gun is cold. Another clearance probiem stems from the transport condition, where road clearances may be critical with the recoil mechanism attached to the top of the gun tube.

## C. OTHER O-TYPE CRADLES

17. Another form of O-type cradle is the concentric recoil mechanism type (see Figure 4b). In outward appearance, it resembles the conventional type but, unlike the conventional type, the cradle forms the outer recoil cylinder
and fits concentrically around the gun tube. The internal elements of the recoil mechanism fit between outer diameier of the gun tube and inner dismeter of the cradle. Due to the compactness of the assembly, this type cradle is usuaily found in tanks where space is at a premium. A big advantage offered by this type is that the recoil mechanism is on the axis of the gun bore which is also the line of action of the recoiling parts and can readily be made the location of the trunnions. Consequently, reactions to the moments at the trunnion bearings are negligible. Frictional forces are minimal, produced only by the normal component of the weight of the recoiling parts.

## IV. DESIGN PROCEDURES

A. STRUCTURE TO CARRY THE VARIOUS FORCES
18. In its role of supporting structure for the other tipping parts, the cradle is subjected to a number of forces which it must transmit to the carriage. The predominant one is the recoil force. Others include the equilibrator force, the elevating gear reaction due to tipping moments, and the reaction on the key or guides due to the rifling torque. During the early stages of design, approximate loads are adequate and are readily available. When the design is in its final stages, the loads should be accurate. However, first approximations, in all likelihood, will be close enough to the final values so 'that only minor revisions in the structure will be necessary.

## 1. Recoil Attachment to the Cradle

19. When the recoil mechanism housing is integral with the cradle, the recoil forces are applied through it and no additional supporting structure is necessary. If it is merely attached to the cradle, appropriate yokes or similar structures are needed to carry its force to the cradle. If the recoil mechanism is attached to and moves with the recoiling parts, the recoil rod is fixed to the cradle, sometimes by an adapter or sometimes by a nut threaded to the end of the rod. The rod, in this case, is attached to the front of the cradle where local
reinforcement of the structure may be necessary to carry the load.
20. The method for calculating the approximate recoil force is found in Reference 2. This force comprises the sliding frictional resistance of the resoiling parts and the resistance provided by the recoil mechanism. Sketches in Figure 5 show how the applied loads and corresponding recoil force are distributed on the U-type and O-type cradies. Figure 5 a has those of a single recoil system and Figure 5b has those of a double recoil system. The definitions of the symbols in Figure 5 follow.
$C G=$ the center of gravity of the recoiling parts
$F_{n}=$ inertia force of the recoiling parts in a single recoil system, or of the primary recoiling parts in a double recoil system
$F_{y}=$ propellant gas force
$F_{1}=$ inertia force of primary recoiling parts due to secondary recoil acceleration
$f_{1}=$ frictional resistance of front bearing
$f_{2}=$ frictional resistance of rear bearing
$K_{h}=$ force provided by recoil mechanism
$R_{1}=$ normal reaction of front bearing
$R_{2}=$ normal reaction of rear bearing
$W_{1}=$ weight of recoiling parts in a single recoil system or of the primary recoiling parts in a double recoil system
$\theta=$ angle of elevation
$K=$ total resistance to recoil (recoil force)
$\mu=$ coefficient of friction

$$
\begin{gather*}
f_{1}=\mu R_{1}  \tag{1a}\\
f_{2}=\mu R_{2}  \tag{1b}\\
K=K_{l l}+f_{1}+f_{2} \tag{2}
\end{gather*}
$$

$$
\begin{equation*}
F_{a}=F_{y}+W_{1} \sin \theta-K-F_{1} \cos \theta \tag{3}
\end{equation*}
$$

The force $F_{1}$ occurs in double recoil systems where

$$
\begin{equation*}
F_{1}=\frac{W_{1}}{g} a_{2}=m_{1} a_{2} \tag{4}
\end{equation*}
$$

and the acceleration of the secondary recoiling mass is

$$
\begin{gather*}
a_{2}=\frac{F_{2}}{m_{2}}  \tag{5}\\
F_{2}=\frac{K \cos \theta-W_{1} \cos \theta \sin \theta-R}{1+\left(m_{1} / m_{2}\right) \sin ^{2} \theta}  \tag{6}\\
* \text { Obtained from Reference 2, Equation } 88 .
\end{gather*}
$$



Figure 5. Forces on Recoiling Parts
where
$F_{2}=$ inertia force of secondary recoiling parts $m_{1}=W_{1} / g$, mass of primary recoiling parts $m_{2}=W_{2} / g$, mass of secondary recoiling parts $R=$ secondary recoil force

The terms double recoil system, primary recoiling parts, and secondary recoiling parts are defined in the glossary. The forces applied to the secondary recoiling parts, $F_{2}$ and $R$, are not applied to the cradle. Each appears merely as the means for determining $F_{1}$, a
primary force found only in double recoil systems. A detailed discussion of these forces appears in Reference 2.

It is assumed that the reactions $R_{1}$ and $R_{2}$ are uniformly distributed on the bearings but, if the mating bearing surfaces are continuous, a triangular loading distribution is assumed, with distance between load centers equal to two-thirds of the total length. After $f_{1}$ and $f_{2}$ are written in terms of $R_{1}$ and $R_{2}$, there remain three unknown values, namely, $R_{1}, R_{2}$, and $K_{R}$. These can be obtained by solving the three equations of static equilibrium.

(a) equilibrator attachment on cradle structure

(b) EOUILlbrator attachment on elevating arc

Figure 6: Equilibrator Attachments, Cantilever Type

$$
\begin{align*}
& \Sigma V=0  \tag{7a}\\
& \Sigma H=0  \tag{7b}\\
& \Sigma M=0 \tag{7c}
\end{align*}
$$

Assume that the axis of the bore is horizontal and take the moments at a convenient point such as the intersection of $f_{2}$ and $R_{2}$.

## 2. Equilit:rator Attachment

21. Each equilibrator, whether one or two to the weapon, pivots on a shaft attached to the cradle. If two are used, the shafts are usually cantilevered from each side of the structure
(Figure 6a) or the elevating arc (Figure 6b). If one is used, each end of the shaft is supported by the structure (Figure 7).
22. For any given angle of elevation, the equilibrator force is the one required to produce the moment which balances the weight momeni of the tipping parts. Although actual equilibrator forces do not always equal the theoreticaliy' required ones, differences are small enough that ihe structure is not affected. Hence, for design purposes, the theoretical value will be used to simplify the load analysis. From Figures 8 or 9 the equilibrator force is

$$
\begin{equation*}
F_{\varepsilon}=\frac{W_{1} r_{1} \cos \left(\theta+\phi_{1}\right)+W_{2} r_{4} \cos \left(\theta+\phi_{2}\right)}{r} \tag{8}
\end{equation*}
$$

where
$W_{1}=$ weight of recoiling parts
$W_{r}=$ weig it of cradle.
In Figure 8, $\phi_{2}$ is negative.
It is apparent that, before the preliminary design of the cradle is completed, the equilibrator geometry, at least a preliminary one, must be determined in order to compute its force (see Reference 3).

## 3. Elevating Arc

23. The attachment of the elevating arc to the cradle should be through a well-fitting, rigid joint because the meshing of gear teeth is involved. Improper meshing of the gears will prove detrimental in one or all of three ways; poor load distribution may overstress the teeth, excessive wear may occur, and gear efficiency may decrease, thus requiring an increased torque at the handwheel. The attach-


Figure 7. Equilibrator Altachment, Simple Beam Type

$F_{Q}=$ INERTIA FORCE OF RECOILING PARTS
$F_{E}=$ EQUILIBRATOR FIRCE
$F_{q}=$ PROPELLANT GAS FCOCE
$F_{A}=$ TRUNNION REACTION PARALLEL TO BORE
$F_{N}=$ TRUNNION REACTION NORMAL TO BOPE
$\bar{r}_{g}=$ ELEVATING GEAR LOAD
$R_{p}=$ PITCH RADIUS OF ELEVATING GEAR
$W_{1}=$ WEIGHT OF RECOILING PARTS
$W_{c}=$ WEIGHT OF CRADLE
$W_{t}=$ WEIGHT OF TIPPING PARTS
$\beta=$ PRESSURE ANGLE OF GEAR
$\theta=$ ANGLE OF ELEVATION
figure 8. Exfernal Loads on Cradle, Single Recoil System

$F_{E}=$ EQUILIBRATOR FORCE
$R_{q}=$ elevating gear load
$R_{\text {RH }}=$ SHEAR REACTION ON KEY $R_{\text {RL }}=$ LEFT BOLT REACTION
Rnf $_{\text {n }}=$ RIGHT BOLT REACTION
Figure 10. Elevating Arc
ment then must be secured in a manner which will preclude objectionable misalignment under load. This requires close fitting machined surfaces held together by shear connections such as body-bound bolts, keys, pins, or shafts. Figure 10 illustrates the use of a key and bolts. It also shows the loads on the attachments between arc and cradle.
24. The action of equilibrators practically eliminates the gear load except during elevation and recoil. The largest load usually occurs during recoil. Only if the trunnions and the center of gravity of the recoiling parts lie on the bore axis will no additional gear load be applied during recoil. Figure 8 illustrates the applied external loads on the cradle for a single recoil system. In a double recoil system, the additional inertia loads $F_{1}$ and $F_{r}$ are produced by the secondary recoil acceleration* and are applied horizontally at the centers of gravity of the primary recoiling parts and the cradle (Figure 9). The reaction on the gear, $R_{v}$, is computed by taking moments about the trunnions. The trunnion reactions, parallel and normal to the bore axis, are found by bringing the force system into equilibrium.

## 4. Atiachment for Transnitting Rifling Torque

 25. In addition to supporting the tube, the[^2](a) CRADLE

(b) O-TYPE Cradle


Figure 11. Forces Due to Rifing Torque
cradle must also transmit the rifling torque to the top carriage. $\vec{F}$ igure 11 shows the torque reactions on the cradle and the distances between load centers.
The approximate rifling torque equation

$$
\begin{equation*}
T_{r}=\frac{0.6 \pi^{2} R_{b}^{3} P_{a}}{N_{r}} \tag{9}
\end{equation*}
$$

is derived from the basic torque equation

$$
\begin{equation*}
T_{r}=I_{p \alpha} \tag{9a}
\end{equation*}
$$

The derivation includes the following symbols
$A_{b}=$ bore area (less rifling groove area)
$a=$ linear acceleration of projectile
$a_{1}=$ peripheral acceleration of projectile at the bore
$F_{g}=$ propellant gas force
$I_{\mathrm{n}}=$ mass moment of inertia of projectile
$k=$ radius of gyration of projectile
$m_{p}=$ mass of projectile
$N_{r}=$ twist of ritling, calibers per turn
$P_{u}=$ propellant gas pressure
$R_{u}=$ radius of bore
$\alpha=$ angular acceleration of projectile
$\theta_{r}=$ helix angle of the rifling

$$
\begin{align*}
& A_{b}=\pi R_{b}^{2}  \tag{9b}\\
& F_{y}=A_{\iota} P_{y} \tag{9c}
\end{align*}
$$

From the general expression, $F_{\mu}=m_{1} a$

$$
\begin{gather*}
a=\begin{array}{c}
F_{y} \\
m_{p}
\end{array}  \tag{9d}\\
\tan \theta_{r}=\stackrel{\pi}{N_{r}}  \tag{9e}\\
a_{t}=a \tan \theta_{r} \tag{9f}
\end{gather*}
$$

Substituting the appropriate terms of Equations $9 b, 9 c, 9 d$, and 9 e into Equation $9 f$ and collecting terms, we have

$$
\begin{gather*}
a_{t}=\frac{\pi^{2} R_{5}^{2} P_{b}}{m_{p} N_{r}}  \tag{9~g}\\
\alpha=\frac{a_{t}}{R_{b}}=\frac{\pi^{2} R_{b} P_{g}}{m_{p} N_{r}} \tag{9h}
\end{gather*}
$$

The value, $k^{2}=0.6 R_{b}$ is generally accepted as an approximate value. Then

$$
\begin{equation*}
I_{p}=m_{p} k^{2}=0.6 m_{p} R_{i,}^{2} \tag{9i}
\end{equation*}
$$

Equation 9 is obtained by substituting the terms of Equations 9h and 9i into Equation 9a. From Figures 11a and 11b, the load on the trunnions due to rifling torque is

$$
\begin{equation*}
F_{r}=\frac{T_{r}}{d_{t}} \tag{10}
\end{equation*}
$$

and, correspondingly, the load on the rails (Figure 11a) or on the key (Figure 11b) is

$$
F_{r}^{\prime}=\frac{T_{r}}{\ddot{d}_{r}}
$$

Note that the maximum torque occurs when the propellant gas pressure is maximum. For the U-type cradle, the torque is transmitted directly to the guides through the rails or the sleigh in the form of vertical forces having a moment arm equal to the distance from their lines of action to the bore axis (Figure 11a). For the O-type cradle, the tube is keyed to the cylindrical portion of the cradle. The torque is transmitted through the key and the contacting surface between tube and cradle (Figure 11b). The moment arm extends
from the key to the center of the assumed triangular distributed load on the projected diameter. For either type cradle, the loads induced by the rifling torque are eventually transmitted to the top carriage by the trunnion.

## B. SLIDING SURFACES OF U-TYPE CRADLE

## 1. Load Analysis

26. There are two types of force on the slides: the normal forces and the frictional forces derived from the normal forces. The normal forces are obtained as reactions to the rifling torque, to the couple created by the recoil forces and the inertia of the recoiling parts, and to the weight of the recoiling parts. Thus, the shorter the distances between the forces parallel to the bore axis and the closer the center of gravity is to the midspan of the slides, the smaller will be the forces on the slides. Actually, in single recoil systems, the weight contributes little to the maximum forces; therefore, the center of gravity may fall outside the slides without deleterious effects. However, in double recoil systems, the inertia force due to secondary recoil may produce appreciable loads on the slides, and it becomes desirable not to have the center of gravity overhang the sliding surfaces.
27. When calculating the loads on rails and guides, the distribution of bearing pressure should be considered. If the two mating sliding surfaces are continuous, a triangular load distribution is assumed. Triangular load distribution implies zero clearance and linear compliance of rails and guides. The assumption of triangular load distribution is subject to change for unusual constructions. If the sliding surfaces are discontinuous, a trapezoidal distribution is assumed or, if the pads are spaced sufficiently far apart, uniform load distribution is assumed. Figure 12 illustrates these effects. The diagrams represent the reactions to the couple, $M_{n \prime}$, of the recoiling parts. The reactions $R_{1}$ and $R_{2}$ are calculated by assuming that they are concentrated at the center of the area that represerts the distributed load. After the reactions to the couple are found, those resulting from the rifling torque and the normal component of the


Figure 12. Load Distribution on Rails, U.Type Cradle
weight are added algebraically as uniformly distributed loads.
28. The maximum bearing pressure is then determined from the completed load diagram. A bearing pressure of 200 to $300 \mathrm{lb} / \mathrm{in}^{2}$ is recommended for continuous motion but since motion is not continuous, bearing pressures as high as $500 \mathrm{lb} / \mathrm{in}^{2}$ are permissible. If pressures exceed this limit, there is danger of se-
vere wear on the sliding surfaces, requiring early replacement. This condition must be tolerated if no other design resource is available but it usually means added maintenance and should be avoided if at all possible.

## 2. Construction

29. The sliding structure which supports
the recoiling parts consists of male and female members. The male members are called rails or slides and the female members are called Euides. These latter are similar to channels in cross section, so that bearing surfaces will


Figure 13. End View of Sliding Structure Showing Bearing Liner
support the rails against either upward or downward loads. Bronze liners with surface finishes between 32 and 63 microinch rms cover the rai's and slides to provide the bearing surface (Figure 13). The guides are unlined but their surfaces are machined to the same finish as the rails and slides.
30. Rails are usually secured to the tube. The front attachment is to a sleeve or flat ring, either clamped or shrunk on the tube. The rear attachment is to a similar ring or it may be the breech ring. Figure 14 shows a typical installation. The rails or guides may be continuous or discontinuous. If the guides are discontinuous, they are sometimes called clips. Discontinuity in the sliding surfaces is not recommended if contact between them is broken during the recoil stroke because of the difficulty in re-entering the guides during counterrecoil. The present trend in design is to have continuous rails on the gun tube.
31. When a sleigh is used, the gan tube is held securely to it by collars or yokes as shown in Figure 15. Figure 16 shows another type sleigh. This one has the recoil cylinders


Figure 14. Tube Assembly Showing Rails


Figure 15. Sleigh With Aftached Recoil Cylinders, Gun Tube Secured With Yokes


Figure 16. Sleigh With Integral Recoil Cylinders
integral with the sleigh body. Close itting mating lugs on tube and sleigh preclude tipping while locking wedges preclude relative longitudinal motion. The wedges move laterally and fit into recesses machined in the tube structure. The structure forming the sliding surfaces may be bolted or welded to the sleigh. Sometimes they form an integral part, being machined from the sleigh or cradle structure.
32. The strength of the rail, slide, or guide is determined by the following conservativa method of analysis. Assume that the maximum distributed load is constant for a distance of one inch. Isolate a one-inch length of structure with this load and investigate its strength. Thus at Section A-A of Figure 17a, which is one inch deep,
$w=\mathrm{lb} / \mathrm{in}$, unit load, maximum intensity
$F=1 \times w, \mathrm{lb}$, total load
$M=F d$, lb-in, bending moment
$A=1 \times a, \mathrm{in}^{2}$, area of section in tension
$\sigma_{t}=\frac{M c}{I}+\frac{F}{A}, \mathrm{lb} / \mathrm{in}^{2}$, tensile stress in section
33. In addition to strength requirements, the sliding members must have ample contacting surfaces to insure a reasonable bearing pressure (Paragraph 28). But large areas alone may not achieve a reasonable pressure if ensuing deflections cause a reduction in

(a) LOADING DIAGRAM

(b) CANTILEVER BEAM ANALOGY

Figure 17. Slide and Guide Showing Assumed Defiections
contact area, thereby intensifying the pressure. Two types of deflection may occur. One involves bending of the vertical members which causes the contacting areas to rotate and thus overload the edges of the bearing. This is illustrated by the phantom member in Figure 17a. However, if both members are identically constructed, angular deflections will be equal and the whole contact area remains intact. The other deflection concerns the equivalent cantilever beam of the bearing members as illustrated by the sectional view in Figure 17b. This defleciion poses a difficult problem for, as the beam deflects, the load immediately becomes redistributed over a smaller space with accompanying higher pressures. Since the mating parts are of similar construction, they deflect similarly with the result that the contact area becomes progressively smaller, theoretically approaching a line. Actually, line contact never material:zes but pressure
will peak excessively because of the defiections. A means of circumventing peak pressures employs the practice of providing enough flexibility in the structure to enable the deflection of one mating surface to conform to the deflection of the other, thus maintaining the original contact area. But this type of structure is not always feasible and may not be applicable to cradle design. If not, one must resort to approximation methods that are available for determining the required bearing area. One such method assumes a uniform load distribution with the maximum design pressure limited to 500 psi (see Paragraph 28).

## C. SLIDING SURFACE OF O-TYPE CRADLE

34. The general discussions on the sliding surfacee of the U-type cradle apply as well to those of the O-type cradle. It is advantageous to have the center of gravity of the recoiling parts located as near as possible to the bearings. For the load analysis, the bearings are usually far enough apart to assume a uniform load distribution. The tipping moment during recoil produces the largest loads on the bearings. A bearing pressure of 200 to $300 \mathrm{lb} / \mathrm{m}^{2}$ is desirable but pressure should not exceed $500 \mathrm{lb} / \mathrm{in}^{2}$ (see Paragraph 28). The O-type differs in that the gun tube is keyed to the cradle and in that the reaction to the rifling torque is transmitted by the key, not by the sliding surfaces (see Figure 11). Because the structure must be held to reasonable proportions, and the sliding surface offered by the key is limited in area, the bearing pressures here may be much higher than on other sliding surfaces. However, due to the extremely short duration of the rifling torque, allowable bearing pressures may be high. After incorporating the factor of safety, they may approach the bearing strength of the material.
35. The construction of the sliding surfaces of the 0 -type cradle is relatively simple. The primary structure is a cylinder in which the gun tube slides during recoil. A bronze bearing at each end and a straight portion of the gun tube, machined to a 32 rms finish, provide the sliding surfaces. Thus, the gun tube serves as its own slide and the bearings serve as guides. The key is usually secured to the gun tube and
slides in the keyways of each bearing. Although the rifling torque is applied for only a short recoil distance, the key should be long enough to maintain contact with the bearings at all positions of the recoil stroke so that no difficulty in alignment develops during counterrecoil.

## D. EFFECT OF FRICTION ON SLIDING SURFACES

36. Frictional forces, as such, are not a serious design criterion with respect to the structural strength. In other aspects, they present serious problems. For design analysis, the present practice is to use a coefficient of friction of $0.15^{*}$. Friction resists recoil and thus forms part of the recoil force. Theoretically, it does not matter whether the recoil force is generated by friction or by the recoil mechanism. However, it is desirable to keep friction to a minimum by proper lubrication because wear and eventual damage to the sliding surfaces are less likely to occur. Also, frictional forces are somewhat of an unknown value on exposed surfaces and may vary considerably. If their maximum value is small, it will constitute only a small part of the total recoil force and will have only a slight effect on the functioning of the recoil mechanism.

## E. EFFECT OF TEMPERATURE VARIATION ON SLIDING SURFACES

37. Artillery is (mployed in both desert and arctic climates. Changes in the ambient temperature will shrink or expand the structure. If made of like material, all components will be affected equally, causing no relative displacement among them. However, if the structural members are of unlike material, their rates of expansion will differ and this may prove deleterious sinply by reducing clearances between moving parts to the extent where binding impends. The sliding surfaces of cradles must be of different materials because two mating surfaces of like material seldom provide compatible sliding properties. Bronze makes an excellent sliding surface for steel. Its coefficient of linear expansion is

[^3]$3.5 \times 10^{-6} \mathrm{in} / \mathrm{in} /{ }^{\circ} \mathrm{F}$ larger than that for atecl. This value is computed from the coefficients in the centigrade scale of $16.8 \times 10^{-8}$ for phosphor bronze and $10.5 \times 10^{-6}$ for $1.2 \%$ carbon steel.* Based on an ambient temperature of $70^{\circ} \mathrm{F}$, the extreme ranges will show a difference in dimensions for bronze and steel of 0.00033 in/in at $165^{\circ} \mathrm{F}$ and of -0.00047 in/in a.t $-65^{\circ} \mathrm{F}$. The lower limit of the temperature extremes because of climatic conditions hes the greater effect. However, neither extreme requires excessively loose fits to comperisate for the thermal activity.

## F. LOCATION AND DESIGN OF TRUNNIONS

38. The location of the trunnions in the vertical plane directly infiuences the reaction on the elevating gear during the recoil cycle. In single recoil systems, the reaction is due solely to the moment about the trunnions produced by the propellant gas force and the inertia force of the recoiling parts. Figure 18 shows these forces and the perpendicular distance from their lines of action to the trunnions. The position of the trunnions with

$F_{a}=$ INERTIA FORCE OF RECOILING PARTS
$F_{g}=$ PROPELLANT GAS FORCE
$K=$ RECOIL FORCE
$M_{T}=$ MOMENT ABOUT TRUNNIONS
Figure 18. Trunnion Location With Respect to Recoil Forces

[^4]respect to these lines of action determines the moment. Thus, from Figure 18,
\[

$$
\begin{equation*}
M_{T}=b F_{p}-a F_{z} \tag{12}
\end{equation*}
$$

\]

Where $a, b=$ moment arms
$F_{a}=F_{g}-K$, inertia force at $C G$
$F_{0}=$ propellant gas force
$K=$ recoil force
$M_{r}=$ trunnion moment
If $F_{a}^{\prime}$ and $F_{p}^{\prime}$ represent the maximum izertia and propellant gas forces respectively, the ideal trunnion location lies within the limits of $b \leq a \leq\left(F_{g}^{\prime} / F_{a}^{\prime}\right) b$ although it is not always feasible to have this arrangement. If $a$ extends beyond these limits, or if $c$ and $b$ lie on opposite sides of the trunnion, then the moment will increase, varying 93 the distances. When $a=\left(F_{o}^{\prime} / F_{a}^{\prime}\right) b$, the moment becomes zero when the gas ior:e becomes niaximum. It gradually increases as the gas force üminishes and reaches the maximum of

$$
\begin{equation*}
M_{T}=-a F_{a}=-a K \tag{12a}
\end{equation*}
$$

when the gas force becomes zero.
39. In double recoil systems, the inertia force of the tipping parts caused by the acceleration of the secondary recoiling parts becomes a factor when determining the trunnion moment and, subsequently, the elevating gear reaction (see Figure 9). $\dagger$ The trumion location with respect to the center of the bore has little influence with this additional moment because the component of force perpendicular to the bore center line has a moment arm considerably larger than that for the parallel component.
40. The trunnion loads are composed of five components which are derived from the weight of tipping parts, recoil force, equilibrator force, force due to the rifling torque, and elevating gear reaction. The first four do not vary with trunnion location but form the bulk of the maximum trunnion loads; consequently, any change in the elevating gear reaction will not materially affect the trumnions but small shifts in trunnion location may greatly influence elevating gear reactions.
41. Sometimes the location of the trunnion in the vertical plane is adjusted to satisfy
$\dagger$ Reference 2, Chapter XI.
some structural requirement. For example, when located below the bore center line, more space will be provided for an underslung recoil mechanism. Or, if they are located on the center line, structural symmetry is preserved. Also, if the trunnions are on the center line, the sighting equipment will not have to be corrected for discrepancies due to an off-center location.
42. In the horizontal plane, it is advantageous to have the trunnions located equidistant from the center line of the gun bore. Here, the object is mainly one of symmetry. If symmetry cannot be achieved, the cradle will be subjected to a direct load and a couple equal to the recoil force times the offset. Its vertical component 'ends to turn the weapon on its side. Its horivontal component tends to rotate the cradle and top carriage. However, the base supporting the top carriage is symmetrical with respect to the bore and the loads revert to a symmetrical condition at this point. If any residual horizontal moment persists, it is resisted at the traversing gear. This discussion does not include the rifling torque which is transmitted through the structure while the projectile travels in the bore.
43. When the distances from the bore to the trumnions are anequal, the cradle must be made larger to offset the effects of the unsymmetrical loads and this eventually leads to a heavier structure. In considering defections, symmetry becomss definitely desirable. If both sidea of the cradle deflect squally, compensation ior misalignment during firing presents lesser problems in fire control than if the two sides of the cradle deffected unequally. In the first case, the gun tube would remain essentially in line, while in the latter, it would turn slightly askew.
44. The size of the trunnion is usually dictated by required bearing dimensions. However, it should be investigated to determine its strength in bending and shear. As a rule, the trunnion may be considered a short beam and the stresses calculated according to the formulas below, which can be found in any text on strength of materials. The bending stress is

$$
\begin{equation*}
\sigma=\frac{M c}{I} \tag{13}
\end{equation*}
$$

and the shear stress, $\tau$, at any line $q$.either on, or at a distance from, the neutral axis of the total section, is

$$
\begin{equation*}
\tau=\frac{F_{\tau} A_{n} \bar{d}}{I t} \tag{14}
\end{equation*}
$$

where $A_{n}=$ area above the line $q$
$t=$ thickness of section at $q$
$\bar{d}=$ distance between the neutral axis of the section and the neutral axis of $A_{n}$
$F_{r}=\sqrt{\overline{F_{N}^{2}}+F_{\lambda}^{2}}$, resultant shear at the section (see Figures 8 and 9)
1 = moment of inertio of the section
$L=$ moment arm of trunnion measured to center of bearing
$M=F_{r} L$, bending moment

## G. STRENGTH REQUIREMENTS

45. Stresses are calculated for a cradle which is assumed to be completely isolated from all other components of the gun. This approach is conservative because the stiffness associated with gun tube and structural members which ordinarily would lend strength to the cradle is ignored.
46. The general stresses of the main cradle structure are due to bending and direct shear. However, at each point of load application, local stresses are present which may be greater than or may augment the general stress. The local areas are loaded by the recoil mechanism, the trunnion, and the elevating mechanism through their attachments to the cradle. After the principal stresses have been found, usually by conventional methods of stress analysis, the equivalent stress is determined. An accepted method for computing the equivalent stress comes from the maximum-shear-stress theory of Tresca and Saint Venant* which states that yielding begins when the largest difference of two principal stresses equals the yield strength of the material, or

$$
\begin{equation*}
\sigma_{1}-\sigma_{2}=\sigma_{y} \tag{15}
\end{equation*}
$$

To be compatible with other components of the gun carriage, a factor of safety of 1.5 is rec-

[^5]ommended for the cradle. Now, if $\sigma_{1}=\sigma_{t}$ and $\sigma_{2}=\sigma_{c}$ the equivalent stress is
\[

$$
\begin{equation*}
\sigma_{t}=\sigma_{t}-\sigma_{c} \tag{15a}
\end{equation*}
$$

\]

The factor of safety is

$$
\begin{equation*}
S_{f}=\frac{\sigma_{y}}{\sigma_{e}} \tag{15b}
\end{equation*}
$$

## 1. Trunnion Huts

47. The hubs or sockets holding the trunnions are either welded or bolted to the cradle. Reinforcements at the hub are sometimes necessary to distribute the loads and prevent local failure. If the trumion shank fits the hub freely, the latter is stressed in shear and bending. But, if the joint is a shrink fit, the interface pressure produces hoop stresses in the trunnion shank and the hub. This pressure is found by equating the interference to the total deflection of the concentric cylinders at their interface.
Thus

$$
\begin{equation*}
p=\frac{E \Delta}{r} /\left(\frac{r_{2}^{2}+r^{2}}{r_{2}^{2}-r^{2}}+\frac{r^{2}+r_{1}^{2}}{r^{2}-r_{1}^{2}}\right) \tag{16}
\end{equation*}
$$

where $\quad E=$ inodulus of elasticity
$p=$ pressure at interface due to shrink fit
$r=$ radius at contact surfaces of concentric cylinders
$r_{1}=$ inner radius of inner cylinder
$r_{2}=$ outer radius of outer cylinder
$\Delta=$ radial interference
48. The strength of the gussets supporting the hub is based on the loading arrangement shown in Figure 19, the hub being assumed to be supported by the gussets only. The load distribution of the reactions $R_{G}$ and $R_{v}$ is assumed triangular and is based on the load paraliel to the canzon bore.

Each structural member, whether gusset or hub, must be statically balanced. Therefore, by isolating the hub

$$
\begin{gather*}
R_{a}=\frac{F_{s}}{n}  \tag{17}\\
r_{\lambda} R_{k}+\frac{2}{3} c R_{\sigma}=a \frac{F_{A}}{n} \tag{17a}
\end{gather*}
$$

where $\quad F_{A}=$ urunnion load parallel to bore
$n=$ number of guseets parallal to $F_{A}$

$F_{n}=$ TRUNNION LOAD PARALLEL TO 4 bORE
$R_{c}=$ ShEAR bETWEEN GUSSET AND CRADLE BODY
$R_{G}=$ HORIZONTAL REACTION OF GUSSET, DISTRIBUTED
$R_{h}=$ SHEAR BETWEEN GUSSET AND HUB
$\mathrm{R}_{\mathrm{V}}=$ VERTICAL REACTION OF GUSSET, DISTRIBUTED
Figure 19. Load Distribution on Trunnion Structure

When a gusset is isolated

$$
\begin{align*}
R_{v} & =R_{h}  \tag{17b}\\
R_{c} & =R_{G}  \tag{17c}\\
b R_{v} & =c R_{G} \tag{17d}
\end{align*}
$$

Substitute the value of $R_{G}$ of Equation 17 into Equation 17a and solve for $R_{3}$

$$
\begin{equation*}
R_{\lambda}=\frac{a-\frac{2}{3} c}{r_{\lambda}} \frac{F_{A}}{n} \tag{17e}
\end{equation*}
$$

Sinco $\mathbf{R}_{p}=\mathbf{R}_{h}$, substitute the expressions in Equations 17 and 17 e for $R_{C}$ and $R_{r}$ in Equation $17 d$ and solve for $b$

$$
\begin{equation*}
b=\frac{r_{k}}{a-\frac{z_{3}}{3} c} c \tag{17f}
\end{equation*}
$$

This is the gusset length required to substantiate the assumption of the triangular load distribution. If $R_{G}$ represents the area of a triangle, then

$$
\begin{equation*}
w=\frac{2 R_{G}}{c}, \text { maximum unit load on the gusset } \tag{18}
\end{equation*}
$$

and the maximum direct tensile or compressive stress becomes

$$
\begin{equation*}
\sigma=\frac{w}{t} \tag{19}
\end{equation*}
$$

The elastic stability is checked by assuming that the gusset is a rectangular plate loaded in compression on the two opposite edges. This assumption, although approximate; is conservative. The critical compressive stress is*

$$
\begin{equation*}
\sigma_{c}=K_{s} \frac{E}{1-\nu^{2}}\left(\frac{t}{b}\right)^{2} \tag{20}
\end{equation*}
$$

where $b=$ width of loaded edge
$t=$ thiclness of gusset
$K_{\mathbf{t}}=$ fixity factor determined from the width to length ratio
$E=$ modulus of elasticity
$\nu=$ Poisson's ratio
49. Gussets should be arranged so that the transmitted loads will not unduly aggravate the stresses and deflections of the cradle wall. For example, chose gussets that are tangent to the cradle generally produce less stress and deflection than those that are not tangent. This characteristic often leads to appreciable

[^6]
(b) Lomeituomal qussets in cmonone onectron

Figure 20. Gusset Reinforced Trunnion Housings
savings in weight as the cradle wall is less able to sustain the induced radial load of the latter. Figure 20 shows two types of construction, both having the same origin on the trunnion housing. Since the gusset of tangential direction is essentially larger than the one of chordal direction, its moment arm to the weld seam is also larger, thereby inducing a larger load perpendicular to the weld. However, the low stress and deflection affected by the tangential component may more than compensate for the larger load. If both types of construction are feasible, each should be investigated to determine which is preferable.
50. One method for determining the influence of the gusset load involves isolating a section of the cradle wall and treating it as a ring. Its width is assumed equivalent to the length of the gusset at the weld. This approach is conservative as the analysis does not utilize the stiffness provided by the adjacent cradle wall, thus yielding bending moments and deflections somewhat larger than their true values. The gussets of the trunnion housings provide loads equivalent to those on the diagrams shown as Conditions (a) and (b) of Figure 21. The equations that follow are specific applications of Case 8 from Reference 7, Table VIII, and define the bending moment and deflection at critical points on the ring.

CONDITION (a)


CONDITION (b)


CONDITION (c)


CONDITION (d)


Figure 21. Equivalent Ring Loading Conditions

When loads are tangent, equations are obtained by superimposing the expressions for their horizontal and vertical components.
$M_{x}=$ moment at the $x$-axis
$M_{\nu}=$ moment at the $y$-axis
$M_{s}=$ moment at the load
$\Delta D_{z}=$ diametral deflection on $x$-axis
$\Delta D_{y}=$ diametral deflection on $y$-axis
$I=$ moment of inertia of ring cross section
$E=$ modulus of elasticity
$R_{m}=$ mean radius of ring
$W$ = concentrated ring load
Positive deflections indicate increase in diameter.
Condition (a): four equal tangential loads symmetrical about the $x$ - and $y$-axes.

$$
\begin{gather*}
M_{z}=W R_{m}\left[\frac{2 \theta}{\pi}-\sin \theta\right]  \tag{21}\\
M_{v}=W R_{m}\left[\frac{2 \theta}{\pi}+(\cos \theta-1)\right]  \tag{22}\\
\Delta D_{x}=\frac{W R_{m}^{3}}{E I}\left[\frac{4 \theta}{\pi}+\theta \cos \theta-2 \sin \theta\right]  \tag{23}\\
\Delta D_{\mu}=\frac{W R_{m}^{3}}{E I}\left[\left(\frac{4 \theta}{\pi}-2\right)+2 \cos \theta+\left(\theta-\frac{\pi}{2}\right) \sin \theta\right] \tag{24}
\end{gather*}
$$

Condition (b): four equal and parallel chordal loads symmetrical about the $x$ - and $y$-axes.

$$
\begin{gather*}
M_{x}=W R_{m}\left[\frac{2}{\pi}(\theta \sin \theta+\cos \theta)-1\right]  \tag{25}\\
M_{v}=W R_{m}\left[\frac{2}{\pi}(\theta \sin \theta+\cos \theta)-\sin \theta\right]  \tag{26}\\
M_{\theta}=M_{\mu} \tag{27}
\end{gather*}
$$

$\Delta D_{\mathrm{z}}=\frac{W R_{m}^{3}}{E I}\left[\frac{4}{\pi}(\theta \sin \theta+\cos \theta)-\sin ^{2} \theta-1\right]$

$$
\begin{align*}
& \Delta D_{y}=\frac{W R_{m}^{3}}{E I}\left[\frac{4}{x}(\theta \sin \theta+\cos \theta)\right.  \tag{28}\\
& \left.+(\cos \theta-2) \sin \theta+\left(\theta-\frac{\pi}{2}\right)\right] \tag{29}
\end{align*}
$$

nd

## 2. Recoil Mechanism Attachment Bracket

51. The recoil méchanism cylinder may be integral with the sleigh or it may be attached to the sleigh or cradle by brackets, one at the rear near the trunnions, the other farther
toward the front. One bracket, usually the front, serves merely as a stabilizing structure while the other transmits the reccil rod force to the cradle. These forces produce shear and bending stresses at the joint between bracket and cradle body. One method for calculating the stresses and converting these stresses to applied loads on the cradle tody is demonstrated in the sample problem in Section VI. Local bending moments and deflections are present at the attachments. These are found according to the methods of Paragraph 50 but, in this case, only two loads are involved. Cases 2 and 25 of Reference 7, Table VIII, and Case 25 were used to derive the equations for Conditions (c) and (d) of Figure 21 respectively.

Condition (c): two equal tangential loads symmetrical about the $y$-axis.

$$
\begin{align*}
& M_{x}=W R_{m}\left[\frac{\theta}{\pi}-\frac{1}{2} \sin \theta\right]  \tag{30}\\
& M_{y}=W R_{m}\left[\left(\frac{\theta}{\pi}-1\right)-\left(\frac{\theta}{\pi}-1\right) \cos \theta+\frac{3}{2 \pi} \sin \theta\right] \tag{31}
\end{align*}
$$

$\Delta D_{y}=\frac{W R_{m}^{3}}{E I}\left[\left(\frac{2 \theta}{x}-1\right)+\left(\frac{\theta}{2}-\frac{\pi}{4}\right) \sin \theta+\cos \theta\right]$
Condition (d): two equal and parallel chordal loads symmetrical about the $y$-axis.

$$
\begin{align*}
& M_{x}=W R_{m}\left[\frac{1}{\pi}(\theta \sin \theta+\cos \theta)-\frac{1}{2}\right] \\
& M_{\imath}=W R_{m}\left[\frac{3}{2 \pi}+\frac{1}{\pi}(\theta \sin \theta+\cos \theta)\right.  \tag{34}\\
& \left.-\sin \theta-\frac{1}{\pi} \cos ^{2} \theta\right]  \tag{35}\\
& M_{\theta}=W R_{m}\left[\left(\frac{5}{2 \pi}-\frac{1}{\pi} \cos ^{2} \theta\right) \cos \theta\right. \\
& \left.-\left(1-\frac{2 \theta}{\pi}\right) \sin \theta\right]  \tag{36}\\
& \Delta D_{x}=\frac{W R_{m}^{3}}{E I}\left[\frac{2}{\pi}(\theta \sin \theta+\cos \theta)\right. \\
& \left.-\frac{1}{2}\left(\sin ^{2} \theta+1\right)\right]  \tag{37}\\
& \Delta D_{y}=\frac{W R_{m}^{3}}{E I}\left[\left(\frac{\theta}{2}-\frac{\pi}{4}\right)+\left(\frac{2 \theta}{\pi}-1\right) \sin \theta\right. \\
& \left.+\left(\frac{2}{\pi}+\frac{1}{2} \sin \theta\right) \cos \theta\right] \tag{38}
\end{align*}
$$

52. Pressure is created by the deflection of the cradle wall and the resistance to it provided by the inner sliding member. When excessive, this pressure causes galling of the bearing. The object is to maintain this pressure within the acceptable limits defined in Paragraph 28. Equation 39 provides an approximate pressure*

$$
\begin{equation*}
\Delta D_{v}=-0.467 \frac{w R^{4}}{E I} \tag{39}
\end{equation*}
$$

where
$I$ = moment of inertia of ring cross section
$R_{n^{\prime}}=$ mean radius of ring
$w=$ load per linear inch on the periphery
The bearing pressure induced by the defiection can be solved by setting Equation 39 equal to either Equation 33 or 38, whichever is appropriate, and solving for $w$. The bearing pressure becomes

$$
\begin{equation*}
p_{b}=\frac{w}{b} p s i \tag{40}
\end{equation*}
$$

where $b=$ length over which the gusset load is applied.

## V. DESIGN PRACTICE

## A. STRUCTURE

53. The structure should be simple and symmetrical. Simplicity and symmetry offer several advantages. Fabrication is easier. They tend to keep weight down. A stronger, more compact, and efficient unit is the ultimate result. If a material of large strength-toweight ratio is needed, high strength is indicated but if rigidity is also essential, low weight must be sacrificed and the necessary strength derived from a bulkier structure. Cradles must be rigid to insure an accurate weapon, therefore the overall design should be directed towaid this end.
54. The choice of whether forgings, castings, or weldments should be used is usually determined by the nature of the structure. If high strength-weight ratios are needed, forgings are used. However, forgings are costly. If weight is not important, castings may be applicable. They provide large fillets, thus decreasing stress concentrations at re-en-

[^7]trant angles. Forgings and castings are less susceptible to warpage than weldments although all should.be stress relieved to insure dimensional stability. The main disadvantages of castings include bulkiness and a lengthy manufacturing process. Welded assemblies should be used where applicable. The built-up structure is relatively simple and light. Joints are permanent, providing a more rigid structure than if bolted or riveted. Weldments can be made from available stock material permitting construction at low cost in a relatively short time. Although weldments are prone to warp, this tendency is overcome by tress relieving through heat treatment.

## B. SUGGESTED MATERIALS FOR CRADLE

55. The predominant requirements for the cradle are strength and rigidity. For its main structure, an inherently strong material with a high modulus of elasticity is preferred. This suggests steel although it does not exclude other materials having the required physical properties. For the sliding surfaces, hardness and compatibility are necessary, hardness to preclude scoring and compatibility to preclude galling of the contacting surfaces. Steel slides, rails, and guides, as components of the main structure, provide strength ard rigidity and, as sliding members, provide a hardened surface. Hard bearing bronze, covering one member, also provides a hardened surface and, in conjunction with the bare steel of the other member, constitute two adjacent materials which can provide the compatibility requirement. Bronze is preferred to brass because of the tendency of the latter to form zinc oxide, a substance that promotes galling.

## C. MANUFACTURING PROCEDURES

56. Standard production practices are followed in constructing cradles regardless of whether castings, forgings, or weldments form the besic structure. If this practice deviates, it is only in handling. 'Basic fabrication activities remain undisturbed. If necessary, machines are adapted for convenient operation. Warpage is corrected by stress relieving
through heat treatment. Those members of the cradle which require finished surfaces are made oversize so that residual irregularities may be removed when the pari is machined to size.

## D. mainitenance

57. A well designed structure embodies good maintenance features; hence ease of maintenance, both preventive and corrective, begins on the drawing board.* Inspecting, cleaning, and lubricating are activities usualiy associated with preventive maintenance, with lubrication being the most important because it not only reduces friction and the accompanying wear or galling but it also protects the sliding surfaces from corrosion. A.good lubricant for sliding surfaces is Spec MIL-G-10924A grease which lubricates effectively through the temperature range of $-65^{\circ}$ to $125^{\circ} \mathrm{F}$. Lubrication should be a simple task requiring only a short time to perform. Therefore, fittings must be readily accessible oin the assembled weapon but should not be located in highly stressed regions of the cradle because small holes cause stress concentrations. If this is unavoidable, then the lubrication holes should be heavily bossed for reinforcement.
58. A cradle functions best when clean. Any dirt or other foreign substance on slides or trunnions will impede recoil and elevation. Maintenance here means continuous effort in keeping the cradle and its attachments clean. Sand, mud, water, snow, or ice must not accumulate in it. Pockets created by structural members should have drain holes or should be easily reached for cleaning, otherwise water, from rain or melting snow, accumulating in these pockets may later freeze and damage even otherwise well designed equipment. Dirt must be kept off sliding surfaces. Cover plates are effective seals at the trunnion. Wipers, located where the initial contact begins between sliding surfaces, remove dirt and grit from the exposed portions of the slides.
59. Corrective maintenance is a repair or replacement activity which may require the disassembly of the cradle. In many instances this
*The subject of maintenance is covered in detail in Reference 8.
work must be performed in the field where regular handling equipment is normally not available, thus increasing the burden of maintenance crews. If feasible, each subassembly should be designed so that it will not interfere with the dismantling of other components. When this practice is followed, only those parts requiring attention need be removed, leaving the undamaged ones undisturbed. This will expedite maintenance in the field particularly from the handling viewpoint.
60. Failure of the primary structure can often be repaired by welding. Hence, the selection of a weldable material while the cradle is in the design stage may prove to be an asset. Other repairs involve sliding surfaces. Scored or galled surfaces can be scraped and hand polished until smooth. If damage is too extensive, they must be replaced which is relatively easy if bronze liners are used. But if the damage is on the steel surface of a slide integral with the main structure, the entire cradle may have to be scrapped. This emphasizes the need for good design practice with respect to maintenance. Those members of a structure which have a critical function and which are prone to damage should be anade detachable.

## VI. SAMPLE PROBLEM, O-TYPE CRADLE

## A. LOAD ANALYSIS

61. An O-type cradle is selected for the sample problem involving a double recoil gun carriage. Figure 5b represents the loading diagram for the analysis. Except for the center of gravity, all forces and their respective locations are as shown. The center of grevity lies on the line of action of $R_{1}$.

$$
\begin{array}{ll}
a=80 \mathrm{in} & d=0.10 \mathrm{in} \\
b=0 & e=7 \mathrm{in} \\
c=16 \mathrm{in} & h=7 \mathrm{in}
\end{array}
$$

$W_{1}=10,000 \mathrm{lb}$, primary recoil weight
$W_{2}=14,000 \mathrm{lb}$, secondary recoil weight
$F_{g}=1,810,000 \mathrm{lb}$, propellant gas force
$K=150,000 \mathrm{lb}$, primary recoil resistance
$R=40,000 \mathrm{lb}$, secondary recoil resistance
$\mu=0.15$, ccefficient of friction
$\theta=60^{\circ}$, angle of elevation
62. There are five unknown quantities: the vertical reactions $R_{1}$ and $R_{2}$, the frictional forces $f_{1}$ and $f_{2}$, and the recoil rod force $K_{R}$. These are found by balancing the loads and moments; but first, the inertia forces $F_{a}$ and $F_{1}$ must be calculated. From Equation 6

$$
\begin{gathered}
F_{2}=\frac{K \cos \theta-W_{1} \cos \theta \sin \theta-R}{1+\frac{m_{1}}{m_{2}} \sin ^{2} \theta} \\
F_{2}=\frac{30,700}{1.535}=20,000 \mathrm{lb}
\end{gathered}
$$

From Equation 5

$$
a_{2}=\frac{F_{2}}{m_{2}}=\frac{F_{2}}{W_{2} / g}=\frac{20,000}{14,000} g=1.43 g
$$

From Equation 4

$$
F_{1}=\frac{W_{1}}{g} a_{2}=\frac{10,000}{g} 1.43 g=14,300 \mathrm{lb}
$$

From Equation 3

$$
\begin{aligned}
F_{a} & =F_{0}+W_{1} \sin \theta-K-F_{1} \cos \theta \\
& =1,810,000+8700-150,000-7200 \\
& =1,661,500 \mathrm{lb}
\end{aligned}
$$

All information is now available to solve for the five unknowns.

$$
\Sigma V=0
$$

$$
R_{1}-R_{2}-F_{1} \sin \theta-W_{1} \cos \theta=0
$$

$$
R_{2}=R_{1}-17,400
$$

From Equation 2

$$
\Sigma H=0
$$

but

$$
\begin{aligned}
& K_{R}+f_{1}+f_{2}-K=0 \\
& f_{1}=\mu R_{1}=0.15 R_{1} \\
& f_{2}=\mu R_{2}=0.15 R_{1}-2600
\end{aligned}
$$

and
therefore

$$
\begin{gathered}
K_{R}=K-\left(f_{1}+f_{2}\right)=152,600-0.30 R_{1} \\
\Sigma M_{R_{2}}=0
\end{gathered}
$$

$(c-e) K_{R}+a\left(F_{1} \sin \theta+W_{1} \cos \theta\right)+e F_{0}$
$-(e-d)\left(F_{a}+F_{1} \cos \theta-W_{1} \sin \theta\right)$
$-(a-b) R_{1}-(e+h) f_{1}=0$
$9 K_{R}+80 \times 17,400+7 \times 1,810,000-6.9$
$\times 1,660,000-80 R_{1}-14 \times 0.15 R_{\mathrm{t}}=0$
Substituting for $K_{R}$ and solving for $R_{1}$

$$
\begin{aligned}
84.8
\end{aligned} \begin{aligned}
R_{1} & =3,981,000 \\
R_{1} & =46,900 \mathrm{lb} \\
R_{2} & =29,500 \mathrm{lb}
\end{aligned}
$$

$$
\begin{aligned}
& \hat{f}_{1}=7,000 \mathrm{lb} \\
& f_{2}=4,400 \mathrm{lb} \\
& K_{R}=138,600 \mathrm{lb}
\end{aligned}
$$

The cradle liners have a diameter of 14 inches and are 10 inches long

$$
\begin{aligned}
& A_{b r}=10 \times 14=140 \mathrm{in}^{2}, \text { bearing area } \\
& \sigma_{b r}=\frac{R_{1}}{A_{b r}}=366 \mathrm{lb} / \mathrm{in}^{2}, \text { bearing pressure }
\end{aligned}
$$

This pressure is acceptable according to Paragraph 28.
63. Calculate the equilibrator force, $F_{E}$, by balancing the weight moment of the tipping parts about the trunnions. Referring to Figure 9,

$$
\begin{array}{lll}
M_{w}= & r_{1} W_{1} \cos \left(\theta+\phi_{1}\right) & +r_{c} W_{c} \cos \left(\theta+\phi_{2}\right) \\
=424,000 \mathrm{lb}-\mathrm{in} & \\
\text { where } & r_{1}=75 \mathrm{in} & \theta=60^{\circ} \\
& r_{c}=25 \mathrm{in} & \phi_{1}=0^{\circ} 04^{\prime} \\
W_{1}=10,000 \mathrm{lb} & \phi_{2}=0^{\circ} \\
& W_{c}=4000 \mathrm{lb} &
\end{array}
$$

The equilibrator force at elevation $\theta=60^{\circ}$ is found by equating the equilibrator moment to the weight moment.

$$
r F_{B}=M_{w}=424,000 \mathrm{lb}-\mathrm{in}
$$

when $r=12$ in

$$
F_{s}=35,300 \mathrm{lb} \text {, equilibrator force }
$$

64. The reaction on the elevating gear arc, $R_{g}$, is found by balancing the moments about the trunnions. Before continuing, the forces at the center of gravity of the cradle should be resolved into components parallel and perpendicular to the bore. Again referring to Figure 9 , the inertia force caused by secondary acceleration is

$$
\begin{aligned}
& F_{\mathrm{r}}=\frac{W_{c}}{g} a_{2}=\frac{4000}{g} 1.43 \mathrm{~g}=5700 \mathrm{lb} \\
& H_{r}=W_{c} \sin \theta-F_{r} \cos \theta=600 \mathrm{lb}, \\
& \quad \text { parallel to bore } \\
& V_{r}=W_{r} \cos \theta+F_{c} \sin \theta=6900 \mathrm{lb}, \\
& \quad \text { perpendicular to bore }
\end{aligned}
$$

Additional dimensions for Figure 9 are

$$
\begin{aligned}
& \lambda=20^{\circ} \\
& \gamma=25^{\circ} \\
& \beta=20^{\circ}, \text { pressure angle of gear tooth } \\
& R_{p}=36 \text { in, pitch radius of elevating arc }
\end{aligned}
$$

The applied loads and dimensions are those used in the previous sample problem. With
reference to Figure 5 b , the trunnions are located 5.0 inches to the left of $R_{2}$.

$$
\Sigma M_{T}=0
$$

$R_{s} R_{g} \cos \beta+r F_{E} \cdots r_{c} \cos \phi_{2} V_{r}$
$-d\left(F_{a}+F_{1} \cos \theta-W_{1} \sin \theta\right)-r_{c} \sin \phi_{2} H_{c}$ $-r_{\mathrm{l}} \cos \phi_{\mathrm{l}}\left(F_{\mathrm{l}} \sin \theta+W_{\mathrm{l}} \cos \theta\right)=0$
$R_{p} R_{g} \cos \beta=36 \times .940 R_{g}=33.8 R_{g}$ $r F_{E}=12 \times 35,300=424,000$ $r_{i} \cos \phi_{2} V_{c}=25 \times 6900=172,000$

$$
r_{c} \sin \phi_{2} H_{c}=0
$$

$d\left(F_{a}+F_{1} \cos \theta-W_{1} \sin \theta\right)$

$$
=0.10 \times 1,660,000=166,000
$$

$r_{1} \cos \phi_{1}\left(F_{1} \sin \theta+W_{1} \cos \theta\right) \ldots$

$$
=75 \times 17,400=1,305,000
$$

$33.8 R_{q}=1,2 \times 9,000 \mathrm{lb}-\mathrm{in}$
$R_{g}=36,100 \mathrm{lb}$
65. The trunnion reactions $F_{A}$ and $F_{N}$ are found through the summation of forces parallel and perpendicular to the center line of the bore

$$
\begin{aligned}
F_{N}= & \left(F_{1} \sin \theta+W_{1} \cos \theta\right)+V_{r} \\
& +F_{\varepsilon} \sin (\theta+\lambda)-R_{g} \cos (\theta+\gamma-\beta) \\
= & 17,400+6900+0.985 \times 35,300 \\
& -0.423 \times 36,100=43,800 \mathrm{ib} \\
F_{A}= & F_{a}-\left(F_{a}+F_{1} \cos \theta-W_{1} \sin \theta\right)+H_{c} \\
& -F_{\varepsilon} \cos (\theta+\lambda)-R_{a} \sin (\theta+\gamma-\beta) \\
= & 1,810,000-1,660,000+600- \\
& 0.174 \times 35,300-0.906 \times 36,100 \\
= & 111,800 \mathrm{lb}
\end{aligned}
$$

## B. ELEVATING ARC

66. With reference to Figure 22, the loads at the attachments of elevating arc to cradle are calculated by resolving the equilibrator and gear tooth loads about these attachments. The key provides the shear resistance for the resultant horizontal load. Take moments about the intersection of $R_{R R}$ and $R_{R \|}$ and solve for $R_{R L}$.

$$
\begin{gathered}
32 R_{R L}=25.85 R_{g} \cos 25^{\circ}+13.14 R_{g} \sin 25^{\circ}- \\
1.15 F_{B} \sin 80^{\circ}+9.0 F_{B} \cos 80^{\circ}=1,061,000 \\
25.85 R_{g} \cos 25^{\circ}=25.85 \times 32,700=845,000 \\
13.14 R_{g} \sin 25^{\circ}=13.14 \times 15,300=201,000 \\
1.15 F_{E} \sin 80^{\circ}=1.15 \times 34,800=40,000 \\
9.0 F_{E} \cos 80^{\circ}=9.0 \times 6100=55,000 \\
R_{R L}=33,200 \mathrm{lb} \\
R_{R L}=R_{R L}+F_{E} \sin 80^{\circ}-R_{\bullet} \sin 25^{\circ}=52,700 \mathrm{lb}
\end{gathered}
$$



Figure 23. Recoil Bracket and Loading Diagram
angular. The compressive load is represented by the area of a triangle, thus,

$$
R_{e}=\frac{1}{2} c t \sigma_{v}=159,000 \mathrm{lb}
$$

where

$$
\begin{aligned}
t \sigma_{c} & =56,100 \mathrm{lb} / \mathrm{in}, \text { altitude of triangle } \\
t & =2 \times 0.75=1.5 \mathrm{in}, \text { total thickness } \\
c & =5.46 \mathrm{in}, \text { base of triangle } \\
R_{t} & =159.000 \mathrm{lb} \text { (must equal } R_{c} \text { ) }
\end{aligned}
$$

The span between the two reactions is

$$
s=\frac{M}{R_{t}}=\frac{1,000,000}{159,000}=6.28 \mathrm{in}
$$

69. Figure 24 shows the dimensions and the
applied loads of the cradle body. The data are now complete for the shear and moment diagrams appesring in Figure 25. Loads upward and to the right are positive; counterclockwise moments are positive. The trunnions are located at Station 0.
$x=$ distance between stations
$y=$ distance from cradle center line to horizontal load
$H=$ horizontal load at a station
$V=$ vertical load at a station
$\Sigma V=$ tocal shear at a given station
$M_{x}=x \Sigma V_{(n-1)}$, moment due to vertical shear
$M_{v}=y H$, moment due to horizontal shear
$M=\Sigma M_{x}+\Sigma M_{y}$, moment at a given station

$F_{A}, F_{m}=$ aXAL ANO MOPMAL REACTIONS ON TRUWHONS
$f_{1}, f_{k}=$ FRICTIONAL FORCES ON FRONT ANO REAR EEARNGS
$H_{c}, V_{c}=$ horizontal ano vertical forces of cradle mass
$K_{R}=$ RECOK ROD FORCE
$R_{E}, R_{t}=$ VERTICAL LOADS DUE TO RECOL ROD OFFSET
$R_{1}, R_{2}=$ NORMAL LOACS ON FRONT AND REAR BEARTMGS
$R_{m+1} R_{n}, R_{m}=$ HORIZONTAL ANO VERTICAL LOADS OF ELEVATNG AATC

Figure 24. Cradlo Body Showing Applied Loads and Reactions
Shear and Moment Chart

| Station | $x$ | $V$ | гV | $H$ | $y$ | $M_{\text {s }}$ | $M_{y}$ | $M$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 75 | 0 | -469 | -469 | 70 | 7 | 0 | 49 | 49 |
| 25 | 50 | -69 | -538 | 6 | 0 | 2345 | 0 | 2394 |
| 22 | 3 | 332 | -206 |  | 0 | 161 |  | 2555 |
| 12 | 10 | 0 | -206 |  |  | 206 |  | 29\%1 |
| 12 |  |  |  | -388 | 10 |  | -388 | 2373 |
| 0 | 12 | 438 | 232 | -1118 | 0 | 247 | 0 | 2620 |
| -5 | 5 | 1885 | 2117 |  |  | -116 |  | 2504 |
| -5 |  |  |  | 44 | 7 |  | -31 | 2473 |
| -7.5 | 2.5 | 0 | 2117 |  |  | -529 |  | 1944 |
| -10 | 2.5 | -527 | 1590 |  | 0 | -529 |  | 1415 |
| -11.3 | 1.28 | -1590 | 0 | 1386 | 8.78 | -204 | -1218 | -7 |

To resume the analysis, assume the cylinder to have an ID of 14.5 inches and an OD of 15.5 inches. The area is

$$
A=\frac{\pi}{4}\left(\overline{15.5}^{2}-\overline{14.5}^{2}\right)=23.56 \mathrm{in}^{2}
$$

The moment of inertia is

$$
I=\frac{\pi}{64}\left(\overline{15.5}{ }^{4}-\overline{14.5^{4}}\right)=662 \mathrm{in}^{4}
$$

The bending and shear stresses are
$\sigma=\frac{M \mathrm{c}}{I}=\frac{2,761,000 \times 7.75}{662}=32,400 \mathrm{lb} / \mathrm{in}^{2}$
$\tau=\frac{\Sigma V}{A}=\frac{211,700}{23.56}=9,000 \mathrm{lb} / \mathrm{in}^{2}($ not critical $)$
With the material having a yield strength of $60,000 \mathrm{lb} / \mathrm{in}^{2}$, the bending stress shows a factor of safety of almost 2 , thus making a wall of $\frac{1}{2}$-inch thickness more than ample for the general bending stresses. However, local stresses present another problem and the wall must be


Figure 25. Shear and Moment Diagram, Cradie Body


Figure 26. Isolated Ring Section of Cradle
reinforced where they appear. Rigidity is also needed to avoid local overloading and eventual galling of the bearing. Assume a ring at the front end of the cradle body with a cross section

10 inches in length with flanges at each end as shown in Figure 26. It carries a radial load $R_{1}$ which is assumed to be uniformly distributed along the diameter of its centroida]

From Equation 35

(a) Chordal gussets
(b) taxgential gussets

Figure 27. Recoil Bracket With Gusset Reinforcement axis. According to Equation 209 of Reference $9^{*}$
$M=\frac{1}{8} R_{1} r_{n}=\frac{1}{8} \times 46,900 \times 8.09=47,400 \mathrm{lb}-\mathrm{in}$ where $r_{n}=r+.59=7.5+0.59=8.09 \mathrm{in}$, the radius to the neutral axis of the section

$$
\sigma=\frac{M c}{I}=35,000 \mathrm{lb} / \mathrm{in}^{2}
$$

where $I=1.58 \mathrm{in}^{4}$

$$
c=1.164 \mathrm{in}
$$

70. The two types of gusset construction discussed in Paragraph 51 are illustrated in Figure 27. Sketch (a) shows the gussets parallel to chords of the ring. From Paragraph 68, the total load on two gussets is

$$
\begin{aligned}
& R_{c}=159,000 \mathrm{lb} \\
& W=\frac{1}{2} R_{c}=79,500 \mathrm{lb}, \text { load per gusset }
\end{aligned}
$$

This condition is represented by Condition (d) of Paragraph 51 and Figure 21. The dimensions from Figures 23 and 27 are

$$
\begin{aligned}
a & =3.625 \text { in } & \theta & =0.440 \text { radian } \\
h & =7.22 \text { in } & \sin \theta & =0.426 \\
R_{m} & =8.5 \text { in } & \cos \theta & =0.905
\end{aligned}
$$

From Equation 34

$$
\begin{aligned}
M_{x} & =W R_{m}\left[\frac{1}{\pi}(\theta \sin \theta+\cos \theta)-\frac{1}{2}\right] \\
& =79,500 \times 8.5 \times(-0.152)=
\end{aligned}
$$

$-102,800 \mathrm{lb}-\mathrm{in}$

[^8]\[

$$
\begin{aligned}
M_{y}= & W R_{m}\left[\frac{3}{2 \pi}+\frac{1}{\pi}(\theta \sin \theta+\cos \theta)\right. \\
& \left.\quad-\sin \theta-\frac{1}{\pi} \cos ^{2} \theta\right] \\
= & 79,500 \times 8.5 \times 0.139=94,000 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$
\]

From Equation 36
$M_{\theta}=W R_{m}\left[\left(\frac{5}{2 \pi}-\frac{1}{\pi} \cos ^{2} \theta\right) \cos \theta\right.$

$$
=79,500 \times 8.5 \times 0.177=119,700 \mathrm{lb}-\mathrm{in}
$$

From Equation 37

$$
\begin{aligned}
\Delta D_{x}= & \frac{W R_{m}^{3}}{E I}\left[\frac{2}{\pi}(\theta \sin \theta+\cos \theta)\right. \\
& \left.-\frac{1}{2}\left(\sin ^{2} \theta+1\right)\right] \\
= & \frac{79,500 \times 614}{29 \times 10^{6} I} \times 0.105=\frac{0.177}{I} \mathrm{in}
\end{aligned}
$$

## From Equation 38

$\Delta \mathrm{D}_{1}=\frac{W R_{m}^{3}}{E I}\left[\left(\frac{\theta}{2}-\frac{\pi}{4}\right)+\left(\frac{2 \theta}{\pi}-1\right) \sin \theta\right.$

$$
\begin{array}{r}
\left.+\left(\frac{2}{\pi}+\frac{1}{2} \sin \theta\right) \cos \theta\right] \\
=\frac{79,500 \times 614}{29 \times 10^{6} I} \times\left(-0.104=\frac{0.175}{I} \mathrm{in}\right.
\end{array}
$$

71. Lower bending moments and deflections become available by having the gussets tangent to the ring as shown in Sketch (b) of Figure 27. Then, according to Paragraph 51 and Condition (c) of Figure 21, the bending moments and deffections are relieved despite the larger gusset load which increases by the ratio of $d / h$, the distance of the applied load to the points of attachment of the two types of gusset. Thus

$$
W=\frac{d}{h} \times \frac{R_{r}}{2}=112,800 \mathrm{lb}
$$

where

$$
\begin{aligned}
& d=10.23 \mathrm{in} \\
& h=7.22 \mathrm{in}
\end{aligned}
$$

Other dimensions of Sketch (b), Figure 27, are

$$
\begin{array}{ll}
a=3.625 \text { in } & \sin \theta=0.954 \\
R_{m}=8.25 \text { in } & \cos \theta=0.300 \\
\theta=1.268 \text { radian } &
\end{array}
$$

From Equation 30

$$
\begin{aligned}
M_{x} & =W R_{m}\left[\frac{\theta}{\pi}-\frac{1}{2} \sin \theta\right] \\
& =112,800 \times 8.25 \times(-0.074)=-68,800 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

From Equation 31

$$
\begin{aligned}
M_{\nu} & =W R_{m}\left[\left(\frac{\theta}{\pi}-1\right)-\left(\frac{\theta}{x}-1\right) \cos \theta+\frac{3}{2 \pi} \sin \theta\right] \\
& =112,800 \times 8.25 \times 0.038=35,400 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

From Equation 32

$$
\begin{aligned}
\Delta D_{x} & =\frac{W R_{m}^{3}}{E I}\left[\frac{2 \theta}{\pi}-\sin \theta+\frac{\theta}{2} \cos \theta\right] \\
& =\frac{112,800 \times 562}{29 \times 10^{5}} \times 0.044=\frac{0.096}{1} \mathrm{in}
\end{aligned}
$$

From Equation 33

$$
\begin{aligned}
\Delta D_{y} & =\frac{W R_{m}^{3}}{E I}\left[\left(\frac{2 \theta}{x}-1\right)+\left(\frac{\theta}{2}-\frac{x}{4}\right) \sin \theta+\cos \theta\right] \\
& =\frac{112,800 \times 562}{29 \times 10^{6} I}(-0.037)=-\frac{0.081}{I} \mathrm{in}
\end{aligned}
$$

72. A comparison of the required wall thickness for the two conditions shows that the tangential gussets are preferred. For the chordal gussets, the maximum equivalent stress is at $\theta$ (see Sketch (a) of Figure 27). The dimensions of the cradle wall at this location are
$b=5.46 \mathrm{in}$, assumed ring width (Figure 23)
c $=8.78 \mathrm{in}$, chordal distance to gusset
$R_{i}=7.5$ in, inside radius
$R_{\text {. }}=9.5$ in, outside radius
$t=2.0 \mathrm{in}$, wall thickness
$M_{\rho}=119,700 \mathrm{lb}$-in (see Paragraph 70)
$Z=\frac{1}{6} b t^{2}=3.64 \mathrm{in}^{3}$, section modulus
$\sigma_{c}=-M_{\theta} / \mathrm{Z}=-32,800 \mathrm{lb} / \mathrm{in}^{2}$
$I=\frac{\pi}{4}\left(R_{o}^{4}-R_{i}^{4}\right)=3910 \mathrm{ini}^{4}$
The general bending moment is taken from the Shear and Moment Chart (Paragraph 69). At Station 5
$M=2,473,000 \mathrm{lb}-\mathrm{in}$
$\sigma_{t}=\frac{M c}{I}=\frac{2,473,000 \times 8.78}{3910}=5,600 \mathrm{lb} / \mathrm{in}^{2}$
From Equations 15a and 15b
$\sigma_{t}=\sigma_{t}-\sigma_{c}=38,400 \mathrm{lb} / \mathrm{in}^{2}$, equivalent stress $S_{f}=\frac{\sigma_{y}}{\sigma_{*}}=\frac{60,990}{38,400}=1.56, \begin{array}{r}\text { factor of } \\ \text { safety. }\end{array}$
For the tangential gussets, the maximum equivalent stress is located on the $x$-axis where the general bending stress is zero. Thus

$$
\begin{aligned}
Z & =\frac{1}{6} b t^{2}=2.05 \mathrm{in}^{3} \\
\sigma_{e} & =-\sigma_{r}=\frac{-M_{x}}{Z}=33,500 \mathrm{lb} / \mathrm{in}^{2}
\end{aligned}
$$

where
$M_{x}=-68,800 \mathrm{lb}$-in (see Paragraph 63)
$b=5.46 \mathrm{in}$, assumed ring width (Figure 23)
$t=1.5 \mathrm{in}$, wall thickness
$R_{0}=9.0 \mathrm{in}$, outer radius of cradle wall
The wall thickness of 1.5 inches is preferred over that of 2.0 inches, thus demonstrating the adivantage of tangential gussets.
73. The bearing pressure induced by the gusset load is found according to the method discussed in Paragraph 52. By equating the expressions for $\Delta D_{y}$ in Equation 39 and Paragraph 71, we have

$$
(-0.467) \frac{w R_{m}^{4}}{E I}=\frac{W R_{m}^{3}}{E I}(-0.037)
$$

so that the peripheral load is
$w=\frac{0.043 W}{0.467 R_{m}}=\frac{0.037 \times 112,800}{0.467 \times 8.25}=1080 \mathrm{lb} / \mathrm{in}$ and from Equation 40, the induced bearing pressure is

$$
p_{b}=\frac{w}{6}=\frac{1080}{5.46}=198 \mathrm{psi}
$$

The pressure is less than the limit stated in Paragraph 28.

Similarly, according to $\Delta D_{y}$ in Paragraph 70, the bearing pressure induced by the chordal gusset load is
$P_{b}=\frac{0.104 W}{0.467 R_{m} L}=\frac{0.104 \times 79,500}{0.467 \times 8.5 \times 5.46}=382 \mathrm{psi}$
Although this pressure is greater than that for the tangential gussets, it is also less than the manimum allowable.

## E. TRUNNION ANALYSIS

74. The trunnions support the normal and axial forces during firing plus the couple introduced by the rifling torque. From Equation 9
$T_{r}=\frac{0.6 \pi^{2} R_{b}^{3} P_{a}}{N_{r}}=545,000 \mathrm{lb}-\mathrm{in}$, rifing torque where
$N_{r}=25 \mathrm{cal} / \mathrm{turn}$, twist of rifing
$P_{g}=36,000 \mathrm{psi}$, maximum propellant gas pressure
$R_{b}=4.0 \mathrm{in}$, radius of bore
$F_{r}=\frac{T_{r}}{d_{t}}=\underset{\text { torque }}{19,500 \mathrm{lb}, \text { trunnion load due to }}$
where
$d_{t}=28 \mathrm{in}$, span of trunnion bearings
The maxirnum load on a trunnion bearing is

$$
F_{T}=\sqrt{\left(\frac{F_{A}}{2}\right)^{2}+\left(\frac{F_{N}}{2}+F_{r}\right)^{2}}=69,500 \mathrm{lb}
$$

(see Paragraph 65 for values of $F_{A}$ and $F_{N}$.)
75. The trunnion in the hub is shown in Figure 28. Assume triangular distributions for the reactions in the housing cylinder. Then, according to the dimensions shown

$$
I=\frac{\pi}{64} \times 2.76^{4}=2.85 \mathrm{in}^{4}
$$



Figure 28. Trunnion Loads and Reactions

$$
M=1.66 \mathrm{~F}_{T}=116,000 \mathrm{lb}-\mathrm{in}
$$

$$
\sigma=\frac{M c}{I}=\frac{116,000 \times 1.38}{2.85}=56,300 \mathrm{lb} / \mathrm{in}^{2}
$$

The trunnion is made of steel with a yield strength of $90,000 \mathrm{lb} / \mathrm{in}^{2}$

$$
S_{j}=\frac{90,000}{56,300}=1.60
$$

This is a short beam, therefore, the horizontal shear stress may be severe

$$
\begin{aligned}
& \text { د. }=\frac{F_{T} A \bar{y}}{I t}=15,500 \mathrm{lb} / \mathrm{in}^{2} \\
& S_{f}=\frac{0.6 \times 90,000}{15,500}=3.48 \text { (not critical) } \\
& A=\frac{1}{2} \times \frac{\pi}{4} \times 2.76^{2}=2.99 \mathrm{in}^{2} \\
& t=2.76 \mathrm{in} \\
& \bar{y}=0.586 \mathrm{in}
\end{aligned}
$$

76. The trunnion housing shown in Figure 19 is a weldment. The principal stresses occur in the welds of the gussets and in the joint between hub and cradle body. Four gussets on each housing are parallel to the cradle axis and carry components of the axial force $F_{A}$. Four other gussets on each housing are perpendicular to the axis and carry components of the normal forces $F_{k}$ and $F_{1 /}$. The analysis of the former will be shown. Figure 29 shows the isolated gusset with the applied loads.

The numerical values of Figure 19 are
$a=4.7$ in $n=4$, number of gussets
$c=3.45 \quad r_{h}=2.75$
$F_{\mathrm{A}}=111,800 \mathrm{lb}$ (see Paragraph 65)


Figure 29. Gussat Lood Diagram
According to Equation 17, $R_{G}=28,0100 \mathrm{lb}$, the total horizontal load on each gusset. From Equations 17b and 17e
$R_{h}=R_{v}=\frac{\left(a-\frac{3}{3} c\right)}{r_{A}} \frac{F_{A}}{n}=\frac{2.4 \times 111,800}{2.75 \times 4}$
$=24,400 \mathrm{lb}$
the vertical load and reaction on the gusset. According to Equation 18,
$w=\frac{2 \times 28,000}{3.45}=16,200 \mathrm{lb} / \mathrm{in}$, maximum linear load
For a thickness of $t=0.5 \mathrm{in}$, the direct tensile or compressive stress between gusset and hub is

$$
\sigma=\frac{w}{t}=32,400-\mathrm{lb} / \mathrm{in}^{2} \quad \text { (see Equation 19) }
$$

The direct shear stress is

$$
\tau=\frac{R_{h}}{0.5 \mathrm{c}}=\frac{24,400}{0.5 \times 3.45}=14,100 \mathrm{lb} / \mathrm{in}^{2}
$$

The combined shear stress is

$$
\tau_{\max }=\sqrt{\left(\frac{\sigma}{2}\right)^{2}+\tau^{2}}=21,500 \mathrm{lb} / \mathrm{in}^{2}
$$

The combined tensile or compressive stress is

$$
\sigma_{\max }=\frac{\sigma}{2}+\tau_{\max }=37,700 \mathrm{lb} / \mathrm{in}^{2}
$$

With a tensile yield strength of 60,000 $\mathrm{lb} / \mathrm{in}^{2}$, the shear and tensile factors of safety are, respectively,

$$
S_{j}=\frac{.6 \times 60,000}{21,500}=1.67
$$


(b) TANGENTIAL GUSSETS

Figure 30. Trummion Housing With Gused Reinforcement

$$
S_{f}=\frac{60,000}{37,700}=1.59
$$

From Equation $17 f$

$$
b=\frac{r_{k}}{a-\frac{7}{3} c} c=\frac{2.75}{2.4} \times 3.45=3.96 \mathrm{in}
$$

the required gusset length for the attachment to the cradle body. The stresses here are obviously less than those between gusset and trunnion hub.
77. The effects of the gussets on the cradle wall are discussed in Paragraph 50. In Figure 29, $R$, represents the load W of Coindition (b), Paragraph 50, and the chordal gussets in Figure 30a.

$$
\begin{aligned}
& a=2 \text { in } \\
& \theta=0.237 \text { radian } \\
& h=3.45 \text { in } \quad \sin \theta=0.235 \\
& R_{m}=8.5 \text { in } \\
& \cos \theta=0.972
\end{aligned}
$$

## From Equation 25

$$
\begin{aligned}
M_{x} & =W R_{m}\left[\frac{2}{x}(\theta \sin \theta+\cos \theta)-1\right] \\
& =24,400 \times 8.5 \times(-0.346)=-71,700 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

## From Equation 26

$$
\begin{aligned}
M_{y} & =W R_{m}\left[\frac{2}{\pi}(\theta \sin \theta+\cos \theta)-\sin \theta\right] \\
& =24,400 \times 8.5 \times 0.119=87,000 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

From Equation 28

$$
\begin{aligned}
\Delta D_{x} & =\frac{W R_{m}^{3}}{E I}\left[\frac{4}{\pi}(\theta \sin \theta+\cos \theta)-\sin ^{3} \theta-1\right] \\
& =\frac{24,400 \times 614}{29 \times 10^{\circ} I}(0.253)=\frac{0.131}{I} \mathrm{in}
\end{aligned}
$$

From Equation 29

$$
\begin{aligned}
\Delta D_{\nu} & =\frac{W n_{m}^{2}}{E I}\left[\frac{4}{\pi}(\theta \sin \theta+\cos \theta)+(\cos \theta-2) \sin \theta+\left(\theta-\frac{\pi}{2}\right)\right] \\
& =\frac{24,400 \times 614}{29 \times 10^{4} I}(-0.268)=-\frac{0.139}{I} \mathrm{in}
\end{aligned}
$$

78. When the gussets are made tangent to the cradle body at the mean radius as shown in Figure 30b, they tend to develop smaller bending moments and deflections. The load $W$ increases over the above value by the ratio of $d / h$ for the reason presented in Paragraph 71.

$$
W=\frac{d}{\hbar} 24,400=44,900 \mathrm{lb}
$$

where $d=6.35 \mathrm{in}$

$$
h=3.45 \mathrm{in}
$$

Other dimensions are

$$
\begin{array}{ll}
a=2 \mathrm{in} & \sin \theta=0.859 \\
R_{m}=8.25 \mathrm{in} & \cos \theta=0.512 \\
\theta & =1.033 \text { radian }
\end{array}
$$

## From Equation 21

$$
\begin{aligned}
M_{x} & =W R_{m}\left[\frac{29}{\pi}-\sin \theta\right] \\
& =44,900 \times 8.25(-0.201)=-74,500 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

## From Equation 22

$$
\begin{aligned}
M_{y} & =W R_{m}\left[\frac{2 \theta}{\pi}+(\cos \theta-1)\right] \\
& =44,900 \times 8.25 \times .170=63,000 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

From Equation 23

$$
\begin{aligned}
\Delta D_{x} & =\frac{W R_{m}^{3}}{E I}\left[\frac{4 \theta}{\pi}+\theta \cos \theta-2 \sin \theta\right] \\
& =\frac{44,900 \times 562}{20 \times 10^{6}} \times 0.127=\frac{0.1108}{I}
\end{aligned}
$$

From Equation 24

$$
\begin{aligned}
\Delta D_{y} & =\frac{W R_{m}^{3}}{E I}\left[\left(\frac{4 \theta}{x}-2\right)+2 \cos \theta+\left(\theta-\frac{\pi}{2}\right) \sin \theta\right] \\
& =\frac{44,900 \times 562}{29 \times 10^{6} I}(-0.122)=\frac{-0.1062}{I} \mathrm{in}
\end{aligned}
$$

A comparison of results obtained for the two types of gussets shows that, although the tangential type has almost twice the load, the bending moments and deflections produced in the cradle wall are less than the maximum corresponding values of the parallel gussets.
79. To compute the induced bearing pressure, the procedure used in Paragraph 73 is followed

$$
(-0.467) \frac{w R_{n}^{d}}{E I}=\frac{W R_{m}^{3}}{E I}(-0.122)
$$

$w=\frac{0.122 \times 44,900}{0.467 \times 8.25}=1,422 \mathrm{lb} \mathrm{in}, \begin{array}{r}\text { peripheral } \\ \text { load }\end{array}$
From Equation 40, the bearing pressure is

$$
P_{b}=\frac{w}{b}=\frac{1422}{3.96}=360 \mathrm{psi}
$$

where $b=3.96$ in (see Figure 29, Paragraph 76).

The pressure meets the conditions stated in Paragraph 28.
80. The critical compressive stress obtained from Equation 20 indicates that the possibility of the gussets buckling is extremely remote.

$$
\sigma_{c}=K_{:} \frac{E}{1-\nu^{2}}\left(\frac{t}{b}\right)^{2}=540,000 \mathrm{lb} \mathrm{in}^{2}
$$

where $a=3.96 \mathrm{in}$

$$
b=3.45 \mathrm{in}
$$

$$
E=29 \times 10^{6} \mathrm{lb} / \mathrm{in}^{2}
$$

$$
\begin{aligned}
K, & =1.06 \\
t & =0.5 \mathrm{in} \\
\nu & =0.3
\end{aligned}
$$

## VII. SAMPLE PROBLEM, U-TYPE CRADLE

## A. LOAD aNALYBIS

81. A U-type cradle is selected for the sample problem involving a single recoil gun carriage. Figure 5a represents the loading diagram for the analysis

$$
\begin{array}{ll}
a=80 \mathrm{in} & d=0.1 \mathrm{in} \\
b=20 \mathrm{in} & e=8 \mathrm{in} \\
c=16 \mathrm{in} & h=7 \mathrm{in}
\end{array}
$$

$W_{1}=10,000 \mathrm{lb}$, weight of recoiling parts
$F_{0}=1,810,000 \mathrm{lb}$, propellant gas force
$K=150,000 \mathrm{lb}$, total recoil resistance
$\mu=0.15$, coefficient of friction
$\theta=60^{\circ}$, angle of elevation
82. The values of $R_{1}, R_{2}, f_{1}, f_{2}$ and $K_{R}$ are unknown. These are found by balancing the loads and moments.
From Equation 3, since $F_{1}$ is zero, the inertia force is

$$
\begin{aligned}
F_{a} & =F_{q}+W_{1} \sin \theta-K \\
& =1,810,000+8700-150,000 \\
& =1,668,700 \mathrm{lb} \\
\Sigma V & =0 \\
R_{1} & -R_{2}+W \cos \theta=0 \\
R_{2} & =R_{1}+5000 \\
\Sigma H & =0
\end{aligned}
$$

From Equation 2

$$
K_{n}+f_{1}+f_{2}-K=0
$$

but from Equations 1 and $1 a$

$$
\begin{aligned}
& f_{1}=\mu R_{1}=0.15 R_{1} \\
& f_{2}=\mu R_{2}=0.15\left(R_{1}+5000\right)
\end{aligned}
$$

therefore

$$
\begin{gathered}
K_{R}=K-\left(f_{1}+f_{2}\right)=149,200-0.30 R_{1} \\
\searrow M_{R_{2}}=0 \\
(c-e) K_{R}+e F_{a}-(e-d)\left(F_{a}-W \sin \theta\right) \\
-a W \cos \theta-(e-h) f_{1}-(a-b) R_{1}=0 \\
(a-b) R_{1}=60 R_{2} \\
(e-h) f_{1}=1.0\left(0.15 R_{1}\right)=0.15 R_{1} \\
(c-e) K_{R}=8\left(149,200-0.30 R_{1}\right) \\
=1,194,000-2.40 R_{1} \\
e F_{a}=8 \times 1,810,000=14,480,000 \\
(e-d)\left(F_{a}-W \sin \theta\right)=7.9 \times 1,660,000 \\
=13,114,000 \\
a W \cos \theta=80 \times 5000=400,000 \\
62.55 R_{1}=2,160,000 \\
R_{1}=34,500 \mathrm{lb} \\
R_{2}=39,500 \mathrm{lb} \\
f_{1}=5200 \mathrm{lb} \\
f_{2}=5900 \mathrm{lb} \\
K_{R}=138,900 \mathrm{lb}
\end{gathered}
$$

## 1. Rails and Slides

83. The reactions $R_{1}$ and $R_{2}$ are carried equally by two rails, each 90 inches long. These reaccions have an assumed triangular distribution, but the reaction due to rifling torque is distributed uniformly. The rails are 3.0 inches wide and their center-to-center distance is 17 inches. From Paragraph 74, $T_{r}=545,000 \mathrm{lb}-\mathrm{in}$

$$
\begin{aligned}
& F_{r}^{\prime}=\frac{T_{r}}{d}=\frac{545,000}{17}=32,000 \mathrm{lb} \\
& w_{r}=\frac{F_{r}^{\prime}}{90}=355 \mathrm{lb} / \mathrm{in}
\end{aligned}
$$

Let $\frac{1}{2} R_{2}$ represent the triangular portion of $R_{2}^{\prime}$ (see Figure 12).

$$
\begin{aligned}
& \frac{1}{2}\left(\frac{L}{2}\right) w_{2}=\frac{1}{2} R_{2} \\
& w_{2}=\frac{39,500}{45}=875 \mathrm{lb} / \mathrm{in} \\
& w=w_{2}+w_{r}=1230 \mathrm{lb} / \mathrm{in}
\end{aligned}
$$

The maximum bearing pressure becomes

$$
\sigma_{b r}=\frac{w}{3}=410 \mathrm{lb} / \mathrm{in}^{2}
$$

This pressure is acceptable according to Paragraph 28.

The strength of the rail or slide is determined according to Paragraph 32 and Figure 17: Assume that rail and slide have identical crosssectional dimensions

$$
\begin{aligned}
a & =b=0.75 \mathrm{in} \\
d & =e=2.25 \mathrm{in} \\
F & =1.0 w=1230 \mathrm{lb} \\
A & =1.0 \times 0.75=0.75 \mathrm{in}^{2} \\
Z & =\frac{I}{c}=\frac{1}{6} \times 1.0 \times 0.75^{2}=0.0937 \mathrm{in}^{3} \\
M & =e F=2770 \mathrm{lb}-\mathrm{in}
\end{aligned}
$$

From Equation 11
$\sigma_{\mathrm{t}}=\frac{M}{Z}+\frac{F}{A}=29,600+1600=31,200 \mathrm{lb}$ in:
$S_{s}=\frac{60,000}{31,200}=: 1.92$

## 2. Equilibrator Load

84. Caiculate the equilibrator force, $F_{E}$, by balancing the weight moment of the tipping parts about the trunnions. Referring to Figure 8
$M_{w}=r_{1} W_{1} \cos \left(\theta+\phi_{1}\right)+r_{r} W_{c} \cos \left(\theta+\phi_{2}\right)$ $=461,000 \mathrm{lb}-\mathrm{in}$
where
$r_{1}=75$ in $\quad \theta=60^{\circ}$, angle of elevation
$r_{c}=25$ in $\quad \phi_{1}=0^{\circ} 04^{\prime}$
$W_{1}=10,000 \mathrm{lb} \quad \phi_{2}=-30^{\circ}$
$W_{c}=4000 \mathrm{lb}$
The equilibrator force is found by equating the equilibrator moment to the weight moment.

$$
r F_{t:}=M_{\omega}=461,000 \mathrm{lb}-\mathrm{in}
$$

when $r=12$ in

$$
F_{\bar{k}}=38,400 \mathrm{lb}, \text { equilibrator force }
$$

85. The equilibrator attachment to the cradle is similar to the trumnion housing of Figure 31. Its location is designated in the force diagram of Figures 8 and 32. The loads on the primary cradle structure are

$$
\begin{aligned}
R_{z} & =\frac{8.91}{12} F_{E} \cos (\theta-\lambda)+\frac{1}{2} F_{E: \sin (\theta-\lambda)} \\
& =\frac{3.91}{12} \times 37,200+\frac{1}{2} \times 10,000=17,100 \mathrm{lb} \\
R_{4} & =\frac{3.91}{12} F_{E} \cos (\theta-\lambda)-\frac{1}{2} F_{\varepsilon} \sin (\theta-\lambda) \\
& =12,100-5000=7100 \mathrm{lb}
\end{aligned}
$$



Figure 31. Trunnion Housing and Cross Section of Cradle


Figure 32. Applied Loads on Cradle
$R_{E}=F_{E} \cos (\theta-\lambda)=37,200 \mathrm{lb}$
where $\lambda=45^{\circ}$ (see Paragraph 86).

## 3. Elevating Gear Load

86. The reaction on the elevating gear arc, $R_{a}$, is found by balancing the moment about the trunnions. Additional dimensions for Figure 8 are:

$$
\begin{array}{ll}
\lambda=45^{\circ} & a=0.10 \mathrm{in} \\
\gamma=50^{\circ} & b=0.2 \mathrm{in}
\end{array}
$$

$\beta=20^{\circ}$, pressure angle of gear tooth
$\boldsymbol{R}_{\boldsymbol{p}}=36 \mathrm{in}$, pitch radius of elevating arc
The applied loads and dimensions are those previously used in this sample problem. With reference to Figure 5a, the trunnions are located 5.0 inches to the left of $R_{2}$.

$$
\Sigma M_{r}=0
$$

$R_{p} R_{g} \cos \beta-r F_{E}+r_{c} W_{c} \cos \left(\theta+\phi_{2}\right)$

$$
+a\left(F_{n}-W_{1} \sin \theta\right)-r_{1} W_{1} \cos \left(\theta+\phi_{1}\right)
$$

$$
-b F_{v}=0
$$

$R_{p} R_{0} \cos \beta=36 \times .940 R_{d}=33.8 R_{u}$
$r F_{E}=12 \times 38,400=461,000$
$r_{\mathrm{c}} W_{\mathrm{c}} \cos \left(\theta+\phi_{2}\right)=25 \times 3460=87,000$
$a\left(F_{a}-W_{1} \sin \theta\right)=0.10 \times 1,660,000=166,000$
$r_{1} W_{1} \cos \left(\theta+\phi_{1}\right)=75 \times 10,000 \times .499$

$$
=374,000
$$

$b F_{\nu}=0.20 \times 1,810,000=362,000$
$33.8 R_{g}=196,000 \mathrm{lb}-\mathrm{in}$
$R_{g}=5800 \mathrm{lb}$
87. With reference to Figures 8 and 33 , the loads at the attachments of elevating arc to cradle are calculated by resolving the gear tooth load about these attachments. Take moments about the intersection of $R_{R R}$ and $R_{R I I}$, the shear reaction on the key, and solve for $R_{k l}$.

$$
\begin{aligned}
20 R_{R l .} & =4.3 R_{g} \cos (\theta+\beta+\gamma) \\
& +14 R_{g} \sin (\theta+\beta+\gamma) \\
& =4.3 \times 3700+14 \times 4400=77,500 \\
R_{R L} & =3900 \mathrm{lb} \\
R_{R / R} & =R_{R L}+R_{g} \cos (\theta+\beta+\gamma)=7600 \mathrm{lb} \\
R_{R / I} & =R_{g} \sin (\theta+\beta+\gamma)=44 \mathrm{Colb}
\end{aligned}
$$

The bolts and key size required to transmit


Figure 33. Loads on Elevating Arc
the above loads are determined as shown in Paragraph 66.

## 4. Trunnion Loads

88. The trunnion reactions, $F_{A}$ and $F_{N}$, are found by summation of forces parallel and perpendicular to the center line of the bore (Figure 8).

$$
\begin{aligned}
F_{N}= & F_{B} \sin (\theta-\lambda)+R_{g} \cos (\theta+\beta+\gamma) \\
& -W_{1} \cos \theta-W_{c} \cos \phi_{2} \\
= & 10,000+3700-5000-3500 \\
& =5,200 \mathrm{lb} \\
F_{A}= & F_{\theta}-F_{a}+W_{1} \sin \theta+W_{c} \sin \phi_{2} \\
& +F_{B} \cos (\theta-\lambda)+R_{\theta} \sin (\theta+\beta+\gamma) \\
= & 1,810,000-1,668,700+8700+2000 \\
& +37,200+4400=193,600 \mathrm{lb}
\end{aligned}
$$

89. The maximum load on the trunnion housing bolts is applied when the rifling torque is maximum.

$$
\begin{aligned}
& T_{r}=545,000 \mathrm{lb}-\mathrm{in} \text { (see Paragraph 74) } \\
& F_{r}^{*}=\frac{T_{r}}{18.5}=29,400 \mathrm{lb} \\
& \frac{F_{N}}{2}=\frac{5200}{2}=2600 \mathrm{lb} \\
& \frac{F_{A}}{2}=\frac{193,600}{2}=96,800 \mathrm{lb}
\end{aligned}
$$

With reference to Figure 31, the maximum bolt load is

$$
\begin{aligned}
R_{t}^{\prime} & =\frac{5.8}{18} \times \frac{F_{A}}{2}-\frac{1}{2}\left(\frac{F_{N}}{2}-F_{r}^{0}\right) \\
& =31,200+13,400=44,600 \mathrm{lb}
\end{aligned}
$$

There are four 5/8-11 NC bolts at $R_{t}^{\prime}$
$A=4 \times 0.202=0.808 \mathrm{in}^{2}$, total root area
$\sigma=\frac{R_{t}^{\prime}}{A}=55,300 \mathrm{lb} / \mathrm{in}^{2}$, tensile stress
$S_{f}=\frac{100,000}{55,300}=1.81$
$R_{t}=\frac{F_{A}}{2}=96,800 \mathrm{lb}$, key load
$A_{1}=4.5 \times 0.75=3.375 \mathrm{in}^{2}$, shear area
$\tau=\frac{R_{s}}{A_{\mathrm{a}}}=28,700 \mathrm{lb} / \mathrm{in}^{2}$, shear stress
$S_{f}=\frac{0.6 \times 100,000}{28,700}=2.09$
$A_{b r}=4.5 \times 0.375=1.6875 \mathrm{in}^{2}$, bearing area
$\sigma_{b r}=\frac{R_{s}}{A_{b r}}=57,400 \mathrm{lb} / \mathrm{in}^{2}$, bearing stress
The cradle material has the lesser strength. At a yield of $60,000 \mathrm{lb} / \mathrm{in}^{2}$

$$
S_{f}=\frac{1.4 \times 60,000}{57,400}=1.46
$$

90. The trunnion bearings are based on the loads that are present when the rifling torque is maximum. This is the only condition to be investigated here. Other conditions may be more critical and should be checked. From Paragraph 74

$$
F_{r}=19,500 \mathrm{lb}
$$

## From Paragraph 88

$$
\begin{aligned}
& F_{N}=5200 \mathrm{lb} \\
& F_{A}^{\prime}=193,600 \mathrm{lb}
\end{aligned}
$$

Maximum trunnion load

$$
F_{r}=\sqrt{\left(\frac{F_{A}}{2}\right)^{2}+\left(\frac{F_{N}}{2}+F_{r}\right)^{2}}=100,000 \mathrm{lb}
$$

The trunnion bearing load of the single recoil type is over 45 per cent more than that for the double recoil type illustrating one advantage of having the latter type system (see Paragraph 74). The remaining analyses of trunnion and hub follow procedures similar to those of Paragraphs 74 through 76.
91. The reactions produced by the rifling torque are transmitted directly from the slides to the trunnion housing and therefore do not enter into the analysis of the ( U -shaped) primary cradle structure. All the remaining normal and axial loads and reactions are considered. With reference to Figures 31 and 32, the total reactions now become
$R_{t}=\frac{5.8}{18} F_{A}-\frac{1}{2} F_{N}=62,400-2600=59,800$
$R_{c}=\frac{5.8}{18} F_{A}+\frac{1}{2} F_{N}=65,000 \mathrm{lb}$
$R_{1}=F_{A}=193,600 \mathrm{lb}$
B. CRADIE BODY

1. Shear and Moment Chart

| Station | $x$ | $V$ | こV | H | $y$ | $M_{z}$ | $M_{\gamma}$ | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 59.35 | 0 | 171 | 171 | 0 | 0 | 0 | 0 | 0 |
| 55.00 | 4.35 | 345 | 516 | 52 | 6.35 | -74 |  | -74 |
|  |  |  |  |  |  |  | -33 | -107 |
| 53.5s | 1.65 | 0 | 516 |  |  | -35 |  | -192 |
|  |  |  |  | 372 | 7.35 |  | -273 | -465 |
| 47.35 | 6.00 | -71 | 445 | 0 | 0 | -310 | 0 | -775 |
| 21.65 | 25.70 | -35 | 410 |  |  | -1143 | -1 | -1918-1919 |
|  |  |  |  | 20 | 65 |  |  |  |
| 12 | 9.65 | -39 | 371 | 0 | 0 | -396 | 0 | -2315 |
| 9 | 3.00 | -650 | -279 | 0 | 0 | -111 | 0 | -2426 |
| 2. | 7.00 | 0 | -279 |  |  | 195 |  | -2231 |
|  |  |  |  | 44 | 6.65 |  | 29 | -2202 |
| 0 | 2.00 | 0 | -279 |  |  | 56 |  | -2146 |
|  |  |  |  | -1936 | 7.35 |  | 1420 | -726 |
| -5 | 5.00 | -395 | $-674$ |  |  | 139 |  | -587 |
|  |  |  |  | 59 | 5.35 |  | $-2.9$ | -619 |
| -8 | 3.00 | 76 | -598 | 0 | 0 | 202 |  | $-417$ |
| -9 | 1.00 | 598 | 0 | 0 | 0 | 60 |  | -357 |
| -15 | 6.00 | 0 | 0 | 1389 | 2.65 |  | 368 | +11 |

Unite of $x$ and $y$ are given in inches; $V, \Sigma V$ and $H$ in $100 \mathrm{lb} ; M_{z}, M_{y}$ and $M$ in $1000 \mathrm{lb}-\mathrm{in}$.
The maximum bending moment occurs nine inches in front of the trunnion. The moment of inertia is based on the dimension of the cradle cross section shown in Figure 31.

## 2. Stress and Deflection

92. The bending stress of $10,200 \mathrm{lb} / \mathrm{in}^{2}$ falls far below the stress that would yield a factor of safety of 1.5 . However, rigidity is a property of higher priority inasmuch as large deflections eventually mean poor accuracy Thus, rather than decrease the section and increase its structural efficiency stress-wise, it is bettc to maintain its rigidity to promote beiter accuracy. Both structures, cradle and gun tube, combine their stiffness although they are treated as two parallel beans with no horizontal shear connection betwee., them.
93. The deflections are determined by the moment area method (refer to Figure 34). First, the deflection is determined for the structure at the breech end by computing the moment of the $M^{\prime} E I$ area at this point, Station -15.0. This deflection is normal to the tangent of the elastic line at Station 59.35. Referring to the Shear and Moment Chart of Paragraph 91 , the value of $M I$ is computed for each station and drawn to scale in Figure 34.

$$
I=I_{r}+I_{T}=4600 \mathrm{in}^{4}
$$



The moment of inertia at the base line ( $B L$ ) is
$I_{B L}=\Sigma A d^{2}+\Sigma I_{O}=4200 \mathrm{in}^{4}$
$\bar{d}=\frac{\Sigma A d}{\Sigma A}=6.65 \mathrm{in}$, distance from base line to neutral axis
$I_{c}=I_{B L}-\Sigma A d^{2}=1590 \mathrm{in}^{4}$, moment of
$c=14 \quad \bar{d}=7.35 \mathrm{in}$
$\sigma=\frac{M c}{I}=\frac{2,426,000 \times 7.35}{1590}=\frac{11,200 \mathrm{lb} / \mathrm{in}^{2},}{\text { bendingstress }}$
In the above table
Dimension $=$ base $\times$ height of parts of the section
$A=$ area of each part
d = distance from base line to neutral axis of part
system, the recoiling parts equivalent to those of a single recoil system, i.e., tube, breech assembly, guides or sleigh, and those parts of the recoii mechanism which move with the tube.
recoiling parts, secondary. In a double recoil system, the cradle, those parts of the primary recoil mechanism which do not. move with the tube, the top carriage, and all those parts attaclied to it.
recuperator. The equipment that stores some of the energy of recoil for counterrecoil.
ring, breech. Breechblock housing, screwed or shrunk on the rear of a cannon.
sleigh. The housing of a gun tube that slides in a U-type cradle during the recoil cycle.
slide. Same as rail.
tipping parts. The assembled structure of a
weapon which moves in elevation or depression about the trunnions.
torque, rifing. The reaction on the gun tube of the angular accelerating forces on the projectile.
traversing gear. A gear rigidly attached to the traversing parts and meshed with the traversing mechanism.
trunnion. The cylindrical structural component of the cradle which serves as the pivot for the tipping parts and which transmits the recoil forces to the top carriage.
trunnion bearing. The beariag that supports the trunnion.
tube, gun. A hollow cylinder, usually of steel, in which a rcund of ammunition is fired and directed.
tube whip. The flexing of the gun tube due to accelerating forces nozmal to the tube axis.

## REFERENCES

1. ORDP 20-340, Ordnance Engineering Deaign Handbook, Carriages and Mounts Series, Carriages and Mounts, General.
2. ORDP 20-342, Ordnance Engineering Design Haadbook, Carriages and Mounts Series, Recoil Systems.
3. ORDP 20-345, Ordnance Engineering Design Handbook, Carriages and Mounts Series, Equilibrators.
4. A Report on the Analysis of Gun Carriage, 175 mm , T76, The Franklin Institute, Final Report F-2240, Part I.
5. Charles D. Hodgman, et al, Ed., Handbook of

Chemistry and Physics, 40th Ed., Chemical Rubber Publishing Company, Cleveland, 1958-1959.
6. Hoffman and Sachs, Introduction to the Theory of Plasticity for Engineers, McGraw-Hill Book Co., Inc., New York, 1953.
7. R. J. Roark, Formulas for Stress and Strain, 3rd Ed., McGraw-Hill Book Co., Inc., New Yori, 1954.
8. ORDP 20-134, Ordnance Engineering Deeign Handbook, Maintenance Engineering Principles for Design Engineers.
9. F. B. Seely and J. O. Smith, Advanced Mechanics of Materials, John Wiley and Sons, Inc., New York, 1952.

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[^0]:    * Prepared by Martin Regina, Laboratories for Research and Development of The Franklin Institute. $\dagger$ Reference 1. References are found at the end of this handbook.

[^1]:    * Reference 3

[^2]:    * Reference 2, Chapter XI.

[^3]:    * Reference 4.

[^4]:    * Reference 5, page 2239.

[^5]:    *Reference 6, Page 39.

[^6]:    * Reference 7, Page 312, Conditional A, Case 4. Reprinted by permission from Formulas for Stress and Strain, 3rd Ed., by R. J. Roark, Copyright 1954, McGraw-Hill Book Co., Inc.

[^7]:    * Reference 7, Page 142, Case 18. Reprinted by permission from Formulas for Stress and Strain, 3rd Ed., by R. J. Roark, Copyright 1954, McGraw-Hill Book Co., Inc.

[^8]:    * Reprinted by permission from Advanced Mechanics of Materials by B. F. Seely and J. O. Smith, Copyright 1952, John Wuiey \& Sons, Inc.

