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## ABC PAMPHLET

ENGINEERING DESIGN HANDBOOK

BALLISTICS SERIES

## INTERIOR BALLISTICS OF GUNS



AMCP 706-150, Interior Ballistics of Guns, forming part of the Ballistics Series of the Army Materiel Command Engineering Design Handbook Series, is published for the information and guidance of all concerned.
(AMCRD)
FOR THE COMMANDER:

SELWYN D. SMITH, JR.
Major General, USA
Chief of Staff

## OFFICIAL:



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## PREFACE

The Eingineering Design Handbook Series of the Army Materiel Command is a coordinated series of handbooks containing basic information and fundamental data useful in the design and development of Army materiel and systems. The handbooks are authoritative reference books of practical information and quantitative facts helpful in the design and development of Army materiel so that it will meet the tactical and the technical needs of the Armed Forces. Several of these handbooks give the theory and experimental data pertaining to interior, exterior and terminal ballistics. The present handbook deals with the interior ballisties of guns.

This handbook, Interior Ballistics of Guns, presents fundamental data, followed by development of the theory and practice of interior ballistics, with application to rifled, smooth-bore and recoilless guns. Included in the presentation are studies pertaining to heat transfer, temperature distribution and erosion, together with standard and experimental methods of measurements. Finally, ignition, flash and other special topics are explored.

This handbook has been prepared as an aid to scientists and engineers engaged in military research and development programs, and as a guide and ready reference for military and civilian personnel who have responsibility for the planning and interpretation of experiments and tests relating to the performance of military materiel during design, development and production.

The final text is the result of the joint writing efiorts of 1 . N. Jones, H. 1P. Hitchcock and D. R. Villegas, of the staff of John I. Thompson and Company, for the Engineering Handbook Office of Duke University, prime contractor to the Army Research Office-Durham. Many valuable suggestions were made by the Interior Ballistics Laboratory and Development and Proof Services at Aberdeen Proving Ground, Picatinny Arsenul, Frankford Arsenal and Springfield Armory. During the preparation of this handbook Government establishments were visited for much of the material used and for helpful diseussions with many technical personnel.

Elements of the U.S. Army Materiel Command having need for handbooks may submit requisitions or official requests directly to Publications and Reproduction Agency, Letterkenny Army Depot, Chambersburg, Pennsylvania 17201. Contractors should submit such requisitions or requests to their contracting officers.

Comments and suggestions on this handbook are welcome and should be addressed to Army Research Office-Durham, Box CMI, Duke Station, Durham, North Carolina 27706.

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## CHAPTER 1

## LIST OF SYMBOLS

B Burning rate coeflicient

1. Mass of propedling charge
('. Specific heat of propellant gas at constant volume
© ${ }^{\circ}$. Average specifie heat of propellant gas at constant volume
r Mass of propellant burned
I) Outer diamoter of propedlant grain
d Diameter of the perforatiois of propellant grain
E: Specific energy or potential ei solid propsllant
F Forec of propellant
$f$ Faction of weblenamaci through
$I$ Internal eneigy of propellant gas
$K \quad$ Energy expended by propellant gas in doing work and heating tubs
.1/ Mass of profectile
$N$. Number of grains in the change
$n$. Number of moles of gas per unit weight of propellant gas
1 P Pressure
R Molar gas constant
s. Area of burning surface of propelling charge
$S_{0} \quad$ Initial surface area of propelling charge
I' T'emperature of propellant gas
To Adiabatic flame temperature of propellant
$t$ Time
$\dot{I}^{\circ}$ Volume of the propellant gas
" Travel of projectile
1 Velocity
$w \quad$ Wel) thickness
$\alpha \quad$ Burning rate pressure exponent $\gamma \quad$ Factor analogous to the ratio of specific heats at constant pressure and volume Specific covolume of propellant gas
$\rho \quad$ Specific mass of the propellant

## CHAPTER 1

## DISCUSSION OF THE PROBLEM

## 1-1 INTRODUCTION

The imparting of high velocities to projectiles requires tremendous fores. The source of the energy which supplies these forees must be readily manufactured, easy to transport, and capable of being safoly applied. At various times, proposals have been made for utilization of energy provided by means other than explosives, such as compressed air, electromagnetic force, and centrifugal force. Thus far, however, no results have been attained from any of these sources which approach those realized from chemical explosives.

Interior ballistics of gums (that branch of ballisties dealing with motion imparted to a projectile by a gun) comprises a study of a chemical energy source, a working substance, and the aceessory apparatus for controlling the reloase of energy and for directing the activity of the working substance. Of allied interest is the mechanical functioning of guns and acessories. (ieneral information on the types of guns and their construction and functions is given in Reference 1. References applicable to each chapter of this Handbook are given at the end of the chapter.

Since unnecessary weight is an unjustified logistical extravaganee, weapons are designed to operate under greater extremes of temperature and pressure than are usually eneountered in the use of nommilitary ragines. Because the time eyche involved is quite small, there is not sulficient time for the consummation of slow processes such as heat transfor. Consequently, it is necessary that the chemical energy source also furnish the gascous products which in themselves constitute the working substance. This energy souree may be a solid propellant, as in most gans, or a ligaid fuel and oxidizer source, such as is sourtimes used in rocket propulsion.

Propellants are studied from several aspects. Thermodymanic properties indicate the release of as much energy por unit woight as may be consistent with other demands. Studies of the mechanism of decomposition indicate the effects of unconfrollable parameters such as ambient temperature. Dymamies of the gases are meeresarily a subject of invertigntion beranse the kinetie mergy of the
propelling gases is an important part of the total energy of the process. The study of motion of a projectile inside the gun tube is not a matter of simply applying Newton's laws to the motion of the projectile regarded as a point mass, but a complicated study of the rate at which the high temperature gas is evolved from the propellant; the motion of the gas so produced; and the effeet of this gas on the motion of the projectile itself. The passage of the projectile stresses the tube mechanically and subjects the interior of the barrel to siding friction. The passage of high temperature gases, in addition to the high pressures generated, heats the barrel to the extent that chemical interaction with the metal itself occurs.

Interior ballistics is defined as the branch of applied mechanics which deals with the motion and behavior characteristics of projectiles while under the influence of the gases produced by the propellant. As an applied science it is still much of an art and largely empirical. The phenomena with which it deals are explicable in terms of well established physical and chemical principles. Unfortunately, the phenomena are complex and related in subtle and obscure ways so that considerable experience and judgment are necessary in the application of the principles if trustworthy theoretical results are to be derived. There occur in the formulation of the theory quantities which are difficult to determine by independent measurement because their proper yalues for particular cases depend in obscure ways on the particular circumstances of the case considered. They have the nature of empirical correction factors whose values can frepuently be estimated only from the results of numemus examples involving comparison of the theory used with the records of actual firings. The begimer is, therefore, forewarned to be on his guard. All theoretieal results should be as fimly backed up by comparison with actual firings as is possible. In this sense the theory serves as a means of interpolation between, or extrapolation from, existing designs.

The subject of interior ballisties of guns has been investigated through more than 200 years, starting with the invention of the ballistic pendulum in 174.

I very extensive litemature has bern bill up, and many cxedent fexts ame avalable. For gemomal background, the fexts prepared hy (omore and Homet are rerommended. More apereifie treatments hawe berom made hy Bemett' and Taylor and Yagi". A consolidated XI ) a ( report, writton by (urtiss and Wremeh", covers the work done daring Word War II. A remeral teatment of the problem with applications to guns is given in liaforener 7 .

## 1-2 GUN:

## 1-2.1 Defin:tion

The ter:n gn" in this heandmook, mbless otherwise indieated, may be taken in its general semse, that is, a projeretile-throwing deviere consisting essentially of a projectila-guiding tube, with an incorporate or comeded reaction chamber in which the chemical cuergy of a propedlant is rapidly converted into hoat and the hot gases produced expand to exped the projeretile at a high velority.

## 1-2.2 Classification

For rompenimere of disenssiong guns are ratasified ateoding to their salient features, funetions, modes of operation, etc.' The bomdarios of these classificafons ate not anways dody defined, and the elassifications and momenclature are olten traditional. The chassilieations are useful, howerer, and are in common use. The primeipal one is based roughly on size and portability and classifies gums as small arms and artillery. Simall arms are in general hess tham :30mm in caliber and are minally portable by foot soldiers. Artillery comsists of the larger weapons usmally momed on campures and moved by other than human power. small arms are more variable in design and function. Thery include such weapons as rifles, machine gums, pistols, ate, Artillery weapoms inelude guns (spereific). howityers and mortars. (imes (spercifie) indelude those fiting usually at lower elevation and higher veloeity, and howitzors include those which operate in gemeral in a lower vedocity range. The latter ran bre fired at high angles and mese
zoned chatres, that is, charges which arre baded in separate increnacots and can be varied withan limits by the gumere. Mortars opserate at high angess like howitzers but operate at still bower volocitios and are gemerally loaded from the muzale. They are simple in design and can be broken down and transported by foot soldicers.

## 1-2.3 Action Inside the Gun

A gun is mesontially a heat emgine. Its artion resembles the pewer stroke of an atomobile catgine with the expansion of hot gases driving the projectile instead of a piston (Figure 1-1). When the charere is ignited, gases are crolved from the surface of emeh grain of propellant, and the pressure in the chamber inereases rapidly. Resistaner to initial motion of the projectile is great and relatively high chamber pressites ate attaned before mueh motion of the projectile takes place. In the solution of the interior ballisties problem, fietitions starting pressures are assumed, which work well in practiere.

The chamber volume is inereased by the movement of the projectile, which has the efleet of derreasing the pressure; howewer, the rate of homing of the chatren inereases. The bet affere is a rapid inerease in the propellant peresure until the point of maximum pressure is reached. This oremes at a relatiody short distane from the owing of rifling. Beyond that point, pressume drops and, at the muzale, reaches a value comsiderably less than maximmen pressure, probably of the order of 10 er to 30);, thereor, depending upon the wapon design and the propellant. This muz\%le pressure eontimes to act on the projertile for a shont distaner berond the mazale. Thus, the projeretile confimes to ace ederate beyond the mazzle.

A spereial form of this method of propulsion is represented by the recoilless system (Figure 1-2). Here recoil forecs are rountered be the discharge of mases through a nozale at the breeth. The rate of disehauge of gases can be controlled be controlling propellant burning, thus permitting a batamer of the momentum of the gimeproperiant gas-projectile




FIGURE $1 \because$. Recoilless Gun System.
system. The interior ballistic problen here is not only one of combustion but of balancing the orifice diameter againse thrust required to maintain a mean recoil velocity of the weapon at zero. The propellant weight in this case exceeds that for a comparable recoil gun by a factor of 2 to 3 . The pressure-travel curve is designed for minimum muzzle velocity consistent with satisfactory exterior ballistic performaner, thus permitting the use of a thin gum tube which is necessa:y to maintain the characteristic light weight of this weapon. The subject of lecoilless weapons and other leaking guns is covered more fully in (hapter 2 of this handbcok.

## 1-3 PROJECTILES

Projectiles, like gans, exist in a great variety of designs, depending upon the intended use. Since most of the design characteristics do not affect the interior ballisties, we shall consider only a few. The most important of these factors is the mass of the projectile. This must always be taken into account in the formulation of interior ballisties theory, as it has a major offect on aceeleration and velocity of the projectile, as well as on the propellant pressure at all points.
Another very important characteristic is the design of the rotating band on those projectiles which are to be spin stabilized. The band is slightly lagere than the tube diameter and must be swaged to the tube diamoter and engraved by the rifling. The result of this process is a high initial resistance to motion of the projectile, which means that the gases must build up a relatively large starting pressure before the projectile has moved appreciably. This has an important effect on the interior ballisties, partieularly on the maximum pressure reached and the time at which it oceuss. This varable is largely rliminated in recoilless weapons in which the rotating band of the projectile is prengraved to fit the rifling. It is also climinated in smooth bore wompons which fire fin-stabilized projertiles. Here an important
factor is the amount of clearance between the projectile and the tube, as this determines the leakage of gas around the projectile. The principal weapon having this problem is the mortar. Here, with muzzle loading, the clearance must be sufficient to permit the escape of air so that the projectile will slide down the bore and strike the n̈ring pin with the impact energy required to initiate the primer.

Only one other characteristic of the projectile need be mentioned and that is the axial moment of inertia for spir-stabilized projectiloc. And here the effect on interior ballistics is quite small, as the energy of rotation normally represents only a fractional percent of the energy of translation of the projectile.

## 1-4 DISTRIBUTION OF ENERGY

As an indieation of the relative magnitude of the factors involved in utilizing the energy developed by the burning of the propellant in a medium caliber recoil gun, the following possible distribution is given:

| Eneryy Absorleed | of Total |
| :---: | :---: |
| Translation of projectile | 32.0 |
| Frictional work on projectile <br> (Due to engraving of rotating <br> bands and wall friction) |  |
|  |  |
|  |  |
| Trunslation of propellant gases | 3.0 |
| Heat loss to gun and prajectile | 20.0 |
| sensible and latent heat losses in propellant gases | +2.0 |
| Rotation of projectile and transation of revoiling parts (ench about 0.1 <br> and residuals in approximations total) |  |

Propellant potential, $\boldsymbol{E}$ ( (O), (K)
Distribution of the available energy of the propellant charge is discussed in Chapter 2, as basie to the solution of the interior ballistics problem.

## 1-5 PRESSURE-TRAVEL CURVES

In order that the projectile may acquire the designated muzale volocity, and that the pressures
developed to accomplish this do not damare the weapon, all tubes are designed in accordance with a desimable pressure-traved curve for the proposed weapon."

The pressure-travel curves (Figure 1-3) indicate the pressure (or force if pressure is multiplied by the cross-sectional area of the bore) existing at the lase of the projectile at any point of its motion. Hence, the area under any of the curves represents the work done on the projectile per unit cross-sectional area, by the expanding gases.
If the areas under curves $A$ and $B$ are equal, then the work performed in cach of these cases will be erfual, and the muzzle velocities produced by eath of these propellants will be the same, sine

$$
W O R K=K E=\frac{1}{2} M I^{\circ}
$$

The fact that curve A exeeeds the permissible pressure curve camot be tolerated.
should it be desired to increase the mozzle velocity of a projectile, the work performed, or the area under some new carve, must be greater than the area under a curve giving a lower muzzle velocity. Such an increase in volority is indicated by curve ${ }^{\prime}$ whose maximum pressure is equal to that of curve $B$, but whose area is greater tha: that under $B$. It appears that the iden! pressure-travel curve would be one which would concide with the curve of permissible pressure; however, if it were possible to design a propellant capabli: of producing such a result, many objectionable oceurenees would take
place. In addition to producing execseive erosion (a factor which would materially decrease the accuracy life of the gun), brilliant flashes and nonteniform velocities due to high muzzle pressure would result. Moreover, the chamber would have to be materially increased and this would affect the weight and hence the mobility of the gun. As a result of experience, the velocity prescribel for a particular gun is always soncwhat below the maximum which it is pasible to obtain; and the propellant grain most suitable for producing this result is the one which will give the preseribed velocity uniformly from round-to-round without execeding the permissible pressure at any point in the bore.

## 1-6 CONTROL OF INTERIOR BALLISTIC. PERFORMANCE

Consideration of the desired relationships between gas pressure and projectile velocity necessary to mect the demands imposed for the achicevement of desired ballistic performance has been diseussed in a general sense; however, it remains a fundamental problem of interior ballisties to determine and evaluate the influence of all variables of the problem. The solution may be based on theoretical analysis, established empirical relationships, or detailed, meticulous experimentation.

The variables basie to the problem include the following:
a. Variation in chemical composition of the propellant.




FIC:CRE 1-1. Pressure-Trard lidlutionship.

1. Variations in rate of reaction.
c. Variations in ignition characteristics.
d. Variation in grain geometry (surface factors).
c. Variation in charge weight (density oil loading).
f. Envirommental factors.

## 1-7 EFFECTS OF PROPELLANT GRAIN CHARACTERISTICS

Assuming proper ignition of all propellant grains, the characteristic shaping of pressure-travel or pres-sure-time relationship, for the gun system is dependent on such variables as grain composition (guickness), grain size, grain configuration, and density of loading. Although in a final design all factors may be involved, it is of basie importance to note first the independent effects of such variables.

Propellant compositions (single-base, double-base, nitrognanidine, ete.) and definitions of configurations (degressive, neutral and progressive burning propellants) are discussed in subsequent paragraphs of this chapter. Performance of gum systems is usually demonstrated using pressure (I')-travel (11) coordinates, although pressure-time relationships are often used in experimental investigations.
in cach case discussed in this paragraph, initial burning rates are directly related to area exposed for the total number of grains per charge; bence, it is difficult to consider the influenee of single factors without making allowance for the total area initially exposed to kindling temperatures. For any pressure-travel curve, the shape of the curwe is affected by the variables shown in Figure 14. For a given presure-traved eure (ligure 1-1) the slope of the eure in the region (1) to (2) is dietated by ignition chametroisties and total area initially.
exposed to burning. The region (3) to (t) will be governed primarily by the grain configuration. The methods of manufacturing propellants and determining and maintaning the desired configuration of the propellant grains are covered in Referenees 7 and 9.

## 1-7.1 Grain Configuration

Exposed burning arer as a function of "percent grain consumed" (ligure 1-10) offers a key to the effects of configuration on pressure-travel relationships. As indicated in Figure $1-i$, changing configuration to a more progressive burning design (employing grains of the same initial surface area, composition, and total charge weight) results in lowered peak pressures (with peak pressure oceuring later in the cycle) and in higher muzzle pressur: when compared with degressive grains. For identical charge weight, areas under the curve are approximately equal. In order to meet requirements for equal initial surface areas for the total charge, the degressive grains must be the smallest of the designs considered.

## 1-7.2 Grain Size

For a fixed weight of charge of similar composition and contiguration, shaping of pressure-travel relationships may be accomplished by varying the initial area exposed to burning hy varying grain size. Similar offects illustrated in ligure 1-i) result as grain size is inereased (Figure 1-(i).

Similarly, comparative results of independentle: rarying composition (quickness) or web thickness (a combination of size and configuration parameters) can be demonstrated. In adapting such relationships
to speceific gum systems, a compromise of their characteristics must be utilized. Hand and shoulder weapons require pressure-travel relationships that minimize muzzle blast at the expense of reaching high peak pressures and, characteristically, utilize "quick", degressive, small-grained propellant design. High peak pressures, avoided in larger guns because of design problems of the gun tubes, are minimized by propellant designs based on "slow," progressive or neutral burning configurations of large size.

## 1-7.3 Density of Loading

The various typers of gums, with different calibers and lengths, and each with its own muzele velocity requirement, present special problems for the propellant designer. The lengths of travel of the projectile in the bore and, consequently, the times of its travel, differ greatly. In addition, the volume of the chamber and the weight of the projectile introduce elements which must enter into the selection of a propellant for a gum.

Since muzzle energy is directly dependent on the amount of charge burned, it becomes necessary to consider feasible means for increasing the total amount of energy made available to perform useful work done on the projectile. It is possible, by choosing increasingly large charges of slow propellants, to obtain increased velocity without exceeding the maximum allowable pressure. Efficiency will be correspondingly lowered; hence, it is not advantageous to fire slow propellant in a gun not designed for it. Irregularity in muzzle velocity is closely associated with overall efficiency. If the burning rate is lowered enough, unburned propellant is expelled in varying amounts, increasing irregularity, muzzle blast, and flash. With slower propellants, the point of maximun pressure occurs later, thus demanding stronger, and therefore heavier, construction over the length of the tube. Conversely, increasing the weight of charge of propellant of given quickness increases the maximum



$u$
FIGCRE: 1-6. Effrets of Indepentenlly Vorrying Grain Size. (Charge we:yht is ryuul in earh case.)
pressure attained and causes it to occur sooner in the travel of the projectile.

## 1-8 BLACK POWDER*

Black powder, once the only available propellant, is no longer used for that purpose. It is still of interest because of other military uses. It is manufactured as small, shiny black grains. The ingredients are usually finely pulverized potassium or sodium nitrate, charcoal, and sulfur which are incorporated into an intimate mechanical mixture. The charge is pressed into a cake and pressed or extruded to the desired grain size ahd shape. The grains are glazed with graphite to prevent coking and accumulation of static clectricity. The potassimm or sodium nitrate (about $75 \%$ ) acts as an oxidizing agent, while charcoal (about $15 \%$ ) and sulfur (about $10 \%$ ) are combustibles. Sulfur also lowers the ignition temperature of the mixture from $340^{\circ} \mathrm{C}$ to $300^{\circ} \mathrm{C}$.
Black powder is no longer considered suitable as a propellant because of its many objectionable features and because of the development of newer propellants in which the undesirable qualities have been overcome or improved. It is difficult to control accurately the burning speed of black pewder. Consequently, the range of a projectile propelled by it may vary. Black powder is too easily ignited, being extremely sensitive to heat and friction, and therefore, must be handled very carefully. It is hygioscopic, which requires that sealing precautions be taken to retain stability. Its strength is relatively low and the large amount of solid residue which it leaves makes smoke reduction difficult. Flash reduction is also a problem with black powder.

Black powder, in its several grades, is still used for the following military purposes:
a. Propellant igniters in artillery ammunition.
b. Delny elements in fuzes.

[^0]

FIGCIRE 1-7. Typical Sherpes of Propellant Grains.
c. Saluting and blank charges.
d. Spotting charges for practice ammunition.
c. Safety fuse (burning rate, 1 ft in $30-10$ seconds).
f. Quickmatch (burning rate, 9-120 ft per second).

## 1-9 GUN PROPELLANTS*

## 1-9.1 Present Gua Propellants

Present gun propellants are forms of nitrocellulose explosives with various organic and inorganic additives. They may be divided by composition into ciasses of which two, the single-base and double-base, are the most common. Both classes are manufactured in quantity in a variety of shapes including flakes, strips, sheets, pellets, or perforated cylindrical grains (Fizure 1-7). The cylindrical grains are made in

[^1]various diameters and lengths, depending on the size of the gun. Figure 1-8 shows, to approximately $\frac{7}{3}$ full size, grains for various calibers of guns. The grains for a caliber . 30 cartridge are 0.0 .32 inch in diameter and 0.08.i inch long, while those for a 16 -inch round are 0.947 inch in diameter and $2 \frac{7}{16}$ inches long. The perforations shown in Figure 1-7 are for the purpose of controlling the rate of gas liberation as well as burning time.

## 1-9.2 Burning Time

The burning time can be controlled by the following means:
a. The size and shape of the grains including the number of perforations (ligure 1-7).
b. The web thickness or amount of solid propellant between burning surfaces; the thicker the web, the longer the burning time (Figure 1-9).


FIGURE: 1-8. Sies of Some Timpicul Graine.


FI(iURI: 1-!). W'el Thickness and Routc of Burning Progress through a l'rogressively Burning Grain.
c. The quickness or rate of burning of the propellant.
d. The percentages of volatile materials, incert materials, and moisture present. A $1 \%$ change in volatiles in a low volatile content propellant may cause as much as a $10 \%$ change in burning rate.

## 1-9.3 Burning Action

Cnconfined smokeless propellant burns with little ash or smoke. When confined, its rate of burning increases with temperature and pressure. In order not to exceed the permissible chamber pressure of the weapon, the time of bu:ning of the propellant is controlled. At constant pressure the time of burning is proportional to the amount of exposed propeliant surface. Therefore, a propellant charge is made up of accurately sized grains of specificd shape.

Since the grains burn only on exposed surfaces, the rate of gas evolution for a given propellant will depend upon the area of the burning surface. For a given weight of propellant the initial burning surface will depend upon the form and dimensions of the grains. As burning continues, the rate of combustion and of pressure variation will depend upon how the area of surface changes, that is, upon the rate of area increase or decrease. ligure 1-10 shows, for typical grain configurations, the relation between percent of grain consumed and area of burning surface.

The rapidity with which a propellant will burn depends upon the chemical composition, pressure, and area exposed to burning. The quickness of a propellant is a relative term only, expressing its rate of burning compared with others. A quick propellant will burn more rapidly and produce a higher pressure in a given gun than a slow one. Propellants of fixed weight, chemical composition,
and grain geometry may be made quicker by decreasing size, thus increasing burning arca.

## 1-9.4 Degressive Burning

The total surface of a propellant grain changes with burning, and on cord and strip forms the surface ares of the grain decreases. The burning action of these grains is classified as degressive.

## 1-9.5 Neutral Burning

As a single-perforated grain burns, the outer surface decreases and the inner surface inereases. The result of the two actions is that the net burning surface remains approximately the same. The burning of this type of grain is known as neutral.


FICUURL' 1-10. Relntise Areas of Burning ds a Function of Percent of Indinidual Grain Consumed, for Scureal Typical Grain Shropes.

## 1-9.6 Progressive Burning

When the multiperforated grain burns, the total surface area increases since the perforated grain burns from the inside and sutside at the same time. This type of burning is called progressive (Figure 1-9). When a multiperforated grain is not completely consumed, as may be the case when a reduced charge is used, portions of the grain remain in the form of slivers and may be ejected as such from the weapon. The rosette grain (ligure 1-7) was designed to reduce the formation of slivers.

## 1-9.7 Single-Base Propellants

Single-base propellants are cesentially gelatinized nitrocellulose to which various organic substances are added cither to produce improved cualities or for special purposes. Single-base propellants are amber, brown, or black in color, depending on the additives present.

Single-base propellant is rather insensitive. In fact, it is dillicult to ignite, recquiring a powerful primer and additionally, in large ammunition, a black powder igniter. It ignites at $315^{\circ} \mathrm{C}$. In the open, single-base propellant burns very much like celluloid. sommingly, this explocive is very safe but the fact should not be overlooked that, although it is used as a low explosive, it is an organic nitrate and may detonate if burned in large quantities. It may also detonate syimpathetically from the detonation of other explosives, although in actual pactice this rately oceurs. Single-base propetlant is more powerfal than black powder, giving off 1000 calorices and 900 eubic centimeters of gas per gram, compared with 700 calorics and :300 cubic centimeters per gram of black powder. It has a burning speod of 0.1 to 18 eentimeters per second at pressures up to 00,000 pounds per spuare inch.

Simplo-base propeliant is mistable and decomposes in hot moist storage. It is hygroseopic, although not as hygroseopic as hack powder. Nitrocellulose in the presence of moisture hydrolizes to free acid, which takes the form of oxides of nitrogen. These oxides aceelerate the decomposition, building up heat to an ignition temperature, and spontancous combustion may result. Addition of a chemical stabilizer brings the stability to acerptable limits.

To summarize; the characteristies of single-base propellants are:
a. C'mentrolled bu:ming. The burning time of singlebase propediants can be controlled to a point where the masimum propedling colle et is obtained.

1. S'msitivily. Ignition is diflicult, and the propollant is remsombly safe.
c. Stability. The propellant is unstable, but this can be controlled to within acceptable limits by the addition of stabilizers.
d. Residue. There is some residue and smoke.
e. Manufacturc. This is complicated but safe. Rew materials are plentiful.
f. Erosive action. Single-base propellant erodes the bore, but not quite as much as black powder. Its isochoric adiabatic flame temperature is $2400^{\circ} \mathrm{K}$ to $3000^{\circ} \mathrm{K}$.
g. Flash. This is caused by hot gases which ignite when they come into contact with oxygen at the muzzle. It can be controlled by adding cooling materials to the propellant.
Single-base propellant can be produced in a form lacking most of the objectionable features.

The propellant grains for small arms are usually glazed with graphite to facilitate machine loading and to prevent the accumulation of static electricity, and thus present a black, polished apper nee. Since the grains are small, they ignite more readily and burn morc frecly than cannon propellant. When moisture is, present or abuormal temperatures prevail, they are subject to more rapid deterioration tinan the larger grains.

## 1-9.8 Double-Base Propellants

This form of propellant is cssentially a combination of nitroglycerin and nitrocellulose with certain additives to give special properties. The nitroglyeerin inereases the potential and reduces hygroscopicity, the latter improving the stability. The color of the grains is gray-green to black, and the forms are the same as for single-base propellant.

Double-base propellant is more sensitive than single-base propellant, igniting at $1: 50^{\circ} \mathrm{C}$ to $160^{\circ} \mathrm{C}$. It detonates more readily than single-base propellant and can be made to yilld a higher potential and liberate more heat, but produce a smaller volume of gas. The burning rate, generally faster than that of single-base propollant, can be controlled similarly.

The characteristics of double-base propellants are, in summary:
a. Combrolled burning. Burning can be controlled, as with singlo-base propellants.
b. Sensitivity. This is greater than for siagle-base propellant, slightly increasing hazard.
c. Stability. Double-base propellants can be made stable by the addition of stabilizing ingredients.
d. Residur. Siner there is not so much inert material, there is little solid residue. Smoke can be controlled.
r. . Manufarlure. Not as sale as single-base pro-
pellant due to presence of nitroglycerin. Raw materials are readily available.
f. Erosice action. High temperature and heat of explosion from the higher potential dotible-base propellants cause more crosion than results from use of single-base propellants.
g. Plash. As in the case with single-base propellants, flash can be controlled to a certain extent by the use of additives. The presence of nitroglyeerim aceentuates the tendency to flash by increasing the flame temperature.

Double-base propellants have limited use in artillery weapons and in small arms in the U. S. They are widely used in mortans, where erosion is not an important factor. However, they are used as the standard propellants in most other comutries. The ('. S. Army and Navy both evaluated single-base and double-base propellants in guns prior to World War I and decided in favor of the former due to their lesser erosive effect.

## 1-9.9 Nitroguanidine (Triple-Base) Propellants

A propellant containing nitroguanidine in addition to nitroglyecrin and nitrocellulose as principal ingredients is commonly referred to as a triple-base propellant. This type of propellant was developed in Creat Britain during World War II as a result of researeh directed toward obtaining a propellant with desirable properties such as cool burning, low crosion, and flashlessness, without decrease in stability or potential. The British have designated their nitroguanidine propellant as Cordite N. The nitroguanidine propellant, designated $M-15$, developed by the Conited States, represents an interim solution for selected rounds of ammunition where the flash or obscuration problem is critical and where its special properties are particularly necded.

The M-1.5 propellant has a ballistic potential comparable to single-base propellants currently in use but with a lower erosive effect and less tendency to flash.

## 1-9.10 Solvent Emulsion Propellant (Ball Powder)

A radically different manufacturing process uses a volatile solvent to form propellant in small grains of spherical shape, designated Ball Powder.* The sizes of the grains are appropriate for use in small arms. The propellant is produced by dissolving wet nitrocellulose in a solvent (ethyl acetate) with additives. When a protective colloid is added and the solution is agilated, small globules are formed.

[^2]When the volatile soivent is removed by evaporation the globules solidify, and when coated, dried, and graphited, become balls or spheres, $\Lambda$ wide variety of double-base and siagle-base compositions may be produced by this technique. Because of the economy and speed with which this powder can be mannfactured, this propellant has promise in future applications not limited to small arms.

## 1-9.11. Characteristics of Standard Propellants

The compositions of some of the standard and experimental (XI and T designations, respectively) propellants, and some of the thermodynamic and calorific values of them are given in Table 1-1. The practice, as illustrated therein, of specifying certain additives, coatings and residues as percentages of the total of the principal constituents. resulting in over $100 \%$ total contents, is standard in the explosives field.

## 1-9.12 The Rate of Burning

Since the burning of a propellant occurs only on the exposed surfaces, the smallest dimensions between the exposed surfaces become the critical dimensions, as it determines in general when the propellant will be completely consuncd. This critical dimension is called the web. As was explained in discussion of the progressive, multiperforated grain, burning through of the web is followed, in this case, by burning of the slivers. A correeted form of multiperforated grain is the rosette, illustrated in ligure 1-7. With the exception of multiperforated grains, all forms of propellant grains are completely consumed when the web is burned through.

In the multiperforated grains, having seven symmetrically located perforations, Figure 1-9, the web may be calculated f:om the formula

$$
\begin{equation*}
w=0.2 ;(D-3 d) \tag{1-1}
\end{equation*}
$$

where
$w=$ web thickness
$D=$ outside diameter of the grain
$d=$ diameter of the perforations.

Experimental measurements show that the rate of burning of a propellant is primarily dependent on the pressure under which the reaction proceeds and this dependence may be expressed approximately by the pressure to some power.

The mass rate at which gas is produced may then be expressed as

$$
\begin{equation*}
\frac{d c}{d l}=p_{1} S B P^{\prime \prime} \tag{1-2}
\end{equation*}
$$

wher
r is the mass of propellant bumed
$l$ the time
$I$ the pressure
Is the burning rate coceliciont
ar the buming rate pressure expoment
$\rho$ the sperific mass of the propellant
is the atrea of the bumbing surface
The deperndence of the buming surface area on the remaining webl as the charge is consumed ean be colloulated from the geometey of the graia by assuming that this geometry docs not change during huming, i.e., that the lincar buming rate is the samme at all points on the burning surface loor propellant in thin sheres, the area of the edges is nergligible and the surface is eonstant during burning. For single perforated cylinders, if $S_{0}$ is the initial surfare and $f$ the fraction of the web harned throagh

$$
\begin{equation*}
\left.S=S_{11}-2 \pi \cdot V / w(I),+l_{1}\right) \tag{1-:i}
\end{equation*}
$$

 The sereond term on the right arises from the combincel cfiect of the change in the area of the cond surfaces and the raduction in longth of the grain. Otherwise, the decrease in the otster diameter i: compensated by the in:erease in the inner dianneter. If the second teim can be neglected, the bimning surfare is constant. lion single perforated grains of the usual propertions, the meglect of this second term gives a surface area, at bumont of the charge. Whirh is approximately ten pereent tow high. That is, the charge is actually sonsowhat degressive. loor seven perforaterl grains, the charge is progressivo until the wel) is burned through; after which it is degrexvive.

I rolation such as liguation $1-3$, which takes acconnt of the colfect on the rate of gas coolution of the changing buming surface area, is called the form function of the granulation. Tlis function is simple for shorets, cords and long single porforuted grains or tubs. For seven perforated grains, it is complex, especially after splintering. lior the simple shapes it is expressed as a polynomial in the remaining wol, but for seven perforated grains it is often given in tabular form. Fommas for the stafaces for complex shapes are given in Reference 10. lior other formutations of the form function sere Reforconcos $2,: 3$ and 6 .

In pratiee the differone in the interior hallistics for single and multiple porforated grains is not as great as theory indicates. In calculating the surfaces from the geometry, the assumption is made that
the ignition is simbltabeons over all surfaces. 'This is never the case in practiere. For the seven perforated changes the degressive haming of the slivers remaining after the web is burned through tends to reduce the progressive chanteter of the carly buming. Also the buming rate is influeneed not only by the pressure but also by the flow of the gas over the grain and within the perforations. Whe shape of the zamins is bot caactly maintaned during buming. Dixerpt, thew fore, for highly degressive grains such as cords, the assumption of a constant burning surface is aderpuate.

With this assumption, the buminer surfitere of the chatge may be calculated by the formula

$$
\begin{equation*}
S=\frac{2(!}{\rho w} \tag{1-4}
\end{equation*}
$$

whore
(' is the mass of change
In practier, the rate at whicin gas is crolved depends on the detailed comditions mader which the charge is burmed. is standad method for dotemmining buming rates is to burn the charge in a closed chamber at co: ${ }^{\text {stant }}$ volume and mousure simultaneously the pressure and its time derivative. 'Then, if the relation betwere! $r$ and $I$ ' is known, a value for the buming rate corlliciont, $B$, can be: derived by the use of liduations $1-2$ and $1-4$. The cocflicient, $B$, so detormined, is also called the closed chamber buming rate conliciont, and it is usod mainly for comparative purposes and for standardizing propellant lots. The details of the mothod are given in IReferencos $: 3$ and 7.
'The elosed chamber burning rate coceliciont rarely yields grod agrecoment with observation if used in interior ballistic calculations for gans. 'The conditions in the gen are very diforent, and the burning rate eocfliciont must be determined by adjustment to the results of actual firings. By ohserving the results of numerous firings when fitted to a given formulation of the thoory, the user can estinate a burning rate coefficient for a particular case which then can be adjusted to the actual case in fuestion.

The burning rate pressure index, $\alpha$, varies for different propellants, but the latest experiments indicate that it lies betweon 0.8 and 0.9 . A figure as low as $\frac{1}{2}$ has been used by some authors, and frequently it is assumed to be equal to umity. In the: latter case, the soletion of the erpuations of the theory can be given analytically, otherwise this is not possible and numerical methods must be used. With the development of high speed eomputing

TABLE 1-1 CALCULATED THERMOCHEMICAL
VALUES FOR STANDARD PROPELLANTS
(INCLUDING RESIDUAL VOLATILES)
(Located in the back of this handbook)
nachines, this is not the disadrantage it once was.
The value of a may be determined from closed thamber measurements. The meti:ods are deseribed 11 Reference 3.

## 1-9.13 Energy of Propellants

The propellant gas is a complex mixture of several cases und for the misture to have the same properties of independence of energy from density) all changes n equilibrium which occur must be equivoluminar and independent of the density. This in effect estricts the theory to "cool" propellants,* that is, o those for which the temperature is not high mough to produce significant dissociation of the nain constituents of the gas mixture. To the approxmation of the assumed equation of state, each ropellant formulation has a definite explosion temcrature. Thus, the decomposition of a unit mass if propellant always liberates the same amount of nergy which then heats the product gases to the ame temperature independent of the density. For nost propellants, the most important equilibrium ; the water gas equilibrium and since this equilibium is equimolar, the assumed equation of state s sufficiently accurate for use in the interior ballistic heory of guns. The use of a more accurate equation f state would greatly complicate the theory and ;ould not be justified in view of other simplifying ssumptions and approximations which are always art of any formulation of the theory. In treating he thermochemistry of propellants, however, a more ccurate equation of state must be used. Fxtended reatments of the thermochemistry of propellants re given in References 1 and :\%.
It is standard practice in the formulation of iterior ballistic theory to assume an equation of tate of the simple covolume type. This is an equaion of state of the Van der Waals type with the $a$ " term omitted but not the " $b$ " term and is nown as the Abel eguation of state. lior a gas beying such an ecquation of state, the internal nergy depends only on the temperature and not n the density. The Abel equation is expressed as

$$
\begin{equation*}
I^{P}\left(I^{\cdot}-\eta\right)=n R T \tag{1-i}
\end{equation*}
$$

n interior ballistics it is usually written in terms f a unit weight of gas, so that $V$ and $\eta$ have dimenions volume per unit weight and $n$ is the number $f$ moles per unit weight. Many authors also define i as the gas constant per unit weight so that $n$

[^3]does not appear explicitly in the conation. If wo defined is not constant tanless $n$ is also.

If $T_{0}$ is the adiabatic flame temperature, the energy released by the decomposition of enit weight of propellant, called the "force" of the propehant, although it has the dimensions of energy per unit weight, i.e., length, is defined by

$$
\begin{equation*}
F=n R T_{0} \tag{1-f}
\end{equation*}
$$

The force can be determined experimentally by burning a charge of propellant in a closed chamber (i.e., at constant volume) and measuring the maximum pressure produced and using Liquation 1-5, along with suitable cooling corrections. To do this reguires a knowledge of $\eta$ which can be dotermined simultancously by firing a series of charges of different masses and measuring the corresponding maximum pressures.

Table 1-1 includes values of the foree for a number of standard and experimental gun propellants. Foree and other thermodynamic parameters of propellants can be calculated theoretically if the necessary thermochemical data are available. Results of extended calculations of this sort are given in Reference 11. The subject is also covered briefly in Reference 9.

During the operation of the gum, the gas is produced at temperature, $T_{0}$, and falls to a lower temperature, $T$, due to the loss of heat to the tube and the performance of work during the expansion. The change in internal energy per unit mass of gas can be expressed as $\bar{C},\left(T_{0}-T\right)$ where $\bar{C}$, is ant average value of the specific heat of the gas at constant volume averaged over the temperature range, ( $T_{0}^{\prime}-7$ ). This energy is used in heating the gun and in imparting kinetic energy to the projectile, the gas, and moving parts of the weapon. The quantity $\bar{C}, T_{0}$ is called the specific energy or potential of the propellant.

It is assumed in interior ballistics that the average specific heat at constant volume, $\bar{C}$, bears the same relation to $\rho$, the gas constant, as the specific heat at constant volume for a perfect gas docs, so that it may be stated $n / R=\bar{C},(\gamma-1)$. However, $\gamma$ is not now the actual ratio of specific heats, but is analogous to it, its value being adjusted for best fit to the theory used. In effect, $\bar{C}$, is replaced by $n R / \gamma-1$. Then, from lequation 1-(i), if $E$ denoter the potential, $\bar{C}, T_{0}$

$$
\begin{equation*}
E=\frac{F}{\gamma-1} \tag{1-7}
\end{equation*}
$$

E' represents effectively the total energy available
from unit mass of propellant. It is equal approximately to the internal energy of unit mass of the propellant gas at the adiabatic flame temperature, $T_{0}$, which is given by $\int_{0}^{T \cdot} C, d T$.
The equation of state of the gas in the gun is written as

$$
\begin{equation*}
P\left(U_{v}-c \eta\right)=c n R T \tag{1-8}
\end{equation*}
$$

where $c$ is the mass of propellant burned, equal to the mass of gas, and $U_{1}$ is the actual volume of the gas. Actually the pressure is not uniform throughout the mass of gas, nor is the temperature, so both $I^{\prime}$ and $T$ are unknown average values consistent with the equation as written, and $P, U_{0}, c$ and $T$ are rapidly varying functions of the time.
$\eta$ and $n$ are also variable, but less so, so that average values can be used.

If it is assumed that the internal energy, $I$, of the gas can be represented by $c \bar{C}_{0} T$ with sufficient accuracy, then

$$
\begin{equation*}
I=\frac{P\left(U_{0}-c \eta\right)}{\gamma-1} \tag{1-9}
\end{equation*}
$$

and the general energy equation of interior tallistics can be written

$$
\begin{equation*}
\frac{c F}{\gamma-1}=\frac{P\left(U_{\theta}-c \eta\right)}{\gamma-1}+K \tag{1-10}
\end{equation*}
$$

where $K$ is the energy expended by the gas in the doing of work and in heat conducted to the gun.

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## CHAPTER 2

## LIST OF SYMBOLS

A Area of the cross section of the bore, int ${ }^{2}$
A Area of the cross section of a nozzle, in ${ }^{2}$
$A_{1} \quad$ Leakage area, in ${ }^{2}$
a Accelcration, $\mathrm{in} / \mathrm{sec}^{2}$
a Throat arca of nozzle, in ${ }^{2}$
$a_{10} \quad$ Sonic velocity in gas, in/sec
$B \quad$ Burning rate coefficient, in/sec-P"
$B \quad$ Momentum index, dimensionless
C Weight oî the propellant, lb
$C_{0} \quad$ Weight of propellant for ideal riffe, lb
$C_{r} \quad$ Thrust coefficient, dimensionless
c Weight of propellant burnt, lb
c. Specific heat at constant volume, in-lb/12 slugs- ${ }^{\circ} \mathrm{K}$
d Caliber of the gun, diameter of the projectile body, in
E. Specific energy of the propellant, in-lb/lb
$E_{n} \quad$ Standard. specific energy of the propellant, in-lb/lb
e Ballistic efficiency, dimensionless
F Specific force of the solid propellant, in-lb/lb
$F$. Engraving force, lb
$F_{r}$ Thrust force, lb
$f$ Similarity factor, dimensionless
f Momentum factor, dimensionless
/ Heat loss ratio, dimensionless
g Gravitational acceleration, in/ $\mathrm{sec}^{2}$
$I$ Internal energy of the gas, in-lb
$I_{s} \quad$ Axial moment of inertia of the projectile, $\mathrm{lb}-\mathrm{in}-\mathrm{sec}^{2}$
$K \quad$ Work done by the gas, in-lb
$l \quad$ Axial radius of gyration of the projectile, in
$l$ Leakage factor, dimensionless
$l$ Reduced chamber length, dimensionless
.II Effective mass of the projectile, 12 slugs
$M_{1} \quad$ Modified effective mass of the projectile, 12 slugs
$m \quad$ Log. 10 (approximately 2.3026 )
$m \quad$ Momentum of gun, lb-sec
$N$ Angular velocity of the projectile, rad/sec
$N$ Proportion of the propellant that is in a recoilless rifle in gaseous form, dimensionless
$n \quad$ Lead of rifling, dimensionless
I' Space average pressure, $\mathrm{lb} / \mathrm{in}^{2}$
p $\quad P / \pi$, dimensionless
in Reduced pressure, dimensionless

Q Incat loss, in-1b
$q$ Quickness (Bemnett), dimensionless
$q$ Rate of flow, $\mathrm{lb} / \mathrm{in}^{2}$-sec
$q_{1} \quad$ Empirical quickness factor, (in $\left.{ }^{3} / \mathrm{lb}\right)^{\frac{1}{3}}$
$R \quad$ Weight of gun and recoiling parts, lb
R. Gas constant, in-lb/ $/ \mathrm{lb}-{ }^{\circ} \mathrm{K}$
$r$ Lincar rate of regression, in/sec
$r$ Ratio of actual to tabular velocity, dimensionless
$r \quad$ Ballistic parameter, dimensionless
$r_{1} \quad$ Empirical velocity factor, ( $\left.\mathrm{lb} / \mathrm{in}^{3}\right)^{\frac{1}{2}}$
$S \quad$ Surface area of the grains, in ${ }^{2}$
$s \quad$ Space ratio (expansion), dimensionless
$T$ Energy of the fraction of the charge burined, in-lb
T' Temperature, ${ }^{\circ} \mathrm{K}$
$T^{\prime} \quad$ Ratio of the gas temperature at any time after burnt to the mean value during burning, dianensionless
$T_{0} \quad$ Adiabatic flame temperature, ${ }^{\circ} \mathrm{K}$
$t$ Time, sec
$\tau$ Reduced time, dimensionless
$l$ Dimensionless time
$U \quad$ Free volume, in ${ }^{3}$
$U$ Volume from nozzle throat to base of projectile in recoilless riffes, in ${ }^{3}$
$V_{c h} \quad$ Chamber volume, in ${ }^{3}$
$U_{0} \quad$ Volume of the propellant gas, in ${ }^{3}$
$u \quad$ Specific volume of the solid propellant, $\mathrm{in}^{3} / \mathrm{lb}$
$u_{0} \quad$ Specific volume of the gas, in ${ }^{3} / \mathrm{lb}$
$u_{10} \quad$ Specific volume of water: $27.68 \mathrm{in}^{3} / \mathrm{lb}$
$V$ Velocity of the projectile, in/sec
$V_{1} \quad$ Leakage velocity coefficient, in/sec
$V$. Velocity of the recoiling parts of the gun and carriage, in,'sce
V. Sonic velocity in air, in/sec
$\overline{-} \quad$ Dimensionless projectile velocity
$\bar{V} \quad$ Average projectile velocity after burnt, in/sec
$v \quad$ Velocity of gas, in/see
IV Weight of the projectile, Ib
IV. Effective projectilc weight (Strittmater), lb
$W^{\prime} \quad$ Effective projectile weight, Ib (Mayer and Hart; and Bemett)
$w \quad$ Web thickness, in
$\lambda$ Travel of the projectile, in
$x$ Axial coordinate of the projectile, in
$\alpha_{1}$ biscape sperd of gas, in see
(*) Differential cocflicient: $\left.\left(C_{1} \partial I_{r}\right)^{\prime}\right)^{\prime}\left(I_{r} \partial C C^{\prime}\right)$, dimenvionless
$\therefore \quad$ Buming rate coeflicient (linearlaw), in'/ Ib-sec
3 Jifferential codficient: $\left(V_{c h} \partial V_{m}\right){ }^{\prime}\left(V_{m} \partial C_{c h}\right)$, dimensionless
$\sigma_{1} \quad$ Differmitial coefliciont: $\left(l_{c h}{ }_{c h} \partial J_{n}\right),\left(I_{\nu} \partial C_{c h}\right)$, dinumsionk(:ss
$\gamma$ Ratio of specific heats of the gas (assumed effective value: 1.30 )
Differritial coofficicut: $\left(\mathrm{X}_{\mathrm{m}} \partial \mathrm{I}_{m}\right)^{\prime}\left(\mathrm{V}_{m} \partial \mathrm{X}_{m}\right)$, dimensionless
$\%$ Ratio of specific heats of the gas, adjusted to taki account of loss of heat to the gum, dimensionless
1 Irensity of loading. specific gravity of loading, dimmensionless
Differential corfliciont: ( $\left.\Delta \partial\right|^{\circ}$. $)\left(I_{m} \partial \Delta\right)$, dimensionless
Pidduck-Kent constant, dimensionless
Differential coofficient: $\left(\Delta \partial I^{\prime},\right)_{\prime}^{\prime}\left(I^{\prime}, \partial \Delta\right)$, dimensionless
Interior ballistic parameter, in-lb
Interior ballistic parameter: $\psi a w^{\prime} / \beta C \lambda^{\frac{1}{2}}$, dimensionless
Specific covolume of the gas, in ${ }^{3}$ /lb
Differential coefficient: ( $\left.\mathrm{IV}^{\prime} \partial \Gamma_{m}\right)^{\prime}\left(\Gamma_{m} \partial W^{\prime}\right)$, dimensionless
Differential cocfficient: ( $\left.\mathrm{IV}^{2} \partial P_{\nu}\right) \cdot\left(I_{r}, \partial \mathrm{IV}\right)$, dimensionless
lactor accounting for rotational energy and frictional resistance (assumed value: 0.05 )
Fraction of total energy available to projectile: Katsanis factor, dimensionless Differential cocfficient (wor ${ }^{\prime}$ ) ${ }^{\prime}\left(\mathrm{I}^{\circ} \not{ }^{\circ} \partial w\right)$, dimensionless
$\kappa_{1} \quad$ Jifferential coefficient: $\left(w \partial I_{p}\right) /\left(I_{\nu}^{\prime} \partial w\right)$, dimensionless
$\lambda$ Differential coofficiont: $\left(E^{\prime} \partial V_{m}\right) /\left(V_{m} \partial E\right)$, dimensionless
$\lambda_{1}$ Differential cocflicient: $\left(E_{\partial} I_{p}{ }_{p}\right) /\left(P_{p} \partial E\right)$, dimensionless
$\mu(\omega)$ Pressure function, dimensionless
$\mu \quad$ Ratio of throat area to bore area: $A, A$, dimensionless
$\nu(\omega)$ Travel function, dimensionless
$\xi \quad$ Dependent variable: $\omega_{i} /\left(d K^{\prime} / d T\right)$, dimensionless
$\pi \quad \operatorname{Pressure}$ factor, $\mathrm{lb}, \mathrm{in}^{2}$
$\rho \quad$ Specific weight of propellant, $1 \mathrm{~b} / \mathrm{in}^{3}$
$\sigma \quad$ Ballistic parameter: $F \rho S B / A(\gamma-1)$, di-
$\tau(\omega)$ Time function, dimensionless
$\tau$ Time unit, sec
$\phi$ Independent variable: $\log (T / \mathrm{E})$, dimensionless
$\phi \quad$ Proportion of the propellant burned, dimensionless
$\psi \quad$ Ballistic parameter, function $\lambda$, dimensionless
$\omega \quad$ Dependent variable $\left(K^{/} / T\right)^{\frac{1}{2}}$, dimensionless

## Subscripts

$0 \quad$ Initial value: when $t=0$
1 Characteristic of fast propellant in dual granulation charge
2 Characteristic of slow propellant in dual granulation charge
Of atmosphere
At end of burning: "burnt" value
Chamber value: at breech
At nozzle exit
Bither 1 or 2
Muzzie value: when base of projectile is at muzzle
. .ozzle opening
At peak or theoretical maximum
In reservoir
Space maximum at any instant
At base of projectile
Tabulated value (Bemett)
At nozzic throat

## CHAPTER 2

## THEORY AND PRACTICE OF INTERIOR BALLISTICS

## 2-1 INTRODUCTION

There are numerous systems of interior ballistics. Different ballisticians have formulated the theory in various ways. Their systems, if they are not purely empirical, do not differ essentially since they are treatments of the same thermodynamical and mechanical phenomenon. They differ in the simplifying assumptions made, that is, mainly in degree of complexity and sophistication of treatment and in the details of the mathematical procedures. For many practical problems, very simple formulations are adequate and these are much used. However, with the widespread and increasing availability of high speed automatic computers the more complicated formulations can be used without too much expenditure of time and effort.
There are five general equations which are used in the formulations of interior ballistic theory. They are: (1) the equation of state of the propellant gases; (2) the equation of energy; (3) the equation of motion; (4) the burning rate equation; and (5) the equation of the form function. The first two of these are related, as the first is involved in the formulation of the second; therefore, only four equations are basic to any particular formulation of the theory.

The form of the equation of state of the propellant gases generally used in interior ballistics has been discussed in Chapter 1 and is given in Equation 1-8. The equation has bren shown, by experience, to be sufficiently accurate for the purpose.

The equation of energy has also been discussed in Chapter 1 and its form is given in Equation 1-10. The equation of energy is a statement of how the energy released by the combustion of the propellant is distributed during the operation of the gun.

The equation of motion is the formulation of Newton's second law as applicable to the interior ballistic problem. It relates the forces due to the gas pressure to the acceleration of the projectile.

The burning rate equation takes account of the rate at which new gas is being generated in the gun by the combustion of the charge. This rate is assumed to be a function only of the pressure, under which the combustion takes place, and the area of the
reacting surface. The form of the equation used here is given in Equation 1-2.

If the reacting surface is not constant, it is necessary to introduce the form function to account for the effect of the changing burning surface on the rate of generation of gas in the gun. Except for very degressive granulations, the assumption of constant burning surface is generally sufficiently accurate and this assumption is made in the explicit treatments which follow.

## 2-2 STATEMENT OF THE EQUATIONS

## 2-2.1 The Energy Equation

There is presented here first the formulation due to Taylor'. The fundamental units used in the Taylor system are the inch, pound (weight) and second. This makes mass a derived unit with dimensions weight over gravitational acceleration. With the length unit the inch, the unit of mass is equal to 12 slugs.
The energy equation may be stated sinsply as

$$
\begin{equation*}
T=K+I \tag{2-1}
\end{equation*}
$$

where
$T$ is the energy released by the amount of charge which has been burned
$K$ the work done by the gas, plus energy lost by heating the barrel
$I$ the internal energy of the gas
By Equation 1-10

$$
\begin{equation*}
T=\frac{c F}{\gamma-1} \tag{2-2}
\end{equation*}
$$

where $F$ is now defined as the energy per unit weight of propellant (specific force) and by Equation 1-9

$$
\begin{equation*}
I=\frac{P\left(U_{\theta}-c \eta\right)}{\gamma-1} \tag{2-3}
\end{equation*}
$$

where $C_{0}$ is the volume occupied by the gas. $K$, the work done by the propellant gas, consists of several parts.
a. The principal part is the translational kinctic
energy of the projectile, equal to $\mathrm{WI}^{2} / 2 y$ where $W$ is the weight of the projectile, $V$ its velocity and $g$ the gravitational acceleration.
b. Treating the gases and unburned propellant as a fluid consisting of the gases and burning solid grains thoroughly mixed, and supposing the tube to be cylindrical (that is, neglecting chambrage), an approximation of Iagrange in which the density of the fluid is assumed independent of position may be used for ordinary velocities. Then the velocity of this fluid increases linearly from 0 at the breech to $1^{\circ}$ at the base of the projectile; the average velocity is $V / 2$, and the mean square velocity is $r^{2} / 3$. Hence, the kinetic energy of the unburned propellant and the gas is $C V^{2} /(\mathrm{Kg}$, where $C$ is the weight of the propellant charge.
c. The projectile, propellant, and recoiling parts of the gun and carriage may be considered a system whose initial momentum is zero and remains so providing there is no recoil mechanism or shoulderof the user to prevent free recoil. Under this assumption, if $R$ is the weight of the recoiling parts and $-I^{\circ}$, their velocity, the momentum equation is

$$
\begin{equation*}
\frac{1}{g}\left(W V+\left(V^{\prime} / 2-R V^{F}\right)=0\right. \tag{2-4}
\end{equation*}
$$

whence

$$
\begin{equation*}
\mathrm{r}_{r}=\frac{(W+(i / 2) V}{R} \tag{2-i}
\end{equation*}
$$

Therefore, in free recoil, the kinetic energy of the recoiling parts is

$$
\begin{equation*}
\frac{\left(W+(C / 2)^{2} V^{2}\right.}{2 g R} \tag{2-5a}
\end{equation*}
$$

For standard weapons, the ratio (II $+C / 2$ ) $/ R$ is of the order of 0.02 . The energy of recoil is, therefore, a very small fraction of $K$. The recoil velocity is much less than that given by Equation 2-j; as the energy is absorbed in cannoa by the recoil mechanism. In nonautomatic small arms fire the energy is transferred to the body of the man firing the weapon. The energy of recoil can be neglected without scrious effect.
d. The rotational kinetic energy of the projectile is

$$
\begin{equation*}
\frac{I_{x} N^{2}}{2} \tag{2-6}
\end{equation*}
$$

where
$I_{s}$ is the axial moment of inertia and
$N$ is the angular velocity.

The axial moment of inertia may be expressed as

$$
\begin{equation*}
I_{s}=h^{2} \frac{W}{g} \tag{2-6a}
\end{equation*}
$$

where
$k$ is the axial radius of gyration
The angular velocity is

$$
\begin{equation*}
N=2 \pi V / n d \tag{2-7}
\end{equation*}
$$

where
$n$ is the lead or twist of rifling in calibers, that is, the number of calibers in which the land makes one complete turn and
$d$ is the caliber
By substitution, it is found that

$$
\begin{equation*}
\frac{I_{x} N^{2}}{2}=\left(\frac{2 \pi k}{n d}\right)^{2} \frac{W V^{2}}{2 g} \tag{2-8}
\end{equation*}
$$

For most guns, $n$ is between 18 and 32 calibers, say 2.). For a solid cylinder, $(k / d)^{2}$ is $0.12 \bar{i}$; but for high explosive projectiles, it is about 0.14 . Therefore, the factor $(2 \pi k / n d)^{2}$ is approximately 0.01 .
$e$. The work done against the frictional resistance to the motion of the projectile, including the engraving of the rotating band, is equal to a small proportion of the translational kinetic energy, say

$$
\frac{\theta_{1} W V^{2}}{2 g}
$$

If $\theta_{1}$ is taken to be constant, this is equivalent to assuming that the resistance is proportional to the pressure. The value of $\theta_{1}$ is usually of the order of 0.04 .
$f$. The heat transferred from the hot gas to the gun is denoted by $Q$. Adding the contributions $a$. to $/$.

$$
\begin{align*}
K=\frac{W V^{2}}{2 g} & +\frac{C V^{2}}{6 g}+\frac{(W+C / 2)^{2} V^{2}}{2 g R} \\
& +\left(\frac{2 \pi k}{n d}\right)^{2} \frac{i W V^{2}}{2 g}+\frac{\theta_{1} W V^{2}}{2 g}+Q \tag{2-9}
\end{align*}
$$

Equation 2-9 is complicated but can be simplified in the following manner. Drop the third term on the right as negligible, and let

$$
\begin{equation*}
\theta=(2 \pi k / n d)^{2}+\theta_{1} \tag{2-10}
\end{equation*}
$$

In the calculations, $\theta$ will be taken equal to $0.0 \%$. Also, let an "effective" mass be defined as

$$
\begin{equation*}
M=\frac{(1+\theta) W+C / 3}{g} \tag{2-11}
\end{equation*}
$$

Then, if $Q$ is neglected or accounted for in some other manner, $K$ takes the simple form

$$
\begin{equation*}
K=\frac{M V^{2}}{2} \tag{2-12}
\end{equation*}
$$

It is customary to take account of the heat loss Q by adjusting the value of $\gamma$ upward so that the estimated total available energy of the gas is reduced. The effect of the heat loss is to :educe the kinetic curergy produced by the gas and has an effect similar to an increase in the effective mass. Assuming that the dffect can be taken account of by simply inercasing the effective mass by a constant factor,

$$
\begin{equation*}
Q=\int \frac{. u V^{2}}{2} \tag{2-1:3}
\end{equation*}
$$

Then, omitting the covolume term for simplicity

$$
\frac{1}{\gamma-1}\left(f F-P U_{y}\right)=\frac{M V^{2}}{2}(1+f)
$$

and if $\bar{\gamma}$ is the adjusted value of $\gamma$

$$
\frac{1}{\bar{\gamma}-1}\left(c l-P^{\prime} U_{y}\right)=\frac{M V^{2}}{2}
$$

ss) that

$$
\begin{equation*}
\frac{\bar{\gamma}-1}{\gamma-1}=1+\rho \tag{2-14}
\end{equation*}
$$

In practice $\rho$ is about 0.1. but may be considerably laterer than this especially in small arms. To take areount of hat loss then, one simply substitutes the value of $\dot{\gamma}$ for $\gamma$ wherever the latter occurs. Taylor assumed an adjusted value of $\gamma$ ecpual to 1.36. In what follows it will be assumed that $\gamma$ has been adjusted so that, in the subsecquent text, $\gamma$ represents the adjusted value, unless the heat losis, $Q$, oceurs explicitly in the equation. For more detailed diselssion of the heat loss problem, see References 2 and $\mathbf{i}$, also ('oner', page 141 .

The interna! energy of the propellant gas is then rexprosised as

$$
\begin{equation*}
I=\frac{P\left(U_{0}-r \eta\right)}{\gamma-1}=\frac{P U}{\gamma-1} \tag{2-1.5}
\end{equation*}
$$

where
$I$ is an arerage pressure consistent with the equation of state and
$r^{\circ}$, the volume oecupied by the gas
The complete expression for the free volume is

$$
\begin{equation*}
U=.1 . X+c_{c n}-c u+c u-c \eta \tag{2-16}
\end{equation*}
$$

where
$A$ is the area of the cross section of the bore
$X$ the travel of the projectile
$U_{c h}$ the chamber volume
$u$ the specific volume of the solid propellant and
$\eta$ the specific covelume of the gas.
Initially $\mathbf{X}=0$, the free volume is then

$$
\begin{equation*}
U_{0}=U_{c h}-C u+c_{0}(u-\eta) \tag{2-17}
\end{equation*}
$$

where $c_{0}$ is the weight of the small amount of charge burncd before start of projectile motion. Substituting $U$ from Eq. 2-16 in Eq. 2-15 the equation for the internal energy of the gas is
$I=\frac{P}{\gamma-1}\left[A X+U_{c \mathrm{~A}}-C u+c(u-\eta)\right]$
The specific volume of the propellant (u) is about $17 . \mathrm{j}^{\mathrm{j}} \mathrm{in}^{3} / \mathrm{lb}$. The specific covolume ( $\eta$ ) (the volume apparently occupied by the molecules in $\Omega$ unit weight of gas) is approximately $27.7 \mathrm{in}^{3} / \mathrm{lb}$. The specific covolume of the gas is thus about 1.5 times the specific volume of the solid propellant. It is assumed here that they are equal, $\eta=u$ for the sake of simplicity. This assumption is often made in interior ballistics, but its validity becomes questionable for high ratios of charge to chamber volume.
With this assumption, Equation 2-17 becomes

$$
\begin{equation*}
U_{u}=U_{t A}-C u \tag{2-17a}
\end{equation*}
$$

and from Eqs. $2-16$ and 2-18

$$
\begin{gather*}
U=U_{0}+A X  \tag{2-16a}\\
I=\frac{P}{\gamma-1}\left(A X+U_{r h}-C u\right)
\end{gather*}
$$

Substituting from Jiquations 2-2, 2-12 and 2-18a in Eifuation $2-1$ the energy equation becomes

$$
\begin{equation*}
\frac{F c}{\gamma-1}=\frac{M V^{2}}{2}+\frac{P}{\gamma-1}\left(A X+U_{c h}-C u\right) \tag{2-19}
\end{equation*}
$$

## 2-2.2 The Equation of Motion

The equation of motion expresses the relation between the acceleration of the projectile and the pressure on its base, $l^{\prime}$. Since the unburned propellast and the gas are being aceelerated along with the projectile and there is also friction at the bore surface, there is a pressure gradient in the gas. The result is that the average pressure, $l$, occuring in Equation $2-19$ is not the same as the base pressure, $I_{1}$. Furthermore, it is customary to measure the pressure in a grom at a location at or near the brecel. To solve the erputions theoretically and to express
the result in terms of the measured pressure, $P_{c}$, some assumption must be made concerning the relations between the different pressures. An assumption commonly used is that
$P_{c}: I^{\prime}: P_{0}=1+\frac{C}{2(1+\theta) W}: 1+\frac{C}{3(1+\theta) W^{\prime}}: 1$

This :clation is an approxitiation based on a special solution of the Lagrange problem which is diseussed more fully in Chapter 5 . Its use is restrieted to artillery weapons firing at moderate velocities. When applied to treatments of high velocity weapons or to small arms, it yields poor results. V'sing Equations 2-11 and 2-20

$$
\begin{equation*}
I_{0}=\frac{(1+\theta) W}{(1+\theta) W+C / 3} P=\frac{(1+\theta) W}{M g} P \tag{2-21}
\end{equation*}
$$

If the friction of the bore surface is neglected, the cquation of motion of the projectile is

$$
\begin{equation*}
A P_{0}=\frac{(1+\theta) W}{g} \frac{d V}{d t} \tag{2-2:2}
\end{equation*}
$$

Substituting lial. $2-21$ in E(1. 2-22, gives

$$
\begin{equation*}
A I^{\prime}=. M \frac{d V}{d l} \tag{2-23}
\end{equation*}
$$

which expresses the equation of motion in terms of the average pressure and the effective mass, . $1 /$, given by liquation 2-11.

Since

$$
\begin{equation*}
V=\frac{d N}{d l} \tag{2-24}
\end{equation*}
$$

the equation of motion may be written

$$
\begin{equation*}
A P^{\prime}=M V^{\prime} \frac{d V}{d X} \tag{2-2i}
\end{equation*}
$$

## 2-2.3 The Burning Rate Equation

The burning has been described in Chapter 1. The area of the burning surface, which is here assumed constant, may be calculated by the formula

$$
\begin{equation*}
S=\frac{2 C}{\rho w} \tag{2-26}
\end{equation*}
$$

where
$S$ is the surface area of the propellant grains $w$ the web thickness and
$\rho$ the specific weight of the propellant, that is, the weight per unit volume.
The rate of regression of the surface is assumed to
be uniform over the entire surface. l'ropellant gas is cvolved at a rate

$$
\begin{equation*}
\frac{d c}{d t}=\rho S B P^{a} \tag{2-27}
\end{equation*}
$$

where
$B$ is the burning rate coefficient and
$\alpha$ is the burning rate pressure exponent, which is here assumed to be 0.8 .
$B$ should be determined for each type of propellant under actual conditions of use; that is, by adjusting its value for best fit of the theory to actual firing records. $B$ is frequently determined from closed chamber measurements but these values usually yield poor results when used in gun ralculations because the conditions in the gun are very different from those in the closed chamber. The closed chamber values are, however, of great value in determining relative burning rates of different types and lots of propellant.

## 2-2.4 Elimination of Variables

The preceding equations involve about a dozen variables. By straightforward manipulation, they can be reduced to three equations involving only four variables: $K, I^{\prime}, X$ and $T . T$ will be treated as the independent variable. Differentiating Equation 2-12, gives

$$
\begin{equation*}
d K=M V d V \tag{2-28}
\end{equation*}
$$

By substitution in Eq. 2-25.

$$
\begin{equation*}
d K=A P d X \tag{2-29}
\end{equation*}
$$

Since $A d X=d C^{\prime}$ by differentiation of E(1. 2-16a

$$
\begin{equation*}
d K=P d U \tag{2-30}
\end{equation*}
$$

Finally, since $d X=V d t$ by definition

$$
\begin{equation*}
d K=A P V d l \tag{2-31}
\end{equation*}
$$

The right members of these four equations are all equal. Furthermore, by differentiating Eq. 2-1,

$$
\begin{equation*}
d T=d K+d I \tag{2-32}
\end{equation*}
$$

Now, by Eqs. 2-2 and 2-31,

$$
\begin{equation*}
\frac{d K}{d T}=\frac{A P V d t}{F d c /(\gamma-1)} \tag{2-33}
\end{equation*}
$$

which, by Eq. 2-27, may be expressed

$$
\begin{equation*}
\frac{d K}{d T}=\frac{(\gamma-1) A P V}{r^{\prime} \rho S B P^{a}} \tag{2-3-4}
\end{equation*}
$$

Defining a constant

$$
\begin{equation*}
\sigma=\frac{F \rho S B}{A(\gamma-1)} \tag{2-3.5}
\end{equation*}
$$

and substituting Eq. 2-12, Equation 2--3t becomes

$$
\begin{equation*}
\frac{d K}{d T}=\sqrt{\frac{2 K}{l l}} \frac{p^{1-\alpha}}{\sigma} \tag{2-36}
\end{equation*}
$$

Combining Eqs. 2-32 and 2-30

$$
\begin{equation*}
1=\sqrt{\frac{2 K}{M}} \frac{p^{1-\alpha}}{\sigma}+\frac{d I}{d T} \tag{2-37}
\end{equation*}
$$

Substituting Eif. 2-30 in the differential of Eq. 2-1; gives

$$
\begin{equation*}
d I=\frac{d K}{\gamma-1}+\frac{U d P}{\gamma-1} \tag{2-38}
\end{equation*}
$$

Combining the last three equations yields

$$
\begin{equation*}
\gamma-1=\gamma \sqrt{\frac{2 K}{M}} \frac{P^{1-\alpha}}{\sigma}+\frac{U d I^{\prime}}{d I^{\prime}} \tag{2-39}
\end{equation*}
$$

Eliminating $I$ from Eqs. 2-1 and 2-1;

$$
\begin{equation*}
U=\frac{(T-K)(\gamma-1)}{P} \tag{2-40}
\end{equation*}
$$

Substituting this in E(1. 2-39 and rearranging produces

$$
\begin{equation*}
\frac{d P}{d T}=\frac{P}{(\gamma-1)(T-K)}\left[\gamma-1-\gamma \sqrt{\frac{2 K}{M}} \frac{P^{1-\alpha}}{\sigma}\right] \tag{2-41}
\end{equation*}
$$

Eliminating (' from Eqs. 2-16a and 2-40 gives

$$
\begin{equation*}
U_{0}+A X=\frac{(T-K)(\gamma-1)}{P} \tag{2-42}
\end{equation*}
$$

Equations 2-30 and 2-41 form a system of two first order differential equations in $K$ and $I^{\prime}$ with $T$ as independent variable, and $\mathrm{E}_{1} .2-42$ relates $X$ to these variables.

## 2-3 SOLUTION OF THE EQUATIONS

## 2-3.1 Reduction to Normal Form

Let the system of equations that was just derived be rewritten:

$$
\begin{gather*}
\frac{d K}{d T}=\sqrt{\frac{2 K}{M}} \frac{P^{1-\alpha}}{\sigma}  \tag{2-36}\\
\frac{d P}{d T}=\frac{P}{(\gamma-1)(T-K)}\left[\gamma-1-\gamma \sqrt{\frac{2 K}{M I}} \frac{P^{\prime-\alpha}}{\sigma}\right] \tag{2-41}
\end{gather*}
$$

$$
\begin{equation*}
U_{0}+A X=\frac{(T-K)(\gamma-1)}{P} \tag{2-42}
\end{equation*}
$$

In order to obtain solutions which are compatible with observed results, initial conditions must be imposed that represent gun conditions as closely as fcasible. The resistance to the motion of the projectile during the engraving of the rotating band is much greater than it is at any later stage. Closer agreement between computed and observed results is frefuently obtained at small cost in added complexity by imposing an added resistance corresponding to engraving resistance, which prevents the projectile from moving until the pressure reaches a certain value, $P_{0}$, called the starting pressure. This has already been taken into account in that a resistance proportional to pressure varies somewhat like a combination of engraving resistance and later bore friction, being large in the carly stage and small later.

The initiai conditions are then

$$
\begin{gather*}
V=0, \quad K=0, \quad P=P_{11}, \quad X=0 \\
T=T_{11}=\frac{U_{1} P_{11}}{\gamma-1}, \quad U=U_{11} \tag{2-43}
\end{gather*}
$$

where
$L_{0}$ is the initial free volume as given by Equation 2-17a namely,
$U_{0}=U_{c h}-C^{\prime \prime} u$ and $T_{y}$ (the adiabatic flame
temperature) is derived from Equation 2-43.
In order to facilitate the solution of this system by machine methods, three new variables, $\phi, \omega$ and $\xi$ are defined by the following relations:

$$
\begin{align*}
& T=\epsilon 10^{\star}  \tag{2-44}\\
& \omega=\sqrt{K / T}  \tag{2-45}\\
& \frac{\omega}{\xi}=\frac{d K}{d^{\prime} T^{T}} \tag{2-46}
\end{align*}
$$

where $\epsilon$ is a constant to be defined later (Eq. 2-61) and $\phi$ is the new independent variable.

In accordance with these definitions,

$$
\begin{equation*}
\frac{d \omega}{d \phi}=\frac{m}{2}\left(\frac{1}{\xi}-\omega\right) \tag{2-47}
\end{equation*}
$$

where $m=\log , 10$ (approximately 2.3026).
Eliminating $d K / d T$ from $E q_{[1} .2-36$ and 2-46, and differentiating logarithmically, produces

$$
\begin{equation*}
\frac{d \omega}{\omega}-\frac{d \xi}{\xi}=(1-a) \frac{d P}{l^{\prime}}+\frac{1}{2} \frac{d K}{K} \tag{2-48}
\end{equation*}
$$

【sing Liç. $2-41$ and $2-4.5$, this may be expressed
$-\frac{d \xi}{\xi}=\frac{d T}{T}\left\{\frac{1}{\underline{2}}+\frac{1-\alpha}{1-\omega^{2}}\left[1-\frac{\gamma}{\gamma-1} \sqrt{\frac{2 K}{M I}} \frac{p^{1-\alpha}}{\sigma}\right]\right\}$

Finally, using lids. 2-36, 2-4( and $2-4+$, this becomes $\frac{1}{m \xi} \frac{d \xi}{\frac{1}{\phi} \phi}=-\left\{\frac{1}{2}+\frac{1-\alpha}{1-\omega^{2}}\left[1-\frac{\gamma}{\gamma-1} \frac{\omega}{\xi}\right]\right\}$
From Ex: 2-16, 2-36 and 2-4in, there results
$\frac{1}{\xi}=\frac{1}{\omega} \frac{d K}{d T}=\frac{P^{1-\alpha}}{\omega \sigma} \sqrt{\frac{2 K}{M I}}=\sqrt{\frac{2 K}{M}} \frac{P^{1-\alpha}}{\sigma} \sqrt{\frac{T}{K}}$
L.et $T \rightarrow T_{0}$ and $\xi \rightarrow \xi_{0}$ when $\omega \rightarrow 0$; then $i^{\prime} \rightarrow I_{0}^{\prime}$ and

$$
\begin{equation*}
\xi_{n}=\sqrt{\frac{1}{2}} \frac{\sigma}{P_{0}^{1-a} \sqrt{T_{0}}} \tag{2-52}
\end{equation*}
$$

Substituting $T_{"}$ from E(1. $2-43$, this becomes

$$
\begin{equation*}
\xi_{1}=\sqrt{\frac{I I}{2}} \frac{\sigma \sqrt{\gamma-1}}{P_{1}^{1-a} \sqrt{U_{0} P_{n}^{\prime}}} \tag{2-i;3}
\end{equation*}
$$

Since, by dofinition,

$$
\sigma=\frac{Y \rho S B}{A(\gamma-1)}
$$

this may be written
$s_{n}=\frac{1}{\sqrt{2(\gamma-1)}} \frac{\rho \cdot S B F}{A} \sqrt{\frac{\Lambda I}{L_{0}^{\prime}}}\left[\frac{1}{P_{n}^{\prime}}\right]^{(3-2 a) / 2}$
L. $\cdot \mathrm{t}$

$$
\begin{equation*}
\mathrm{II}=\left[\frac{\rho S B F}{A} \sqrt{\frac{1 I}{L_{0}^{\prime}}}\right]^{2 /(3-2 a)} \tag{2-8;}
\end{equation*}
$$

and substitute in Eq. 2-5t,

$$
\begin{equation*}
\xi_{n}=\frac{1}{\sqrt{2(\gamma-1)}}\left[\frac{11}{P_{n}^{\prime}}\right]^{(3-2(n) / 2} \tag{2--56}
\end{equation*}
$$

Next define:

$$
p=\frac{P}{I I}, \text { so that } p_{1 \prime}=\frac{P_{11}}{I I}
$$

and let

$$
\begin{equation*}
\phi_{n}=\log p_{n}=\log \frac{P_{n}}{I I} \tag{2-i8}
\end{equation*}
$$

From E:c s. $2-44$ and 2-43,

$$
\begin{equation*}
T_{0}=\epsilon 10^{\circ} \cdot=\frac{U_{n} P_{0}}{\gamma-1} \tag{2-59}
\end{equation*}
$$

This gives

$$
\begin{equation*}
\phi_{0}=\log \frac{U_{n} I_{01}}{t(\gamma-1)} \tag{2-60}
\end{equation*}
$$

Identifying this with Eq. 2-is8, produces, from Eq. 2-in)
$\epsilon=\frac{\Pi U_{0}}{\gamma-1}=\frac{U_{0}}{\gamma-1}\left[\frac{\rho S F B}{A} \sqrt{\frac{\lambda I}{U_{0}}}\right]^{2 /(3-2 \alpha)}$
or; from E(1. 2-35
$\epsilon=C_{0}^{(2-2 \alpha) /(3-2 a)} M M^{1 /(3-2 \alpha)}(\gamma-1)^{(2 \alpha-1) /(3-2 \alpha)} \sigma^{2 /(3-2 \alpha)}$

Also from Eis. 2-i56 and 2-58 it follows that

$$
\begin{equation*}
\log \xi_{y}=-\frac{1}{2} \log 2(\gamma-1)-\frac{3-2 \alpha}{2} \phi_{0} \tag{2-6.3}
\end{equation*}
$$

Thus the system of Equations 2-36, 2-41 and 2-42 has been changed into the normal form

$$
\begin{align*}
\frac{d \omega}{d \phi} & =\frac{m}{2}\left(\frac{1}{\xi}-\omega\right)  \tag{2-47}\\
-\frac{d \xi}{d \phi} & =m \xi\left\{\frac{1}{2}+\frac{1-\alpha}{1-\omega^{2}}\left[1-\frac{\gamma}{\gamma-1} \frac{\omega}{\xi}\right]\right\} \tag{2-i0}
\end{align*}
$$

$\log \xi_{n}=-\frac{1}{2} \log 2(\gamma-1)-\frac{3-2 \alpha}{2} \phi_{n}$
Here, the independent variable is

$$
\begin{equation*}
\phi=\log \left(T^{\prime} \epsilon\right) \tag{2-4+a}
\end{equation*}
$$

where $\epsilon$ is defined by Eif. 2-61, and the dependent variables are

$$
\begin{align*}
& \omega=\sqrt{K / T}  \tag{5}\\
& \xi=\omega \frac{d T}{d K} \tag{2-46}
\end{align*}
$$

The nume: ical solution of the normal form and its use in computing trajectory data will be discussed next. After that, some simple methods of estimating trajectory data will be given, and the effects of variations in the parameters will be explained.

## 2-3.2 Numerical Integration of the Normal Form

The system of equations in normal form represents a one-parameter family of trajectorics. The parameter is the initial value $\phi_{0}$ of the independent variable $\phi$. This system has been integrated numerically by computer for values of $\phi_{0}$ ranging from $-2 . i$ to +1.8 . It was observed that the set of curves for $\phi_{0}$ less than -2.5 tends asymptotically to the solution for the case of zero starting pressure, which may be called the "limiting trajectory". From the solutions working charts have been constructed.

## 2-3.3 Interior Ballistic Trajectories During Burning

The principal problem of interior ballistic theory is to determine the pressure history and the travel of the projectile in the gom. To obtain this information, the pressure, $P$, and the velocity, $V$, as functions of the time, $t$, or travel, $X$, are needed, and also $X$ as a function of $t$. This may be obtained from the trajectories, which are conveniently divided into two phases: (1) during burning and (2) after burring. The first phase will now be considered.
lirom Equation 2-12, the work done by the gas is expressed as

$$
K=\frac{M V^{2}}{2}
$$

Combining this with the definitions in Wels. $2-44$ and 2-45, yiclds

$$
\begin{equation*}
V / \omega=\sqrt{2_{\epsilon} / M} 10^{\delta / 2} \tag{2-64}
\end{equation*}
$$

From Eqs. 2-36, 2-4c and 2-12,

$$
\begin{equation*}
\frac{V}{\omega}=\frac{\sigma}{\xi P^{1-\omega}} \tag{3}
\end{equation*}
$$

Equating the right members and simplifying gives

$$
\begin{equation*}
P^{1-\alpha}=\frac{\sigma}{\xi} \sqrt{\frac{I I}{2 \epsilon}} 10^{-\sigma / 2} \tag{2-66}
\end{equation*}
$$

If the values of $P$ are calculated for a fixed $\phi=\phi_{0}$, trajecto:y, the point at which the peal pressure, $l_{p}$, occurs can be located. By taking values from all the trajectories, $I_{p}$ can be obtained as a function of the normal variables $\omega$ and $\xi$.

In Equation 2-40 the free volume was expressed as

$$
\zeta=\frac{(T-K)(\gamma-1)}{P}
$$

Substituting the values of $T, K$ and $I$ from liquations 2-44, 2-4.) and 2-6is makes

$$
\begin{equation*}
U=\frac{\left(1-\omega^{2}\right)(\gamma-1)_{\epsilon} 10^{\phi}}{\left[\frac{\sigma}{\xi} \sqrt{\frac{1 I}{2}} 10^{-\phi / 2}\right]^{1 /(1-\alpha)}} \tag{2-67}
\end{equation*}
$$

.s can be seen from Eifuation 2-16a,

$$
x-\frac{U-U_{n}}{A}
$$

Hence,

$$
\begin{equation*}
I=\frac{\left(1-\omega^{2}\right) \epsilon^{(3-2 a) /(2-2 a)} 10^{(3-2 a) /(2-2 \alpha) \phi}(\gamma-1)}{A\left[\frac{\sigma}{\xi} \sqrt{\frac{1 I}{2}}\right]^{1 /(1-a)}}-\frac{U_{0}}{A} \tag{2-68}
\end{equation*}
$$

For $\alpha=0.8$ and $\gamma=1.30$, this becomes

$$
\begin{equation*}
X=\frac{1.6908\left(1-\omega^{2}\right) \epsilon^{3.5} 10^{3.5 \phi} \xi^{5}}{A \sigma^{5} M I^{2.5}}-\frac{U_{0}}{A} \tag{2-68a}
\end{equation*}
$$

Equations 2-64 and 2-68 give $V$ and $X$ as functions of $\phi$. These quantities can be obtained by routine calculation. However, the computation of pressure and time can be facilitated by the use of a reduced pressure and a reduced time.

## 2-3.4 Reduced Variables

The reduced pressure, $\ddot{p}$, is defined by the formula

$$
\begin{equation*}
\beta=\left(\xi 10^{\phi / 2}\right)^{1 /(1-\alpha)} \tag{2-69}
\end{equation*}
$$

Then Eq. 2-66 becomes

$$
\begin{equation*}
P^{\prime}=\frac{[\sigma \sqrt{M I / 2 \epsilon}]^{1 /(1-\alpha)}}{p} \tag{2-70}
\end{equation*}
$$

or, using the definitions of $\sigma$ and $\epsilon$, Equations 2-35 and 2-61

$$
\begin{equation*}
P=\frac{2^{1 /(2(\alpha-1))}(\gamma-1)^{(1-2 \alpha) /(2(1-\alpha))_{\epsilon}}}{U_{0} \bar{p}} \tag{2-71}
\end{equation*}
$$

For $\alpha=0.8$ and $\gamma==1.30$, Equations 2-69 and 2-71 become

$$
\begin{equation*}
\tilde{p}=\left(\xi 10^{\Delta / 2}\right)^{;} \tag{2-69a}
\end{equation*}
$$

and

$$
\begin{equation*}
r=1.076 \frac{\epsilon}{U_{0} \eta} \tag{2-71n}
\end{equation*}
$$

By liquation 2-2 the released energy is

$$
T=\frac{c F}{\gamma-1}
$$

and by Equation $2-27$ the rate of burning is

$$
\frac{d c}{d l}=\rho S B P^{a}
$$

Substituting the derivative of 7 from Equation 2-2 and using the definition of $\sigma$ from Equation 2-3: leads to

$$
\begin{equation*}
d T=A \sigma P^{a} d t \tag{2-72}
\end{equation*}
$$

Hy diffrentiation of Equation 2-44

$$
\begin{equation*}
d T=m \in 10^{4} d \phi \tag{2-73}
\end{equation*}
$$

where $m=\log , 10$
Equating the right members gives

$$
\begin{equation*}
d t=\frac{m \epsilon 10^{\phi}}{A P^{a} \sigma} d \phi \tag{2-74}
\end{equation*}
$$

The reduced time is defined by the formula

$$
\begin{equation*}
I=\int_{\phi \phi}^{\phi} \xi^{n /(1-\alpha)} 10^{(2-n) / 12(1-\alpha) / \phi} d \phi \tag{0}
\end{equation*}
$$

Then, with the help of Eif. 2-66 the integral of Eq. 2-74 may be expressed

$$
\begin{equation*}
t=\frac{m}{A}\left[\frac{2}{I i}\right]^{\alpha / 12(1-a) 1} \frac{\epsilon^{(2-a) /(2(1-a) 1}}{\sigma^{1 / 1-a}} i \tag{i}
\end{equation*}
$$

For $\alpha=0.8$,

$$
\begin{gather*}
I=\int_{\infty}^{\infty} 10^{\log \xi^{\prime}} d \phi  \tag{2-7.5a}\\
l=\frac{4 \epsilon^{3} m}{\lambda \sigma^{3} . \|^{i}} I \tag{2-76a}
\end{gather*}
$$

## 2-3.5 Pressure Ratio Chart

The ratio of initial pressure, $P_{\text {n, }}$ to peak pressure, $P_{r}$, as a function of

$$
p_{15}=\frac{P_{n}}{I I}
$$

where 11 is defined by liquation 2-5.5 is shown in ('hart $\stackrel{2}{2}$. This chart permits a determination of $l_{p}$, the thereretieal maximm pressure, to be expected for any choice of starting pressure for any given gun, propellant and projectile systeiii.

## 2-3.6 Interior Ballistic Trajectory Charts

On (harts $2-2 a,-2 b$ and -2c several sets of curves give data from the interior ballistic trajectorics. The abseissa is $\omega$ and the ordinate is $\log \xi$.
$\phi_{\text {. }}$ trajectorics are solid curves, starting with the labeled values of $\phi$ and showing the coordinates along each trajectory. (Chart 2-2a)
$\phi$-curves are dashed curves joining the points of konstant $\phi$ on all trajectorics. The value of $\phi$ on mach of these curves is the labeled value. (Chart $2-2 a$ )
$l_{p}$-curve crosses all trajectorices at the points where the pressure is a maximum. (Chart 2-2a)
$l$-curves join points of constant reduced time on wach trajectory, and are so labeled. (Chart 2-2b)
p-curves join poiats of constant reduced pressurr bu each trajectory. They are labeled with the values bf $\log \bar{p}$. (Chart 2-2c)

The charts reproduced here are for illustration purposes only. For use in calculations they should be reproduced on a much larger seale.

The use of the trajectory charts is as follows. Fistimate a starting pressure, which depends on the gun and projectile. Bemett' assumed a value of 2:00 psi for his tables, but it may be anywhere from 1000 to $: 000$ psi. Compute ${ }^{\prime}$ ' $1, N, S, \sigma$ and
© from Liquations 2-17a, 2-11, 2-26, 2-35 and 2-62, respectively. Setting $\gamma=1.30$ and $\alpha=0.8$ the latter two equations become

$$
\begin{equation*}
\sigma=\frac{\rho S B I F^{\prime}}{0.3 A} \tag{2-3.9}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon=\left[0.027 U_{n .2}^{2} / l^{5} \sigma^{\prime \prime \prime}\right]^{1 / \tau} \tag{n}
\end{equation*}
$$

Then compute

$$
\begin{equation*}
\phi_{\Delta}=\log \frac{U_{n} I_{n}}{0.3 \epsilon} \tag{f0a}
\end{equation*}
$$

This identifies the o, trajectory (the set of values of $\omega, \xi$ and $\phi$ along the solid line from $\phi=\phi_{0}$ on Chart $\because-2 a)$ which is applicable until the end of burning. If "burnt" values are denoted by the subseript $l, \phi$ becomes

$$
\begin{equation*}
\phi_{t}=\log \frac{F C}{0.3 \epsilon} \tag{2-77}
\end{equation*}
$$

The intersection of the $\phi$-curve for $\phi=\phi_{b}$ (dotted line on (hart 2-2a) and the $\phi_{0}$-trajectory indicates the point on the trajectory at which the charge was all burned, that is the point ( $\omega_{l,}, \xi_{L}, \phi_{l}$ ).

The intersections of the $p$-curves and the $l$-curves with the $\phi_{-}$-trajectory give the values of the reduced pressme and reduced time. (To do this using the three Charts $2-2 \mathrm{a}, 2-2$ ) and $2-2 \mathrm{c}$ recpuires that the $\phi$, trajectory be transposed from Chart 2-2a to ('hart 2-2h for $l$, and to Chart 2-2c for $\pi$.) (The three charts have been combined into one so that this transposition is not necessary. The combined chart however, is complicated and hard to read unless made very large. A combined chart was produced on a large scale at Ballistic Rescarch laboratories and copies can be obtaincd. A reduced copy is published in Reference 1.) Then the pressure and time can be computed by Equations 2-71a and 2-7 (ia. In particular, the "burnt" values, $p_{c}$ and $T_{L}$, can be found. Also, the peak pressure, $I_{r}$, can be calculated from $p_{p}$, which is the value of $p$ at the intersection of the $P_{2}$-curve (Chart 2-2a.) and the $\phi_{D}$-trajectory. ${ }^{\prime}$ ', is the theoretical peak pressure which is not necessarily the same as the actual maximum pressure. It will be the same only if the charge doess not burn out before $P^{\prime}$, is reached, that is, the time to reach the theoretical peak pressure must be less than the time to burnt. If this is not the case, the actual maximum pressure is the pressure at burnt, $l_{1}$.

After $\phi_{b}$ has been calculated from Eq. 2-77, $\omega_{b}$ has been obtained from Chart 2-2a, and $l_{6}^{\prime}$ has been computed, the free volume, $U_{\iota}$, the travel, $X_{0}$ and the volocity, $V_{i}$, at "burnt" can be calculated by

# CHART 2-2a. INTERIOR BALLISTICS TRAJECTORIES <br> (Located in the back of this handbook) 

# CHART 2-2b. INTERIOR BALLISTICS <br> TRAJECTORIES <br> (Located in the back of this handbook) 

# CHART 2-2c. INTERIOR BALLISTICS TRAJECTORIES <br> (Located in the back of this handbook) 

the formulas

$$
\begin{align*}
& L_{i}=\frac{\left(1-\omega_{b}^{2}\right) V_{C} C_{c}}{P_{b}^{\prime}}  \tag{2-78}\\
& X_{n}=\frac{V_{1}-U_{n}}{A} \\
& r_{b}=\omega_{l} \sqrt{\frac{2_{\epsilon}}{I /}} 10^{0 \pi / 2} \tag{2-80}
\end{align*}
$$

## 2-3.7 Conditions After Burnt

After all the propellant is burned, $c=(?$. With this substitution, Jipuation ?-2 shows that the released mergy is

$$
\begin{equation*}
T=\frac{F(:}{\gamma-1}=\text { constant } \tag{2-81}
\end{equation*}
$$

Is before, the work done by the gas is, Equation 2-12

$$
K=\frac{. / l^{2}}{2}
$$

and the internal energy is, Equation 2-1.5

$$
I=\frac{P^{\prime} U}{\gamma-1}
$$

Hence, the energy equation may be expressed

$$
\begin{equation*}
F C=P C+\frac{\gamma-1}{2} M I^{2} \tag{2-8:2}
\end{equation*}
$$

Taking differentinls produces,

$$
\begin{equation*}
0=P d U+U d P^{\prime}+(\gamma-1) M V^{*} d V^{*} \tag{2-83}
\end{equation*}
$$

By Equations 2-28 and 2-30

$$
\begin{equation*}
M V d V=P d U \tag{2-8t}
\end{equation*}
$$

so that $\quad \frac{d P}{P}=-\gamma \frac{d U}{U}$
Integrating with $P_{b}$ and $U_{b}$ as initial values, yields

$$
\begin{equation*}
P U^{\gamma}=P_{0} L_{6}^{\gamma} \tag{2-86}
\end{equation*}
$$

Since the right member is a constant, the expansion is adiabatic; actually, the value of $\gamma$ is adjusted to account for the loss of heat.

From Ifquations 2-40 and 2-45, the energy equation may also be expressed

$$
\begin{equation*}
P U=(\gamma-1)\left(1-\omega^{2}\right) T \tag{2-87a}
\end{equation*}
$$

At bume,

$$
\begin{equation*}
P_{b} U_{b}=(\gamma-1)\left(1-\omega_{b}^{2}\right) T_{b}^{\prime} \tag{2-87~b}
\end{equation*}
$$

But after burnt, $T=T_{\iota}$, so that

$$
\begin{equation*}
\frac{1-\omega^{2}}{1-\omega_{b}^{2}}=\frac{P U}{P_{b} U_{b}} \tag{2-88}
\end{equation*}
$$

Then, by lic. 2-86

$$
\begin{equation*}
\frac{1-\omega^{2}}{1-\omega_{t}^{2}}=\left[\frac{U}{U_{t}}\right]^{1-\gamma} \tag{2-89}
\end{equation*}
$$

Hence

$$
\begin{equation*}
U=\frac{U_{b}}{\left(1-\omega_{i}^{Z}\right)^{1 /(1-\gamma)}}\left[1-\omega^{2}\right]^{1 /(1-\gamma)} \tag{2}
\end{equation*}
$$

Substituting Eff. 2-90 and 2-81 in Eq. 2-87a, gives

$$
\begin{equation*}
l^{\prime}=\left[1-\omega_{i .}^{2}\right]^{1 /(1-\gamma)} \frac{F C}{U_{b}}\left[1-\omega^{2}\right]^{\gamma /(\gamma-1)} \tag{2-91}
\end{equation*}
$$

By ligs. 2-82 and 2-80,

$$
\begin{equation*}
V^{2}=\frac{2}{(\gamma-1) . M}\left(F C-P_{l} U_{l}^{\gamma} U^{1-\gamma}\right) \tag{2-92}
\end{equation*}
$$

Ľsing Ef(s. 2-81, 2-87b and 2-90, produces

$$
\begin{equation*}
V=\sqrt{\frac{2 F C}{(\gamma-1) . M}} \omega \tag{2-93}
\end{equation*}
$$

The travel is, from Equation 2-16a

$$
X=\frac{U-U_{n}}{A}
$$

From Equation ?-2:3

$$
d l=\frac{M I}{A P} d V
$$

With $t_{b}$ and $V^{\circ}$, as initial values, the integral of this is

$$
\begin{equation*}
t=t_{b}+\frac{M}{A} \int_{V_{b}}^{V} \frac{d V}{P} \tag{2-94}
\end{equation*}
$$

Substituting Eqs. 2-91 and 2-53 in Eq. 2-94, gives

$$
\begin{array}{r}
t=t_{b}+\frac{U_{b}}{A} \sqrt{\frac{2 . M}{(\gamma-1) F C}}\left[1-\omega_{b}^{2}\right]^{1 /(\gamma-1)} \\
\cdot \int_{\omega_{b}}^{\omega} \frac{d \omega}{\left[1-\omega^{2}\right]^{\gamma /(\gamma-1)}} \tag{2-95}
\end{array}
$$

## 2-3.8 Time, Pressure and Travel Functions

To facilitate the determination of time, pressure and travel after burnt, Chart 2-3 has curves of the following functions of $\omega$ :

$$
\begin{align*}
& \tau(\omega)=\int_{1}^{\omega}\left(1-\omega^{2}\right)^{-13 / 3} d \omega  \tag{2-96}\\
& \mu(\omega)=\left(1-\omega^{2}\right)^{13 / 3}  \tag{2-97}\\
& \nu(\omega)=\left(1-\omega^{2}\right)^{-11 / 3} \tag{2-98}
\end{align*}
$$

Using these functions with $\gamma=1.30$, Equations 2-95, 2-91 and 2-90 may be expressed

$$
\begin{align*}
t=t_{b}+\frac{U_{b}}{A \nu\left(\omega_{b}\right)} & \sqrt{\frac{2 . M}{0.3 F C}}\left[\tau(\omega)-\tau\left(\omega_{b}\right)\right]  \tag{2-99}\\
P & =\frac{F C \nu\left(\omega_{b}\right)}{U_{b}} \mu(\omega)  \tag{2-100}\\
U & =\frac{U_{h}}{\nu\left(\omega_{b}\right)} \nu(\omega) \tag{2-101}
\end{align*}
$$

Then $X$ can be calculated by Eq. 2-1Ga and $V$ by Eic. 2-93, with $\gamma=1.30$.


CHART 2-3. Time, Pressure and Travel F'unctions

These equations apply from the position at burnt, denoted by the subseript, $b$, to the muzzle, denoted by the subscript, $m$. If $\bar{X}_{m}$ is known, $l^{\circ}$, can be found,

$$
\begin{equation*}
\zeta_{m}=l_{n}+A X_{m} \tag{2-102}
\end{equation*}
$$

and then

$$
\begin{equation*}
\nu\left(\omega_{\ldots}\right)=\frac{\dot{C}_{m}}{U_{b}} \nu\left(\omega_{1}\right) \tag{2-103}
\end{equation*}
$$

then $\omega_{n}$ can be determined from the chart.
Finally, $l_{m}, I_{m}^{\prime}$ and $I_{m}$ can be calculated as explained above. In practice $\Gamma_{m}$ is divided by 12 to obtain the muzzle velocity in feet per sccond.

## 2-3.9 Examples

a. Determine the maximum pressure; the time, pressure and travel at burnt; and the time, pressure and velocity at the muzzle for the 105 mm Howitzer M4, firing the High Explosive Projectile M1, propelled by 10 ounces of multiperforated propellant M1 lot ___*.
*A "lot" of propellant, is the produrt of one set of manufacturing operations such that its characteristics are essentially uniform, or a combination of such products obtained by blending so that the grains are thoroughly mixed and statistical uniformity among charges may be expected, even though slight variations exist among individual grains. The manufactu ?r furnishes the characteristies of each lot.

| Characteristic | Symbol | Value | Unit | lemarks |
| :---: | :---: | :---: | :---: | :---: |
| Area of bore | A | 13.4 | in ${ }^{2}$ | Given |
| Chamber volume | $\mathrm{V}_{\mathrm{ch}}$ | 153.8 | in ${ }^{3}$ | Given |
| Travel (to muzzle) | $\lambda_{m}$ | 80.4 | in | Given |
| Weight of projectile | W | 33.0 | 1 l | Given |
| Starting pressure | $P_{1}$ | 4000 | psi | Given |
| Weight of propellant | C' | 0.625 | 1b | Given |
| Burning rate coefficient | 13 | 0.0011 | $\frac{\mathrm{in} / \mathrm{sec}}{(\mathrm{psi})^{0.8}}$ | Given |
| Force of propellint | $F$ | $3.73 \times 10^{8}$ | in- $\mathrm{lb} / \mathrm{ll}$ ) | Given |
| Specific weight ${ }^{+}$ | $\rho$ | 0.0511 | $1 \mathrm{l} / \mathrm{in}^{3}$ | Given |
| Web thickness | $w$ | 0.0140 | in | Given |
| Friction factor | $\theta$ | 0.05 |  | (iiven |
| Adjusted ratio of sperific heats | 9 | 1.30 |  | Given |
| Burning surface | $S$ | 1564 | $\mathrm{in}^{2}$ | Ec. 2 -20 |
| Initial free volume | ${ }_{0}$ | $1+2.9$ | $\mathrm{in}^{3}$ | Eq. 2-17a |
| Effective mass | . 1 | 0.0903 | 12 slug: | E.l. 2 -11 |
| Constant | $\sigma$ | $9.115 \times 10^{1}$ |  | Eq. 2 -35 |
| Constant | 6 | $5.38+\times 10^{6}$ |  | Eq. 2-62a |
| $\phi_{0}$-trajectory | $\phi_{0}$ | -0.451 |  | Eq. 2-60a |
| ф-curve | $\phi_{5}$ | 0.160 |  | Eq. $2-77$ |
| Abscissa of intersection | $\omega_{6}$ | 0.323 |  | Chart 2-2a |
| Ordinate of intersection | $\log \xi$, | 0.056 |  | Chart 2-2a |
| Reduced time curve | ${ }_{\text {f }}$ | 1.87 |  | Chart 2-2b |
| Reduced pressure curve | $\log p_{b}$ | 0.670 |  | Chart 2-2c |
| Redured peak pressure curve | $\log i^{\prime}$ | 0.660 |  | Charta 2-2a, -20 |
| Muxinum pressure | $P_{r}$ | 8870 | jxi | Eq. 2-71a |
| Pressure at burnt | $P_{b}$ | 8665 | psi | Eq. 2-71n |
| Time at burnt | ${ }_{\text {b }}$ | $3.91 \times 10^{-3}$ | sec | Eq. 2-76a |
| Free volume at burnt | $l_{6}$ | 240.9 | $\mathrm{in}^{3}$ | Eq. 2-78 |
| Travel at burnt | $\lambda_{8}$ | 7.31 | in | Eq. 2-79 |
| Free volume at muzzil- | $U_{\text {m }}$ | 1220 | in ${ }^{2}$ | Eq. 2-102 |
| Travel function | $\nu\left(\omega_{b}\right)$ | 1.435 |  | Chart 2-3 |
| Travel function | $\nu\left(\omega_{m}\right)$ | 7.27 |  | Eq. 2-103 |
| Abscissa | $\omega_{m}$ | 0.670 |  | Chart 2-3 |
| Time function | $\tau\left(\omega_{m}\right)$ | 1.970 |  | Chart 2-3 |
| Time function | $\tau\left(\omega_{b}\right)$ | 0.380 |  | Chart 2-3 |
| Pressure function | $\mu\left(\omega_{m}\right)$ | 0.075 |  | Chart 2-3 |
| Time at muzzle | $t_{m}$ | $14.02 \times 10^{-3}$ | sec | Eq. 2-90 |
| Pressure at muzzle | $P_{\text {m }}$ | 1040 | psi | Eq. 2-100 |
| Velocity at muzzle | $V_{m}$ | 8780 | in/ser | Eq. 2-93 |
| Veiocity at muzzle | $V_{m}$ | 732 | $\mathrm{ft} / \mathrm{sec}$ | Eq. 2-93 |

$\dagger$ The manulacturer customarily given the apecific gravity of the propellant. To obtain the apecific weight in pounds per cubic inch, this must bo divided by the specific volume of water, which is $27.08 \mathrm{in}^{3}$, lb.

If desired, the foregoing process can be repeated for selected values of $\phi$ less than $\phi_{i}$, and selected values of $\omega$ betwern $\omega_{0}$ and $\omega_{\omega} .$. Interpolated ralues of time, pressure, travel and velocity corresponding to the selected points can then be determined from which pressure-time, pressure-trawe and velocitytravel cures can be drawn. Figure $2-1$ is the pres-arre-tine curve for the preceding example.
b. Determine the weight of the same lot of propellant that will give a maximum pressure of 10,000 psi in the same gum, firing the same projectile as in the first example (par. 2-3.9a), and compute the resulting muzzle velocity.
For this example, some of the formulas need revision. lirst, "fuate the right members of Lepua-tions:-.is and 2 -(60, and obtain

$$
\begin{equation*}
\epsilon=\frac{U_{11} P_{n}}{0.3 p_{11}} \tag{2-104}
\end{equation*}
$$

Min, Dipuation $\because$-iba may be expressed

$$
\begin{equation*}
\sigma=\left[\frac{\epsilon^{\bar{\top}}}{0.02 \bar{u} L_{0,2}^{2,} l^{5}}\right]^{1 / 11} \tag{3}
\end{equation*}
$$

Finally, combining Equations $2-26$ and $2-3 ;$ yiclds

$$
\begin{equation*}
r=\frac{0.1 .5 w_{1}}{\sqrt{1} B} \sigma \tag{2-106}
\end{equation*}
$$

Since $P_{0}=4,000 \mathrm{psi}, P_{0} P_{\nu}=0.400$. Entering Chart $?_{-1}$ with this ratio, find $p_{0}=0.2935$.

Now, a trial and enror method should be used. Using an estimated value of $C$, compute $U_{0}$ and .$/ /$ as in the first example then calculate $\epsilon, \sigma$ and C' by the fommas above. If the two values of $C$ ' are not the same, use the calculated $C$ as the second estimate, and repeat the calculations. If necessary, repeat again. When both values agree, this gives the desired maximum pressure; the muzzle velocity can then be found as in the first cexample.
Since 0.62.5 pound of propellant gave a maximum pressure of 8,870 psi, (' must be greater in the present problem. Paragraph $2-9$ will explain how this increase can be estimated; however, for the present, a charge of 0.800 pound will be assumed.
The following tables give the results of the computations.
Thus, a charge of 0.709 pound will give a maximum pressure oit 10,000 psi and a muzzle velocity of 780 fps .
c. Determine the weight and web thickness of a charge of Ml propellant that will give a maximum pressure of $1 \mathrm{i}, 000 \mathrm{psi}$ and a muzzle velocity of 1000 fps in the same gun, firing the same projectile as in the preceding examples.



FIGURE 2-I. Theoretical Pressure-Time C'urve for 105mm Howizer Ustug T'aylor's Theory.

Solving licquation $\because-10$; for w gives

$$
\begin{equation*}
w=\frac{F B C}{0.15 A \sigma} \tag{-2-106a}
\end{equation*}
$$

Entering Chart 2-1 with the ratio $I_{" 1}^{\prime} I_{n}=0.2607$, find $p_{11}=0.170$.

Assume a series of values of $C^{\prime}$; calculate $\epsilon, \sigma$ and $w$ by Equations $2-104,2-10$ ) and 2-106a; and compute the muzzle velocity as in the first example. The value of $C$ that will produce a muzzle velocity of 1000 fps and the corresponding weh thickness can then be found by interpolation. The results of such a calculation are tabulated below.

| ( 'hatrge, ll | Wrob tuirkness, in | Muzzle Velority, fps |
| :---: | :---: | :---: |
| 1.(M) ${ }^{\text {( }}$ ) | .11:37 | 927 |
| 1.100 | .015: | 969 |
| 1.20) | .0164 | $1(X)+$ |
| 1.188* | .0165; | 1000 |

* Interinilated.

1. When the IIL: Projectile M1 was propelled by 1 pound of propellant MI with a web thickness of 0.0140 inch in the $10: \mathrm{mm}$ Howitzer MIt, a maximum pressure of 14,200 pounds per square inch and a muzzle velocity of 920 feet per second were observed. Idjust the values of the factor $\theta$ and the burning rate coefficient, $B$, sonshat the calculated results will agree with the observed ones.

With the known values of $P_{0}$ and $I_{r}$, Chart 2-1 can be used to find $p_{0}$. With this value of $p_{11}$, c can be computed by Equation 2-104. Then $\omega_{b}$ and $\log p_{b}$ can be founc with the help of Chart 2-2c, and $\omega_{m}$ with the he!, of Chart 2-3. With the known value of $I_{m}$, the adjusted value of $M$ can be computed by the formula

$$
\begin{equation*}
M=\frac{\omega_{m}^{2} F C^{\prime}}{0.15 V_{m}^{2}} \tag{2-107}
\end{equation*}
$$

Solving lifuation 2-11 for $\theta$, gives

$$
\begin{equation*}
\theta=\frac{M g-C / 3}{W}-1 \tag{2-11b}
\end{equation*}
$$

Using the adjusted values of $\epsilon$ and $N, \sigma$ can be calculated by Equation 2-105. Finally, solving Equation 2-106; for B, yields

$$
\begin{equation*}
B=\frac{0.15 A w \sigma}{F^{\prime} C^{\prime}} \tag{2-10Gb}
\end{equation*}
$$

Thus the factor $\theta$, which accounts for rotational energy and frictional resistance, is 0.062 insterd of

The results of these computations are given below.

| Symbol | $V$ alue | Remarks |
| :---: | :---: | :---: |
| $d$ | 13.4 | (iven |
| $U_{\text {ch }}$ | 153.8 | Given |
| $X_{m}$ | 80.1 | (iiven |
| IV | 33.0 | Given |
| $r_{0}$ | ( $\mathrm{MOH}^{(1)}$ | (iiven |
| (: | 1.00 | (iiven |
| F | $3.73 \times 10^{6}$ | (iiven |
| $\rho$ | 0.0571 | (iiven |
| $w$ | 0.01-40 | (iiven |
| $P^{1}$, | $14,2(3)$ | (iven |
| $V_{m}$ | 11,0.40 | (iven |
| $P_{0} / I^{\prime}{ }_{p}$ | 0.2817 |  |
| $\boldsymbol{p}_{0}$ | $0.18: 3$ | Chart 2-1 |
| $\phi$ | -0.738 | 1\%. $2-58$ |
| $\mathrm{Cu}_{0}$ | 136.3 | 14.2-17: |
| e | $0.93 \times 10^{0}$ | E4. 2 -10.4 |
| $\phi_{b}$ | 0.0975 | Ec. $2-76$ |
| $\omega_{\text {b }}$ | 0.320 | Chart 2 -2a |
| $\log \xi$, | 0.102 | Chart $2-2 \mathrm{a}$ |
| $\log \mu_{0}$ | 0.7 .17 | Chart 2-20 |
| $\log p_{1}$ | 0.737 | Charts $2-2 \mathrm{il}$, -20 |
| $P_{P}$ | 14,360 |  |
| $P^{\prime}$ | 11,0.10 | 14. 2-71: |
| $1 \%$ | 239 | Eq. 2 --8 |
| $i_{m}$ | 121.4 | 14. 2-102 |
| $\nu\left(\omega_{l}\right)$ | 1.12 | Chart 2-3 |
| $\nu\left(\omega_{m i}\right)$ | 7.21 | J:\%. 2-10:3 |
| $\omega_{m}$ | 0.670 | Chart $2-3$ |
| $M$ | 0.0916 | lig. 2-107 |
| $\theta$ | $0.06 ; 2$ | Jı. 2-111 |
| $\sigma$ | $1.402 \times 10^{2}$ | E\%. 2-105 |
| /3 | 0.00106 |  |

the assumed value 0.0:0; the burning rate coeflicient, $B$, is 0.00106 , which is practically the same as the closed chamber valuc, 0.00110 .

## 2-3.10 Dual Granulation Charges

In some guns, especially howitzers and mortars, two or more charges are used so as to obtain various muzzle velocities and thus vary the angle of fall. The lowest velocity is obtaiied with a base charge; and the higher velocities, by adding increments.
The same velocity can be obtained with different types of propellant or different web thicknesses by adjusting the weight of charge. A charge with a faster burning rate or a smaller web thickness will produce a higher pressure along the first part of the travel, and therefore a higher maximum pressure, and a lower pressure along the latter part of the travel, including the muzzle. It has been found that the round-to-round dispersion in muzzle velocity for a given charge is large when the max-
imum pressure is very low. Therfore, it is desirable to use a fast propellant for the low velocities. However, the use of such a fast prepellant at the high velocities is likely to make the maximum pressure exceed the pressure that the gion can stand without damage.
This difficulty could be solved by using fast propellant throughout for the low charges and slow propellant throughout for the high charges. However, it is more feasible to use fast propellant for the base charge and low inerements, and slow propellant for the additional increments. For howitzers, it is customary to use small, single-perforated grains for the low charges and add increment charges of large, multiperforated grains for the high charges. More than two granulations could be used, but this is not customary.

The fundamentai theory is the seme for dual granulation as for single granulation, but some of the equations have to be modified to make them applicable to dual granulation. It is here assumed that both kinds of propellant are ignited at the same time, but the faster propellant is burnt sooner than the slower propellant.

The subscript 1 will be used to refer to the characteristics of the fast propellant, or to the trajectory during the simultaneous burning of both propellants. The subscript 2 will be used to refer to the characteristics of the slow propellant, or to the trajectory during the burning of the slow propellant alene. The subscript $i$ will denote either 1 or 2 .

The characteristies of the gun and projectile are the same as before: the area of the cross section of the boee, $A$; the chamber volume, $U_{e n}$; the travel, $X$ ( $X_{m}$ at the muzzle); weight of the projectile, $\mathrm{W}^{\text {; }}$; and starting pressure, $P_{0}$. The characteristics of the propellants are: the burning rate confficient, $B_{i}$; the foree, $F_{i}$; the specific weight, $p_{i}$; and the web thickness, $w_{i}$.

Obviously, the total charge is

$$
\begin{equation*}
C=C_{1}+C_{2} \tag{2-108}
\end{equation*}
$$

The surface area of the grains may be calculated by the formuia

$$
\begin{equation*}
S_{i}=2 C_{i} / \rho_{i} w_{i} \tag{2-109}
\end{equation*}
$$

The initial free volume is

$$
\begin{equation*}
U_{13}=U_{c n}-\left(\frac{C_{1}}{\rho_{1}}+\frac{C_{2}}{\rho_{2}}\right) \tag{2-110}
\end{equation*}
$$

As before,

$$
M=\frac{1.05 W+C / 3}{386}
$$

While both propellants are burning, the constants

$$
\begin{equation*}
\sigma_{1}=\frac{(F \rho S B)_{1}+(F \rho S B)_{2}}{0.3 A} \tag{2-111n}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon_{t}=\left(0.0 \pm T U_{u}^{2} 1 I^{3} \sigma_{1}^{17}\right)^{1 / 7} \tag{2-112a}
\end{equation*}
$$

The trajectory starts at

$$
\begin{equation*}
\phi_{n, 1}=\log \frac{P_{10} U_{n 1}}{0.3 \epsilon_{1}} \tag{2-113}
\end{equation*}
$$

It the burnt position of the fast propellant,

$$
\begin{equation*}
\phi_{1 b}=\log \frac{F_{1} C_{1}\left[1+\left(F_{\rho} S B\right)_{2} /(F \rho S B)_{1}\right]}{0.3 \epsilon_{1}} \tag{2-114a}
\end{equation*}
$$

On Chart 2-2a, at the intersection of the $\phi_{01}$-trajectory and the $\phi_{11}$-curve, the coordinates $\omega_{16}$ and $\log \xi_{16}$ are found.

While only the slow propellant is burning,

$$
\begin{equation*}
\sigma_{2}=\frac{(F \rho S B)_{2}}{0.3 \mathrm{~A}} \tag{2-111b}
\end{equation*}
$$

The second phase starts at the coordinates $\omega_{16}$ and $\operatorname{iog} \xi_{2, n}$; and

$$
\begin{equation*}
\log \xi_{2 n}=\log \left(\xi_{11}, \sigma_{2} / \sigma_{1}\right) \tag{2-115}
\end{equation*}
$$

On Chart 2-2a, the $\phi_{n-2}$-trajectory and the $\phi_{20}$-curve that cross this point are found. Then, since the weight of burnt propellant and the pressure are the same at the beginning of the second phase as at the end of the first phase,

$$
\begin{equation*}
\epsilon_{2}=\epsilon_{1}\left(10^{\delta_{1} \phi} / 10^{\delta, n}\right) \tag{2-112b}
\end{equation*}
$$

It the burnt position of the slow propellant,

$$
\begin{equation*}
\phi_{2 b}=\log \frac{F_{1} C_{1}+F_{2} C_{2}}{0.3 \epsilon_{2}} \tag{2-11+b}
\end{equation*}
$$

On Charts 2-2a, -2c at the intersection of the $\phi_{0}-$-irajectory and the $\phi_{2 \iota}$-curve, $\omega_{2 b}$ and $\log p_{2 b}$ are found. At all burnt, then, the pressure is

$$
\begin{equation*}
P_{2 b}=\frac{1.076_{\mathrm{e}_{2}}}{U_{0} \hat{p}_{2 b}} \tag{2-116a}
\end{equation*}
$$

and the free volume is

$$
\begin{equation*}
U_{3 b}=\frac{\left(1-\omega_{2 b}^{2}\right)\left(F_{1} C_{1}+F_{2} C_{2}\right)}{P_{2 b}} \tag{2-117}
\end{equation*}
$$

If the reduced peak pressure curve crosses the used part of either the $\phi_{0}$ :-trajectory or the $\phi_{01}-$ trajectory, the intersection indicates the value of $\tilde{p}_{1 p}$. Then, the maximum pressure can be calculated by the formula

$$
\begin{equation*}
P_{p}=\frac{1.076 \epsilon_{1}}{U_{0} \tilde{p}_{i p}} \tag{2-116b}
\end{equation*}
$$

If the $p_{\nu}$-curve does not cross cither trajectory, $P_{2 b}$ is the actual peak pressure.

At the muzzle, the free volume is

$$
\begin{equation*}
U_{m}=A X_{m}+U_{0} \tag{2-102}
\end{equation*}
$$

Mitw finding $\quad\left(\omega_{2}\right.$, ）from Chart 2－3，compute

$$
\begin{equation*}
\nu\left(\omega_{m}\right)=\frac{U_{m}}{U_{2 k}} \nu\left(\omega_{3 n}\right) \tag{2-118}
\end{equation*}
$$

and find $\omega_{\mathrm{m}}$ from the chart．Then the velocity at the muzzle is

$$
\begin{equation*}
r_{n}=\sqrt{\frac{2\left(r_{1} c_{1}+F_{2} C_{2}\right)}{0.3 .1 /}} \omega_{m} \tag{2-119}
\end{equation*}
$$

The modifications of the formulas for time，pres－
sure，velocity，and travel at arbitrary values of $\omega$ are obvious；they will not be given here．

## 2－3．11 Example for Dual Granulation Charges

Determine the maximum pressure and muzzle velocity for the 10.5 mm Howitzer $\mathrm{M}[4$ ，firing the HE Projectile MI，propelled by 2 ounces of single－ perforated propellant 218 lot $\qquad$ and 8 ounces of multiperforated propellant MI lot $\qquad$

|  | Chararteristic | Symbol | $V$ Vhue | Unit | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Are：a of lore | ． 1 | 13.1 | in ${ }^{2}$ | Given |
|  | （hamiker vohume | $1{ }_{\text {ch }}$ | 15：3．8 | in ${ }^{3}$ | （iiven |
|  | Travel（to muzzle） | $\lambda_{m}$ | 80.4 | in | （iiven |
|  | Weight of projertil． | $1{ }^{\circ}$ | 33．0 | ll | （iiven |
|  | starting pressure | $P_{0}$ | H000 | pri | （iven |
|  | Charge：M8 | P1 | 0.125 | II， | （iven |
|  | ． 11 | $C$ | 0.50 ） | 115 | Given |
|  | ？${ }^{\text {a }}$ ：$:$ ！ | \％ | 0.625 | 11） | 1：ィ．2－108 |
| ， | Burning rate corfliciont | $B_{1}$ | 0.0023 | $\frac{!n / \mathrm{nct}}{(\mathrm{psi})^{0 . s}}$ | （iven |
|  |  | $B_{2}$ | 0.0011 |  | （iven |
|  | Forre | $F_{1}$ | $4.60 \times 10^{6}$ | i．i－ll $/$／ll | （iiven |
|  |  | $F_{2}$ | $3.73 \times 10^{6}$ | in－ll $/ 1 \mathrm{ll}$ ， | （iven |
|  | Sixucilie wright | $p_{1}$ | 0.0571 | $11 . / \mathrm{in}^{3}$ | （iiven |
|  |  | $\rho \cdot$ | 0.0571 | $1 \mathrm{l} / \mathrm{in}^{3}$ | （iiven |
|  | Wi．h，thiokness | $w_{1}$ | 0．00－1i） | in | （iven |
|  |  | $u$ ： | 0.0140 | in | Siven |
| ， | Burning Surfare | $S_{1}$ | 1095 | in ${ }^{2}$ | 1：4．2－109） |
|  |  | $S_{2}$ | 1251 | $\mathrm{in}^{2}$ | 14．4．2－109 |
|  | Initial free volumu | $L_{0}$ | 142．！ | in＇ | 1：4．2－110 |
|  | 1：ffertive mass | M | 0.09031 | 12 sluss | E¢．2－11a |
|  | Constant | $\sigma$ | $2518 \times 10^{5}$ |  | 1：\％．2－111a |
|  | Constant | ${ }_{6}$ | $2.299 \times 10^{7}$ |  | 1：4．2－112a |
|  | $\phi_{01}$－trajectory | $\phi_{u 1}$ | $-1.081$ |  | 1\％4．2－113 |
|  | фı－rurve | $\phi_{16}$ | －0．930 |  | 14． 2 －ilta |
|  | Aloscissa | $\omega_{1 s}$ | 10.023 |  | Chart 2－2a |
|  | Ordinate | $\log \xi_{1,}$ | 0．78： |  | Chart $2-2 \mathrm{n}$ |
|  | Constant | $\sigma$ \％ | $7.291 \times 101$ |  | leq．2－111b |
| ： | Ordinate | $\log 5:$ | 0.24 |  | E¢．2－115 |
|  | $\phi_{n}=$ trajuctory | ¢．： | －0．225 |  | Chart 2－2a |
|  | $\phi=-$－urve | $\phi_{: ~}^{\prime}$, | －0．190 |  | Chart 2－2n |
|  | Constant | ： | $4.184 \times 10^{6}$ |  | E¢．2－1121， |
|  | ¢tt－rurve | $\phi ⿻ 土 ㇒$ | 0.293 |  | E¢．2－114） |
|  | Abscissa | $\omega_{0 \prime \prime}$ | 0.369 |  | Chart 2－2a |
|  | Redured l＇reswure | $\log 71 \%$ | 0.616 |  | Chart $2-2 \mathrm{c}$ |
|  | Reduced prak prensure | losk $\bar{\sim}$ | 0.517 |  | Charts $2-2 \mathrm{ia},-2 \mathrm{c}$ |
|  | Maximum pressure | $P_{P}$ | 854\％ | 1 pi | E（f．2－116b） |
|  | Pressure at burnt | $P_{z h}$ | 7 （i33） | pxi | E4．2－116： |
|  | Frece volume at burnt | $\underline{V}$ | 275 | in ${ }^{3}$ | Eq．2－117 |
|  | Frie volume at muzale． | $1{ }_{n}$ | 120 | in＇ | 1：1．2－102 |
|  | Travel function | $\nu\left(\omega_{0}\right)$ | 1．64） |  | Chart 2－3 |
|  | Travel function | $r\left(\omega_{m}\right)$ | 7.05 |  | E！f．2－118 |
|  | Abscissa | $\omega_{m}$ | 0.665 |  | Chart 2－3 |
|  | Velority at muzzle | $V_{m}$ | 8924 | $\mathrm{in} / \mathrm{sec}$ | 1：4．2－119 |
|  | Velocity at muzzle | $V_{1}$ | 74 | $\mathrm{ft} / \mathrm{sec}$ | Esf．2－11！ |

## 2-4 THE HIRSCHFELDER SYSTEM

${ }^{11}$ the. Mirschfelder System it is assumed that the buming rate is linearly propertional to the pressure, that is, $\alpha=1$. With this assumption the equations are solvable analytically. Hischfelder does not assume $\eta=u$ nor that the burning surface is constant. If the burning surface is not constant a fourth equation, the form function equation, is introduced into the system of equations to be solved simultancously. The form of this equation depends on the shape of the grains and in effect takes account of the change in the burning surface as the web bums away. The system also assumes a starting pressure taken as the pressure produced when one percent of the charge has been consumed. This can be obtained from closed chamber data since antil the projectile muves the burning is at constant volume. The I!irschfelder System is covered completely and in detail in leeference 6 of Chapter 1 (Report No. 1) and is presented together with charts and working tables for use in gun design problems. There is also included a discussion of the thermodynamic properties of propellants.

## 2-5 "SIMPLE" INTERIOR BALLISTIC SYSTEMS

## 2-5.1 General

By neglecting all, or most, secondary effects which are difficult to evaluate, such as heat and friction losses and starting pressures, and by making all or most of the usual sinplifying assumptions, a very simple analytical treatment can be given. Examples of this are the British RD-38 System (Cf. Comer ${ }^{3}$ ) and the Mayer and Hart System ${ }^{3}$. These systems result in simple analytic formulas for the variables and, if the adjustable parameters are adequately evaluated by numerous comparisons with firing records, can yield results valid to a few percent for standard guns, especially for the larger calibers. They are also useful in that the interrelations between the more important parameters are more obvious than in the more involved systems, so that they allow a more direct feeling for the interrelations to develop.

## 2-5.2 The Mayer and Hart System

The Mayer and Hart System employs the following simplifying assumptions:
(1) the starting pressure and engraving pressure are zero, that is, the projectile starts to move as sonn as the propellant begins to burn;
(2) the covolume of the gas is equal to the origimal charge volume, that is, $\eta=u$;
(3) the burning rate is linearly proportional to the pressure, that is, $\alpha=1$;
(t) the buming surface is constant throughout the burning;
(i) all terms in the kinetic energy expression are negligible except those for the kinetic energy of the gas and the projectile; and
(i) energy losses due to friction and heating are also negligible.
dssumptions (i) and (i) do not affect the form of the theory, since the neglected encrgies are taken account of in practice by adjusting the values of the weight of the projectile and the specific heat ratio, $\gamma$, to some effective values. Consistent with the practice of other interior ballistics methods, the kinetic energy of the propellant gas is accounted for by adding one-third of the gharge weight to the projectile weight, producing an adjusted projectile weight, $\|^{\prime \prime}$, equal to projectile weight plus ( $/ 3$. The symbol $W^{\prime \prime}$ will be used in the remainder of this discussion for the adjusted weight. The value of $\gamma$ may be adjusted according to the judgment or experience of the user. In the preceding method, according to Taylor, ari adjusted value of $\gamma$ of 1.30 is assumed. This value may not be valid in the Mayer and Hart method, because of assumption in the latter method of linear proportionality of the burning rate to the pressure.

In the following, the original notation of Mayer and Hart has been changed to conform to that exhibited in the List of Symbols at the beginning of Chapter 2. Mayer and Hart also introduced numerical factors in their formulas to adjust for the units used. These have been omitted, leaving it to the user to express the quantities in a consistent set of units.

The three fundamental equations are then

$$
\begin{gather*}
\frac{d c}{d l}=\frac{2 C B P}{w}  \tag{2-120}\\
P^{\prime}=c F^{\prime}-(\gamma-1) \frac{W^{\prime}}{2 g} V^{\prime 2} \tag{2-121}
\end{gather*}
$$

and

$$
\begin{equation*}
\frac{W^{\prime}}{g} \frac{d V}{d t}=P A \tag{2.122}
\end{equation*}
$$

Maycr and Hart define two ballistic parameters having the dimensions of pressure

$$
\begin{equation*}
P_{n}=\left(\frac{2 B C F^{\prime}}{A w}\right)^{2}\left(\frac{W^{\prime}}{G U_{v}}\right) \tag{2-123}
\end{equation*}
$$

and

$$
\begin{equation*}
l_{r}=\frac{C F}{U_{b}} \tag{2-124}
\end{equation*}
$$

where $l_{"}$ is the initial free volumes, "qual to the chamber volume minus the volume of the charge. $P$. is thr pressure which would be developed in the ehamber if there were no motion of the projectile. If it is assumed that the pressures occuring in Equations $2-120$ and $2-121$ are the same, that is, that the pressure drop in the gas can be neglected
$I^{\prime}\left(\frac{C^{\prime}}{l_{n}}\right)=P_{r}^{\prime}\left(\frac{c}{c_{i}}\right)\left[1-\underset{\underline{2}}{\gamma_{-}-1}\left(\frac{P_{r}^{\prime}}{P_{a}^{\prime}}\right)\left(\frac{c}{c}\right)\right]$
and also

$$
\begin{align*}
& \frac{C}{C_{11}^{\prime}}=\left[1-\frac{\gamma-1}{2}\left(\frac{P_{c}^{\prime}}{P_{u}^{\prime}}\right)\left(\frac{C}{C}\right)\right]^{-2 /(\gamma-1)} \\
& \frac{c}{C}=\left[\frac{2 P_{u}^{\prime}}{P_{c}^{\prime}}(\gamma-1)\right]\left[1-\left(\frac{U}{C_{n}^{\prime}}\right)^{-(\gamma-1) / 2}\right] \tag{2-127}
\end{align*}
$$

which state the relations between the fraction of the charge burned and the expansion ratio. Since $r^{\circ}=c_{0}+. N$, bination $\because-126$ yields also the relation betwen the fraction of charge burted and the travel.

By Eas. -120 and $2-127$

$$
\begin{equation*}
r=\frac{O P B}{\gamma-1}\left(\frac{U}{l^{\prime}}\right)^{-\gamma}\left[\left(\frac{U}{U_{1}}\right)^{i r-1 / 2}-1\right] \tag{2-28}
\end{equation*}
$$

The pressure is, theoretically, a maximum when

$$
\frac{c}{l_{n}}=\left[\frac{2 \gamma}{\gamma+1}\right]^{2 /(\gamma-1)}
$$

and

$$
\frac{c}{c}=\frac{P_{n}}{\gamma I_{c}^{\prime}}
$$

and has the theoretical value

$$
\begin{equation*}
i_{\text {max }}=P_{n}\left[(\gamma+1)^{(\gamma+1)} \gamma^{\left.\left.-2 \gamma \cdot 2^{-(\gamma+1)}\right]^{1 /(\gamma-1)}\right)}\right. \tag{2-131}
\end{equation*}
$$

This value will be reached only if $P_{w i} P_{c} ₹ \gamma$, that is, if the theoretical maximum pressure is reached before the charge is all burned. Otherwise $P_{\text {max }}$ occurs when $c / C$ is ecqual to unity and the maximum pressure is then the pressure at burnt.

The energy of the projectile up to burnout is given by

$$
\begin{equation*}
\frac{\|^{-\rho^{-}} I^{-2}}{2 g}=\frac{2 P_{\mu} U_{n}}{(\gamma-1)^{2}}\left[1-\left(\frac{U}{U_{0}}\right)^{-(1-1) / 2}\right]^{2} \tag{2-132}
\end{equation*}
$$

The conditions at burnt can be obtained by setting c. $C=1$ in Eq. $2-126$ to derive $U$ at burnt, and substituting $C^{*}$ at burnt in Eqs. 2-128 and 2-132 to derive $i^{\prime \prime}$ and $\mathrm{I}^{\prime}$ at burnt. The travel at burnt is derived from ! $=l_{0}+A X$.
The corresponding values at burnout are

$$
\begin{align*}
& C_{b}=L_{0}\left[1-\frac{1}{2}(\gamma-1)\left(\frac{P_{c}}{P_{b}}\right)\right]^{-2 /(\gamma-1)} \\
& P_{b}=P_{c}\left[1-\frac{1}{2}(\gamma-1) \frac{P_{c}^{\prime}}{P_{b}}\right]^{(\gamma+1) /(\gamma-1)}  \tag{2-132~b}\\
& I_{b}=P_{\cdot}\left(\frac{U_{0} g}{I^{\prime \prime} I_{b}^{\prime}}\right)^{1 / 2}  \tag{2-1:32c}\\
& \Gamma_{t}=\frac{U_{b}-U_{n}}{A} \tag{2-132~d}
\end{align*}
$$

After burnout the gas expands adiabatically so that, when the projectile is at the muzzle, the pressure is given by ( $\left.C_{b} / L_{m}\right)^{2} l_{b}$. The pressure when the projectile is at the muzzle is therefore given from Equations $9-13 \sum_{a}$ and $2-1 ; 2 b$ by

$$
\begin{equation*}
I_{m}^{\prime}=I_{c}\left(\frac{U_{u}}{U_{m}}\right)^{\nu}\left[1-\frac{1}{2}(\gamma-1) \frac{P_{0}}{P_{u}^{\prime}}\right]^{-1} \tag{2-1;33}
\end{equation*}
$$

The muzzle energy is given by
$\frac{W^{\prime \prime}}{2 g} V_{m}^{2}=\frac{\left(F^{\prime}\right.}{\gamma-1}\left\{1-\left(\frac{U_{0}}{U_{m}}\right)^{\gamma-1}\left[1-\frac{1}{2}(\gamma-1) \frac{P_{r}}{P_{r} n}\right]^{-1}\right\}$

## 2-6 THE EFFICIENCY OF A GUN-AMMUNITION SYSTEM

There are two "efficiencies" commonly used to estimate the effectiveness of a given gun-propellant system in imparting energy to a projectile. These are thermodynamic or ballistic efficiency and the piezometric efficiency.

The former is defined as the ratio of the translational energy of the projectile at the muzzle to the total energy of the charge as defined by Equation 2-2, that is
Themodynamic Eificiency $=\frac{(\gamma-1) W V_{m}^{*}}{2 C V g}$

The piezometric efficiency is defined as the ratio of the nean base pressure, which acting during the travel to the muzzle, would produce the muzzle velocity, to the maximum pressure. It is related to the flatuess of the pressure-travel relation. A high piezometric efficiency means a higher muzzle pressure and the projectile position at burnt further toward the muzzle. This will result in inereased muzzle blast and greater round to round variation in muzzle velocity.

I high ballistic efliciency results when the charge is completely barned as carly as possible in the projectile travel. Thus a high thermodynamic or ballistic eflicinney comesponds to a low piezometrie aflicioner.

## 2-7 COMPARISON WITH EXPERIMENT

## 2-7.1 General Considerations

Whatever system of interior ballisties one makes use of, when faced with a practical problem in the analysis of interior ballistic measurements or in the design of a gun, one must assign values to certain of the patameters oceuring in the theory which ammot be determined by independent means. The most important of these parameters is the buming rate coerliciont which will oceur in any formulation of the theory. Othess are the heat loss ratio which determines the adjusted value of $\gamma$, the starting pressure, and the buming rate pressure exponent. Proper values of these parameters must be do-
 of actual gun firings. 'To mateh the theory to , herervation by simultancous adjust ment of all such uncertain parameters would be very tedions, and, in view of the simplify ying assumptions made in the formulation of the theory, unwartanted. One must make a judgment as to the most uncertain parameters and those to which the solution is most sensitior and assign approximate values to the least important ones and adjust the others for best fit." By a process of trial and error one arrives at the best set of values to fit the theory to the experimental data. It is often possible to estimate approximate or limiting values of eertain parameters theoretically or by independent mowsurements. For example, the thermodynamic value of $\gamma$ can be calenlated and the effective value will be greater than this. The heat loss could be calculated approximately by the method of Chapter:3. Relative values of the burning rate constant can bre determined for diferent propellants from closed chamber measurement.

If one has to deal with guns of menconventional
design such as the so called light gas guns or other guns operating outside the usual range of pressure and velocity a greater dependence on theory becomes necessary. A major difficulty in treating high performance guns such as modern tank guns is the proper treatment of the pressure gradient in the gas. This problem is discussed in some detail in Chapter i.

## 2-7.2 Experimental Evaluation of the Parameters

a. Cicneral I'rucedures. The interior ballistic quantities that can be measured with modern instrumentation are described in Chapter 4. To try to furnish data on all of these would not be warranted except possibly for firings carried out in connection with research in the subject. C'sually the data supplied will be much less extensive and complete. This will influence the complexity of the theory used to analyze the data and the procedures followed. If all that is available are erusher gage values of the peak pressure and the muzzle velocity a very simple theory such as that of Mayer and Hart may lse aderiuate.

For a more detailed falalysis of the interior ballistic trajcetories, the firing records will furnish breech pressure and projectile travel as functions of the time. The records will be provided with a common time seale and fiducial marks to adjust the time scales to a common zero time (sec Chapter 4). These records are read by means of an optical comparutor capable of measuring both horizontal and vertical displacements on the record. Csually the travertime data are derived from an interferometer record which furnishes values of the time for erual intervals of distance. By interpolation, these data are converted to intervals of distance for equally spaced intervals of time so that one finally has the beech pressure and projectile travel presented on a common time seale. The travel data is then differentiated twien to provide values of the velocity and acceleration of the projectile also as functions of the time. This will recpuite considerable smoothing and other mathematical manipulation. For details and references to the mathematical literature underlying the process see Referenee 7 .

To reduce the data in this manner and to make eomparisons with the theoretical formulas in detail for many records, is a very time consuming process. To analyze the records from a single firing may take two or more man weeks if carried out with desk calculators. Where the necessary automatic recordmeasuring and computing equipment is available, the procedures can be antomated and coded for high sperd computers so that the data from a gum


HItiClll: 2-2. Result of the Analysis of a Firing Record for a 105mm Howitzer Round (Measured Values of Pressure and Displacement. V'elocity and Acceleration Delermined by Numerical Differentiation of Displacement)
fring can be processed in a matter of hours. ${ }^{7}$ A plot of the result of such a record analysis, for a typical case, is shown in Figure 2-2.
b. The Conditions in the Early Stages of Burning; the Slarting Pressure. From such a record, the pressure at the actual start of motion can be read directly. From the record shown, it is about 750 psi . It $s^{\text {bould }}$ be emphasized, however, that the time of start of motion is difficult to determine accurately because it is difficult to determine from the record just where the displacement actually begins. This would be true even if the graph were precisely determined. Error of reading and reduction will increase the uncertainty.

The "starting pressure" used in the formulation of interior ballistic theories is a quantity quite different from the actual pressure at the start of motion. Immediately after the projectile starts to move, the rotating band engages the rifling and a large resisting force is developed. After engraving, the resisting force drops rapidly to a much lower talue. One might expect this to show as an irtegularity in the graph of the acceleration in Figure 2-2. It does not do so because the resolutio: of the apparatus and the data reduction procedures are
not sufficient to show it. This is usually the case unless special methods are used to study the motion during engraving. A description of one such method is given in paragraph 4-4.3.
If the acceleration is adequately determined, the effective pressure, $P_{c}$, on the base of the projectile, that is the difference between the actual base pressure and the pressure necessary to balance the forces of engraving and friction, can be determined from the relation

$$
\begin{equation*}
P_{\cdot}=\frac{W a}{A g} \tag{2-136}
\end{equation*}
$$

If a measurement of the base pressure, $P_{0}$, is available or if it can be calculated by the use of a formula such as Equation 2-20, an estimate of the engraving and frictional force can be calculated from the difference $P_{,} A-P_{,} A$. A graph of this force for a typical 105 mm Howitzer round is shown in Figure 2-3. The observed maximum force of about 4:500 pounds, divided by the nomimal bore arca of 13.4 square inches gives a starting pressure of 3350 pounds per square inch. This compares reasonabiy well with the value indicated on Figure 2-1 of 4000 psi .

In assigning starting pressures some average value of the engraving force should determine the assigned value. The engraving forec will evidently be quite variable from round to round. It will be sensitive to manufacturing tolerances in the size of the rotating band, how elosely the band fits the projectile and possibly in the thickness of the projectile wall.

When comparing their theories with experiment, most authors assign a starting pressure in an arbitrary manner. It is hardly possible to do it in any other way. The experimental values of engraving foreses can serve only to set some limits on it. In the Hirschfelder system, for example, the starting pressure is assumed to be the pressure existing when one percent of the charge has been consumed. Since the projectile has presumably not moved up to this time, the starting pressure is that which would be
produced in a closed chamber under similar loading conditions.
c. T'he Rate of Charge Consumption. The amount of propellant burned up to any specified time can be determined from the energy balance equation. This equation, as developed in paragraph 2-2.1, can be written as

$$
\begin{array}{r}
\frac{c F}{\gamma-1}=\frac{P\left[A X+U_{c s}-C u-c(\eta-u)\right]}{\gamma-1} \\
+\frac{M V^{2}}{2}+Q \tag{2-137}
\end{array}
$$

where $\gamma$ is the calculated thermodynamic ratio of specific heats and $l$ is the average pressure consistent with the equation of state. $Q$ is mainly heat loss to the tube, but may contain other losses not taken care of by the definition of the effective mass, $M$.


FI(iCRE 2-3. E'nyraving Force for " Typical 10inmm Howilzer Round
$l$ ' ean be converted to breech pressure by the use of the Pidduck-Kent solution for the pressure ratios determined from paragraph i-1.1 or by the use of Equation 2-20 for the case of low velocity weapons. The value of the effective mass, $I /$, may also be determined more accurately by the use of the Pidduck-Kent solution for the kinetic energy of the unburned propeliant and the gas.

Assuming that $P$ is correctly related to the breceh pressure and that $.1 /$ is determined with sufficient accuracy, $c$ can be determined from Equation 2-137 provided some knowledge of the value of $Q$ is available.
$Q$ is often neglected entirely because of the difficulty of incorporating it in the analysis. This may lead to error, as the heat loss can be a considerable fraction of the available energy, especially in smallerwapons or when hot propellants are used. The use of a nominal value of from:) to 20 percent of $M V^{2} / 2$ should improve the analysis; the lower value to be used for standard cammon, and the upper range for high velocity gtus and small arms.

Allowing for $Q$ in this manner and substituting measured values of $I, X$ and $V$ in Equation 2-137 $c$ can be calculated for specified values of $t$ and a graph of $c$ against $t$ plotted. This graph should approach a limit wheu $c=C$ since the charge docs not burn out discontinuously, as the charge is not ignited simultancously over its entire surface and the burning rate is not the same everywhere. The burning surface, and therefore $d c / d t$ go to zero at burnout. This may not be obvious on the graph depending on the shape of the grains and the cffectiveness of the ignition as well as the time resolution of the data.

Because of errors in the values of $I^{\prime}$ and in the assigned value of $Q, c$ will not, in general, equal $C$ when de'dl becomes equal to zero. Since it should do so a further adjustmenit must be made in $P$ or $Q$ or both to bring it about. As $Q$ is a correction term its value does not affect the adjustment as strongly as does that of $P$ and since the formulas for the pressure ratios are admittedly uncertain the adjustment should be brought about mainly by changes in the values of $P$.

After burnout, the right-hand side of Equation 2-1:37 should remain constant, equal to $C F / \gamma-1$. As $V^{\prime}$ and $X$ increase, $P$ declines, as the gas continues to do work and lose heat to the barrel. The graph for $c$ should, therefore, remain at the value $C$. This can be brought about by continuing to make the necessary adjustments in $P$.

In the interval before burnt, $I^{\prime}$ can be adjusted
using the same ratio of $l^{\prime}$ to $l_{a}^{\prime}$ ( the adjusted value of $P$ ) as was necessary to make $c$ equal to $C$ at burnt. This is in accord with the usual formulas which state that the pressure ratios are constant during burning. This, however, is hardly possible at the start of motion where the ratio ought to be equal to unity. A procedure which has been used in practice is to adjust the ratio lincarly with time from its experimental value at burnt to unity at the start of motion. A graph of $c$ versus $t$ constructed in this way, for a 37 mm gim firing, is shown in Figure 2-4.

Once the graph of $c$ versus $t$ has been constructed in this way an experimental value of the time to burnt can be read off immediately and experimental values of the travel and velocity at burnt will be known from the associated values of $X$ and $V$.

The graph can also be differentiated and the rate of charge consumption determined as a function of time. The surface area, $S$, of the burning grains can be calculated from the geometry of the grains ${ }^{8}$ and the linear burning rate, $r$, at any specified time determined from the relation

$$
\begin{equation*}
r=\frac{1}{\rho S} \frac{d c}{d t} \tag{2-138}
\end{equation*}
$$

Since the pressure, $P$, at the same instant is also known, a graph of the linear rate of burning versus $P$ can be constructed. In general this graph will not be a straight line when plotted on a log-log scale indicating that the burning rate dependence on the pressure cannot be represented by a simple power relation. This is to be expected because the burning in the gun takes place at different pressures in different parts of the tube and the average burning rate will depend on the pressure distribution and also on how the unburned propellant is distributed. The rate at any time during the burning depends also on factors other than the pressure, such as the velocity of the gas. The graph, however, can be used to estimate an effective value of the pressure exponent. A graph of linear burning rate versus pressure contructed in this way for the 105 mm Howitzer is shown in Figure 2-5. The closed chamber burning rate for the same propellant is also shown. Above about 1700 psi the pressure exponent for the gun burning rate is lower than that for the closed chamber and below 1700 psi the exponent for the gun is considerably larger than for the closed chamber. It is also considerably greater than unity. There is no theoretical reason why this should not be possible.

0.00
0.03

保

theory such as that of Mayer and Hart and that their specification does not involve a knowledge of unknown quantities such as starting pressures or burning rates. Charts or nomograms are then constructed showing the relation between the chosen parameters. The charts or nomograms are adjusted to their final form by fitting to numerous firing records. In solving a practical problem the given data will permit the evaluation of certain of the parameters and from the charts or nomograms the proper values of the others can be read off. The parameters are known functions of the desired var-
iables such as maximum pressure and muzzlc velocity so that the latter can be determined once the proper values of the parameters are known.
Such a scheme is that published by Strittmater ${ }^{11}$ which is presented in a single working chart (Chart 2-4). The theory used is that of Mayer and Hart supplemented by a further assumption that bore friction is proportional to chamber pressurc. This assumption is used to improve the agrecment between the chart and experiment by adjusting the effective projectile weight.

The resulting effective weight is defined as

TABLE 2-2. VALUES OF $\eta, \beta, \eta_{1}$, AND $\beta_{1}$ FOR STANDARD WEAPONS


| Wiapon, Giun | Prujectile |  |  | Proprellant |  |  |  | \%one | $\begin{aligned} & \mathrm{Vel} \\ & \mathrm{fps} \end{aligned}$ | Crusher Press, psi | Coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Velocity | Pressure |  |  |  |  |
|  | Moxtel | Type | Wt, H |  |  |  |  | Lot |  |  | TYpe | Web, in | Wt | " | $\beta$ | $\eta 1$ | $\beta_{1}$ |
| Cimm M1 | M19. 11 | HL: | 12.80 | 60105-3 | MP, M6 | . 0365 | 55.72 oz |  |  | 2700 | 37100 | $-.31$ | -. 21 | . 65 | $-1.17$ |
|  | M6ed | APC-T | 15.40 | 176336-R | MP, M6 | . 0419 | 61.17 oz |  | 2600 | 42300 | $-.35$ | -. 22 | . 69 | $-1.18$ |
|  | M1352 | HE | 15.00 | 60105-S | MP, M6 | . 0365 | 55.01 oz |  | 2400 | 28.400 | $-.38$ | -. 10 | .69) | $-.82$ |
| 76mm M3: | 11339 | AP-'T | 14.50 | 63848 | MP, M17 | . 0565 | 85.40 oz |  | 3200 | 44800 | -. 30 | -. 21 | . 66 | $-1.18$ |
| ! 0 min M1 | 1171 | HE | 23.40 | 17665 | MP, Mi | . 0496 | 115.13 \%\% |  | 2700 | 36630 | -. 30 | $-22$ | . 68 | $-1.01$ |
| $\begin{aligned} & 9 \mathrm{Mmm} \mathrm{M} 36, \\ & \mathrm{M}, 41 \end{aligned}$ | M71E: | HE-T | 23.40 | 63439 | M1', MI | . 03340 | 85.38 \%\% |  | 2.400 | 35800 | -. 39 | -. 21 | . 67 | -. . 9 \% |
|  | T?1 | HE-T | 18.00 | 38740-s | MP, MI | . 0268 | 72.46 oz |  | $2-400$ | 20500) | -. 38 | -. 21 | . 66 | $-.78$ |
|  | M1318.11 | $\mathrm{Al}^{\text {P-T }}$ | 2.4 .10 | 3871-S | MP, M17 | . 0784 | $1+1.58 \mathrm{cz}$ |  | 3000 | +4000 | -. 2.4 | -. 23 | . 65 | $-1.22$ |
| $120 \mathrm{~mm} \mathrm{M1}$ | M\%3 | HE | 50.00 | 38470-S | MP, M6 | . $\mathrm{O} \times 7.1$ | 23.38 lb |  | 3100 | 37200 | -. 30 | -. 2.4 | . 06 | $-.97$ |
| 120mm M58 | $\begin{aligned} & 11: 35 ; \\ & 11358 \end{aligned}$ | $\begin{gathered} H E \\ A P^{\prime}-T^{\prime} \end{gathered}$ | 50.40 00.85 | $\begin{aligned} & 3872: 3-12 \\ & 3: 3879-\mathrm{S} \end{aligned}$ | $\begin{aligned} & \mathrm{SP}, \mathrm{M15} \\ & \mathrm{MP}, \mathrm{M17} \end{aligned}$ | $\begin{aligned} & .03+4 \\ & .1140 \end{aligned}$ | $12.24 \mathrm{lb}$ $2!.43 \text { If }$ |  | $\begin{aligned} & 25(0) \\ & 35(0) \end{aligned}$ | $\begin{aligned} & 385(0) \\ & 48(0) \end{aligned}$ | -.39 | -. 21 | . 67 | $\begin{aligned} & -.82 \\ & -1.21 \end{aligned}$ |
| 15.7mm M2 | .1101 | HE: | 95.00 | 30:348-5 | MP, M6 | .0559 | 20.68 ll, IRedurer! 30.86 Full | 2100 <br> 2800 |  | $\begin{aligned} & 17: 3(1) \\ & 38600 \end{aligned}$ | $\begin{aligned} & -.35 \\ & -.37 \end{aligned}$ | $\left\lvert\, \begin{aligned} & -.21 \\ & -.24 \end{aligned}\right.$ | $\begin{aligned} & .6 .4 \\ & .67 \end{aligned}$ | $\left\lvert\, \begin{aligned} & -.0! \\ & -. .00 \end{aligned}\right.$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Sin M1 | M103 | HE | 240 | 16697-S | $\left\lvert\, \begin{gathered} \text { MP. Mo } \\ \text { Mo Charge } \end{gathered}\right.$ | .0839 | 80.89 lb Redured (12.27 ll Normal |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 2850) | 37000 | -.t2 | $-.20$ | . 88 | - . 80 |
|  |  |  |  | 8501-S |  | . 0601 | 53.00 lb Itedured |  | 2100 | 18700 | -. 34 | -. 22 | . 0.4 |  |
|  |  |  |  |  | In Charge |  | $74.50 \mathrm{lb}$ |  |  |  |  |  | . 0.4 |  |
|  |  |  |  |  |  |  | Normal |  | 2600 | 33500 | $-.34$ | -. 2.5 | . 66 | -. 78 |
| 280mm 11:31 | M124 | ILE | 600 | $\begin{array}{\|l} 39370 \\ 60580 \end{array}$ | MI', M6 | $\begin{aligned} & .0688 \\ & .100 \end{aligned}$ | 52.83 1 l | 1 | 1:380 | 8800 | $-.39$ |  | . 60 |  |
|  |  |  |  |  |  |  | 90.15 lb | 2 | 1780 | 15400) | -. 3 9 |  | . 60 |  |
|  |  |  |  |  |  |  | 118.70 lb | 3 | 2100 | 22000 | -. 39 |  | . 64 |  |
|  |  |  |  |  |  |  | 156.92 lb | 4 | 2500 | 334400 | $-.38$ |  | . 65 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## 2-8 SIMILARITY AND SCALING

When the solution of the equations for a theory of interior ballistics is attaincd, the solutions for particular cases are found to be characterized by certain ballistic parameters which are combinations of the quantities specifying the details of the gun and charge. What these parameters are and their form will depend on the way the theory is formulated. If the characteristics of two gun-ammunition systems lead to the same numerical values for the ballistic parameters, the theoretical solutions for both will have the same form and the actual solutions can be transformed one into the other by simple changes of scale.

If now one starts from a standard gun-ammunition system which has been well studied experimentally so that the gun-ammunition parameters occurring in the theory are properly adjusted to match the theory to experiment and numerical values of the ballistic parameters are known, the similarity can be used to predict the trajectorics for scaled models of the standard gun system.

It follows also that in tabulating solutions the extent of the tables can be much reduced by tabulating solutions for only certain values of the ballistic parameters. Solutions for other values can then be obtained by interpolation. The tables can be converted also into graphs or nomographs from which


FIC:LRI: 2-5. Linrar Burning Rate versus Pressure for 10:5mm Howitzer, MI Propellant

TABLE 2-2. (Continued)


$$
\begin{equation*}
W^{-}=W+i \times \times 10^{i} d X_{m} / V_{m}^{2} \tag{2-160}
\end{equation*}
$$

The parameters are presented on Chart $\geq-t$. They are in reduced form and are the following:
f thermodynamic efficiency
$z$ piezometric efficiency
.r volume expansion ratio
$r$ cnergy ratio
! pressure ratio
They are defined by the following equations

$$
\begin{equation*}
e=\frac{(\gamma-1)\left(W_{0}+(1 / 3) V_{m}^{2}\right.}{2 g V^{\prime}( } \tag{2-161}
\end{equation*}
$$

$$
\begin{align*}
& z=\frac{\left(I V_{0}+(!/ 3) V_{m}^{2}\right.}{2 g \bar{P}_{p} U_{1 m}}  \tag{2-162}\\
& x=\frac{U_{m}}{U_{0}}  \tag{63}\\
& r=\frac{P_{1}(!}{\bar{\Gamma}_{1} U_{1 "}^{\prime}}  \tag{2-164}\\
& y=\frac{\Gamma_{m}^{\prime}}{\bar{\Gamma}_{n}}
\end{align*}
$$

lifuation $2-162$ is in accord with the definition in paragraph $\dot{2}-6$, when $V_{m}$ is defined as the product of the bore area multiplied by the projectile travel
different solutions can be obtained with little auxilliary computation.

As an example, the effect of inereasing all linear dimensions of a gun-ammunition system will be investigated using Bemett's theory and tables.' The ballistic parameters and the scaling factors of Bemett's theory are the quantitios $q, \perp, r$, and $\ell$ as defined below, and to take account of changes in propellant type a standard specific energy, $E_{0}$, is used equal to $1.5 \times 10^{16}$ in- -lh Ih. The effective projectile weight is taken as

$$
\begin{equation*}
\left\|^{\prime \prime}=\right\|^{\prime}+(1 / 3 \tag{2-139}
\end{equation*}
$$

and wright ratio is defined as

$$
\begin{equation*}
e=\frac{\ddot{H}+(\%)}{R} \tag{-140}
\end{equation*}
$$

where $R$ is the weight of gum and recoiling parts. The quickness, 4 , is then defined as

$$
\begin{equation*}
q=q_{1} \frac{\left(E^{\prime} E_{1}\right)^{j / n} C_{1 / n}^{1 / 2}\left(V^{-\prime}\right)^{1 / 2}}{w \cdot 1(1+e)} \tag{2-141}
\end{equation*}
$$

where $\mu_{1}$, which depends on the type, temperature aud moisture content of the propellant, is determined empirically.
$\Delta$, the density of loading, is defined by

$$
\begin{equation*}
\Delta=U_{*} C^{\prime} / l_{c h}^{\circ} \tag{2-142}
\end{equation*}
$$

where $C_{-}$, is the specific volume of water. $\Delta$, so defined, is actually a specific gravity of loading and is dimensionless. It is numerically equal to density of loading in grams per cubic centimeter. The projectile travel, $X$, is given in terms of the expansion, $s$, as defined by

$$
\begin{equation*}
s=1+\left(i / C_{c k}\right) \cdot N \tag{2-143}
\end{equation*}
$$

Three of Bennett's tables tabulate values of pressure, $I_{1}$, velocity, $I_{l}$, and time, $l_{1}$, as functions of $q, \Delta$ and $s$. The actual values of $I, V$ and $t$ are related to the tabulated values by the following relations

$$
\begin{equation*}
P=\left(E / E_{0}\right) P \tag{2-144}
\end{equation*}
$$

so that if the propellant is not changed the actual pressure is the tabulated one.

$$
\begin{equation*}
V^{\circ}=r V_{t} \tag{2-145}
\end{equation*}
$$

where $r=r_{1}\left(E_{;}^{\prime} ; E_{0}\right)^{\prime}\left(C_{c a}^{\prime} / V^{\prime}\right)^{\prime}$ and $r_{1}$ is again a factor to be determined empirically.

$$
\begin{equation*}
t=(t / r) t \tag{2-146}
\end{equation*}
$$

where

$$
\begin{equation*}
\ell=\frac{U_{r A}}{A(1+e)} \tag{2-1+7}
\end{equation*}
$$

Now consider the effects of changing all linear dimensions of the gun-ammunition system by a factor, $f$. The composition of the propellant will not be changed so that $E$ is not changed but the web and other dimensions of the propellant will change by a factor, $f$. A will change by $f^{\prime} . U_{\text {ch }}, R$, If and $\left(\right.$, will all change by $f^{3}$.

It follows from Equations $2-1+1$ and $2-1+2$ that $q$ and $\Delta$ remain unchanged so that the tabulated values of pressure, volocity and time remain unchanged for the same value of the expansion, $s$. The actual value of the pressure is the tabulated value and since $r$ is also unchanged the velocity is the same for the same expansion. The displacement, $X$, will be changed according to E(y. 2-1+3, the second term on the right being divided by $f$. Since $\ell$ is changed by a factor, $f$, the time sexle will be multiplied by $f$ so that it will take $f$ times as long to reach the same expansion.

## 2-9 EFFECTS OF CHANGES IN THE PARAMETERS

To be considered chicfly are the effects on muzzle velocity, $I^{\prime}$. and maximum pressure, $P_{p}$, due to changes in $C^{\prime}, U_{c h}, X_{m}, I^{\circ}, w$ and $E$. A change in $C$ and/or $C_{\text {en }}$ may be expressed as a change in $\Delta$. Equations 2-26 and 2-27 show that the effects of a change in $B$ is equal to the same proportional change in $w$, hut in the opposite sense; that is, a 10 percent increase in $B$ produces the same effect as a 10 percent decrease in $w$.

The effects are usually expressed as differential coefficients, which are defined by the formulas (Sec: List of Symbols)

$$
\begin{align*}
\frac{d \Gamma_{m}}{\Gamma_{m}}=\alpha \frac{\partial C^{\prime}}{C}+\beta \frac{\partial U_{-h}}{C_{C \Lambda}} & +\gamma \frac{\partial X_{m}}{I_{m}}+\eta \frac{\partial U^{\circ}}{W} \\
& +\kappa \frac{\partial w}{w}+\lambda \frac{\partial E^{\prime}}{L^{\prime}} \tag{2-1+8}
\end{align*}
$$

$$
\begin{equation*}
\delta=\frac{\Delta \partial V_{m}}{V_{m} \partial \Delta} \tag{2-140}
\end{equation*}
$$



$$
\begin{equation*}
\delta_{1}=\frac{\Delta \partial P_{r}}{P_{p} \partial \Delta} \tag{2-1;0}
\end{equation*}
$$

These differential coefficients may be determined either experimentally or theoretically by changing one of the parameters at a time. In testing a new lot of propellant or a new projectile, it is customary

TABLE 2-1. DIFFERENTIAL COEFFICIENTS FOR ARTILLERY WEAPONS

| Propellant Composition | No. of <br> V:alues <br> Considered | Caliber of Weapons Cunsidered |  | Velocity Coefficients |  |  |  |  |  | Pressure Coefficients |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\alpha$ | $\beta$ | $\gamma$ | $\eta$ | $\times$ | $\lambda$ | . $\alpha_{1}$ | $\beta_{1}$ | $\eta_{1}$ | $\kappa_{1}$ | $\lambda_{1}$ |
| M1 Simgle-I3:se. | $3!1$ | $3 \mathrm{mmm-8-inch}$ | Mean | . 57 | -. 17 | .21 | $-.36$ | -. ${ }^{\text {b }}$ | . 8. | 1.42 | -.79 | . 62 | -1.2) | 2.50 |
|  |  |  | Max | . 66 | -. 26 | . 33 | -.ti | -.5!) | 1.19 | 1.83 | $-1.62$ | .88 | $-1.58$ | 2.84 |
|  |  |  | Min | . ${ }^{7}$ | -. 05 | . 13 | $-20$ | -. 06 | . 57 | . 88 | -. 30 | . 47 | -. .00 | 2.12 |
| Mt single-has. | 36 | $57 \mathrm{mm-240mm}$ | Mean | . 62 | -. 21 | . 21 | $-32$ | -. 30 | . 85 | 1.79 | - . 98 | . 67 | $-1.51$ | 2.76 |
|  |  |  | Max | . 2 | -. 27 |  | -. 42 | --. 69 | 1.30 | 2.02 | $-1.28$ | . 88 | -1.64 | 2.91 |
|  |  |  | Min | . 2.2 | -. 15 |  | $-.15$ | -. 16 | . 69 | 1.23 | $-.55$ | . 60 | $-1.29$ | 2.51 |
| M2 1omble-13:as | 11 | 37 mm -90mm | Mean | . 59 | -. 18 | . 21 | -. 3 s | -. 29 | . 84 | 1.61 | -. 91 | . 03 | -1.37 | 2.60 |
|  |  |  | Max | . 66 | $-.23$ | . 25 | -. 48 | -. 55 | 1.14 | 1.88 | -1.22 | .67 | -1.48 | 2.73 |
|  |  |  | Min | . 33 | $-.07$ | . 18 | $-.20$ | $-.10$ | . 6.2 | 1.25 | $-.63$ | . 60 | $-1.22$ | 2.42 |
| M1\% Triphe-I3:4e | 9 | $76 \mathrm{~mm}-120 \mathrm{~mm}$ | Mean | . 67 | -. 23 | . 23 | -. 2.4 | -. 44 | 1.00 | 2.04 | -1.19 |  | -1.48 | 2.73 |
|  |  |  | Max | .73 | -. 28 | . 27 | -. 29 | -. 60 | 1.10 | 2.33 | $-1.40$ | . 68 | -1.51 | 2.76 |
|  |  |  | Min | . 64 | $-.20$ | . 19 | $-.16$ | $-.36$ | . 02 | 1.64 | - . 00 | . 39 | $-1.35$ | 2.58 |

to use a series of charges of increasing weight, plot the observed values of muzzle velocity and maximum pressure, and draw smooth curves to fit the points. Although this is done primarily to determine the charge that will give the recuired muzzle velocity and to see whether the maximum pressure exceeds that for which the gun was designed, the curves also indicate the effects of changes in weight of charge. The effects of a change in projectile weight can be determined by comparative frings of projectiles of the same model but different weight.

Taylor's charts have not yet been used to calculate the differential coefficients. The coefficients for single perforated grains have been computed by means of Röggla's charts," averaging the effects of a 10 percent increase and a 10 percent decrease in each parameter. The coefficients for multiperforated grains have beell computed by means of Bennett's tables, averaging the effects of one tabular interval increase and decrease in $q, \Delta$ and $s$ to find $-\kappa, \delta$, $\gamma_{1}-\kappa_{1}$ and $\delta_{1}$, then calculating the other coefficients by the formulas ${ }^{10}$,

$$
\begin{equation*}
\alpha=D_{1}(-\kappa)+\delta-\left(\% / 6 \|^{\prime}\right. \tag{2-1;2}
\end{equation*}
$$

wher

$$
\begin{align*}
D_{1} & =\frac{C}{6 I^{\prime}}-\frac{C}{2 R^{\prime}} \\
W^{\prime \prime} & =W^{\prime}+C / 3 \\
R^{\prime} & =R+W^{\prime}+C / 2 \\
\beta & =0 . \pi(-\kappa)-\delta-\gamma+0 . \pi  \tag{2-153}\\
& =D_{2}(-\kappa)-\Pi^{\prime} / 2 W^{\prime \prime} \tag{2-1:4}
\end{align*}
$$

where

$$
\begin{align*}
D_{2} & =\frac{W}{2 W^{\prime}}-\frac{W}{R^{\prime}} \\
\gamma & =\frac{7}{6}(-\kappa)+0.5  \tag{2-1;5}\\
\alpha_{1} & =D_{1}\left(-\kappa_{1}\right)+\delta_{1}  \tag{2-156}\\
\beta_{1} & =0.5\left(-\kappa_{1}\right)-\delta_{1}  \tag{2-157}\\
\eta_{1} & =D_{2}\left(-\kappa_{1}\right)  \tag{2-158}\\
\lambda_{1} & =\frac{7}{8}\left(-\kappa_{1}\right)+1 \tag{2-159}
\end{align*}
$$

Table 2-1 gives the average values of the differential coefficients for artillery weapons. Table 2-2 gives the values of $\eta, \beta, \eta_{1}$ and $\beta_{1}$ for standard weapons.

Following are some estimated values for recoilless riffes:

$$
\begin{aligned}
\alpha & =1.0 \\
\eta & =-0.6 ; \\
\alpha_{1} & =2.4 \\
\eta_{1} & =0.62
\end{aligned}
$$

## 2-10 SIMPLE GRAPHICAL METHODS

Numerous schemes have been devised by interior ballisticians for making rapid approximate calculations of certain interior ballistic variables especially of maximum pressure and muzzle velocity. These schemes are formulated in terms of a set of parameters chosen so that their form and interrelations can be determined by the use of some simplified


CHART 2-4. Chart for Interior Ballistic C'alculations by the Scheme of Strilimater
to the muzzle and $\bar{P}_{1}$ is defined as the space mean pressure at the time of maximum pressurn. The latter may also be called the space mean peak pressure.

With Equation $9-160$ substituted in Eis. ${ }^{2}-161$ and $2-162$ using the numerical values 1.30 and $: 386$ for $\gamma$ and $g$, the ballistic and piezometric efficiencies may be expressed

$$
\begin{equation*}
e=3.89 \times 10^{-1} \frac{\left(11+(/ / 3) I_{m}^{-2}+.5 \times 10^{5} d X_{m}^{-}\right.}{F C} \tag{2-161a}
\end{equation*}
$$

$$
==\frac{\left(1 I^{-}+(3) I_{m}^{2}+\pi \times 10^{5} d X_{m}\right.}{\pi I \bar{P}_{p} C_{m}}
$$

Solving the former for $\mathrm{I}_{\mathrm{m}}^{2}$ gives
li the maximum breech pressure, $P_{r p}$, is given, the space mean poak pressure, $\bar{P}_{b}$, is calculated by the formula

$$
\bar{P}_{r}=\frac{\|+(\% / 3}{\|+} r_{c u}
$$

The initial free volume is defined by lequation - -17a as

$$
r_{i n}=l_{c h}-c^{\prime} u
$$

The free volume at the muzzle is defined by Equation $2-102$ as

$$
l_{n}=l_{n}+A X_{m}
$$

Theoretically, if any two of the reduced paramctens are known, the other three can be evalnated by means of the chart.
The use of the chart will be illustrated by an example. The data were taken from the firing record for a caliber .30 gum , firing a $1: 50 . \mathrm{i}$-grain bullet, propelled by :0 grains of a certain lot of propellant. The characteristics of the gun and charge are:

| Charateristio | symbel | Value | Unit | Remark: |
| :---: | :---: | :---: | :---: | :---: |
| Propellant weight | $1 \cdot$ | 0.00714 | II, | Civen |
| Sperific furco | $r$ | +.023 $\times 10^{6}$ | in-lb/ll | Given |
| Specifie volume | 11 | 17.5 | in ${ }^{3} / \mathrm{ll}$, | Given |
| Chamixer volunu. | $1 \%$ | 0.258 | in ${ }^{3}$ | Given |
| Area of bore | . 1 | 0.07:35 | in: | (iiven |
| Bore diameter | 1 | 0.30 | in | Given |
| Travel to muzale | .$^{\prime \prime}$ | 21.80 | in | Given |
| Projertile weight | II' | 0.021:\% | II) | Civen |

To calculate at least two of the parameters one also needs to know cither the muzzle volocity or the space mean peak pressure. One or both of these will normally be specified in any gun design problem. In the present exa:mple the maxinum breceh pressure will be assabield to be given sud equa! to the measured value $3.5,890 \mathrm{lb}$. $\mathrm{in}^{\prime}$, so that the space mean peak pressure can be calculated by Equation ?-167.

Now suppose the following are calculated:

| Charameristio | Sym | Value | Vnit | Remarks |
| :---: | :---: | :---: | :---: | :---: |
| Initial free vokame | 10 | 0.133 | : $1^{3}$ | F.!. 2-17a |
| Muzzle free volume | $\Gamma_{n \prime}$ | 1.735 | in ${ }^{3}$ | E¢. 2-102 |
| sipare mean peak presure | $\vec{P}_{p}$ | 3-1, 180 | $11 / \mathrm{in}^{2}$ | 1:4. $2-167$ |
| Volume expansion ratio | $\boldsymbol{r}$ | 13.05 |  | Eq. $2-163$ |
| Bathistic parameter | $r$ | (6.32 |  | 1.4. 2-16-4 |

Then from Chart $9-4 e, z$ and $y$ can be read as:
Hallistic efficiency, $c=0.352$
Piezometric efficiency, $z=0 . i 66$
Pressure ratio, $y=0.316$
from which mazzle pressure is calculated as 10,800 Ib in', and the muzzle velocity as 2,572 feet per second. The observed value of the muzzle velocity was $2, \% 6, \%$ feet per second for the firing used.

If the theory represented by the chart were exact, the lines representing the five different parameters for any gun-ammunition system would all intersect at a point. When experimental values for the cuantities defining the parameters are substituted in the corresponding efuations the lines so determined do not cross at a single point but form a polygon. If the experimental values are not subject to serious error, the dimensions of this polygon are a measure of the discrepancies involved in using the chart. The triangles shown on the chart are the result of using experimental values (inserted into Equations 2-1(61, $2-163$ and $2-164$ ), to determine $e, x$ and $r$ for the weapons indicated. For the eximple, the predicted value of $\rho$, using the values for $x$ and $r$ in the preceding table, is 0.352 which the triangle indicates is too high by about 0.006 ; the amount corresponding to the height of the triangle.

A set of nomograms constructed by similar methods, for the rapid determination of muzzle velocities for artillery weapons has been constructed by Kravitz. These are published in usable form with instructions for their use in Reference 12.

## 2-11 EMPIRICAL METHODS

For many practical problems of gin design, theory is used as a guide to the selection of a set of dimer:sionless patameters in terms of which the scheme to be used is formudated and to define the chosen parameters in terms of the interior ballistic variables themselves. If then sufficiont firing data are a a ailable from a group of weapons, not too dissimilar from cach wher, fitted graphs or charts can be prepared comecting pertinent variables with the chosen ballistic parameters which will vary for different members of the reference group. The most commonly used parameters are propellant weight per unit projectile weight, expansion ratio and density of loading. From such a set of graphs the parameters and ballistic variables of a new but similar weapon system can be determined by simple interpolation. Such a set of graphs, prepared at Frankford Arsenal for we in suall arms design, are presented on ('harts $2-5,9-6,9-7$ and $2-8$. Thess normalized graphs were obtained by reducing experimental data from firing eleven different small arms weapon systems. Least squares curves were drawn to give the best fit to the data. They are used to relate maximum pressure, propellant weight, projectile weight, expansion ratio, muzzle velocity and chamber pressure at a given projectile travel.

Example: (iven the Cartridge, Caliber .30, Ball MO data as follows:

| Projectile weight | -1.0 grains |
| :--- | :--- |
| Propellant weight | -49.9 grains |
| Bore arca | -0.0732 in |
| Case volume | -0.2 .3 in |
| Bullet travel | -21.9 in |
| Maximum pressure -31.2 kpsi |  |

Find the muzzle velocity.
Calculation:

$$
\begin{aligned}
\frac{C}{I^{\prime}} & =\frac{49.9}{150}=0.333 \\
\frac{V_{m}}{V_{11}} & =\frac{0.0732 \times 21.9+0.2 .}{0.2 ;}=7.41 \\
V^{-} & =V_{3.6 i 1} \times \frac{V^{\prime}}{I_{3}} \times \frac{V_{m}}{V_{s i 0}} \\
& =2610 \times 1.106 \times 0.981=28.32 \mathrm{ft} / \mathrm{scc}
\end{aligned}
$$

where: 2610 is read from Chart $2-5,1.106$ is read from Chart $2-6$ and 0.981 is read from Chart 2-7.
For comparison, the muzzle velocity was actually recorded as $2832 \mathrm{ft} / \mathrm{sec}$.

It should be noted that these graphs work well
for a marly optimum selection of propellant and primer. In order to sellect the best propellant for a given system, use is made of the relation:

$$
\operatorname{web} \alpha \frac{\| V}{A}
$$

where If and . 1 have the usual meanings and $V^{\circ}$ b is the projectile velocity at all burnt. This gencrally oceurs in small arms systems at an cexpansion ratio of about 33.5. For purposes of estimation, the velocity at burnout may be replaced by muzzle velocity, and a new system compared to an existing woll performing onc.

## 2-12 THE ATTAINMENT OF HIGHER VELOCITIES

## 2-12.1 General

In many tactical situations great advantage is derived from the use of gums with higher muzzle velocities. The use of such guns moans shorter time of flight to the target and a flatter trajectory thus improving the probability of hitting the target, especially a moving one. Against an armored target such as a tank, high velocity is necessary to penetrate the armor with a projectile depending on striking energy for penetration. The using services therefore demand higher velocity guns.

## 2-12.2 The Optimum Gun

a. If the geometry of the glun, the mass and geometry of the projectile, the propellant to be used and the maximum pressure are specified, the density of loading is varied there is a density of loading for which the muzzle velocity is a maximum. To avoid exceeding the maximum pressure one must vary the web, increasing the web) as the density of loading is increased. Increasing the web will cause the travel of the projectile at burnout to shift toward the muzzle and perhaps inercase the variation of the muzzle velocity. The thermodynamic efficiency will also decrease because the muzzle pressure and temperature will be higher, as the gas formed late in the cycle does less work on the projectile. As stated previously, the piezonetric efficiency will increase.
$b$. If the total volume of the gum, the density of loading and the maximum pressure are fixed, and the expaision ratio is varied by changing the chember volume, the muzzle velocity will attain a maximum value for a certain chamber volume, that is, for a certain charge.
c. When these two conditions are simultanconsly




satisfied, for a gim of specified total volume and maximum pressure, the gum should give the maximum muzzle velocity which can be imparted to the projectile with the propellant used. That is, the gum is operating at its maximum efficiency.
To satisfy these conditions may require a gun having unsatisfactory muzzle velocity regularity or other characteristies that would prohibit its use in pactice. lew standard gun systems are optimum in the theoretical sense but employ compromises among the many factors involved.

## 2-12.3 The Conventional Procedure to Attain Higher Velocities

In order to increase muzzle velocity the available anergy must be increased. This involves using a larger charge of hotter propellant with a lower value of $\gamma$. The larger charge at optimum density of loading requires a larger chamber. If the expansion ratio is to be near optimum a larger tuie volume is repuired for the gron. In general, the length of the gum must be limited for practical reasons, so that the inerased volume must often be attained by increasing the tube diameter. The muzzle velocity for a given projectile traved can be incerased further by inereasing the permitted operating pressure of the gun by using a stronger tube. Then a smaller web can be used for the optimm density of loading. This will permit the travel at burnt to be smaller and increase the effieiency. To attain higher veloeities, therefore, by conventional means, one must use a larger gum operating at higher pressure.
The optimem solution for a specified operating pressure may be such that the caliber of the gun respuired to give the muzzle velocity wanted is larger than the diameter of the projectile. This means that the projectile must be provided with some type of sleeve, called a sabot, to fit properly in the bore. The saloot forms an integral part of the projectikwhile in the bore but is designed to be discarded immediately after exit. From an interior ballistic point of view the sabot permits the use of a larger bore which, for a given pressure, increases the forec on the projectile and hence the aceleration, so that a higher velocity is reached for the same travel. The extra mass of the sabot is useless and must be kept to a minimum. Design of the sabot is, therefore, a very important feature.

The increase in the operating pressure of the gron i.: limited by the yield strength of the stecl available and also by weight considerations which limit the wall thicknesses wiich can be used. The strength of the bared cannot be increased indefinitely by
increasing the wall thickness. There is a limiting ratio of inside to outside diameter of the tube beyond which no increase in the tube strength results. Tubes can be made stronger however, by special fabrication methods. ${ }^{13}$ The operating pressure will also be limited by the design of the projectile and sabot which may not be able to withstand the resulting base pressures and set back forees due to the larger accelerations. The development of modern high strength steels have made it possible to design gun tubes to withstand higher pressures and the operating pressures of high velocity guns are being constantly increased with a consequent increase in muzzle velocity.

The use of hot propellants is limited by the rapid increase in erosion of the tube that accompanies increase in the temperature of the propellant gas. Since high pressure and velocity also increase the erosion high velocity guns usually have relatively short lives. Special methods for reducing crosion have been de. .loped, however, so that the erosion problem is not as serious as it once was.

## 2-12.4 Unconventional High Velocity Guns

Several unconventional schemes have been proposed to attain higher muzzle velocities from guns. None of these, however, shows much promise of resulting in a practical field weapon and only one has been much developed as a laboratory device. Two such schemes that have been seriously considered are incorporated in the so called traveling charge gun and the light gas gum. They are both designed to circumvent the limitation imposed on the attainment of ligh velocity by the perssure gradient which must exist in the gas during expansion and to reduce the amount of kinctic energy in the gas. The more rapidly this expansion must take place the greater will be the drop in pressure between the breech and the projectile base. This limits the acceleration that can be produced and hence the velocity attained in a given travel.
a. The Traveling Charge Gun. The traveling charge gun attempts to surmount this difficulty by having the charge attached to and move with the projectile and burn at the rear surface at such a rate as to form gas just fast enough to maintain a constant pressure in the gas column. The gas then is at rest up to exit of the projectile and a larger fraction of the available energy is transferred to the projectile. Although this ideal situation camot be realized in practice, some increase in gun efficiency could possibly be attained by partial application of the principle. The main difficulties encountered
in attempts to apply the principle are designing propellants with sulficiently high burning rates and arranging the charge so that the burning rate is sufficiontly controllable over the burning period. Ewen if the difficultias could be overcome, poor muzzh volocity uniformity probably would result. Laboratory studies of the travelling charge principle arr deseribed in Referener 14 and work on the dovelopment of rapidly burning charges in Refwernes: $1 . \operatorname{sand} 16$.

1. Ther Lieghe Giens (iun. The light gas gen is so designed that the expanding gas which does work on the projectile has a low molecular weight and a high value of $\gamma$. This assures that for a given projectile veloeity the fraction of the cmergy appearing as kinetic energy of the gas is reduced. The presture gradients: in the gas will also be reduced bereanse these depend on the velocity at which rarefaction waves can propagate in the gas; that is, on the weloeity of sound in the gas. If the projectile had zerom mass, the gas would expand frecty. Thero is a limit to the free rate of expansion determined be what is called the cesape spered of the gas given be the relation

$$
\begin{equation*}
\alpha_{n}=\frac{\underline{2} a_{n}}{\gamma-1} \tag{2-11;8}
\end{equation*}
$$

where an, is the ersape spered and $a_{1}$, is the verocity of somul. For a pertere gas $a_{n}$ is given be

$$
\begin{equation*}
a_{11}=(\gamma, / R T)^{1 / 2} \tag{2-169}
\end{equation*}
$$

Sianera large csuape spered means a reduced pressure gradient for a given expansion rate, the working gate of a gen should have a low molecular weight and a high value of $\gamma$. The working gas most often nsed in light gas guns is helimen for which $1 /=1$ allid $\gamma=\overline{3}$.
l.ight gas gillis hatre undergome mud development as tabouatore derieres for lamerhing small projectiles of satums shapes to be used in experiments on high whority impart and for lameling models for the stuly of the arrolymamies of projectiles at very high verocities. These developments are continuing and a mumber of selames have beromporosed and at edied. The most sureressul have bere the combustion hated light gats gun and the piston compressor type in which the light gas is heated adiabatically by compression with a propollant driven piston".

## 2-12.5 Extension of Interior Ballistic Theory to High Velocity Weapons

The formations of the lheory of interion hallisties so far presemted are limited in their usefulness of
applied to very high velocity guns mainly becaus: they lack an adequate incorporation into the theory of the hydrodynamies of the propellant gas. The use of unrealistic expressions for the pressure ratios is one of the chicf factors which limit their usefulness when extended to guns with muzzle velocities above about 3000 feet per second (Cf. par. i -1.1). The development of light gas guns partieularly has demanded an extension of interior ballistic theory to permit its application to high velocity weapons ${ }^{\prime \prime}$. A general discussion of the pressure ratios and their dependence on projectile velocity and chamber grometry is given in Reference 10 of Chapter: :

## 2-13 THE HIGH-LOW PRESSURE GUN

Ender some cireumstances it may be desirable to fire a projectile from a standard gun at much lower than the standard velocity. This means that the gun must operate at a much reduced pressure. The pressure may be so low that it becomes difficult to ignite the charge effectively and the charge may not burn with suflicient uniformity from round to round. To circumvent this difficulty and maintain a high pressure in the chamber and a lower pressure in the bore, a plate piered by one or more nozzles or holes can be interposed between the chamber and the bore, that is, at the mouth of the cartridge case. The arca of the nozzles is adjusted to maintain adequate perssure in the chamber to assure stable ignition and burning and also to provide sufficient mass flow of gas through the nozzles to adjust the pressure in the bore to the required limits. Such a gum is called a high-low pressure gum. A theory of the high-low pressure gums is given in Reference 3. The theory of the burning in the high pressure chamber of such a gin is given by Vintiand Kravitz."

An application of the high-low pressure principle appears in the design of mortar eartridges where part of the charge is ignited and burned inside a perforated chamber. The principle could also have application in special low pressure cartridge actuated thrusters and ejectors such as are used to ejoet apparatus from aireraft and rockets"

## 2-14 RECOILLESS RIFLES ${ }^{13}$

## 2-14.1 Theory of Efflux of Gas Through Nozzles.

f. Imtrenturtion. In order to diminate the nered for heave recoil mechanisms, some gous are built with a mozale in the breceh, so that part of the propollant gas can flow hackward and comenter bataner the momentam of the projectile and the
part of the propellant that moves forward. Such gums are called reooilless guns or recoilless rifles.

The theory of efllux of gas through nozzles will first be discussed in simple form and then applied to the interior ballisties of recoilless rifles. This mesentation is csencially that developed by Comer:

It is assmod that the gas originates in a large reservoir; that the cross-scetional area of the nozzle decreases to a minimum, called the throat; and then the area increases to the exit, where the gas flows into the atmosphere. It is also assumed that the condition of the gas is a function of the coordinate.$x$, measured along the axis of the nezzle, and is uniform acmes ach nomial coos section; that loss of heat to the walls, turbulence, and surface resistance can be neglected; and that the fluid does not separate from the walls. The assumption that the reservoir is large means that the conditions in the reservoir ho not change appreciably in the time required for in element of the gas to pass through the nozzle; Whe flow is then said to be quasi-steady; that is, t is assumed that the equations for steady flow apply at cach instant of time and may be applied to the non-steady flow in the recoilless rifle. It is also assumed that the nozzle is so designed and that the flow conditions are such that the gas undergoes it continuous expansion through the nozzle so that the velocity of the gas increases steadily and the pressure falls steadily between the reservoir and the csit of the nozzle This will occur if the reservoir pressure is always considerably higher than the pressume at the exit and if the flow entering the bozzle is subsonic and becomes sonic at the throat of the nozzle. Except at the begiming and possibly the end of the firing cycle of a recoilless gun the pressures and temperatures of the gas are such as to satisfy these conditions.
2. Theory. As stated the following equations are strictly true only for a steady state, but they are approximately tiue for slowly varying flow. The expansion is adiabatic. The effects of the covolume of the gas are neglected, they amount to only a few percent theoretically, and can be compensated for by using empirical coefficients. The discussion, therefore, is for perfect gases and for adiabatic flow. The fundamental units are as before the inch, pound (ireight), second.

The rate of gas flow, q (weight per second), at any section of area, $A$, is a constant and is given by

$$
\begin{equation*}
q=A \rho v==A_{1} \rho_{1} v_{1} \tag{2-170}
\end{equation*}
$$

where $\rho$ is the weight of the gas per unit volume and $r$ its velocity. Sinee the expansion is adiabatic,

$$
\begin{equation*}
I^{\nu} \rho^{-\gamma}=I_{1} \rho_{l}^{-\gamma}=I_{r}^{\gamma} \rho_{r}^{-\gamma} \tag{2-171}
\end{equation*}
$$

The subscripts $r$ and $t$ refer to conditions in the rescrvoir and at the throat, respectively. The equation of state of a perfect gas is

$$
\begin{equation*}
P / \rho=R T \tag{2-172}
\end{equation*}
$$

where $R$ is in inch-pounds per pound weight per degree. The equation of conergy may be expressed as

$$
\begin{equation*}
\frac{\gamma R\left(T_{r}-T\right)}{\gamma-1}=\frac{v^{2}}{2 g} \tag{2-173}
\end{equation*}
$$

Since

$$
\begin{equation*}
\left(\frac{d \cdot 1}{d x}\right)_{1}=0 \tag{2-174}
\end{equation*}
$$

by logarithmic differentiation of Equation 2-170

$$
\begin{equation*}
\left(\frac{1}{\rho} \frac{d \rho}{d x}\right)_{t}+\left(\frac{d v^{\prime}}{v^{\prime} d x}\right)_{t}=0 \tag{2-175}
\end{equation*}
$$

From Equations 2-171, 2-172 and 2-175 it can be shown that

$$
\begin{gather*}
\frac{T_{1}}{T_{r}}=\frac{2}{\gamma+1}  \tag{2-176}\\
\frac{P_{1}}{P_{r}}=\left[\frac{2}{\gamma+1}\right]^{\gamma /(\gamma-1)}  \tag{2-177}\\
\frac{\rho_{1}}{\rho_{r}}=\left[\frac{2}{\gamma+1}\right]^{1 /(\gamma-1)} \tag{2-178}
\end{gather*}
$$

and from Equations 2-173 and 2-176

$$
\begin{equation*}
u_{i}^{2}=\frac{2 \gamma g R T_{r}}{\gamma+1} \tag{2-179}
\end{equation*}
$$

Lect

$$
\begin{equation*}
\psi=\gamma^{1 / 2}\left[\frac{2}{\gamma+1}\right]^{(\gamma+1) /(2(\gamma-1))} \tag{2-180}
\end{equation*}
$$

Then the rate of How may be expressed

$$
\begin{equation*}
q=\psi \rho_{r} A_{t}\left(g R T_{r}\right)^{1 / 2} \tag{2-181}
\end{equation*}
$$

or

$$
\begin{equation*}
q=\psi P_{r} A_{1}\left(R T_{r}\right)^{-1 / 2} g^{1 / 2} \tag{2-182}
\end{equation*}
$$

For most propellants, $\gamma$ is approximately 1.25 , and, according to Equation 2-180, $\psi$ is close to 0.66 . Actually, the covolume correction and other variations from standard conditions bring the empirical value of $\psi$ a few percent below the theoretical value.

For $\gamma=1.25$, Equations $2-178$ and $2-179$ show that the specific weight and relocity at the throat are

$$
\rho_{t}=0.62 \rho_{r}, \quad v_{t}=1.05\left(g R T_{r}\right)^{1 / 2}
$$

The pressure at any expansion ratio, $A, \prime^{\prime} A$, may be found from the relation

$$
\begin{align*}
{\left[\frac{P}{P_{r}}\right]^{2} } & -\left[\frac{P}{P_{r}}\right]^{r+\cdots r} \\
& =\frac{\gamma-1}{\underline{2}}\left[\frac{2}{\gamma+1}\right]^{n+1}\left[\frac{A-1}{A}\right] \tag{2-18:3}
\end{align*}
$$

For $\gamma=1.2$. , this may be expressed

$$
\begin{equation*}
\left(P_{1}^{\prime} P_{r}\right)^{1,4}-\left(P_{r}^{\prime} P_{r}\right)^{1, N}=0.043: 30\left(A_{1}^{\prime} A\right)^{2} \tag{2-18:3a}
\end{equation*}
$$

The pressure ratios that satisfy Equation $2-183 \mathrm{Ba}$ may be read from Chart $2-9$. If desired, the temperature may be found from the relation

$$
\begin{align*}
& {\left[\frac{T}{T_{r}}\right]^{2(\gamma-n)}-\left[\frac{T}{T_{r}}\right]^{(\gamma+1) /(-1)}} \\
& \quad=\frac{\gamma-1}{2}\left[\frac{2}{\gamma+1}\right]^{(\gamma+n /()-1)}\left[\frac{\Lambda_{1}}{A}\right]^{2} \tag{2-184}
\end{align*}
$$

The corresponding gas velocity is given by

$$
u^{2}=\frac{\underline{2} \gamma}{\gamma-1} g R T,\left[1-\left(P^{\prime} \cdot P_{r}\right)^{(\gamma-1) / \gamma}\right]
$$

The ratio
$\left[\frac{L_{1}}{v_{t}}\right]^{*}=\frac{\gamma+1}{\gamma-1}\left[1-\frac{2}{\gamma+1}\left(\frac{A_{1} v_{2}}{A_{0}}\right)^{\gamma-1}\right]$
can be solved numerically by successive approximations.
3. Thrust. The thrust on the nozzle is the sum of the rate of change of momentum and the force exerted by the excess pressure at the exit:

$$
\begin{equation*}
F_{T}=\frac{q}{g} v_{r}+A_{d}\left(P_{r}-P_{a}\right) \tag{2-187}
\end{equation*}
$$

In applications to recoilless rifles, the atmospheric pressure, $P_{a}$, is negligible compared to the exit pressure, $P_{\text {. }}$.

The thrust coefficient, ( ${ }_{r} r$, is defined as the ratio

$$
\begin{equation*}
C_{\tau}=\frac{F_{\tau}}{A_{i} P_{r}} \tag{2-188}
\end{equation*}
$$

If there were no expansion, the exit would be at the throat, and the thrust would be

$$
\begin{equation*}
\left(F_{\tau}\right)_{t}=\frac{q}{g} v_{t}+A_{t} P_{t} \tag{2-187a}
\end{equation*}
$$

By substituting Equations 2-177, 2-179 and 2-182 in Eq. 2-187a, it is found that the thrust coefficient for this case is

$$
\begin{equation*}
\left(C_{r}\right)_{t}=(\gamma+1)\left[\frac{2}{\gamma+1}\right]^{\gamma /(\gamma-1)} \tag{2-189}
\end{equation*}
$$

For

$$
\begin{equation*}
\gamma=1.2 \bar{J}, \quad\left(C_{r}^{\prime}\right)_{1}=1.248 \tag{2-189n}
\end{equation*}
$$

This formula can be used only if the throat area is small compared to the cross section of the resservoir; for, if the system were a pipe of miform section, closed at one cud, the momentum term in Equation 2-187a would vanish, and $\left(C_{r}\right)$, would be cipal to 1 .

In the usual cise of an expanding nozzle, the formula for the thrust coefficient becomes
$C_{r}=\left[\frac{\underline{g}}{\gamma+1}\right]^{\gamma /(\gamma-1)}\left[\gamma\left(\frac{v_{c}}{r_{1}}\right)+\left(\frac{A_{t}}{A_{r}}\right)^{\gamma-1}\left(\frac{v_{1}}{r_{r}}\right)^{\gamma}\right]$
(2-190)
which can be solved with the help of Equation 2-186. Table 2-3 gives the values of $C_{r}$ as a function of $\gamma$ and $A, A_{1}$. Linear interpolation can be used in this table. The effect of covolume on thrust is small and in the forgoing discussion it has been entirely neglected.

TABLE 2-3. THRUST COEFFICIENT, $C_{T}{ }^{*}$

| $=1 . / A$, | $\gamma=1.20$ | $\gamma=1.30$ |
| :---: | :---: | :---: |
| 1.0 | 1.242 | 1.255 |
| 1.2 | 1.318 | 1.327 |
| 1.4 | 1.369 | 1.374 |
| 1.6 | 1.408 | 1.409 |
| 1.8 | 1.439 | 1.438 |
| 2.0 | 1.466 | 1.461 |
| 2.5 | 1.516 | 1.505 |
| 3.0 | 1.554 | 1.537 |
| 3.5 | 1.607 | 1.562 |
| 4.0 | 1.644 | 1.582 |
| 5 | 1.673 | 1.612 |
| 6 | 1.713 | 1.635 |
| 8 | 1.742 | 1.667 |
| 10 |  | 1.689 |

[^4]
## 2-14.2 Application to Recoilless Rifles

1. Assumptions. It is assumed that no unburned propellant is lost through the nozzle, and that the flow out of the gun can be represented by the equations for quasi-steady flow through nozzles, which have been derived in paragraph 2-14.1, begiming instantancously with a nozzle-start pressure. The nozzle or cartridge case is originally sealed with a rupture closure which ruptures at the nozzle start pressure. It is also assumed that the burning law is

$$
\begin{equation*}
\frac{d c}{d l}=\rho S B P^{\prime} \tag{2-191}
\end{equation*}
$$



- but the sufface of the grains is not necessarilyconstant.

2. L:quatime a 1 Ithem. The erpation of state for unit wright of the meoolded products of explosion is expressed as

$$
\begin{equation*}
I^{\prime}(i,-\eta)=R T_{n} \tag{2}
\end{equation*}
$$

The sperific forer is

$$
\begin{equation*}
r=R T_{n}, \tag{2-19:3}
\end{equation*}
$$

The relations connecting the space mean pressure, $I^{\prime}$, the brecech pressure, $I^{\prime}$, and the projectile pressure. $P_{\text {, , discussed in paragraph 2-2.2, are affected }}$ by the flow through the nozzle, because this changes the veloeity distribution. It any instant, there is a maximum pressure at some place in the gum; this is identified with the "reservoir pressure", I'.. The propellant weight, (', must here be replaced by l: $\alpha$, , where $N$ is the proportion of the charge that has butaed bet remans in the gum, and $k$ is an empirical factor (taken as a constant, although it actually varies during the motion of the projectile). When there is no backward flow, $k=1$; but with backward flow, $t$ is appreciably less than 1. The pressure relations now become (Cf. l:quation 2-20)

$$
\begin{align*}
& P=\frac{(1+\theta) W}{(1+\theta) W+k C V / 6} P  \tag{-2-194}\\
& P=\frac{(1+\theta) W}{(1+\theta) W+k(N / 2} P \tag{2-19.3}
\end{align*}
$$

On the basis of Equation 2-19.), the modified effective mass of the projectile is (Cf. Eequation 2-11)

$$
\begin{equation*}
M_{1}=\frac{(1+\theta) U+k \cdot N / \underline{0}}{g} \tag{Ni}
\end{equation*}
$$

The specific volume of the gas, $u_{\text {, }}$, is assumed to be uniform along the gum. Then the volume occupied by the gas in the gun is

$$
\begin{equation*}
C D u_{2}=U_{c \Lambda}+A X-C^{\prime} u+c u \tag{2}
\end{equation*}
$$

From Equation 2-194, the equation of state at the reservoir temperature, $T_{r}$, may be written

$$
\begin{equation*}
P_{r}\left(u_{u}-\eta\right)=R T\left[1+\frac{k C N}{6(1+\theta) W}\right] \tag{2-198}
\end{equation*}
$$

Multiplying by $C N$ and substituting $E(1.2-197$ makes this

$$
\begin{align*}
& P_{r}\left(C_{\cdot L}+A N-C u+c u-\left(N_{\eta}\right)\right. \\
& =C N R T,\left[1+\frac{k C N}{6(1+\theta) \|}\right] \tag{2-109}
\end{align*}
$$

During burning, this equation may be approximated by

$$
\begin{equation*}
P_{r}\left(C_{, A}+A N-(u)=\left(N R T_{r}\left[1+\frac{K C N}{\theta(1+\theta) W^{\prime}}\right]\right.\right. \tag{2-109a}
\end{equation*}
$$

Ifter burning, this approximation is inadecquate, and Byuation 2-1!9 must be used.
The erfuations of motion of the projectile are now

$$
\begin{equation*}
A M_{1}=M_{1} \frac{d l^{v}}{d t}, \quad V^{V}=\frac{d \Gamma}{d t} \tag{2-200}
\end{equation*}
$$

with $M_{1}$ defined by J:quation 2-196;
3. Nozzle Fome and Eincryy. In order to solve these equations, $1 /{ }_{1}$. which involves ('N (weight of gas remaining in the gum), must be evaluated. If ( $\phi$ denotes the weight of propellant burned, the rate of flow through the nozzle is

$$
\begin{equation*}
q=\left(\cdot \frac{d \phi}{d!}-\left(\frac{d N}{d \prime}\right.\right. \tag{2-201}
\end{equation*}
$$

Substituting E\&. 2-18: in Lía. 2-201

$$
\begin{equation*}
C \frac{d N}{d l}=C^{\prime} \frac{d \phi}{d l}-\psi g^{1 / 2} P_{r} I_{t}\left(R T T_{r}\right)^{-1 / 2} \tag{2-202}
\end{equation*}
$$

So the reservoir temperature, I', must be determined. To do this, divide the temperature differential into three parts:

$$
\begin{equation*}
d T_{r}=d T_{1}+d T_{2}+d T_{3} \tag{2-20.3}
\end{equation*}
$$

liist, in providing the kinetic energy, $1 / I^{\prime}, N$, the gas loses temperature by the amount

$$
\begin{equation*}
d T_{1}=-\frac{g \cdot t P_{r}}{C \cdot V c_{r}} d N \tag{2-204}
\end{equation*}
$$

where $c$, is the specific heat at constant volume, that is, energy per unit mass per degree. But

$$
\begin{equation*}
c_{r}=\frac{g R}{\gamma-1} \tag{-20.0}
\end{equation*}
$$

s)

$$
\begin{equation*}
d T_{1}=-\frac{(\gamma-1) A P_{r}}{(N R} d N \tag{2-206}
\end{equation*}
$$

Second, if $E$ is the specific internal energy, the gas accuires energy at the rate

$$
\begin{equation*}
C E\left(T_{11}\right) d \phi=C \cdot E\left(T_{r}\right) d \phi+\frac{C N c_{r}}{g} d Y_{2} \tag{2-207}
\end{equation*}
$$

If $c$, is constant from $T$, to $T_{0,}$

$$
\begin{equation*}
d T_{2}=\frac{T_{n}-T_{r}}{N} d \phi \tag{2-208}
\end{equation*}
$$

Third, since the gas that escapes through the nozzle at the rate, $q$, expands adiabatically

$$
\begin{equation*}
\frac{d T_{3}}{T_{r}}=\frac{(\gamma-1) d \rho}{\rho} \tag{2-209}
\end{equation*}
$$

anglecting the eovolume. Also

$$
\begin{equation*}
\frac{d \rho}{\rho}=\frac{d N-d \phi}{N} \tag{2-210}
\end{equation*}
$$

Hener

$$
\begin{equation*}
d T_{s}=(\gamma-1) \frac{d N-d \phi}{N} T_{r} \tag{2-211}
\end{equation*}
$$

Tdding l:quations 2-206, 2-208 and 2-211, multiplying by $N$, and substituting in Eq. 2-203 gives

$$
\begin{array}{r}
\therefore d T_{r}=-(\gamma-1) \frac{A P_{r}}{C R} d X+\left(T_{\prime}-T_{r}\right) d \phi \\
+(\gamma-1) T_{r}(d N-d \phi) \tag{2-212}
\end{array}
$$

Since

$$
\begin{equation*}
d\left(N T_{r}\right)=N d T_{r}^{\prime}+T_{r}^{\prime} d N \tag{2-213}
\end{equation*}
$$

tiquations $2-202$ and $2-212$ yicld

$$
\begin{align*}
\frac{(N T)}{d l}= & -(\bar{\gamma}-1) \frac{A P_{r} d M}{d R} \frac{d M}{d l} \\
& +T_{0}^{\prime} \frac{d \phi}{d l}-\frac{\gamma \psi A_{1} P_{r}}{(g R}(g R T)^{1 / 2} \tag{2-214}
\end{align*}
$$

where $\bar{\gamma}$ denotes the ratio of specific heats adjusted to take account of tiev loss of heat to the gun, as explained in paragraph !-2.1.f.
In terms of $\dot{\psi}$ and $l_{r}$, Lquation 2-191 may be expressed

$$
\begin{equation*}
\frac{d \phi}{d l}=\frac{\rho S S B P_{r}^{a}}{C} \tag{2-191a}
\end{equation*}
$$

The differential liguations 2-200, 2-202, 2-214 and -191a can be solved numerically with the help of Eduations 2-196 and 2-199 or 2-199a. Before $P^{\prime}$, is high enough to open the nozzle, $A_{1}=0$. Before it is high enough to start the projectile, $X=0$. Before the propellant is all burned, $\phi$ is less than 1, and Equation 2-199a may be used; after it is all burned, $\phi=1$ and Equation $9-199$ must be used.
4. Recoil Momentum. While the projectile is moving in the bore, there is a force, $A I^{\prime} r$, tending to move the riffe backward. While the gas is flowing through the nozzle, there is a thrust, $F_{r}$, tending to move the rifle forward. Their difference is the rate of change of momentum:

$$
\begin{equation*}
\frac{d m}{d l}=A I_{r}^{\prime}-F_{r}^{\prime} \tag{2-21i}
\end{equation*}
$$

if $A P_{r}>F_{r}$ the momentum, $m$, is positive to the rear. Using the definition of the thrust encfficient, $C_{T}$ (Eq. 2-188), and letting

$$
\begin{equation*}
\mu=A_{1} / A \tag{2-216}
\end{equation*}
$$

Equation 2-215 may be written

$$
\begin{equation*}
\frac{d m}{d l}=\left(1-\mu C_{T}\right) A I_{r}^{\prime} \tag{2-217}
\end{equation*}
$$

Note that $A$ denotes the cross-sectional arca of the bore, not the nozzle.
If the nozzle flow starts at the same time as the projectile begins to move, it is theoretically possible to find a throat area that makes the resultant force vanish at all times, including the post-cjection period. The rifle would then be truly recoilless. However, this condition is difficult to achieve. Furthermore, if the nozzle start time, $t_{n}$, is different from the projectile start time, $t_{0}$, it is theoretically impossible. Therefore, there is no alternative but to make the total integrated momentum zero. Since the force acts for only a short time, the riffe will recoil only a short distance and return to its original position.
Before the projectile is cjected, at time, $t_{m}$, the pressure, $P$ ', is evaluated in solving the interior ballistic erpations. Thereafter, according to IHgoniot's theory of efllux of gases from a reservoir

$$
\begin{equation*}
I_{r}=\frac{\left(N_{m} R \eta_{m}\right.}{U_{m}}\left[1+\frac{l-t_{m}}{\tau}\right]^{-2 \gamma /(\gamma-1)} \tag{2-218}
\end{equation*}
$$

where
$\tau=\frac{2}{\gamma-1}\left(\frac{U_{m}}{A+A_{t}}\right)\left[\frac{1}{\gamma g R Z^{\prime}}\left(\frac{\gamma+1}{\underline{2}}\right)^{(\gamma+1) /(\gamma-1)}\right]^{1 / 2}$
(2-219)
13y integrating lifuation 2-217 it is found that the momentum at piojectile ejection is

$$
\begin{equation*}
m_{m}=A \int_{l_{0}}^{l^{-}} I_{r}^{\prime} d l-\mu C_{r}^{\prime} A \int_{l n}^{l m} P_{r} d l \tag{2-220}
\end{equation*}
$$

Similarly, by substituting Equation 2-218 in Eq. 2-217, the additional momentum after projectile cjection is found

$$
\begin{equation*}
\Delta m=\left(1-\mu C_{r}^{\prime}\right) A \frac{\left(: N_{m} R T_{m}\right.}{U_{m}}\left(\frac{\gamma-1}{\gamma+1}\right) \tau \tag{2-221}
\end{equation*}
$$

Approximately, if $\gamma$ is about 1.2;)

$$
\begin{equation*}
\Delta m=1.3 \cdot \frac{1-\mu C_{r}}{1+\mu}\left(N_{m}\left(\frac{R T_{m}}{g}\right)^{1 / 2}\right. \tag{2-2912}
\end{equation*}
$$

The total momentum, then, is

$$
\begin{equation*}
m=m_{m}+\Delta m \tag{2-222}
\end{equation*}
$$

It is theoretically possible to determine a ratio, $\mu$, of throat area to bore area that will make $m=0$.
$\bar{j}$. Ballistic Efficiency. The ballistic efficiency of a conventional weapon is defined as the ratio of the linear kinetic energy of the projectile at the muzzle to the energy of the solid propellant

$$
\begin{equation*}
e=\frac{W V_{m}^{2}}{2 g C^{\prime}} \tag{2-223}
\end{equation*}
$$

The specific energy of the propellant $E$, defined as $F / \gamma-1$, is discussed in paragraph 1-8.13. The modification of the projectile weight, considered in paragraph 2-2.1, is neglected here.

In a recoilless rifle, part of the energy is used in preventing recoil. Katsanis has proposed a new definition of its ballistic efficiency ${ }^{22}$

$$
\begin{equation*}
e=C_{0} / C \tag{2-224}
\end{equation*}
$$

where $C_{0}$, is the propellant weight for an ideal recoilless rifle. If $C$. is the weight of propellant gas that leaves the nozzle, the total energy for the ideal recoilless rifle is

$$
\begin{equation*}
C_{1,}^{\prime} E_{\prime}^{\prime}=\frac{C_{1}, v_{j}^{2}}{2 g}+\frac{W V_{m}^{2}}{2 g} \tag{2-22:5}
\end{equation*}
$$

where $r$, is an effective exit velocity. If $\kappa$ is the fraction of the total energy available to the projectile,

$$
\begin{equation*}
\kappa C_{1}, E=\frac{W V_{m}^{2}}{2 g} \tag{2-226}
\end{equation*}
$$

Then the weight of the propellant gas that balances the recoil in the ideal rifte is

$$
\begin{equation*}
C_{0}=(1-\kappa) C_{1}=C_{n}^{\prime}-\frac{W V_{m}^{2}}{2 g E^{\prime}} \tag{2-227}
\end{equation*}
$$

The actual recoilless rifle may have a small momentum, which is designated $f / V V_{m} / g$. The factor, $/$, is positive if the riffe moves backward; negative if it moves forward. Then the momentum equation is

$$
\begin{equation*}
C, v_{\mathrm{c}}+J W V_{m}=W V_{m} \tag{2-228}
\end{equation*}
$$

The exit velocity is then

$$
\begin{equation*}
v_{0}=\frac{(1-f) W V_{m}}{C_{0}} \tag{2-229}
\end{equation*}
$$

Squaring and multiplying by $C_{0} / 2 g$ with the help of Equation 2-227 produces

$$
\begin{equation*}
\frac{C_{1}^{\prime}, v_{i}^{2}}{2 g}=\frac{(1-f)^{2} E W^{2} V_{m}^{2}}{2 g C_{0} E^{\prime}-W V_{m}^{2}} \tag{2-2:30}
\end{equation*}
$$

Substituting this in Equation 2-225 and rearranging yields

$$
\begin{equation*}
C_{n}=\frac{W V_{m}^{2}}{2 g E^{\prime}}\left[1+\frac{(1-\rho)^{2} W}{C_{0}^{\prime}-W V_{m}^{2} / 2 g L^{\prime}}\right] \tag{2-231}
\end{equation*}
$$

Solving as a quadratic equation in $C_{0}$ gives, as the positive solution

$$
\begin{equation*}
C_{0}=\frac{W V_{m}^{2}}{2 g E^{2}}\left[1+\sqrt{(1-f)^{2} 2 g L^{\prime} / V_{m}^{2}}\right] \tag{2-232}
\end{equation*}
$$

Therefore, according to the definition, Eq. 2-224, the ballistic efficiency of a recoilless rifle is

$$
\begin{equation*}
e=\frac{W V_{m}^{2}}{2 g C E^{2}}\left[1+\sqrt{(1-1)^{2} 2 g L^{2} / V_{m}^{2}}\right] \tag{2-233}
\end{equation*}
$$

The ballistic efficiences of the 57 mm M18, 75 mm M20, $10: \mathrm{mm}$ M 27 , and 106 mm M40 Rifles, firing High Explosive, High Explosive Antitank and White I'hosphorus I'rojectiles, vary from 0.44 to 0.54 .

## 2-14.3 Graphical Methods for Recoilless Rifles

To avoid the large amount of computation necessary if a general theory such as that given above is used in the design of conventional recoilless guns, Katsanis has developed a simplified semiempirical treatment and presented it in the form of graphs and nomograms that can be used to determine the interior ballistic trajectories for recoilless rifles of standard characteristics. The method is explained and the graphs and nomograms are presented in usable form in Reference 23.

## 2-15 SMOOTH BORE MORTARS AND WORN GUNS

Smooth bore mortars are loaded by dropping the fin-stabilized projectile into the muzzle. Therefore, there has to be an appreciable clearance between the bore and the projectile body with the result that some of the propellant gas escapes past the projectile. Thus, the space between the bore and the projectile is iike a nozzle, and the theory of efflux through nozzles can be applicd to smooth bore mortars. Since the flow is forward, the leakage factor $k$ is equal to unity, (Cf. par. 2-14.2).

In rifled guns, thero is no appreciablo lcakago when they are new; but after they are badly worn, there is considerable leakage between the rotating band and the bottom of the grooves. Therefore, the same theory can be applied to a worn rifled gun as to a smooth bore mortar.
For simplicity, the linear law of burning will be used.

$$
\begin{equation*}
w \frac{d z}{d l}=\beta l \tag{2-2:3t}
\end{equation*}
$$

Let

$$
\begin{equation*}
\Psi=\frac{\psi A_{1} w}{\beta C^{\prime} \lambda^{i / i}} \tag{2-23;}
\end{equation*}
$$

where
$\psi$ is the function of $\gamma$ defined in Equation $\because$ - 180
$A$ is the leakatge area
$\beta=\frac{B}{\omega}$
(orner' shows that a leaking gun behaves almost like an orthodox gin with the effective charge

$$
\begin{equation*}
r^{\prime \prime}=(\cdot(1-\Psi) \tag{2-236}
\end{equation*}
$$

. Nso. the muzzle velocity, $I_{m}$, and maximum pressure, $P_{r}$, vary approximately according to the relations

$$
\begin{align*}
& I_{m} \alpha 1-\epsilon \Psi  \tag{2-2:37}\\
& I_{»} \alpha 1-2 \Psi \tag{2-238}
\end{align*}
$$

The coeflicient, $e$, depends on the details of the gun and charge, but is usually about 0.7 .

Equation -2 -is hay also be cexpressed

$$
\begin{equation*}
\frac{\Delta I^{\prime}}{I_{w}}=-\epsilon \Psi \tag{3!}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\Delta I^{\prime}}{l^{\prime}}=-\frac{\epsilon \psi w}{\beta C F^{1 / 2}} A_{1} \tag{2-23!a}
\end{equation*}
$$

With some finther approximations, Comer finds that the variation in muzale velocity is

$$
\begin{equation*}
\Delta V_{m}=-V_{1} A_{1} / A \tag{2-240}
\end{equation*}
$$

Where $A$ is the cross-sectional area of the bore, and the cocfficient, $I_{1}$, is about $24,000 \mathrm{in}$ 'sec ( $2,000 \mathrm{ft}$ 'sec). If $d$ is the caliber of the mortar;
$\Delta d$ the diametral clearance between mortar and projectile

$$
\begin{equation*}
\Delta V_{m}=-2 V_{1} \frac{\Delta l}{d} \tag{2-2+1}
\end{equation*}
$$

It is thes seen that the clearance should be small, not only to increase the efficiency of the mortar, but also to decrease the dispersion in muzzle velocity, and hence in range. Besides, a small clearanee makes the projectile fly nearly straight after ejection, so that the air resistance is minimized.

## 2-16 THE USE OF HIGH SPEED COMPUTING MACHINES

Iarge high speed automatic computing machines are becoming increasingly available. Their use will greatly facilitate the solution of the equations of interior ballistics and the reduction of experimental data so that more sophisticated treatments of the theory and more elaborate instrumentation for experiment and testing can be used without too great an expenditure of time and labor. Once a formulation of the theory has been properly prepared for machine computation, the effect of changes in the parameters can be determined very rapidly. The parameters to which the solution is most sensitive can be readily selected and the adjustment for best fit to firing records made with relatively little expenditure of time and labor. These machines will not, however; entirely supercede the use of the simple analytical formulas or the use of charts or tables of solutions as exemplified carlier. I'or many problems, especially of preliminary design, the simple methods are sufficiently accurate and are rapid and easy to use and do not involve a large computing group.

For treatment of interior ballistic theory devised especially for solution by high speed digital computers the reader is refered to References $2 t$ and $2 \%$.

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## CHAPTER 3

## LIST OF SYMBOLS

| . 1 | (ross-scetional area 'bore |
| :---: | :---: |
| B | Burning rate coeffic, |
| B | Quantity of heat necessary to raise a unit volume of the metal to the melting point and melt it |
| $1 \cdot$ | Weight of propellant |
| ${ }^{\prime}$ | specific heat of propellant gas at constant temperature |
| 1. | Specitic heat of steel |
| (', | Specific heat of gas at constant volume |
| c | Weight of burnt propellant |
| $d$ | Diameter of the bere: the coliber |
| E: | Specific energy of the propellant |
| $E$ | Burning paramoter |
| LFC | Equeivalent full charge factor |
| $F$ | S'pecifie foree of the propellant |
| $F(1)$ | Function of time |
| $F_{m}$ | Maximum value of $P(t)$ |
| $f(r)$ | Heat transfer function |
| $g$ | Gravitational acecleration |
| II | Dinmensiouless heat transfer coefficient |
| II | Instantancous rate of heat input to the hot spots per unit area |
| $H^{\prime}$ | Time rate of heat flow |
| $h$ | Heat transfer cocfficient |
| I | Heat transfer integral, defined by Equation 3--92 |
| j | Wright ratio: $\mathrm{IV}^{\prime \prime}$, ${ }^{\text {c }}$ |
| $\kappa$ | Empirical constant |
| l: | Thermal conductivity of steel |
| I. | Heating parameter |
| I. | Heat of fusion of the sterl tube |
| " | Number of moles of gas formed by burning one gram of propellant |
| P | Pressure |
| I' | Maximum chamber pressure |
| $Q$ | Heat input |
| $R$ | Molar gas constant |
| S | Expansion ratio |
| $T$ | Temperature of the gas |
| $T_{u}$ | Temperature of the gas |
| T, | Adiabatic flame temperature |
| $T_{i}$ | Initial temperature of the tube |
| $7{ }_{m}$ | Average temperature of the gas when the projectile is at the muzzle |
| $T_{\text {m }}$ | Ifelting temperature of the tube |

$t$ Time
$t^{\prime} \quad$ Time after cjection: $t-t_{m}$
$U$ Free volume
$U_{c h} \quad$ Chamber volime
1 Velocity of projectile
$I^{\bullet}$ Muzzle velocity
1., Speed of surface regression
$v \quad$ Velocity of the gas
$r$ ' Velocity factor
II Wear per round
II' $^{\prime \prime} \quad$ Effective weight of projectile
w Web thickness
X Travel of projectile
$x \quad$ Coordinate along the axis of the bore:
$x \quad$ invel of proinctile plus reduced chamber
lougth
: Reduced coordinate along the axis of the hore: $l$. 1
\% Empirical constant
$z \quad$ Coordinate normal to surface
$\alpha \quad$ Time factor
$\beta \quad$ Distance factor
$\gamma \quad$ Ratio of specific heats
$\triangle \quad$ Density of loading: $C_{1}^{\prime} U_{\text {ch }}$
$\delta \quad$ Density of solid propellant
$\zeta$ Dimensionless distance from the inmer surface of the barrel
Specific covolume of the gas
Temperature rise
Temperature of the gas
$\theta$. Temperature of the metal surface
$\lambda$ Friction factor
$\rho \quad$ Density of the gas
$\rho$. Density of the stecl tube
$\tau$ Dimensionless time
$\phi \quad$ Factor: ${ }^{\prime} C^{\prime} g F^{\prime} ; w .1$

## Sulscripts

Initial value: at time of projectile start
P'ertaining to ammunition for which $E F C=1$
Begimning of erosion interval
End of crosion interval
At time when propellant is all burnt
Property of the gas
At time when the projectile is at the muzzle
At time of peak pressure

## CHAPTER 3

## HEAT TRANSFER, TEMPERATURE DISTRIBUTION AND EROSION OF GUN TUBES

## 3-1 HEAT TRANSFER

## 3-1.1 General Discussion

The transformation of the propellant from the solid to the gaseous state produces a large amount of heat and the difference in temperature between the gas and the surface of the gun bore is always wey large. This, combined with the fact that the bondary layer is very thin, leads to a large temperature gradient to the surface. This results in a high rate of heat transfer to the surface and appreeibhle heating of the barrel, in spite of the short time during which the hot gas is in contact with the wall. The heating effect is most marked in rapid-fire weapons, such as machine guns, where the temperature attained limits the number of rounds that may be fired continuously. In the larger caliber shower-fired weapons the heating of the bore surface is a major cause of the erosion of the bore which liaits the useful life of the barrel.

The flow of the gas in a gun is highly turbulent arid the heat transfer is by forced convection. In treating the problem theoretically, it has been the universal practice to assmme that the equations of heint transfer for steady, fully developed flows in pipes or over flat plates can be taken over into the therey for guns, despite the existence of a highly unistrady state, and the lack of a fully developed flow. Due to these conditions the heat transfer varies rapidly with time and the boundary layer has not reached its final form, since the theory must be applied at positions too close to the entrance of the bore. The bomdary layer is always very thin and has not reached its final form for steady flow anwhere in the barrel.

The theory staris by assuming the well kiown heat transfer equation

$$
\begin{equation*}
I^{\prime}=h\left(\theta_{2}-\theta_{n}\right) \tag{3-1}
\end{equation*}
$$

where

[^5]$\theta$, the temperature of the surface and
$h$ the heat transfer coefficient

## 3-1.2 Heat Transfer Coefficient

Nordheim, Soodak and Nordheim made an extensive study of themal effects of propellant gases. ${ }^{1}$ They assumed that lifuation $3-1$ is valid, in the sense that it is valid at each instant of time. Although this assumption is doubtful, the results appear to be reasonable. The following discussion is based on their work.
There exists an extensive literature on the specification of $h$. A very simple formula for $h$ which is adecpuate for the heat transfer problem in gums was adopted by the investigators, namely:

$$
\begin{equation*}
h=0 . \dot{\mathrm{i} \lambda\left({ }^{\prime}, \rho l^{\prime},\right.} \tag{3-2}
\end{equation*}
$$

where (', is the specific heat of the gas at constant pressure, and $\rho$ and $r$ its density and velocity, respectively. The dimensionless factor, $\lambda$, is called the friction factor. It is related to the frictional force on the surface due to the flow of the gas and hence on the momentum transfer to the surface. Efpuation 3-2 cxpresses the analogy between the momentum transfer and the heat transfer and is called the Reynolds analogy. $\lambda$ depends on the surface condition, particularly on the roughness. It is not possible to specify the roughness, or its effect upon $\lambda$, in any simple way so that $\lambda$ must be determined by comparison with experiment.
The only experimental data on guns available at tlic time were data derived from calorimetric measurements made, just beyond the foreing cone on a ca::ixer . 00 machine gum, by Machler." Based on these measurements a value of $\lambda$ for this weapon, equal approximately to : $\times 10^{-3}$, was derived. Two assumptions were made concerning the distribution of the unburnt propellant; (a) that it was uniformly distributed in the gas, and (b) that it remomed in the chamber, so that the value of $\lambda$ depends on the assumption used. Assumption (a) is the better assumption and is assumed in all the systems presented in this handbook.

To evaluate $\lambda$ for other calibers a formula based
on heat transfer in pipes was modified and stated in the form

$$
\begin{equation*}
\lambda=\left(\%++\log _{11} d\right)^{-2} \tag{;-;:3}
\end{equation*}
$$

whered is the caliber and $Z$ is an empirical constant. When 1 is expressed in centimeters, $Z$ has the value 1:3.D. This formula asserts that $\lambda$ duponds only on d. Later experiment has shown that this is not so, as $\lambda$ is also a function of position along the tube. It will also depend on the condition of the bore surface. The validity of Equation :3-3 is doubtful, but it will be used here.

## 3-1.3 Calculation of the Rate of Heat Input

a. Itrat Transfor ('orflicicint. Consider the example of paragraph e-3! 3 , pertaining to the $10 . \mathrm{mm}$ Ilowitzer fiyng a ligh Explosive Projectile MI propelled by propellant MII. To calculate the rate of heat input, $I l^{\prime}$, the heat transfer coefficient, $h$. must be dotermined by licuation :3-2.

By. lipuation :3-:i, with

$$
\begin{aligned}
& \%=1: 3: 3 \\
& 1=10 . i \cdot 111
\end{aligned}
$$

the friction factor is

$$
\lambda=1300
$$

The effective chamber
longth is

$$
\begin{aligned}
V_{c h} . U_{1} & =11 . \pi \mathrm{in} \\
A_{m} & =80.4 \mathrm{in} \\
& =91.9 \mathrm{in}
\end{aligned}
$$

'The total trated is
Distance from the beecel to the muzzke
The meszale velocity is

Therefore, assmming the gas velocity to vary linearly from the breech to the projectile when the projectile is at the muzzle, the estimated gas velocity at the front end of the chamber is

$$
r=\frac{11 . \pi}{!11 .!} 878!=1100 \mathrm{in} \text { 'sice }
$$

Cosiug a value of ( 0 , egual to 199 cal ${ }^{\prime} \mathrm{lb}-{ }^{\circ} \mathrm{K}$ and substituting these values in lequation $3-2$ gives

$$
\begin{aligned}
h & =0.187 \text { cal } \mathrm{in}^{3}-\mathrm{sec}-{ }^{\circ} \mathrm{K} \text { at the front } \\
& \text { cud of the chamber } \\
h & =1.49: 3 \text { cal in:-sec- }{ }^{\circ} \mathrm{K} \text { at the muzale }
\end{aligned}
$$

b. Termperature Brjore lijection. The temperature of the propellant gas needs also to be known. With substitution of Equation 2-14 in Equation !) of Reference 3 and the assmmption that the specific covolume of the gas is erfual to the specific volume of the solid propellant, the erfuation of state may be written

$$
\begin{equation*}
\frac{c \cdot W_{u}}{T_{n}^{\prime}}=P \tag{:;-4}
\end{equation*}
$$

where
$F$ is the specific fore of the propellant
$T_{\sim}$ the absolute temperature of the gas
$T_{0}$ the adiabatic flame temperature
$U$ the free volume
$r$ the pressure and
$c$ the weight of burnt propellant
It was shown in Chapter 1 that

$$
\begin{equation*}
r_{n}=\frac{F}{(\gamma-1)( } \tag{3-i}
\end{equation*}
$$

where (', is an appropriate average value of the specific heat at constant volume. Henee,

$$
\begin{equation*}
T_{v}=\frac{P V^{\circ}}{(\gamma-1)(c, c} \tag{i}
\end{equation*}
$$

In the example chosen,

$$
\begin{aligned}
& I^{\prime}=I^{\prime}=10=10 \mathrm{psi} \\
& r^{\prime}=l_{m}=1 \underline{2} 0 \mathrm{in}^{\prime \prime} \\
& \gamma-1=0.2 \text {; } \\
& \text { (', = .ssti in-ll. } 1 \mathrm{ll})^{\circ} \mathrm{K} \\
& r=r^{\prime}=0 .(\dot{O} ; i n
\end{aligned}
$$

By substitution in Equation: $;$ - ; wen the projectile is at the mazale, the absolute temperature is

$$
r_{,}=r_{\prime \prime}^{\prime}=1: 3.3 ; ;^{\circ} \mathrm{K}
$$

If the inner surface of the gum is taken as: :00 ${ }^{\circ}$ 位 then

$$
\theta_{v}-\theta_{1}=T_{v}-: 3(K)=10: 30^{\circ} \mathrm{K} \text { and } \mathrm{b}_{\mathrm{y}} \mathrm{E} \text { E. . : } 3-1
$$

the heat input rate

$$
\begin{aligned}
& J^{\prime}=0.187 \text { ( } 1033 i \text { ) }=19+\text { cal } / \mathrm{in} \text {-sec at } \\
& \text { the front cud of the chamber and } \\
& I^{\prime}=1.4!3 ;(10: 3 i)=1 ., 4 \mathrm{cal} / \mathrm{in}^{2}-\mathrm{sec} \\
& \text { at the muzzle }
\end{aligned}
$$

c. Densily, Velucily and Trimperalure A fler l:jertion. After the projectile is cjected, the hat gas remaning in the bore continues to tronsmit heat to the barrel. The density, velocity and tenpromatue can be determined during this phase by the following.
I.ct

$$
\begin{equation*}
x=x+r, \ldots \tag{3.7}
\end{equation*}
$$

The density, $\rho$, pressure, $l^{\prime}$, and temperature, $T_{0}$ should be nearly independent of $x$; but the evoeity, $r$, may be assumed to be a linear function of $r$ :

$$
\begin{equation*}
r^{\prime}=r^{\prime} \cdot r \tag{;-8}
\end{equation*}
$$

where the factor, $t^{\prime}$, is a function of the time, $t$, only.
Since the motion of the gas is assumed to be one-dimensional, the equations of continuity, motion, and energy may be expressed

$$
\begin{array}{r}
\frac{\partial \rho}{\partial t}+\rho \frac{\partial r}{\partial r}=0 \\
\frac{d r}{d t}=\frac{\partial r}{\partial t}+r \frac{\partial r}{\partial r}=0 \\
\text { (. } \rho \frac{\partial T}{\partial t}+r^{\prime} \frac{\partial r}{\partial r}=0 \tag{3-11}
\end{array}
$$

linder the abowe assumptions, these erquations become

$$
\begin{array}{r}
\frac{d \rho}{d t}+\rho r^{\prime}=0 \\
\frac{d r^{\prime}}{d t}+r^{\prime 2}=0 \\
r_{\cdot \rho} \frac{d I}{d!}+P^{\prime \prime}=0 \tag{:3-1+4}
\end{array}
$$

Let the subseript $m$ denote values at ejection, and lit

$$
\begin{equation*}
t^{\prime}=t-t_{m} \tag{:3-1.0}
\end{equation*}
$$

Then integration of Liguation :3-1:3 leds to

$$
\begin{equation*}
r=\frac{r_{m}^{\prime}}{1+r_{m}^{\prime} t^{\prime}} \tag{3-16}
\end{equation*}
$$

Hence, by liguation :3-8, the velocity is

$$
\begin{equation*}
r=\frac{r_{m}^{\prime}, r^{\prime}}{1+r_{m}^{\prime} m^{\prime}} \tag{3-17}
\end{equation*}
$$

It is crident that

$$
\begin{equation*}
r_{m}=r_{m,}, \quad r_{m}^{\prime}=V_{m, r} r_{m} \tag{3-18}
\end{equation*}
$$

where $I_{m}$ is the muzzle velocity of the projectile.
Integration of Equation 3-12 gives

$$
\begin{equation*}
\rho^{\prime} \rho_{m}=r^{\prime}, r_{m}^{\prime} \tag{3-19}
\end{equation*}
$$

Hence, by substituting Equation 3-16; the density is found

$$
\begin{equation*}
\rho=\frac{\rho_{m}}{1+r_{m}^{\prime} l^{\prime}} \tag{:-20}
\end{equation*}
$$

The erination of state is

$$
\begin{equation*}
I P(1 ; \rho-\eta)=n R T \tag{:3-21}
\end{equation*}
$$

where
$\eta$ is the specilic eorolume of the gas
$n$ the number of moles of gas formed by burning one gram of propellant and
$R$ the molar gas constant

Substituting this in Equation 3-14 gives

$$
\therefore \frac{d T_{o}}{I t}=\frac{n R T_{u} r^{\prime}}{1-\eta \rho}
$$

Assuming

$$
\begin{equation*}
n R=C_{r}^{\prime}(\gamma-1) \tag{3-2?3}
\end{equation*}
$$

and with the help of Equations 3-10 and 3-20,

$$
\begin{equation*}
\frac{1}{T_{y}} \frac{d T_{u}}{d l}=\frac{(\gamma-1) r_{m}^{\prime}}{1+r_{m}^{\prime} I^{\prime}-\eta \rho_{\mu}} \tag{3-2-4}
\end{equation*}
$$

The integral of this is

$$
\begin{equation*}
T_{v}=T_{m}\left[\frac{1-\eta \rho_{m}}{1+r_{m}^{\prime} t^{\prime}-\eta \rho_{m}}\right]^{1-1} \tag{3-2-5a}
\end{equation*}
$$

or

$$
\begin{equation*}
T_{v}=T_{n}\left[\frac{1 \rho_{m}-\eta}{1 \rho-\eta}\right]^{r-1} \tag{3-25~b}
\end{equation*}
$$

which is the adiabatic relation for an imperfect gas that obeys the equation of state (E(1. 3-21). Here, the ratio of specific thats, $\gamma$, should be adjusted to take account of the loss of heat; as in paragraph 2-2.1. Let us assume $\gamma=1.30$.
By idurations : $3-20,3-21$ and $3-25$, the pressure is

$$
\begin{equation*}
I^{\prime}=I_{m}^{\prime}\left[\frac{1 / \rho_{m}-\eta}{1 / \rho-\eta}\right]^{\gamma} \tag{3-26a}
\end{equation*}
$$

or

$$
\begin{equation*}
I^{\prime}=P_{m}\left[\frac{1-\eta \rho_{m}}{1+r_{m}^{\prime}!^{\prime}-\eta \rho}\right]^{r} \tag{3-26b}
\end{equation*}
$$

This example can be continued by calculating the rate of heat input at the muzzle 0.1 second after the projectile is cjected.

| Simbol | Vilue | Unit | Remarks |
| :---: | :---: | :---: | :---: |
| $\ell^{\prime}$ | 0.1 | sers: | .tssumed |
| $s=r m$ | 91.9 | in | (iiven |
| $1{ }^{*}$ | 878! | in/ser | (iiven |
| ค.. | 5. $12 \times 10^{-4}$ | $\mathrm{lh} / \mathrm{in}^{3}$ | Given |
| $T{ }^{\prime \prime}$ | 13:36 | ${ }^{\circ} \mathrm{K}$ | Given |
| $P_{\text {r }}$, | 1040 | 11 /in ${ }^{2}$ | Given |
| $r^{\prime}$ | 190 | (al/l)-0\% | Given |
| $\lambda$ | 1/300) |  | (iiven |
| $\eta$ | 27.7 | in ${ }^{3} / \mathrm{lb}$ | Par. 2-2.1 |
| 7 | 1.30 |  | Given |
| $\mathrm{r}^{\prime \prime}$ | 95.6 | Nor-1 | bq. 3-18 |
| $1+r^{\prime}, l^{\prime}$ | 10.5; |  |  |
| $v$ | 8:32 | in/sec | Eq. 3-17 |
| $\rho$ | $4.85 \times 10^{-5}$ | $\mathrm{lb} / \mathrm{in}^{3}$ | Eq. 3-20 |
| $h$ | 0.013 .4 | ral/in²-sect- ${ }^{\circ} \mathrm{K}$ | Eq. 3-2 |
| $T "$ | (93) | ${ }^{\circ} \mathrm{F}$ | Eif. 3-2\%a |
| $\theta_{\text {g }}-0$. | 354 | ${ }^{\circ} \mathrm{K}$ |  |
| $H^{\prime}$ | 4.88 | eal/in ${ }^{2}$-see | Eq. 3-1 |
| $r$ | 48.1 | li)/in ${ }^{2}$ | Eic. 3-26) |

## 3-1.4 Nondimensional Heat Transfer Coefficient

In order to apply the calculation of heat input to all calibers, it is convenient to define a nondimensional heat transfer coefficient.
A simple interior ballistic theory similar to that of Mayer and Hart (paragraph 2-i,2) is used. The notion in the following has been changed where necessary from that used in Reference 1 to conform to the notation used in Chapter 2 .

The burning rate coefficient, $B$, is defined by the weight buming rate law

$$
\frac{d c}{d t}=\frac{B C P}{w}
$$

The position of the projectile is defined by the coordinate

$$
\begin{equation*}
y=r^{\prime} \cdot A \tag{3-27}
\end{equation*}
$$

Its initial position is then $y_{n \prime}=\zeta_{c h} / A$. The relation between the position of the projectile and its velocity is given, up to the time the charge bums out, by

$$
\begin{equation*}
\frac{!}{y / n}=\left[1-\frac{1}{1-\frac{\gamma-1}{2 \phi} r}\right]^{2 /(\gamma-1)} \tag{3-28}
\end{equation*}
$$

where $\phi$ hats the dimensions of velocity and is given by

$$
\begin{equation*}
\phi=\frac{B C \cdot F}{u \cdot l} \tag{3-29}
\end{equation*}
$$

Equation : $3-28$ holds up to $y=y, y$, the position of the projectile at the time of charge burnout, when it becomes:

$$
\begin{equation*}
\frac{!}{!n}=\left[\frac{1}{1-1}\right]^{? \because, \cdots} \tag{3-30}
\end{equation*}
$$

where $l:$ is given by

$$
\begin{equation*}
E=\frac{\gamma-1}{2} \frac{\mu F}{j \phi^{2}} \tag{3-31}
\end{equation*}
$$

and $j$ is cqual to $I^{\prime \prime}: C . E$ is called the burning parameter. It is dimensionless and specifies the gun-ammunition system. Systems with the same $E$ have similar ballisties. From Fequation 3-28 $\mathrm{F}^{-}$is given by

$$
\begin{equation*}
r=\frac{\partial \phi}{\gamma-1}\left[\left(1-\frac{y!y}{y}\right)^{\gamma-11: 2}\right] \tag{3-3;2}
\end{equation*}
$$

before the chatge burns out. After bumout it is given by

$$
\begin{equation*}
r=\left(\frac{2!}{j(\gamma-1)}\right)^{\prime}\left[1-\left(\frac{11}{y}\right)^{\prime} \cdot\left(\frac{!11}{!}\right)^{\gamma-1}\right]^{1 / 2} \tag{:;-:3:3}
\end{equation*}
$$

The pressures before and after burnout are given respectively, by

$$
\begin{equation*}
I=\frac{F \Delta}{1-\Delta / \delta} \frac{1}{E}\left(\frac{!/ 2}{y}\right)^{(\gamma+1) / 2}\left(1-\frac{\eta_{1}}{!}\right)^{(\gamma-1) / 2} \tag{3-3;3}
\end{equation*}
$$

and

$$
\begin{equation*}
P=\frac{F \Delta}{1-\Delta \delta}\left(\frac{y_{1}}{y_{n}}\right)^{(\gamma-1 / 2)}\left(\frac{y_{11}}{y_{j}}\right)^{\gamma} \tag{;;-3;:;1}
\end{equation*}
$$

The maximum pressure comes at

$$
\begin{equation*}
\frac{y_{0 \text { mar }}}{y_{0}}=\left(\frac{\underline{ }}{\gamma+1}\right)^{2 /(\gamma-1)} \tag{:3-34}
\end{equation*}
$$

unless $y_{p \text { max }}$ is not greater than $y_{b}$, when it comes at $y_{b}$. The value of the maximum pressure is given by

$$
\begin{array}{r}
P_{m+a r}=\frac{j \Delta}{1-\Delta / \delta} \phi^{2} \frac{1}{\gamma}\left(\frac{\gamma+1}{2 \gamma}\right)^{(\gamma+1) /(\gamma-1)} \\
\text { for } E^{\prime} \geqq(\gamma+1) / 2 \gamma \\
P_{m a x}=\frac{j \Delta F}{1-\Delta / \delta}\left(1-E^{\prime}\right)^{(\gamma+1) /(\gamma-1)} \\
\text { for } E^{\prime}<(\gamma+1) / 2 \gamma \tag{3-3,5a}
\end{array}
$$

The gas temperatures are given by

$$
\begin{equation*}
T=T_{n}\left(\frac{y / 1}{y}\right)^{(\gamma-1) / 2} \text { before burnout } \tag{3-3-3i}
\end{equation*}
$$

$T=T_{n}\left(\frac{y_{n}}{y_{n}}\right)^{(\gamma-1) / 2}\left(\frac{y_{0}}{y_{j}}\right)^{\gamma-1}$ after burnout
In using the system, the values of $\phi$ and, hence, $E$ are determined from Equations 3-35 and 3-3:~a from the observed maximum pressure. The ballisties can then be cross checked by comparing the muzzle velocity calculated from Equation 3-32a with the observed valuc. The calculation of $\phi$ from Equation $3-29$ leads to poor results because the value of $B$ is not well known.
It is to be noted that in Equations 3-32 to 3-3i the dimensions of the gun do not appear. Also since $V / y_{0}=d / d l\left(y^{\prime} / y_{0}\right)$, an examination of E(quations $3-32$ and $3-32 \mathrm{a}$ shows that if a reduced time defined by

$$
\begin{equation*}
\tau=\frac{2 \phi}{y_{1}(\gamma-1)} t=\alpha l \tag{3-37}
\end{equation*}
$$

is introduced, the velocity-time curves for all guns are the same on the reduced scale. The result is that for a given gun class defined by $E$, a change in the size of the gun means simply a change in the time scale. All ballistic curves are the same execpt for multiplicative factors when expressed as functions
of the reduced time. This misult is true, of comse, only to the approximation of the simple theory.

It follow: from the abowe discussion that, to this approximation, the heat transfer coofficiont, $h$, is a universal function of the redueed time variable, r, execept for the multiplicative factor. $\lambda$, so that

$$
\begin{equation*}
h=\lambda_{1} \tag{:3-38}
\end{equation*}
$$

where $h$ applies to all eases if expressed in the $\tau$ scale.

## 3-2 TEMPERATURE DISTRIBUTION

## 3-2.1 The Equations of Temperature Distribution in Reduced Variables

To calculate the temperature distribution in the gun oue must solve the Fourier equation of heat conduction subject to the proper bomdary condition. The curvature of the bore surface is neglected and the equation is restrieted to one dimension. This approximation is also used by Comer (Roference 1 of (Chapter 1) and is probably sufficient in view of the other approximations in the theors. lariations in the themal propertios of the barrel material are also nowereted and constant average values are assumed.

The fommer equation in one dimension is

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{l_{1}}{\sigma_{A} \rho_{A}} \frac{\partial^{2} \theta}{\partial z^{2}} \tag{3:3}
\end{equation*}
$$

where $l$, $f^{\prime}$, and $\rho$, are the themal conductivity, the speceifer heat and the density, respeetioedy, of the material in which the heat is being conducted; in the ease of the gum, the steel of the harrel wall. The boundary condition at the wall expressing the consinvation of heat flux is

$$
\begin{equation*}
h\left(\theta_{0}-\theta_{.}\right)+l_{i} \frac{\dot{\partial}}{\dot{\partial z}}=0 \tag{3-40}
\end{equation*}
$$

Where $z$ is a coordinate nomal to the surface in the direetion of heat flow. The initial condition is

$$
\begin{equation*}
\theta(0, z)=0 \tag{3-41}
\end{equation*}
$$

(6) that $\theta$ represents temperature above the initial trmperature of the hared.
Substituting the reducerd variables $\tau=\alpha d$ and
 and :;-4 become, respectieny,

$$
\begin{gather*}
\frac{\partial \theta}{\partial \tau}=\frac{1}{2} \frac{\partial^{2} \theta}{\partial \zeta^{2}}  \tag{3;-39}\\
H\left(\theta,-\theta_{n}\right)+\frac{\partial \theta}{\partial \zeta}=0  \tag{:;-40:1}\\
\theta(0, \zeta)=0 \tag{;;-41:1}
\end{gather*}
$$

where

$$
\begin{equation*}
H=\frac{\sqrt{2} h}{\sqrt{\text { clit }} l_{n} \rho_{*}}=\frac{\sqrt{2} \lambda h_{n}}{\sqrt{\alpha l_{i} C_{n} \rho_{0}}} \tag{:3-1+2}
\end{equation*}
$$

is the reduced heat transfer corfliciont.
If it is assumed that the unburnt propellant is uniformly distributed in the gas the mass flow of the gas is given by
where $x$ is the axial coordinate of position along the barrel defined by

$$
\begin{equation*}
r=1 \therefore, .1 \tag{:3-44}
\end{equation*}
$$

where $l$, is the volume between the position, $x$, and the breech. $x$, is the position at which pr is determined, $x$ is the position of the projectile and dre'd its velocity. $\rho_{, 1}$ is the density of the propellant.

The coeflicient // can be ceppressed as a function of the reduced distance down the bore $x$, $x$, (where $x_{n}=\left(r_{n,}^{\prime}, 1\right)$, and the reduced time. $r$, in the form

$$
\begin{equation*}
I I=\left(x, r_{n}\right) / / /(\tau) \tag{:3-4.5}
\end{equation*}
$$

L, which is called the heating parancter, is given by

where $\Delta$ is here defined as ( $\%$ le $1, L$ is a function only of sperefie intantitios related to the gum system. The time dependenee is given her f(r) which is expressed as
before hurnout, and

$$
\begin{equation*}
f(r)=\frac{L^{a / 2}\left[1-\frac{1}{-e^{\prime}}\left(y_{n}^{\prime}(y)^{)^{-1}}\right]^{1 \cdots}\right.}{\left[1-\frac{\Delta}{\rho_{r}}\right]\left[\frac{!}{!_{n}}+\frac{\Delta \rho}{1-\frac{\Delta}{\rho_{r}}}\right]^{\prime}} \tag{0,-17a}
\end{equation*}
$$

after burnout.
$f(\tau)$ is tabulated in Tables $3-1$ and 3-2.
The theory so far developed holds up to the time of exit of the projectile. After exit the hot gases continue to flow from the tube and so continue to transfer heat by twbulent foresd convection. For location near the muzzle most of the heat is so transterred. If the gases in the barrel undergo a uniform adiabatic expansion, from Equations 3-16, :3-17 and :3-2 lia, $^{2}$
where the subseript $m$ indieates values when the projectile is at the muzzle and $b=V^{\prime},{ }^{\prime} \cdot \alpha_{1} r_{m}$. The problem is now eompletely specified.

Equations :3-39a, 3-40a and 3-41a camot be solved analytically: They have beren solved numerically for a limited range of the parameters and tables of the solutions are published in Reference 1. The theory has been eoded for machine computation at the Ballistic Researeh Labomatories.

## 3-2.2 Heat Input

. ffer the temperature distribution for any specified time has been computed, the heat input, $Q$, up to the specified time can be found by means

## TABLE 3-1. HEAT TRANSFER FUNCTION, $f(\tau)$,

 FOR GUNS DURING BURNING| $r$ | 10K0) $f(\tau)$ | $\tau$ | $1000) f(r)$ | $\tau$ | $1000) \int(r)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.0 \% 8 | 12 | 1.51 | 2.1 | 3.(\%) |
| 1 | 0.100 | 1:3 | 1.88 | 25 | 3.88 |
| $\because$ | 0.1:3 | 11 | 2.04 | 20 | 3.88 |
| 3 | 0.173 | 15 | 2.31 | 27 | 3.88 |
| 4 | 0.230 | 16 | 2.62 | 28 | 3.8 .1 |
| 5 | 0.310 | 17 | 2.89 | ?!) | 3.88 |
| 1 | 0.40 .4 | 18 | 3.15) | 30 | 3.72 |
| 7 | 0.59 .4 | 19 | 3.35 | 31 | 3.6 .4 |
| 8 | 0.661 | 20 | 3.54 | 32 | 3.54 |
| $!$ | 0.8:40 | 21 | 3.68 | 33 | 3.43 |
| 10 | 0.090 | 21 | 3.75 | 3.1 | 3.31 |
| 11 | 1.29 | 23 | 3.86; | 35 | 3.20 |

TABLE 3-2. HEAT TRANSFER FUNCTION, $f(r)$, FOR GUNS AFTER ALL BURNT FOR $r_{6}=24$ AND 28

| $\tau$ | $\tau_{3}=24$ |  |  | $\tau_{b}=28$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1000) \int(r)$ | $\boldsymbol{r}$ | $1000)(\%)$ | $\tau$ | $10(0) f(r)$ | $\tau$ | $1000 /(5)$ |
| 25 | 3.80) | $5: 3$ | .675 | 2!) | 3.80 | (i) | .552 |
| 26 | 3.60 | 57 | . 550 | 30) | 3.60 | (i) | . 116 |
| 27 | 3.40 | 61 | . 450 | 31 | 3.3!) | 70 | .367 |
| 28 | 3.20 | 65 | . 386 | 32 | 3.18 | 7.1 | . 308 |
| 29 | 3.0 ) | (6) | . 32.2 | 33 | 2.99 | 78 | 26: |
| 30 | 2.81 | 73 | . 285 | 34 | 2.80 | $8:$ | : 2 ( |
| 31 | 2.68 | 77 | .248 | 35 | 2.6 | 86 | .197 |
| 32 | 2.16 | 81 | .219 | :36 | 2.46 | 102 | .122 |
| 333 | 2.30 | 85) | .19.4 | 37 | 2.30 | 118 | (0835) |
| 3.4 | 2.11 | $8!$ | . 175 | :38 | 2.16 | 13.1 | (1)(10) |
| 35 | 2.01 | 105 | . 117 | 3!) | $2 .(0)$ | 150 | .0.158 |
| 36 | 1.88 | 121 | . 0844 | 40 | 1.88 | 168 | .0.358 |
| 37 | 1.76 | 137 | .033: | 41 | 1.70 | 18:2 | .028!) |
| 38 | 1.65 | 15:3 | . 0.40 .4 | $+3$ | 1.6.4 | 198 | . $0 \times 37$ |
| $3!$ | 1.54 | 169) | .03!)6 | . 46 | 1.31 | 214 | .0199) |
| 40 | 1.45 | 185 | .0:325 | 50 | 1.0.4 | 230 | . 01168 |
| 41 | 1.36 | 201 | . 0270 | 54 | 0.850 | 2.16 | . 01.15 |
| 42 | 1.28 | 217 | (0)30 | 58 | 0.609 |  |  |
| 43 | 1.21 | $2: 3: 3$ | . 0197 | Pro | jectile |  |  |
| 4 | 1.14 | 249 | .0171 | lo:a | ves muzale |  |  |
| 15 | 1.07 |  |  |  |  |  |  |
| 46 | 1.01 |  |  |  |  |  |  |
| 17 | 0.961 |  |  |  |  |  |  |
| 18 | (0.910 |  |  |  |  |  |  |
| $4!$ | 0.860 |  |  |  |  |  |  |
| 1'rojectile |  |  |  |  |  |  |  |
| leaves muzale |  |  |  |  |  |  |  |

'of the integral

$$
\begin{equation*}
Q=\int_{11}^{\infty}\left({ }_{1} \rho_{*} \theta(z, \tau) d z\right. \tag{;--i0}
\end{equation*}
$$

In terms of the nondimensional distance, $\zeta$

$$
\begin{equation*}
Q=\left(2 l\left(C^{\prime}{ }_{\mathrm{N}} \rho_{\mathrm{s}} / \alpha\right)^{1 / 2} I\left(\tau,!/ / y_{n}\right),\right. \tag{3-;1}
\end{equation*}
$$

where

$$
\begin{equation*}
J\left(\tau, y / y_{n}\right)=\int_{0}^{\omega} \theta(\zeta, \tau) d \zeta . \tag{:-5i2}
\end{equation*}
$$

Two values of $Q$ are of specibl interest: at the time when the projectile leaves the muzzle, and at an infinite time. Therefore, the heat tramser integral, $I$, is tabulated in Table $3-3$ as a function of $y /!/$, and the heating parameter, $L$, for two values of the burning time, $\tau_{h}$, at muzale time corresponding to two values of $!y_{m} / y_{n}$, and also at $\tau=\infty$.

## 3-2.3 The "Thermal Analyzer"

The solution of the loowier equation is quite complieated, even for a single round, and even mone
so for a series in rapid suecession since the solution must be repeated for cach round using as initial conditions the temperature distribution resulting from: the previous rounds. For points a short dis*ance below the surface, however, the temperature history is insensitive to the details of the heat transfer and the problem can be simplified by assum-

| $\cdots$ | 11. !" | " ! ! | 1. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 28.4 | 19.4 | $8(6.5$ | 148 |
| $\because 1$ | $\therefore: 3$ | 1.0 | 18:31) | 2930 | 3 Sc 0 | 4910 |
|  |  | 1.2 | 1970 | 2880 | 3910 | 4910 |
|  |  | I.fi | ?(F) | $\underline{2810}$ | 370 | +500 |
|  |  | 2.1 | 18.10 | $\because 600$ | 3510 | +1.40 |
|  |  | 2.4 | 16i 0 | 2030 | 3050 | 3710 |
|  |  | 1.7 | ! (\%) | 1:3:30 | 18.40 | $2: 350$ |
|  |  | 8.3 | 11 | 0 | 0 | 0 |
|  | 10.: | 1.0 | 12:31 | 20 | 3!8: | 50:30 |
|  |  |  | 20:30 | 2!\%行 | $10: 0$ | 50:30 |
|  |  | 1.1 | 20 | 20 | 3850 | 47.40 |
|  |  | $\because .1$ | 1:101 | 290 | 35.0 | +3:30 |
|  |  | 2.7 | 17.0) | $\because 4.0$ | 3210 | 31910 |
|  |  | 1.7 | 1050 | 1500 | $\cdots 2(6)$ | 2760 |
|  |  | s:3 | .110 | (i.\%) | 020 | 1230 |
|  | $r=\cdots$ | 1.11 | $\because 100$ | $3(\mathrm{Kk})$ | 120 | 5:330 |
|  |  | $1 \because$ | 2000 | : $3:(6)$ | 4:310 | 3:350 |
|  |  | 1.10 | $2: 310$ | 3190 | 1210 | 5120 |
|  |  | $\because .1$ | 2:10 | : 1 c 0 | 41010 | 48.10 |
|  |  | 2.7 | -20il | 2!10 | 3810 | Hi:30 |
|  |  | 1.7 | 1.590 | $2: 300$ | 3010 | 3 STO |
|  |  | s.: | 13:9) | $1!80$ | 2750 | 3150 |
| $\cdots$ | S! | 1.1 | 2030 | (0)0 | 13:311 | $\because 2$ |
|  |  | 1.6 | $\cdots$ | :31:0 | +1010 | +190 |
|  |  | ?.i) | 2(\%) | -iso | 3 BSO | 12.40 |
|  |  | 3.1 | 1900 | 2310 | 31120 | 3460 |
|  |  | \%.i) | $111: 30$ | 1180 | 1090 | $2+20$ |
|  |  | s.3 | 1 | 0 | 0 | 0 |
| $11 .:$ |  | 1.1 |  | :3070 | +280 | 83:30 |
|  |  | 1.10 | -30 | :3:31 | 19.40 | 5070 |
|  |  | 2.: | $\because 1010$ | 29110 | 3780 | 1.150 |
|  |  | :3.1 | 15010 | ב.aio | 3:3:30 | 3960 |
|  |  | B.i | 1:310 | 15:\% | 2.410 | -930 |
|  |  | s.: | SN0 | Si;) | 1211 | 1.3 .00 |
| $\tau=0$ |  | 1.10 | 20: 2 | :3:0 | 1:30 | 8380 |
|  |  | 1.15 | 200 | 3.310 | 1.300 | $5 \mathrm{f}(\mathrm{K})$ |
|  |  | 2.5 | $\because 101$ | :3310 | 12(x) | 4950 |
|  |  | 3.1 | 2:30 | 30110 | 38\%0 | 16:3) |
|  |  | -i. | 18.0 | 2610 | 33880 | .1100 |
|  |  | s.3 | 1.150 | 200 | $28(8)$ | 3(1i) |

ing that the heat is transferred in a series of instantancous pulses. This assumption is usually made in treating the problem of heating in machine guns.
l'urdue C'niversity has built an electrical analog computer or "thermal analyzer" to determine the temperature distribution in machine guns. ${ }^{\text {a }}$

## 3-2.4 Comparison With Experiment

A detailed and extensive comparison of the results of the theory of Reference 1 with the experimental measurements available at the time is given in Reference 5 . A more recent comparison with measurements made in a 37 mm gun can be found in Reference 6. The general conclusion from these studies is that the theory yields results which are fairly reliable, perhaps better than one might expect in view of the drastic assmmptions underlying it. In most cases the agreement is within 20 pereent using values of $\lambda$ for different calibers derived from Equation 3-3. If $\lambda$ is fitted to the experimental data, it turns out that $\lambda$ is a finction not only of caliber but of position in the bore as well as propellant type and probably other factors. It would be expected to depend on position in the bore because when a gun is used the bore always becomes roughened preferentially near the breech and nete: the muzzle. In general the fitted values of $\lambda$ seem to follow a corresponding pattern.

Exeept on the outer surface accurate measurements of barrel temperature are difficult to make. This is especially true of the bore surface temperature (sere paragraph $4-9.1$ ). In fact it is doubtful that a uniform bore surface temperature actually exists. The measurements usually show large round to round variations so that the fitted value of $\lambda$ depends on the particular firing. The total heat input, $Q$, is much less variable and it is better to fit $\lambda$ to $Q$. When this is done and the resulting values of $\lambda$ used to calculate the temperatures, the agreement with the measured values is usually within the uncertainty in the measured values. For further details the reader should consult the original papers.

## 3-3 EROSION

## 3-3.1 General Discussion

The phemomemon deseribed as erosion is the progressive wearing away of the bore surface as the gom is used. It is greatest on the surface of the lands and near the origin of the rifling so that the bore tends to become collarged preferentially in this region. The effect is to lower the cmgraving forces and to shift the forming eone somewhat toward the
muzzle. This is cquivalent to increasing the effective chamber volume and to lowering the engraving resistance and the starting pressure. The result on the interior ballisties is to lower the maximum pressure and the muzzle velocity, an effect called pressure and velocity drop. These effects were studied by Nobel ${ }^{7}$.
The details of the erosion proess are not understood. The process is extremely complex and involves mechanical, chemical and thermal effects which are interrelated in unknown ways and no doubt interrelated differently depending on the particular circumstances. It is a fact of observation, however, that crosion is very sensitive to the heating of the barrel. Low energy weapons using cool propellants crode very slowly. As the muzzle velocity is increased the crosion per round inereases rapidly so that a high velocity tank gun crodes at a rate many times that of a howitzer of similar caliber. The crosion is particularly severe if hot propellants are used. As the flame temperature is increased, for constant ballistics, the erosion rate increases much more rapidly than the rate of increase of the flame temperature, so much so that the thermal effects become dominant. That the erosion is intimately related to the heating is indicated by Figure :3-1 which is a plot of the observed erosion per round versus the heat input $Q$ calculated by the method of Reference 1.
There are other forms of damage to the tube due to firing, some of them due to thermal effects. When a gun is fired, the firing cyele is very short, of the order of milliseconds. During this interval, the tube is subjected to very large thermal and mechanical stresses. The most characteristic result of this is heat checking. The bore surface develops a characteristic pattern of cracks which lead to a developing roughness which increases the heat transfer. These eracks crode locally so that the surface eventually becomes quite rough and gas tends to leak past the rotating band which causes large local crosion. For more details on erosion and other types of damage of gun tubes and methods of dealing with them, reference should be made to another handbook in this series, Refercnce 8.

## 3-3.2 Estimation of the Erosion of Gun Tubes

A general theory of erosion of guns has not been formulated. The crosion rate decreases as the gun is used due to changes in the interior ballistics resulting from the erosion. It does not seem possible to formulate a complete theory from first principles in any general way. Jones and Breitbart ${ }^{9}$ developed
 crosion :acar the commencement of ribing ion a new gun for slow rates of fire. In discussing crosion by propellant gases, it is usually assumed that the surface is first brought to the melting point and then removed in the molten form by the frictional forees of the gas flowing over the surface. A mathematical treatment of this problem has been given by Landau. ${ }^{10}$ Jones and Breitbart could not fit this pieture to the observed data for guns and were led to assume that, due to the roughness, the surface reaches the melting point only locally so that crosion occurs at "hot spots". The heat insolved in the erosion is only a small fraction of the total input. The hot spots occur for short times and shift about on the surface. The instantancous rate at which material is being removed can be averaged over the surface and will define an instantaneous average rate of surface regression, $V_{r}$. The surface will move back on the average in one round an amount

$$
\begin{equation*}
V^{V}=\int_{1_{1}}^{t_{3}} V_{r} d t \tag{3-i,3}
\end{equation*}
$$

where $t_{1}$ and $t_{2}$ specify the beginning and end of the crosion interval, respectively. If
$I I$ is the instantancous rate of heat input to the hot spots per unit area and
$B$ is the quantity of heat necessary to raise a unit volume of the metal to the meltiny point and melt it,

$$
\begin{equation*}
\mathrm{r}_{r}=H^{\prime} B \tag{3-54}
\end{equation*}
$$

Evidently,

$$
\begin{equation*}
B=o_{s}\left(C_{0}\left(T_{m}-T_{i}\right)+L\right] \tag{i}
\end{equation*}
$$

where
$\rho$. :s the density of the tube material
C. the specific heat of the tube material
$T_{m}$ the melting temperature of the tube material
$T_{i}$ the initial temperature of the tube material and
$L$ the heat of fusion of the tube material
Taking

$$
\begin{aligned}
\rho_{n} & =7.8 \mathrm{gm} / \mathrm{cm}^{3} \\
C_{0} & =0.13 \mathrm{cal} / \mathrm{gm}^{\circ} \mathrm{K} \\
T_{m} & -T_{i}=1+00^{\circ} \mathrm{C} \\
L & =60 \mathrm{cal} / \mathrm{gm}
\end{aligned}
$$

one finds that $B=1.9 \times 10^{3}$ cal, $\mathrm{cm}^{3}$. L'sing Equations $3-1$ and $3-2$ it is assumed that an crosion function, $A$, can be defined by the equation

$$
\begin{equation*}
I=A\left({ }^{\prime}, \operatorname{NPI}^{\prime}\left(T_{v}-T_{m}\right)\right. \tag{3-96}
\end{equation*}
$$

Downloaded from http://www.everyspec.com


FIGGLIE 3-1. Observed Radial IVear per Rownd at the C'ommencement of Rifing versus C'alculatel Heat Input per cmí per Round.
where $\%=$ temperature of the propellant gas. This effectively detemines the fraction of the heat input responsible for the erosion. Based on general arguments regarding the behavior of hot spots it is further assumed that

$$
\begin{equation*}
. I=K t_{1} F_{m} \tag{3-57}
\end{equation*}
$$

where $r_{m}$ is the maximum value of the function

$$
F(l)=\rho r\left(T_{s}-T_{m}\right)
$$

and $\mathcal{K}$ is an cmpirical constant. latuation $3-\pi$-3 then beomes

$$
\begin{equation*}
\mathbb{W}^{*}=\frac{K\left(\sigma_{1}, F_{m}\right.}{B} \int_{1}^{t_{3}} F(t) d t \tag{8-5-3}
\end{equation*}
$$

The values of $t$, and $t$, the times at which erosion starts and ends, are taken as the times at which
$l^{\prime}(t)$ for the weapon in cquestion, rises above and falls below the value of the maximum value of $F(t)$ for a lew velocity gun like a howitzer for which the erosion is negligible. A standard value, based on a study of such low velocity weapons, of $200 \times 10^{4}$ cgs units was chosen. The interval during which the value of $F(t)$ for the weapons under consideration was above the standard value was taken as the interval over which the integral in Equation 3-is8 was to be evaluated.
The function $F(t)$ was evaluated using the formulas of Reference 1 and then plotted and the integral determined from the graph. $K$ was then evaluated by fitting to the observed rate of wear at the origin of rifling for 29 guns of various types, and an average value of $K$ determined. The value of $K$ so determined was equal to $3.28 \times 10^{-8} \mathrm{~cm}^{2} / \mathrm{gm}^{\circ} \mathrm{K} / \mathrm{rd}$. The wear

TABLE 3-4. WEAR OF GUNS

| (ian | $\begin{aligned} & \text { Pror } \\ & \text { pull:ant } \end{aligned}$ | $\begin{aligned} & \text { l'ru- } \\ & \text { jectile } \\ & \text { Typr } \end{aligned}$ | $\begin{aligned} & \text { Nomi- } \\ & \text { nal } \\ & \text { M.V., } \\ & \text { fps } \end{aligned}$ | $\begin{gathered} \mathrm{IF}(\mathrm{u}) \mathrm{s}), \\ \mathrm{rm} \times 10^{-3} \end{gathered}$ | $K \times 10^{8}$ | $\begin{gathered} 11(\text { malr }), \\ \sin \times 10^{-3} \end{gathered}$ | $\left.\\|^{\prime}(\text { calle })-\\|^{(o b s}\right)$ | $\frac{U^{*}(c a l v)-\\|^{\circ}(u b s)}{U^{*}(a h s)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 1 'm! |  |  |  |  |  |  |  |  |
| :5mm M3 | M | AP | 2!061 | . 11 | 3.74 | .097 | -.013 | $-12$ |
| M1 | M | HF: | 2870 | . 04.3 | 2.79 | . 050 | . 01 | 18 |
| 5inm M11 | Mis | AP' | 2700 | . 28 | 3.85 | .239) | $-.041$ | $-10$ |
| Simm M1 | . 16 | HE: | 2700 | . 13 | 1.71 | .250) | .120 | !2 |
|  | M1 | HE: | 2300 | . $04+9$ | 3.33 | . 048 | -. (\%) 1 | - 1 |
|  | M6; | AP | +(\%) | . 38 | 3.96 | . 315 | -. (His | $-17$ |
| 3-inch Mi | M6; | APC | 2600 | .19 | 3.24 | .102 | (k)? | 1 |
| ! 0 mm \I3 | Mis | HE: | 2700 | . 30 | 4.04 | .244 | -.03\% | $-1!$ |
| ! 0 mm ¢ T! | M12 | $\mathrm{Al}^{1} \mathrm{C}$ | 2650 | 1.10 | 3.18 | 1.137 | .0:37 | 3 |
| 90mm m | Mis | APC | 3300 | 1.90 | 4.27 | 1.462 | -. 138 | $-23$ |
| 90mm Tob | T12 | APC | 33300 | 2.00 | 3.41 | 1.920 | . 080 | 4 |
| !0mm Tratio | . 16 | APC | 3200 | 1.20 | 8.40 | .469) | $-.731$ | -6i1 |
| $120 \mathrm{~mm} \mathrm{M1}$ | M6 | H1: | 3100 | 1.00 | 2.99 | 1.100 | .100 | 10 |
| (i-inch | M6; | T ${ }^{\text {P }}$ | 2800 | 1.20 | 3.28 | 1.202 | . 012 | . 1 |
| 155 mm M? | Mi; | HE | 2800 | . 36 | 1.57 | . 528 | . 168 | 4 |
| 159\%m M1 How. | M1 | HE | 1850 | . 058 | 3.07 | . 062 | (\%) 4 | 7 |
| 8 -inch MI | M6; | HE: | 2800 | 2.00 | 3.44 | 1.011 | -. 08 ! | $-+$ |
| $240 \mathrm{~mm} \mathrm{MI} \mathrm{How}$. | M1 | HE | 2300 | . 4 | 3.50 | . 382 | -. 058 | -13 |
| Sury |  |  |  |  |  |  |  |  |
| 3-inch/50 Mk? | - H |  | 2700 | . 15 | 1.77 | .279 | .129) | 86 |
| 5-inch/38 Mkl-1 | NC |  | 2600 | . 34 | 1.61 | . 408 | -. 158 | $-4 i$ |
| 5-inch 51 MkT心8 | N'0 |  | 3150 | . 86 | 3.84 | . 735 | -.125 | $-14$ |
| 5-inch/54 Mk16 | - ${ }^{\text {c }}$ |  | 2650 | . 51 | 2.87 | .58:3 | .073 | 14 |
| (6-inch/47 Mklf | - ${ }^{\text {c }}$ |  | 2500 | .83 | 2.25 | 1.212 | . 382 | 41 |
| 8 -inch/55 Mk1: | NC |  | 2500 | 1.27 | 2.60 | 1.605 | .335 | $2($ |
| 12-inch/50 Mk8 | $\cdots$ |  | 2500 | 2.21 | 2.58 | $\underline{2.815}$ | .605 | 27 |
| 14-inch/45 Mk12 | $\cdots$ |  | 2660 | 3.95 | 3.37 | 3.85 2 | -.098 | 2 |
| H-inch/50 Mkll | NC |  | 2700 | 3.55 | 3.55 | 3.287 | -.263 | -7 |
| 16-inch/45 Mk; | Y( |  | 2300 | 3.25 | 3.12 | 3.424 | .17.4 | i) |
| 1G-inch/50 Mk7 | NC |  | 2500 | 5.13 | 3.84 | 4.390 | -.740 | -14 |



FIGCRE: 3-9. Chan!e in l'ertical Land Diameler al 0.1 inch from ('ommenrement of Rifling versus
Number of liounds in the S-inch Gun, .IFI.
per round as calculated using this average value is presented in Table 3-4 along with other pertinent data and compared with the measured values. It is shown that the theory correlates the data with the interior ballistics fairly well over a wide range of different guns and ammmition. The measured values themselves are subject to considerable uncortainty, because they vary for different tubes of the same model and with the amount of use the particular tube has undergone (Cl. Figures $3-2$ and $3-3$ ). The agrecment is also, at least in part, a reflection of the fact that the gons studied are approximately scale models of each other. The crosion rate depends on minot differences in the guns such as the design of the forcing cone and the rifling which are not
taken account of in the theory. Some of the scatter may, therefore, be due to such factors.

The theory applies only to crosion at the origin of rifling since the specification of $A$ by Equation $3-57$ is not possible at other locations.

The evaluation of If from Equation 3 -is is time consuming. By making a number of quite drastic approximations, Breithart ${ }^{11}$ recast the procedure in analytical form and derived a simple algebraic expression for $I^{\prime}$ which yiolds results in almost as good agreement with observed values as bepuation :3-i8, namely:

 (Data from U. S. .Varal W'eapons Lahoratnry, Dah!gren, V'iryinia)
where $\triangle$ is dimensionless and is equal to $27.08 C_{i} C_{c h}^{\circ}$. Numerically it corresponds to the density of loadins in gm 'ce. $X_{m}$ is the travel to the muzale (in), $S$ the expansion ratio and $I_{\text {'. }}$ the maximum pressure (psi).

Sreitburt ewaluated the empirical constant, $K$, by fitting the formula to the measined wear per round at the commencement of rifling in the 8 -inch gun 311. If If represents the increase in the diancter per round in inches and the units of the other quantities are as indieated above, $K$ has the value $4.29 \times 10^{-2}$.

The fomula as written applies to standard gums. Breithart showed that it can be applied to howitzers by introducing an empirical correction factor equal
 howitzers

$$
\begin{equation*}
W^{\prime}=\frac{K^{\prime}\left(A^{2} A^{2}\right.}{S^{2} \|^{-i}}\left[\frac{P^{2}-16000^{2}}{P_{m}^{3}}\right] \tag{3-5,-9a}
\end{equation*}
$$

where (' is the charge, $d$ the caliber and $k^{\prime \prime}=$ $K+85=8.81 \times 10^{-3}$.

## 3-3.3 Life of Gun Tubes

a. E'stimation of trith Life. P'aragraph 3-3.2 gives formulas for the rate of vear of gun tubes, derived by Jones and Breitbart. Eventually, a tube wears so much that it camot be used, either because the mazale velocity is so low that it camot be properly allowed for in firing, or because the spin of the projectile is too low to stabilize it properly. The insta!ility of the projectile can usually be traced to shearing of the engraved part of the rotating band while it is still in the tube. Examination of the bands of reensered projectiles will usually inslicate the expected remaining accuracy life.

After studying data pertaining to camon from 37 mm to 203 mm ( 8 -inch) caliber, Jones and Breitbart found that the useful life can be correlated rather well with the wear at the commenement of rifling.'" The commencement of rifing is defined as the point at which the full height of the land is first encountered, in contrast to the origin of riffing, which is the point at which the land starts to rise. At the end of the useful life of these guns, the wear at the commencement of rifling was between 3.j and is pereent of the original diameter between the lands. Therefore, a wear of is percent of the original bore diameter may be taken as the tolerable limit and having calculated the rate of wear, the life of the gun can be estimated.
b. Equiralent Full Charge Factors. In order to estimate the uscful life of guns that fire different kinds of projectiles with different charges, it is
necessary to determine a factor that represents the relative crosiveness of the ammunition. Such a factor is called the equivalent full charge ( $E F C$ ) factor.
. Pt er studying all available data, Riel found an empirical formula that satisfies the experimental results quite well. ${ }^{13}$ This; formula is

$$
\begin{equation*}
E F^{\prime}=\left(I^{\prime} P_{1}\right)^{\prime \prime}\left(C^{\prime}\left(C_{1}\right)^{\prime \prime}\left(I_{;} \Gamma_{1}\right)\left(L_{1}^{\prime} E_{1}\right)\right. \tag{3-60}
\end{equation*}
$$

where

> EFC is the equivalent full charge factor
> $l^{\prime}$ the maximum chamber pressure
> $C^{\prime}$ the weight of propellant
> $V^{\prime}$ the muzzle velocity
> $E^{\prime}$ the specifie energy of the propellant
and the subseript 1 denotes the value pertaining to the ammunition for which the $E F C$ is chosen to be unity. Of couse, the data for both types of ammunition must be expressed in the same units; if two types of propellant are used in a mixed charge, their average specific energies may be substituted for $E$ :

Riel has tabulated the estimated life and $E F C^{C}$ for most of the present artillery ammunition. ${ }^{14}$

## 3-3.4 Erosion in Vents

Many experiments, going back over many years have been conducted to study the crosion of materials by propellant gases. These experiments have been conducted to study the basic processes involved as well as to develop materials more resistant to gas erosion. The technique most often used is to bun the propellant in a combustion chamber and allow the gases to flow out through a nozzle or vent and to study the effect upon the nozzle surface. There is an extensive literature on the subject some of which will be reviewed here.
$a$. Greaves, Abram and Rees used three chambers of different volumes and tested several different propellants with different adiabatic flame temperatures. ${ }^{15}$ They measured only peak pressures, using copper crusher gages. They rated their materials on a relative scale of "erodability": the slope of the linear part of the graph of weight loss versus peak pressure.

They established a strong dependence of weight loss on the "calorific value" of the propellant. In their experiments, for a given maximum pressure, the weight loss per unit charge is independent of the chamber volume. Their data confirm the expected result that the weight loss tends to go up as the melting point of the vent material goes down. In the case of steels, the milder the steel, the less


FI(iCRE: 3-4. The General Shape of Vent Erosion versus Charge
the crosion. They conchude that the principal factor of crosion in both guns and vents is generally the heating of the stuface with its consequent melting. They arr convineed that direct chemical reaction of the gases with the surface material plays a minor role, if any, under the conditions of their experiments.

1. Evans, Hom, Shapiro and Wagner studied the crosion by the gases produced by the explosion of carbon monoxide and oxygen. ${ }^{11}$ " Their apparatus was fitted with a blowout seal. Several blowont pressures and chamber volumes were used. The weight loss of the rents was measured and correlated with the number of moles of the product gases, the explosion temperat.re, and the ratio of CO to $\mathrm{CO}_{2}$ in the product.

For an equal number of moles, the weight loss increased with the gas temperature, but in a nonlinear manner: the plots were concave upwards with a curvature that was quite sharp initially, but


1. Fintrance ladius Tangent to Bxit Cone.
2. Inmer Surfare Mast be Smooth and Free of Tool Matis. 3. All Toleranees $\pm .001$ Dxerpt as Noted, Seale $2 / 1$.

FIGCRE: 3-i. Early Design of Erovion Tret lient





A-C/D C'onstant
B- r' C'onstant


FIC:LRL: :3-S. Diameter Increase versus Number of Rounds
Nozzle Type: E1
Material: AR .1/(') Iron
Charge: io G.MS. of M: I'ropellant

$\times$ Nozzle Diameter 0.500" -Total Throat .1 rena $=.1!83^{\prime \prime} i n^{2}$
( . os: les represented by $\odot$, were fired together and were subjected to the same pressure-time gas flow as the single nozzle represented by $X$.)
decreased almost to linearity. At a given calculated temperature, the weight loss increased rapidly as the ratio of CO to CO , decreased. The addition of small amounts of sulfur, nitrogen, and hydrogenbearing compounds to the gas mixtures as well as hydrogen itself usually caused a large increase in erosion. This was interpreted as a catalytic effect upon the reaction of CO with the iron of the steel vent to form iron pentacarbonyl. Evans et al saggested that two fundamental phenomena underlie the erosion of the vents: at low temperatures and large $\mathrm{CO} / \mathrm{CO}$, ratios, material is removed by convetting iron to volatile iron carbonyl; as the tempperature increases, direct melting of the surface sets in and increases until it becomes predominant.
c. The Ballistic Research Laboratories have conducted several experiments on vent erosion. Wis-
gand ${ }^{17}$ found that their carly data and also those of Greaves, Abram and hes produced better correlation when the weight loss was plotted against charge weight rather than against peak pressure. A representative curve is shown in ligure 3-4. This indicates a region of low severity, where the removal of material is related to some chemical reaction of one or more constituents of the gas with the material of the channel wall, followed by a region of high severity, wherein the material is removed predominantly by melting of the surface. In the latter region, the points can be fitted closely with a straight line, whose intercept with the axis of abscissas roughly divides the two regions. The slope of the curve at any point depends upon the experimental arrangement; that is, upon chamber volume, vent diameter, material and shape of the
but, typeand granulation of the propellant, method of ignition, and any other factor that affects the rate of heat transfer.
The apparatus consists of the breech and chamber of a 3 3mm gun with the barrel cut off just before the foreing cone, and an adapter to hold the vent in plare. In the early experiments ${ }^{14}$ vents like that of ligure $: 3-\pi$, with various throat diameters and an repanding cone beyond the thioat, were used. It was found that the crosion varied along the vent s) as to rhange its shape. The result was that the wosion per round was not constant but depended an the roume mumber. That is, the plot of the
integrated erosion, as measured by total weight loss, against round number was not a straight line and no valid crosion rate per round could be determined. To circumvent this difficulty the shape of the vent was changed to conform to Figure $3-6$ and 'also steps were taken to adjust the charge to maintain constant maximum chamber pressure. It was found that this could be done over a considerable change in vent diameter by simply keeping the ratio of charge to vent diameter constant. This would ensure that the characteristics of the gas flow were nearly constant from round to round. The type of data obtained is illustrated is. Figere 3-7. The results


Nozzle T!ype: E'1
Material-A R.1/CO Iron
('haryr: io G.MS. of . 12 I'ropellant


(.ios:le: represented by $\odot$, were fired together aml were subjected to the sumir prexsurr-time !us forr as the single nozele rupressented by $X$.)



FIGIDE 3-10. If eight Lows and Diameter Increase versus Number of Rounds.
Nozzle Type -E1
Material-Gun Steel ( $: 11: 10)$
Charge-ss G. ISS. of ML Propellant
Maximum Chamber Pressure-22,600 psi.
© $\triangle$. Nozzle . .o. J41-3. Diam. $=0.411^{\prime \prime}$ Throat Area $=0.136 \%^{\circ} \mathrm{in}^{2}$ - $\triangle$ Nozzle No. J2!-3. Diam. $=0.2866^{\prime \prime}$ Throat Area $=0.009 .5 \mathrm{in}^{2}$
also show that when these procedures are used, the weight loss per round is proportional to the diameter increase per round so that either may be used as a measure of erosion.

Figure : $3-8$ shows the increase in throat diameter versus round number for three different vents of the later type. The two small ones were fired simultaneously, using a manifold attached to the chamber. Their combined throat area was approximately the same as that of the larger one, which was fired alone from the same chamber with the
same charge so as to produce the same flow conditons. With all vents tested, the rate of diametral increase was independent of the diameter.

Figure 3-9 shows the weight loss versus round number for the same series of firings. The fact that the weight losses of the two small vents are nearly the same proves the reproducibility of the data. With vents of different sizes, the ratio of the weight losses is larger than the ratio of diameters ligure $3-10$. Since the weight loss is a measure of the integrated erosion over the entire imper surface of


FIGCRE 3-11. Dependence of Erosion on Initial Wrall T'emperature
the vent, its relatively greater variation indicates that, at upsteam locations, the thickness croded in the larger vent increases relative to that in the smaller one. This may be partly ?aused by the fact that, as one proceeds upstream, the relative increase in cross section is smaller for the larger vent, so that the gas velocity will not decrease as rapidly from its sonic velocity at the exit and consequently will be higher in the larger vent than at the corresponding location in the smaller vent. Experiments indicate that the erosion rate is very sensitive to the gas velocity and increases rapidly with it. ${ }^{18}$

The temperature of the vent before firing also affects the rate of crosion. To determine the tem-
perature, the thermocouple was imbedded in the vent so that the junction was about $\frac{1}{16}$ th inch from the surface. The vent was well insulated from the mount and heated by means of an electiical resistance heater inserted into the opening. The heater was withdrawn immediately before firing. The results for a 70 -gram charge of M 2 propellant are shown in Figure 3-11.

It should be noted that, in both vents and guns, the rate of crosion is affected by the roughness of the surface.'" With a given propellant, a particular combination of the gas velocity and heat transfer rate can be duplicated only under conditions where the nature of the surface roughess is similar.

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## CHAPTER 4

## LIST OF SYMBOLS

. 1 Area of the bore
.1. Area of the piezoelectric gage piston
a Acecleration of projectile or piston
(c) Ballistic co.fficient

1) Distance between sky screcens
$f$ Borc friction
. $/$ Mass of projectile
II Mass of piston
I) Breech pressure
$I_{i}$ Base pressure
$P_{v} \quad$ (iage pressure
$l$ Time of passage of projectile over measured distance, $D$
I. Distance from the muzale to the midpoint of sky screens
\% Reduced distance: $X / C$
$\delta \quad$ lhase difference
$l$ Difference in distance traveled by two beams
$\lambda \quad$ Wave length of radiation

## CHAPTER 4

## EXPERIMENTAL METHODS

## 4-1 INTRODUCTION

The preeding chapters have been, in the main, roneroned with the theoretical aspects of interior hallisties. From the carliest days of the use of firearms, attempts have bern made to measure ballistic quantities. Because the quantities to be measured have values outside the range normally experienced in other fields, the apparathes neerssary to measure them is rather sperialized. Because the values are extreme, exist for very short times and vary with great rapidity, they are diflicult to measure by mechanical mems. The temdency in recent times is to depend more and more on sophisticated electrical and optieal devieres. Piring ranges are often claborately instrumented, and for work in the field much of the apparatus is installed permancutly in vehieles which ean be tramsonted to the site.

A great deal of interior ballistic measurement. that related to rontine testing of gums and ammmition and also a considerable amount of development work, does not repuite more than the measurement of the maximum pressure and the muzale velocity, but these must often be made repeatedly to derive a statistical result. Simple deviees which will make these measurements quickly, without refuiring instrumentation attached to the gun, are essentials. Experimental work of this kind has been called by (omer "practical ballisties."

Research in the subject and more sophistirated development procedurs, however, require a nowe daborate instrmuntation. The maximum pressure and muzzle velocity are the resilts of complex phemomena going on inside the tube which the ballistician would like to rolate to the theoretical approach. To make measurements within the thickwalled tube often requures that openings be made in the tube wall to accommodate the measuring deviecs. This makes the tube useless for any other serviec and such instrunentation camot be used for routine testing. Some measurements can be made by apparatus designed to look down the tube from the muzzle and others by placing instruments in the projectile and bringing the signal out on wires or by telemetering; that is, by modulating an electromagnetie wave with the signal from the device in
the projectile and transmitting the modulated wave to an external detecting apparatus.

In recent yours there has been a rapid development. of apparatus sultable for interior ballistic moasurement and much of it is available commereially. The ballistician uereds only to fit it to his special problems. Examples of such apparatus are high spered motion picture cameras, rotating drum and mirror cameras, cathode ray oseilloseopes, electronic chronographs, ete. It will be assumed that the reader is familiar in general with most standard deviees which are in wide use and generally available.

Details of experimental studies of the interion ballistics of eertain guns covering a great many of the measurable parameters have been publishel. ${ }^{\text {. }}$ Since these gemeral experiments were done, developmont has continued to improve apparatus and mothods and to develop devices to measure directly quantities previonsly not possible to measure: such as for example, the motion of the propellant grains. In what follows, measuming deviees and associated apparatus will be deseribed mainly in prinepipe. There are usually several models extant of the different momaning deviees and the associated epuipmont and procedners vary at different placess and at different times. The referenees should be eonsulted for details and for further refernees to pertinent literature. It should be pointed out also that rapid development of instrtmentation of all sorts is taking place and the tendency is teward more and more antomation of mothods and proecdures.

## 4-2 PRESSURE MEASUREMENTS

## 4-2.1 General Principles

Pressure gakes are of two classes, (a) those which measure maximum pressure only, called erusher gages, and (b) those which measure the pressure as a function of time. (iages of class (h) are of two types, those which make use of the mechanical strain produced by the pressure, and those which depend on the piegoclectric effect. The earliest gages of class (b) were of the mechanical type. The strain cement was usually a diaphragon subject to the gas pressure on the inside, and having the outside


FIGURL: 4-1. Quartz Piezoelectric Pressure Gate.
roupled mechanically to a small mirror to form an optical fever to defiect a spot of light onto a moving film. These gages are quite accurate and reliable but camot be used on guns because of the recoil. They have been much used on closed chambers. For use of: guns, strain type gages have a esistance strain wite conpled to the strain element so that the strain appears as an electrical signal which can the displayed on a cathode ray oscilloscope.
The pie\%ollectric gages depend on the fact that ertain erystals develop a surface charge when sub)jected to an external pressure. Quartz erystals have been used most frequently. Gages have also been made using toumaline. Tommaline has the adrantage that it responds to hydrostatic pressures so that the erystal need only be immersed in a medium such as a grease to protect it from the hot gases, and the pressure applied directly to the surface of the grease. The response of guartz, on the other hand, depends on the direction of the stress with respect to the crystal structure. For optimum effect, the erystal plates must be eut with their faces properly oriented with respect to the erystal axes
and the forees must be applied to the crystal by means of a piston or anvil. Tourmaline crystals of sufficiently high quality are not readily available, however, and, as they are considerably more fragile than quartz, they have been infrequently used in the linited States. Quartz is readily available and has the highest breaking stress of all the more commonly used piezoelectric erystals. The use of the piston complicates the gage but it has the advantage that the range of the gage can be adjusted by varying the ratio of the piston area to the area of the erystal plates.

## 4-2.2 The Quartz Piezoelectric Gage

The piezoelectric element of one model of this gage is a stack of $X$ cut* erystal plates in the form of dises. The plates are stacked so that contiguous faces will generate charges of the same sign when pressure is applied. Metal foil charge-collectors are

[^6]

FICillRE +2. Dinyram of Recorling System for l'iezalectric I'ressure Gaye.
placed between the piates and so comected that the total charge of each sign appears at the electrodes: at opposite ends of the stack. The cond surfaces are in contact with electrodes of hemispherical shape. This shape, which fits into corresponding sockets on the element through which the pressure is applied. tends to assure that the pressure will be uniformily distributed over the surface of the erystals to minimize the possibility of cracking. To further assure that the pressure will be uniformly distributed the surfaces of the crystal plates and metal parts in contact with them must be optically ground and lapped. The details of the construction and momenting of the gage element are shown in ligure $t-1$ for a model much used at Ballistic Rescarch Laboratorics.

The recording circuit and apparatus are illustrated in Figure $4-2$. The charge developed by the gage is shared with a ballast capacitor in parallel with the gage and the voltage developed across the gage and capacitor fed through a high gain direct current amplifier to a cathode ray oscilloscope. The cathode spot is then photographed by a ruming film or rotating drum camcra. . 1 time scale is simultancously placed on the record by photographing an intermittent light source. This can also be done by blocking the cathode spot intermittently to make breaks in the record. A typical record is shown in Figure $4-3$.

To determine the pressure from the record recquires that the gage and recording apparatus be calibrated. The gage is calibrated separately in the laboratory because it is not practicable to calibrate the gage when mounted in the gun. This is done using a dead weight hydraulic pressure apparatus, Figure t-i. The gage is mounted in a hydraulic chamber provided with a piston which is attached through a mechanical linkage to a scale platform carrying a series of weights. Oil is then pumped into the hydranlic chamber until the weights are lifted and
the perssure in the chamber determined from the piston area and the weight lifted. The hydraulic chamber is provided with a quick release valve which releases the pressure very rapidly. To make a calibration, the charge developed by the gage under. pressure is first removed by shorting the gage and then the pressure is suddenly released. An equal charge of opposite sign is generated by the gage. This charge is immediately sent through a ballistic galvanometer and its magnitude determined from the deflection of the galvanometer. The recording apparatus is calibrated by applying a known charge across the ballast capacitor and observing the deflection of the cathode ray spot.

Quartz piezo gages are rugged and tend to hold their calibration well. By varying the number of plates in the stack the sensitivity can be adjusted. The crystals will not withstand stresses much above 1.0000 psi but the working stress on the erystals can be varied by adjusting the piston area. The piston size has a practical lower limit, below which it will deform and bind. This imposes an upper limit on the pressures that can be measured. Practical gages can be made to measure pressures between 10,000 and $70,000 \mathrm{psi}$. The gage has a high impedance and problems of grounding and shielding have to be dealt with, especially in field use where the leads from the gage to the recording equipment may be


FICiCRE: l-3. Typical Pressure-Time Recorls from Quart: Piesoolectric Gage (1:5mm Gun).
long, wen with mobile equipment. Piezoclectric gages are not suitable for measuring low pressures of long duration such as exist in rocket motors because the gage discharges too rapidly, mainly through the input impedance of the amplifier. Also, for use at low pressures, the gage would have to be
large to generate sufficient charge for case of measurement.

## 4-2.3 Strain Type Pressure Gages

The sensitive element in these gages is a short tube or ferrule closed at one end and so meunted as


FI(:URE 4-4. Drad Weight . 1 pparalus for C'alibration of Pressure Gapes.
to be subjected to the gas pressure either on the inside or outside surface. In the earlier models the pressure is exerted on the inner surface, the ferrule being filled with a hydraulic medium such as grease or oil. A strain wire is wound on the outside of the ferrule and cemented to the surface or applied in the form of a commercial strain patch cemented to the surface in the usual way. The strain wire or patch forms one arm of a Wheatstone bridge circuit. When the pressure is applied, the bridge is unbalanced; the degree of unbalance being a measure of the pressure. The emf developed across the bridge by the unbalance is fed to a suitable amplifier and then to a cathode ray oscilloscope. The deflection of the cathode spot is then photographed with a ruming film or drum camera.
One model in which the pressure is applied to the
outside of the ferrule has been referred to as a "hat" gage because of the shape of the ferrule. In this model the strain wire or patch is on the inner surface. The operation of the gage is the same as for the other models. Some typical models are shown in Figures $4-5,4-6,4-7$, and $4-8$, which are self-explanatory.

A typical recording circuit is shown in Figure 4-9. The circuit is calibrated by a suitable variable resistor in the gage arm of the bridge which establishes the relation between bridge output and resistance change in the gage. The gage itself is calibrated in the laboratory on the dead weight calibrator to establish the relation between the resistance change in the gage and the applied pressure.

Strain type gages can be used to measure lower pressures of long duration. The pressure range of the




FI(iURL: 4-6. An Improved Strain T'ype Pressure Gage Using Cemented Foil Sirain P'atches to Permit the U'se of a Smaller Ferrule to Reduce Dimensions of the Gage.


FIGERE: A-7. Internal Strain T'ype Pressure Gaye Mountra in C'artridye Case to Measure Breech I'ressure.


gage can be adjusted by design of the ferrule. They can also be made smaller than piezoclectric gages of the same pressure range and can be used in locations where space is restricted.

All pressure gages possess natural frequencies of oscillation. They will, therefore, overshoot and oscillate when subjected to rapid pressure changes. These oscillations will damp out more or less rapidly depending on the design of the gage and its mount. If too much damping is designed into the gage. however, it will be sluggish in response and will not aremately follow rapid pressure changes. Com-


WHEATSTONE 日RIDGE
FICillll: f-!. I'ipical Input Circuit of Strain Type Prossure Gayes.
promises must therefore be introduced. A high natural frequency is desirable for rapid response. The gage should be small and rigid. The hat gage shows up well in this respect. Figure $4-10$ shows the response of the different types of gages when subjected to an almost instantancous rise in pressure due to the impact on the gages of a shock wave generated in a high pressure shock tube.

For further information on pressure-time recording gages and their use the reader is referred to References $3,4,5$ and 6 . Reference 4 gives a brief review of pressure gage development at Ballistic Research Laboratories and is the source of the figures used in paragraphs $4-2.2$ and $4-2.3$.

## 4-2.4 Crusher Gage ${ }^{7}$

For routine proof firing and most developmental firing, the pressure-time relation is not needed, but the maximum pressure is desired. For this purpose, crusher gages are used. A crusher gage consists of a steel cap, a copper gas-check cap, and a stecl housing that contains a stecl piston and a copper or lead cylinder. For recording very low pressures, the lead cylinder is recquired. An illustration of the crusher gage is given by Figure $4-11$.
The copper gages are made in three sizes; their uses and the mean dimensions of their cylinders


FI(:ERRI: +-10. lrequency Response Curves for Different Types of Iressure Gafes when Subjected to a Stepuise Pressure Signal in a High Pressure Shock T'ube.
are tabulated below. The medium and major caliber gages use the same copper cylinders but have different size housings.

| Si\% | (imms | Diameter | Iength |
| :---: | :---: | :---: | :---: |
| Minor caliber | Small arms | .220 | . 400 |
| Medium malior | Simall and |  |  |
|  | medium cannon | .2525 | . 500 |
| Major caliker | Major c:mmon | . 25.25 | .500) |

For work on small arms, one minor caliber gage is inserted in the wall of a gun that is set aside for this purpose. For medium calibers, two medium caliber gages are placed in the chamber. Two major caliber gages are placed in depressions on the inner side of the breechblock for recording pressures in separate loading, major caliber, weapons.

The propellant gas pressure is exerted against the gas-check cap and transmitted to the piston, which compresses the cylinder against the cap. The length of the cylinder is measured in ten thousandths of an inch with micrometer calipers, both before and after firing. A table, which relates the compression to the static pressure that produces it, is based on values obtained by subjecting representative samples of eylinders from a lot to various pressures in a hydraulic press for 1.5 seconds. The pressures
taken from the table are recorded and corrected, since they are less than the dynamic pressures corresponding to the same compression. Comparisons with piezoclectric gage pressures show that copper crusher gage pressures, when determined from copper crushers calibrated statically, should be multiplied by 1.20 to obtain the true chamber pressure, which is used in interior ballistic calculations. For the lead crusher gages, used for low pressures, the static pressure obtained is less than half the dynamic pressure.

Recent developments in dynamic calibration techniques have made it possible to reduce the difference between the maximum pressure as determined by a crusher gage and by a piezoelectric or strain type gage. The crusher elements are calibrated under pressures applied at rates approximating those occurring in guns. These techniques have been developed to the point where, for the larger caliber guns, crusher gages can be made to yield maximum pressures equivalent to those given by the pressure time measuring gages.

## 4-3 MEASUREMENT OF MUZZLE VELOCITY

## 4-3.1 General Principles

Before the invention of chronographs capable of recording very short iniervals of time and of cameras capable of photographing projectiles in flight, there
was no way of making direct measurements of muzzle velocity by the simple method of measuring the time it took a projectile to turerse a previously measured distance or the distance traversed in a previonsly determined time.

After the invention of modern chronographs, muzze velocity determination no longer posed a difficulty. In the method now commonly used, the projectile is timed over a measured distance by recording its passage as it enters and leaves the measured course. A device which determines the time between these two ceents is called a chronograph.

In the other method, the projectile is photographed against a distance seale by means of a high speed motion picture camera whose frame rate is known or by taking two or more photographs of the projectile against a distance scale using a serics of fixed film cameras and a set of flash lamps with fixed time delays between them. Photographic methods are usually used when observations on the behavior of the projectile are desired, as well as determining its velocity. They cannot be classed as standard methods of muzzle velocity measure-
ment. A discussion of these methorls and the details of some applications are given in Reference 8.

## 4-3.2 Chronographs

There are two types of chronographs in general use; those which display the time interval directly, known as counter chronographs, and those which record the passage of the projectile as it enters and leaves the measured distance, known as camera chronographs. The former type makes use of electronic cycle counters which are started and stopped by the passage of the projectile and arranged to display the clapsed time on an indicating panel which is read directly. In the camera type of chronographs the output of the detecting device is fed to an oscilloscope and the face of the scope photographed with a drum or ruming film camera. The elapsed time, therefore, is determined from the distance on the film between the indications of the passage of the projectile by the detecting devices. This reduires that a time scale be simultancously recorded on the film. ligure $4-12$ is a photograph of a standard model of camern chronograph using a drum camera.





The camera chronograph provides a permanent photographie record, which can be checked at any time: but about 20 minutes is reguired to develop, fix, and dey the film before it can be read. With the counter chronograph, about one minute is required to compute the velocity and report it : but the reading is erased after the time interval is noted. A permanent ereord can be provided by photographing the display pancl or the process can be simplified be hating the elapsed time printed out automatically.
some of the carlier models of counter chronographs were rather eritical in operation and difficulty arose berause they could be activated by noise on the limes. In newer models, the operating conditions are less eritical and these difficulties have been largely diminated. The present tendency is to increasing use of the eounter chronograph because of its speed and portability. For field wese it has the great adrantage that it does not refuire photographic processing facilities.

## 4-3.3 Detecting Devices

Commonly used deviees to detect the passage of the projectile are of two types. One depends on the inductive effect as the projectile, which has been previously magnetized, passes through a coil of wire, and the other by the variation in the intensity of the light falling on a photoelectric cell caused by the passage of the projectile. The coil is the simpler devies." It requires little care and ran be used in the open without special protection or attention. Two coils can be casily monnted at opposite ends of a frame and the assembly elevated on a tower at an angle so that the gun can be fired at its normal operating elevation. This is more difficult to do with the more elaborate and fragile photochectric devies. The coils, however, camot be used with nommagnetizable projectiles, such as small arms bulletes and the other deviees must be used. Nomuagnetizable, developmental projectiles used in interior ballistic research have been provided with small imbedded permanent magnets to permit the use of coil detectors.

The design of the coil detector is important." As the projectile approaches the coil the inereasing magnetic flux through the coil induces an emf in it which increases and reaches a maximum value when the projectile is in such a position that the rate of change of flux is a maximum. The emf the in begins to decline as the rate of change of flux declines and becomes equal to zero, at which time the emf is zero. The flux then temds to decrease as the projectile starts to leave the eoil and the emf reverses

sign and follows a course similar to the approach but reversed in direction. The coil should be so designed that the signal reversal is sharp so that the record crosses the axis steeply. The film distance between the crossing points is then accurately detemined and serves to determine the time accurately. Figure $4-13$ shows a drum camera chronograph record from coil detectors.

There are two types of photoclectric detectors, lumiline sereens and sky screens. Lumiline sereens use a light source incorporated in the apparatus and sky sereens use the light from the sky. In the latter the photocell is screened so that it is illuminated by the light from a narrow region of the sliy transuerse to the trajectory. When the projectile crosses this region of the sky, the illumination of the cell is reduced sufficiently to induce a voltage change in the cell circuit which defleets a cathode ray oscilloscope and the deflection is photographed on a moving film camera or the signal can start or stop a counter chronograph. Sky screens have been developed for fiold use and are fully deseribed together with the associated comiter chronograph in Reference 10.
The light sourec in the lumiline sereen is a long filament electric lamp mounted in a metal frame behind a narrow slit transverse to the trajectory. A photocell is mounted behind another slit on the opposite side of the frame. A projectile passing through the frame reduces the illumination on the cell in a manner similar to that of the sky sereen. Lumiline sereens are used mainly in indoor ranges for moasuring the velocity of small caliber projectiles such as bullets from small arms. Screens in use are shown in Figure t-14.

## 4-3.4 The Calculation of the Muzzle Velocity

Once the time of passage, $t$, of the projectile over the measured distance, $D$, is known, the ratio, $D$. $t$, gives the average velocity over the distance, $D$. This average volocity will ocew at the midpoint of $D$ provided the aceeleration of the projectile is constant. The latter romdition will hold if the air
resistance to the motion of the projectile does not change appreciably over the distance $D$. This is closely the case so that the velocity furnished by the instrumentation can be assumed to occur at the midpoint of $D$.

On account of the air resistance, the velocity
decreases between the muzzle and the midpoint of the detecting devices. The small correction required to give the velocity at the muzzle is proportional to the ratio

$$
\begin{equation*}
Z=X / C \tag{t-1}
\end{equation*}
$$



FIGURE 4-14. Lnmiline Srreens in Jse in an Indoor Ranye.
where
$X$ is the distance from the muzzle to the midpoint and
(! is the ballistic coofficient
$C$ is a function of the weight, caliber and shape of the projectile, the density of the atmosphere, and the range wind. The correction factor, which depends on the velocity and the drag function, has been tabulated by the Instrument Laboratory of the Development and Proof Services at Aberdeen Proving (iround. ${ }^{\text {'". }}$ "

For a short distance in front of the muzzle, the projectile is surrounded by propellant gas. Since the muzzle blast increases the projectile velocity, another corvection must br subtracted from the apparent mazale velocity to obtain the true one. From the measumements of the spin of several projectiles by a radiosonde, extrapolated to the muzzle, Hitchcock found that the increase in velocity due to the muzzle blast is about 1.2 pereent of the apparent muzzle volocity for grus having a nomal expansion ratio."

## 44 TRAVEL-TIME MEASUREMENTS

## 4-4.1 Barrel Contacts

The position of the projectile in the bared as a function of tinn cail be determined by inserting insulated probes through holes bored in the barrel wall which make contact with the projectile as it passes. liach probe is part of an electrical circuit which is eompleted through the projectile and the barrel. The projectile acts as a switch which closes the rirenits momentarily. The current in the circuits can be detected and displayed on a cathode ray oscilloseope and the face of the oscilloscope photographed on a moving film or drum camera in the usual way and related to a time scale on the film. The aro of time is usually indicated on the film by detecting the initiation of the primer or by a contact at the muzzle.

Bancel contacts have to be designed and used with care if the signal is to occur when the projectile has the same position with respect to the contact at each location. This can be assured in part by machining a noteh in the forward part of the projectile. The shape of the noteh is such that a vertical surface is presented to the contact. If care is taken to insert the contacts only in the rifling grooves of the gun, they will then establish contact with the rotating band only.' For smooth bore weapons and when firing jacketed bullets, the noteh in the projectile beomers neressary. ${ }^{2}$ If the electrical poten-
tial of the contacts is too high the current may start before mechanical contact is made. This is especially likely if the gas ahead of the projectile becomes ionized. This can happen if hot gas leaks past the projectile or if the air in the tube is compressed by the motion of the projectile itself.

Barrel contacts are extensively used in interior ballistic research, the exact form varying with the user, often as check points against other methods of measuring projectilo displacement. ${ }^{\text {: }}$

When it is desired to derive from the measurements an accurate travel-time relation, a rather large number of contacts must be used. This involves considerable damage to the barrel thus making it useless for other purposes. Considerable time and expense is involved in doing the necessary machine work.

## 4-4.2 Microwave Interferometer

The mierowave interferometer is the microwave analog of the well known Michelson moving mirror interferometer which is used to measure small distances using light of optical wave lengths. In both instruments a beam of radiation is divided into two beams. One beam is sent to a reflector at a fixed distance and the other bean to a reflector which can be moved. The reflectors return the radiation to the point of separation where the beams are superimposed and combinc to form a beam of radiation whose intensity depends on the amplitudes of the reflected beams and their phase difference. The phase difference depends on the difference in the distance traveled by the two beams from the point of separation and back again according to the relation

$$
\begin{equation*}
\delta=\frac{2 \pi l}{\lambda} \tag{4-2}
\end{equation*}
$$

Where $\delta$ is the phase difference, $($ the difference in distance and $\lambda$ the wave length of the radiation. Now, if the movable reflector is displaced parallel to the direction of the reflected beam, $($ will be changed and $\delta$ will be different. This will change the intensity of the combined beam. If $\ell$ is changed continuously $\delta$ changes continuously and by one cycle ( $2 \pi$ radians) every time $\ell$ changes by one wave length. The intensity of the combined beam changes cyclically, going through one cycle every time $\delta$ does. Since $\ell$ changes by one wave length when the movable reflector moves one-half wave length, one cycle of intensity change indicates a displacement of the reflector of one-half the wave length of the radiation.


FIGURE 4-15. Typical Micronave Interferometer Record of Projectile Travel versis Tine (Caliber .50)

For the wave length range used in the microwave interferometer, the radiation is propagated in a wave guide. The source of the radiation is a Klystron oscillator. The radiation is divided at a "magic tee". One part is led by wave guide and directed down the barrel of the gun to the projectile which constitutes the moving reflector. The other part is led to the fixed reflector and in this arm of the circuit there is provided an attenuator and a phase shifter so that the intensity of the output signal can be adjusted to a convenient value initially and the phase difference adjusted to zero.

The combined output is detected by a crystal detector and fed to a cathode ray oscilloscope
through suitable amplifiers. The face of the oscilloscope is then photographed on moving film along with a time scale. A typical record is shown in Figure $4-15$ and a block diagram of the apparatus in Figure 4-16. For further details of the construction and use of the interferometer the reader is referred to References 14 and 15 .
The microwave interferometer has obvious advantages over the barrel contacts. It requires no modification of the tube and yields a continuous record of the travel of the projectiie so that a more accurate travel-time curve can be derived. The relative accuracy for the two methods is discussed in Reference 13 where it is shown that even with
amere nnocr-ar mom


FIC:IRI: A-16. Block Diagram of the Microurar Interfcromeler for Mensuring Projectile Wravei versus T'ime.





FIGUIRE 4-18. Foil Contactor Assembly for Measuring T'ravel During the Engraving Process, 105 mm Howitzer.
great care in the design and placement of the contacts they exhibited some erratic behavior although there was no systematic error in the results derived from their use. There were no systematic errors in the results ior either system, however.

## 4-4.3 Measurement of Projectile Travel Near the Start of Motion

Neither the barrel contacts nor the interferometer gives a sufficiently detailed and accurate account of the motion of the projectile at the start of motion and during engraving to permit entirely satisfactory conclusions to be drawn about engraving forces and starting pressures.

If extra tubes are available, for research purposes it is often permissible to cut the tube off so that the nose of the projectile is visible and then take a high speed motion picture of the projectile as it moves. Another method ${ }^{2}$ is to machine two slits on opposite sides of the tube so located that the nose of the projectile projects slightly beyond the breech end of the slits. The slits are then backlighted and an image of the illuminated slits formed on the film of a moving film camera so that the film runs perpendicular to the image. As the projectile moves the slits are progressively covered and the boundary between the illuminated and unilluminated part of the film is the locus of the travel-time curve, Figure 4-17. The tube must be cut off just beyond the end of the slits since the slits relieve the pressure when the projectile base passes and the projectile may stop in the tube.

If damage to the tube is not permissible other methods must be used. One of these which has been used for measurements on a $10: \mathrm{mmm}$ Howitzer is described in Reference 16. In this method, a rod is inserted down the barrel and fixed at the muzzle. At the other end, the rod holds a set of foil contactors inside the hollowed out projectile. An internal con-
tact is provided in the projectile, arranged so that the projectile movement causes the internal contact to touch the foil contactors in succession. The arrangement is shown in Figure 4-18. The rod and contactor assembly are insulated from the barred electrically and an electrical circuit is compleied through the projectile and the barrel when contact is made with the foils. After a travel of about four inches the base of the projectile comes in contact with the end of the rod and forces it out of the barrel. In the 105 mm Howitzer the rod comes out smoothly. It may not do this if used in higher velocity weapons where the accelerations are larger and the force on the rod greater.

The results of the measurements were differentiated twice to yield the acceleration, and from a simultaneous measure of the pressure the resistance to the motion is determined from the relation

$$
P A-f=M a
$$

where $f$ is the resistance to the motion. A typical result is shown in the consolidated plot, Figure 4-19.

## 4-5 IN-BORE VELOCITY AND ACCELERATION MEASUREMENT

### 45.1 Differentiation of the Travel-Time Data

The velocity and acceleration in the bore can be determined by differentiating the travel-time data provided by the interferometer or barrel contacts. To do this requires a complicated data reduction process and if the differentiations are to yield accurate results, not only must the records be read with great accuracy but the records must be very precise to begin with. In no case is the original data obtained from the records sufficiently accurate to be used directly. It must be first smoothed and the smoothed data differentiated. The result of the


first differentiation must also be smoothed before the second differentiation, and the final result will usually require some final smoothing. The amount of work required to reduce the data and make the calculations for a single round is considerable ${ }^{2,17}$ and the results may be subject to unknown errors. It is desirable to measure the velocity and acceleration directly.

## 4-5.2 The Measurement of Velocity

A standard method of measuring the velocity of brojectiles makes use of the Doppler effect; that is, the change in frequency which occurs when a radar beam is reflected from the projectile. There are a number of ways, differing in detail, which theoretically can be used to measure the Doppler shift.

One method is to use two stabilized Klystrons differing in frepuency by a fow megacycles. The tadiation from one is led down the barrel of the grom and reflected from the projectile. The second Klystron is provided with a contrel system which locks its frequency to the signal from the first after reflection, which serves as a reference signal. When the projectile is not moving, the control voltage required to lock the second Klystron to the radiation of the first is constant. Another method of providing
the reference signal, using only one Klystron, is to lead part of the signal from the Klystron out and shift its fiequency, thus taking the place of the signal from the second Klystron. if now the projectile starts to move, the reference signal will be increased in frequency and the control voltage will increase or decrease depending upon whether the difference in frequency between the two lilystrons is incressed or decreased. The change in the control voltage is proporticual to the frequency change and hence to the change in velocity. The control voltage is recorded as a measure of the velocity. It has been found that the controlled Klystron has a locking range of about 200 ke which corresponds to projectile velocities of about 3000 feet per second.

These methods for measuring the velocity directly simplify the determination of displacement and acceleration. Not only will the tedious smoothing and differentiating processes be reduced but the accuracy of the final results should be improved. The direct measurement of the velocity by the Doppler effect is simple in principle and from an accurate measurement, the displacement can be determined by integration, which is a more accurate process than differentiation, and the acceleration determined by a single differentiation which will climinate the errors due to the second differentiation.


FIGLRE: 4-20. Diagram of Quartz Piezoelectric .Icceleration Gage Assembled in the Projectile.

## 4-5.3 The Measurement of Acceleration

To measure the acceleration directly requires instrumentation in the projectile. The earliest acceleration gages for mounting in projectiles depended upon the pressure developed on a quortz crystal plate by the inertial reaction to the acceleration of a weight bearing against the crystial plate. One model is shown in Figure $4-20{ }^{14}$
To record the signal from the gage one must transmit the signal out of the barrel. This is done by leading a wire down the barrel and connecting it to one terminal of the gage, the circuit being


FI(:CIIL: $4-21$. Diagram of the Variable ('apacitaner .Iccelcration Gage.
completed by grounding the other terminal to the projectile wall which makes contact with the barrel through the rotating band. As the projectile moves the wire is gathered up in a cup-shaped receptacle at the forward end of the projectile.

Considerable difficulty is usually encountered with this system of measurement. It requires direct carrent operation and it is difficult to eliminate the noise generated by the sliding contact at the rotating band. In the process of gathering up the wire, it may shatter at the higher velocities which will generate noise as will leakage of ionized gases past the projectile.

In an attempt to circumvent these difficulties, apparatus has been developed ${ }^{10}$ using an acceleration gage depending on the change in capacitance when subjected to acceleration. The design of one model of the gage is shown in Figure $4-21$. When the projectile is accelerated, the body of the gage is slightly flattened which decreases the separation between the metal plated surfaces and hence increases the capacitance. In use the gage is part of an oscillator circuit. If the capacitance of the gage is changed the frequency of the oscillator is shifted; the frequency shift being a measure of the accelcration.
This scheme permits the use of alternating current


operation so that much of the unwanted noise can be blocked out of the recording circuits by proper filtering. It introduces other difficulties, however, in that the instrumencation in the projectile is much more complicated, and as it must function while mader large accelcration, the design requiements are very stringent. One is also never quite certain that the frecuency of the oscillator will be stable under the conditions of firing. Models have been used successfully in the $10 . \mathrm{mm}$. Howitzer where the conditions are not too drastic. There seems to be no difficulty in designing apparatus rugged enough to withstand high velocity gun accelerations. The question of stability will remain.
Trouble also arises with the uncertainty of the calibration of the capacitance gage. It is obviously not possible to subject the gage to a known acceleration of the required magnitude. Recourse has, therefore, been made to subjecting the gage to a mechanical force applied externally with a press. Under acceleration, however, the force system acting on the gage is not the same as that used in the calibration. The gage has a complex shape and any distortion due to acceleration cannot be predicted accurately by theory so that it could be allowed for. The piezoclectric crystal gages, discussed in paragraph $4-2.2$, could be calibrated much more con-
fisently as the effect of the acceleration in the erystal itself was considered negligible, the charge developed therefore being entirely due to the inertial pressure of the weight.

### 4.6 THE MEASUREMENT OF BASE PRESSURE

The methods deseribed to measure accelderation can be used to measure base pressure. ${ }^{\text {ix }}$. All that is needed is to modify the gages and their installation in the projectile to withstand and be subjected to the pressure of the gases and to make the effects of the acceleration negligible with respect to those due to the gas pressure. The design of a piezoclectric hase pressure gage is shown in Figure 4 -22. It is similar to the acceleration gage except that the erystal is now compressed by a piston subject to the gas pressure through an opening in the base of the projectile. Because the piston also has inertia, the pressure on the gage, $l_{0}$, is not equa! to the base pressure, $l^{\prime}$, but is given by

$$
\begin{equation*}
P_{1}-\frac{m a}{I_{1}}=P_{\nu} \tag{4-4}
\end{equation*}
$$

where $m$ is the mass of the piston and $A_{1}$ its area.
A variable capacitance base pressure gage dosigned for alternating current operation in a mamer


similar to the variable capacitance acceleration gage is illustrated in ligure $4-23$. It does not make use of a piston; the gas pressure being applied directly to the outside of the pressure element which serves also as the outer electrode. The pressure reduces the imer diameter of the pressure element and hence reduces the clearance from the inner electrode. This increases the capacitance of the gage.

## 47 THE MEASUREMENT OF BORE FRICTYON

If the acceleration and the base pressure are known one can derive the bore friction, $f$, from the relation,

$$
\begin{equation*}
f=P_{l, A} 1-M a \tag{4-i}
\end{equation*}
$$

where $I_{b}$ is the base pressure, $a$ the acceleration, I/ the mass of the projectile, and $A$ the bore area.


FIt;UIRE 4-24. Recort Produced by the Aulomatic-Recording Bore Gaye. Three Complete Scans in Both Directions to Test Repeatabilit!.



In cxamination of Equations $4-4$ and $4-5$ shows that if the mass of the piston, $m$, is so chosen that m $A_{p}=M A$, then $f=I_{v} A$ so that under these circumstances the output of the gage is proportional to the bore friction. By use of this property, a base pressure gage can be designed to yield a simultancous measure of the bore friction.

## 4-8 THE MEASUREMENT OF BARREL EROSION

## 4-8.1 General

The standard deviees used to measure changes, in brere diancter are the star gage and the pullover gage. They are both hiechanical micrometers designed experially to measure both the diameter of the tube at a certain location and the distance to the point of measwement. The measuring head is attached to a loug staff. The staff serves to manipulate the head to make the diamster measurement and is also provided with a scale to measure the distanee to the point of measurement. Both gages are made in a number of sizes for use with the vari hes calibers. The details of their construction and use are given in Reference 20.

## 4-8.2 The Star Gage

There are several types of star gages but the principal ones are the levar and small bore gage; used for large and small calibers. They do not differ in principle, the difference being only in the mechanism used to manipulate the measuring head. In both types, the head is provided with contactors which are forced out radially to make contact with the surface of the bore by advancing a cone shaped piece upon whieh the immer ends of the contactors ride. When the cone is retracted the contactors disengage from the sulface. The position of the cone when eontart is made with the bore surface is an
indication of the diameter, and is read on a scale at the operating end of the staff.

## 4-8.3 The Pullover Gage

This gage functions in a manner similar to a telescoping inside micrometer. The head which is constructed so that it will telescope and retain its minimum size is initially set larger than the bore diameter and inserted with its staff into the barrel to the distance at which the measurement is to be made. This requires that it be set at an angle with respect to the staff. It is then "pulled over" which forees it to telescope until its length is equal to the imer diameter of the tube. A vernier scale is provided on the head so that the diameter can be read off when the gage is withdrawn.


FIG:DRF; $4-26$. Diayram of Bore Surfare Thremomouple and Housin!!, Blil, Model.


Fl(il'RE: + -27. Block Diagram of i pparatus for Ohserving Molion Durin! Firing of
a Radioactive Source Imbedded Inilially in a Propellant Grain. a Radioactive Source Imbedded Initially in a Propellant Grain.

## 4-8.4 The Automatic Recording Bore Gage

Bore gages have been constructed which depend on the strain produced at the surface of a cantilever beam. The beam is mounted in a frame which fits into the barrel and carries one or more contactors which bear on the bore surface. When the frame is forced into the bore, the beam is bent from its initial position by the force on the contactors. The strain produced in the beam is recorded by a system of two strain gages cemented to opposite surfaces of the beam and forming two arms of a Wheatstone bridge so arzanged as to maximize the bridge output. When calibrated by applying known deflections to the beam the imbalance of the bridge is a measure of the bore diameter.
A gage of this type was developed ${ }^{21}$ to record automatically a continuous measurement of the bore diameter (or radius, depending on the model) when it was pulled through the barrel by a motor driven mechanism. The gage is provided with supports which engage the rifling so that the contact stays on the same land throughout the travel. The output of the gage is fed to an automatic recorder which traces on a moving paper a record of the bore diameter as a function of distance. A calibration is
placed on the record by pulling the gage through a calibration tube provided with stepwise diameter changes or by a micrometer calibrator which can be varied continuously by a micrometer serew.

These gages have been found to be consistent in operation and to give reproducible results. The records can be read to .000 inch and the precision is about of this order. By reducing the radius of curvature of the tip of the contactor they can be made to record the major roughnesses of the surface and yield knowledge of the condition of the surface. The type of record produced is shown in Figure 4-2t and a diagram of one model of the gage in Figure 4-2.).

## 4-9 BARREL TEMPERATURE MEASUREMENTS

## 4-9.1 Thermocouples

A variety of themocouples have been used for barrel temperature measurement. On the outside surface where the temperature variations are low, no special difficulty is encountered; all that is neeessary is a good bond to the surface which can be obtained by soldering or welding.
To make mensurements within the wall, holes most
be bored to the desired depth and an insulated thermocouple wire inserted to make good thermal and electrical contact with the bottom of the hole. This usually requires weldirg although mechanical pressure can be made to work. Such contact may be unreliable, however, in a gun during firing. Electric welding of the contact requires care. If too much metal is melted, the exact location of the contact is uncertain. Just enough and no more should be
melted. This usually requires an automatically controlled switching mechanism and the time to just make the weld is determined by trial in a separate test sample.

The hole and wire should be small and the thermal diffusivity of the wire and weld should io as close to that of the barrel material as possible. This is to minimize the disturbance to the heat distribution brought about by the presence of the hole and


Pl(iUlRL: 4.28. Photograph of Apparatus for the Sludy of Propellanl Motion During Firing L'sing ladionctive 'Trarer T'rhnique, Simm Gun with Four Scintillation C'ounters on Eurh Sile of the Barrol.

 Sourer Poxilion ersus Time.
thermocouple wire. The thermojunction formed by a single wire inserted in this mamer in a steel bared is that betwern iron and the material of the wire. Nickel is a good material for the wire siner its thermal properties are similar to those of iron.

Two wire themocouples can be used for in-wall temperatures but these require a larger hole and would be expected, in general, to disturb the temperature distribution more than the single wire. For very rapidly varying temperatures they would also be expected to follow the temperature changes less accurately then the welded single wire.

Thermocouples have been developed especially for bore surface temperature measurements. The origimal model was devoloped in (iemany*. Various other models have been developed in different laboratories but they do not differ in principle from the origimal. Figure $1-2$ es is a diagram of one model.":

The therincouple consists essentially of an insulated nickel wire inserted through a hole in the gun steel mounting plug. The wire and plug are then faced off and ground and polished. A thin layer of known thickness ( $2-i \mu$ ) of niekel is then deposited on the polished surface. The deposited nickel bridges the thin layer of insulation on the wire and a thermojunction is formed between the nickel and the stecl. The emf generated is that for nickel and iron. In the original model the nickel wire was insulated by heating in air to form a layer of nickel oxide. Later models use aluminum oxide ( $\mathrm{Al}_{3}\left(\mathrm{O}_{3}\right)$ as an insulating material because it stands up better at higher temperatures. These thermocouples require care in fabrication. They are fragile and last only a few rounds in high performance guns.

In use the themocouple is so mounted that the surface of the nickel plate forms part of the bore surface. It has been found not to be necessary to match the surface of the nickel layer exactly to the surface. It is all advantage to withdraw the thermocouple surface somewhat to protect it and this can be done up to a millimeter or so without apparenily reducing the indicated temperature. The temperatures indicated by these thermocouples show large round to mound variations ${ }^{23}$. This may be partly due to the deposition of variable layers of contamination from the propellant which is observed to occur. Much of the variation is, however, real. The temperature of the propellant gases is not uniform. (ias produced carly in the cycle has expanded more than gas produced later and is cooler. The reading of the themocouple will record this fact by random round-to-round variations in response. It has been shown experimentally ${ }^{\text {ma }}$ that if rounds are fired with charges of mixed hot and cool propellants the respenses of the thermocouples are much more variable. Jn a gum, the firing evele is so bricf that there is not time for parts of the gas at different temperatures to mix and come to a uniform average temperature.

## 4-9.2 Resistance Type Temperature Measuring Gages

These gages measure surface temperature changes by noting the change in clectrical resistance of a fime wire in contact with the surface. One model, which is available commereially, resembles in appearance an ordinary comented-on straingage. They are very convenient for measuring extemal hamed tomperatures. They are mounted in a manner similar to a strain gage and form an arm of a Wheatstome bridge eireuit. When the gage is heated the
conf appearing across the bridge is an indication of the temperature rise if multiplied by the proper calibration factor.

## 4-10 MOTION OF THE PROPELLANT DURING BURNING

In any theory of interior ballistics some assumption must be made about the distribution of the umburnt propellant. It is usual to assume cither one of two limiting situations, namely; that the propellant remains in the chamber during burning and burns at the chamber pressure; or is uniformly distributed in the gas column behind the projectile and burns, on the average, at the average pressure in the gas. The actual situation is neither of these extremes but until recently there was no way of measuring evell approximately the actual motion of the grains of propellant. This difficulty has been overeone, at least to a large extent, by the availability of radioactive tracer technicues."

The method is to incorporate in one of the grains of the eharge a gamma radioactive source of sufficient strength that a measurable amount of the gamma radiation from the souree will penetrate the barrel wall and activate gamma ray detectors placed along the outside of the barrel. The detectors are shielded with lead shielding except for a narrow opening on the side toward the barrel. When the source passes in front of the opening the detector is activated and the fact recorded as a displacement of the spot on the sereen of a cathode ray oseilloseope. The motion of the spot is recorded with a moving film canera and related to a time scale so that one derives from the observations a displace-ment-time curve for the activated grain. By repeated firings, with the activated grain initially at differont distances from the breech end of the chamber, a composite picture of the motion of the charge can be derived.
 millicuries in strength. They were sheathed in stain-

a

less steod and imbedded in a small hole drilled in the center of the grain. The activated grain was about 30 perecent heavier than the normal grain because of the higher density of the source.

The motion of the grain was followed by cight collimated seintillation comnters placed along the barrel. The individual seintillation pulses from the scintillation counters have a time duration of approximately 0.1 microsecond. The possible counting rate is, therefore, several million per second. The counting rate must be high to obtain the necessary statistical accuracy because the time interval over which the measurements are made is of the order of a few milliseconds. The individual pulses are amplified and fed to pulse height discriminator circuits. The uniform output pulses of these circuits are then fed to integrator circuits with time constants small compared to the time intervals to be measured and large compared to the individual pulse widths. $A$ block diagram of the apparatus is shown in Figure $4-2 \overline{6}$ and a photograph of the apparatus and the gun in Figure $4-28$.

The apparatus was checked by firing a metal pellet containing the source through the barrel with an air gun. Sereen contacts were placed at the breech and muzzle to detect the passage of the pellet. Data from the screens and the counters are shown in Figure $4-2$ ?) which shows that the counters record faithfully the position of the source.

Figure $4-30$ is a typical record. The upper and lower parts were recorded on different oscilloscopes on a common time seale. The numbers $1-6$ indicate the times of the maxima of the detector pulses, starting at the breech. The ietter $a$ indicates firing pin contact, $b$ the time when the projectile had moved one-half an inch, $c$ the time when the projectile had moved $9 \frac{1}{2}$ inches and $d$ the time when the projectile was at the muzzle. The trace $D$ indicates the pressure.

Figure $4-31$ is a consolidated plot of the data for four different initial positions of the activated grain. The carly motion of the source could not be followed because of the excessive thickness of the walls of the chamber.

Figures $4-32,4-3.3$ and $4-34$ derived from Figure $4-31$ exhibit the relation between the motion of the activated grain as compared with the motion of the gas as predicted by the Lagrange approximation. The grains will always lag behind the gas because of the higher density of the grain. The activated grain will lag even more because of its still higher density and this effect will be enhanced as the grain burns away leaving the source itself. A great deal


FIGURE 4-33. Distance Traveled by Radioactive Source as a Function of Initial Position Compared with the Displacement of the Gas Given by the Lagrange .Ipproximation.
of the lag probably occurs carly when the velocity of the gas is low and the gas density also low. The Lagrange approximation does not hold in the carly stages when the charge is still burning. The results show in general that the propellant grains follow the gas motion as predicted by the Lagrange approximation more closely the farther forward they are initially and tend to approach it more closely in the later stages of the burning.

## 4-11 ROTATING MIRROR CAMERA

When using the microwave interferometer to measure projectile displacement, a long continuous record is usually desired, and for higher projectile velocities the film speed must be large to resolve the oscillations sufficiently. This can be done with the commercially available running film cameras up to about 2000 fps at which velocity the oscillation frequency is 100 ke per second.

For use at higher velocities a specially designed
rotating mirror cameral wats built. ${ }^{25}$ Rotating mirror cameras sweep the image over a stationary film by means of a rotating mirror. They are available commereially but the design is not advantageons for use with the interferometer. In most models the image is on the film for only a fraction of the time of one revolution of the mirror and the length of the film is too short for a long record.

In the Ballistic Research Laboratories model use is made of a four-sided mirror which, as it rotates, divides the incident beam of light from the lens of the camera so as to form two images, at least one of which is always on the film. The record, therefore, is contimoous.

Figure 4-3.5 shows the design of the basic optical system. The two images are always $180^{\circ}$ apart, and as the film is carried around the focal surface more than $180^{\circ}$, one inage always comes on the film before the other leaves it. When photographing the spot of a cathode ray oscilloseope the spot can be stepped across the sereen and a record several times the length of the film can be recorded. Since the axis of rotation of the mirror is not at the reflecting
surfaces, the focal curve is not a circle. Also the image of an extencod object is not everywhere in exact focus on the film so that as the spot changes position on the sereen the sharpness of focus of the spot image on the film will change. These effects are not large in the present model where the distance from the lens to the film is about 40 inches. The instrument works at approximately one to one magnification so that the inage of the spot is the same size as the spot itself and any fuzziness is reproduced in the image. The instrument uses an ordinary ballopticon projection lens. The mirror is made of stainless steel and is driven by a one-half horse power motor. The faces of the mirror are 3 by $t$ inches. The film is held on the focal curve by being wrapped on a plexiglass surface, emulsion side toward the mirror, the plexiglass surface being formed to the focal curve. The film is loarled in daylight from a built-in film supply and loading arrangement. A photograph of the camera with the cover removed is shown in l-igure $4-36$. This is a prototype model in which no attempt was made to optimize the design cither mechanically or optically.

 the Gus Girrn by the Lagrange .Ipproximation.


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FIGURE 4-36. Pholograph of BRI, Four-Surface Rotating Mirror Camera for Recording Interior Ballistic Trajectories (Cover Removed).

The intensity of the image changes with position on the film because of the way the mirror divides the light between the two images but this has been found not to be serious. Under the best of conditions of spot intensity, color and focus, and using the most
sensitive film, interferometer frequencies up to 1200 ke per second have been resolved and recorded. Projectile velocity for this frequency would be about $20,000 \mathrm{fps}$ for the wave length used in the present interferometer.

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## CHAPTER 5

## LIST OF SYMBOLS

| $a_{10}$ | P'arameter characteristic of the Pidduck-Kent solution | 11 | Ileating time |
| :---: | :---: | :---: | :---: |
|  |  | 1 | Reduced temperature variable |
| $C$ | Mass of charge | 1 | Velocity of the projectile |
| $i$ | A function of $\epsilon$ and $n$ | ${ }^{*}$ | Distance from gas-solid interface |
| $i^{\prime}$ | Heat capacity of the propellant per unit volume | $\begin{aligned} & \alpha \\ & \beta \end{aligned}$ | Initial slope of $(2 n+3) / \delta$ versus $n$ Ratio (final slope/initial slope) of $(2 n+3) / \delta$ |
| $E$ | Setivation energy for the reaction |  | versus $n$ |
| $e$ | Base of natural logarithm | $\gamma$ | Effective ratio of specific heats |
| f | Frequency factor | $\delta$ | l'idduck-hent constant |
| $h$ | Heat transfer coefficient | $\boldsymbol{\epsilon}$ | Ratio: C/M |
| $K$ | Thermal conductivity of the propellant | $\mu$ | Dummy variable |
| $K$ | Kinctic energy of the gas and unburnt propellant | $\begin{aligned} & \xi \\ & \tau \end{aligned}$ | Reduced distance variable Reduced time variable |
| $K$ $M$ $M$ | Finetic energy of the projectile Effective mass of the projectile | Subscripls |  |
| $n$ | Polytropic index: $1_{\text {' }}(\gamma-1)$ | $g$ | Gias phase |
| 1 | Average pressure | $i$ | Ignition |
| $P_{\text {c }}$ | Breech pressure | ia | Idiabatic ignition |
| $P$. | Base pressure | ili | Minimum ignition |
| $Q$ | Heat evolution per unit volume | 0 | Shut-off or end of heating |
| $q$ | Heat of reaction per unit volume | $0 c$ | Critical heating |
| $R$ $T$ | Universal gas constant Temperature at any point in the propellant | Superscripls |  |
| ' | Gas temperature | (0) | Initial value |
| $t$ | Tuis | (1) | A subsefuent value |

## CHAPTER 5

## SPECIAL TOPICS

## 5-1 THE HYDRODYNAMIC PROBLEMS OF INTERIOR BALLISTICS

## 5-1.1 Pressure Distribution and Kinetic Energy of the Propellant Gases

In most formulations of the thenry of interior ballisties, it is customary to account for the effects of the motion of the gas and the unburnt propellant by means of certain simple formulas. These prupoit to define (a) the relations between the different pressures necurring in the basic equations of the theory; nanely, the breech pressure, $l_{\text {}}$, the pressure on the base of the projectile, $l_{\text {, }}$, and the average space pressure, $r$, consistent with the equation of state of the gas, and (b) the amount of kinetic energy to be attributed to the gas and unburnt propellant, ( $(\uparrow$. paragraphs $2-2.2$ and $2-2.3$ ). If $\in$ denotes the ratio of the mass of the charge, $C$, to that of the projectile, $1 /$, where $.1 /$ may be an effective mass as in Chapter 2, these relations can be stated as:

$$
\begin{align*}
& r_{r}=I^{\prime}\left(1+\frac{\epsilon}{2}\right)  \tag{5-1}\\
& r^{\prime}=I^{\prime}\left(1+\frac{\epsilon}{3}\right)  \tag{i,-2}\\
& K_{0}=\frac{\epsilon}{3}\left(\frac{1}{2}, 1 / I^{2}\right)=\frac{\epsilon}{3} K_{u} \tag{:-3}
\end{align*}
$$

where $K_{g}$ is the kinetic energy of the gas and unburnt propellant and $K_{r}$, the kinetic energy of the projectile.

These relations are very approximate and relate rather remotely to the actual situation in a gum. They can be justified under certain conditions by appeal to solutions of what is called the Lagrange Ballistic Problem.

This problem was first formulated by Lagrange in 1793 and is hased on the following simpic model. The propellant is all burnt instantaneously so that one deals only with the gas. For the black powder used as a propellant in Lagrange's day this is not too bad an assumption. The gas is then initially at uniform pressure, density, and temperature, and at rest. It is assumed also that the bore and chamber are of uniform cross section so that they are parts of a uniform tube closed at one end. At the origin
of time the prejectile is released. The prohlem then is to find the distribution of pressure, density and velocity of the gas between the breceh and the projectile at all subsecquent times during the travel of the projectile to the muzzie.

The problem was first solved completely using analytical methods by Love and lidduck.' The treatment assumes that the flow is one-dimensional and adiabatic, that is there is no heat loss to the wall, and that gas friction at the wall is negligible.

A general discussion of the Love and lidduck solution as applied to guns is given in Corner. ${ }^{2}$ The rigorous solution is characterized by rarefaction waves traveling back and forth between the breech and projectile base and the ratios $P_{s} / I_{\text {. }}$, and $K_{\sigma} / K_{p}$ oscillate and approach certain limiting values corresponding to a certain special solution of the equation of motion of the gas. This special solution was worked out by lidduck ${ }^{3}$ and by Lient ${ }^{4}$ and is called the Pidduck-Kent special solution of the Lagrange problem. It has not been proved that the rigorous solution approaches the special solution in the limit of large travel, but it is usually assumed that it does. The Pidduck-Kent solution does not satisfy the initial conditions of the Lagrange problem but corresponds to an initially nonuniform distribution of pressure and density.

The Pidduck-Kient solutions for the pressure and kinetic energy ratios can be expanded in powers of e and are given in Corner. ${ }^{2}$ When all the terms in the expansions beyond those in the first power of $\epsilon$ are dropped, Equations 5-1, i-2 and $\bar{i}-3$ result. These equations are, therefore, only valid, even as approximations, for small values of $\epsilon$; that is, for relatively low velocity guns.

The complete Pidduck-Kent solutions for the pressure and kinetic energy ratios can be stated in the form

$$
\begin{align*}
P_{r} & =P_{\cdot}\left(1-a_{n}\right)^{-n-1}  \tag{i-4}\\
P & =P_{\cdot}\left(1+\frac{\epsilon}{\delta}\right)  \tag{i-i-j}\\
K_{r} & =K_{n} \frac{\epsilon}{\delta} \tag{i}
\end{align*}
$$

$\because$ hore $\delta$, the Pidduck-Kent constant, is given by

$$
\begin{equation*}
\frac{1}{\delta}=\frac{1}{2 n+3}\left[\frac{1}{a_{n}}-\frac{2(n+1)}{\epsilon}\right] \tag{i}
\end{equation*}
$$

where $a_{11}$ is a parameter characteristic of the PidduckKent solution, $n=1(\gamma-1)$ and $\gamma$ can be ant effective valuc of $\gamma$ adjusted as in paragraph $9-2.1$ of Chapter 2.

Vinti and Kravitz" prepared tables and graphs

TABLE 5-1.

## TABLE FOR THE PIDDUCK-KENT SOLUTION

In the following table $\alpha$ is given as a function of $e$ for the following vaiues: $\epsilon=0(.05)$ 1 (.1) 4 (.2) 10 . It is formel from the following formula:

$$
\alpha=2, \epsilon[\sqrt{1+2 / \epsilon}\{\ln (1+\epsilon+\epsilon \sqrt{1+2 / \epsilon})\}-2]
$$

The first forward differences of the function are given in the third column marked $\Delta_{1}$. Lincur interponation is premissible.

for calculating numerical values of $1 ; \delta$ for different values of $\epsilon$ and $n$ and these are reproduced in Tables $5-1,5-2,5-3$ and $;-4$ and Figures $5-1$ and $5-2$. In these tables $1 / \delta$ is expressed in terms of new variables $\alpha, \beta$ and $c_{1}$ in the form

$$
\begin{equation*}
1, \delta=\frac{1}{2 n+\pi}\left[1+\alpha n \frac{1+c_{1}, 3 n}{1+c_{1} n}\right] \tag{i}
\end{equation*}
$$

and the tables and graphs are for $\alpha, \beta$, and $c_{1}$ in terms of $\epsilon$ and $n$. These tables and graphs permit theoretical values of the ratios to be calculated over the range of practical values of $C$ and $\gamma$.

Equations $\overline{5}-4, \bar{\pi}-\bar{i}$ and $\pi-6$ assert that the pressure

TABLE 5-2.

## TABLE FOR THE PIDDUCK-KENT SOLUTION

In the following tathe $\beta$ is given as a function of efor the


$$
\beta=1 / \alpha[1 / k-2 / \epsilon]
$$

Where $k$ is the solution of

$$
e=2 k c^{k} \int_{11}^{1} \mu^{-k \mu^{*}} d \mu
$$

The first forward differeners of the function are given in the column marked $\Delta_{1}$. Linmar interpobation is permissible.

| e | $\beta$ | $\Delta_{1}$ | c | $\beta$ | $\Delta_{1}$ | ¢ | $\beta$ | $\Delta_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1.0 ()Y, | (i) |  | 1.1680 | 6:3 | 5.(\%) | 1.3160 | 136 |
|  | 1.0cai | (6!) |  | 1.174! | 12\% | 5.25 | $1.32 ? 46$ | 135 |
|  | 1.0136 | (6:) |  |  |  | 5.50 | 1.34:31 | 13.4 |
| 0.3 | 1.020: - | 69 | 2.8 | 1.1974 | 12.4 | 5.75 | 1.35315 + | $13:$ |
|  | 1.0274 | 71 |  |  |  |  |  |  |
|  | 1.0344 | $6!$ |  | 1.1918 | 122 | (i.)() | 1.360\% | 131 |
| 0.6 | 1.0413 | 70 |  | 1.2120 | 121 | 6.25 | 1.3828 | 129 |
| 0.7 | 1.0483 | 70 |  | 1.22+1 | 119 | 6.50) | 1.3!57 | 128 |
|  | 1.0553 | $6!$ |  | 1.2360) | 118 | (i.75 | $1.4085^{-}$ | 126 |
| 0.9 | 1.0632) | 69 |  | 1.2788 | 117 |  |  |  |
|  | 1.0691 | 6! |  | 1.25!5- | 115 | 7.()) | 1.4211 | 120 |
|  | 1.0760 | $6{ }^{6}$ |  | 1.2710 | 114 | 7.25 | 1.4337 | 124 |
|  | $1.082 ?$ | 68 |  | 1.2824 | 113 | 7.50 | $1 .+461$ | 123 |
|  | $1.08!9$ | 08 |  | 1.20137 | 112 | 7.75 | 1.458 .1 | 121 |
|  | $1.0965+$ | $(38$ |  | 1.3040 | 111 |  |  |  |
|  | 1.1033 | 67 |  |  |  | 8.00 | 1.4705+ | 121 |
| 1.6 | 1.1100 | 67 |  |  |  | 8.25 | 1.1826 | 120 |
|  | 1.1167 | Of |  |  |  | 8.50 | 1.4046 | 118 |
|  | 1.123:3 | 6fi |  |  |  | 8.75 | 1.506 .4 | 118 |
|  | 1.129!9 | 65 |  |  |  |  |  |  |
|  | 1.1364 | C6) |  |  |  | $0 .(0)$ | 1.5182 | 116 |
|  | 1.1430 | 64 |  |  |  | 9.25 | 1.5298 | 116 |
|  | 1.1494 | 65 |  |  |  | 0.50 | 1.5114 | 115 |
|  | 1.1559 | 64 |  |  |  | 0.75 | 1.5520 | 113 |
| $2.41 .162: 3$ |  | 63 |  |  |  |  |  |  |
|  |  |  |  |  |  | 10.00 | 1.5042 |  |

TABLE 5-3.
TABLE FOR THE PIDDUCK-KENT SOLUTION
In the following table $c_{1}$ is given as a function of $a$ and $n$ for all combinations of the following values: $e=0(.2) 1(1) 10$; $n=12,1(1) 5$.

| c | $n$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | !2 | 1 | 2 | 3 | 4 | 5 |
| 0.0 | 1.006 | $1 .(0) 0$ | 1.090 | $1.0 \times 0)$ | 1.000 | 1.000 |
| $\therefore$ | 1.016 | 1.016 | 1.016 | 1.016 | 1.016 | 1.016 |
| . 4 | 1.02!) | 1.029 | 1.02? | 1.029 | 1.02!) | 1.030 |
| .i | 1.0388 | $1033!$ | i.03:! | 1.039 | 1.03: | 1.039 |
| . 8 | 1.045 | 1.046 | 1.0 .46 | 1.047 | 1.047 | 1.0 .47 |
| 1 | 1.051 | 1.051 | 1.05: | 1.052 | 1.053 | 1.053 |
| : | $1.05!1$ | 1.061 | $1 .(1033$ | 1.06it | 1.0)i5 | 1.(4)is |
| 3 | 1.0is ${ }^{\text {a }}$ | 1.057 | 1. MiO | 1.0 Ki 3 | 1.06. 4 | 1.0665 |
| 1 | 1.0.42 | 1.047 | 1.053 | 1.055 | 1.057 | 1.058 |
| 5 | 1.02!) | 1.036 | 1.042 | 1.046 | 1.048 | 1.049 |
| 6 | 1.015 | 1.02:3 | 1.031 | 1.03:3 | 1.037 | 1.03: |
| 7 | $1.06 \%)$ | 1.010 | 1.019 | 1.02 .1 | 1.027 | $1.02!$ |
| 8 | . 986 | . 19 | 1.008 | 1.013 | 1.016 | 1.018 |
| $!$ | .193 | . 88 | \% | 1.002 | 1.06\% | 1.008 |
| 10 | . O (0) | .17\% | .980 | (19\% | .09\% | . 908 |

and kinctic energy ratios are constant for fixed values of $\gamma$ and $\epsilon$ and are independent of the velocity of the projectile. As was mentioned carlier, the general solution predicts that the ratios are initially oscillatory but the oscillations tend to die out. Love and pidduck appled their formulas to the case of a gun for which $\epsilon=0.24$ having a muzzle velocity of about 2.000 feer per second. These results are reproduced in Corner ${ }^{2}$ and show that, for this case, the theory predicts that the pressure and energy ratios, apart from the oscillations, are nearly constant up to exit of the projectile.

Recont interest in the development of high velocity guns, expecially the so-called light gas guns, has stimulated a revived interest in the hydrodynamic problems of interior ballistics. The recent practice is to solve the problems numerically by the method of characteristics rather than analytically as was done by Love and Pidduck. With modern computers, it is probably less work to solve the individual problem numerically from the begimning than to use the analytical formulas which are complicated. Recent treatments of the problem by the method of characteristics for different assumed forms of the equation of state of the gas are given in References $\mathfrak{f}, 7$ and 8 . An experimental investigation of the problem is deseribed in Reference 9.

A treatment of the problem taking account of chambrage and chamber geometry is given in

Reference 10 in connection with a study of light gas gun performance.

## 5-1.2 The Emptying of the Gun

The emptying of the gun after the projectile leaves is a problem of some interest in interior ballistics. The gas flowing from the tube continues to impart recoil momentum to the barrel which can be cstimated by integrating the breech pressure over the time of emptying. This contribution to the recoil momentum can be appreciable especially for the higher velocity guns.

Theoretical treatments of the flow of the gas from the gun have been given by several authors. In general, they assume that the initial conditions for the problem are those given by the solution of the Lagrange problem when the projectile is at the muzzle. Such a treatment is given by Corner," who also gives references to other work on the subject as well as a gencral discussion of the problem.

The gas flowing from the muzzle can be made to redue the recoil forees by attaching a system of baffles just beyond the muzzle. These baffles are designed so as to deflect the muzzle gases sideways and to the rear. The gases, therefore, tend to force the barrel forward and so reduce the recoil forces.

TABLE 5-4.
TABLE FOR THE PIDDUCK-KENT SOLUTION
$V$ :alues of $e$ as a function of $c_{1}$ and $n$.

| $r_{1}$ | $n$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 12 | 1 | 2 | 3 | $t$ | 5 |
| 1.0000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.010 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| 1.020 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| 1.0330 | 0.43 | 0.42 | 0.42 | 0.41 | 0.fi) | 0.40 |
| 1.040 | 0. 0.5 | 0.65 | 0.6.t | 0.64 | 0.6. | 0.60 |
| $1.050)$ | 0.97 | 0.97 | 0.94 | 0.92) | 0.91 | 0.90 |
| 1.060 | -- | 1.69 | 1.50 | 1.4 .4 | 1.40 | 1.35 |
| 1.063 | - | , | 1.98 | 1.75 | 1.63 | 1.60 |
| 1.066 | --- | - | - |  | 2.00 | 1.92 |
| 1.0665 | $\cdots$ | - | - | $\cdots$ | 2.73 | . 208 |
| 1.0003 | - | $\cdots$ | 2.45 | 2.95 | 3.18 | 3.35 |
| 1.060 | - | 2.50 | 3.12 | 3.43 | 3.62 | 3.75 |
| 1.050 | 3:29 | 3.73 | 4.28 | 4.55 | 4.75 | 4.87 |
| 1.0 .40 | 4.15 | 4.60 | 5.20 | 5.52 | 5.73 | 5.8!) |
| 1.030 | 4.92 | 5.44 | (i.OM; | (6.45 | (i.70) | 6.85 |
| 1.020 | 5.65 | 6.2:3 | (\%.\%) | 7.137 | 7.65 | 7.85 |
| 1.010 | (6.34 | 6.08 | 7.80 | 8.30 | 8.60 | 8.83 |
| 1.000 | 7.00 | 7.75 | 8.70 | 9.22 | 9.58 | 9.80 |
| . 90 | 7.72 | 8.57 | 9.65 | - |  |  |
| . 980 | 8.45 | 9.40 | - | - | $\square$ | - |
| . 970 | ! 1.20 | 析 | ---.- | --- | - | ---.. |




I deriece of this type is called a mazale bake. Fon: a general diseussion of the design features and use of muzal brakes, the reader is again refored to Comore."

The emptying of the gun is also of importance in tank applications and automatic weapons. If the brecolh is opened before the breech pressure has berome nempigible, gases will flow from the breech. This can be serious especially when the gun is operated in an enclosed turert. Where the gases can acemulate. The gases can form a combustible mixtwe with air and are also initiating as well as toxic to persomed since they contain ammonia fumes and cathon monoxide.

Exen when the breed pressure has fallen to atmospheric pressure, some propellant gas will reman in the bared. This gas can energe from the open brech. To permit carly opening and to present gas flow from the breeds, a deviee known as a bore evacuator has beon invented. It consists cesentially of a chamber located near the mazale, surrounding the tube and opening into it. so that when the projectile passes the openinge, gas flows into the charaber and fills it to a preswere near that in the tube. When the projectile leaves the muzzle and the pressure in the tube falls below that in the evacuator chamber, the gas in the chamber flows back into the tube. The openings are so designed that the reverse flow has a component toward the muzzle. The effect is to drive the gases in the tube toward the muzzle and so prevent their emergenee from the breech.

## 5-2 IGNITION OF PROPELLANTS

## 5-2.1 General Discussion

When a solid propellant is bumed, the preponderance of experimental evidence supports the conchision that the burning proceeds in two stages. "The first stage takes place in a thin layer of the solid at the surface, and is characterized by a chemical reaction in the solid material. The reaction converts the solid propellant into gaseous products which strean away from the surface in a perpendicular direction. These products continue to react in the gas phase which constitutes the second stage of the process. If the ambient pressure is sufficiently high, the gas phase reactions continue until the final products are produced and, under these ciremmstaners, the temperature of the product gases becomes hig, enough somewhere along the stream that the gass; iweome luminous and a flame appears. The base of the flame is guite sharply defined and its position with respect to the surface is dependent
on the ambient pressure; the base of the flame moving closer to the surface as the ambient pressure increases. It pressures of about 1000 psi, the separation of the flame from the surface becomes too small to be casily observable.

It pressure around an atmosphere or less, it is possible for the propellant to burn without the oceurenes of the flame. This would indicate that the gas phase reaction does not go to completion, leaving combustible products. This condition can be brought about be gentle ignition so that the igniter itself does not start the flame reaction which then would be self-sustaning. For pressures below about 100 atmospheres, the presence of the flame does not appreciably alter the rate of regression of the surface. The solid phase reaction can, therefore, proced without the reception of heat from any outside souree. It will be accelerated, however, by heating from the emviromment, and this accounts for the effect of pressure on the burning rate, which forees the hot flame zone closer to the suface and hence incerases: the rate of heat transfer back to the surface.

When a propellant burns without the presenee of flame it is said to fizz burn. The surface appeas to boil and bubble and a fizzing sound is andible. The nature of the products produced when a propellant fizz burns is not completely known. The absorption spectrum of the dark zone exhibits the absorption bands of nitrous oxide. The products of the thermal decomposition of nitrocellulose have been studied by Wolfram. ${ }^{12}$ 'The products of the fizz burning of propellants probably contain similar fractions but in greater variety and in different proportions.

## 5-2.2 Laboratory Investigations of Ignition

Ignition is brought about by the application of heat to the surface of the propellant. A great deal of research on a laboratory seale has been done in attempts to study the process in detail. In most cases, the procedure has been to apply heat suddenly to a selected sample of propellant and then to obscrve by photographic and photo-electric means the hehavior of the surface and the initiation of the flame as a function of the time. Apart from qualitative description, the experiments yield quantitative measurements of the times necessary to initiate the solid phase and gas phase reactions. The former is assumed to begin when evidence of surface disintegration appears and the latter when the gases become sufficiently luminous for the luminosity to be detected by the measuring apparatus. These limits are obvionsly somewhat ablitrary as is the
shap division of the total combustion reaction into two phases sharply separated in time and space. The flame zone will appear where and when the total reaction has procecded to the point at which the temperature of the products is high enough to stimulate the emission of observable radiation. In studies of ignition, it is ustally assumed that when observable radiation appears effective ignition has taken place. In some experiments, thermocoupler hawe beron imbedded in the propellant sample close to the surface and measurements made of the way the temperature in the surface layer changes dowing ignition and carly burning of the propedlant.

To apply the heat several mothods have been used. They differ in detail but can be classified as (a) contact with a hot solid such as a heated wire, ${ }^{\text {, }}$, (b) contact with or immersion in a ruiescent hot gas. ${ }^{14}$ (e) immersion in a stream of hot gas ${ }^{13}$ and (d) irradiation of the sufface by a source of heat radiation.'" Methods (b) and (c) have been most commonly used. In thesse cases, it is found that the observed time delays depend not only on the temperature and velocity of the igniting gas but also upon its chemical constitution, particularly on the amount of oxygen it contains. The presence of oxygen shortens the time to ignition as evidenced by the appearance of lominosity. Gun propellants are usually oxygen deficient and oxygen in the igniter gases should promote the combustion reactions. One would expect differences in ignition time also when different inert gases are used because the heat transfer coefficients for both conductive and convective transfor are functions of the chemical constitution.

Much of the recent work on ignition, both theoretical and experimental, has been done on composite propellants designed for use in rockets. In rocket propellants, the fuel and oxidizer are usually separate substances mixed more or less intimately. In gun propellants, the constituents camot be classified separately as fuel and oxygen supplying elements. Each constituent carries its own oxygen which forms part of the molecular structure. The present discussion is concerned primarily with gon propellants. Reports of further work on composite propellants are to be found in the publication cited in Refermee 16 .

In most of the laboratory investigations, correlations have been made between the observed ignition delay times and the heat transfer, for different propellant samples, igniter gas composition and temperature, heating times and other pertinent
parameters. . Wthough these correlations often lead to quantitative relations betweren ignition times and such ruantities as heat input rates or total heat input, it camot be said, as yet, that any complete and generally agreed upon un erstanding of the ignition process has been arrived at. The quantitative results are specific to the experimental method and procedures adopted and to the nature of the propellant sample investigated.
In general, it is observed that if heat is applied to the surface of a propellant at a fixed rate and under steady conditions of pressure and other controllable experimental parameters, the propellant will eventually ignite and the sample will be consumed. If the pressure is low enough and the ignition is not too vigorous or if the heat is supplied be a fast moving hot gas, the sample may fizz burn only. If the pressure is an atmosphere or more and the ignition is not too brief or gentle, the flame will appear and what is considered to be effective ignition and nomal combustion is observed. In many experiments, it is possible to observe the onset of the fizz reaction and the flame reaction as separate and nonsimultancous events. ${ }^{14}$

If the time interval over which the heat is supplied is shortened sufficiently, the ignition may or may not go to completion. There seems to be a threshold condition, determined by the manner of heating the surface, below which the propellant will not ignite or above which it always ignites. In the threshold region, the ignition is unstable and effective ignition may or may not take place. If it does take place, it docs so after a variable time delay.

## 5-2.3 Theories of Ignition

Attempts have been made to formulate general theories of ignition based on thermal and chemical considerations. Although the result of these theories can be brought into rough correlation with certain aspects of the ignition process as revealed by observation, they are by no means complete or aderuate for quantitative prediction of the ignition characteristies of a given propellant or as guides for the development of ignition systems for gons. In formulating a theory of ignition, one must of necessity assume a much simplified model of the process.
The purely thermal theory due to Hieks: will serve to illustrate one approach. It is assumed that the ignition and burning of the propellant is dependent on the flow of heat in it and is a function only of its temperature. The propellant is assumed to occupy the half space defined by the coordinate $x$. for $x>0$. It is heated mifomly over its surface
at $x=0$ by gas at temperature, $T_{0}$, and heat is generated within it by a chemical reaction at a rate dependent on the temperature.

The partial differential equation deseribing the heat flow in such a model is

$$
\begin{equation*}
c^{\prime} \frac{\partial T}{\partial T}=K \frac{\partial^{2} T}{\partial x^{2}}+Q: \quad x \geqq 0, \quad t>0 \tag{i}
\end{equation*}
$$

where
$T$ is the temperature at any point in the propellant
1 the time
.r the space coordinate as defieded above
$r^{\prime}$ the heat capacity of the propellant per unit volume
$K$ the themal conductivity of the propellant and $Q$ the heat evolution per unit volume

The reolution of heat due to the reaction is assumed to be of the form

$$
\begin{equation*}
Q=4!\cdot a \cdot k t \tag{9}
\end{equation*}
$$

wher
4 is the heat of reaction per unit volume
$!$ the frerquency factor
$E$ the activation energy for the reaction and
$R$ the gas constant
It the surface, the boundary condition is
$-K \frac{\partial T}{\partial r}=h\left(T_{v}-T\right): \quad x=0, \quad t>0$,
where $h$ is the heat transfer cocflicient.
It an infinite distance, it is assumed that the heat flow vanishes so that the second boundary condition is:

$$
\begin{equation*}
\frac{\partial T}{d x}=0: \quad x \rightarrow \infty, \quad 1>0 \tag{i-11}
\end{equation*}
$$

Initially, the temperature of the propellant is assumed to be a constant and independent of $x$ so that

$$
\begin{equation*}
T=T^{\prime \prime \prime} ; \quad x \geqq 0, \quad l=0 . \tag{i}
\end{equation*}
$$

It is assumed also that at $i=0$ the hot igniter gas at temperature, $T_{v}$, is suddenly brought in contact with the propellant surface and continues to transfer heat to the surface at a constant rate until $l=t_{\text {, }}$ (the heating time) when the heat transfer effectively crases. This is taken account of in the mathematical solution by assuming that $T_{0}$, has a anstant value $T_{v}^{(n)}$ until $1=l_{n}$, when it suddenly
drops to a much lower constant value $T_{g}^{(1)}$ or

$$
\begin{array}{ll}
T_{v}=T_{v}^{(t)} ; & 0<t \leqq l_{u}  \tag{i;-1;3}\\
T_{v}^{\prime}=T_{v}^{(v)} ; & t>t_{0}
\end{array}
$$

The mathematical problem is now defined. The solution yields values of the temperature as a function of $x$ and $t$. The assumption is then made that if the temperature reaches a certain value, $T_{i}$, called the ignition temperature, the propellant will ignite. It is further assumed that when the surface reaches the ignition temperature effective ig, ition occurs.

Because of the exponential form assumed for the: dependence of $Q$ on $T$ ' (Equation $\pi-9$ ), the temperature in a sample of propellant obeying Equation i-8 will always eventually reach the ignition temperature at some value of $x$. The time taken for this to happen when no heat is supplied to the surface is called by IHicks the adiabatic ignition time. This time will, obviously, be dependent on the initial temperature of the propellant. When heat is supplied from the outside, the propellant will, according to the theory, ignite sooner.

The equations are such that the solution can only be given numerically. Hicks solved the problem in the dimensionless variables $U, \tau$ and $\xi$, defined by

$$
\begin{align*}
r & =\frac{R}{R^{\prime}} T  \tag{i-it}\\
\tau & =\frac{R q I}{c^{\prime} E^{\prime}}  \tag{i-1;}\\
\xi & =\left(\frac{R q l}{R E}\right)^{1 / 2} \tag{i}
\end{align*}
$$

IIe also expressed $h$ in the reduced form

$$
I=\left(\frac{E^{\prime}}{R K,}\right)^{1 / 2}
$$

The approximate range of the different parameters occurring in the theory is given in Table i -i).

The nature of the solution at the surface $(\xi=0)$ is shown in ligure :-3 for the indicated values of $U^{(0)}, U_{v}^{(1)}$ and $U_{v}^{(1)}$. The graph shows that under the influence of external heating the temperature of the surface rises monotonically. If the external heating is continued long enough, the surface reaches the ignition temperature ( $U_{i}=0.046$ ) at a time $\tau_{i m}$, the minimum ignition time. If the heating is stopped at a heating time $\tau_{n}$ which is less than $\tau_{i m}$ the surface temperature will then decline at first but will reach a minimum and then increase again and continue to rise until the ignition temperature is reached at an ignition time $\tau_{i}$ greater than $\tau_{i m}$.

| Depth to which reaction jenetrates: | $\Delta x-10^{-6}-10^{3} \mathrm{~cm}$ | $\Delta \xi-10^{2}-10^{10}$ |
| :---: | :---: | :---: |
| Time intervals | $\Delta t-10^{7} \mathrm{sc}$ | $\Delta \tau-10^{3}-10^{20}$ |
| He:ating time | $t_{0}-10^{-6}-10^{3} \mathrm{sec}$ | $\tau_{0}-10^{1}-10^{16}$ |
| Temperature | $r-2(0)^{\circ}-6.6(0)^{\circ} \mathrm{K}$ | $U-0.010-0.050$ |
| (ian tempreature (hot) | $7_{n}{ }^{\prime \prime}{ }^{(0)}$ - $150(1)^{\circ}-30000^{\circ} \mathrm{K}$ | $U_{\prime \prime}{ }^{(0)}-0.18,0.20$ |
| ( ise trmpurature (cowled) $\dagger$ | $T_{4}{ }^{(12}-3000^{\circ}-\left(8000^{\circ} \mathrm{K}\right.$ | $U_{V}{ }^{(1)}$ - 0.0 .021 |
| Initial propullant temperature | $T^{(0)}-200^{\circ}-400^{\circ} \mathrm{K}$ | [(t) - 0.010-0.034 |
| Inmition termprature | $r_{i}-675^{\circ}-12500^{\circ} \mathrm{K}$ | $1_{i}-0.0 .045,0.050$ |
| Heat tramsior rowflicient | $h-10^{-6}-3 \times 10^{-2} \mathrm{cal} / \mathrm{cmin}^{2}$ ser- ${ }^{\circ} \mathrm{K}$ | $H-10^{-10}-2 \times 10^{0}$ |
| Heat enpmity unit volume | $\mathrm{c}^{\prime}-0.5-0.8{\mathrm{cal} / \mathrm{cm}^{3}-{ }^{\circ} \mathrm{C}}^{\text {d }}$ |  |
| Thermal condurtivity | $K-10^{-5}-10^{-2} \mathrm{cal} / \mathrm{cm}$-seco- ${ }^{\circ}$ : |  |
| Heat of reartion/unit volume $\ddagger$ | $4-10^{2}-10^{3} \mathrm{cal} / \mathrm{cm}^{3}$ |  |
| Frumume factor | $f-10^{13}-10^{16} \mathrm{sec}^{-1}$ |  |
| Artivation rimery | E-3.0-5.0) $\times 10^{+}$cal $/$muk . |  |
| (:ats constant | R - 1.989 cal mole- ${ }^{\circ} \mathrm{K}$ |  |

* Thir values in the secomi cohmmare, for the most part, those encontered in practice. The values in the third column are those used in the numerical wort for the brament repert.



Ther graph also shows the adiabatic ignition time $\boldsymbol{T}_{\text {in }}$ and the effect of the surface heating in decreasing ther ignition time from the adiabatic value is evident. . $\mathrm{s}^{\tau_{n}} \tau_{\text {is }}$ increased, $\tau_{n}$ and $\tau_{i}$ becone equal to one another and also to $\tau_{1} .$. .

If $\tau_{\text {, }}$ is derensed below $\tau_{\text {, }}, \tau_{i}$ incerases very rapilly. If $U_{0}^{\prime \prime \prime} \leqq \ell^{\prime}(0, \tau), \tau>\tau_{11}$ (that is, at $\tau=\tau_{n}$ the igniter gas temperature falls to or below the surfaer temperature) then for a sufficiently short heating time $\tau_{\text {. }}, \tau_{\text {, }}$, defined as the time for the surface to rearl the ignition temperature, becomes greater than the adiabatic ignition time. Under these conditions the maximum temperature is not at the surface and the ignition temperature is reached first somewhere inside the solid propellant. The heating tine for which $\tau_{i}=\tau_{, \ldots}$ is called by Ilicks the reitical heating time and designated as $\tau_{n \times}$. It is shown in ligure $\overline{\mathrm{T}}-3$ to be close to $\tau_{i m}$.

If $\zeta_{\because,}^{\because \prime \prime} \geqq C^{\prime}(0, \tau), \tau>\tau_{1,}$, the maximum temperature remains at the surface and there is no critical heating time. $\tau$, is then less than $\tau_{i, n}$. This situation would be most likely to occur in practice because hot gases evolved from the propellant would tend to be hotter than the surface itself and tend to maintain the hoating after the igniter ceased to operate.

It is difficult to chack a theory, such as that of Hicks, by detailed guantitative correlation with the results of experiments. One would not expect any close agreement, quantitatively, hoth because numerical values of the pertinent parameters are not known
for certain and also because the model assumed by the theory is too simple. The theory, however, does in a rough way, account for the critical nature of the ignition process leading to the existence of variable time delays when the ignition is not sufficiently vigorous and extended in time.

Hieks later extended his theoretical work to include chemical effects associated with the production of nitrous oxide. ${ }^{14}$

## 5-2.4 Ignition in Guns

In guns the charge is a bed of propellant grains contained in a tightly soaled chamber. Initially the spaces between the grains are filled with air at atmospheric pressure. As scon as any gas is generated cither by the igniter itself or by combustion of the propellant, the pressure will rise. Ignition in guns, therefore, is always accompunied by an increasing pressure.

In its usual form the igniter produces hot gas which possibly contains hot solid particles. The igniter gas flows more or less freely through the propellant bed and heats the surface of the grains by conductive and convective heat transfer. The igniter gases do not reach all parts of the charge at the same time and may never reach some parts at all. They also cool as they flow through the charge. The grains near the igniter will ignite first. These will produce hot gas which will combine with the igniter gas and aid in the ignition of more remote parts of the charge. The ignition, therefore, will

spread through the charge even alter the miginal igniter has ceased to function. The rise in pressure which areompanies this proecess tends to promote combustion and to make the ignition spread more effectively.

To start the combustion proecse affectively, the igniter gases must initiate the solid phase reaction and remain eflective mitil the combustion proeses has beron established. This rerpuires that the igniter should be so designed as to initiate combustion simultaneonsly over as much of the surface of the entive charge as possible. If the initial region of ignition is ton locatized, it is possible for the moere remote parts of the charge to be heated but not effectively ignited so that the solid phase reaction procerds alone gencrating gases not completaly macted. These gases can accumulate and be subserguently ignited explosively, leading to sporadie sumges of high presimere. Oectasionally, the pressumes so grenerated exered the pressure for which the tube is designed, and it may be permanently distended or ceon ruptured completely. This sort of hehavior is, nowe probable when the ammmition is used at wey low temperature berease the igniter gases are coold nowe rapidly in the cold propellant bed and terome lass eflective in igniting the grons finther away from the igniter.

Poore ignition in guns also results in less mifornity in muzale velocity, pressure waves in the chamber which gencrate variations in burning rate and conserguently rough pressure time curves. Poor ignition also results in variation in the longth of the firing cycle of the gen. In rapid fire automatic weapons, this can canse difficulties because the firing rate of the gen should tre uniform and properly related to the natural vibation rate of the momet." In the development of any particular gun and annmunition system, therefore, the development of the ignition system is a matter of the greatest importance. Although general design principles have beren formulated, the application of these principles is often complicated by the other aspects of the complete round under consideration, so that a certain amount of empiricism and experimentation is reguired.

## 5-2.5 Ignition Systems for Guns

The substance most commonly used to grenerate the igniter gas in guns is black powder, ${ }^{20}$ although other materials have been and are being developed in an attempt to produce more effective ignition systems. The black powder is ignited by gases from a small charge of high explesive, which is initiated
chectrically or by perenssion. The black powder clatere, called the primer charge, is enclosed in a motal tube called a primer tube or in one or more cloth bags. The motal tube is used in cased ammunition and the cloth bags in separately loaded uncased ammmition. In cased ammunition the gas emerges from the primer tube through a system of hokes on vents distributed along the longth of the tube in varions ways. The bokes are closed by a paper liner in the tube. The paper liner ruptures when sufficient pressure develops inside the tube, to permit the efllux of the primer gases.
factors affecting the design of artillery primers are disenssed in Reforenere 2l. Athough a gencral guide for designing a primer can be dedued from experionere and labomatory studies of ignition of propellants, the details of an effective design for any particular case are oftern sperefie to the case in question. A design feature which riminates a centain difficulty in one case may not do so in anothere. The details of the behavior of a primer-properlant. system are usinally not known. The only eriterion is whether satisfactory uniformity in pressure and muzale volocity is ohtained. Laboratory investigations of the functioning of standard artillery primers as well as experimental models designed to investigate the affect of eertain specifice design ferotures have beren conducted. The results of these: (xperiments can be found in Reforones $2 \underline{2}$ and $2: 3$. Further information on the development and "valuation of gron primers and igniters for separate loading ammunition may be found in References 2t, 2.5, 26, 27 and 28 .

## 5-3 FLASH AND SMOKE

## 5-3.1 Flash

The gases issuing from the muzzle of a gin are usually hot enough to be luminous. The luminosity is frepuently very intense and can be ohvious even in broad daylight, and although it exists only momentarily as an intense flash of light, it is very effective in revealing the location of the weapon. It also may impair the vision of the gumner.

A study of the phenomenon reveals that there are three regions of luminosity; ${ }^{30,3 "}$ (a) a rather small hemispherical region of low luminosity at the muzzle sonetimes called the muzale glow, (b) a region of high intensity; just beyond the muzale and separated from the muzzle glow by a dark region, usually called the primary flash, and (c) a rather ill-defined region of high intensity, beyond but usnally not well separated from the primary flash,


(b) Motion l'idur of . Muzele Vlaxh, 1500 Prames/Secont.



calied the secondary flash.* The three luminous zones: are most casily observed in small weapons. In medium and large artillery weapons the secondary flash can be very large and extend many feet beyond the muzzle and persist for relatively long times of the order of 0.1 second or more so that the primary flash is not obvious, ligure i-t. If a small weapon such as a caliber . 30 rifle is fired into an atmosphere of an inert gas such as nitrogen the secondary flash is suppressed and the appearance of the primary flash is clearly evident, Figure i-s. The muzzle glow and primary flash, for a :37mm gm is shown in Figure $\boldsymbol{i}$-(i).

While the phenomenon of muzzle flash is not understood in detail, the general view is that the gases as they leave the muzzle are hot enough to be self-luminous. Immediately after exit, thicy expand rapidly and cool so that the luminosity disappears forming the dark zone. It this point they are over expanded and subsecuently are recompressed adiabatically through a shock. This recompression raises the temperature again almost to the muzzle temperature and the gases are again luminous and form the primary flash. In the meantime the gases have entrained air and a combustible mixture has been formed of the unburned hydrogen and carbon monoxide in the muzzle gases; and if the recompression has raised the temperature above the ignition temperature of this mixture, it will ignite and burn as a diffusion flame forming the secondary flash. Some investigators have also postulated that exeited chemical species play a part in the ignition of the secondary flash. The primary fiash is sma!l and persists for a very short time (a few milliseconds) and is not visible at great distances. The secondary flash because of its extent and longer duration has a high visibility especially for the larger weapons. A phenomenon similar to muzzle flash occurs in the gases issuing from the nozzles of recoilless guns.

An examination of the spectra of muzzle flashes reveals that most of the luminosity is due to the presence of metallic impurities in the propellant gases. The gases produced by pure propellant constituents are mainly $\mathrm{H}_{2} \mathrm{O}, \mathrm{H}_{2}, \mathrm{CO}$ and $\mathrm{CO}_{2}$ along with $\Gamma_{2}$ and $\mathcal{N O}$. These gases are poor emitters of visible radiation. Except for a weak background of continuous radiation, the spectrum of the muzzle gases in the visible region reveals strong radiation from sodium, potassium and calcium and the oxides of calcium and copper. The rediation from sodium

[^7]is the s surce of the yellow tint of the flash. Sodium, potassiam and calcium will be present as they are always present in the materials used in propellant manufacture. Copper comes predominantly from the rotatiag bands.


FIGURE 5-6. Primary Flash and Muzelc Glow From sirmm Gun.

## 5-3.2 Flash Suppression

The tendency of a weapon to flash depends in a complicated way on the design of the weapon and its interior ballistics as well as on the chemistry of the propellant. One would expect that anything which reduced the temperature and probably the pressure of the gases issuing from the muzzle would tend to reduce the tendency to flash. This is illustrated by the following example.

After the troops had complained of bright flashes accompanied by loud noise and streng blast in the 8-inch Howitzer, the High Explosive Projectile Mr106 was fired with experimental charges. Single perforated MI propellant with a web thickness of 0.0161 inch was used for Zones 1 to i., producing muzzle velocities from 820 to 1380 feet per second. Multiperforated M1 propellant with a web thickness of $0.0+14$ inch was used for Zones $\overline{5}$ to 7 , producing muzzle velocities from 1380 to 1950 feet per second. With the single perforated grains, there was little flash and blast; but with the multiperforated grains, there was considerably more flash and blast, especially in Zone i.. Although a combination of black powder and potassium sulfate reduced the flash in Zones is and ( $i$, it produced an intolerable amount of smoke.

In order to explain these phenomena, interior ballistic calculations were made for Zones is and ${ }^{6}$ of the 8 -inch Howitzer. The results are summarized in Table i-(0. The distance to burnout is greatest and the pressure at burnout lowest for the multiperforated propellant in Zone 5 . The muzzle pressure with the multiperforated propellant is 50 percent higher than with the single perforated prepellant in

> TABLE 5-6. INYERIOR BALLISTIC DATA FOR 8-INCH HOWITZER FIRING HE PROJECTILE M106

| \%one | 5 | 5 | 6 |
| :---: | :---: | :---: | :---: |
| Web, in | .0161* | . $0414 \dagger$ | . $0414 \dagger$ |
| Charge weight, It, | 13.0 | 16.6 | 21.8 |
| Velocity, fps | 1350 | 1380 | 1640 |
| Length of Travel, in | 173.83 | 173.83 | 173.83 |
| Approximate Distance to Burnout, in | 10 | 120 | 90 |
| Copper Pressure, psi | 27,000 | 12,100 | 19,800 |
| Approximate Pressure at Point of Burnout, psi | 21,600 | 6,400 | 11,600 |
| Muzzle Pressure, psi | 2700 | $\pm 100$ | 5500 |
| 'remperature of Gases: at Muzzle, ${ }^{\circ} \mathrm{F}$ | 1340 | 1580 | 1505 |

[^8]the same zonc. The temperature of the gas at the muzzle is highest with the multiperforated grains in Zone i. This combination of high pressure and temperature at the muzzle was probably the cause of the flash and blast.

Pressure-time traces of the standard charges showed that the base-ignited Zone 7 charge did not ignite properly, and that the point of bumout was closer to the muzzle than the calculations indicate. To improve the ignition, a long, thin, segmented igniter bag was inclosed in a tube located longitudinally through the center of the charge and an igniter pad assembled to the base of the charge. Also, dual granulation was adopted in order to increase the maximum pressure in the intermediate zones and thus reduce the distance to burnout. This eliminated most of the flash.

Although it has been proposed and investigated experimentally, it does not seem possible to eliminate flash by eliminating the impurities responsible for the luminosity. It has been discovered, largely by trial and crror, that the secondary flash can be greatly reduced and often practically eliminated by two methods, namely; by adding certain chemical substances to the propellant, or by attaching a mechanical device to the muzzle. It is not known exactly in any particular case why either of these methods work.
a. Chemical Flash Suppressors. Numerous chemical compounds when added to the propellant will tend to reduce the tendency to flash. The most studied have been salts of the alkali metals. Research done in Japan during World War II showed that, for the alkali halides, the effectiveness in terms of the relative amount of material necessary to suppress the flash increased with both the atomic number of the alkali and the halide so that cesium iodide was the most effective compound. The tests were made in small weapons, a $60 m m$ mortar, a $2: \mathrm{mm}$ rifle and a 7.7 mm rifle. The results might not hold in larger or different weapons.

Most commonly used in practice are the salts of potassium. The Japanese workers found that some of the most effective compounds were:

> Potassium iodide (KII)
> Potassium bromide ( KBr )
> l'otassium oxalate $\left(\mathrm{K}_{2} \mathrm{C}_{2} \mathrm{O}_{4} \cdot \mathrm{H}_{2} \mathrm{O}\right)$
> l'otassium acid oxalate $\left(\mathrm{KHC}_{2} \mathrm{O}_{1} \cdot 1 / 2 \mathrm{H}_{2} \mathrm{O}\right)$
> l'otassium sulfate $\left(\mathrm{K}_{2} \mathrm{SO}_{4}\right)$.

The number of potassium atoms in the molecule had no appreciable effect on the effectiveness of the compound. In American practice the most com-
monly used compound is petassimm sulfate. ('hemical suppresions do not suppress the primary flash: the primary flash, being due to the adiabatie recompression of the gases, is not influmed be relatively small changes in their chemieal constitution.
b. Wechanical Flash Siuppossoms. The earliest mechanical flath suppressor was in the form of a conc- or frmmel-shaped deviere attached to the muzale. There serems to be some doubt as to whethere this devier was originally intemed to hide the flash from the enemy or to shirld the eves of the grmmer so that his vision was not impaired. In any case it appeared to reduce the flash. This is probably becanse it reduced the amount of orer-expansion of the gas be calusing it to expand more slowly and shemothly and so eliminated or redued the temedeney for a shock to form in the flowing gases with a consegurent sudden increase in haminosity.

In investigating this form of suppresor slits were cut in the colie to permit ohservation of the heminosity inside. It was found that these slits mot only permitted ohservatiom but also had a favorable effect on the flash. Fiuther study reveraled that the eonical
shape could be eliminated entierely and a more effectier suppresese designed be using a system of rods or bats arranged aromud the mazale and paralled to the tube in such a way that they formed effectively a celinder with wide longitudinal slots eut in it.

It is not known in detail why this devier is effertioe but it is believed that the gas expands through the slots which breaks up the continuity of the flow and so prevents shock formation. Mochanical suppressors, therefore, suppress the primary flash as well as the secondary flash.

Mechanical flash suppressors are not used on larger caliter weapons. For the larger weapons dependenee is on chemical methods. Potassimn salt, added to the propedlant greatly inereases the amomet of smoke produced. In execesive amount will reduce the efficieney of the weapon and change its interior ballistics. The use of a chemical suppressor will also result in changes in the interior ballisties of the weapon and may reguire changes in the chemieal composition of the propellant. An cextensive treatment of the problem of gun flash and its suppression is given in the elassified Reference:31.

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| 1 | Artillery |  |  |  |  |  |  |  | Iferrillew |  |  |  | Morts |  |  | Small Ams |  |  |
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| Siratation |  | ${ }_{19}^{13.25}$ | 13.25 <br> 1500 <br> 1 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.15 | 13.25 | ${ }_{13} 13.15$ | ${ }_{13}^{13.15}$ | ${ }_{13.15}^{97.7}$ | ${ }_{30.0}^{13.15}$ |
| narcisur titer |  | 1.40 |  |  |  |  |  |  |  | 10.75 | ${ }^{20.00}$ | ${ }^{25.00}$ | 33.5 | 43.00 | 40.00 |  |  |  |
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| Sithy |  | $\cdots$ | $\ldots$ | ... | ... | $\ldots$ | $\ldots$ | $\cdots$ | 1.0 | $\cdots$ | $\ldots$ | $\ldots$ | $\ldots$ | 3.00 | $\cdots$ |  | $\ldots$ | 0 |
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| Lintmontit | 1.0 |  |  | $1.0^{\circ}$ | $1.0{ }^{\circ}$ | 6.0 | 1.5 | 2.0 | 1.0 |  |  |  | 0 | 0 | 0.75 | 0.70 | 0.80 | 10 |
|  | $\cdots$ | U.:30 | 0.0 |  |  | 6. | 0.11 | 2.0 | $0.10 \dagger$ | ${ }_{0} \mathbf{8 . 3 0}$ | ${ }_{0} 0.00$ | ${ }_{0} 0.00$ | 0.10 | 0.60 | $\ldots$ | $\ldots$ | $\cdots$ | ... |
| crus. | $\ldots$ |  | , | $\ldots$ | $\cdots$ | 0.3 | 0.3 |  |  |  |  |  | 1.20 | $\ldots$ | $\ldots$ |  | ... |  |
|  | 0.75 | 230 | 2:30 | 0.60 | 1.00 | 0.30 | 0.30 | 0.31 | 1.50 | 1.20 |  |  |  |  |  |  |  |  |
| Narravilorl | 0.0 | 0.70 | 0.70 | 0.50 | 0.25. | 0.0 | $0.14)$ | 0.10 | 0.50 | 0.30 | 0.30 | 0.30 | 0.80 0.00 | 0.40 | 0.50 0.00 | ${ }_{100}^{0.00}$ | $1: 10$ | 0.50 |
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|  |  |  |  |  | 20.4 | 31.17 | 20.50 | 30.41 | 27.76 | 29.13 | 28.06 | 28.77 | $\ldots$ | 23.03 | 23.97 | 2088 | 27.91 | 8 |



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table t-l Calculatzd thernochemical values for standard propellarts (ticluding gesmoal volatines)

| Vor | Artilkry |  |  |  |  |  |  |  | Ineoillemp |  |  |  | Mortar |  |  | Small Arme |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Propullant | M11 | 112 | M5 | M6 | M14 | M15 | M17 | T* | 2110 | T18 | T25 | M20 | M17 | M8 | Mo | IMR | M12 | M18 |
| Spriliration | $\underset{: K N R}{ }$ | $\begin{gathered} \text { JAN-P. } \\ 323 \end{gathered}$ | $\underset{323}{\mathrm{JAN}^{2}-\mathrm{P}-}$ | $\begin{gathered} \text { JA.N-P- } \\ 309 \end{gathered}$ | $\begin{aligned} & \text { JAN-P- } \\ & \text { :000 } \end{aligned}$ | $\begin{gathered} \text { PA-PL- } \\ 20 \end{gathered}$ | $\begin{gathered} \text { PA-PI)- } \\ 20 \end{gathered}$ |  | $\begin{gathered} \text { PA-P])- } \\ {[2]} \end{gathered}$ | $\begin{gathered} \mathrm{PA}-\mathrm{PD}- \\ : 50 \end{gathered}$ | $\underset{329}{\text { PA-PD- }}$ | $\begin{gathered} \text { PA-IDD } \\ \mathbf{3 2 0} \end{gathered}$ | JA.N-P- $650$ | JAN-P- $381$ | $\underset{20305}{\text { MIIL_P }}$ | $\begin{gathered} \text { JAN-P. } \\ 733 \end{gathered}$ | $\underset{505}{\text { JA.K.-P.. }}$ | $\begin{aligned} & \text { FA-PU } \\ & 28 A \end{aligned}$ |
| Siltur Malian | 85.0 | 77.45 | 81.45 | 87.0 | 00.0 | 20.0 | 22.11 | 20.0 | ve.o | 72.0 | 78.25 | 67.25 | St. 6 | 58.15 | 87.78 |  |  |  |
| F. Siration | 13.15 | 13.25 | 13.25 | 13.15 | 13.15 | 15.18 | 13.15 | 13.15 | 13.18 | 13.18 | 13.15 | 13.15 | 13.18 | 13.25 | 18.15 | 13.15 | 97.7 18.15 | 60.0 |
| Sierachorrin | ... | 10.60 | 15.00 | ... | ... | 14.0 | 31.5 | 15.0 | ... | 10.78 | 20.00 | 25.00 | 35.5 | 13.00 | 40.00 |  |  |  |
| Barsom nitr:/r | ... | 1.40 | 1.40 |  |  |  |  |  |  | 0.78 | 0.75 | 0.75 |  |  |  | ... | $\cdots$ |  |
|  | . | 0.75 | 0.75 | ... | ... | $\cdots$ | . | $\cdots$ | ... | 0.70 | 0.70 | 0.70 |  | 1.20 | 1.00 | $\cdots$ | $\cdots$ | $\cdots$ |
| Promatinitrabliente |  |  |  | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | ... | . | ... |  | ... | 7.00 | . 2.8 | 1.00 | -•• | $\cdots$ | ... |
| Sitrenemitiow |  | $\ldots$ | $\ldots$ | $\ldots$ | . | 54.7 | 84.7 | 60.0 | $\ldots$ | . | ... | ... | 8.00 | $\cdots$ | . $\cdot$ | ... | . $\cdot$ | ... |
| In:atre thinere | 10.0 | ... | ... | 10.0 | 8.0 | ... | ... |  | $\ldots$ | $\ldots$ | $\cdots$ | ... | $\ldots$ | $\ldots$ | $\ldots$ | $80 \%$ | $\cdots$ | $\cdots$ |
| 1)huly phenamar | 8.0 | $\ldots$ | $\ldots$ | 3.0 | 2.0 | ... | ... | 8.0 | . | $\ldots$ | $\ldots$ | $\ldots$ | . | $\cdots$ | $\cdots$ | ... | $\ldots$ | ¢0" |
| ILinhughthatase | ... | $\ldots$ | . | ... | ... | ... | $\ldots$ | ... | 0 | ... | ... | ... | ... | 3.00 | $\cdots$ | $\ldots$ | $\ldots$ | 0 |
| Wur...invorllatr | $\cdots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | ... | $\ldots$ | $\cdots$ | 1.0 | $\cdots$ | $\ldots$ | $\cdots$ | ... | $\cdots$ | ... | $1.0{ }^{\circ}$ | 0.78 | $\ldots$ |
| Tin |  |  |  | $\ldots$ | $\cdots$ | $\cdots$ | $\ldots$ | $\ldots$ | $\cdots$ | $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | ... | . |  | 0.78 | $\ldots$ |
| Wip!um!onine | $1.0{ }^{\circ}$ | $\cdots$ | $\cdots$ | $1.0^{\circ}$ | $1.0{ }^{\circ}$ | $\ldots$ | $\ldots$ | $\cdots$ | 1.0 | $\cdots$ | $\because$ | $\cdots$ |  | $\cdots$ | 0.78 | $0.7 *$ | 0.80 | 1.0 |
|  |  | O.(i) 0.30 | 0.60 0.30 | ... | .. | 6.0 | 1.5 | 2.0 | 0 | 6.60 | 8.00 | 0.00 | 0.10 | 0.60 | ... | ... | $\cdots$ | $\therefore$. |
| Ciraghar Curloun limek | $\cdots$ | 0.30 | 0.30 | $\cdots$ | $\cdots$ | ... | 0.19 | ... | 0.101 | 0.30 | 0.30 | 0.30 | 120 | $\cdots$ | ... | ... | . | ... |
| Crantio. | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 0.3 | 0.3 | $\cdots$ | $\cdots$ | $\cdots$ | $\ldots$ | $\cdots$ | 1.20 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | ... |
| Ellu! Mrobut $H_{\text {a }}$ eidual | 0.75 | 2.3k | 2.30 | 0.00 | 1.00 | 0.30 | 0.30 | 0.30 | 1.50 | 1.20 | 1.20 | 1.20 | 0.80 | 0.40 | 0.50 | 0.00 | 1:30 |  |
| Nrterill- ilus: | 0.60 | 0.70 | 0.70 | 0.50 | 0.23. | 0.(x) | 0.(x) | 0.00 | 0.50 | 0.30 | 0.30 | 0.30 | 0.00 | 0.00 | 0.00 | 1.00 | 1.10 | 0.00 |
|  | $\cdots$ |  | ... | . |  |  |  | $1.0{ }^{\circ}$ |  |  |  | $\ldots$ |  | ... | ... |  | ... |  |
|  | 2117 | 3314 | $32+5$ | 2570 | 2710 | 2014 | 3017 | 2388 | 3010 | 2938 | 3071 | 3001 | 3734 | 3005 | 3709 | 2527 | 2TM6 | 2577 |
| Fir.e.1.N0. $11 . \times 10^{-1}$ | :115 | 300 | 355 | 317 | 327 | 1380 | 304 | 314 | 838 | 346 | 363 | 356 | 368 | 382 | 382 | 325 | :336 | 319 |
|  | 8.6 | 0 | 1 | 6.8 | 5 | 0.5 | 3.4 | 11.6 | 4 | 3.4 | 1.8 | 2.2 | 0 | 0 | 0 | 1.8 | 6 | $4_{8}$ |
| Cumbectitur.': | 05.3 | 17.2 | 47.4 | 62.4 | 88 | 81.0 | 38.7 | 53.1 | 84.8 | 89.1 |  |  | 33.4 | 37.2 | 32.8 | 80.2 | 53.66 | 06.6 |
| 11.31 "1 "rphovion. cal/rm | 700 | 1650) | 1047 | \%88 | 800 | 740 | 1012 | 812 | 086 | 010 | 002 | 006 | 1258 | 124 | 1248 | 308 | \% $6 \times 3$ | 9.6 7.7 |
| Cise cohrone muc-simm | 010.14513 | 0.041400 | 0.033835 | $0.01+32$ | 0.04 .838 | 0.14645 | 0.04838 | 0.04794 | 0.04088 | 0.04210 | 0.04138 | 0.04157 | 0.03818 | 0.03711 | 0.051818 | 0.04137 | 0.04037 | 0.04457 |
| Wationd ghoiti henta | 1.2543 | 1:29:38 | 1.2288 | 1.2313 | $1.2+103$ | 1.2557 | 1.2412 | 1.2501 | 1.2342 | 1.24 Cl | 1.2373 | 1.2383 | 1.2100 | 1.2148 | 1.2102 | 1.2400 | 1.2528 | 1.2523 |
|  | 1010 0.05687 | 2812 0.0507 | 2647 | 20.50 | 2108 | 22006 | 24.23 | 1807 | 2431 | 2365 | 2482 | $2+48$ | 2086 | 8048 | 8139 | 2280 | 2431 | 2058 |
| Cosrdumr. in'.16 | $30.55^{\text {. }}$ | 27.018 | 27.52 | 29.02 | 20.54 | 31.17 | 20.50 | 30.41 | 0.0608 | 1.0688 29.13 | 20.0886 | 20.0585 | ... | 28.93 | 25.97 | 2887 | 27.90 | $\cdots$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Nem |




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1:0 Experimental Statistics, Section 1, Basic Concepts and Analysis of Measurement Data
111 Experimental Statistics, Section 2, Analysis of Enumerative and Classificatory Data
112 Experimental Statistics. Section 3, Planning and Analysis of Comparative Experiments
113 Experimental Statistics, Section 4, Special Topics
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139 Servomechanisms. Section 4, Power Elements and System Design
170(C) Armor and Its Application to Vehicles (U)
250 Guns--General (Guns Series)
252
Gun Tubes (Guns Series)
Propellant Actuated Devices
290(C) Warheads--Generai (U)
331 Compensating Elements (Fire Control Series)
355 The Automotive Assembly (Automotive Series)
$175 \frac{\text { Ammunition and Explosives Series }}{\text { Solid Propellants, Part One }}$
176(C) Solid Propellants, Part Two (U)
177 Properties of Explosives of Military Interest, Section 1
178(C) Froperties of Explosives of Military Interest, Section 2 (U)
210 Fuzes, General and Mechanical
$211(\mathrm{C})$ Fuzes, Proximity, Electrical, Part One (U)
$212(S) \quad$ Fuzes, Proximity. Electrical, Part Two (U)
<13(S) Fuzes, Proximity, Electrical, Part Three (U)
$214(S)$ Fuzes, Proximity, Electrical, Part Four (U)
215(C) Fuzes, Proximity. Electrical, Part Five (U)
244 Section 1, Artillery Ammunition--General, with Table of Contents, Glossary and Index for Series
245(C) Section 2, Design for Terminal Effects (U)
246 Section 3, Design for Control of Flight
247 Section 4, Design for Projection

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| Number | Title |
| :---: | :---: |
| 281(S-RD) | Weapon System Effectiveness (U) |
| 282 | Propulsion and Propellants |
| 284(C) | Trajectories (U) |
| 286 | Structures |

## Ballistics Series

140 Trajectories, Differential Effects, and Data for Projectiles
150 Interior Ballistics of Guns
160(S) Elements of Terminal Ballistics, Part One, Introduction, Kill Mechanisms, and Vulnerability (U)
161(S) Elements of Terminal Ballistics, Part Two, Collection and Analysis of Data Concerning Targets (U)
162(S-RD) Elements of Terminal Ballistics, Part Three, Application to Missile and Space Targets (U)

Carriages and Mounts Series
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Glass
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Carriages and Mounts-General
Cradles
Recoil Systems
Top Carriages
Bottom Carriages
Equilibrators
Elevating Mechanism:
Traversing Mechanisins

Part Two, Safety, Procedures and Glossary
Part Three, Properties of Materials Used in Pyrotechnic Compositions

Surface-to-Air Missile Series
291 Part One, System Integration
292 Part Two, Weapon Control
293 Part Three, Computers
294(S) Part Four, Missile Armament (U)
295(S) Part Five, Countermeasures (U)
296
Part Six, Structures and Power Sources
297(S) Part Seven, Sample Problem (U)


[^0]:    * Additional information on Wark powder will bo foumd in Reference 1$)$.

[^1]:    * Additional information on gun propellants will lo found in Reference 9 .

[^2]:    * Trademark of Olin Mathieson Chemieal Corporation. Reforence! ! eontains additional information.

[^3]:    * "Cool" propellants may be defined roughly an those: or which the uncosed explosion temperature is not greater
    

[^4]:    * Table 2-3 has been reprinted in part from J. Corner. Theory of the Interior billialics of finns, Copyriglit 1950, with permission from John Wiles and rions. Inc.

[^5]:    I/' is the time rate of heat flow
    $\theta$, the temperature of the fluid in the flow ineyond the boundary layer

[^6]:    * An $X$ cut resstal plate has the normal to its face parallel to the cleetrie axis of the crestal and the optie axis paralled to its face. For a given pressure on its fare, such a plate produces a maximum charge.

[^7]:    *The nomenclature is not standardized. The three rekions are also called primary, intermediate and secondaryflash. Other nomenclature also exists in the literature.

[^8]:    * Single perforsterl.
    † Multiperforated.

